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INTERACTION BETWEEN PROPELLER AND HULL

BY DR. K. E. SCHOENHERR AND A. Q. AQUINO

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Dr. K. E. Schoenherr

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A. Q. Aquino

U.S. Experimental Model Basin
Navy Yard, Washington, D.C.

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It is well known that when the propeller of a ship works in close proximity to the hull - which usually is the case in practice - a mutual interaction between hull and propeller takes place. This interaction has a material influence on propulsive efficiency and also on the most suitable propeller dimensions. For precise analysis it must be determined by self-propelled model tests but for preliminary analysis or design it can be estimated with sufficient accuracy from previous experimental work.

The principal purpose of this report is the presentation of data derived from self-propelled model tests from which the interaction between hull and propeller can be readily estimated. A secondary purpose is a concise restatement of the generally known principles of wake, thrust deduction and relative rotative efficiency with the addition of some not so well known recent developments.

PART I

SECTION I - NATURE AND ORIGIN OF WAKE AND THRUST DEDUCTION

1. WAKE

In its passage through the water the ship or model imparts motion to neighboring fluid particles. The aggregate motion of these particles relative to the ship constitutes what is known as the wake. The wake is usually regarded as positive when it follows the ship and as negative when moving in the opposite direction. Its strength at any point is equal to the difference between the speed indicated by a current meter or pitot tube carried aboard ship and the ship speed through undisturbed water. Thus, if we denote the speed indicated by the meter by v_a and the ship speed by v, the wake speed is $v - v_a$. It is customary in practice to express the wake speed as a fraction of the speed v_a or of the ship speed v. The former method was introduced by R. E. Froude and is used chiefly in Great Britain, while the latter method was introduced by D. W. Taylor and is used in the United States and in countries of continental Europe. According to this practice we have in Froude's notation

$$v - v_a = w_F \, v_a$$

whence,

$$\frac{v_a}{v} = \frac{1}{1 + w_F} \tag{1}$$

In Taylor's notation we have:

$$v - v_a = w \, v$$

whence,

$$\frac{v_a}{v} = 1 - w \tag{2}$$

where:

w_F = Froude wake fraction

w = Taylor wake fraction

Combining Eq. (1) and (2) we get:

$$w_F = \frac{w}{1 - w} \qquad (3a)$$

and

$$w = \frac{w_F}{1 + w_F} \qquad (3b)$$

2. THRUST DEDUCTION

A propeller develops thrust in virtue of the momentum changes occurring in the surrounding fluid. In the theory of propeller action, it is shown that one-half of the final velocity increase is imparted to the fluid before it enters the propeller disc and the other half while it passes through the disc. In accordance with Bernoulli's principle the velocity increase in front of the propeller entails a corresponding reduction in pressure. This pressure reduction extends forward, and, when the propeller is close to the hull, reduces the streamline pressure on the stern of the ship in a manner shown diagrammatically in Fig. 1. Curve A in this figure

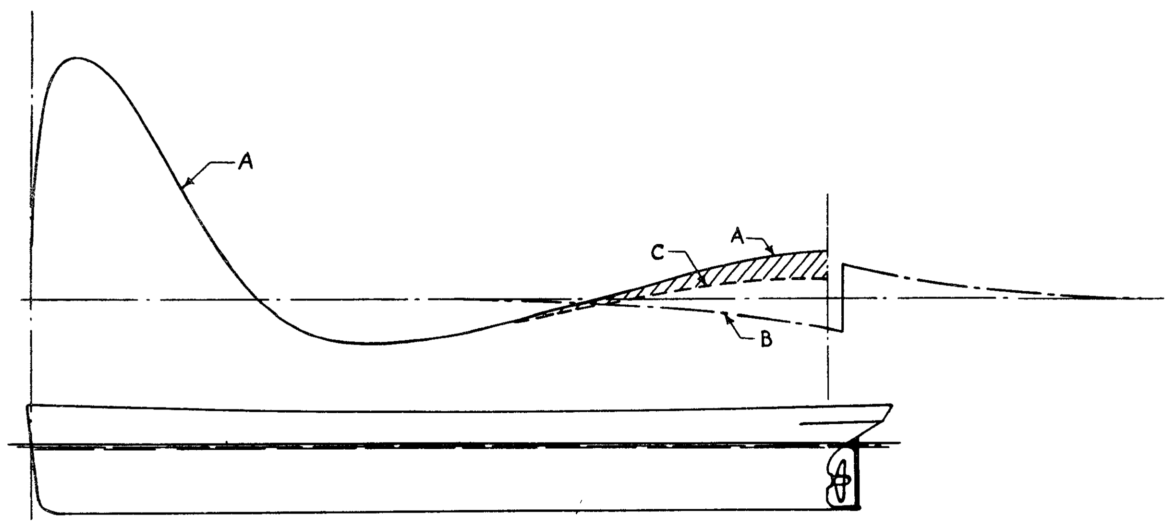

Figure 1

denotes the pressure gradient around the hull without the propeller working (the pressure must be regarded as integrated along the girths of the sections), Curve B denotes the pressure gradient due to the propeller alone and Curve C the combined pressure gradient of hull and propeller. Obviously, as a result of the propeller action, the ship resistance is increased by an amount that is proportional to the shaded area between Curves C and A. This resistance augment must be overcome by a fraction (t) of the propeller thrust (T) so that only the remainder (1 – t)T will be available to overcome the towrope resistance (R) of the hull. The fraction (t) is usually called the thrust deduction coefficient. It is obtained, when the thrust and resistance are known from separate measurements, by the equation

$$\text{thrust deduction} = tT = T - R \tag{4a}$$

or

$$t = 1 - \frac{R}{T} \tag{4b}$$

3. INTERRELATION BETWEEN WAKE AND THRUST DEDUCTION; HULL EFFICIENCY AND RELATIVE ROTATIVE EFFICIENCY

The wake and thrust deduction are not entirely independent quantities but are interrelated. A theoretical expression of this interrelation was first derived by Rankine in 1865 and later was further developed by Fresenius, Telfer, Horn, Helmbold and Dickmann, as will be shown presently. Ordinarily the relation between wake and thrust deduction is expressed in terms of the so-called hull efficiency which arises in a natural manner as follows: Considering hull and propeller as a unit, the useful power output is the effective horsepower (EHP) and the propeller input is the propeller horsepower (PHP). The propeller horsepower differs from the shaft horsepower (SHP) by the friction losses in the shaft bearings and stuffing boxes between the propeller and the torsionmeter. The ratio EHP/PHP is variously called the propulsive efficiency, quasi-propulsive efficiency or quasi-propulsive coefficient. By making use of Eq. (2) and (4) and multiplying numerator and denominator by like quantities, we have

$$e = \frac{EHP}{PHP} = \frac{Rv}{2\pi Qn} = \frac{T\,v_a}{2\pi Qn}\,\frac{1-t}{1-w} = \frac{T_o\,v_o}{2\pi Q_o n_o}\,\frac{1-t}{1-w}\,\frac{v_a/n}{v_o/n_o}\,\frac{TQ_o}{T_oQ} \tag{5}$$

where

- T is the thrust in pounds
- Q is the torque in pound-feet
- n is the revolutions per second
- v is the ship speed in ft./sec.
- v_a is the propeller speed of advance through the wake in ft./sec.
- v_o is the propeller speed of advance in open water in ft./sec.

