On the Kutta condition for the sound transmission through outlet guide vanes

B. FRANÇOIS[a], S. BOULEY[a], M. ROGER[a], H. POSSON[b]

a. École Centrale de Lyon
Laboratoire de Mécanique des Fluides et Acoustique, UMR CNRS 5509
36 avenue Guy de Collongue, 69143 Écully, France
benjamin.francois@centrale-innovation.fr
b. Airbus Operations S.A.S.
306 Route de Bayonne,
Toulouse Cedex 9 31000, France

Résumé :

Ce travail présente une formulation analytique de la diffraction d'une onde acoustique par une grille d'aubes rectilinéaire figurant un stator, en prenant en compte un écoulement porteur et une condition de Kutta au bord de fuite. L'effet de cette dernière sur la forme et l'amplitude du champ diffracté augmente avec le nombre de Mach de l'écoulement. La formulation analytique s'appuie sur une méthode de raccordement modal. La méthode est appliquée en deux dimensions pour une onde acoustique plane monochromatique. Les résultats obtenus illustrent l'effet de la condition de Kutta sur la puissance acoustique diffractée et les champs de pression acoustique pour différents nombres de Mach.

Abstract :

The present work proposes an analytical formulation including a Kutta condition for the acoustic wave-scattering by a linear cascade of outlet guide vanes for turbomachinery applications. The Kutta condition has an increasing effect on the structure and on the magnitude of the acoustic response of the cascade as the Mach number of the mean flow is increased. The present analytical approach is believed a consistent alternative to numerical methods, especially at the early design stage. The analytical formulation relies on a mode-matching technique. The methodology is applied in a two-dimensional context and for an oblique acoustic wave. Results, focusing on the radiated acoustic power and the pressure fields, highlight the effect of the Kutta condition for various Mach numbers.

Key words: Aero-acoustics, wave scattering, Kutta condition

1   Introduction

Axial-flow fan stages are made of a rotor operating with a downstream row of stationary outlet guide vanes (stator). The role of the stator is to recover mechanical energy from the induced swirl. Its aeroacoustic behavior is twofold. Firstly the wakes issued from the upstream rotor impinge on the vanes, inducing unsteady loads which are responsible for what is referred to as wake-interaction noise. Secondly the acoustic waves generated by unsteady aerodynamic phenomena occurring on the rotor blades are restructured by their transmission through the stator before propagating in the exhaust duct. Both mechanisms need being modeled with a reasonable accuracy and at reasonable computational cost for the sake of low-noise design, especially at the preliminary design stage. This is why the modeling task is addressed analytically in the present work, only devoted to the sound transmission problem.

Analytical modeling requires drastic approximations on the geometry and the flow features in order to produce tractable, closed-form solutions. At the same time, it must be extended as far as possible to relevantly address configurations of industrial interest. Because the exit flow from outlet guide vanes must be axial, the vanes assumed with zero camber are modelled as a rectilinear cascade of parallel, axially aligned and zero-stagger plates. Then, the mean flow is considered as uniform and purely axial, of subsonic Mach number M. It has a significant effect on the acoustic propagation of the scattered waves. Firstly, due to the composition of velocities, upstream waves propagate slower and downstream waves faster than in absence of flow with respect to a fixed observer. Wavelengths are also modified accordingly. Secondly, the Kutta condition has an significant effect on the strengths of the scattered waves. According to Howe [1] and Job [2], a part of the acoustic energy of the incident waves is converted into hydrodynamic waves, due to the introduction of viscous effects. Consequently, it is essential to account for it in the analytical formulation. The present study proposes a formulation based on a mode-matching technique (see Mittra-Lee [3]). First attempts by Ingenito and Roger [4], readdressed by Bouley *et al.* [5], showed promising results turbomachinery aeroacoustics.

In the first part, the analytical formulation accounting for the Kutta condition is presented. Simulations are then performed with and without this condition for several Mach numbers to observe its effect. Results focusing on the scattered acoustic pressure field and the acoustic power balance are then discussed.

