Helical vortex systems : linear analysis and nonlinear dynamics

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Abstract :

The near wake behind helicopter rotors, wind turbines and more generally behind rotating devices are dominated by helical vortices. Investigating their stability properties is a necessary step to predict their dynamics. Instabilities in such vortex systems have mainly been studied theoretically (Widnall 1972 [1], Okulov [2] and Sørensen 2007 [3]) in an inviscid framework for small core size vortices. The aim of the present study is to generalize these works to the viscous framework for arbitrary core sizes and vorticity profiles.The base flows considered here are helically symmetric: fields are invariant through combined axial translation of distance Δz and rotation of angle $\theta = \Delta z/L$ around the z-axis, where $2\pi L$ denotes the helix pitch.We first perform a linear temporal stability analysis of these base flows, using an Arnoldi [4] procedure coupled to two different codes : (i) a linearised version of the helical DNS code HELIX [5], (ii) another linear code called HELIKZ, which computes the dynamics of arbitrary perturbations in the vicinity of a helically symmetric base flow. These two codes permit the investigation of different types of instability modes: (i) modes having the same helical symmetry as the base flow which generalize the Okulov modes ; (ii) modes depending on z as $\exp \mathrm{i}kz$ which generalize the Widnall modes. In the first case (i), instabilities are found to be dominated by displacement modes of the type presented in figure 1 for the case of two vortices. In the second case (ii), modes will be compared to those observed in recent experimental work (Leweke et al. 2014 [6]). We then compute the nonlinear dynamics of a basic flow perturbed with a linear mode of type (i) set at a small initial amplitude. In the helical framework, the displacement mode is shown to be responsible for leap-frog dynamics (cf. figure 2) and/or vortex merging (cf. figure 3) with characteristics depending on the various parameters.

Keywords : Helical vortex, Navier-Stokes equations, linear stability analysis, Arnoldi method, nonlinear dynamics

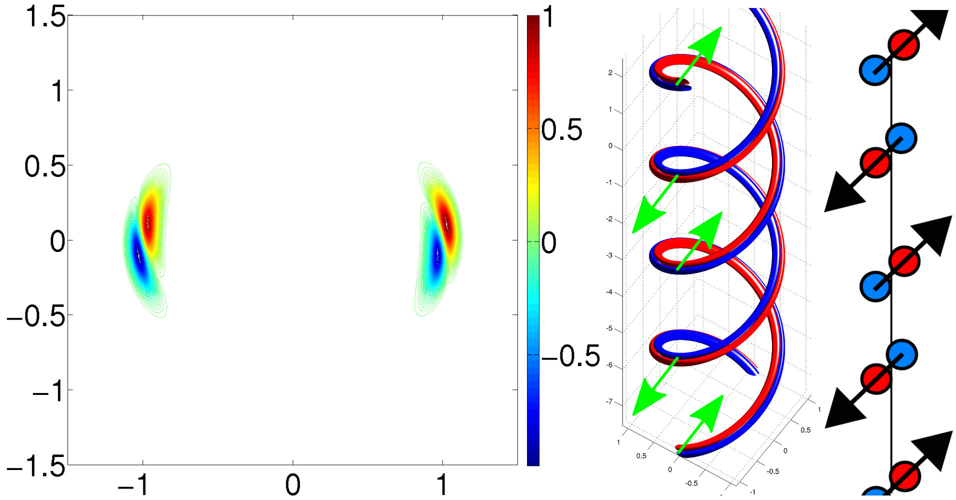

FIGURE 1 – Linear stability analysis : (a) Structure of the most unstable mode. The base flow corresponds to two helical vortices with pitch $2\pi L = 0.3$ and core size $a_b = 0.06$. The mode is represented by the helical vorticity component of the perturbation field in the horizontal (x, y)-plane by colored contours. (b) Mode in a 3-D representation. The arrows indicate the perturbation action on the base flow. The displacement induced by the mode has two components : one along the radial direction and one along the z direction. On the radial direction, one vortex goes inwards while the other goes outwards. (c) Schematic representation : the structure is analogous to the pairing instability mode for an infinite row of point vortices.

