# Multiphysics modelling of transport phenomena within cortical bone treated as a biporous material

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## Résumé :

*Le transport de liquide interstitiel et d'ions au sein des réseaux poreux de l'os joue un rôle majeur dans la mécanotransduction osseuse. Dans cette étude, nous présentons un modèle à trois échelles du transport couplé ayant lieu au sein de la porosité vasculaire et du réseau lacuno-canaliculaire de l'os cortical. Ces deux niveaux de porosité peuvent être caractérisés par des échanges au travers de la paroi perméable des canaux de Havers-Volkmann. Ainsi, les équations couplées du transport électro-chimio-hydraulique sont obtenues de l'échelle nanométrique des canalicules vers le tissu cortical, en prenant en compte l'échelle intermédiaire du tissu intra-ostéonal. Les lois de réciprocité d'Onsager qui régissent le transport couplé sont finalement vérifiées.*

## Abstract:

*Interstitial fluid and ionic transport taking place in the fluid compartments of bone is thought to play a major role in bone mechanotransduction. In this study, we present a three-scale model of the multiphysical transport phenomena taking place within the vasculature porosity and the lacuno-canalicular network of cortical bone. These two porosity levels exchange mass and ions through the permeable outer wall of the Haversian-Volkmann canals. Thus, coupled equations of electro-chemo-hydraulic transport are derived from the nanoscale of the canaliculi toward the cortical tissue, considering the intermediate scale of the intra-osteonal tissue. In particular, the Onsager reciprocity laws that govern the coupled transport are checked.*

**Keywords: Coupled transport; Hierarchical porosity; Three-scale model; Bone cortical tissue**

# 1 Introduction

Interstitial fluid and ionic transport taking place in the fluid compartments of bone is thought to play a major role in bone mechanotransduction. In cortical bone, the two major fluid compartments are the Havers' and Volkmann's canals (HVCs) and the lacuno-canalicular porosity (LCP), see Fig. 1. The HVCs–a few tens of microns in diameter–contain vasculature and nerves and provide a fast way to transmit chemo-mechanical information throughout the cortical tissue. Local messaging takes place within the LCP, a dense network of lacunae and canaliculi hosting the osteocytes. LCP crosses the bone solid matrix and is connected with the HVCs.

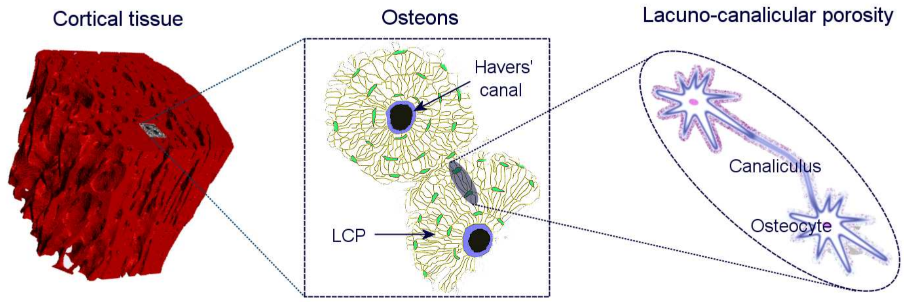


Figure 1: Multiscale structure of cortical bone.

Interstitial fluid and ionic transport in the LCP were extensively studied by our group [3, 4] highlighting their effects on bone physiology [2, 7]. This paper stems from those results and extends them to the osteonal scale, focusing on the fluid and ionic transport along the osteon. First, a unit-cell was identified as the annular fluid compartment of a segment of the osteonal canal. Relevant physical phenomena were described at the unit cell scale, namely: electrostatics (Poisson-Boltzmann equation); fluid flow (Stokes equation including Coulombic forces); ionic flow (Nernst-Planck equation). Boundary conditions for the above equations were obtained by assuming continuity of the hydraulic, ionic, and electric fluxes between the outer and inner walls of the unit cell and the LCP and the vasculature, respectively. Moreover, no-slip conditions for the fluid velocity were assumed at both the walls. Second, an asymptotic homogenisation technique was used to upscale these equations to the osteonal scale [3, 4]. The homogenisation procedure consists in the non-dimensional writing of the problems, the asymptotic expansion, the collection of the slow variables and the proposition of the closure cell-scale problems. The outcome was an explicit description of the velocity and the ionic flux along the osteon which turned out to be ruled by generalized forms of the Darcy's law and convection-diffusion equation, respectively, in response to pressure, osmotic, and electro-osmotic gradients along the osteon.

To the best of our knowledge, this theoretical work is the first attempt to describe the multiphysical transport phenomena taking place within the HVCs and the LCP at once. These two fluid compartments exchange mass and ions through the permeable outer wall of the HVCs. The model developed in this paper accounts for these exchanges by identifying the normal fluxes of mass and ions through the outer walls of the HVCs with the longitudinal fluxes inside the canaliculi. Furthermore, a rewriting of the macroscopic equations when convection and diffusion mechanisms are comparable resulted in checking Onsager reciprocal laws linking the coupled quantities that govern the coupled transport phenomena.