2   Analytical Formulation

Figure 1 shows a 2D unwrapped representation of zero-stagger stator vanes. a denotes the inter-vane distance, c the chord length. Φ_I is the potential associated with the incident acoustic wave while $\Phi_{R,D,U,T}$ denote the potentials of the scattered waves. All waves are monochromatic of pulsation ω.

2.1   Mode-Matching Principles

Consider an oblique incident acoustic wave of angle Θ, the potential of which reads:

$$\phi_I = e^{i\alpha_{i_0}z}e^{iK_{i_0}^+x}, \quad \alpha_{i_0} = \frac{k\sin\Theta}{1+M\cos\Theta}, \quad k = \frac{\omega}{c_0} \tag{1}$$

where α_{i_0} and $K_{i_0}^+$ are respectively the azimuthal and the axial wave numbers. This potential is solution of the convected Helmholtz equation in a uniform axial mean flow. As a result, its axial wave number
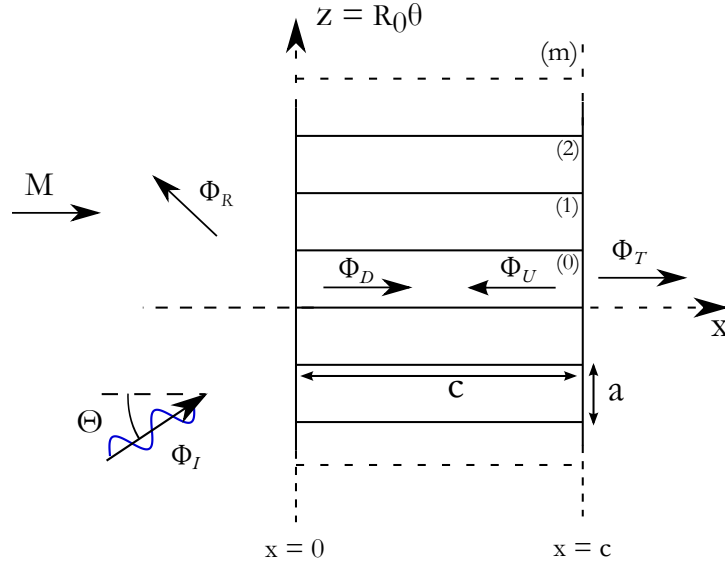
Figure 1: Incident and scattered wave notations. 2D unwrapped representation.

follows:

$$K_{i_0}^+ = -\frac{Mk}{\beta^2} + \frac{\sqrt{k^2 - \beta^2 \alpha_{i_0}^2}}{\beta^2}, \quad \beta = \sqrt{1 - M^2} \tag{2}$$

The principles of the mode-matching technique are explained in the next sections.

2.1.1 Formulation of the Scattered Potentials

Acoustic potentials upstream/downstream the vanes (ϕ_R, ϕ_T) are expressed as infinite sums of plane waves. Their potentials are also solutions of the convected Helmholtz equation and their azimuthal wave number is imposed by the incident wave (see Dowling and Ffows-Williams [6]). As a result, they read:

$$\begin{pmatrix} \phi_R \\ \phi_T \end{pmatrix} = \sum_{s=-\infty}^{+\infty} \begin{pmatrix} R_s \\ T_s \end{pmatrix} e^{i\alpha_s z} \begin{pmatrix} e^{iK_s^- x} \\ e^{iK_s^+ (x-c)} \end{pmatrix} \tag{3}$$

$$\alpha_s = \alpha_{i_0} + s\frac{2\pi}{a}, \quad K_s^\pm = -\frac{Mk}{\beta^2} \pm \frac{\sqrt{k^2 - \beta^2 \alpha_s^2}}{\beta^2}$$

R_s, T_s are the reflected and transmitted modal amplitudes.

In the inter-vane channels, potentials (ϕ_D, ϕ_U) are expressed as infinite sums of duct modes that fulfill the rigid wall condition on the vanes and propagating on either sides (upstream/downstream). The incident wave imposes a phase-shift between adjacent channels. By virtue of this phase-shift, the acoustic potentials only needs being solved for the reference channel 0:

$$\begin{pmatrix} \phi_d \\ \phi_u \end{pmatrix} = \sum_{p=0}^{+\infty} \begin{pmatrix} D_p^0 \\ U_p^0 \end{pmatrix} \cos\left[\frac{p\pi}{a}z\right] \begin{pmatrix} e^{iK_p^+ x} \\ e^{iK_p^- (x-c)} \end{pmatrix} \tag{4}$$

$$K_p^\pm = -\frac{Mk}{\beta^2} \pm \frac{\sqrt{k^2 - (p\pi/a)^2}}{\beta^2}$$

D_p and U_p are the modal amplitudes of the downstream and upstream travelling waves, respectively.

