MESUREMENT OF AXIAL VELOCITY FIELD OF CLASSIC TAYLOR-COUETTE FLOW

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Résumé:

L'étude des mécanismes de transition du régime laminaire vers le régime turbulent a fait l'objet de nombreux travaux. La géométrie de Couette entre deux cylindres coaxiaux tournants, constitue une géométrie idéale pour le suivi des différentes étapes vers la turbulence. Notre problème concerne l'étude de l'écoulement entre deux cylindres verticaux coaxiaux de rayons R_1 *et* R_2 *dans le cas particulier où le cylindre extérieur est maintenu fixe et le cylindre intérieur « lisse ou ondulé » peut tourner autour de l'axe du système avec une vitesse variable notée* $\Omega_1 \neq 0$*. Nous nous proposons d'examiner expérimentalement la structure de l'écoulement à partir de l'apparition de la première instabilité (tourbillons de Taylor). L'approche expérimentale est basée sur la mesure des profils de vitesse instantanée et moyennée. Il est intéressant de signaler, aux alentours de Ta=600, l'existence d'un accroissement de la longueur d'onde azimutale qui s'accompagne d'une diminution de l'énergie. Par ailleurs l'exploitation spatio*-temporelle directe des champs de vitesses à permis de bien *préciser la topologie des écoulements et notamment le nombre et la taille des cellules, ainsi que la stabilité relative des zones sources et puits.*

Abstract:

The mechanisms of flow transition from laminar to turbulent regime are still preoccupying subject for many practical flow problems. The Taylor-Couette flow between two rotating coaxial cylinders provides an ideal configuration to discover these mechanisms. In this study the instability problem of Taylor-Couette flow between two vertical coaxial cylinders of radius R_1 *and* R_2 *is considered. By keeping the outer cylinder fixed, two cases have been studied, depending on the wall shape "Smooth or Wavy" of the inner cylinder. The inner cylinder is allowed to rotate about the axe of the system at a variable speed* $\Omega_1 \neq 0$*. The examination of flow structures was started from the appearance of the Taylor cells indicating the emergence of the first instability of the flow. The examinations were realized by measuring the instantaneous and the mean velocity profile in the axial. Beyond Ta=600, an increase in the wave length in the azimuthal direction, accompanied by a decrease in energy, has been observed. The analysis of the velocity fields revealed the flow topology specially the number and type of Taylor cells and the relative stability of the sink and source zones.*

Keywards: Taylor-Couette flow, Axial velocity Field.

1. Introduction

In fluid mechanics the analysis of the flow instability which describes the transition mechanisms of the flow to the turbulent regime, is of a great importance. There are many relatively simple flow configurations that can be used to explain the mechanisms that lead to such instability phenomenon. Among these flow configuration, the Taylor-Couette flow between two coaxial cylinders [1]. In any flow problem the velocity of the flow plays a key role in determining when and where the flow becomes instable and transits to turbulent regime. The transition process is marked by the presence of a critical zone which varies depending on the flow configuration. Several experimental and theoretical works have been achieved to determine the conditions of stability of a flow between two coaxial cylinders with one or two rotating cylinder [2]. In laminar regime, the base flow is characterized by concentric smooth stream lines. A classical paper by [1] showed that the laminar profiles are valid until a critical rotation rate. When the rotation rate exceeds a critical value, which corresponds to the appearance of the first type of instability, a secondary flow superimposes to the base flow. This secondary flow is characterized by the apparition of a regularly spaced counter rotating pairs of toroidal vortices along the cylinder axis, known as Taylor cells. By increasing the rotation rate of the inner cylinder, new circumferential waves appear and superimpose to the Taylor cells, and the flow becomes doubly periodic. When the rotation rate is increased gradually, many transition stages successively affect the flow until the damage appears to the Taylor cells leading to the turbulent regime.

