

Pollutant And Sedimentation Dispersion Pattern In The Confluence Of Two Rivers

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Abstract

In addition to river water pollution caused by human activity and industrial activity, also due to influence of natural activities such as volcanic activity that has volcanic ash and cold lava cause river pollution in river water at the same deposition. Besides of community activities Surabaya living along the Brantas River flow contributes to making soil erosion along the river, pollution and deposition also occurs due to discharge of industrial waste or household waste. This was strengthened with the data from the Ministry of Environment, Sindo's magazine on Nopember 7th 2013, "The total number of industries in East Java that has the potential to pollute the river there are about 1,004 industries, of that number as many as 483 industries located along the Watershed (DAS) Brantas river Surabaya and capacity pollution load 35 tons of organic waste a day, but the fact more than 70 tons of organic waste a day ". It becomes negative news for the citizens of Surabaya so that the public and the government of Surabaya more careful in taking precaution because almost 96% of the water supply Surabaya Kali is a tributary of the Brantas river used to PDAM Surabaya, so according to its function, the water must be maintained the quality of the water and do monitoring in order to remain decent for use. In this research focuses on the modeling of pollutant dispersion and sedimentation patterns in the confluence of two rivers by combining elements of hydrodynamics using Finite Volume Method QUICK scheme (upwind Quadratic Interpolation Convective Kinematics) so that the results of this research are expected to helping the parties involved in maintaining the quality of the Brantas River Surabaya to remain fit for consumption and reduce silting of the river.

Keywords: Elements of Hydrodynamics, The spread of pollutants and sedimentation, river Brantas, Finite Volume Method, *QUICK (Quadratic Upwind Interpolation Convective Kinematics)*.

1. Introduction

Many community activities that use water Surabaya Surabaya who is the son of the Brantas River in the fulfillment of everyday life. However, lately many conditions Surabaya and polluted river becomes shallow. Pollution that occurred in Surabaya is a Surabaya impact of community activities as well as many factories along the river. Superficiality rivers also occur as a consequence of water pollutants and sediment from erosion settlement residents along the river which causes sedimentation. The impact of pollution and sedimentation of river water will change the characteristics and patterns of river water, so that the need for supervision and monitoring efforts river water quality. Based on the above description, it will be assessed in this study the spread of pollutants and sedimentation at the confluence of two rivers. The parameters used for pollutants and sediment that TSS (Total Suspended Solid), COD, BOD, and DO. This study will be developed with a method to volume upwind scheme Quadratic Interpolation Convective Kinematics (QUICK). To resolving numerical and visualization of the results will be used MATLAB software.

2. Basic Laws of Hydrodynamics

2.1 The Conservation of Mass

According to Apsley, (2013) and Welty et al, (2007) described the continuum principle that the law of conservation of mass is the mass contained in the Control Volume (CV) coupled with mass flow out through the Control Surface (CS) is equal to the number of generated mass. (Apsley, 2013) and (Welty dkk, 2007).

$$\frac{d}{dt}(\rho V) + \sum_{CS} \rho A u = 0 \tag{2.1}$$

2.2 The Conservation of Momentum

In the continuum principle of conservation of momentum is the average change of momentum in the control volume (CV) coupled with the momentum flow out through the Control Surface (CS) is equal to the rate of change of momentum with respect to time. (Apsley, 2013) and (Welty et al, 2007).

$$\frac{d}{dt}(\rho V)U + \sum_{CS}(\rho u \cdot A)U = F \tag{2.2}$$

2.3 Finite Volume Method

Finite Volume Method was first documented and well used by Evans and Harlow (1957) in Los Alamos and Gentry, Martin and Daley (1966). Volume up method is discretization technique for partial differential equations, especially for issues of conservation laws in physics. Volume method to pendiskritan also often used in fluid dynamics equations. Volume method to use the integral form of the conservation equation so much easier in pendiskritan (Veersteg, 2007). The solution will be divided into a finite number of control volumes, and which has been discretized equations will be applied to each control volume.

