ISBN : 978-602-1037-00-3

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Identifying (Indonesian) Students' Difficulties in Solving Context-Based (PISA) Mathematics Tasks

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Abstract

This paper reports an investigation of the difficulties experienced by (Indonesian) students when solving context-based mathematics tasks. A total of 362 students from 11 schools located in rural and urban areas in the Province of Yogyakarta participated in a paper-and-pencil test. The test items comprised 34 tasks which were selected from the released items of the Programme for International Student Assessment (PISA) tasks. Students' difficulties were examined through an error analysis for which an analysis framework was used. The framework consisted of four error types: comprehension, transformation, mathematical processing, and encoding. The data analysis revealed that the most dominant errors made by the students were comprehension errors (38%) and transformation errors (42%). Of all errors made by students 17% were mathematical processing errors and only 3% were encoding errors. These findings indicate that (Indonesian) students mostly had difficulties in comprehending a context-based task and in transforming it into a mathematical problem.

Keyword: context-based tasks, PISA, error analysis, Indonesia

1. Introduction

The ability to apply mathematics is considered as a core goal of mathematics education in all around the world (see, e.g., Eurydice, 2011; NCTM, 2000). This goal is similar to what in the Programme for International Student Assessment (PISA) is called mathematical literacy, which refers to students' ability "to identify, and understand, the role that mathematics plays in the world, to make well-founded judgments and to use and engage with mathematics in ways that meet the needs of that individual's life as a constructive, concerned, and reflective citizen" (OECD, 2003, p. 24). To develop students' ability to apply mathematics, it is recommended to offer students mathematics problems situated in real-world contexts (De Lange, 2003; NCTM, 2000). In PISA study high value is attached to problems with real-world contexts as a mean to assess mathematical literacy (OECD, 2003). In this paper such problems are called context-based tasks and defined as tasks that are situated in real-world settings and provide elements or information that need to be organized and modeled mathematically.

Similar to many other countries, Indonesia also places a premium on applying mathematics as a core goal of mathematics education and pays attention to the use of context-based tasks (Pusat Kurikulum, 2003). This educational goal is also considered in the newly implemented Curriculum 2013 in which the Indonesian government clearly mandates that education must be relevant to the needs of life and offers students opportunities to apply their knowledge in society (Kementerian Pendidikan dan Kebudayaan, 2012). Nevertheless, there is an apparent discrepancy between this goal and student achievement. The PISA results showed that Indonesian students perform low on context-based tasks. More than three quarters of Indonesian students did not reach the baseline Level 2, which means they could only answer tasks that have familiar contexts and present all relevant information (OECD, 2010). The low performance of Indonesian students on context-based tasks prompted an establishment of a project called "Context-based Mathematics Tasks Indonesia" (CoMTI), which was aimed at investigating how student performance can be improved. This paper reports the first study of the CoMTI project in which we investigated the difficulties experienced by (Indonesian) students when solving context-based (PISA) mathematics tasks. The research question addressed in this paper is: *"What errors do (Indonesian) students make when solving context-based mathematics tasks?"*

2. Theoretical Background

2.1. Solving context-based mathematics tasks

Solving mathematics problem situated in real-world contexts, which in this paper are called context-based tasks, requires an interplay between the real world and mathematics that is often described as a modeling process. According to Blum and Leiss (2007) process of modeling is considered to be carried out in seven steps. The first step is establishing a 'situation model' to understand the real-world problem. Second, developing the situation model into a 'real model' through the process of simplifying and structuring. Third, constructing a 'mathematical model' by mathematizing the real model. After the mathematical model is established, in the fourth step, the solver carry out mathematical procedure to get a mathematical solution. In the fifth and sixth steps, the mathematical solution is interpreted and, then, validated its appropriateness in terms of the real-world problem. The final step is communicating the real-world solution. As the final step, the real-world solution has to be presented in terms of the real-world situation of the problem. This modelling process is similar to what is called 'mathematization' in PISA studies (OECD, 2003). Mathematization involves: understanding the problem situated in reality; organizing the real-world problem according to mathematical concepts and identifying the relevant mathematics; transforming the real-world problem into a mathematical problem which represents the situation; solving the mathematical problem; and interpreting the mathematical solution in terms of the real situation.