Zero subscript designates quantities measured when the propeller is working in open water.

The first ratio on the right hand side of the above equation is by definition the open-water efficiency of the propeller. The second ratio is the hull efficiency referred to in the foregoing. The third ratio is equivalent to the ratio $\frac{1 - s}{1 - s_o}$ where s denotes the true propeller slip ratio behind the model or ship and s_o the corresponding quantity in open water. When slip is evaluated from the propeller force coefficients as explained later this ratio becomes unity. The last term of the equation is a quantity which usually is designated as the relative rotative efficiency. These various quantities will be further discussed in what follows.

SECTION II - THEORETICAL ANALYSIS

1. WAKE

The wake behind the ship is usually not a uniform axial current but varies from point to point in both magnitude and direction. Thus it is found that the wake distribution over the propeller disc of a single-screw ship is such that in zones between two given radii the intensity is greatest at points in line with the stern-post and least at points approximately at right angles to these positions. Furthermore, it is found that the average zonal intensities decrease with increasing radii. This inhomogeneity in wake structure is due to the fact that the stern wake is the resultant of several more or less independent components, each of which has a characteristic distribution of its own. These components are: streamline or potential wake, - hereafter designated by subscript p, - friction wake designated by subscript f, and wave wake designated by the subscript w.

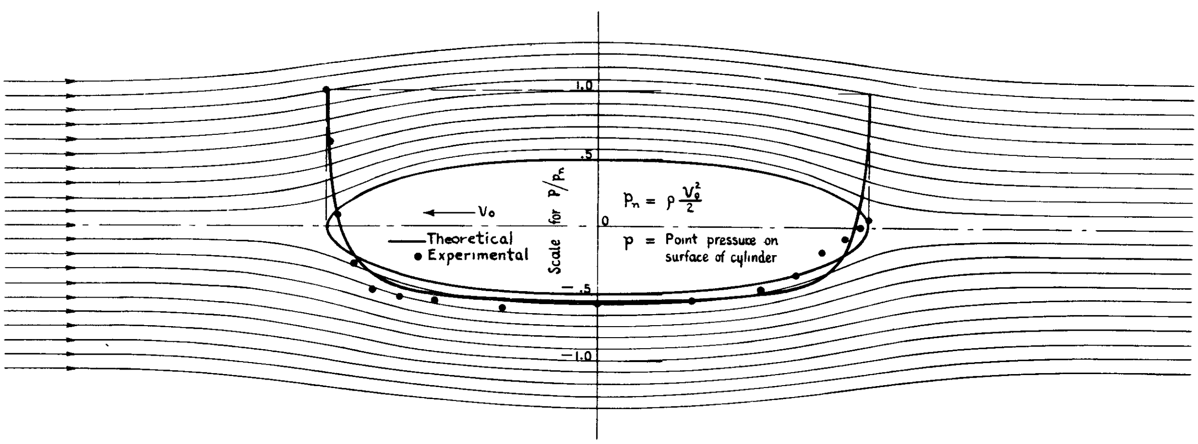

Figure 2

The formation of the streamline wake is best understood by reference to Fig. 2. In this figure the heavy line represents the contour of an elliptic cylinder with its cylindrical axis normal to the plane of the paper. Assume this cylinder to be held stationary in an unbounded frictionless medium which is flowing from left to right with the uniform velocity v_0. The lines in the field surrounding the cylinder are then streamlines, that is, lines traced by individual particles in passing around the cylinder. It will be noticed that these streamlines were drawn so as to be equidistant far ahead of the body where the medium is undisturbed, a construction that enables us to deduce from the figure not only the direction but also the magnitude of the velocity at any point in the field. The direction of the flow at a certain point is found by drawing tangents to the streamlines; the magnitude of the velocity at that point is obtained by multiplying v_0 by the ratio of the streamline spacing in the undisturbed region to the streamline spacing at the point. Keeping this in mind, it appears from the figure that close to the body the streamline wake is a positive maximum at the bow and stern and a negative maximum amidships. At other points the wake velocity lies between these extremes, converging rapidly to zero at some distance from the body.

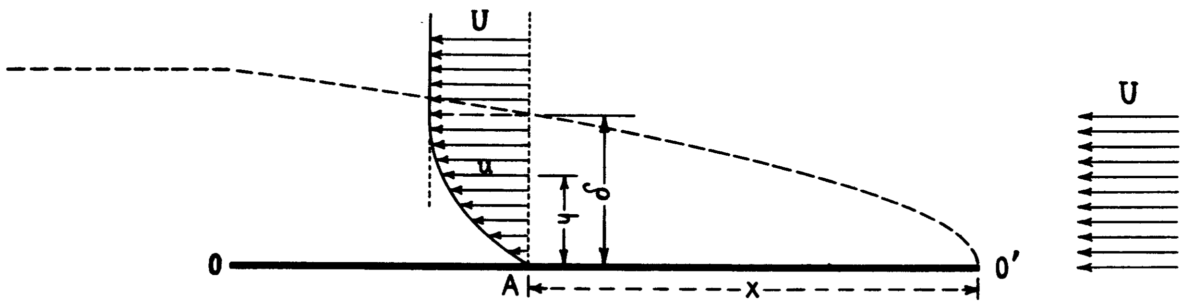

Figure 3

The formation of the friction wake is also best explained by considering a simple case. Thus, consider the thin, straight plank $\overline{OO'}$ in Fig. 3 moving from left to right with the constant velocity U. According to the modern theory of fluid friction originated by Prandtl in 1904, the fluid clings to the solid surface without slipping, but slipping does occur between successive fluid layers. In any YZ-plane normal to the plank the velocity with respect to the wall therefore varies continuously from a maximum U to zero, as shown diagrammatically in the figure. The layer of fluid in which this velocity variation obtains is found by measurements to be relatively thin and is usually called the boundary layer.

The velocity distribution within the boundary layer depends on the type of flow - whether laminar or turbulent - but for a given type of flow is assumed to be the same throughout the length of the plank. Mathematically expressed this means that the ratio u/U is solely a function of y/δ where u is the velocity at any distance y from the wall and δ is the thickness of the boundary layer; that is, the y-value where u is equal to U. When the velocity distribution function is known, a relation between the shear stress at the wall and the mass flow in the boundary layer can be set up by momentum considerations and the thickness δ at any distance x from the leading edge of the plank can be calculated. Thus it is found that δ increases with x proportional to $x^{\frac{1}{2}}$ for laminar flow, and approximately as $x^{\frac{4}{5}}$ for turbulent flow.

From the foregoing discussion it is manifest that the friction wake is always positive and is confined to the region of fluid close to the walls of the solid body. Its intensity is unity at the wall and zero (or nearly zero) at the edge of the boundary layer. Its lateral width increases from zero at the leading edge of the body to a maximum value at the trailing edge, retaining this value for a considerable distance behind the body until its energy is dissipated in the form of heat.

