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Students' Concept Maps in Abstract Algebra

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Abstract

Concept map is evaluation tool in identifying both valid and invalid ideas held by students. It is also learning tool in encouraging students to construct meaningful understanding. The understanding has effect to students' ability in solving problem to proof of Abstract Algebra. It is one of the subjects in Mathematics Education Department. The aim of this study described undergraduate students' concept maps in Abstract Algebra.

The subjects were three mathematics education students that were selected by the following criteria: (a) they had different abilities in Abstract Algebra (good, fair and poor achievements), (b) they were able to communicate their thinking verbally. The ability was based on the students' scores of Abstract Algebra test. First, the researcher asked each student to make a concept map. Then, the researcher interviewed each student to explain the map. The students' concept maps were scored by Bartel's Scoring Rubric (scale 0–12).

One of the subjects got 12 which was good achievement. She showed an understanding of the concepts of Abstract Algebra. She could explain the definition of homomorphism and give an example and a counter-example of the concept. She could indentify all the important concepts and show an understanding of the relationships among them. While the other subjects got 6 (fair achievement) and 3 (poor achievement).

Keywords: concept, concept map, meaningful understanding, homomorphism, abstract algebra

Problem solving is main goal and means of learning mathematics (NCTM, 2000). In everyday situation, students should solve real life problems. Students could acquire ability to solve the problems when they solve mathematical problems in class. Problem solving is the link between real life problems and the ability learned in school. It also helps connect the various topics learned in class. Students can construct new mathematical knowledge through problem solving (Krulik, Rudnick and Milou, 2003).

Students also acquire critical and creative thinking by solving mathematical problems. Krulik, et. al. (2003) divided ablitiy of thinking in four level. They are recall, basic, critical and creative thinking. Critical and creative thinking are high-order thinking. Critical thinking is ability to analyze a problem, determine if there is sufficient data to solve it, decide if there is any extra information in the problem, recognize consistent or inconsistent data, draw conclusion from the data, and determine the conclusion are valid or invalid (Krulik, et. al., 2003). While, creative thinking is synthesis of logical and divergent thinking to produce a new product (Siswono, 2008).

Students' problem solving ablity is affected by their meaningful understanding. Students who have the understanding, have ability to solve a problem (Hudojo, 2005) and to learn a new information (Sutawidjaja and Afgani, 2011). Students have the understanding when they relate a new information to preexisting knowledge in their thinking (Crawford, 2001). The construction form network of concept.The network is called by concept map (Hudojo, 2005).

Furthermore, the concept map is graphical tool to organize and represent knowledge (Novak, 1998; Kane and Trochim, 2007; Novak and Canas, 2007). The map includes concepts and relationships between the concepts were indicated by a connecting line linking two concepts and words on the line. The words are linking words or linking phrases that specify the relationships between two concepts (Novak and Canas, 2007). There are three types of concept map. They are linear map, tree map, system map and spider map (Varghese, 2009). Figure 1 shows the examples of each concept map type.

