ISBN : 978-602-1037-00-3

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Martingale And The Efficient Market Hypothesis (EMH)

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ABSTRACT

This paper will explain how the martingale can be described the efficient market hypothesis. A stochastic process $\{X_n | n \in \mathcal{T}\}$ is a martingale if $E[|X|] < \infty$ dan $E[X_{n+1}|X_0, \ldots, X_n] = X_n$. The martingale property is saying that at each time the expectation of your next gain, conditional on what has happened so far, has mean zero. A martingale is a generalization of the fair game concept. The Efficient Market Hypothesis (EMH) maintains that when investors use all available information in forming expectations of future rates of return, the equilibrium price of the asset equals the optimal forecast of fundamental values based on the available information (i.e., the present value of expected future returns on the asset). In an efficient market, all information currently available is reflected in the asset price. Thus, the best forecast of tomorrow's stock price is today's price. Its implication that EMH is follows a martingale.

Keyword : martingale, fair game, efficient market hypothesis

A. INTRODUCTION

The phenomenons that occurs in the real life can be describe quantitatively by a model called mathematical model. A mathematical model divide into two models, that is deterministic model and stochastic model. In mathematics, deterministic models are explained as a set of states which are predetermined depending on the initial conditions. Thus, as long as the initial conditions never change, the outcome will always be the same. In other words, deterministic models introduce no randomness into the system. Even though these models give seemingly correct outcomes, they go against the nature of real world systems. In order to get an accurate portrayal of a system, the whole system must be modeled accurately, not just certain cases. This is where stochastic models come in. Stochastic models depend on some predetermined and random variables to transition from one state to another.With those random variables comes a way to represent the randomness involved in real world systems.

One of stochastic models is stochastics process. A stochastic process, or sometimes random process (widely used) is a collection of random variables, representing the evolution of some system of random values over time. This is the probabilistic counterpart to a deterministic process (or deterministic system). Instead of describing a process which can only evolve in one way (as in the case, for example, of solutions of an ordinary differential equation), in a stochastic or random process there is some indeterminacy, even if the initial condition (or starting point) is known, there are several (often infinitely many) directions in which the process may evolve. The familiar examples of stochastic proceses that is stock market and exchange rate fluctuations, signals such as speech, audio and video, medical data such as a patient's EKG, EEG, blood pressure or temperature, and random movement such as Brownian motion or random walks.

A stochastic process is defined as sequences of random variables X, indexed by a totally ordered set $\mathcal{T}$ ("time") written by $\{X_n | n \in \mathcal{T}\}$, if $\mathcal{T}$ is a countable set then $\{X_n | n \in \mathcal{T}\}$ is called discrete-time process, but if if $\mathcal{T}$ is an interval then $\{X_n | n \in \mathcal{T}\}$ is called continous-time process. For example, a dice bounded repeatedly, if X_n represent

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an amount of a six number on the dice that appear at the n bounded, then $\{X_n|n \geq 1\}$ is a stochastic process.

Generally, X_n is depend on $X_1, X_2, \ldots, X_n$, in a special case, if the expectation value of X_{n+1} and $X_0, \ldots, X_n$ as a condition equal to X_n, in mathematically $E[X_{n+1}|X_0, \ldots, X_n] = X_n$, then its called martingale process. The martingale discovered by Joseph Doob at the first 1950 from Illionis University. The Martingale have an important role in probability theory and the mathematics of finance.

The martingale analogous with the probability model from the fair game, that is the win expectation value of the next game is equal to the win value at present time. The martingale process have a role at the finance market. Its can be used as tool to make an efficient market hypothesis model. An efficient market hypothesis represent that the price at present time have described every market information of the price at the past time, its mean that the price at present time can be used as the best tool to predict an expectation value at the future. According to that presentation, we have some idea for investigate how martingale can describe the efficient market hypothesis.

B. DISCUSSION

According to the Merriam-Webster Collegiate Dictionary, a martingale is any of several systems of betting in which a player increases the stake usually by doubling each time a bet is lost. The use of the term in the theory of probability derives from the connection with fair games or fair bets; and the importance of the theoretical construct in the world of finance also derives from the connection with fair bets. The martingale property is saying that at each time the expectation of your next gain, conditional on what has happened so far, has mean zero. In the context of gambling, what makes a single bet fair is that you start with a known fortune x_0, you will get a random fortune X_1, and your gain $X_1 - x_0$ has mean zero, that is $E[X_1] = x_0$. For any underlying fair game, if you bet repeatedly, choosing how much to bet each time using some arbitrary strategy of your choice, which you may change and which may depend on what has happened so far, then the progress of your fortune X_n is a martingale. The iconic example of a fair game is to bet on a sequence of coin tosses, winning or losing one dollar each time depending on whether you predict correctly. The fair game hypothesis has two aspects, that is no arbitrage opportunity and unpredictability of security price variations.