The wave component of the wake arises from the absolute velocities possessed by fluid particles in a wave. It is well known from theory and observation that in the crest of a wave fluid particles move in the direction of propagation of the wave and in a wave-hollow in the opposite direction. The wave-wake may therefore be either positive or negative. Its magnitude on slow and moderate speed ships is usually much smaller than that of the other components but on high speed vessels may exceed that of the others. This explains why on vessels of the destroyer type the total wake is often found to be negative.

Turning now to an actual ship, certain differences between the actual flow and the simple flows discussed in the foregoing must be noted. First, in the discussion of streamline flow, the flow was assumed to be two-dimensional, but on an actual ship it is three-dimensional. Second, a ship travelling over a free surface creates waves which change in length and height with ship speed. Hence, in the actual case the flow pattern changes to some extent with speed. Third, in the ideal case the innermost streamline was the contour of the body but in the actual case the innermost streamline is the edge of the boundary layer. Fourth, in the case of the thin, straight plank the pressure in the boundary layer was independent of length but in the case of a ship the pressure varies in accordance with the superimposed streamline pressure. These differences have important consequences as discussed below.

The most important consequence of the above-mentioned differences is the breaking away of the boundary layer from the surface of a ship's hull at some point along the run and the consequent formation of eddies. This phenomenon - which is of course most pronounced on ships with full afterbodies - may be explained as follows: The streamline pressure superimposed on the boundary layer varies approximately as shown in Fig. 2; amidships the pressure is negative but rises sharply at the bow and stern. In a frictionless medium the conversion from pressure head at the bow into velocity head along the sides, and back into pressure head at the stern is complete, as shown by the theoretical pressure curve in the figure. In a real medium, however, fluid particles situated in the boundary layer - having lost some of their kinetic energy by rubbing along the sides - on reaching the stern have not sufficient energy left to penetrate the high pressure region superimposed by the streamline flow and momentarily come to rest forming a "dead water" area. As this area grows in size the whole stagnant mass is detached from the body and when set in rotation by faster moving particles forms an eddy. Once detached, the eddy reacts with other eddies formed in a similar manner and arranges itself in a stable system of equilibrium sometimes referred to as a von Karman "vortex sheet". Behind a ship the wake is thus not the regular smooth flow pictured in the ideal case but a very confused and periodically varying flow.

2. THEORETICAL INTERRELATION BETWEEN WAKE AND THRUST DEDUCTION

As already mentioned, the wake and thrust deduction are interrelated. That this interrelation is a compulsory one was first demonstrated by Rankine (1)* in 1865 and expressed in the following words: "If the stream in which the propeller works would have, in absence of the propeller, a certain velocity impressed on it by the ship, then a force equal and opposite to that required in order to produce that velocity in all the particles of the stream must form part of the action of the water on the ship. But in the absence of the propeller that stream would occupy a greater area than that of the propeller. The propeller in accelerating the stream draws it into a reduced area, and an additional quantity of water flows in to fill the difference between the original and reduced areas; that additional water also assumes the velocity due to the action of the ship in the absence of the propeller, and the force required in order to make it assume that velocity is an addition to the ship's resistance, which has to be deducted from the total thrust of the propeller in order to find the effective thrust."

Expressing the above argument in mathematical form Rankine obtained the following equation:

$$\text{Thrust deduction} = \frac{v - v_a}{v_a} T \qquad (6)$$

where the symbols have the previously defined meaning. By introducing Eq. (1) and (4a) into this expression it reduces simply to

$$t_p = w_F \qquad (7)$$

In the discussion of Rankine's paper, W. Froude took exception to the implication that the wake gain and thrust loss balanced each other under all conditions, which induced Rankine to declare in his reply that his formula applied to streamline flow only and that "frictional wake" had to be excluded. Thus it appears that Rankine and Froude recognized in a general way that streamline and friction wake possess fundamentally different properties. This fact seems to have been forgotten, however, until in 1921 Fresenius (2) rediscussed the fundamental principles of the interrelation between hull and propeller and brought out clearly the different char-

* Figures in parentheses designate references at end of report.

acteristics of wake in a viscous and in a non-viscous medium.

2a. PROPULSION IN VISCOUS FLOW

To simulate the conditions of pure viscous flow, Fresenius imagined a body with large superficial area of such shape that its resistance R_f when advancing with the speed v through a viscous medium would be all frictional resistance and the thrust deduction would be negligibly small; the wake behind the body would then be all frictional wake. Assuming a propeller located in the wake to develop the thrust T, he obtained

$$R_f v = T v_a \frac{1}{1 - w_f} \tag{8}$$

where the relation between v and v_a is that expressed by Eq. (2). In this equation $(R_f v)$ is the useful power output of the system - ship and propeller - while $(T v_a)$ is the useful power output of the propeller alone, the fraction $\frac{1}{1 - w_f}$ being larger than unity. The equation thus expresses that the power output of the system is greater than that of the propeller alone which can be true only if the propeller abstracts energy from the wake. In viscous flow therefore some of the energy lost in forming the wake is recovered by the propeller.

The same argument can be applied for propulsion in a wave wake inasmuch as this case is analogous to that just discussed, in that energy is transferred from the body to the medium. There is this difference, however, that the wave wake can be positive or negative while the friction wake is always positive, so that the term $\frac{1}{1 - w_w}$ (w_w denoting wave wake fraction) is greater or less than unity. From this and previous discussion it follows therefore that propulsive efficiency is increased when the propeller is working in a wave crest and is reduced when the propeller is working in a wave hollow.

2b. PROPULSION IN PURE STREAMLINE FLOW

To investigate the conditions in pure streamline flow, Fresenius assumed a streamline body, such as that shown in Fig. 2 to be advancing with the steady speed v through an ideal frictionless medium. To allow for the fact that the resistance of any body in steady motion through an ideal medium is zero, he imagined a force R - the equivalent of the body resistance in a real medium - to be applied from outside the system in the manner of a towrope pull. Fresenius reasoned then as follows: Since the body has no resistance in travelling through the fluid, no energy is contained in the streamline wake and consequently no energy is available that could be recovered by the propeller. The power output of the system (R v) must therefore be equal to the power output of the propeller ($T v_a$), so that

$$R v = T v_a \tag{9}$$

whence it follows that

$$\frac{R}{T} = \frac{v_a}{v}$$

or, introducing Eq. (2) and (4)

$$t_p = w_p \,* \qquad (10)$$

The conclusion, that in non-viscous flow the wake fraction is equal to the thrust deduction coefficient, plausible as it seemed from its derivation, was never quite accepted by engineers, who reasoned that, after all, the power for propulsion was supplied by the ship's engine and the fraction of the power inevitably lost in the propulsive process could be great or small depending on the flow conditions in which the propeller works. This doubt has since been supported by newer investigations, notably by those of Horn (3), Helmbold (8), Telfer (4) and Dickmann (9). Each of these investigators derived a different expression for the interrelation of wake and thrust deduction which were all similar, however, in that they contained the same elements. Dickmann's expression which is the latest one and is based on the most thorough study is derived by what follows in a somewhat simplified manner.