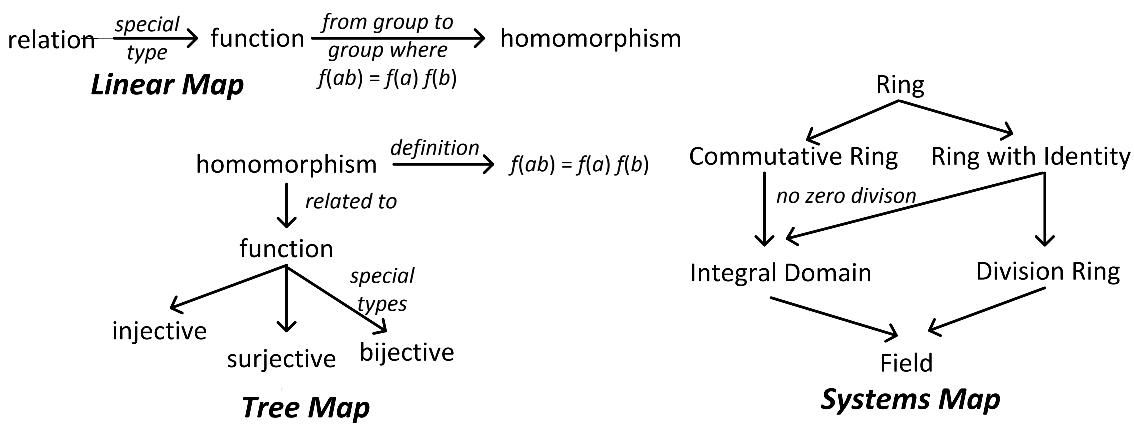

Figure 1. Examples of Each Type of Concept Map

Concept map is powerful tool in mathematics education. The map can be used as a learning tool that help students to construct meaningful understanding. Teacher can use the map to develops lesson plans. The map can also be used as a evaluation tool that help teacher to indentify both valid or invalid ideas held by students (Novak and Canas, 2007; Hudojo, 2005).

There are steps of making a concept. The first step is define the area of knowledge to be mapped by preparing an appropriate focus question or a question that will be answered by the knowledge that was mapped. As example, student constructs a concept map related to homomorphism. The second step is to identify the key concepts of the area of knowledge. The third step is to construct a preliminary concept map. It is always necessary to revise the preliminary map. Other concepts can be added (Novak and Canas, 2007).

University students should have the understanding in solving problem to prove of Abstract Algebra. Main goal of the problem is to determine whether a certain statement is true or false (Polya, 1981). Abstract Algebra is one of the lectures in matematics education department. Example of the problem is as follows (Dummit and Foote, 2004).

> Let $(Z, +)$ are group. Define a function $f: Z \to Z$ by $f(a) = a + 1$ for all $a \in Z$. Is the function f be a homomorfism? Justify your answer.

The aim of the research described the university students' concept maps in Abstract Algebra especially in homomorphism. There are some conccpts which related to homomorphism. The concepts are function (bijection, injection and surjection), group (asosiative rule, commutative rule, identity, invers), and isomorphism. The results of the research could used by the lecturer to improve quality of learning process and students' achievements.

METHOD

The research carried out in the first semester of the academic years 2013/2014 at one department of the University in Palangka Raya, Central Kalimantan, Indonesia. The research subjects were three university students of mathematics education department. They were selected by the following criteria: (a) they had different abilities in Abstract Algebra (good, fair and poor achievements), (b) they were able to communicate their thinking verbally. The ability was based on the students' scores of Abstract Algebra test. The subjects were DK (female), SD (female) and AG (male), consecutively, had good, fair and poor achievements.

The researcher interviewed each student individually to collect the data. The interview was based on a concept map made by the student. The steps were (a) the researcher asked concepts related with homomorphism to the students, (b) the student made the map and (c) the researcher asked the students to explain the map.

The concept maps were scored by Bartel's Scoring Rubric (scale 0–12). The rubric was as follows (Bartels, 1995).

Table 1. Bartels' Scoring Rubric for Concept Map

Indicators		**Codes**
Concept and Terminology		
3 points	shows an understanding of the topic of concepts and principles and uses appropriate terminology and notations.	**C3**
2 points	makes some mistakes in terminology or shows a few misunderstanding of concepts.	**C2**
1 point	make many mistakes in terminology and shows a lack of understanding of many concepts.	**C1**
0 points	shows no understanding of the topic of concepts and principles.	**C0**
Knowledge of the Relationships among Concepts		
3 points	identifies all the important concepts and shows an understanding of the relationships among them.	**K3**
2 points	identifies important concepts but makes some incorrect connections.	**K2**
1 point	makes many incorrect connections.	**K1**
0 points	fails to use any appropriate concepts or appropriate connections.	**K0**
Ability to Communicate through Concept Map		
3 point	Constructs an appropriate and complete concept map and includes examples; places concepts in an appropriate hierarchy and places linking words on all connections; produces a concept map that is easy to interpret.	**A3**
2 points	places almost all concepts in an appropriate hierarchy and assigns linking words to most connections; produces a concept map that is easy to interpret.	**A2**
1 point	places only a few concepts in an appropriate hierarchy or uses only a few linking words; produces a concept map that is difficult to interpret.	**A1**
0 points	produces a final product that is not a concept map.	**A0**

