Formation of band gaps in thin plate with a periodic array of resonators

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Résumé :

L'étude porte sur les caractéristiques vibratoires des plaques en cristaux phononiques constituées d'un réseau périodique bidimensionnel de résonateurs (masse-visse) attachés à une plaque fine homogène. Des simulations numériques basées sur la méthode d'éléments finis ont permis de calculer les diagrammes de bandes des plaques périodiques et d'analyser l'effet des paramètres géométriques sur la variation des bandes d'arrêts. Des calculs de coefficient de transmission d'une onde plane selon la direction OX mettent en évidence la présence des bandes d'arrêts prédites par le diagramme de bande. Ces résultats numériques sont validés expérimentalement par des mesures vibratoires de deux plaques en cristaux phononiques avec périodicités a=40 mm *puis 50* mm. *La plaque testée est encastrée sur l'un de ses cotés entre deux poutres en aluminium dont le déplacement de translation à basses fréquences est imposé par une table vibrante ce qui permet d'imposer une propagation d'onde unidirectionnelle. Cette étude expérimentale montre une atténuation remarquable de l'onde dans les intervalles correspondants aux bandes d'arrêt totale et bande d'arrêt directionnel de résonance.*

Abstract:

Vibration characteristics of locally resonant (LR) phononic plates consisting of two-dimensional periodic array of resonators (mass-screws) attached to a thin homogenous plate are investigated. In the numeric simulations, we employ finite-element method to calculate band structures of the proposed periodic plates and to analyze the effect of geometry parameters on the opening mechanism and evolution behavior of band gaps. Further, vibration transmission is calculated for plane wave propagation along the OX direction and the results are in good agreement with the band gap predicted by the band structure in this direction. The predictions of these theoretical examinations are validated experimentally by two experimental measurements of fabricated LR phononic plates with different lattice constant a=40 then 50 mm. The tested plate is clamped along one side to a shaking table to generate plane wave propagating along the OX direction. The experimental measurements of the vibration attenuation in this direction are in good agreement with the theoretical frequency of complete band gap and directional resonance gap

Keywords: Local resonators, Phononic plate, Band gap, Frequency response, Vibration.

I. Introduction

Phononic crystals are periodic materials with identical unite cells that exhibit the ability to act as mechanical filters of wave propagation. The periodic effect allows to stop wave propagation in a given frequency range [1] [2], called band gap. There are mainly two approaches that generate bands gap. The first approach is based on Bragg scattering mechanism for which wave-lengths are on the scale of the structure's periodicity. The second approach which known as locally resonant (LR) mechanism, utilizes the resonant modes of microstructures in each unit cell. The LR mechanism was first introduced by Liu et al [3]. They fabricated an LR phononic crystal composed of periodic array of coted lead balls immersed in an epoxy matrix, and provided the existing of LR band gaps two orders of magnitude lower than of the Bragg gaps.

Xiao et al [4] proposed an LR phononic plate made of two-dimensional periodic array of spring-mass resonators attached to a thin homogeneous plate. This structure represents a physical model for the understanding of basic wave physics in LR phononic plate. Oudich et al [5] presented numerically and experimentally the existence of a low-frequency LR band gap which is created by the low frequency local resonance of the soft rubber pillars coupled with the plate's modes. A similar idea was studied by Yu et al [6] who considered rubber matrix plate with periodic steel stubs. Recently, li et al [7] show that, with the introduction of composite plate-type acoustic metamaterial, locally resonant band gap shifts to lower frequency, and a significant enlargement of the relative bandwidth by a factor of 3 can be obtained, compared to one-side locally resonant stubbed plates. Hu et al [8] studied a similar LR phononic structure constructed by depositing the heavy cylinder locally resonant stubs squarely onto both sides of a thin elastic composite plate. Nouh et al [9] examined a LR plate system consists of a base structures provided with cavities filled by a viscoelastic membrane that supports a small mass to form a source of local resonance.

In this paper, we propose a novel two-dimensional LR phononic plates which consist of a thin homogenies plate with periodic array of resonators (mass-screws) attached on it. A finite element method (FEM) is used to investigate wave propagation along the proposed periodic structures. Band structures and vibrations transmission are calculated to predict the bands gap characteristics. The predictions of the theoretical examination are validated experimentally when fabricated LR phononic plates with different lattice constant are subjected to directional plane wave excitation. It is observed that there is a good agreement between the theoretical previsions and the experimental results for position and band bandwidth of complete band gap and directional resonance gap.

II. Model and method:

The study focuses on locally resonant (LR) phononic plates consisting of a periodic array of resonators (mass-screws) attached to a thin homogeneous plate (Fig. 1a). Because of the periodicity of the structure, only one unit cell is considered in the calculation. For the band structure of infinite periodic structure, we need to determinate the dispersion relation between the frequency and the wave number of a unit cell by varying the wave vector in the first irreducible Brillouin zone (Fig. 2). Periodic boundary conditions of Bloch-Floquet:

$$U_{destination} = U_{source} e^{-iK_F(r_{destination} - r_{source})} \quad (1)$$

are used for the edges of the unit cell (Fig.1b). In which U_i is the displacement vector, $r_{destination} - r_{source} = a$ is the width of the unit cell and K_F is the wave vector.