2. Stability of Taylor- Couette flow:

Although early work of [Rayleigh 1916] indicated that the rotating inner cylinder flows are unstable for all rotation rates, [Taylor 1923] showed that the laminar profiles are valid until a critical rotation rate. For small clearances,, the critical value for instability is given by the **Taylor number** :

$$Ta_{c1} = R(R_1 - R)^3 \frac{\omega_0^2}{\nu^2} \approx 1700$$

In the case where ($Ta < Ta_c$) the flow is steady and instabilities in the flow are not present and any perturbations to the flow are damped out by viscous forces. According to the value of the Taylor number the Couette-Taylor flow undergoes different kind of stabilities as follows:

2.1 The first instability $Ta_{C1} < Ta < Ta_{C2}$: TVF = Taylor vortex Flow :

When the angular velocity of the inner cylinder is increased above a certain threshold value (i.e. the Taylor number (*Ta*) of the flow exceeds the critical value (Ta_{C1})), the flow becomes unstable without being turbulent and a secondary steady state emerges characterized by the presence of a toroidal vortices in the flow stacked one on the top of the other. This flow configuration is called the axisymmetric Taylor vortex flow and remains independent of time until the second instability appears.

2.2 The second instability $Ta > Ta_{C2}$: WVF = Wavy Vortex Flow:

Subsequently as the rate of rotation is increased beyond > , the system undergoes a new kind of instabilities known as a non-axisymmetric instabilities, which lead to a states of great spatio-temporal complexity, known as wavy vortex flow.

3 Experimental apparatus and flow investigation techniques:

The experimental apparatus used, the same as in the work [4] consists of two coaxial cylinders as shown in (Fig. 1):

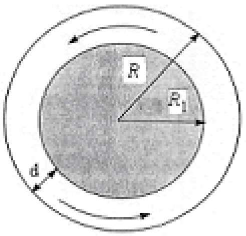


Fig.1- Top view cross section of the main part

A transparent outer cylinder of Plexiglass with radius *R=65.3* mm, thickness of 5mm and height *H=287* mm is used. An inner coaxial rotating cylinder is fitted inside the outer cylinder. Inner cylinders with different shape geometry were used. For the classical Couette-Taylor flow a smooth inner cylinder of a constant radius $R_1=51.3$ mm is used, where the clearance between the outer and inner cylinder is (*d=14*) mm. The parameters determining the flow regime are the dimensionless amplitude ($\bar{R}_1 = R_1/R$) with the Taylor number, (*Ta*), which is defined as:

$$Ta = Re^* (\bar{R}_1)^{1/2} *(1-\bar{R}_1)^{3/2} \qquad \text{where,} \quad Re = (\rho\omega R_1^2)/\mu.$$

where ($\bar{R}_1 = R_1/R$) is the dimensionless average radius of the inner cylinder, μ is the fluid viscosity which varies according to the results shown in Fig. 2, as a function of the fluid temperature, $\rho = 1023$ kg/m^3 is the density of the liquid, ω is the angular velocity of the inner cylinder, and Re is the Reynolds number. The entire device is mounted vertically on two Plexiglas plates of *25* mm thickness and a dimension of (*250x250*) mm^2. Therefore a removable enclosure lid containing an O-ring sealing is added with a necessary bearing for the centering and guiding of the inner cylinder. The rotation of the inner cylinder is guaranteed by a manually controlled driving motor with a maximum applied torque of 60 N.cm, which imposes a rotation speed varying from *50* to *2000* rpm with an error of less than ±*5* rpm. To avoid working with very low speeds, which could lead to inaccuracies in the measurement, we used an aqueous solution of PLuracol (EMCAROX, ICI product). During experiments the temperature of the liquid was gradually changing. Therefore, for exact determination of the Taylor number (Ta), the temperature of the fluid column is constantly recorded by a thermocouple inserted in the wall of the outer cylinder.