2.1.2 Matching Equations

To determine the modal amplitudes, matching equations on the acoustic pressure and axial velocity are set up at the leading-edge interface ($x = 0$) and at the trailing-edge interface ($x = c$). These equations are inherited from the basic conservation laws of fluids dynamics applied to a stator (see Roger *et al* [7]). As an example, matching equations at the trailing-edge interface read:

$$p'_D(c, z) + p'_U(c, z) = p'_T(c, z), \quad \forall z \tag{5}$$

$$v'_{x,D}(c, z) + v'_{x,U}(c, z) = v'_{x,T}(c, z), \quad \forall z \tag{6}$$

The expression of the acoustic pressure and velocity are reminded here:

$$p' = -\rho_0 \left(\frac{\partial \phi}{\partial t} + \vec{W}.\nabla\phi \right), \quad \vec{v'} = \nabla\phi \tag{7}$$

Equations 5-6 give rise, after mathematical projections, to a linear system (see Bouley *et al.* [8] for further details) in which modal amplitudes U^0 and T are the unknown variables, D^0 being already known from a previous solving iteration at the leading-edge interface. This linear system, being well conditionned, is then solved by matrix inversion. The same procedure applies at the leading-edge interface.

2.2 Kutta Condition

The Kutta condition is introduced to account for the viscosity effect at the trailing-edges: the pressure jump between both sides of a vane must be zero at the trailing-edge. Considering the mth channel, this gives:

$$p'^{m-1}_D(c, ma) + p'^{m-1}_U(c, ma) = p'^m_D(c, ma) + p'^m_U(c, ma) \tag{8}$$

The solving of the matching equations (Eqs. 5-6) with the Kutta condition (Eq. 8) leads to an over-determined linear system. Only an approximate solution can be found using a least-square minimization approach. But this implies that Eqs. 5-6 and Eq. 8 are not properly fulfilled.

Due to viscous effects artificially introduced by the Kutta condition, vorticity perturbations are produced downstream of the vanes. Howe [1] suggests that these vorticity perturbations can be concentrated on lines (in a 2D approach) at $z = ma$, $x \geq c$, when the length scale of the vorticity is large compared to the vane and wake thickness. Consequently, the vorticity field can be modeled as a Dirac comb, the magnitude of each Dirac function being phase-shifted between adjacent trailing-edges. These vorticity waves are convected by the mean flow. Their expression is given as:

$$\Omega_K(x, z).\vec{y} = \Omega_0 e^{i(\omega/W_x)x} \sum_{m=-\infty}^{+\infty} e^{imu} \delta(z - ma)\vec{y}, \quad u = a\alpha_{i_0}, \quad x \geq c \tag{9}$$

Ω_0 represents the magnitude of the vorticity and is a new unknown variable. The Dirac comb of vorticity can be expanded into a Fourier series. As a result, the vorticity field is written as an infinite sum of plane waves:

$$\Omega_K(x, z).\vec{y} = \frac{\Omega_0}{a} \sum_{q=-\infty}^{+\infty} e^{i(\omega/W_x)x} e^{i\alpha_q z}\vec{y}, \quad \alpha_q = \alpha_{i_0} + q\frac{2\pi}{a}, \quad x \geq c \tag{10}$$

The associated velocity field $\vec{v}_K$ follows the same plane-wave expansion. The definition of the vorticity, $\Omega_K(x, z).\vec{y} = \vec{rot}\, \vec{v}_K$ and the incompressible property of the velocity field enables to determine each

velocity component. The expression for the axial velocity is given as:

$$v_{x,K}(x,z) = \sum_{q=-\infty}^{+\infty} V_q e^{i(\omega/W_x)x} e^{i\alpha_q z}, \quad V_q = \frac{i\Omega_0 \alpha_q}{a\left(\alpha_q^2 + \frac{\omega^2}{W_x^2}\right)} \tag{11}$$

Then, it can be included into the matching equation for the axial velocity:

$$v'_{x,D}(c,z) + v'_{x,U}(c,z) = v'_{x,T}(c,z) + v_{x,K}(c,z), \quad \forall z \tag{12}$$

Due to the vorticity variable Ω_0, Eqs. 5,8 and 12 become, after mathematical projections, a square linear system. This system admits an unique solution.