To develop Dickmann's argument, again consider the streamline body shown in Fig. 2 to be advancing with the velocity v_0 through a non-viscous medium. Following Fresenius, it is assumed that a force R - the equivalent of the body resistance in a real medium - is applied from outside the system. This force is counterbalanced by part of the propeller thrust and, since the system is assumed to be conservative, must reappear as the momentum of the propeller race.

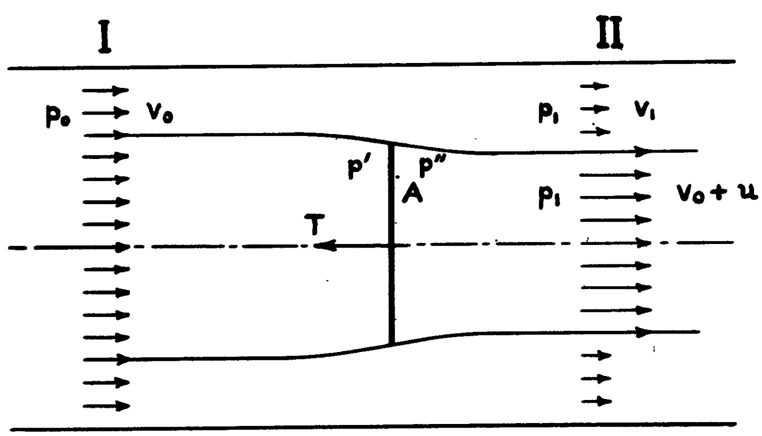

Figure 4

* (Note: The last equation is similar but not equal to that deduced by Rankine.)

Assuming now that the propeller is simply an actuator disc, we have the theoretical flow conditions shown diagrammatically in Fig. 4. In this figure, T denotes the thrust, v_o the speed of advance through the undisturbed medium and u the ultimate velocity increase imparted by the propeller to the fluid. Further denoting the disc area of the propeller by A and the density of the medium by ρ, the following equation is obtained from momentum considerations, as proved in many textbooks.

$$T = \rho A\left[v_o(1-w_p)+u'\right]u \tag{11}$$

where $A\left[v_o(1-w_p)+u'\right]$ is the volume of fluid passing through the propeller disc per second.

u' is the velocity increase imparted to the fluid before reaching the propeller disc.

To eliminate the unknown u', we make use of Bernoulli's Theorem. Let p_o be the pressure in the undisturbed medium, p' the pressure immediately in front of the propeller and p" the pressure immediately behind the propeller; then, by applying Bernoulli's Theorem twice - once to a streamline extending from the propeller to the right and once to the continuation of this streamline extending from the propeller to the left - we get the following pair of equations

$$\begin{aligned} p_o+\frac{\rho}{2}(v_o+u)^2 &= p''+\frac{\rho}{2}\left[v_o(1-w_p)+u'\right]^2 \\ p_o+\frac{\rho}{2}v_o^2 &= p'+\frac{\rho}{2}\left[v_o(1-w_p)+u'\right]^2 \end{aligned} \tag{12}$$

Subtracting, reducing and multiplying both sides of the resulting equation by A, we get

$$A(p''-p') = \rho A\left(v_o+\frac{u}{2}\right)u \tag{13}$$

The left-hand side of this equation is equal to the propeller thrust, so that we finally have

$$T = \rho A\left(v_o+\frac{u}{2}\right)u = \rho A v_o^2\left(1+\frac{s'}{2}\right)s' \tag{14}$$

where $s' = u/v_o$ is the theoretical slip ratio.

The remarkable fact about the final equation is that the thrust depends on v_o and u but is entirely independent of the wake fraction w_p.

Previously it was mentioned that the force R must reappear in the form of momentum imparted to the propeller race. By the Impulse Equation we have therefore for unit time

$$R = \rho a(v_o+u)u \tag{15}$$

where a is the cross-section area of the propeller race. The other quantities have the previously defined meaning.

In this equation, the term $a(v_0 + u)$ is the volume of fluid passing through the cross section of the race in unit time, which, by the condition of continuity of flow must be equal to the volume of fluid passing through the propeller disc in unit time. We can write therefore

$$R = \rho A\left[v_o(1-w_p)+u'\right]u \tag{16}$$

Combining this equation with Eq. (14) and (4a) we get

$$\rho A\left(v_o+\frac{u}{2}\right)u(1-t_p) = \rho A\left[v_o(1-w_p)+u'\right]u \tag{17}$$

Reducing, this equation becomes

$$1-t_p = \frac{v_o(1-w_p)+u'}{v_o+\frac{u}{2}} \tag{18}$$

The last expression contains both the velocity increase in the propeller disc u' and the final velocity increase u. For a propeller working in free water it is found that $u' = u/2$. That this same relation holds for a propeller working in the wake was proved by Dickmann by an independent method involving calculation of the propeller thrust by a source and sink method. Substituting this relation in the last equation, it becomes therefore

$$t_p = w_p\frac{2v_o}{2v_o+u} = w_p\frac{2}{2+\frac{u}{v_o}} = w_p\frac{2}{2+s'} \tag{19}$$

Combining further the last of these expressions with Eq. (14), the inter-relation between streamline wake and thrust deduction can be expressed in the form

$$t_p = w_p\frac{2}{1+\sqrt{1+2\tau}} \tag{20}$$

where $\tau = \frac{T}{\rho A v_0^2}$ is the load coefficient of the propeller.

With the expressions just derived, the theoretical propulsive efficiency and propeller efficiency in pure streamline flow can be calculated. The useful power output of the system considered above is obviously Rv_0. The power input must be equal to the sum of this output and the kinetic energy losses remaining in the fluid. We have therefore

$$e_i = \frac{R v_o}{R v_o+\frac{a}{2}(v_o+u)u^2} \tag{21}$$

Substituting for R expression (16) with $u' = u/2$, we get

$$e_i = \frac{v_o}{v_o+\frac{u}{2}} = \frac{2}{2+s'} \tag{22}$$

which is easily transformed into

$$e_i = \frac{2}{1 + w_p + \sqrt{2\tau_r + (1 - w_p)^2}} \tag{23}$$

where $\tau_r = \frac{R}{\rho A v_o^2}$

Finally, since propeller efficiency (e_{pi}) is equal to propulsive efficiency divided by hull efficiency ($\frac{1 - t_p}{1 - w_p}$), we get by making use of Eq. (19) and (22)

$$e_{pi} = e_i \frac{1 - w_p}{1 - t_p} = \frac{2}{2 + s'} \frac{1 - w_p}{1 - w_p \frac{2}{2 + s'}} = \frac{2}{2 + \frac{u}{v_o(1 - w_p)}} \tag{24}$$

The interesting part of this analysis is, first, the result expressed by Eq. (23) that the propulsive efficiency is increased by a low positive or high negative streamline wake and, second, the result expressed by Eq.(24) that the efficiency of the propeller behind the model is equal to that of a propeller in free water developing the same slip velocity u but advancing with the slower velocity $v_a = v_o(1 - w_p)$.