2.4 Discretization of QUICK

Each domain has a control grid in the face and control nodes. Controls for the two-dimensional face consists of $\phi_e, \phi_w, \phi_n, \phi_s$, while for the control node to consist of a two-dimensional $\phi_E, \phi_W, \phi_N, \phi_S$. QUICK discretization is done based on the magnitude of the velocity (v) is

$$\begin{cases} \phi(i, j) + g_1(\phi(i, j + 1) - \phi(i, j)) \\ + g_2(\phi(i, j) - \phi(i, j - 1)), \text{ for } v > 0 \\ \phi(i, j + 1) + g_3(\phi(i, j) - \phi(i, j + 1)) \\ + g_4(\phi(i, j + 1) - \phi(i, j + 2)), \text{ for } v < 0 \end{cases}$$

with:

$$g_1 = \frac{(x_e - x_p)(x_e - x_w)}{(x_E - x_p)(x_E - x_w)}$$

$$g_2 = \frac{(x_e - x_p)(x_E - x_e)}{(x_p - x_w)(x_E - x_w)}$$

$$g_3 = \frac{(x_e - x_E)(x_e - x_{EE})}{(x_p - x_E)(x_p - x_{EE})}$$

$$g_4 = \frac{(x_e - x_E)(x_P - x_e)}{(x_E - x_{EE})(x_P - x_{EE})}$$

$$\begin{aligned}\phi_e(i, j) &= -\frac{1}{8}\phi(i-1, j) + \frac{3}{4}\phi(i, j) + \frac{3}{8}\phi(i+1, j) \\ \phi_w(i, j) &= -\frac{1}{8}\phi(i-2, j) + \frac{3}{4}\phi(i-1, j) + \frac{3}{8}\phi(i, j) \\ \phi_n(i, j) &= -\frac{1}{8}\phi(i, j-1) + \frac{3}{4}\phi(i, j) + \frac{3}{8}\phi(i, j+1) \\ \phi_s(i, j) &= -\frac{1}{8}\phi(i, j-2) + \frac{3}{4}\phi(i, j+1) + \frac{3}{8}\phi(i, j)\end{aligned}\tag{2.3}$$

3. Mathematical model River Stream

Conservation laws of physics which meet river flow model are

1. The conservation of mass
2. The conservation of momentum
3. The conservation of energy

(Governing Equation) used only the continuity equation of the law and the law of conservation of mass due to temperature changes are ignored in this study.

Governing Equation of Conservation of Mass

$$\frac{du}{dx} + \frac{dv}{dy} = 0\tag{3.1}$$

3.1 Governing Equation Marmoyo River

$$\frac{d}{dt}(u + q_b) + u \frac{du}{dx} + v \frac{du}{dy} = -\frac{dgh}{2dx} - \frac{dgh}{2dy} + gh(S_x + S_y)\tag{3.2}$$

3.2 Governing Equation Surabaya River

$$\begin{aligned}\frac{d}{dt}(u + q_b) + \frac{du}{dx}(u + v \cos \theta) + v \frac{du}{dy} \sin \theta \\ = -\frac{dh}{dx} \left(\frac{g}{2} + \frac{g}{2} \cos \theta \right) - \frac{dgh}{2dy} \sin \theta + gh(S_x + S_y)\end{aligned}\tag{3.3}$$

3.3 Governing Equation of Confluence of Two River

$$\begin{aligned}\frac{d}{dt}(u + q_b) + \frac{du}{dx}(u + v \cos \theta) + v \frac{du}{dy} \sin \theta \\ = -\frac{dh}{dx} \left(\frac{g}{2} + \frac{g}{2} \cos \theta \right) - \frac{dgh}{2dy} \sin \theta + gh(S_x + S_y)\end{aligned}\tag{3.4}$$

dengan:

$$q_b = c_m [(s - 1)g]^{0,5} d_{50}^{1,5} (\mu\theta_b - \theta_c)^{1,5} \tag{3.5}$$

$$\tau_b = \frac{1}{2} \rho \left(\frac{0.06}{\left(\log \left(\frac{12h}{2.5d_{50}} \right) \right)^2} \right) u^2 \tag{3.6}$$