The analysis procedures were reduce the data, display of the data and make conclusions (Miles and Huberman, 1992). The data reduction was done by giving code to transcript of interview. The data was displayed by organizing the code in natural sequence. Conclusions were made by giving meaning and explanation of the data.

RESULTS AND DISCUSSION

Results

Based on the research steps, the researcher interviewed each student individually. The results of interviewing was transcribed by the researcher. The reseacher gived codes in the transcript and students' concept maps. The data analysis was based on the codes. The results of the analysis were as follows.

DK's Concept Map

DK was good achievement student in Abstract Algebra. She could construct an appropriate and complete concept map and includes examples, place linking words on all connections, and produce a concept map that was easy to interpret (Figure 2). Thus, her concept got 3 points in ability to communicate through concept map.

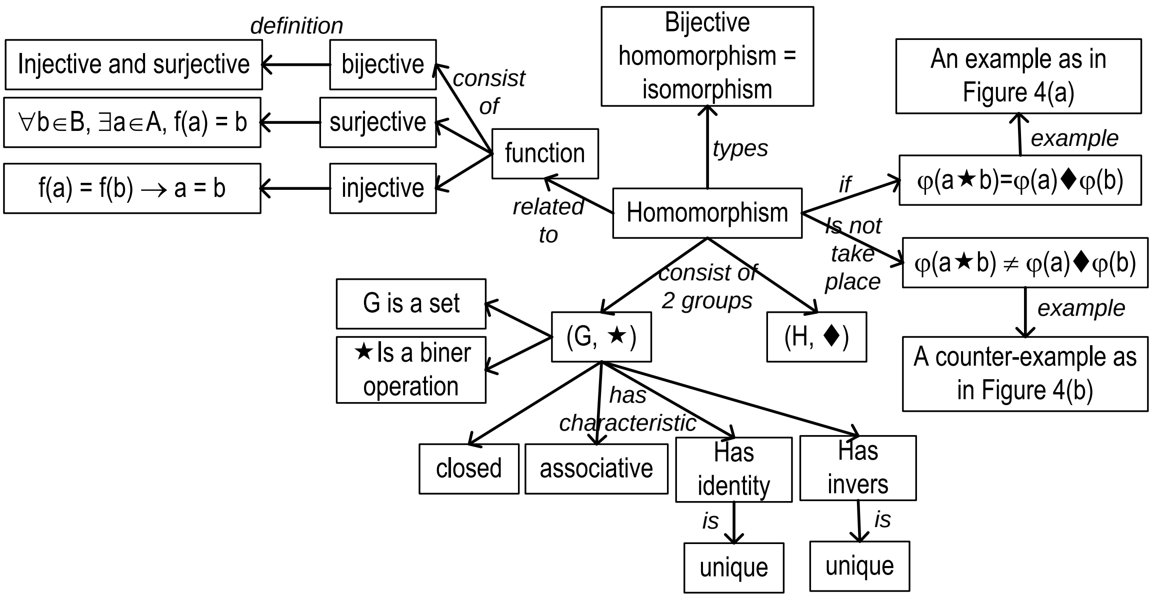

Figure 2. DK's Concept Map (Redrawn into English)

She could write the definition of homomorphism (Figure 3). She also could give an example and a counter-example of homomorphism (Figure 4). Therefore, she showed an understanding of the topic of homomorphism and used appropriate terminology and notations. The concept map got 3 points in Concepts and Terminology based on Bartels' Scoring Rubric.