3. RELATIVE ROTATIVE EFFICIENCY

The relative rotative efficiency which arises as explained in Sec. I,Art.3 has been much discussed and interpreted in various ways. Some aspects of its nature will become clear from the following argument. Consider two identical propellers presumed to be working at the same average slip ratio, Propeller A working in a uniform wake and Propeller B in a non-uniform wake. To be definite, let us assume that the average wake strength is the same in the two systems but that in the non-uniform system the intensity is greater than the average value for the inner part of the propeller disc (that is, the part near the hub) and smaller than the average value for the outer part of the disc. As the result of this assumed non-uniformity of flow, the inner blade sections of Propeller B work at greater slip angles and the outer sections at smaller slip angles than the corresponding sections of Propeller A. Manifestly, the flow patterns around the sections of the two propellers at the same radii are not similar.

From the general law of dynamic similitude we know now that the conditions which must be fulfilled for two systems to be dynamically similar are, (a) similitude of flow patterns and (b) constancy of the ratio of corresponding forces. From the special law of similitude for deeply submerged propellers we know that when Condition (a) is satisfied, Condition (b) is also satisfied. By inference it follows that in the latter case when Condition (a) is not satisfied, Condition (b) will in general also not be satisfied.

Applying this reasoning to the two systems considered above, it is manifest that due to the dissimilarity of the wakes the ratios of corresponding elementary forces acting on corresponding blade sections are in general not equal in the two systems and that after integrating the elementary forces over the radii, the ratios of the total forces are in general also not the same for the two systems, that is we have the inequality expressed by

$$\frac{Td}{Q} \neq \frac{T_o d}{Q_o} \tag{25}$$

which can also be written in the form

$$\frac{TQ_o}{T_oQ} \neq 1$$

where T is the propeller thrust

Q is the propeller torque

d is the propeller diameter

subscript zero denotes uniform flow (Propeller A), no subscript, non-uniform flow (Propeller B).

Comparing the last expression with that for the relative rotative efficiency in Eq. (5), it appears that they are identical. The argument shows, therefore, that from one point of view the relative rotative efficiency is a factor that expresses the dissimilarity of flow conditions when the propeller is working in the non-uniform wake behind the ship or model and when it is working in open water. It is probable, however, that this is not the full explanation since the difference in the degree of turbulence and the difference in the Reynolds number that usually prevail in the open water and self-propelled tests also influence the relation between T and Q and, hence, the relative rotative efficiency. In view of this it is not surprising that the relative rotative efficiency in practice behaves quite erratically as will be shown later.

SECTION III - METHODS OF MEASURING WAKE

The wake can be measured in several different ways. Unfortunately, however, the results obtained by the different methods do not always agree and, in fact, the results obtained by any one method are not entirely unique.

The most direct method of obtaining the wake is to measure the water speed by means of a pitot tube or current meter carried on the ship or model, and subtract it from the known ship or model speed through undisturbed water. The pitot-tube method yields spot readings by which it is possible to determine the intensity as well as the direction of the water flow at any point in the wake. The drawback of this method is its tediousness when average results alone are of interest.

The current-meter method is simpler than the pitot-tube method but also is not free from objections. A special type of current meter for wake measurements was developed by Kempf and Hoffmann a few years ago (5). By this meter the average wake strength is measured in a number of narrow annular zones making it possible thereby to evaluate the average radial wake distribution over the propeller disc. Knowledge of the latter usually suffices for the propeller designer since, in any case, peripheral variation of wake cannot be compensated for, while it is possible to allow for radial variation by a suitable distribution of the propeller pitch along the radius. A meter of the Kempf-Hoffmann type designed and constructed at the U.S. Experimental Model Basin is shown in Fig. 16 in the Appendix. In this figure, A is a hub enclosing a contactor whereby an electric circuit is closed every ten revolutions of the wheel. At B are shown the vane-wheels which can be placed interchangeably on the hub. When used on a model the meter is carried in the normal propeller position and successive runs at one or more speeds are made with the different wheels. The wheel revolutions in a given time interval are recorded, whence the water speeds are found by reference to a calibration plot.

The above vane-wheel type of meter was recently criticized by Prohaska and van Lammeren (6) as not giving correct average results in a variable wake flow on account of the inertia effect of the wheels. In place of this meter they proposed to use a different one consisting of a series of circular rings. One ring is placed in position at a time and its resistance measured by a special dynamometer. The resistance, being a function of the speed, serves as a measure of the average wake speed in the region covered by the ring.

For the evaluation of average annular wake from a series of spot readings taken with a pitot tube on the circumference of a circle two methods have been used. In one, the so-called volume integration method, the spot readings are plotted against angular position from an arbitrary origin and a fair curve is drawn through the spots. The area under the curve is measured whence the average water speed through the region is obtained by dividing the circumference of the circle into the area; mathematically this method is expressed by the formula

$$(\overline{1-w_v}) = \frac{\int_0^{2\pi} (1-w)\, d\theta}{2\pi} \tag{26}$$

In the second method - the so-called impulse integration method - the spot readings are plotted as above and in addition the squares of the readings are also plotted. Fair curves are then drawn and the areas under the curves are measured. Dividing the area under the curve of squares by that under the curve of first powers yields the average value sought; mathematically this method is expressed by the formula

$$(\overline{1-w_i}) = \frac{\int_0^{2\pi} (1-w)^2\, d\theta}{\int_0^{2\pi} (1-w)\, d\theta} \; * \tag{27}$$

As already mentioned, the various methods discussed in general yield different results. Prohaska and van Lammeren showed (see Appendix) that the true mean wake (w_m) obtained by an ideal wake meter having only one vane of no mass is connected

*(Note: It will be noticed that in the first method the mean speed is obtained by a simple averaging process while in the second method it is obtained by averaging the moments of the individual masses and dividing by the total mass).

with the wake values obtained by pitot-tube measurements by the formula

$$(\overline{1-w_m}) = \sqrt{(\overline{1-w_v})(\overline{1-w_i})} \tag{28}$$

where $(\overline{1 - w_v})$ and $(\overline{1 - w_i})$ are the values obtained from Eq. (26) and (27). From direct measurements these same investigators concluded that the results obtained with an actual vane-wheel meter usually lie between $(\overline{1 - w_i})$ and $(\overline{1 - w_m})$.

A third method of measuring wake is that introduced by R. E. Froude many years ago, the basis of which is the determination of real propeller slip from the open-water characteristic curves of the propeller. This method is as follows:

Assuming that we have given the thrust, torque, RPM and model speed from a self-propelled model test and also the open-water characteristic curves plotted in the manner shown in Fig. 5, we calculate the C_T and C_Q coefficients (defined in the figure) from the results of the self-propelled test, then enter the open-water curves with the computed values and read off the real slip ratios. The "effective" wake fraction is then obtained by the equation

$$1-w = \frac{1-S_t}{1-S_a} \tag{29}$$

where S_t is the true or real slip ratio, and S_a is the apparent slip ratio of the propeller.