## 2 The three scales of the cortical tissue

In this paper, we focus on a biporous treatment of the interstitial bone fluid transport considering two levels of nested porous networks corresponding to the Havers-Volkmann canals and the lacuno-canalicular network. Thus, in our three scale homogenisation process, the microscale corresponds to the canalicular pore scale (Fig. 1 on the right - characteristic length $\ell_\mu \sim 10^{-8}$ m), the mesoscale corresponds to the intra-osteonal scale (Fig. 1in the middle - $\ell_m \sim 10^{-5}$ m) and the macroscale corresponds to the cortical tissue scale (Fig. 1on the left - $\ell_M \sim 10^{-2}$ m). As a consequence, the micro-to-meso and meso-to-macro characteristic lengths ratios are comparable: $\ell_\mu/\ell_m \sim \ell_m/\ell_M \sim \eta \equiv 10^{-3}$.

## 3 From the canalicular pore to the intra-osteonal tissue

At the microscale, the unit cell is a region of the bone matrix containing a canaliculus segment. The canalicular fluid compartment is seen as the annular space between two concentric cylinders—the cell process and the bone matrix being the inner and outer walls, respectively. The interstitial fluid, which is assumed to be an incompressible Newtonian monovalent electrolyte (typically Na-Cl), occupies the annular space between the canalicular wall and the osteocyte process membrane.

As shown by our group in previous studies [3, 4], in addition to the hydraulic flow, other electrochemical phenomena are generated by the electrolyte movement such as osmosis and electro-osmosis. Thus, the microscopic model combines an electrostatics balance to describe the electrical double layers close to the canalicular walls, Nernst-Planck equations for the ionic species transport that include possible electro-migration effects and a Stokes equation considering the effects of the Coulombic body force. Moreover, the tethering pericellular fibers can sensibly reduce the permeability of the canalicular space. Thus, this friction effect is represented by a Brinkman-like term involving a pericellular permeability.

After a first homogenisation process, the mesoscopic model includes:

**A modified Darcy law.** This law describes the mesoscopic average fluid velocity $\mathbf{V}^m$:

$$\mathbf{V}^m = -\mathbf{K}_P^m \nabla_m p - \mathbf{K}_C^m \nabla_m n - \mathbf{K}_E^m \nabla_m \bar{\psi}, \tag{1}$$

where $\nabla_m$ represents the space derivative operator with respect to the coordinates of the mesoscopic level, and $\mathbf{K}_P^m$, $\mathbf{K}_C^m$ and $\mathbf{K}_E^m$ are the effective permeability tensors at the mesoscale quantifying the Poiseuille, osmotic and electro-osmotic effects in response to the gradients of the pressure $p$, the salinity $n$ and the reduced streaming potential $\bar{\psi}$, respectively. Their expressions are detailed in our previous works.

**Effective diffusion tensors.** The ionic transport at the mesoscale is represented by the ionic effective diffusion tensors $\mathbf{D}_{\pm}^{\mathbf{m}}$ which account for convective and electro-diffusive effects as detailed in [4].

# 4 From the intra-osteonal tissue to the cortical tissue

The purpose of this section is now to describe the flow phenomena and chemical transport at the macroscale of the cortical tissue. At this scale, the porous network corresponds to the Havers-Volkmann canals (named Haversian pores hereafter), whereas the effective solid phase of the porous medium corresponds to the collagen-apatite tissue and the microporosity of the lacuno-canalicular system. Hydro-electro-chemical phenomena occurring across the canalicular network are integrated in the description of transport phenomena across Haversian porosity through the exchanges terms between the different porosity levels.

Since vasculature capillaries and nerves are present in the central part of Haversian pores, the interstitial fluid flows in the annular space between the outer wall of the Haversian canal and the walls of the capillaries and nerves. Again, the pore geometry can be roughly represented by two concentric cylinders symbolizing the Haversian canal (outer cylinder) and capillary and nerves (inner cylinder).

The side walls of the Haversian pores are permeable, allowing the exchanges of matter and ions with the blood vessels (inner wall) and the lacuno-canalicular network (outer wall). To use again periodic homogenisation, we assume that a 3D-periodic representative cell can be derived.

Similarly to the multiphysical approach proposed at the canalicular scale, the description of the transport phenomena at the scale of Haversian porosity should take into account electro-chemical effects in addition to hydraulic flow.

Thus, our transport model combines a Stokes equation involving the Coulombic force with Nerst-Planck equations representing convection, diffusion and possible electro-migration of the ionic species.

The key point here is to connect the boundary conditions at the outer wall of the pores (where the lacuno-canalicular network emerges). On the one side, the (macroscopic) tangent velocity remains zero at the walls (no-slip condition). On the other side, the (macroscopic) normal velocity at the walls is equal to the (mesoscopic) canalicular velocity $\mathbf{V}^m$ determined from the previous upscaling procedure in Eq. (1). Similarly, the normal ionic flux at this pore surface is determined from the analysis at the lacuno-canalicular scale.