- Q_1 = First Streamflow
- Q_2 = Second Streamflow
- v_1 = First velocity
- v_2 = Second velocity
- q_b = amount of bed load sediment
- s = ratio of sediment density to water density
- $d_{50}^{1,5}$ = average diameter of sediment
- c_m = 8.0
- μ = 1.0
- θ_c = 0.047
- $\theta_b = \frac{\tau_b}{(\rho_b - \rho)gd_{50}}$
- ρ = density
- τ_b = shear stress

4. Numerical Solution

Marmoyo river at the x-axis

Node 1:

$$(u_{i,j+1}) = u_{(i,j)} - q_b - \frac{21}{64} Au_{(i+1,j)}u_{(i,j)} + \frac{3}{32} Au_A u_{(i+1,j)} + \frac{7}{32} Au_A u_{(i,j)} - \frac{9}{128} Au_{(i+1,j)}^2 - \frac{49}{128} Au_{(i,j)}^2 + \frac{15}{32} Au_A^2 - \frac{A}{2} gh + ghs_x \Delta t$$

Node 2:

$$(u_{i,j+1}) = u_{(i,j)} - q_b - \frac{3}{32} Au_A u_{(i+1,j)} - \frac{7}{32} Au_A u_{(i,j)} + \frac{3}{64} Au_{(i+1,j)}u_{(i,j)} + \frac{3}{64} Au_{(i+1,j)}u_{(i-1,j)} + \frac{3}{32} Au_{(i,j)}u_{(i-1,j)} - \frac{1}{128} Au_{(i-1,j)}^2 + \frac{13}{128} Au_{(i,j)}^2 + \frac{1}{32} Au_A^2 - \frac{A}{2} gh + ghs_x \Delta t$$

Node 3 and Node 4:

$$\begin{aligned}
 (u_{i,j+1}) &= u_{(i,j)} - q_b - \frac{3}{64} Au_{(i,j)} u_{(i-2,j)} - \frac{3}{32} Au_{(i-2,j)} u_{(i-1,j)} - \frac{9}{32} Au_{(i+1,j)} u_{(i,j)} \\
 &\quad + \frac{3}{64} Au_{(i+1,j)} u_{(i-1,j)} + \frac{3}{8} Au_{(i,j)} u_{(i-1,j)} - \frac{9}{128} Au^2_{(i+1,j)} - \frac{27}{128} Au^2_{(i,j)} \\
 &\quad + \frac{35}{128} Au^2_{(i-1,j)} + \frac{1}{128} Au^2_{(i-2,j)} - \frac{A}{2} gh + ghs_x \Delta t
 \end{aligned}$$

Node 5:

$$\begin{aligned}
 (u_{i,j+1}) &= u_{(i,j)} - q_b - \frac{3}{64} Au_{(i,j)} u_{(i-2,j)} - \frac{3}{32} Au_{(i-2,j)} u_{(i-1,j)} + \frac{9}{32} Au_{(i,j)} u_{(i-1,j)} \\
 &\quad + \frac{9}{128} Au^2_{(i,j)} + \frac{9}{32} Au^2_{(i-1,j)} + \frac{1}{128} Au^2_{(i-2,j)} - \frac{1}{2} Au^2_B - \frac{A}{2} gh \\
 &\quad + ghs_x \Delta t
 \end{aligned}$$

Surabaya river at the x-axis

Node 1:

$$\begin{aligned}
 (u_{i,j+1}) &= u_{(i,j)} - q_b - \frac{21}{64} Au_{(i+1,j)} u_{(i,j)} + \frac{3}{32} Au_A u_{(i+1,j)} + \frac{7}{32} Au_A u_{(i,j)} \\
 &\quad - \frac{9}{128} Au^2_{(i+1,j)} - \frac{49}{128} Au^2_{(i,j)} + \frac{15}{32} Au^2_A + \left(\frac{g}{2} + \frac{g}{2} \cos \theta\right) Ah + ghs_x \Delta t
 \end{aligned}$$

