

## Teacher's Informal Learning Trajectory and Student's Actual Learning Trajectory On Learning Cube And Cuboid Nets

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### Abstract

This article discussed the teachers' informal learning trajectory and students' actual learning trajectories in learning cube and cuboid nets. Informal learning trajectory that is believed to have experienced teachers have taught so as to anticipate all possibilities in interaction. While the actual learning trajectory is the trajectory of student learning derived from real traces of student learning. The writer try to make the observation of teaching and learning in the classroom and try to formalize teachers' informal learning trajectories and students' actual learning trajectory. From the observation of the learning process in fourth class SDI Surya Buana, there is a gap between students' learning actual trajectory and teachers' informal learning trajectory. This seemed to caused props that teachers prepared were inadequate.

**Keyword:** informal learning trajectory, actual learning trajectory

### 1. Introduction

Education should always follow global development. In the era of technology, humans require creative problem solving ability. It required thinking skills of analysis, critical and creative. It is associated with a high level of critical thinking (high order thinking). Therefore it becomes one of the measures of success of a country that is learning mathematics with the PISA (Programme for International Student Assessment). PISA [12] divides the problem into six level mathematics. Minister of Education and Culture [10] explained that on reflection PISA 2011, almost all Indonesian students master subjects only up to level 3 course, while many other countries are up to level 4, 5, even 6. Results of PISA [13] demonstrated mastery of mathematics 75, 7% Indonesian students were level 2 downwards. It decreased from PISA 2011. It also shows that high-level thinking skills (high order thinking) and reasoned Indonesian students are still very weak.

Reasoning is an important aspect in every area of life, not only in the field of science and mathematics. Sriraman and English (2010) stated that human reasoning involves numerous mental acts such as interpreting, conjecturing, inferring, proving, explaining, structuring, generalizing, applying, predicting, classifying, searching, and solving problems. Because reasoning is a fundamental aspect of mathematics, it could be important to understand mathematical concepts. Given the importance of reason, Curriculum 2013 (Permendikbud number 65 in year 2013) make reasoning as one of the activities undertaken to gain math skills.

Curriculum 2013 also requires that through the learning process, students have the experience of learning: 1) trained to think critically and creatively; 2) find the science from the solving of real problems; 3) trained to cooperate in the study group (team) to find solutions to problems; 4) trained to propose an ideas freely and openly; 5) feel the benefits of mathematics in everyday life. Therefore the 2013 curriculum

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emphasizes modern pedagogical dimension in learning, using a scientific approach. Scientific approach (scientific approach) in study referred covering observe, inquire, reason, guess, try, establish connections to all subjects.

How can teachers know that students learn, can be done by observing and studying student's learning trajectories. In mathematics learning, there are many definitions of the learning trajectories. Learning trajectory is defined as the way of certain individuals learning, or fail to learn, as they progress through the training program of an institution [15]. Learning trajectories allows to describe the resources and students' perspectives as they evolve toward a more sophisticated understanding of one or more concepts from time to time.

In [4] Clements and Sarama conceptualize learning as a description of the thinking and learning of children in a particular domain and related mathematics, these allegations through a series of learning tasks. This task was designed to induce mental processes or actions hypothesized to move as well as the children progress through developmental levels of thinking. Learning trajectories can be seen as a way to identify the mathematical key and cognitive components of the concept that evolved over time. One metaphor for thinking about learning trails are conceptual corridor [5]. A trajectory can be seen as a way that allows the student can navigate during instruction and conceptual corridor because all trajectories is possible. A trajectory begins with prior knowledge, instructional task acts as a constraint on students' responses, and the sequence of tasks to encourage student movement through the corridor. Confrey et.al [6] defines learning as researcher's conjecture, descriptions (based on experience) of a series of commands on the discovery of constructing students through instruction (ie, activities, tasks, equipment, and other forms of interaction), to move from the informal ideas through continuous improvement of the representation, articulation, and reflection, the concepts become more complex over time. The key word is learned that the track does not consist of a single trajectory.

According to Wilson [18] learning trajectory can act as a tool to coordinate (1) the student's behavior and verbalization with cognition, (2) various models of thinking among groups of students, and (3) this model with instructional practices. Meanwhile, according Szilágyi, Clements and Sarama (2013) used hypothetical learning trajectory to ensure that the order is consistent with the students' level of thinking that the observed behavior of most students. Learning trajectory obtained from the observed behavior is the student's actual learning trajectory.