Then, the homogenisation procedure is carried out to provide a macroscopic coupled description which can be represented in a matrix form involving the macroscopic fluid flow $\mathbf{v}^M$, the macroscopic total ionic flux $\mathbf{J^M}$ (sum of the cationic and anionic fluxes), the macroscopic electric flux $\mathbf{I^M}$ (difference of the cationic and anionic fluxes), the coupling matrix, the macroscopic hydraulic gradient $\nabla_M p$, the macroscopic chemical gradient $RT\,\nabla_M \ln n$ ($R$ being the gas constant and $T$ the absolute temperature) and the macroscopic electrical gradient $\nabla_M \psi$:

$$\begin{pmatrix} \mathbf{v}^M \\ \mathbf{J^M} \\ \mathbf{I^M} \end{pmatrix} = \begin{pmatrix} -\mathbf{L}^{PP} & -\mathbf{L}^{PC} & -\mathbf{L}^{PE} \\ -\mathbf{L}^{CP} & -\mathbf{L}^{CC} & -\mathbf{L}^{CE} \\ -\mathbf{L}^{EP} & -\mathbf{L}^{EC} & -\mathbf{L}^{EE} \end{pmatrix} \begin{pmatrix} \nabla_M p \\ RT\,\nabla_M \ln n \\ \nabla_M \psi \end{pmatrix} \tag{2}$$

The diagonal terms of the matrix represent the direct parameters whereas the off-diagonal terms quantify the coupling phenomena. Their expressions are detailed in [5]. In this paper, we also explicitly show that this coupling matrix is symmetric, proving in this case the Onsager reciprocity principle.

# 5 Example of application: recovery of the streaming potentials at the macroscale

The use of this model makes it possible to provide an estimation of the macroscopic streaming potentials induced by the stress-generated fluid flow within the lacuno-canalicular pores. This idea was first proposed by [6] to validate hydromechanical models of bone.

Considering homeostasis and so neglecting chemical gradient effects, we put forward the Onsager reciprocity relations to obtain the streaming potential distribution from the macroscopic fluid velocity $\mathbf{v}^M$. The direct phenomena described by the coefficients $\mathbf{L}^{PP}$ (hydraulic permeability) and $\mathbf{L}^{EE}$ (electric conductivity of the fluid) are respectively the Darcy and Ohm laws, whereas the coupled phenomena are the electro-osmosis and streaming currents described by the coefficients $\mathbf{L}^{PE}$ and $\mathbf{L}^{EP}$, respectively. The Onsager reciprocity relations indicate that these two coupling coefficients are equal.

When steady state is reached, there is no net charge transfer, implying that $\mathbf{I^M} = \mathbf{0}$. This is equivalent to setting the Ohmic and convective currents equal and opposite. Thus, the streaming potential is linearly connected to the macroscopic pressure field as shown in [3].

In Fig. 2, streaming potentials derived from this model is compared with the *in vivo* recording of stress-generated potentials during walking measured by Cochran *et al.* [1]. The model parameters and the macroscopic pressure field are those presented in [3].

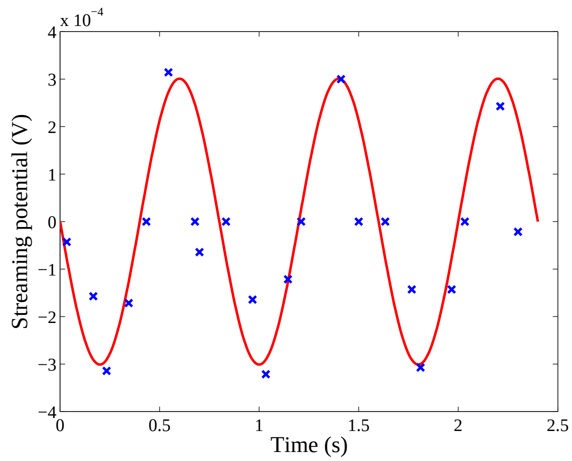


Figure 2: Comparison between the simulated (solid line) and experimental data (××) of streaming potentials.

Notwithstanding the coarse approximations in this model (idealized annular geometry of the pores for instance), this illustration indicates the strength of the multiscale treatment developed in this study.

# 6 Conclusion

A multiscale model of the transport phenomena occurring at different scales of cortical tissue was developed adopting a biporous approach. It takes into account the hydro-electro-chemical phenomena occurring at the different scales of pores (Haversian porosity and canaliculi). Through this three-scale model, the appearance of new exchange terms between the different porosity levels in the macroscopic equation of ionic transport has been put into relief.

Furthermore, when convection and diffusion mechanisms are comparable, we are able to check Onsager reciprocity relations linking the coupled quantities that govern the coupled transport phenomena. These reciprocity properties are fundamental when validating hydro-mechanical models of bone using streaming-potentials measurements.

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