Node 2:

$$\begin{aligned}
 (u_{i,j+1}) &= u_{(i,j)} - q_b - \frac{3}{32} Au_A u_{(i+1,j)} - \frac{7}{32} Au_A u_{(i,j)} + \frac{3}{64} Au_{(i+1,j)} u_{(i,j)} \\
 &\quad + \frac{3}{64} Au_{(i+1,j)} u_{(i-1,j)} + \frac{3}{32} Au_{(i,j)} u_{(i-1,j)} - \frac{1}{128} Au^2_{(i-1,j)} \\
 &\quad + \frac{13}{128} Au^2_{(i,j)} + \frac{1}{32} Au^2_A + \left(\frac{g}{2} + \frac{g}{2} \cos \theta\right) Ah + ghs_x \Delta t
 \end{aligned}$$

Node 3 and 4:

$$\begin{aligned}
 (u_{i,j+1}) &= u_{(i,j)} - q_b - \frac{3}{64} Au_{(i,j)} u_{(i-2,j)} - \frac{3}{32} Au_{(i-2,j)} u_{(i-1,j)} - \frac{9}{32} Au_{(i+1,j)} u_{(i,j)} \\
 &\quad + \frac{3}{64} Au_{(i+1,j)} u_{(i-1,j)} + \frac{3}{8} Au_{(i,j)} u_{(i-1,j)} - \frac{9}{128} Au^2_{(i+1,j)} - \frac{27}{128} Au^2_{(i,j)} \\
 &\quad + \frac{35}{128} Au^2_{(i-1,j)} + \frac{1}{128} Au^2_{(i-2,j)} + \left(\frac{g}{2} + \frac{g}{2} \cos \theta\right) Ah + ghs_x \Delta t
 \end{aligned}$$

Node 5:

$$\begin{aligned}
 (u_{i,j+1}) &= u_{(i,j)} - q_b - \frac{3}{64} Au_{(i,j)} u_{(i-2,j)} - \frac{3}{32} Au_{(i-2,j)} u_{(i-1,j)} + \frac{9}{32} Au_{(i,j)} u_{(i-1,j)} \\
 &\quad + \frac{9}{128} Au^2_{(i,j)} + \frac{9}{32} Au^2_{(i-1,j)} + \frac{1}{128} Au^2_{(i-2,j)} - \frac{1}{2} Au^2_B \\
 &\quad + \left(\frac{g}{2} + \frac{g}{2} \cos \theta\right) Ah + ghs_x \Delta t
 \end{aligned}$$

The confluence of two rivers at x-axis

Node 1:

$$(u_{1,j+1}) = u_{(1,j)} - q_b - \frac{21}{64} Au_{(2,j)}u_{(1,j)} + \frac{3}{32} Au_A u_{(2,j)} + \frac{7}{32} Au_A u_{(1,j)} - \frac{9}{128} Au_{(2,j)}^2 - \frac{49}{128} Au_{(1,j)}^2 + \frac{15}{32} Au_A^2 - \frac{A}{2} gh + (ghs_x + u_1 Q_1 + u_2 Q_2) \Delta t$$

Node 2:

$$(u_{2,j+1}) = u_{(2,j)} - q_b - \frac{3}{32} Au_A u_{(3,j)} - \frac{7}{32} Au_A u_{(2,j)} + \frac{3}{64} Au_{(3,j)}u_{(2,j)} + \frac{3}{64} Au_{(3,j)}u_{(1,j)} + \frac{3}{32} Au_{(2,j)}u_{(1,j)} - \frac{1}{128} Au_{(1,j)}^2 + \frac{13}{128} Au_{(2,j)}^2 + \frac{1}{32} Au_A^2 - \frac{A}{2} gh + (ghs_x + u_1 Q_1 + u_2 Q_2) \Delta t$$