The difficulty with the above method is the frequent occurrence that the real slip obtained from the C_T curve does not coincide with that obtained from the C_Q curve. This lack of agreement arises from the dissimilitude of the flow conditions behind the model and in open water which was discussed in connection with relative rotative efficiency. As a matter of fact, relative rotative efficiency is symbolic of the lack of uniqueness of the method as seen from Eq. (12) which can also be written in the form:

$$e_r = \frac{C_T C_{Qo}}{C_{To} C_Q} \neq 1 \tag{30}$$

This equation shows that when C_T is chosen equal to C_{To} or, which means the same thing, when the real slip is read from the C_T curve, the relative rotative efficiency is unity only when at the same time $C_Q = C_{Qo}$ and, hence, the slip ratios from C_T and C_Q are equal.

To explain the ambiguity that arises in the determination of real slip ratio from the propeller coefficients, it must be realized that two methods are in use at the present time. In one method - used in most European model basins - the slip ratio is read from the C_T curve while that from the C_Q curve is ignored. In the second method - used in the U. S. Experimental Model Basin - the slip ratio is read from

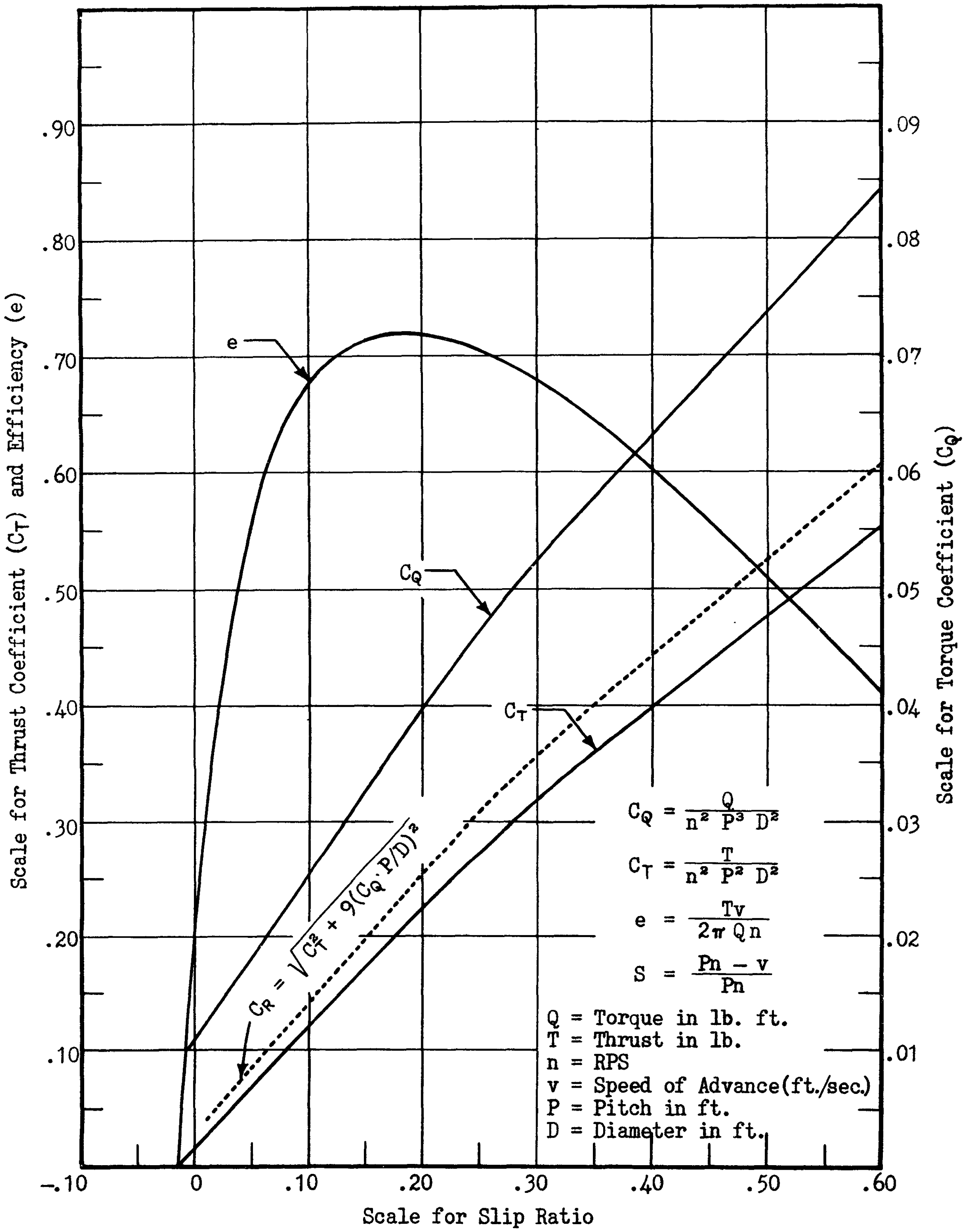

Figure 5 – Characteristic Curves for Propeller of Pitch Ratio (at .7R) = .894

both characteristic curves and the average is formed. Other methods variously proposed but not in general use are: Evaluation of slip ratio from the C_Q coefficient alone, evaluation from a C_T/C_Q curve, evaluation from a slope-curve of the coefficients and, finally, evaluation from the resultant force coefficient C_R which is defined as follows

$$C_R = \sqrt{C_T^2 + k(C_Q a)^2} = \frac{F_r}{\rho n^2 P^2 D^2} \qquad (31)$$

where C_T, C_Q are the thrust and torque coefficients defined in Fig. 5,
a is pitch ratio
k is a constant that depends on the location of the center of pressure on the propeller blades and usually has a value between 8.2 and 9.

Advocates of the foregoing methods base their claims on a great variety of reasons. Thus, the C_T-identity method is based on the contention that the thrust is most directly related to the hull resistance and is the force with which we are mainly concerned. Proponents of the C_Q-identity method take the opposite stand, claiming that power input is the cause and thrust the effect. It was also claimed by Telfer (4) and Horn (3) that torque was less influenced than thrust by extraneous conditions, such as roughness of the propeller blades, which claim is not substantiated, however, by experimental results. The average method used at the U. S. Experimental Model Basin is based on the recognition that neither the C_T nor the C_Q -identity method is supported by a compelling argument and the assumption that both torque and thrust are affected in nearly the same degree by dissimilarity of the flow conditions. The C_T/C_Q-identity method, a recent proposal, is easily shown to be wrong in principle inasmuch as it presupposes that the relative rotative efficiency is unity under all conditions, which not only contradicts the conclusions from the laws of similitude but also does not agree with experimental facts. The slope method was proposed by Telfer (4) who claimed as one of its advantages that the wake thus determined was free from propeller scale-effect influences. Substantiation of this claim is lacking and up to the present the method has found little favor for evaluating wake. The last mentioned method, namely, to evaluate wake from the resultant-force coefficient was proposed by Schoenherr (7) on the basis of the following argument: The forces acting on a propeller blade originate from pressure differences and viscous drag on the blade surfaces. The primary forces are therefore, lift (L) and drag (D) which together form the resultant force (F_r). Thrust and torque forces in turn are components of this resultant force and maintain a constant relation to each other and to the resultant force only for a change in magnitude of the latter, but not for a change in direction.