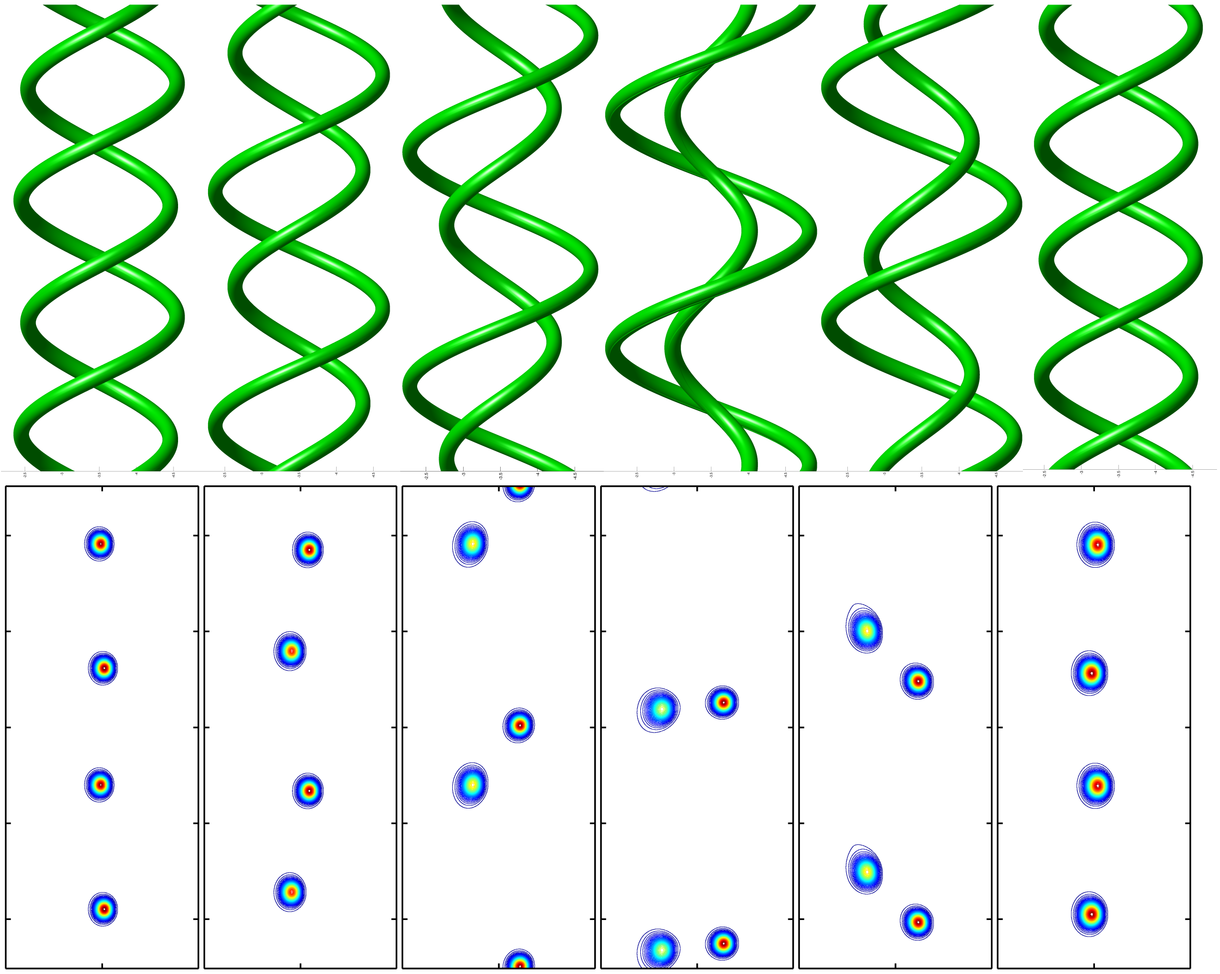

FIGURE 2 – Nonlinear evolution of the baseflow perturbed with the unstable mode described figure 1 : The vortices undergo a leapfrogging process. Time evolution goes from left to right. Top figures : Isovalue of the vorticity component $\omega_B = 10$ represented at different instants for two helical vortices of pitch $2\pi L = 0.3$ and core size $a_b = 0.06$. Bottom figures : vorticity component ω_B in the (r, z)-plane.