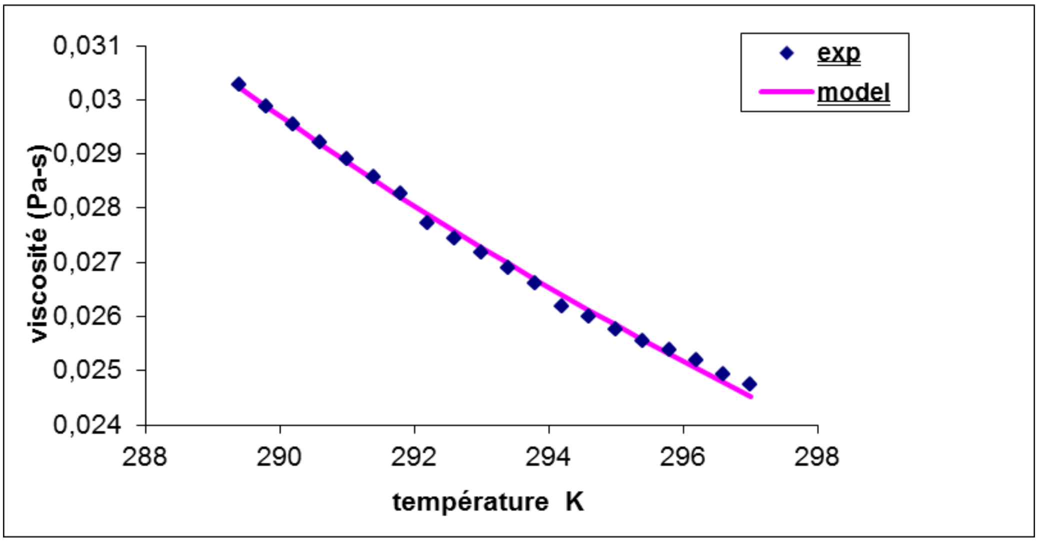


Fig. 3- Evolution of the viscosity as a function of temperature.

The dependence of liquid viscosity on the temperature in a range of *16–24* °C is determined using a separate setup. It is found that the fluid exhibits temperature-dependent Newtonian fluid characteristics in the range of the experiments. Therefore, the appropriate temperature corrections are applied to all measurements. The axial velocity profile has been measured using a Pulsed emission Ultrasound Velocitymeter (UVP in MET-FLOW SA). It is equipped by an emitter/receiver probe of 6 mm diameter and a data acquisition and signal processing system. The probe is placed vertically in the top cover of the device. The distance between the wall of the outer cylinder and the axis of the probe is set to *4* mm. This radial position was chosen to permit measuring high values of the axial flow velocity component without being exposed to any kind of noise. In addition, the probe is kept in direct contact with the fluid to avoid unwanted reflections. The probe transmits waves with a transmitting frequency (f_o) of 4 MHz with a pulsation time of *1* µs. The number of cycles per pulse is fixed as $n = 4$. The propagation velocity of sound waves in the fluid is considered close to that in the pure water (i.e. *1480* m/s). The repetition rates were adapted to the measured velocities. Indeed, for a given repetition rate, the maximum speed that can be measured is given by:

$$V_{max} = \frac{cF_{prf}}{4\,f_0}$$

The maximum depth at which the measurement can be performed is related to the repetition rate by the following equation:

$$P_{max} = \frac{c}{2F_{prf}}$$

The UVP device measures the axial velocity component in 128 points distributed along the measuring window. The size of the measuring window should not exceed the maximum depth ($P_{\max}$). The resolution of the velocity measurements is given by:

$$\Delta V = \frac{V_{max}}{127}$$

The experimental investigations consist of: visualizing and measuring of flow patterns in the (r, z) -plane using the Kalliroscope particles of size *10–20*µm. These white particles reflect light and gave a velocity value to its instantaneous position. Then the visualization of the Taylor vortices and that of the second instability become feasible and can be monitored on the device screen.