To put the implications of this argument on a definite basis let us again consider two propellers, - Propeller A working in a uniform wake and Propeller B in a non-uniform wake, and assume the non-uniform wake distribution to be such that the intensity is greater than the average near the propeller hub and less than the average near the propeller blade tips. With this assumed wake distribution the blade sections of Propeller B near the hub work at greater slip angles and those near the tips at smaller slip angles than the corresponding sections of Propeller A. We know from tests with airfoils and waterfoils that the lift coefficient $C_L = L/\rho A v_0^2$ is a linear function of the angle of attack over quite a long range; moreover, that for airfoils of different thickness-to-chord ratios the lift-coefficient curves in the linear portions are practically parallel to each other (see Ref. 7). Thus the difference in lift of corresponding blade sections of Propellers A and B may be expressed with good approximation by the equation

$$\Delta C_L = m \Delta \alpha \qquad (32)$$

where m is the slope of the lift-coefficient curve, and α is the angle of attack.

Similarly, when m is nearly the same for the various blade sections, we have with good approximation for the whole propeller blade

$$\sum \Delta C_L = m \sum \Delta \alpha \qquad (33)$$

The lift coefficient C_L can now be replaced by the previously defined resultant-force coefficient C_R, since for airfoils of normal shape and at small angles of attack the lift is always much greater than the drag. Hence Eq. (33) expresses a linear relation between the change in average propeller slip caused by the non-uniformity of water flow and the corresponding change in the resultant-force coefficient. Such a linear relation must exist if the previously discussed Froude method of evaluating wake is to give accurate results. It is important now to note that, if the relationship between $\sum \Delta C_R$ and $\sum \Delta \alpha$ is linear, this is not the case for the corresponding changes in thrust and torque coefficients, since the following relations hold, as shown by Fig. 6:

$$\left.\begin{aligned} \sum \Delta C_r &= \sum \Delta C_T \sec\beta = m \sum \Delta \alpha \\ \sum \Delta C_r &= \sum \Delta C_Q \csc\beta = m \sum \Delta \alpha \end{aligned}\right\} \qquad (34)$$

whence it follows that neither $\sum \Delta C_T$ nor $\sum \Delta C_Q$ are proportional to $\sum \Delta \alpha$.

Summing up the above discussion it seems that the last-mentioned method of evaluating wake in a non-uniform flow has a more rational foundation than the others. Whether it will yield more consistent results in practice than the method in use now and hence should supersede these methods, remains to be determined by future investigations.

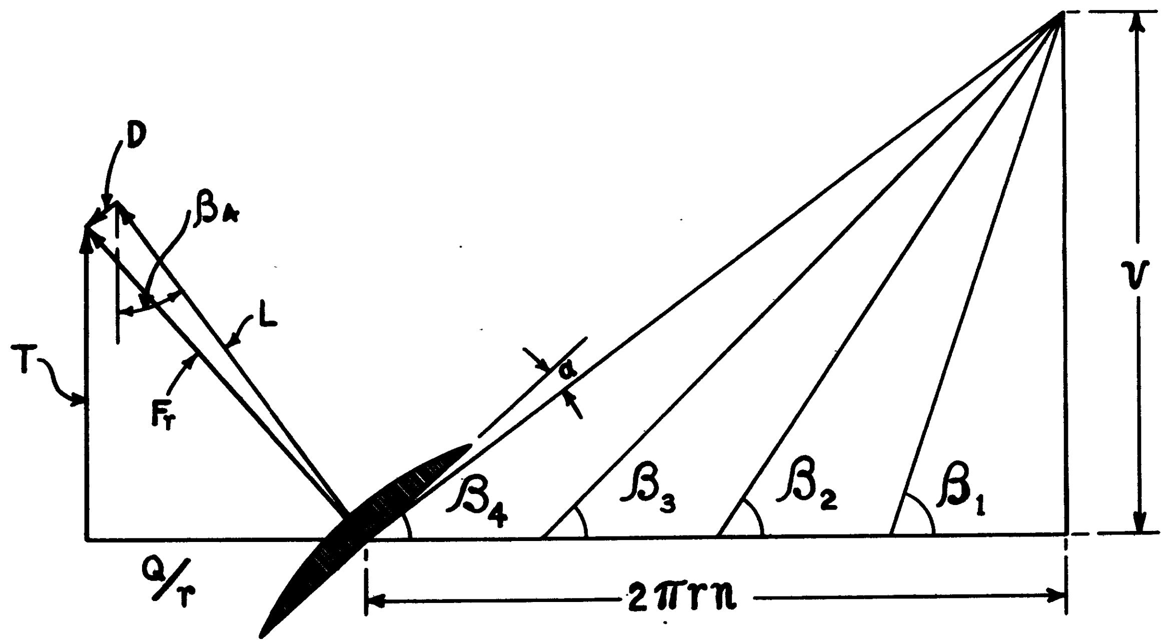

Figure 6

2. HELMBOLD'S METHOD OF SEPARATING STREAMLINE FROM FRICTION WAKE

As already mentioned, Helmbold devised both an approximate and an exact method whereby the streamline wake can be separated from the friction wake. The exact method is based on the following considerations (8). Let the velocity of the model be v_o and the pressure in the undisturbed medium be p_o; then the total pressure ($\overline{p_o}$) measured by a pitot tube at some distance from the model is equal to

$$\overline{p_o} = \frac{\rho}{2} v_o^2 + p_o \qquad (35)$$

where ρ is the density of the medium.

If the assumption is now made that the fluid is frictionless, then, in accordance with Fresenius' argument, no energy loss occurs, so that when p denotes the static pressure in the stern wake and v_p the wake velocity, we have

$$p_o + \frac{\rho}{2} v_o^2 = \frac{\rho}{2} (v_o - v_p)^2 + p \qquad (36)$$

From this equation we find, when the abbreviation $\Delta p = p - p_o$ is introduced, the following expression:

$$v_p = v_o - \sqrt{v_o^2 - \frac{2}{\rho}\Delta p} \tag{37}$$

Considering now the actual medium we have an energy loss due to viscosity which must appear as the difference in the total pressure measured by a pitot tube in open water and behind the model. Denoting this pressure difference by $\Delta\bar{p}$, and the velocity of the friction wake by v_f, we have therefore

$$\frac{\rho}{2}v_o^2 + p_o = \frac{\rho}{2}(v_o - v_p - v_f)^2 + p + \Delta\bar{p} \tag{38}$$

whence we find

$$v_p + v_f = v_o - \sqrt{v_o^2 - \frac{2}{\rho}(\Delta p + \Delta\bar{p})} \tag{39}$$

All the quantities appearing on the right-hand side of Eq. (37) and (39) are known, so that v_p and v_f, and consequently also the wake fractions w_p and w_f, can be calculated.