From the research studies on learning trajectories, there is a wide path of learning that can be formulated that are: hypothetical learning trajectory, informal learning trajectory, as well as actual learning trajectory. Hypothetical learning trajectory is determined by the teacher prior to the learning process. Informal learning trajectory is believed to be owned by teachers who have experience teaching so as to anticipate all possibilities in interaction. While the actual learning trajectory is the trajectory of student learning derived from real traces of student learning. The writer tried to make the observation of teaching and learning in the classroom and tried to formulate informal learning trajectories of teachers and students' actual learning trajectory

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## 2. Theory

Learning Trajectory has been defined as a description of the thinking and learning of children in certain mathematical domains, and the suspect is related through a series of learning tasks designed to induce mental processes or actions hypothesized to move children through the development of growth level thinking (Clements & Sarama, 2004, p. 83). Confrey defined LTs as "researcher conjecture, empirical description of supported command sequence construction meeting students through instruction (ie activities, tasks, tools, forms of interaction and evaluation methods), in order to move from the informal notion of representation through successive improvements, articulation, and reflection, the concepts become more complex over time "(Confrey et al., 2009, p. 347). This definition establishes the impossibility of separating the students learn from instruction in a school setting; focus the teacher's role in the growth of students' understanding of mathematics. Many studies in developing LTs makes the difference between logic and logic discipline the student or learner's cognitive development (Corcoran et al., 2009). These differences suggest that, rather than set of mathematical topics and learning experiences for children based on logical analysis of disciplinary knowledge, LTs allows learning of mathematics should be based on "research on how students learn really took place" (p. 8). This difference shifts the focus of the discipline of organizing mathematics instruction to students. Thus, the fundamental characteristics of the LTS is attention to the ways learners mature logic into the logic of the discipline.

The development of learning trajectories have contributed significantly in the field of mathematics education. Nevertheless, it is equally important is what is done by the teacher with the learning trajectory in learning practice. Many mathematics education researchers have studied and developed a learning trajectory (or some closely related to the construction of learning trajectories) mathematics students (Battista, 2004; Clements & Wilson, 2004; Gravemeijer, 2004; Lesh & Yoon, 2004; Simon & Tzur, 2004; Steffe, 2004).

In many studies, discussed trajectory study made by researchers. Learning trajectory made or designed by curriculum specialists and researchers called the hypothetical learning trajectory (Hypothetical Learning Trajectory / HLT). Rhodes [14] makes a difference in researching learning trajectory. He researched 'informal learning trajectory high school teacher for the purpose of identifying what teachers noticed about students' mathematical thinking during classroom interactions. Chen [2] formulate a hypothetical learning trajectory (HLT) of the statement of the arguments about geometric images. Students are required to assess whether the statement is true or not, not only their own needs confirmation or refutation requested, but also reveal an informal image or proposition about image. From an analysis of students' explanations obtained HLT argue that consists of four levels. In addition, the difference between the different levels in the HLT form the basis for designing experimental teaching.

Hypothetical learning trajectory is an instrument used in the study design that can bridge between theory and experiment. Hypothetical learning trajectory used by Simon (1995) with the term Hypothetical Learning Trajectory (HLT). HLT is a hypothesis or conjecture made by researchers about the learning process that will occur during the implementation of learning in the classroom. This hypothesis was made to anticipate all the possibilities that can arise in the classroom, so that researchers can minimize the things that are not desirable. Another thing that is important is that the LBH built on theories that have been studied previously.

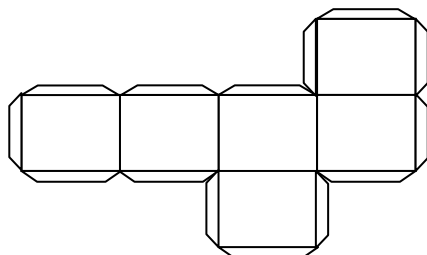
HLT provide daily planning for teachers and researchers of the series of activities in carrying out experiments in the classroom. In the HLT, the researchers describe the learning activities to be carried out. The learning activities are sorted according to the stage of understanding concepts that are usually contained in the activity (Gravemeijer, 2004). HLT consists of three main components, namely objectives, learning activities, as well as allegations of the learning process. Goal, a goal of mathematics learning activities undertaken to achieve that is what the students in the learning activities. Learning activities, carefully designed so that learning objectives can be achieved. Furthermore, researchers make initial assumptions about how the activities will take place and the main learning is the learning process of students during these activities. With these allegations, researchers can anticipate all possibilities in the field.

HLT has been made by many researchers. But there are difficulties in obtaining to use the limited availability and the model is complex. Teachers do not have access to a learning path created by the researchers, and therefore they can not use it. In addition, the structure does not provide for a teacher teaching time to read long technical report detailing the learning path made by the researcher. Description of the learning trajectory should be concise and easily accessible to teachers so that they can reap the benefits (Rhodes, 2007). According to experienced teachers who have developed a learning trajectory "informal" to their students through their interactions with students. This informal learning trajectories inform their instructional practices and guide their interactions with students. Learning trajectories developed and modified through interaction with students doing math.