3   Results and Analysis

The methodology is applied for an oblique plane wave incident on a two-dimensional cascade. The main parameters are (see Fig. 1):

- direction of the incident wave: $\Theta = 30°$

- non-dimensioned incident wavenumber : $k \times c = 5$

- solidity of the vanes: $\sigma = c/a = 1.25$

These parameters are selected as a test case for illustration. In practical applications, they would be tuned to ensure the 2π-periodicity of the original stator.

3.1   Acoustic Pressure Fields

Figure 2 presents the instantaneous acoustic pressure fields obtained with the mode-matching approach with (right side) and without Kutta condition (left side) for several Mach numbers. From Fig. 2-(a,b) to (e,f), the effect of the Mach number is visible: adjacent wavefronts of the downstream propagating waves become more distant while those of the upstream waves get closer to each other. With increasing Mach number, additionnal modes become cut-on: for instance, results at $M = 0.6$ show that at least two modes propagate downstream the vanes whereas only one exists for the other Mach numbers ($M = 0.2$, $M = 0.4$).

Figure 2 also highlights the increasing effect of the Kutta condition with the Mach number. At $M = 0.2$, the magnitudes of the reflected and transmitted wave are slightly higher when this condition is not active. However, the structure of the upstream and downstream pressure fields remains quite similar. At $M = 0.4$ and $M = 0.6$, the magnitude of the scattered pressure fields become significantly different whether the Kutta condition is imposed or not. This implies as well different structures of the scattered pressure fields. Figure 2 shows also that the upstream field is much more sensitive to the Kutta condition than the downstream field.