2.2 Analyzing students' errors in solving context-based mathematics tasks

Analysis of students' errors has long been considered as a powerful source to diagnose learning difficulties (see, e.g. Batanero, Godino, Vallecillos, Green, & Holmes, 1994; Seng, 2010) because errors provide access to students' reasoning (Brodie, 2014). With respect to analysing students' difficulties in solving mathematical word problems, Newman (1977) developed a model that is known as Newman Error Analysis. Newman proposed five categories of errors, i.e. *reading* (error in simple recognition of words), *comprehension* (error in understanding the meaning of a problem), *transformation* (error in transforming a word problem into an appropriate mathematical problem), *process skills* (error in performing mathematical procedures), and *encoding* (error in representing the mathematical solution into acceptable written form).

Word problems are rather different with context-based tasks, i.e. word problems mostly use contexts that can be neglected in the solving process and often explicitly provide the required procedures (see, e.g. Maass, 2010). Therefore, it is necessary to check whether Newman Error Analysis is applicable for analyzing students' difficulties in solving context-based tasks. Table 1 shows the association between Newman's error categories with the stages of modeling process and PISA's mathematization. It is shown that in general Newman Error Analysis could be used to analyze students' errors when solving context-based tasks. Of Newman's five error categories, only the first category that does not match to modeling process or mathematization.

Table 1. Newman's error categories and stages in solving context-based mathematics tasks

Newman's error categories	Stages in solving context-based mathematics tasks	
	Blum and Leiss' Modeling	PISA's Mathematization
Reading: Error in simple recognition of words and symbols	--	--
Comprehension: Error in understanding the meaning of a problem	Understanding problem by establishing situational model	Understanding problem situated in reality
--	Establishing real model by simplifying situational model	--
--	--	Organizing real-world problems according to mathematical concepts and identifying relevant mathematics
Transformation: Error in transforming a word problem into an appropriate mathematical problem	Constructing mathematical model by mathematizing real model	Transforming real-world problem into mathematical problem which represents the problem situation
Process skills: Error in performing mathematical procedures	Working mathematically to get mathematical solution	Solving mathematical problems
Encoding: Error in representing the mathematical solution into acceptable written form	Interpreting mathematical solution in relation to original problem situation	Interpreting mathematical solution in terms of real situation
	Validating interpreted mathematical solution by checking whether this is appropriate and reasonable for its purpose	
--	Communicating the real-world solution	--

3. Methods

3.1. CoMTI Test

A so-called 'CoMTI test' was administered to collect data about students' errors when solving context-based mathematics tasks. The test items were 19 PISA mathematics tasks consisting of 34 questions, which were selected from PISA's (OECD, 2009) released mathematics tasks. The questions were equally distributed over four different booklets based on the difficulty level of the tasks, as reflected in the percentage correct answers found in the PISA 2003 survey (OECD, 2009). Six of the questions were used as anchor tasks and were included in all booklets. Every student took one booklet consisting of 12 to 14 questions.

3.2. Participants

A total of 362 students from 11 schools located in rural and urban areas in the Province of Yogyakarta, Indonesia participated in the CoMTI test. After the test we checked whether the results of our sample were comparable with those of Indonesian students who participated in the PISA surveys. For this purpose, we compared the percentages of correct answers of Indonesian students participated in the PISA 2003 survey (OECD, 2009) with those of students in our sample for 17 PISA mathematics tasks. A significant correlation was obtained, r (15) = .83, $p < .05$, which indicates that the tasks that were difficult for Indonesian students in the PISA 2003 survey were also difficult for the students in the present study.