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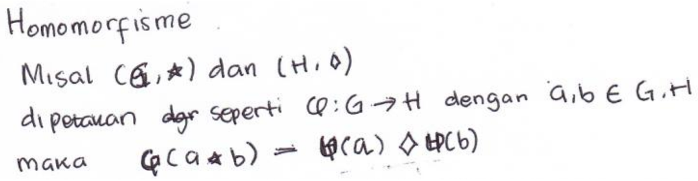
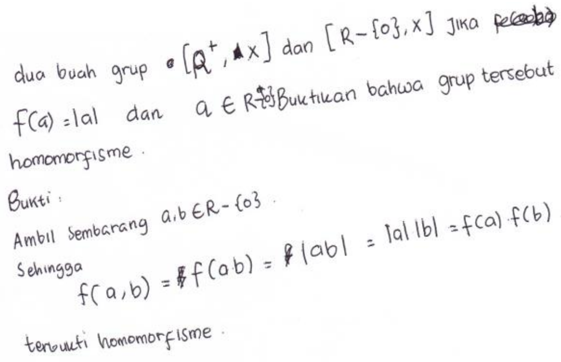

Homomorfisme
Misal $(G,\star)$ dan $(H,\diamond)$
dipetakan dgr seperti $\varphi: G \to H$ dengan $a,b \in G,H$
maka $\varphi(a \star b) = \varphi(a) \diamond \varphi(b)$

Translated in English
Homomorphism
Let $(G,\star)$ and $(H,\diamond)$ and $\varphi: G \to H$ where $a, b \in H$, then

$$\varphi(a \star b) = \varphi(a) \diamond \varphi(b).$$

Figure 3. The Definition of Homomorphism by DK

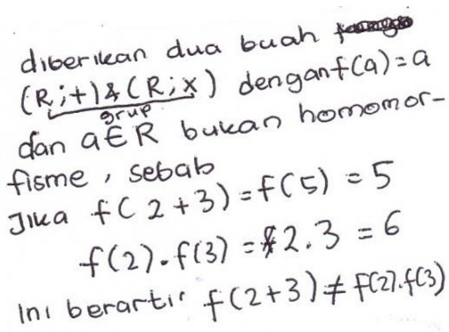

dua buah grup $[R^+, \times]$ dan $[R-\{0\}, \times]$ jika $f(a) = |a|$ dan $a \in R^+$ Buktikan bahwa grup tersebut homomorfisme.
Bukti:
Ambil sembarang $a,b \in R-\{0\}$.
Sehingga $f(a,b) = f(ab) = |ab| = |a||b| = f(a).f(b)$
terbukti homomorfisme.

Translated into English
Let $[R^+,\times]$ and $[R^+ - \{0\},\times]$ are groups. If $f(a) = |a|$ for all $a \in R^+$. Prove the groups are homomorphism.
Proof
Let an arbitrary $a, b \in R^+ - \{0\}$. So

$$f(a,b) = f(ab) = |ab| = |a||b| = f(a)f(b).$$

Thus, homomorphism

(a) Example of Homomorphism


diberikan dua buah grup $(R;+)$ & $(R;\times)$ dengan $f(a) = a$ dan $a \in R$ bukan homomorfisme, sebab
Jika $f(2+3) = f(5) = 5$
$f(2).f(3) = 2.3 = 6$
Ini berarti $f(2+3) \neq f(2).f(3)$

Translated into English
Let $(Z,+)$ and $(Z,+)$ are groups where $f(a) = a$ for all $a \in Z$ is not homomorphism.
since
If $f(2+3) = f(5) = 5$
$f(2).f(3) = 2.3 = 6$
It means $f(2+3) \neq f(2).f(3)$.

(b) Counter-example of Homomorphism

Figure 4. DK's Example and Counter-example of Homomorphism

Furthermore, she could identify all the important concepts related to homomorphism. The concepts were group (set, binner operation, axioms of group), function (injection, surjection, bijection, and each definition of the type of function), the definiton of homomorphisn, and isomorphism. She also could explain the relationships among the concepts. The citation of the transcript of interview was as follows (translated in English).