3.2   Acoustic Power

Figure 3 completes the analysis by quantifying the deviation from the exact energy balance between incident, reflected and transmitted waves. The ratio of acoustic power per unit span in dB obtained with
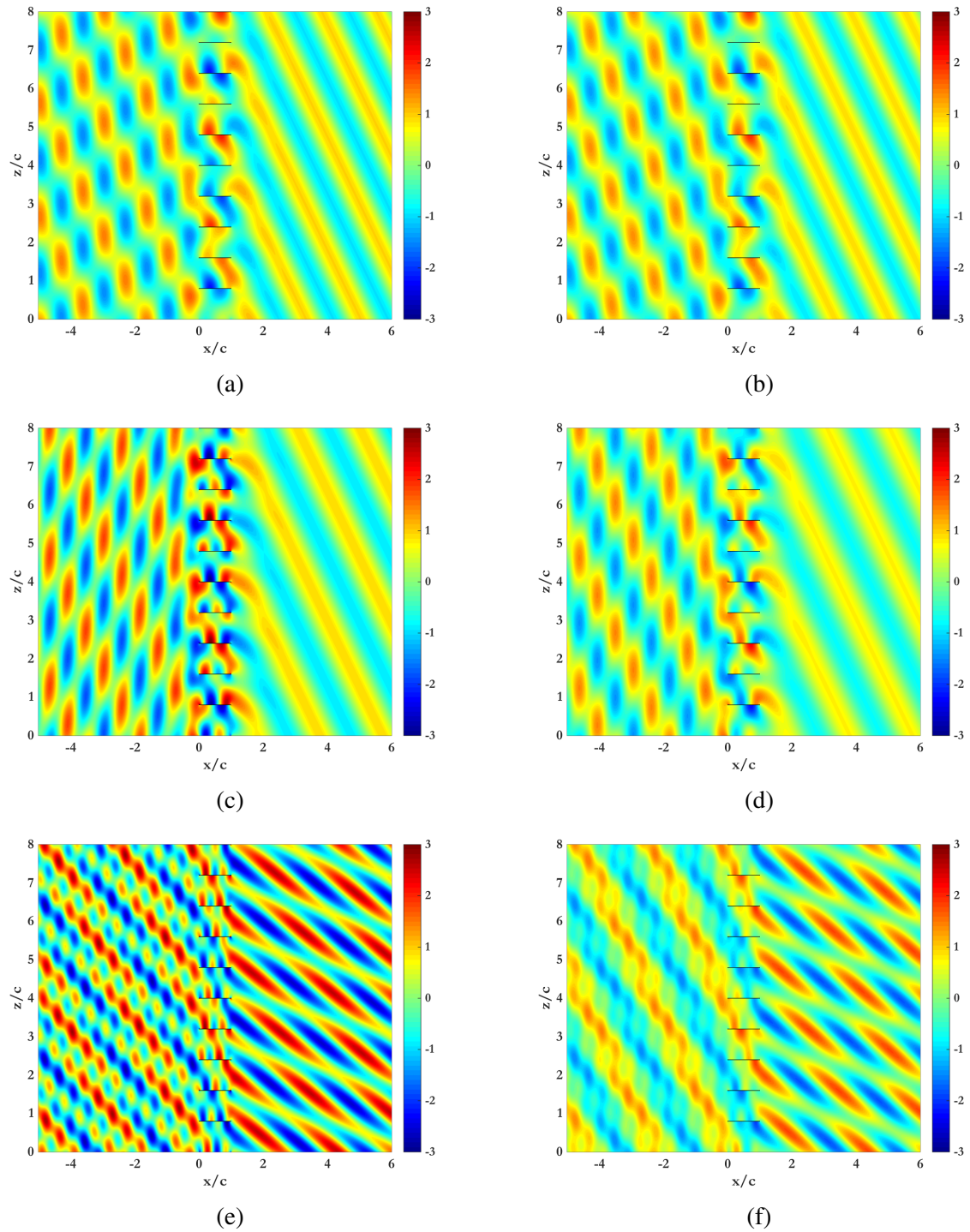
Figure 2: Instantaneous acoustic pressure fields obtained with the mode-matching technique for $M = 0.2$ (a-b), $M = 0.4$ (c-d) and $M = 0.6$ (e-f). Results (b-d-f) account for the Kutta condition. The flow goes from left to right.
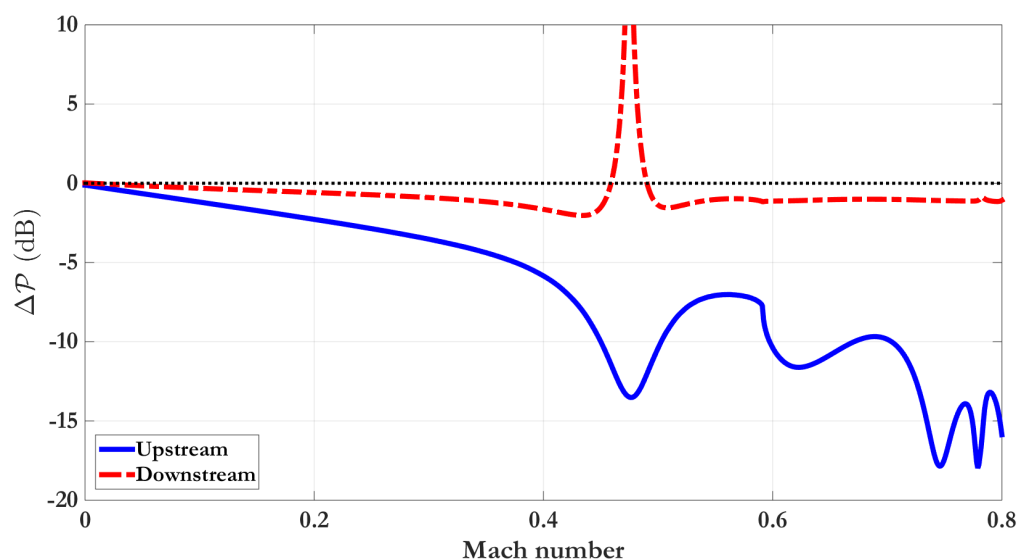
Figure 3: Power ratio in dB between results with and without Kutta condition (see Eq. 13).

and without Kutta condition is introduced as:

$$\Delta \mathcal{P} = 10 \, \log \left(\frac{\mathcal{P}^K}{\mathcal{P}} \right) \tag{13}$$

where $\mathcal{P}$ and $\mathcal{P}^K$ denote the acoustic power without and with Kutta condition, respectively.