3 The analysis of investigation results

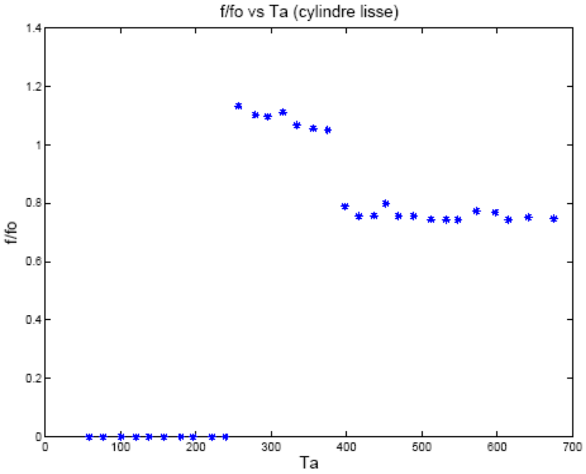
Experiments performed for different angular velocities of the inner cylinder to cover the flow regimes in the range from (*Ta=0 to 650*). The analyses of velocity signals are performed for all variants of the rotor geometry using MATLAB software. The experimentally observed and signal analysis of axial velocity field shows a good qualitative and quantitative agreement for stationary regimes both.

3.1 Spectral aspects

The spectral analysis, widely used for the study of turbulence, is a means for tracking the evolution of the flow representation of the main frequencies on a global frequency-diagram summarizing the complete evolution of the flow. The details of a system frequency are analyzed for different numbers of Taylor. Here we present analysis results for classical Taylor Couette flows "smooth cylinder”. We give the spectral diagrams summarizing the state of the flow and results of fields that axial velocities obtained for different flow regimes.

3.1.1 signal analysis

In Couette flow, the axial velocity component equal to zero, the ultrasonic signal issued by sensor equal to zero also. Reverse against beyond the Taylor regime, it is then possible to measure the spatial variation of the velocity and its changing in time point by point, whose the spectral analysis allows to obtain the frequency of the main signal.



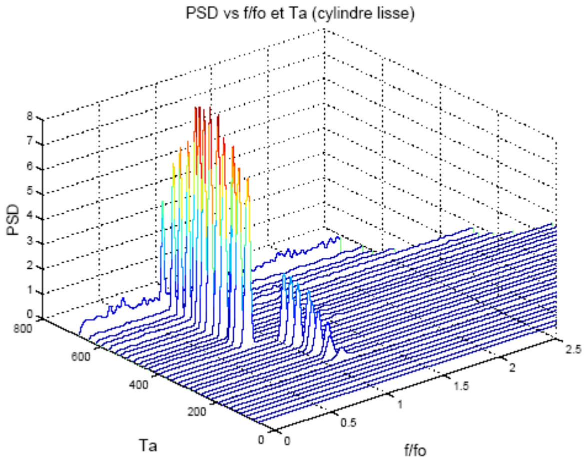


Fig.3 The evolution of the frequency ratio (*f/f_o*) vs Taylor. **Fig. 4 Spectrum of the energy density**

In the figures (3 and 4), we have plotted the results of this analysis in which the frequency is reduced by the frequency of the rotational speed of the inner cylinder. For numbers of Taylor greater than $Ta_{c2} = 42$ and less than $Ta_{c2} = 220$, no frequency in the spectrum, Taylor cells are stable over time. In the range of Tac2, the azimuthal wave appears with a frequency above (f_o) and decreases slightly when the Taylor number increases. The energy of the wave is monotonically grown in this region (Fig. 3).

At $Ta_{c2}=375$, the frequency jumping from ratio (f/f_o) greater than *1* to *0.8*. This frequency hopping is accompanied by a jump of the wave energy (Fig. 3).

