Wave propagation in the framework of strain gradient continua: the example of hexachiral materials

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Résumé :

La chiralité est une propriété générale d'asymétrie qui est d'importance primordiale dans de nombreux domaines de la physique et de la biologie modernes. Un objet, ou un système, est dit être chiral s'il n'est pas identique à son image miroir. Beaucoup de matériaux présentent une chiralité à différentes échelles. À l'échelle microscopique la chiralité peut être observée, par exemple, en chimie, dans des molécules biologiques (acides aminés, protéines, sucre), dans des cristaux, dans des cristaux liquides. Une organisation chirale est également observée à plus grande échelle dans les os, les composites renforcés par des fibres, les structures mécaniques. L'effet de la chiralité sur la propagation d'onde en optique est connu depuis la première moitié du 19ème siècle, avec les travaux sur l'activité optique des cristaux par, entre autres, Arago, Biot et Pasteur. Contrairement à l'électromagnétisme, la théorie classique de l'élasticité n'est pas sensible à la chiralité, bien que les preuves mécaniques de son importance ont été rapportés dans des situations nombreuses et variées. En conséquence, au cours des dernières décennies, différentes tentatives ont été faites pour étendre le cadre classique d'élasticité afin de le rendre dépendant chiralité. Comme montré en [1], une description continue de la chiralité peut être obtenue en utilisant les théories de milieux continus généralisés.

Abstract :

Chirality, which means handedness, is a general property of asymmetry that is of prior importance in many fields of modern physics and biology. An object, or a system, is said to be chiral if it is not identical to its mirror image. Many materials exhibit chirality at different scales. At the microscopic scale chirality can be observed, for example, in chemistry, biological molecules (amino acids, protein, sugar), crystals, liquid crystals. Chiral organization is also observed at larger scales in bones, fiber-reinforced composites, mechanical structures. Effect of chirality on optical wave propagation is known since the first half of the 19th century with the work on crystal optical activity by, among others, Arago, Biot and Pasteur. But contrary to electromagnetism, the classical theory of elasticity is not chiral sensitive, although mechanical evidence of its importance has been reported in numerous and various situations. As a consequence, during the last few decades, different attempts have been made to extend the classical framework of elasticity in order to make it chirality-dependent. As shown in [1] a continuum description of chirality can be achieved using generalized continua theories.

Mots clefs : Strain gradient, Chirality, Wave propagation, Dispersion

1   Introduction and model

In the strain-gradient theory of linear elasticity [2], and for centrosymmetric media, the constitutive law gives the symmetric Cauchy stress tensor σ and the hyperstress tensor τ in terms of the infinitesimal strain tensor $\varepsilon = \nabla u$,

where u is the displacement vector, and strain-gradient tensor $\eta = \nabla \varepsilon$ through the two linear relations:

$$\sigma_{ij} = C_{ijlm}\varepsilon_{lm}, \qquad \tau_{ijk} = A_{ijklmn}\eta_{lmn}. \tag{1}$$

where $\mathbf{C}$ is the classical fourth-order elastic tensor and $\mathbf{A}$ is the sixth-order second-order elastic tensor. The motion equation of a strain-gradient media subjected to body forces f_i reads :

$$\left(\sigma_{ij} - \tau_{ijk,k}.\right)_{,j} + f_i = \rho \ddot{u}_i - \frac{1}{3}\left(\kappa_{ijmn}\ddot{u}_{m,n}\right)_{,j} \tag{2}$$

where κ_{ijmn} is the micro-inertia tensor [2] . From equation 2 , through the definition of a *generalized acoustic tensor*, it is possible to compute the dispersion curves for the hexachiral material, in the long wave approximation. In particular, the strain gradient model captures the first two acoustic modes.
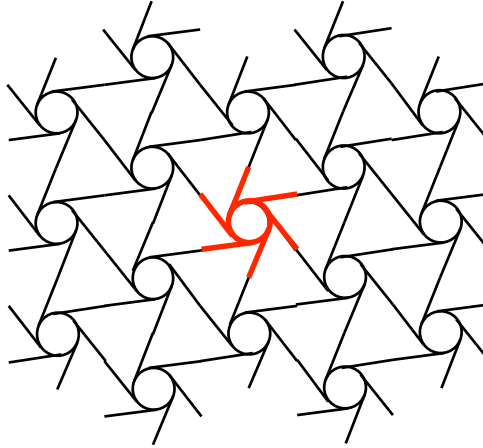

Figure 1: Hexachiral 2D pattern.