Node 3 and 4:

$$(u_{3,j+1}) = u_{(3,j)} - q_b - \frac{3}{64} Au_{(3,j)}u_{(1,j)} - \frac{3}{32} Au_{(1,j)}u_{(2,j)} - \frac{9}{32} Au_{(4,j)}u_{(3,j)} + \frac{3}{64} Au_{(4,j)}u_{(2,j)} + \frac{3}{8} Au_{(3,j)}u_{(2,j)} - \frac{9}{128} Au_{(4,j)}^2 - \frac{27}{128} Au_{(3,j)}^2 + \frac{35}{128} Au_{(2,j)}^2 + \frac{1}{128} Au_{(1,j)}^2 - \frac{A}{2} gh + (ghs_x + u_1 Q_1 + u_2 Q_2) \Delta t$$

Node 5:

$$(u_{5,j+1}) = u_{(5,j)} - q_b - \frac{3}{64} Au_{(5,j)}u_{(3,j)} - \frac{3}{32} Au_{(3,j)}u_{(4,j)} + \frac{9}{32} Au_{(5,j)}u_{(4,j)} + \frac{9}{128} Au_{(5,j)}^2 + \frac{9}{32} Au_{(4,j)}^2 + \frac{1}{128} Au_{(3,j)}^2 - \frac{1}{2} Au_B^2 - \frac{A}{2} gh + (ghs_x + u_1 Q_1 + u_2 Q_2) \Delta t$$

5. Simulation Results and Discussion

The simulation was performed based on the numerical solution obtained with the help of MATLAB. The analysis is performed using the selected parameters. Speed data used in this simulation is the data rate for the year 2013.

Distribution of Pollutants Marmoyo River

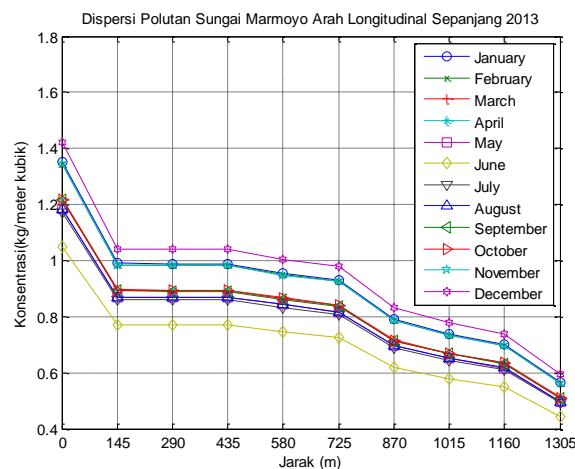


Figure 5.1: Dispersion of Pollutants River Marmoyo Longitudinal Direction

In Figure 5.1., seen that the distribution of pollutant concentrations Marmoyo River in the longitudinal direction decreases. Distribution of Pollutants lowest occurred in June and highest pollutant distribution in December. The decline in the distribution of pollutants contained in June due to a decline in the volume of river water due to the dry season, while the increase in the spread of pollutants occurs in November through February due to the rainy season. At $x = 0$ the distribution of pollutants is high because there are waste from factories and industrial waste on the river Marmoyo then a decline in the distribution of pollutants from $x = 0$ to $x = 145$ then constant and the distribution of pollutants decreases until $x = 1305$.