3.3 Procedure of coding the errors

An error analysis was performed on the basis of students' incorrect responses to investigate the kinds of difficulties experienced by students. For this purpose, an analysis framework was developed based on Newman's error categories that were associated with the stages of modeling process and PISA mathematization. The analysis framework comprised four types of errors: comprehension, transformation, mathematical processing, and encoding. Newman's reading error was not used in our analysis because this error category refers to the technical aspect of reading and does not match to any stage of modeling process or PISA's mathematization. Furthermore, to make the coding more fine-grained the four error types were specified into a number of sub-types, which was done on the basis of a first exploration of the data and a further literature review (see Table 2 for the sub-types of comprehension errors, Table 3 for the sub-types of transformation errors, and Table 4 for the sub-types of mathematical processing errors. For the encoding error we did not specify into sub-types).

The coding was carried out by the first author and afterwards the reliability of the coding was checked through an additional coding by an external coder. This extra coding was done on the basis of 22% of students' incorrect responses which were randomly selected from all mathematics units. In agreement with the multiple coding procedure, the interrater reliability was done for each error type, which resulted in Cohen's Kappa of .72 for comprehension errors, .73 for transformation errors, .79 for errors in mathematical processing, and .89 for encoding errors, which indicate that the coding was reliable (Landis & Koch, 1977).

3.4 Statistical analyses

As an addition to the error analysis a statistical analysis was performed to investigate the pattern of errors made by students with different ability level. We applied a Rasch analysis to obtain scale scores of the students' performance. The reason for choosing this analysis is that it can take into account an incomplete test design (different students got different test booklets with a different set of tasks). A partial credit model was specified in ConQuest (Wu, Adams, Wilson, & Haldane, 2007). The scale scores were estimated within this item response model by weighted likelihood estimates (Warm, 1989) and were categorized into four almost equally distributed performance levels where Level 1 indicates the lowest performance and Level 4 the highest performance. To test whether the frequency of a specific error type differed between performance levels, we applied an analysis of variance based on linear mixed models (Bates, Maechler, & Bolker, 2011). This analysis was based on all responses where an error could be coded and treated the nesting of task responses within students by specifying a random effect for students

4. Results

4.1 Students' errors when solving context-based (PISA) mathematics tasks

In total, there were 4707 responses (number of tasks done by all students in total) which included 2472 correct responses (53%), 1532 incorrect responses (33, and 703 missing responses (15%). The error analysis was carried out on the basis of the 1532 incorrect responses. The analysis of these responses revealed that a total of 1718 errors were made by the students. The total number of errors exceeded the total number of the incorrect responses because a multiple coding was applied, which means an incorrect response could be coded with more than one error type. Of all errors made by the students 38% were comprehension errors, 42% were transformation errors, 17% were mathematical processing errors, and only 3% were encoding errors. A closer examination was carried out to identify the sub-types of comprehension, transformation, and mathematical processing errors.

4.1.1 Comprehension errors

It was found that a half of the comprehension errors were errors in selecting relevant information (see Table 2). Students tended to use all numbers provided in a task (see Figure 1a). Figure 1a shows an error made by a student when solving the Staircase task. This task is about finding the height of each step of a staircase consisting of 14 steps. The student has deduced correctly that to solve the task he had to divide the height of the staircase by the number of steps. However, in the calculation he included the depth of the staircase although this information was irrelevant for solving the task.

Another kind of error in selecting information is related to students' inability to connect information from different sources (see Export task in Figure 1b). Students were asked to calculate the value of fruit juice exported in 2000 for which they needed to use the data from the pie chart, for the percentage of fruit juice, and the data from the bar diagram for the total annual exports in 2000. A student, whose work is shown in Figure 1b, already used the correct mathematical procedure. However, he used incorrect data in the calculation. Instead of taking the data of total annual exports in 2000 from the bar diagram (i.e. 42,6 million zeds), he used '360', which seems to be derived from the total angle of a circle.