Beyond this transition due to a jump in the number of azimuthal wave, the wave energy increases monotonously with the Taylor number, and passes to maximum at Ta_{c2}=570 before to decreasing when the Taylor number is growing. It should be noted that the decrease in energy is accompanied by the appearance of very small perturbations of random type.

Spatial analysis of axial velocity signals allow to specify changes in the wavelength (Fig. 5). This analysis shows that a wavelength (λ=26.5 mm) appears with the onset of Taylor cells at $Ta=42$. This wavelength is also changed when frequency hopping with a slight delay. As against the decrease in the wave energy $Ta > 600$, there is a jump passing wavelength from 25.8 mm to 37 mm. This result is not mentioned in the literature to our knowledge

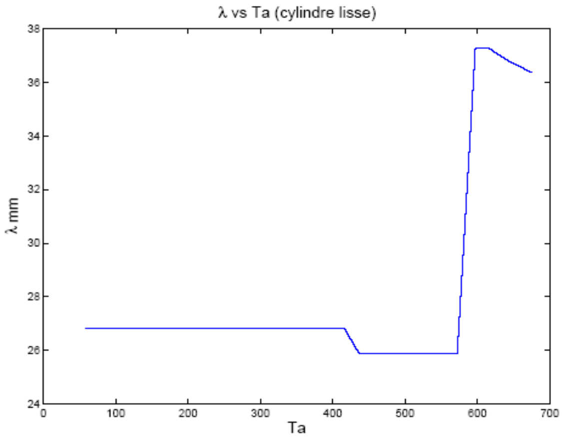


Fig. 5- changing in wavelength according to the number of Taylor.

3.2 Axial velocities fields

Using UVP, we get the axial component of the velocity field according to the position (128 measurement points), in function of time (*1024* profiles). Direct path of a measure by UVP then gives the evolution of the axial velocity of Taylor vortices or geometric of several wavelengths and its evolution over time with spatial resolution (*0.7* mm) and temporal variant from 23 * 10^{-3} to 50 *10^{-3} s. Direct representation *3D* results clearly shows the changes in flow regimes.

3.2.1 Velocity field analysis

To better observe these different variations in the structure of the flow, we have represented the raw measurement results in 3D. On (Fig. 6), we have shown the case of Taylor flow without azimuthal wave *Ta=77*.

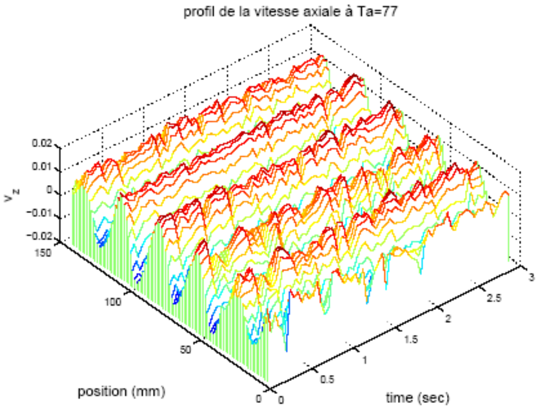


Fig. (6), profile of the axial velocity (*m/s*); *Ta = 77*.

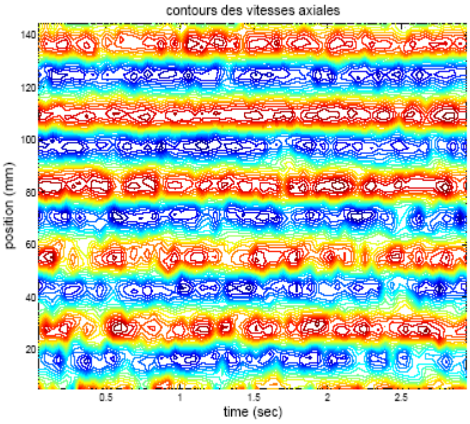


Fig. (7) - Contours of iso- axial velocity; *Ta = 77*

Similarly, Figure (7) we postponed the contours of axial iso-velocity. It is seen that each wavelength is composed of two equal-sized cells with lines of separation constants in time. The flow is stable with small disturbances that appear as premises of the azimuthal wave. By increasing the rotational velocity, the Taylor Couette flow transits to the second instability " established azimuthal waves" and at Taylor number *Ta=295*, the velocity axial is ten times greater than that is at *Ta=77*. At this time, we see the oscillation signal becomes very clear, with larger amplitude. Figures (8 and 9) clearly show the azimuthal waves with the contours of iso-axial velocities oscillatory significantly over time.