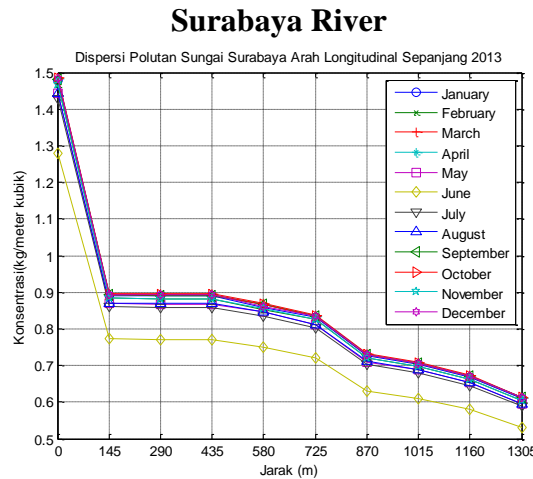


Figure 5.2: Dispersion of Pollutants Surabaya River Longitudinal Direction

In Figure seen that the distribution of pollutant concentrations in the Surabaya River longitudinal direction decreases. Distribution of Pollutants lowest occurred in June and highest pollutant distribution in December. Decreased the distribution of pollutants contained in June due to a decline in the volume of river water due to the dry season, while the increase in the spread of pollutants occurs in November through February due to the rainy season. At $x = 0$ the distribution of pollutants is high because there are waste from factories and industrial waste in Surabaya River and then a decline in the distribution of pollutants from $x = 0$ to $x = 145$ then constant and the distribution of pollutants decreases until $x = 1305$.

The Confluence of two rivers

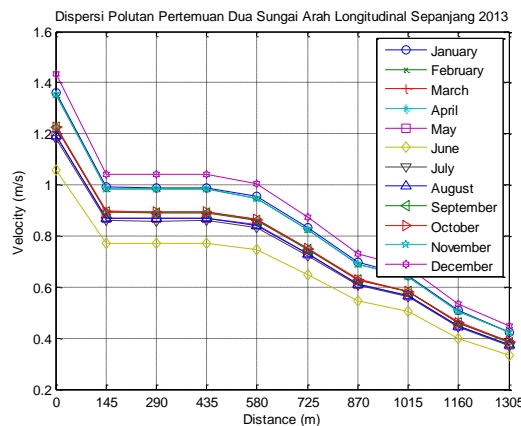


Figure 5.3: Dispersion of Pollutants Meeting of Two Rivers Longitudinal Direction

In Figure seen that the distribution of pollutant concentrations Meeting of Two Rivers in the longitudinal direction decreases. Distribution of Pollutants lowest occurred in June and highest pollutant distribution in December. Decreased the distribution of pollutants contained in June due to a decline in the volume of river water due to the dry season, while the increase in the spread of pollutants occurs in November through February due to the rainy season. At $x = 0$ the distribution of pollutants because there are high concentrations of pollutant mixtures Surabaya River and River Marmoyo then a decline in the distribution of pollutants from $x = 0$ to $x = 145$ then constant and the distribution of pollutants decreases until $x = 1305$.

Distribution of Sediment Marmoyo river

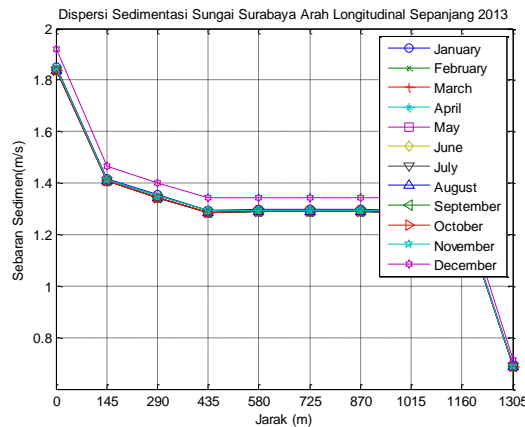


Figure 5.4: Dispersion Marmoyo River Sedimentation meeting Longitudinal Direction

In Figure seen that the distribution of sediment in the longitudinal direction of Surabaya River is declining because the pollutants TSS follow along streams resulting sedimentation. Distribution of low sedimentation occurred in June and the highest distribution of sedimentation occurred in December. There is a Decreased sedimentation distribution in June, while increase in the distribution of sedimentation occurs from November to February. At $x = 0$ the distribution of high pollutant because it follows the TSS pollutants along the river there are waste from factories and industrial waste in Surabaya River and then a decline in the distribution of sedimentation from $x = 0$ to $x = 145$ then tends to a constant and the distribution of sedimentation decreases until $x = 1305$.