Table 2 Frequencies of sub-types of comprehension errors

Sub-type of comprehension error	n	%
Misunderstanding the instruction	227	35
Misunderstanding a keyword	100	15
Error in selecting information	326	50
Total of observed errors	653	100

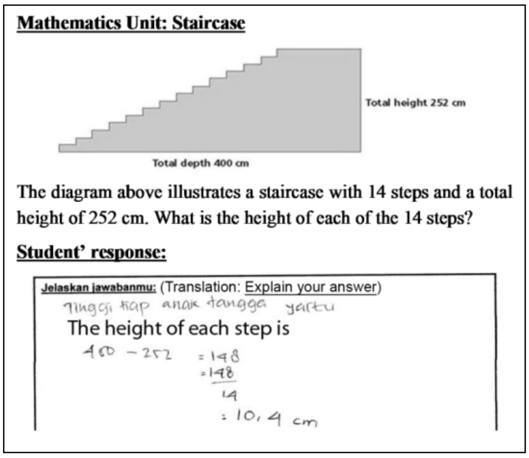


Figure 1a. Example of an *error in selecting information*: using all numbers provided in the task

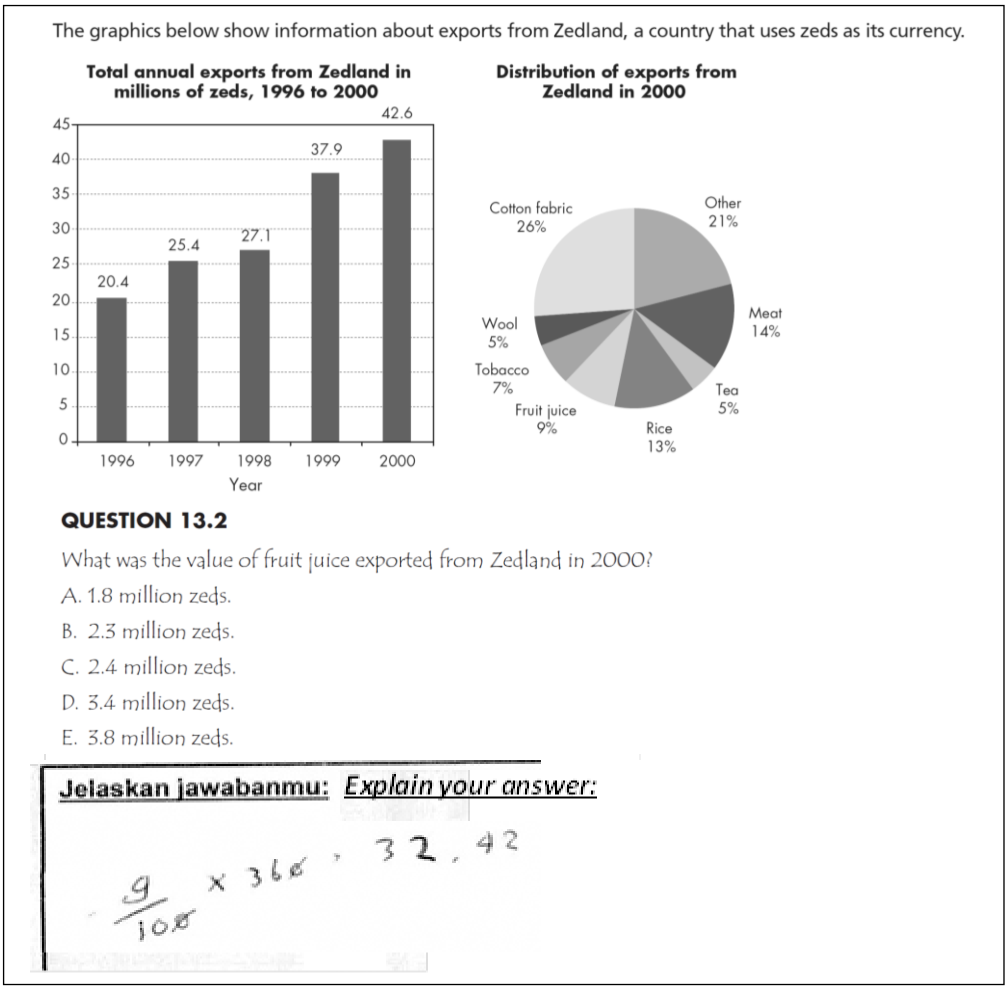


Figure 1b. Example of an *error in selecting information*: inability to find information from different sources