P : Now, please explain the concept map begin with homomorphism.
DK : ... there is type of homomorphism. It was bijective homomorphism. It called by isomorphims. Then we called isomorphims if psi a star b is equal to psi a diamond psi b. We called non-homomorphism if psi a stard b is not equal to psi a diamond psi b. This is an example of non-homomorphism.

Therefore, her concept map got 3 points in knowledge of the relationships among concepts. In general, her concept map got 12 points.

SD's Concept Map

SD was a fair achievement student in Abstract Algebra. She placed almost all concepts related to homomorphism in her concept map and produced a concept map that was easy to interpret (Figure 4). Thus, her concept map got 2 points in ability to communicate through concept map.

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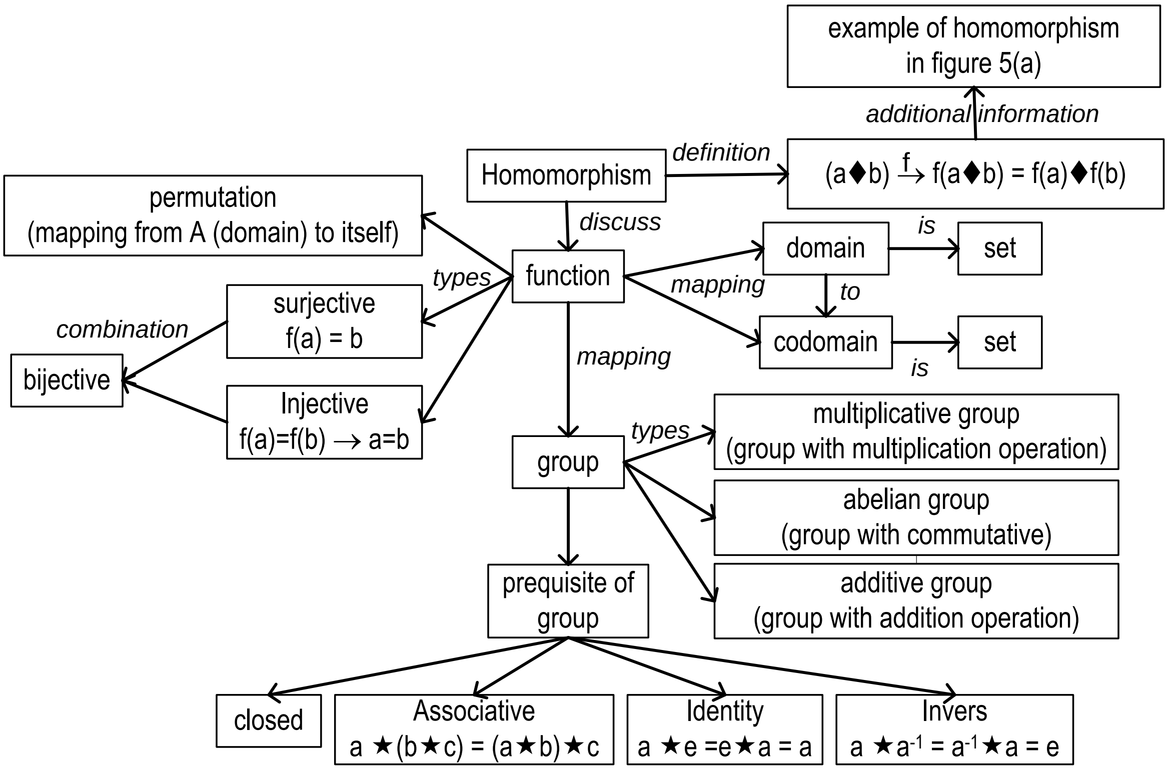
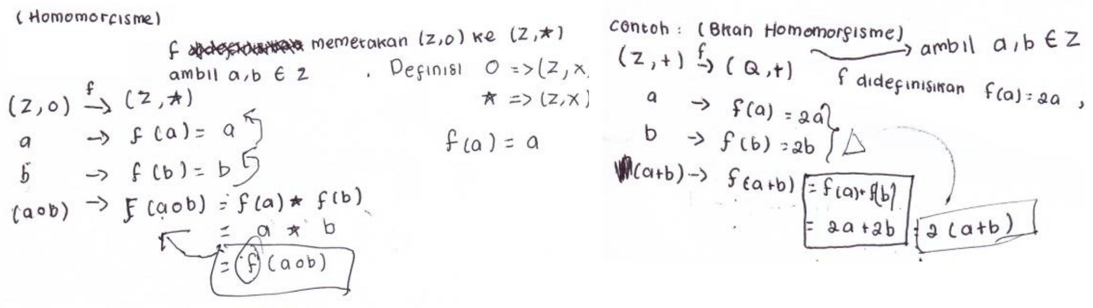