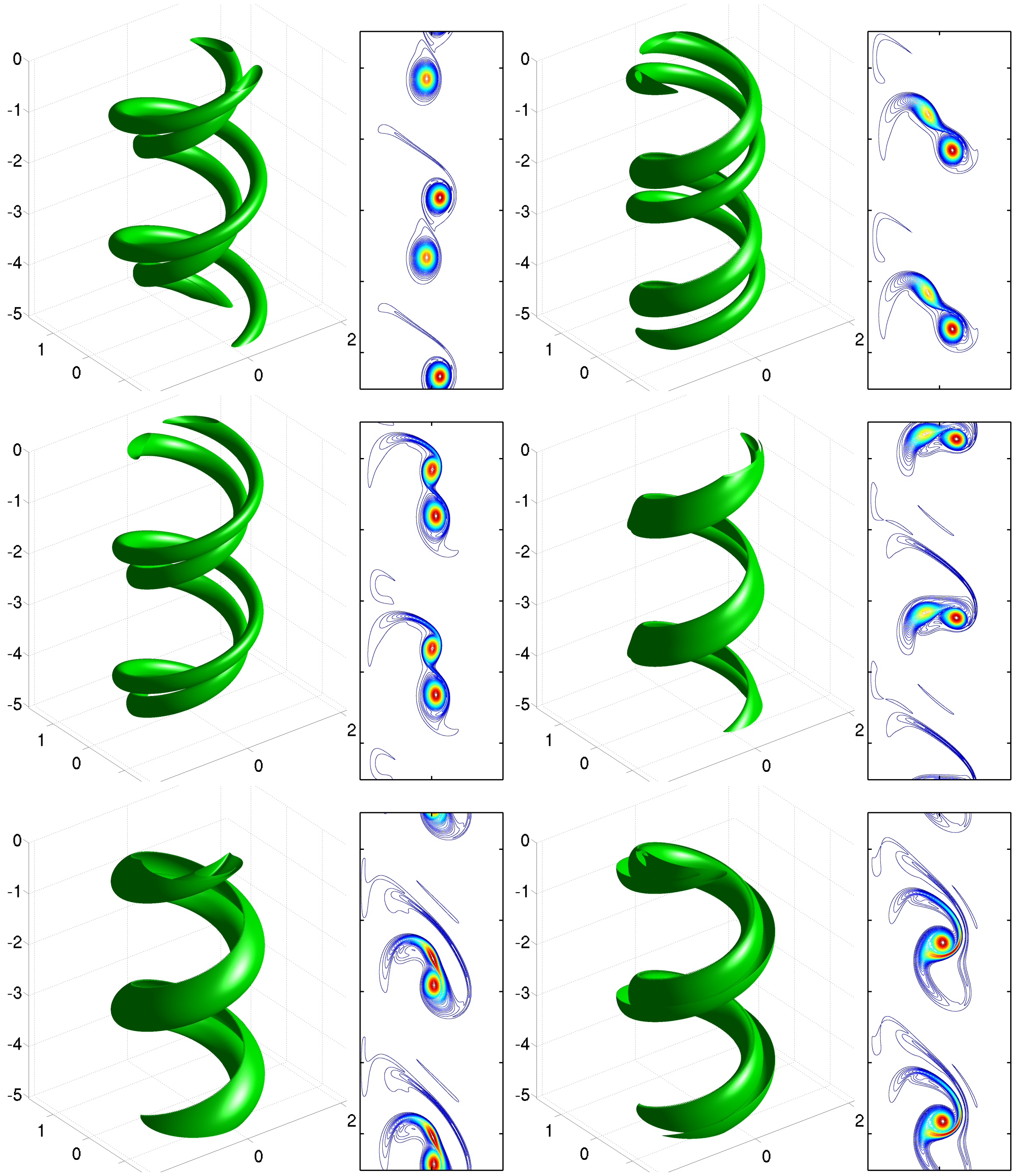

FIGURE 3 – Nonlinear evolution of the baseflow perturbed with the unstable mode described figure 1 : After a given number of leapfrogs the vortices merge due to viscous diffusion. Isovalue of the vorticity component $\omega_B = 4$ of two helical vortices of pitch $2\pi L = 0.3$ and core size $a_b = 0.06$ represented at different instants. Time evolution goes from left to right and top to bottom.

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