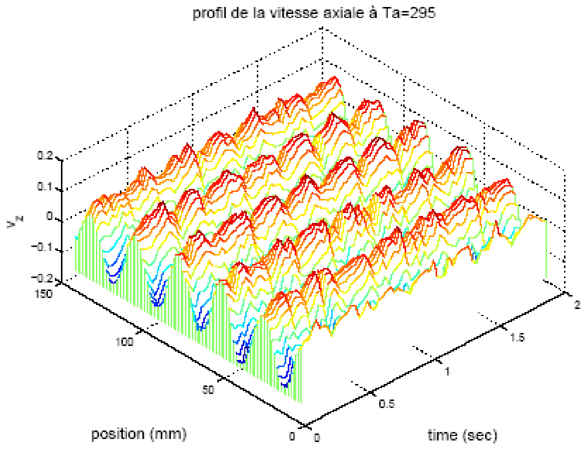


Fig. (8), profile of the axial velocity (m/s); Ta = 295.

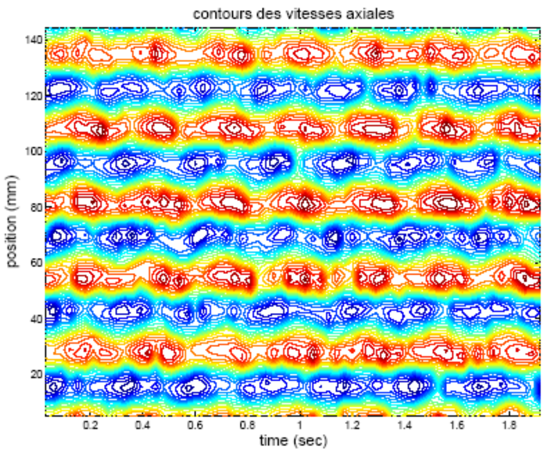


Fig. (9) - Contours of iso- axial velocity; Ta = 295.

(Fig.10) (after the jump frequency) shows that the value of the axial velocity becomes of the order of twice that where *Ta=295*. Azimuthal waves are well established with a lower temporal frequency if we look at the peak maximum velocities over time at a given position. On the contours of the velocities the wavys appears sharper with more marked between positive and negative values of the asymmetry rate. The velocity profile shown in (figure *12*) (*Ta=615*) is the case where a wavelength jump is observed. This figure shows a wavelength greater than in the previous case length. The temporal evolution clearly shows that the crests are wider maxima in space and are formed by two peaks. The flow appears very disturbed and this time we can think premise of chaos.

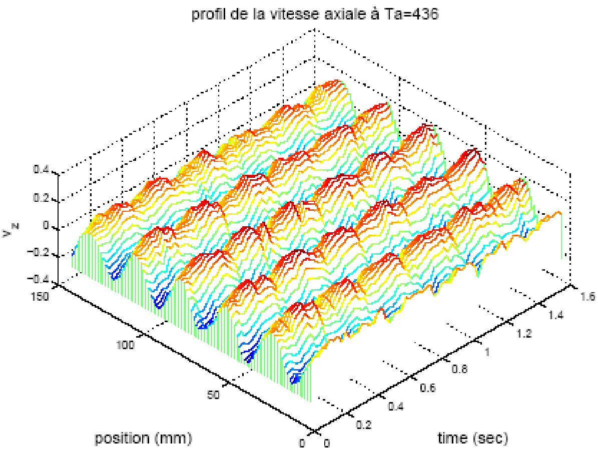


Fig. (10), profile of the axial velocity (m/s); Ta = 436.