Formally, Let $\mathcal{T}$ be a subset of $[0, \infty)$. A sequence of random variables $\{X_n|n \in \mathcal{T}\}$, indexed by $\mathcal{T}$, is called a stochastic (or random) process. When $\mathcal{T} = \mathbb{N}$, $\{X_n|n \in \mathcal{T}\}$ is said to be a discrete-time process, and when $\mathcal{T} = [0,1)$, it is called a continuous-time process. A stochastic process defined on some probability space $(\Omega, \mathcal{F}, P)$ and indexed by some ordered set $\mathcal{T}$, whose predicted value at any future time is the same as its present value at the time of prediction, its called martingale. In other words, the directions of the future movements in martingale are impossible to forecast.

Mathematically we can said that, a sequence of random variables $\{X_n|n \in \mathcal{T}$, with finite expected values and defined on the same probability space, is called a martingale if and only if $\mathrm{E}[\mathrm{X}_{n+1}|\mathrm{X}_0, \ldots, \mathrm{X}_n] = \mathrm{X}_n$. Similarly, such a sequence is called a sub-martingale if and only if $E[X_{n+1}|X_0, \ldots, X_n] \geq X_n$, and is called a super-martingale if and only if $E[X_{n+1}|X_0, \ldots, X_n] \leq X_n$. One important property of Martingales is that $E[X_n] = E[X_0]$ for all $n \geq 1$.

Suppose $\{X_n\}$ is martingale then the decomposition of a martingale $\{X_n\}$ can written by a partial sum process $X_n = X_0 + \sum_1^n \xi_j$ where $\xi_j = X_j - X_{j-1}$. Thats gain another propotition as follow : if $\{X_n\}$ is martingale and $\xi_{n+1} = X_{n+1} - X_n$ then $E[\xi_{n+1}|X_0, \ldots, X_n] = 0$. The process $\{\xi_{n+1}\}$ is called a martingale difference sequence (MDS).

A market in which prices always 'fully reflect' available information is called 'efficient'. The efficient markets hypothesis (EMH) maintains that market prices fully reflect all available information. Developed independently by Paul A. Samuelson and Eugene F. Fama in the 1960s, this idea has been applied extensively to theoretical models and empirical studies of financial securities prices, generating considerable controversy as well as fundamental insights into the price-discovery process.

The Efficient Market Hypothesis (EMH) applies Rational Expectations to the pricing of assets: When investors use all available information in forming expectations of future rates of return, the equilibrium price of the asset equals the optimal forecast of fundamental values based on the available information (i.e., the present value of expected future returns on the asset). In an efficient market, all information currently available is reflected in the asset price.

When economists speak of markets as being *efficient*, they usually consider asset prices and returns as being determined as the outcome of supply and demand in a competitive market, peopled by rational traders. These rational traders rapidly assimilate any information that is relevant to the determination of asset prices or returns (e.g. future dividend prospects) and prices adjust accordingly. Hence, individuals do not have different comparative advantages in the acquisition of information. It follows that, in such a world, there should be no opportunities for making a return on a stock that is in excess of a fair payment for the riskiness of that stock. In short, abnormal profits from trading should be zero. Under the EMH, the stock price already incorporates all relevant information, and, the only reason for prices to change between time t and time t +1 is the arrival of 'news' or unanticipated events.

If changes in stock prices caused by arrival of new information is random, then the level of stock prices follows a random walk $X_{t+1} - X_t = \xi_{t+1}$. To make full use of past stock price data, we can consider a more general form of EMH, that is, stock prices follows $E[\xi_{t+1}|X_0, \ldots, X_t] = 0$. In the other word, the level of stock price is a martingale difference sequence (MDS).

Any change in market price from one period a head will be completely accounted for by new information on market fundamentals which arrives between time t and $t + 1$. If the predicted price ($E[X_{t+1}|X_0, \ldots, X_t]$) based on publicly available information is different from the current price (X_t), there exists an arbitrage opportunity. If the market is efficient, investment will rapidly bid on the stock so that the arbitrage opportunity will soon disappear. For example, if an announcement, such as a merger, is already publicly made, the news has already been reflected in the prices. By the time the merger actually takes place, there will be little impact on the stock price. Stock prices respond to announcements only when the information is new and unexpected. Thus, the best forecast of tomorrow's stock price is today's price or $\mathrm{E}[\mathrm{X}_{t+1}|\mathrm{X}_0, \ldots, \mathrm{X}_t] = \mathrm{X}_t$. Its implication that EMH is follows a martingale.

C. CONCLUSION

Martingales are random variables whose future variations are completely unpredictable given the current information set. In other words, the directions of the

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future movements in martingales are impossible to forecast. In an efficient market, all information currently available is reflected in the asset price. Thus, the best forecast of tomorrow's stock price is today's price. Its implication that EMH is follows a martingale.

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