Atypical dynamics of materials with periodic microstructure and local resonance

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Abstract :

This work investigates the dynamic behavior of periodic unbraced frame structures made up of interconnected beams. Two types of microstructures are especially studied: non-orthogonal unbraced frame and honeycombs. The microstructure being much stiffer in compression than in shear, a great variety of behaviors can occur. Assuming the condition of scale separation is respected, the dynamical behaviors at the leading order are approached by the homogenisation method of periodic discrete media. In the studied ranges, the local elements behave ever in quasi-statics, ever in dynamics. For studied materials, the elastic laws are given in function of the elements properties. These laws correspond to upgraded materials as double gradient media or meta-material. To illustrate their atypical properties, propagations of 'shear' and 'compression' waves are studied. In the presence of the local resonance, the form of the equations is unchanged but the mass depends on the frequency and, as a result, frequency bandgaps appear.

Keywords : microstructured material, dynamics, metamaterial

1 Introduction

Two considerations explain the great number of studies devoted to the dynamic properties of periodic reticulated (or cellular) structures, namely structures obtained by repeating a unit cell made up of interconnected beams (or plates). First they are frequently encountered: in sandwich panels, stiffened plates, truss beams used in aerospace and marine structures. Second, the periodic materials can generate complex behavior (Cauchy or generalized media, meta-material). This work analyzes the propagation of plane waves in two-dimensional periodic materials (Fig 1), constituted of elements oriented in two or three directions. These elements being more flexible in bending than in compression, the macroscopic properties of such materials can present a great variability in function of the direction and of the frequency of the solicitation. The homogenization method of periodic discrete media (HPDM) is used [2] [3]. This method has already given interesting results on the dynamic behavior of frame structures [4], [5]. Its advantages are: (1) The equivalent continuum is derived rigorously from the properties of the cell. The only assumption is the scale separation, i.e. two scales with very different characteristic lengths can be defined : the macroscopic (or global) scale is given by the wave propagation and the microscopic (or local) scale by the size of the cell. (2) The method is completely analytic. This provides a clear understanding of the mechanisms governing the behavior and of the role of each parameter. Such a knowledge is desirable for the design of new materials with prescribed properties. (3) The global behavior being identified, it is always possible to come back to the local scale to determine the deformations and the efforts in the elements. (4) Superior orders of the expansions are obtained relatively easily. This is particularly interesting for the frame structures because the shear and the compression stiffnesses do not have the same order of magnitude. Since the method of multiple parameters and scale changes is generally limited to the leading order, it misses the shear properties and the coerciveness of the macroscopic description is lost [1].

2 Studied materials and homogenisation method of periodic discrete media

The studied materials (Fig. 1) are infinite and periodic in the plane (x; y). In the first case named inclined lattice, the fundamental cell is constituted by two elements of different lengths and in the second case named regular honeycomb, the fundamental cell is constituted by three identical elements. Elements are beams or plates behaving as Euler beams in the plane (x; y). They are linked by perfectly stiff and massless nodes of coordinates. Moreover, all elements have similar material and geometric properties.
The study is conducted within the framework of the small strain theory, the linear elasticity and in harmonic regime.

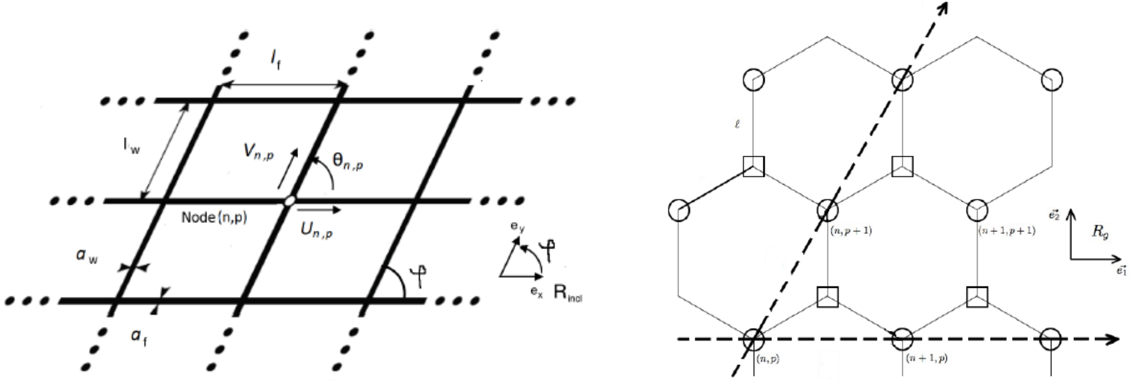

Figure 1 : Studied materials (inclined lattice and honeycomb)

The homogenisation method of discrete periodic media is applied to find the continuous description at the leading order of the dynamic behaviour of these materials. Extensive description of this method can be found in [5] [6] [7].

3 Inclined Lattice

The continuous description is used to described the ‘shear’ and ‘compression’ waves. From the point of view of the shear waves, the material is very anisotropic, and (only two directions of propagation are possible) and non dispersive (Figure 2).