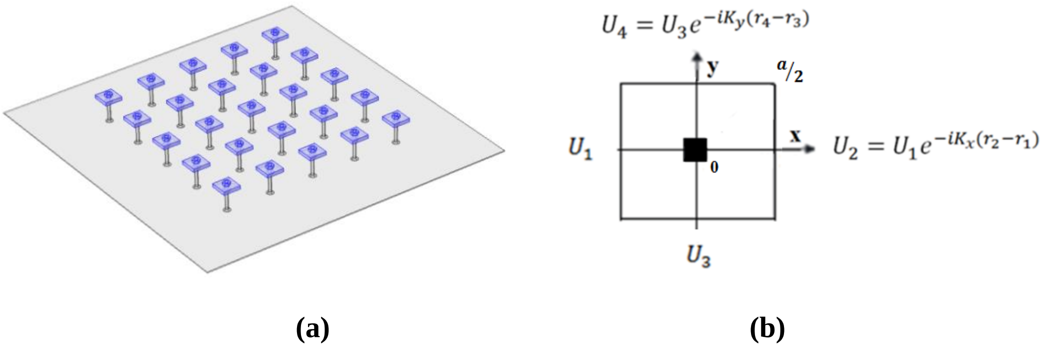

Figure 1 : (a) Configuration of locally resonant phononic plate with periodically attached resonator (mass-screws), (b) Periodic boundary conditions for a square unit cell

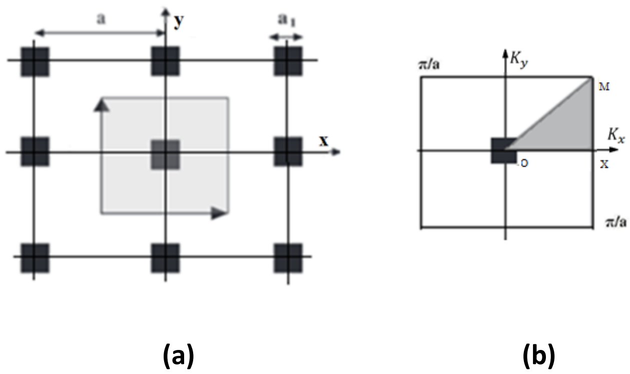

Figure 2 : (a) Cross section of a square unit cell, (b) First Brillouin zone of a square unit cell. The grey area in (b) is the irreducible region of the Brillouin zone.

II.1. Effect of the geometry parameters on the band gap

Three examples of LR plate systems are analyzed with the same choices of the plate, lattice constant and added mass. The geometrical parameters are chosen as follows: the lattice constant a=40 mm, the plate thickness is e=1 mm, the width and the thickness of the square steel fillers are respectively a_1=15 mm and e_1=3 mm. the square fillers are made of steel (Young's modulus E=205 GPa, density ρ=7850 kg/m^3, Poisson's ratio ν=0.28) being the same of the nuts M3. The host thin plate is made of isotropic aluminum (E=70 GPa, ρ=2700 kg/m^3, ν=0.33). The conical screw head is made of nylon (E=2 GPa, ρ=1150 kg/m^3, ν=0.4). As shown schematically in Fig. 3, between two nuts the square filler is attached to the nylon screw M3x40 at the extremity (Fig. 3a) and in the middle (Fig. 3b). The third configuration (Fig. 3c) the square filler is attached to nylon screw M3x25 at the extremity.

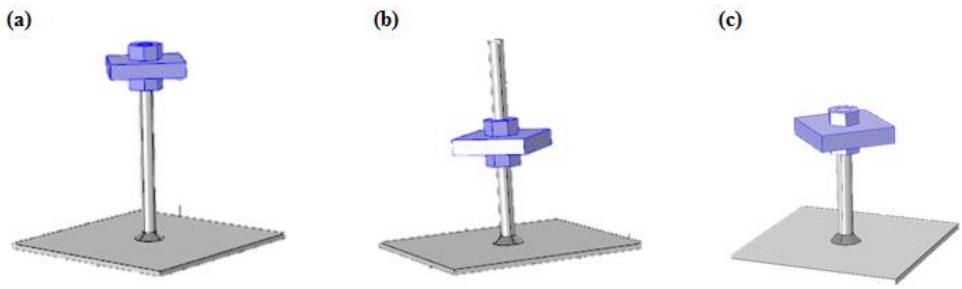

Figure 3 : Different configurations of resonators (mass-screws): (a) screw M3x40 with mass fixed to the extremity, (b) screw M3x40 with mass fixed in the middle, (c) screw M3x25 with mass fixed to the extremity.

The LR phononic plate is constituted of repetitive unit cells. This periodic nature of the plate induces that it acts as a mechanical filter for wave propagation. Consequently, waves are blocked along the periodic structures over ranges of frequency called bands gaps and can propagate only within other ranges of frequency called pass bands.

In order to investigate the propagation characteristics of elastic waves in the proposed LR phononic plate, a series of dispersion relations are calculated using a finite element method [Comsol 5.0]. This finite element method has been proven to be efficient for obtaining the phononic crystal dispersion curves in previous works [7] [10]. For infinite structure, the governing filed equation to calculate the dispersion relation of elastic wave propagation in solids given by:

$$\rho \frac{\partial^2 U_i}{\partial t^2} = \sum_{j=1}^{3} \frac{\partial}{\partial x_j}\left(\sum_{l=1}^{3}\sum_{k=1}^{3} C_{ijkl}\frac{\partial U_k}{\partial x_l}\right),\ (i = 1,2,3) \quad (2)$$

In which ρ is the mass density, U_i is the displacement, t is the time, C_{ijkl} are the elastic constant and $x_j (j = 1,2,3)$ represent the coordinate variables x, y, z respectively.