4.1.2 Transformation errors

With respect to the transformation errors, it was revealed that two thirds of them were the sub-type 'wrong mathematical operation/concept' which means errors in selecting the required mathematical procedures (see Table 3). Figure 2 shows an incorrect response of a student that contained a transformation error. The task shown in Figure 2 is about the concept of direct proportion that is situated in the context of money exchange. The student was asked to convert 3900 ZAR to Singapore dollars with an exchange rate of 1 SGD = 4.0 ZAR. Instead of dividing 3900 by 4.0, the student multiplied 3900 by 4.0. This means the student chose the wrong mathematical procedure for solving the task.

Table 3 Frequencies of sub-types of transformation errors

Sub-type of transformation error	n	%
Procedural tendency	90	12
Taking too much account of the context	56	8
Wrong mathematical operation/concept	489	68
Treating a graph as a picture	88	12
Total of observed errors	723	100

Mathematics Unit: Exchange Rate (question 2)

On returning to Singapore after 3 months, Mei-Ling had 3900 ZAR (South African rand) left. She changed this back to Singapore dollars, noting that the exchange rate had changed to:

1 SGD = 4.0 ZAR

How much money in Singapore dollars did Mei-Ling get?

Student' response:

Jelaskan jawabanmu: (Translation: Explain your answer)

3900 x 4,0

= 15600

Figure 2. Example of *transformation error*

4.1.3 Mathematical processing errors

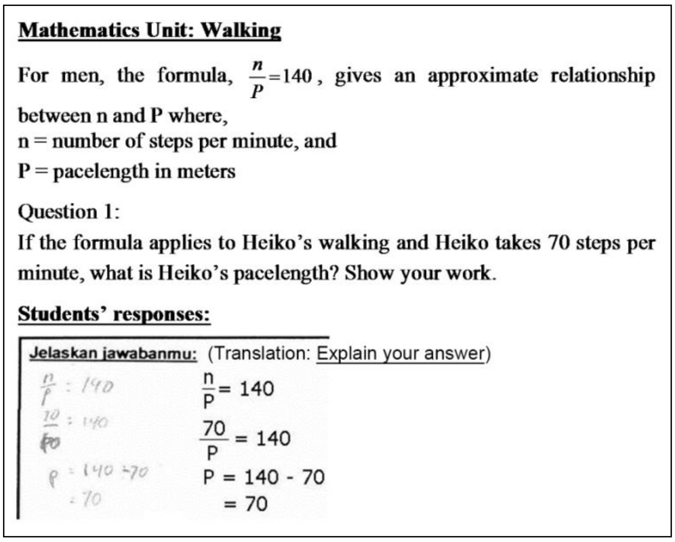
The sub-types of mathematical processing errors are dependent on the mathematical topic addressed in a task. For example, errors in interpreting a graph do not occur when there is no graph in the task. Consequently, the frequencies of the sub-types of mathematical processing errors were calculated only for the related tasks, i.e. tasks in which such errors may occur (see Table 4).

ISBN : 978-602-1037-00-3

Table 4 Frequencies of sub-types of mathematical processing errors

Sub-type of mathematical processing error	Related tasks	All errors in related tasks	Mathematical processing errors in related tasks	
	n	n	n	%
Algebraic error	8	243	33	14
Arithmetical error	20	956	94	10
Error in interpreting graph	6	155	43	28
Measurement error	1	74	15	20
Error related to improper use of scale	1	177	49	28
Unfinished answer	26	1125	79	7

An example of a mathematical processing error is shown in a task in Figure 4. The task is about finding a man's pace length (P) by using the formula $\frac{n}{P} = 140$ in which n, the number of steps per minute, is given. The student correctly substituted the given information into the formula; i.e. $\frac{70}{P} = 140$. In the next step, however, instead of dividing 70 by 140 the student subtracted 140 by 70. This response indicates that the student had difficulty to work with an equation in which the unknown was the divisor and the dividend is smaller than the quotient.