Surabaya River

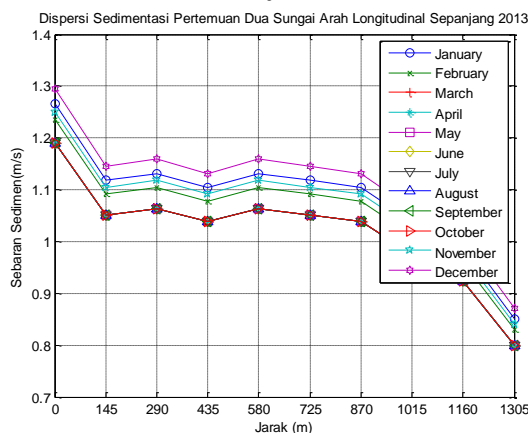


Figure 5.5: Distribution of Sedimentation Patterns In Surabaya River Longitudinal Direction

In Figure seen that the distribution of sediment Meeting of Two Rivers in the longitudinal direction is declining because the pollutants TSS follow along streams resulting sedimentation. Distribution of low sedimentation occurred in June and the highest distribution of sedimentation occurred in December. There is a Decreased sedimentation distribution in June, while increase in the distribution of sedimentation occurs from November to February. At $x = 0$ the distribution of high pollutant because it follows the TSS pollutants along the river there are Mixed quantity Surabaya River Sedimentation and River Marmoyo then a decline in the distribution of sedimentation from $x = 0$ to $x = 145$ then tends to a constant and the distribution of sedimentation decreases until $x = 1305$.

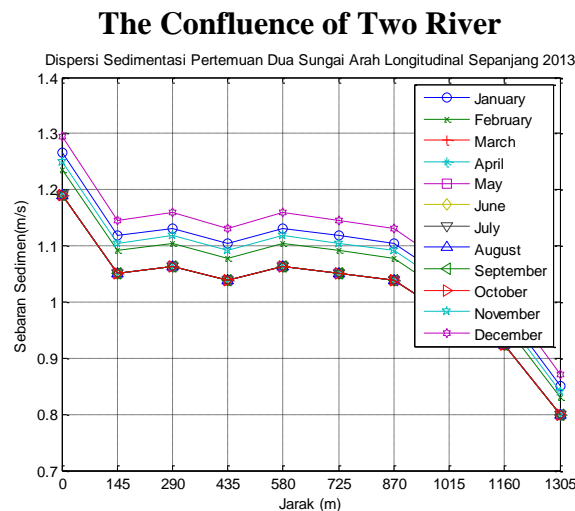


Figure 5.6: Pola Sebaran Sedimentasi Pada Pertemuan Dua Sungai Arah Longitudinal

In Figure seen that the distribution of sediment Meeting of Two Rivers in the longitudinal direction is declining because the pollutants TSS follow along streams resulting sedimentation. Distribution of low sedimentation occurred in June and the highest distribution of sedimentation occurred in December. There is a Decreased sedimentation distribution in June, while increase in the distribution of sedimentation occurs from November to February. At $x = 0$ the distribution of high pollutant because it follows the TSS pollutants along the river there are Mixed quantity Surabaya River Sedimentation and River Marmoyo then a decline in the distribution of sedimentation from $x = 0$ to $x = 145$ then tends to a constant and the distribution of sedimentation decreases until $x = 1305$.

6. Conclusion

1. It was found mathematical model of the equation of conservation of mass, conservation of momentum, the equation of sediment, and the scalar transport equation. Conservation of mass, conservation of momentum, and the equation used to calculate the quantity of sediment distribution and sediment transport equations used to calculate the scalar quantity distribution of pollutants.
2. During the year 2013, the spread of pollutants and sedimentation velocity is highest due to occur in December rainy season, and the speed of the spread of pollutants and sedimentation lowest occurred in June.

3. At $x = 0$ to $x = 145$ quantity of sediment distribution and quantity of pollutants to the direction of the longitudinal distribution decreases.
4. Pollutants and Sedimentation spread throughout the longitudinal and lateral direction and thoroughly spread on the direction of the longitudinal follow the river.
5. Quantity distribution of pollutants and sediment decreases with increasing distance.

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