Fig .4a-c shows the calculated dispersion relations of the proposed phononic crystal plates with the different unit cells described in figure 3a-c respectively. The vertical axis is the frequency [Hz]. The horizontal axis is the reduced wave vector taken along the first irreducible Brillouin zone MOXM (Fig. 2b). According to the Bloch theorem, this is sufficient to determine the complete and directional frequency band gaps. In the band structures, it can be observed that there are the traditional plate's modes like the shear, symmetric and antisymmetric lamb mode corresponding to the curves which are not horizontal in figures 4a-c. In addition, the horizontal curves correspond to resonant modes with energy localized mainly in the resonators (mass-screws) for which the deformation of the plates are negligible compared to the deformations of resonators.

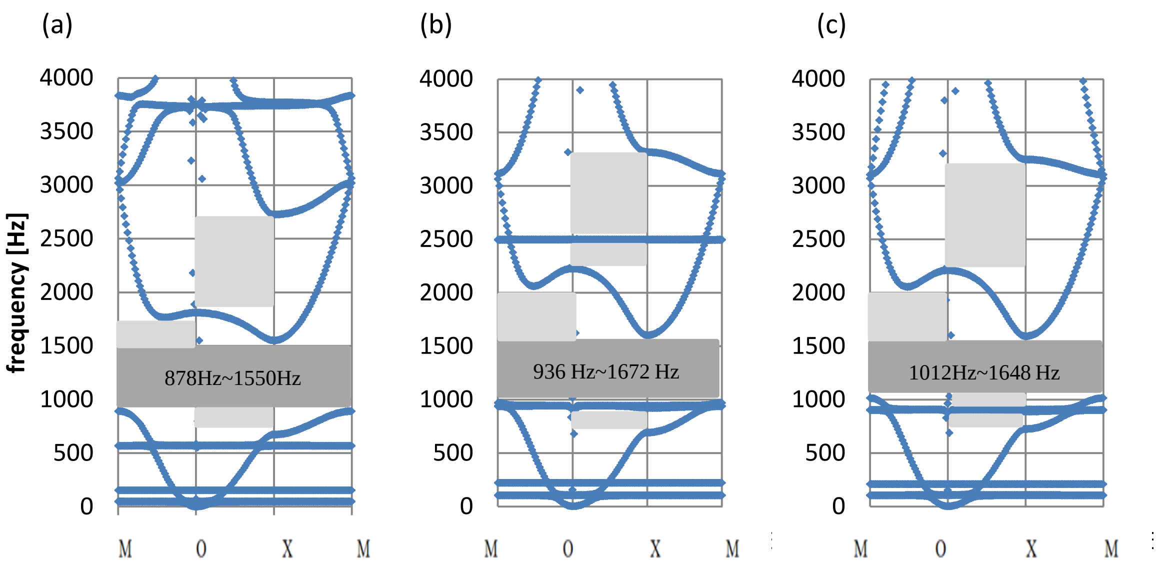

Figure 4 : Band structures of different locally resonant structures: (a) added mass is attached to the screw M3x40 at the extremity (b) added mass is attached to the screw M3x40 in the middle, (c) added mass is attached to the screw M3x25 at the extremity.

Bands structure in Figure 4 shows the existence of large complete and directional frequency band gaps for the three configurations highlighted by the gray areas. The frequency widths of the complete bands gaps are similar. Thus the variation of the stiffness of the resonator that is represented by the mass position in the screws doesn't have an important influence on the frequency width of band gap. In order to achieve a more understanding of the evolution of bad gap, the band structure of the bare aluminum plate is presented by the dotted green lines in figure 5, and the band structure of the resonator is the horizontal dotted red lines, a sign of zero group velocity. It can be clearly observed that the horizontal line near 1900 Hz of the local resonator frequency disappears: a strong coupling with the plate's modes gives the complete band gap between 1000 Hz and 1600 Hz. Moreover, the band gaps opened in these systems are located in the same order of the Bragg scattering mechanism in frequency region. So it is interesting to note that Bragg and resonance bands gaps are coupled in the band structures to create largest band gaps.

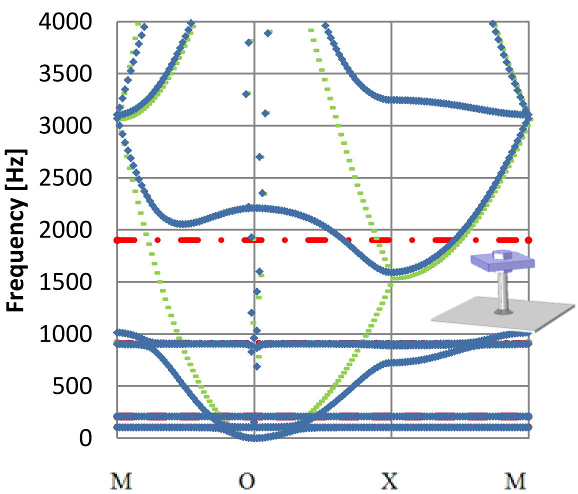

Figure 5 : Band structures of: locally resonant aluminum plate with periodic resonator of fixed mass in the extremity of screw M3x25 (blue), screw M3x25 with mass in the extremity (red), and bare aluminum plate (green)

To illustrate the mechanism of the band gap enlargement in LR phononic crystal plate, the effect of the lattice constant on the band gap is investigated. In the following, we will only focus on the waves' dispersion along the XM direction, because the same physical behaviors are observed for the other propagative direction of the LR phononic crystal structure. Figure 6 a-c show bands structures of LR plates formed with aluminum plate and periodic added mass attached to the screw M3x25 at the extremity with a lattice constant *a*=35 mm, 40 mm and 50 mm respectively. The larger the lattice constant *a*, the smaller the upper edge of band gap and the thinner the band gap.