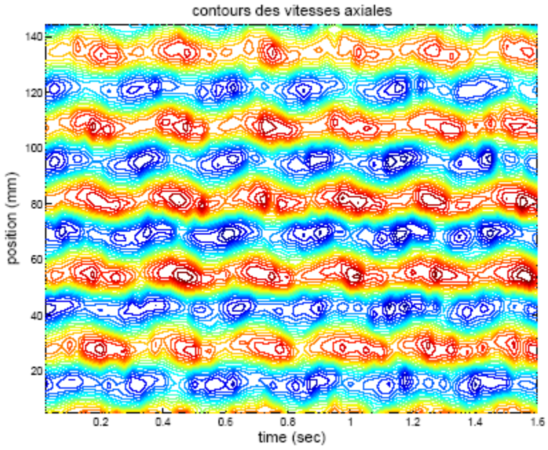


Fig. (11) - Contours of iso-axial velocity;Ta = 436.

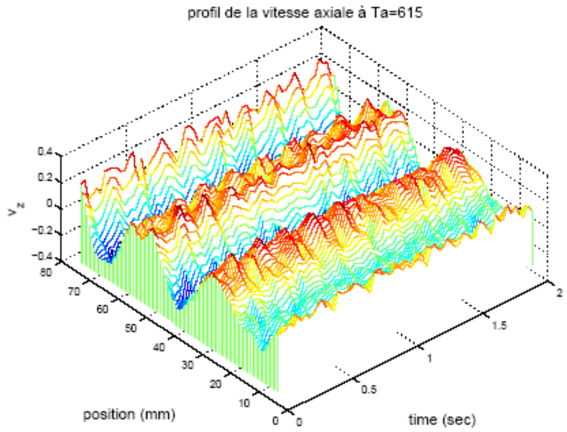


Fig. (12) Profile of the axial-velocity (m/s); Ta = 615.

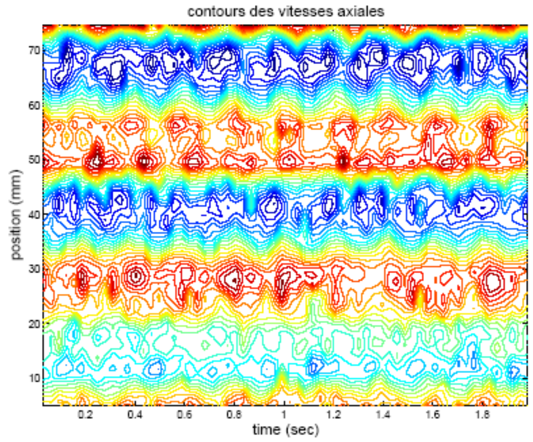


Fig. (13) - Contours of iso-axial velocity;Ta = 615.

4 Conclusion

An experimental approach based on the ultrasound velocimetry (UVP) technique is used to study the instability phenomenon of the classical Taylor- Couette flow. In the case of classical Taylor-Couette flow, a blow up in the frequency of the axial flow velocity field was identified, characterized by an increase in the energy density, and in the same time a decrease in frequency. This result was also reported by [5] and [6] using classical measuring techniques. It is interesting to note that, around Ta = 600, an increase in the azimuthal wave length was observed, accompanied by a decrease in energy . This result is not mentioned in the scientific literature and appears to be new to our knowledge. The analysis of the axial velocity field in space and time throughout all transition processes using the ultrasound technique has permitted to clarify the topology of the flow structures represented by the number and size of the Taylor cells. These analyses clarified also the relative instabilities of the source and sink zones of the flow. Indeed, during the flow oscillations, the source zones appear much less stable in time than the sink zones.

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