Several comments can be made concerning the results from Fig. 3. Firstly, at $M = 0$, acoustic powers are identical for both calculations because the Kutta condition has no effect in the absence of mean flow. Secondly, upstream and downstream acoustic powers are overestimated for all Mach numbers when the Kutta is not imposed, except for the narrow range $M = 0.45 - 0.5$ for the downstream waves. This range of Mach numbers is singular because it corresponds, for the present configuration, to cases in which the transmitted acoustic power is close to zero if the Kutta condition is ignored. Nearly all the incident acoustic energy is reflected. With the Kutta condition, the energy is balanced differently between upstream, downstream acoustic waves and downstream vorticity waves in such a way that part of the sound is transmitted in the range $M = 0.45 - 0.5$. Thirdly, the upstream acoustic power is more sensitive to the Kutta condition than the downstream one. Excluding the singular cases, the acoustic power is overestimated by around 2 dB for the downstream waves in absence of Kutta condition. For the upstream waves, the discrepancies are much higher and can reach up to 15 dB for the highest simulated Mach numbers. The present results demonstrate the necessity to account for a Kutta condition for sound transmission in turbomachinery stages with a mean flow above $M = 0.2$.

4 Conclusions

An analytical formulation based a mode-matching technique and accounting for the Kutta condition has been proposed to formulate the problem of sound transmission in an axial-flow stator of turbomachine. Imposing a zero pressure jump at the trailing-edge is not sufficient and induced vorticity waves need to be considered in the wakes. Simulations on an infinite rectilinear cascade of rigid flat plates impinged by an acoustic plane wave have been performed. The results prove the significant and increasing effect

of the Kutta condition with the Mach number on the scattered pressure fields.

Complementary results [8] indicated that the present model is equivalent to the Wiener-Hopf technique developed by Posson *et al.* [9]. The interest of the mode-matching technique is that it can be extended to annular cascades in cylindrical coordinates. This extension is currently in progress.

References

[1] M. S. Howe. Attenuation of sound due to vortex shedding from a splitter plate in a mean flow duct. *Journal of sound and vibration*, 105(3):385–396, 1986.

[2] S. Job, E Lunéville, and J-F Mercier. Diffraction of an acoustic wave by a plate in a uniform flow: a numerical approach. *Journal of Computational Acoustics*, 13(04):689–709, 2005.

[3] R. Mittra and S.-W. Lee. *Analytical techniques in the theory of guided waves*. Macmillan, 1971.

[4] J. Ingenito and M. Roger. Analytical modelling of sound transmission through the passage of centrifugal compressor. *AIAA Paper*, 3704, 2007.

[5] S. Bouley, M. Roger, and B. François. On a uniform rotor-stator wake-interaction noise model based on a mode-matching technique. In *International Conference on Fan Noise, Technology and Numerical Methods*, April 2015.

[6] A. P. Dowling and J. E. Ffowcs Williams. *Sound and sources of sound*. Horwood, 1983.

[7] M. Roger, S. Moreau, and A. Marsan. Generation and transmission of spiral acoustic waves in multi-stage subsonic radial compressors. In *20th AIAA/CEAS Aeroacoustics Conference*, AIAA Paper 2014-3232, June 2014.

[8] S. Bouley, B. François, M. Roger, H. Posson, and S. Moreau. On a mode-matching technique for sound generation and transmission in a linear cascade of outlet guide vanes. In *21st AIAA/CEAS Aeroacoustics Conference*, June 2015.

[9] H. Posson, S. Moreau, and M. Roger. On the use of a uniformly valid analytical cascade response function for fan broadband noise predictions. *Journal of Sound and Vibration*, 329(18):3721–3743, 2010.