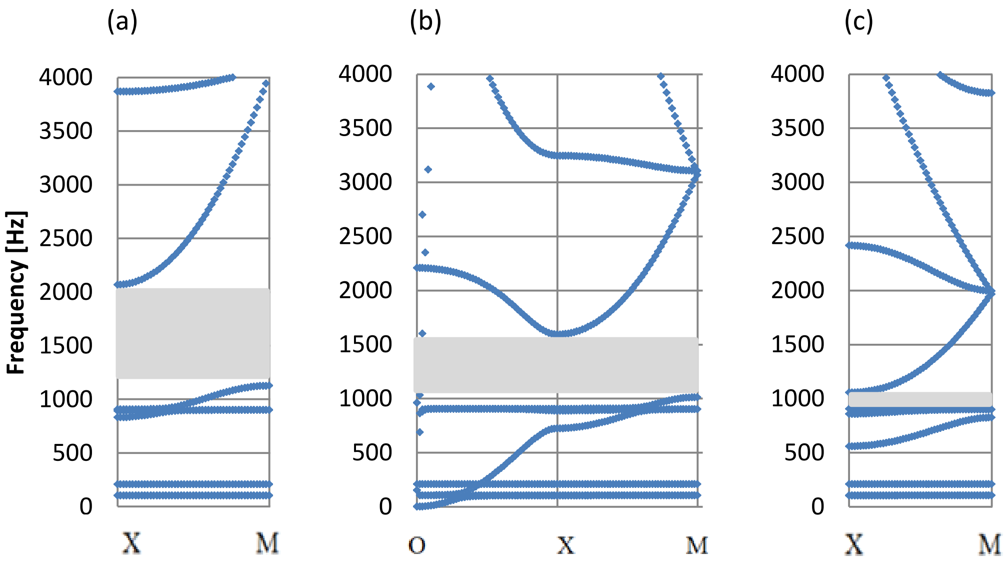

Figure 6 : Band structure of LR structure formed with aluminum fine plate and periodic added mass attached to the screw M3x25 at the extremity for: a) *a*=35 mm, b) *a*=40 mm, and c) *a*=50 mm

II.2. Validation on a periodic set of adaptive cells:

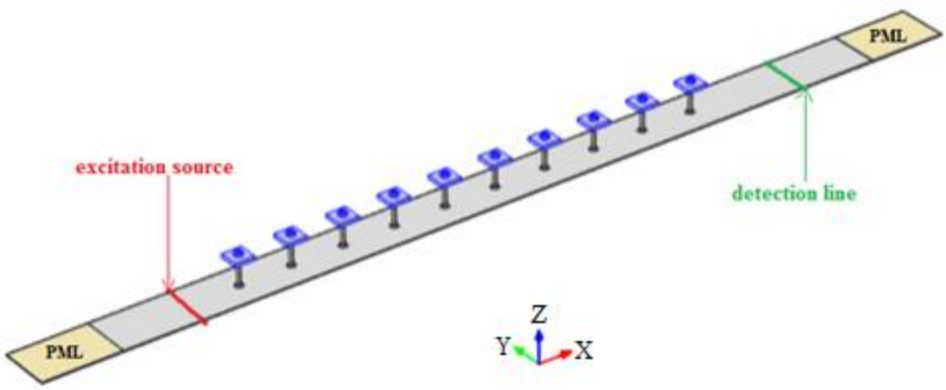

Figure 7 : Calculation schema of the transmission coefficient

To reduce noise, we are particularly interested to out plane mode which induce noise in the surrounding fluid due to vibroacoustic coupling. In this subsection, we focus to represent numerically the experimental test analyzed in the next section. The aim is to calculate the transmission coefficient of an aluminum plate with periodic resonators (configuration described in figure 3c). The excitation of modes out of the plane is done by applying a force perpendicular to the aluminum plate (the Z-direction) on one side of the row of resonators. The line excitation is more efficient to generate plane wave propagating along the OX direction. The displacement component Uz (Eq.3) on a detection line on the other side of the resonators row can be calculated. The transmission coefficient is thus given by (Eq 3.):

$$U_z = \frac{\text{average}(\text{amplitude}(U_{\text{detection}}))}{\text{average}(\text{ amplitude}(U_{excitation}))} \tag{3}$$

Moreover, to prevent the elastic wave reflections at the boundaries of the plate (Fig.7), Bloch conditions have been added in order to introduce the periodicity in the direction Y and absorbent conditions through the Perfectly Matched Layer (PML) are used at the X extremities of the model [11].