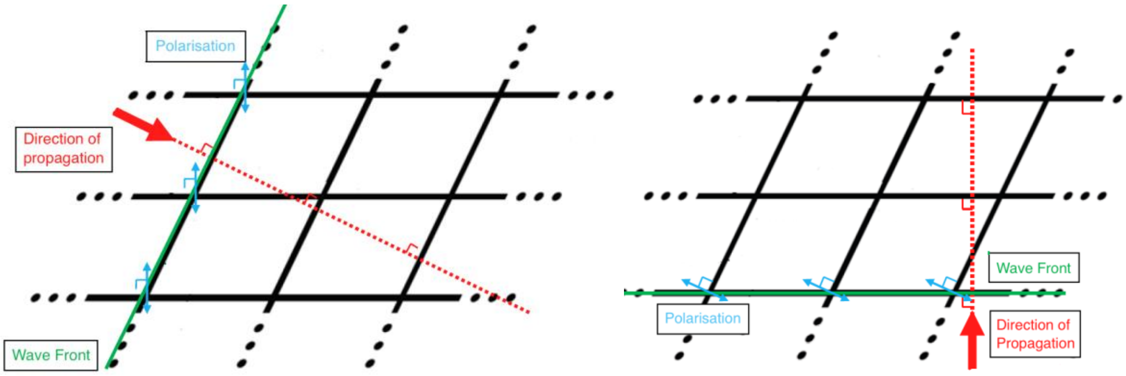

Figure 2 : Shear waves in inclined lattice

For compression waves, two situations can occur, in function of the local properties (implying ever local quasi-statics, ever local resonance). In the first case, the compression waves propagate in all directions, but with two possible polarizations (Figures 3 & 4). The celerity is function of the frequency (dispersive media). In the second case with local resonance, bandgaps appear in addition (Figure 5).

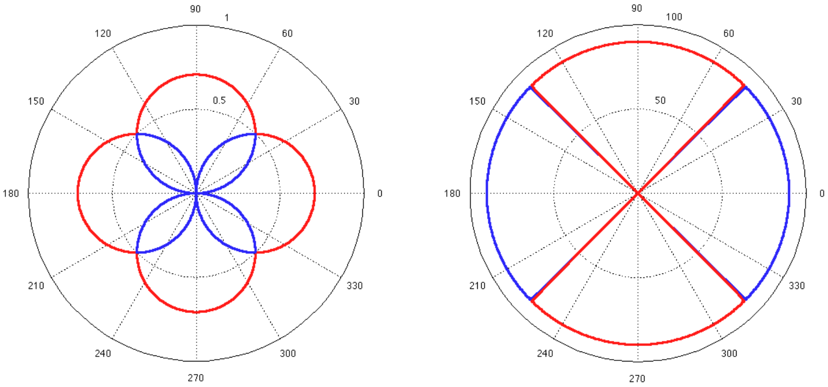

Figure 3 : Compression waves in orthogonal lattice - left the celerity in function of the direction of propagation, right the associated direction of polarisation 0° or 90°.

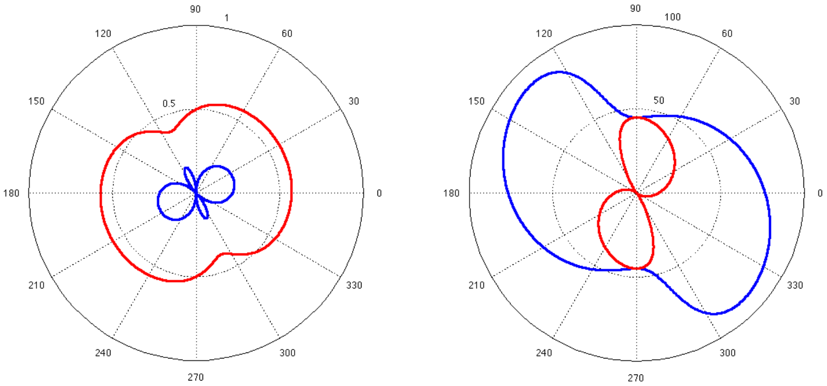

Figure 4 : Compression waves in inclined lattice (45°) - at left the celerity in function of the direction of propagation, at right the associated direction of polarisation.

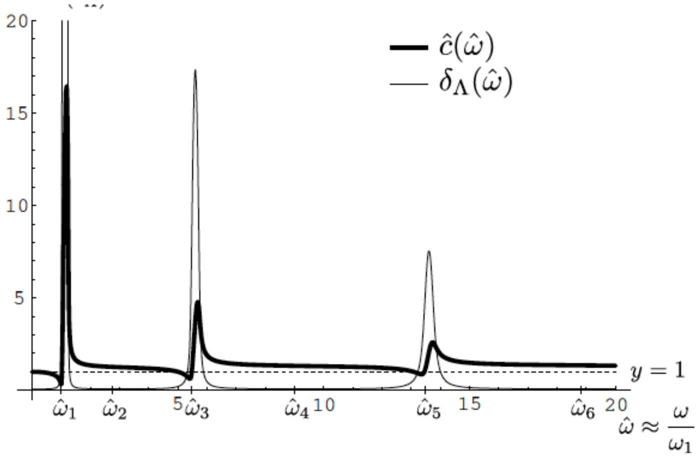

Figure 5 : Compression waves in orthogonal lattice – Bandgaps around the resonance of constitutive elements

4 Honeycomb

The previous approach was conducted on honeycomb materials. The compression waves are the same as in a classical Cauchy media, but shear waves are very atypical, with both a dependance of the frequency (dispersivity) and of the direction of propagation (Figure 6).

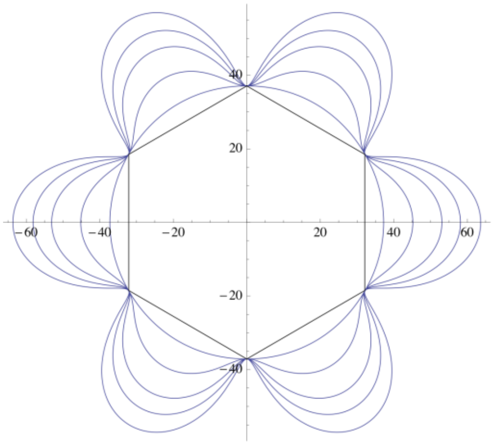

Figure 6 : Shear waves in honeycomb – celerity in function of the frequency and the direction of propagation

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