2 Results

The parameters used in our computation were obtained by performing finite element computations on the unit cell using polynomial boundary conditions. While these values should still be considered as a qualitative approximation of those of the actual material, they still allow us to verify basic properties of the strain gradient model. Once the properties of the material are retrieved, the solution of the eigenvalue problem (**??**) provides the phase velocity and polarization of the two wave solutions, since we are in 2D and the only degree of freedom is the displacement in the plane.

The results of these computations are plotted in Figure 2, where three specific values of the frequencies have been considered : $\Omega_1 = 5 \times 10^3$, $\Omega_2 = 20 \times 10^3$, $\Omega_3 = 30 \times 10^3$. For reading purposes all velocities have been normalized with respect to the low frequency value of the phase velocity of $S-$waves. As can be seen, both modes are isotropic at low frequency Ω_1, while they became more and more anisotropic when increasing the frequency. Chiral behaviour can also be easily noticed, as all polar plots do not posses any mirror symmetry, while the rotational symmetry characteristic of the Z_6 group is preserved. These results are in qualitative agreement with those obtained using Bloch wave analysis. However, it is of major importance to remark that energy and group velocities do not share the same polar plot, thus confirming that they should be treated as two separate quantities.These results are in agreement with those presented in [3].
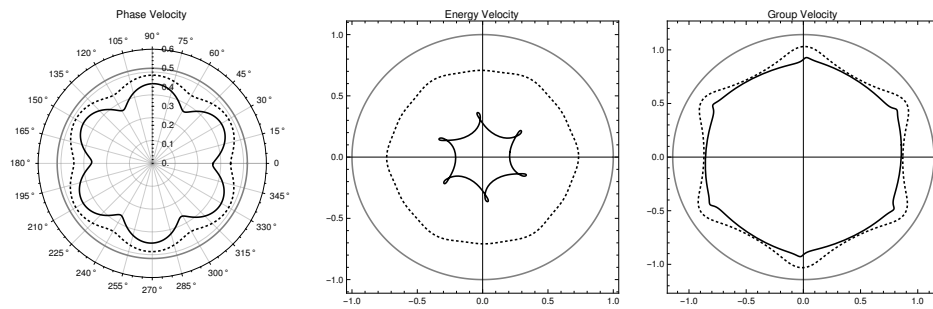
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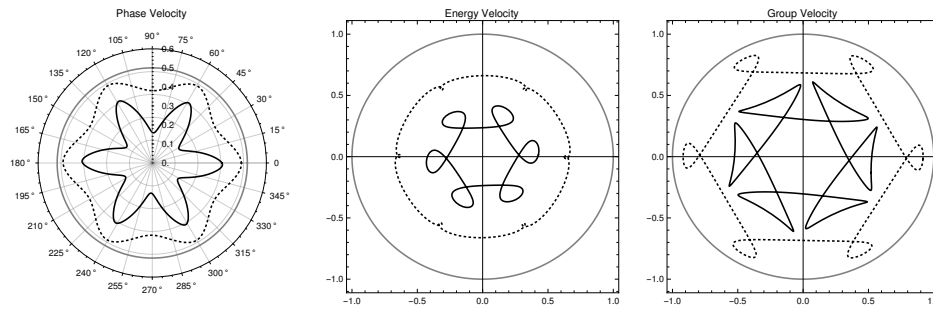
(a) Phase, energy and group velocities the first mode

(b) Phase, energy and group velocities the second mode

Figure 2: Phase, energy and group velocities corresponding to the two modes modes of vibration at 5 kHz (gray), 20 kHz (black dotted) and 30kHz (black solid). The velocities are normalized with respect to the phase velocity of the second mode at low frequency (S-mode).

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