Mathematics Unit: Walking

For men, the formula, $\frac{n}{P} = 140$, gives an approximate relationship between n and P where,
n = number of steps per minute, and
P = pacelength in meters

Question 1:
If the formula applies to Heiko's walking and Heiko takes 70 steps per minute, what is Heiko's pacelength? Show your work.

Students' responses:

Figure 4. Example of *mathematical processing error* and *encoding error*

4.1.4 Encoding errors

Encoding errors were not divided into sub-types. They comprise all errors that are related to students' inability to interpret a mathematical answer as a solution that fits to the real-world context of a task. A student's answer in Figure 4 shows an encoding error. The answer of 70, within the context of this task, does not make sense because a human's pace length of 70 meter is a rather unrealistic answer.

4.2 The relation between the types of errors and the students' performance level

When testing whether students on different performance levels differed with respect to the error types they made, it was found that the low performing students (Level 1 and Level 2) made more transformation errors than the high performing students (Level 3 and Level 4) (see Figure 5). For the mathematical processing errors the pattern was opposite in which the high performing students made more errors than the low performing students. With respect to the comprehension errors there was no such a difference. The low and high performing students made about the same number of comprehension errors.

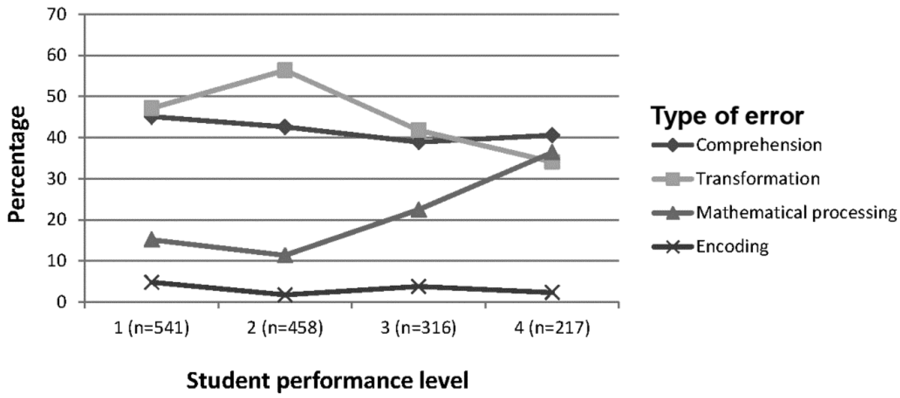


Figure 5. Types of error made by students with different performance levels

4. Conclusion and discussion

The results of the error analysis indicate that (Indonesian) students mostly experienced difficulties in the early stages of solving context-based tasks, i.e. in comprehending a context-based task and in transforming it into a mathematical problem. With respect to the mathematical processing errors it was found that high performing students made more errors than the low performing students. A possible explanation for this result is that the low performing students, in contrast to the high performing students, might get stuck in the first two stages of solving context-based mathematics tasks and therefore are not arriving at the stage of carrying out mathematical procedures. These findings confirm Newman's (1977) argument that the error types might have a hierarchical structure: failures on a particular step of solving a task prevents a student from progressing to the next step.

In addition to these specific results, this study showed how analyzing students' difficulties can be a crucial preliminary step in the process of improving student performance because it sheds light on key aspects of solving context-based tasks that need to be developed. The findings of this study suggest that improving the task comprehension of (Indonesian) students requires a focus not only on students' language competence, but also on the ability to select relevant information. Furthermore, the ability to identify the required procedure or concept was found to be another key competence that needs to be improved.

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