Figure 8 shows the numeric results of finite element calculation of the transmission coefficient for waves in all the directions of the Brilloin area. The above model (Fig. 7) can only simulate wave propagations along the OX direction for transverse input excitations associated with the band structure of the configuration plotted in figure 3c. The transmission curve (Fig 8b) presents three directional band gaps along the OX direction: the transmission coefficient is lesser than 10 dB for the intervals [700 Hz, 1000 Hz], [1000 Hz, 1600 Hz] and [2300 Hz, 3400 Hz]. The frequency of this directional band gaps coincides with those of the band structure in the OX direction (Fig 8a). In order to illustrate the mechanism of band gap in LR plate, several specific resonance eigenmodes are shown in figure 9.

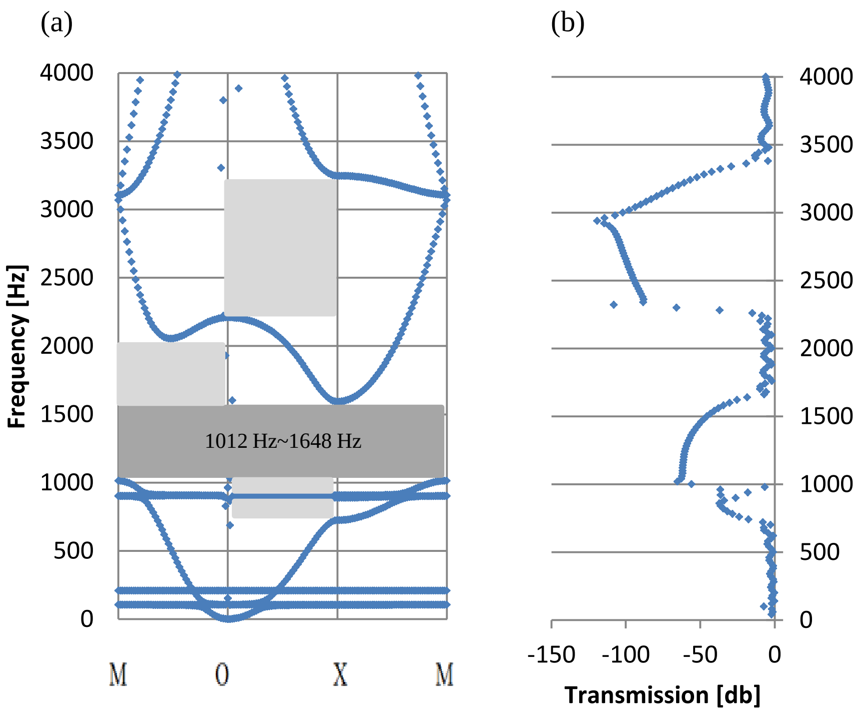

Figure 8 : a) Band structure and b) Amplitude transmission coefficient of elastic waves propagating in a LR structure formed with aluminum fine plate and periodic added mass attached to the screw M3x25 at the extremity.

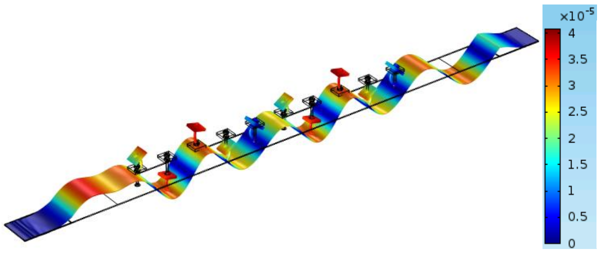

Figure 9 : Vibration shapes: a) f=600 Hz (out of a band gap), b) f=1020 Hz (inside a band gap)

III. Experimental results

To validate the two numerical approaches presented in section 2, the configuration of resonators described in figure 3c is used. The specimen of LR phononic plate are manufactured using an aluminum base plate, which is 600 mm in length (X direction), 500 mm in width (Y direction) and 1 mm thick (Z direction), attached to a square array of 11x6 mass-screw resonators, as shown in figure 10a. The lattice constant of the resonator array is *a*=40 mm. For comparing the behavior of LR plate configuration, a reference aluminum bare plate of same dimension length is tested. To generate plane wave propagating along the X direction, each of the two plates is clamped along one side to a shaking table whose displacement is in the Z direction. The other sides are left free. The imposed displacement along the clamped side of the plate is a white noise over a frequency range between 20 Hz and 2000 Hz. This excitation is equivalent to an inertial uniform surfacic load of the structure. Two accelerometers are used to measure the excitation acceleration: one placed on the shaking table and one rigid piece which clamp the plate (Fig. 10). The out-of-plane response is measured by a scanning laser doppler vibrometer (LDV), with its laser scanned over the face without resonators. The measurements are performed by scanning 11 × 13 points over 0.04 m x 0.04 m (Fig. 10b).

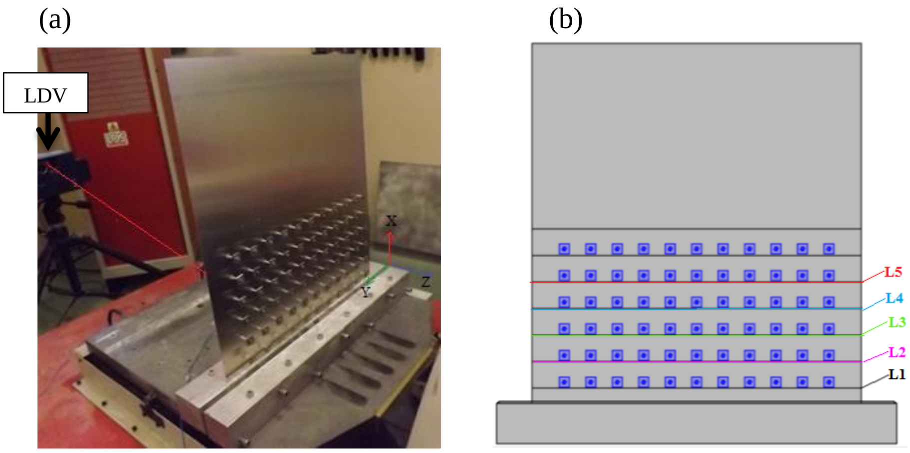

Figure 10 : (a) Experimental prototype of plate with periodic local resonators (*a*=40 mm), (b) Schematic of scanning area.