Figure 4. SD’s Concept Map (Redrawn into English)

She could write the definition of homomorphis, but she made mistakes when gave an example and a counter-example of homomorphism (Figure 5). She showed a few misunderstanding of the concept. Although, she could write and explain axioms of group, definition of some types of group (multiplicative group, additive group, abelian group), definitons of some types of function (injection, bijection, permutation). Therefore, she got 2 points in concepts and terminology.



(a) An Example of Homomorphism **(b) A Counter Example of Homomorphism**

Figure 5. SD’s Example and a Counter-example of Homomorphism

Furthermore, she could indentify important concepts, but make a incorrect connection. It was connection between function and group where the linking word was "in the form of mapping". Therefore, her concept map got 2 points in knowledge of the relationships among concepts. In general, her concept map got 6 points.

AG's Concept Map

AG was a poor achievement student in Abstract Algebra. He placed only a few concepts and produced a concept map that is difficult to interpret (Figure 6). Therefore, his concept map got 1 point in ability to communicate through concept map.

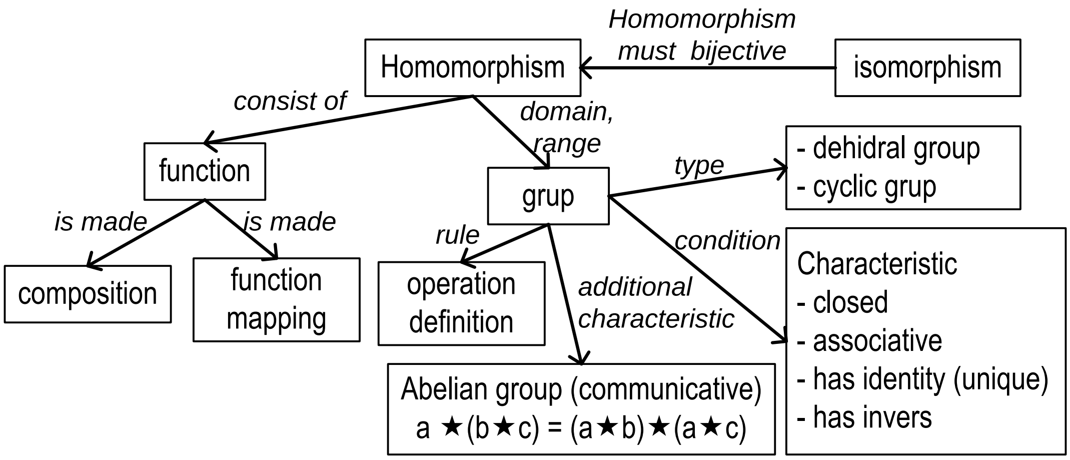


Figure 6. AG's Concept Map (Redrawn into English)

He made mistake in the defintion of homomorphism (Figure 7). He also could not give an example and a counter-example of homomorphism. He said "(silent) I cannot Sir". He also made mistake in the definition of group. He said "group may functions, Sir". Although, he could explain axioms of group. He made many mistakes in terminology and showed a lack of understanding of many concept. His concept map got 1 points in concepts and terminology.