### 3. Results and Discussion

Observations conducted on March 24, 2014 in SDI Surya Buana Malang located at Gajayana Street 4/631. I observed mathematical learning in fourth grade SDI Surya Buana with Mr. Yusuf as teachers. The class discussed about the cube nets. The theme at the time was "cita-citaku". Classes has been divided into 6 groups with each member consists of 4 students. At previous lesson, the students have been asked to create mockups of buildings. From these models, the students brought to the understanding of the shape of cubes and cuboid as well as its properties.

In the Lesson Plan (RPP), noted that the current study was to use the jigsaw method. In practice, the teacher using two stay two stray. First, the teacher distributed cube shape in each group to see the shape of cube nets. Form of nets assigned to each group is set up different from each other. The group was asked to open the cube. The goal is to get the cube nets. Almost all groups experienced difficulty when opening the cube into the nets, because they are not free from where to start opening. The nets are not obtained as expected nets obtained from slicing cube. For example like this:



Similarly, the cuboid netting. Furthermore, the group was asked to draw nets on a piece of paper. Here appears that the group had difficulty to describe their nets. One group was drawing one by one side of the room to get up so that appears not as a whole but the pieces are mounted close together. The group seemed not to understand the nature of the cube because he is still measuring one by one side of the cube so that when he uses a ruler, measure different sides, eg 4.9 cm, 5 cm, 5.1 cm for a cube. Similarly, a group that makes the cuboid nets, they measure with a ruler on each side Netting cuboids that produce different sizes so when described in the paper form into the nets do not like the cuboid again. There are children who realize that the size is different.

S1 : The teacher, the size of the different sides of his logs anyway?

T : No, same size

S1 : But I have a different pack

T : Try to measure again. Should be the same (Teacher closer and see students use a ruler to measure). Tu is the same (but indeed with such Netting allows students one measure and seem less skilled students still use a ruler)

S1: Ooo.

I came to the group

Me : Why do you measure the length of all sides?

S1 : (looks thoughtful)

Me : Try to observe the Cuboid Netting. Do you think is there the same size?

S1 : (little thought) A ... no

Me : Where is it?

S1 : This ... this ... same with. together with this ... (shows the same measures)

S2 : It's the same with this, as with this ..

Me : Is there the same again?

S1, S2: There is a (sure)

Me : Then do you have to measure all sides of the nets had to draw it?

S1 : No

Me : Then now of course you can draw a net-net, right?

S : (nods and follow through with Netting draw more skilled)

Genesis students have difficulty drawing the nets also appears when one of the representatives of the group were asked to copy a drawing nets that have been made of other groups.

From the author's observation, made further informal learning trajectories of teachers as follows.

Purpose: students can find Netting of cubes and blocks

Activities:

1. The teacher divides students in groups
2. Allot 2 blocks and cubes in each group
3. Ask students open the cube and Cuboid respectively to obtain the cube nets.
4. Ask students to draw the nets belonging to each group.
5. Ask the representative group visit to another group to obtain benuk Netting of other groups.
6. Ask each group to presented the their cube nets.

Rationale: Students will be able to obtain a variety of cube nets and cuboid of activities they gain experience and obtained by slicing the cube and the cuboid.

It appears that what is desired by the teacher by providing cube and cuboid that has been set up as different with their nets. Students have a difficulty in opening the cube and the cuboid. This would be easier if students are left alone to explore the net-net with really slicing cuboids and cubes. Thus the child actually gain experience and cube slicing cuboids so as to obtain a net-net without interruption or separately.

With alternatives such activities then it would appear the various possibilities nets are obtained by students:

1. some students make the same nets;
- 2 there are students who do not succeed in making Netting;
- 3 there are students who quickly membuat few Netting for slicing in different ways.

By understanding this possibility, the teacher can think about or prepare for further action if that happens the possibility of 1, 2 or 3 means that the students will also learn different learning trajectory that has been carefully prepared teacher.

#### 4. Conclusion

From the observation and discussion of understanding that there is a gap between teacher's informal learning trajectories and student's actual learning trajectory in the lesson of cube nets and cuboid nets. In this case the teacher has not to take into account all the possibilities that will occur when students carry out planned activities. It appears from the preparation of props and Cuboids cubes prepared teachers. It appears that the measure used is no longer accurate because factors props. From the results of this discussion should bear for teachers to actually prepare the learning so that the tools used can help to prepare students' learning trajectories.

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