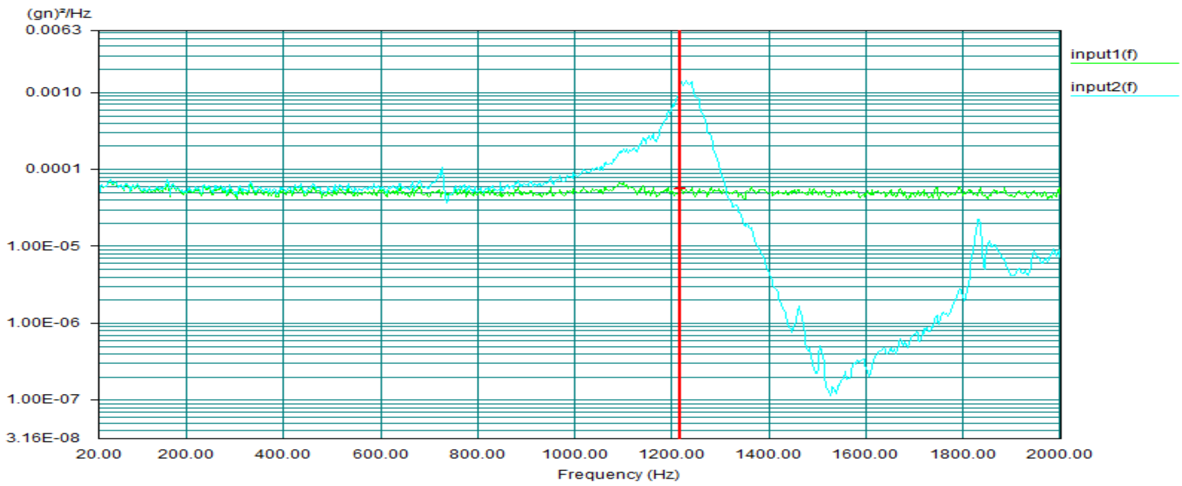

Figure 11 : Excitation acceleration measured in the shaking table (input1) and in the rigid beam (input2)

The figure 11 compares the measure excitations on the shaking table and near the clamping of the plate. This second value is not constant with frequency, maybe due to a local resonance, which demonstrates that the clamping is not perfect. We yet try to elucidate and avoid these phenomena. Nevertheless, the acceleration of the table (input 1) will be used in the following.

Through a fast Fourier transformation (FFT), the LDV achieves the displacement frequency response function (FRF) of each scan point($H1$: $acceration_{Vib}/acceleration_{Ref}$). For each line of the measured scanning points in the OX direction, the average of H is calculated using this equation:

$$H_{line(j)} = \frac{1}{n}\sum_{i=1}^{n} module(H1(i,j)); \qquad j = \{1, 2, 3, 4, 5\} \tag{4}$$

Figure 12 and 13 present the curves of the average H in the first five lines as a function of the frequency for the LR phononic plate and the reference plate respectively. The figure 12 presents a significant decrease of $H_{line(j)}$ for j=1 to 5 between 1000 Hz and 1600 Hz. The vibration attenuation increase as a function of the position of the measured line over the excited side. This decrease is not present in the figure 13 which correspond to the bare plate. It is thus experimentally demonstrated that a band gap take place in the frequency range 1000 Hz~1600 Hz. In that, this is coherent with the theoretical frequency gaps (Fig. 8). The displacement fields of the LR plate are plotted figure 14(a) at a frequency outside the band gap and figure 14(b) inside the band gap. The efficiency of the LR plate is evident on figure 14c: the lower area which is treated with LR systems has a negligible movement compared to the upper area which does not have LR system.