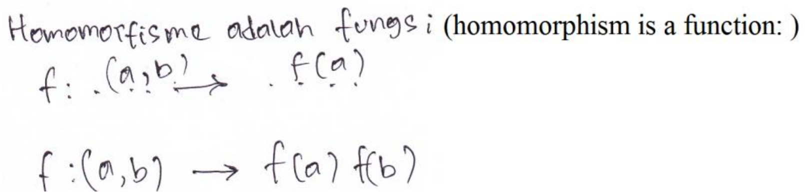


Figure 7. The Definition of Homomorphism by AG

Furthermore, he made four incorrect connection between: (1) function and composition, the linking word "is made by", (2) function and function mapping, the linking word "is made by", (3) group and operation definition, the linking word "rule", and (4) homomorphism and isomorphism, linking word "homomorphism must bijective". Therefore, his concept map got 1 point in knowledge of the relationships among concepts. In general, his concept map got 3 points.

Discussion

Students' achievement was affected by their meaningful understanding of some concepts. Students had the understanding were able to learn a new information and solve mathematical problems. Therefore, the student might good achievement in mathematics. On the contrary, students had lack of the understanding might poor achievement. Therefore, teacher should assess students' understanding to improve their achievement.

Concept map could be used as a tool to assess students' meaningful understanding (Williams, 1998). Understanding of a concept is meaningful if it related meaningfully with other concepts. The relation form schema in student's thinking. Schema itself is concepts network (Hudojo, 2005). Thus, concept map is representation of the schema (Novak and Canas, 2007). If the concept map is meaningful, then the schema is also meaningful. Meaningfulness of the map can be measured by score of the concept map. The score is calculated by using Bartel' scoring rubric for concept map.

Students, who had high score of the map, might have meaningful understanding. The understanding itself had effect to students' achievement. Therefore, scores of the map had positive correlation to the students' achievement. The result of the research showed that the good achievement student (DK) got 12 points, while the fair (SD) and poor (AG) achievement students, consecutively, got 6 and 3 points (the maximum score was 12 points). Ozdemir (2005) stated that there was meaningfulness between concept mapping scores and literature examination scores.

The meaningful understanding of the students also showed by their ability to give an example and a counter-examples of a certain concept. The good achievement student got 3 points in ability to communicate through concept map because the student constructed an appropriate and complete concept map and could give an example and a counter-example of homomorphism. While, the fair and poor achievement students, consecutively, got 2 points and 1 point. The fair achievement student gave a wrong example and a wrong counter-example of homomorphism. Whereas, the poor achievement student could not give the example and the counter example.

CONCLUSIONS

The result showed that the good achievement student was the highest score in other students. DK, the good achievement student, got 12 points of concept map (maximum score). DK was able to construct an appropriate and complete concept map. She also could give an example and a counter-example of homomorphism. She produced the concept map which was easy to interpret. The concept map showed that she had understanding of Abstract Algebra concept and used appropriate terminology and notations. She could identify all the important concepts of Abstract Algebra related to homomorphism and showed understanding of the relationship among the concepts.

SD, the fair achievement student, got 6 points of concept map. She could place almost all concepts in an appropriate hierarchy and assigns linking words to most connection in the concept map. She produced the concept map which was easy to interpret. She made mistakes in terminology or showed a few misunderstanding of concepts. She could give a wrong example and a wrong conter-example of homomorphism. She could indentify the important concepts, but made some incorrect connections.

AG, the poor achievement student, got 3 points. He could place only a few concepts in an appropriate hierarchy or use only a few linking words. He produced the concept map that was difficult to interpret. The concept map showed that he made many mistakes in terminology and showed a lack of understanding of many concepts. She also made many incorrect connections.

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