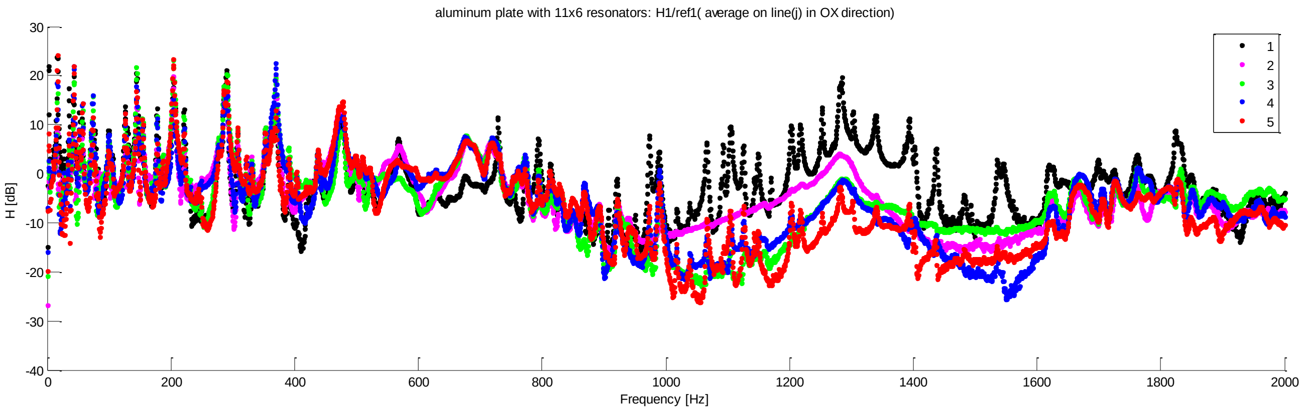

Figure 12 : Measured averaged frequency response $H_{\text{line(j)}}$ for LR phononic plate with periodic resonators (a=40 mm)

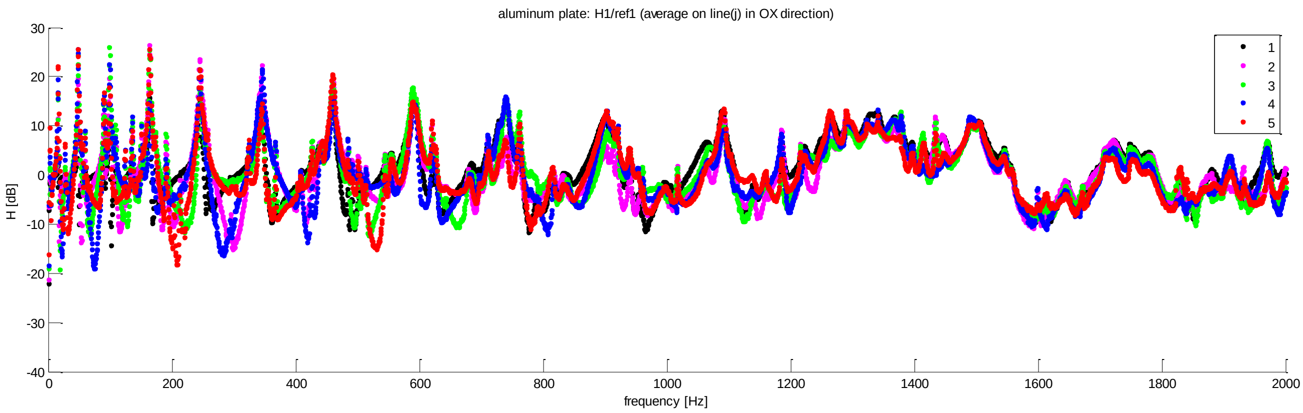

Figure 13 : Measured averaged frequency response $H_{\text{line(j)}}$ for bare aluminum plate

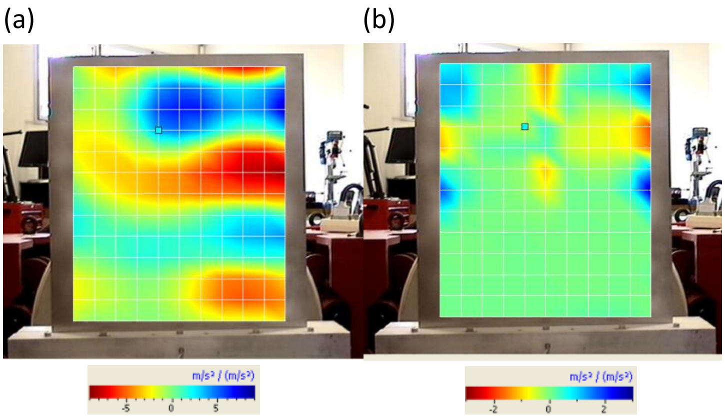

Figure 14 : Measured vibration shape at frequencies: a) f=146 Hz, b) f=1252 Hz

In order to determinate experimentally partials band gaps. A second configuration of LR plate is tested with a lattice constant of the resonator array *a*=50 mm corresponding to the case considered in figure 6c. This specimen of LR phononic plate is manufactured using the same aluminum base plate of the first configuration, with a square array of 9 × 8 mass-screw resonators as shown in figure 15a. The band structure of this configuration is plotted in figure 16. Two partial band gaps (g_1 [556 Hz~858 Hz] and g_2 [1714 Hz~2415 Hz]) and complete band gap (G_1 [842 HZ~1056Hz]) can be observed along the direction OX. It is interesting to note that, for this case g_2 is a resonance gap and g_1 is a Bragg gap which is very narrow of the complete band gap G_1. Experimentally the line excitation is more likely to produce plane waves propagating in the OX direction. Figure 17 show the curves of the average H in the first six lines as a function of the frequency for the LR phononic plate presented in figure 15 a. Comparing to the theoretical frequency gaps (Fig 16) it is seen that $H_{line(j)}$ decrease significantly in the frequency range of the complete band gap G_1 and the resonance bad gap g_2. But no significant wave attenuation can be observed in the frequency range of the Bragg bad gap g_1 (Fig. 17). It can be explained that the attenuation performance of a resonance gap is much higher than a Bragg gap, as has been demonstrated theoretically by Xiao et al [4].

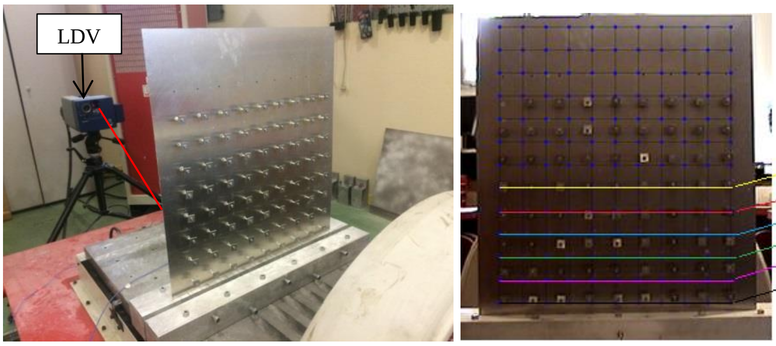

Figure 15 : (a) Experimental prototype of plate with periodic local resonators (*a*=50 mm), (b) Schematic of scanning area.

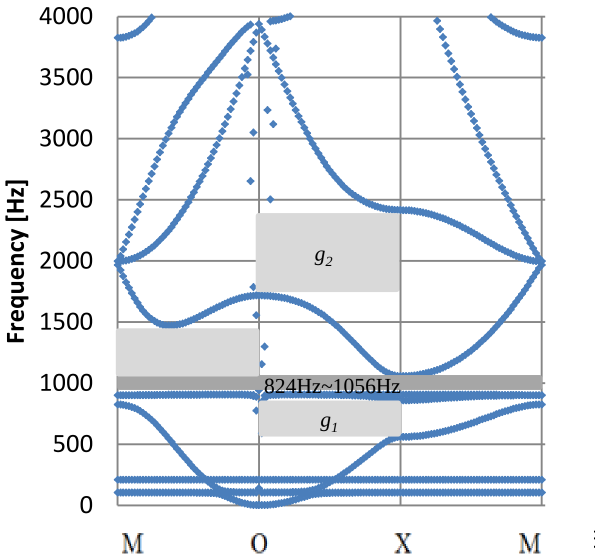

Figure 16 : Band structure of a locally resonant aluminum plate with periodic resonators of fixed mass in the extremity of screw M3x25 (*a*=50 mm)

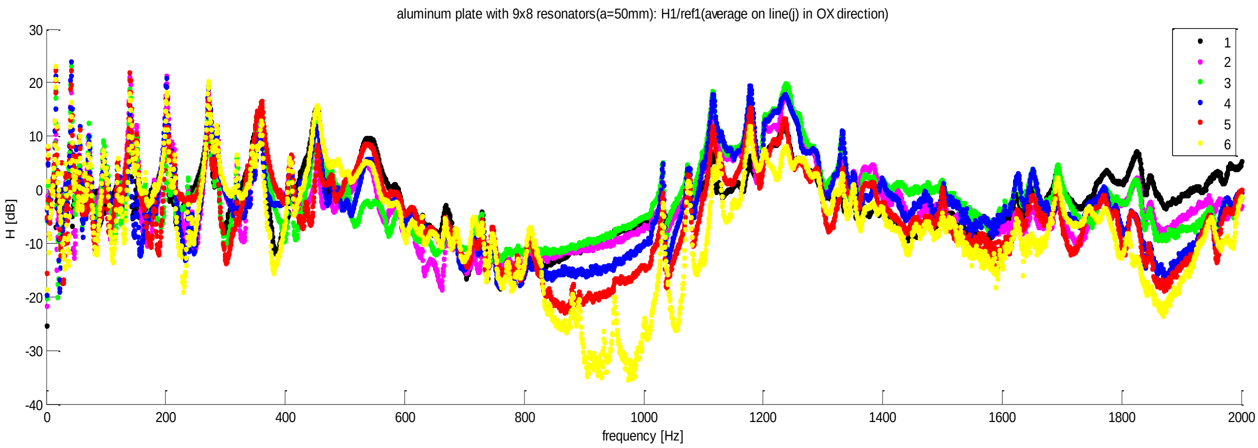

Figure 17 : Measured averaged frequency response $H_{\mathrm{line(j)}}$ for LR phononic plate with periodic resonators (*a*=50 mm).

IV. Conclusion:

In this paper, we report on the theoretical and experimental investigation of the wave propagation and vibration attenuation characteristics of LR thin plate with a 2D periodic array of attached resonators (mass-screws). Band gap behavior is theoretically characterized by plotting band structures of different configuration of resonators with varying the stiffness of the spring: changing the attached location of the mass to the screw. Besides, the effect of the lattice constant on the band gap is investigated. The larger the lattice constant *a*, the smaller the upper edge of band gap and the thinner the band gap. Furthermore, a combination of the resonance gap and the Bragg gap increase the width of the band gap. We further calculate the transmission coefficient in LR aluminum plate with periodic resonators for plane wave propagation along the OX direction. The frequency and bandwidth of this directional transmission band gaps match well with the band gaps properties shown in the band structure in the OX direction. Finally, two experimental measurements was performed with LR phononic plates consisting of a periodic array of resonators (mass-screws) attached to a thin homogeneous plate with different lattice constant of the resonator array *a*=40 then 50 mm. Agreement is observed between the theoretical previsions and the experimental measurements for position and band bandwidth of complete band gap and directional resonance gap. But the attenuation performance of a resonance gap is much higher than a Bragg gap which is difficult to detect experimentally.

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