PART II

SECTION I. - EXPERIMENTAL RESULTS OBTAINED BY DIFFERENT METHODS OF MEASURING WAKE

To illustrate the differences obtained by the various methods of measuring wake which were discussed in PART I, the results obtained on two single-screw merchant ship models are shown in Fig. 7. In this figure the curves at the left pertain to a model with a very full stern while those at the right pertain to a model with a medium fine stern. It is seen that when the spot readings taken with a pitot tube are integrated according to Eq. 26 the results are in general higher than when the integration is carried out according to Eq. 27. It is also seen that the measurements by vane-wheel meter lie in general between the pitot-tube measurements, but not uniformly so.

Comparison of the "nominal" wakes obtained by pitot tube or vane wheel with the "effective" wakes obtained by aid of the propeller coefficients shows that for the model with the fine stern the average nominal wake agrees fairly well with the effective propeller wake, but that for the model with the very full stern the average "nominal" value is much higher than the "effective" wake value. This must mean that the suction of the propeller reduces the dead-water area behind the stern by shifting the point where the boundary layer separates from the hull further aft along the hull.

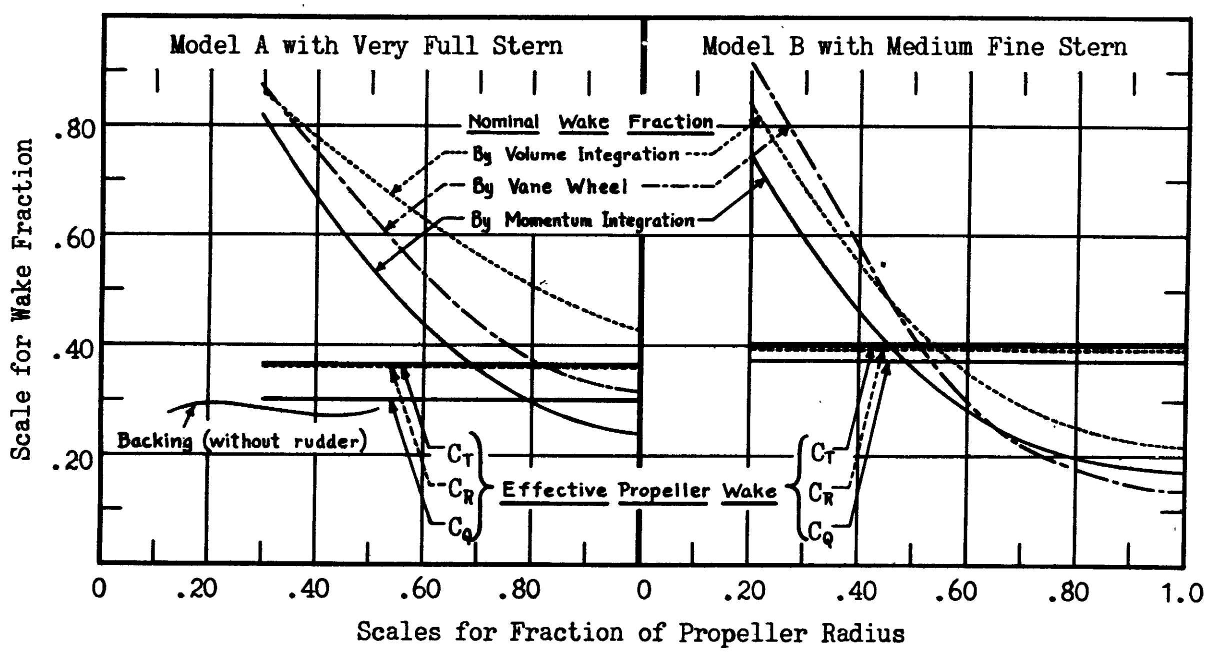

Figure 7 – Comparison Between Nominal and Effective Wakes for Two Single-screw Models

SECTION II – EXPERIMENTAL VALUES OF WAKE AND THRUST DEDUCTION DEDUCED FROM SELF-PROPELLED MODEL TESTS

As stated in the introduction, the principal purpose of this report is the presentation of data derived from self-propelled model experiments whereby the wake and thrust deduction coefficients can be estimated for new ships of average form. The work of deducing such data by statistical methods from routine tests carried out at the U. S. Experimental Model Basin over a period of years was begun by the senior author some ten years ago. Preliminary results of this work were published in chart form in 1934 (7). Subsequently the work was continued and, in fact, is still in progress. Up to the present time, data from 66 tests with 62 single-screw models and from 61 tests with 53 twin-screw models have been tabulated and analyzed. The main dimensions and hull coefficients of the models used in the present analysis, together with the experimental wake and thrust deduction values, are given in Tables 1A and 1B for the single-screw models and in Tables 2A, 2B, 3A, 3B, 4A, and 4B for the twin-screw models. All of the models were of normal merchant-ship form operating at speed-length ratios below unity, this type being the only one considered in the present analysis.

TABLE 1 A

HULL CHARACTERISTICS OF SINGLE-SCREW SHIPS

MODEL NO.	L	B	H	b	m	p	$\frac{D}{B}$	$\frac{E}{H}$	TYPE OF RUDDER
1	345	46	20	.534	.861	.731	.375	.487	PLATE
2	580	59.4	20	.877	.999	.912	.253	.420	FAIR FORM
3	407	55.6	25	.676	.978	.796	.324	.410	PLATE
4	190	36	13.5	.626	.832	.834	.271	.592	PLATE
5	410.5	53.7	24.2	.797	.990	.881	.326	.406	PLATE
6	427	60	25	.652	.979	.793	.317	.427	CONTRA
7	335	44.5	16	.569	.863	.752	.404	.547	FAIR FORM
8	416	53.5	29.54	.776	.984	.869	.341	.351	FAIR FORM
9	200	39	15.06	.604	.955	.721	.256	.398	PLATE
10	480	65.75	26.58	.772	.980	.842	.291	.433	FAIR FORM
11	402	54	24	.766	.981	.872	.301	.417	PLATE
12	143	31.5	10.5	.562	.871	.768	.281	.417	PLATE
13	350	50	23.75	.785	.996	.871	.340	.400	STREAMLINE
14	204	38	14	.627	.934	.785	.289	.464	FAIR FORM
15A	450	61.5	24	.692	.987	.755	.297	.428	CONTRA
15B	450	61.5	24	.692	.987	.755	.297	.428	STREAMLINE
16	450	61.5	24	.690	.979	.762	.297	.428	CONTRA
17	390	54	24.48	.772	.995	.861	.315	.367	FAIR FORM
18	430	60.15	25	.804	.985	.870	.289	.424	FAIR FORM
19	401	54	24	.769	.985	.845	.315	.396	PLATE
20	537.5	70	28.5	.746	.983	.860	.286	.420	STREAMLINE
21	188	35.5	13	.494	.879	.702	.274	.447	FAIR FORM
22	441	62	20	.768	.980	.850	.290	.500	CONTRA
23A	480	65.75	27.33	.767	.979	.856	.297	.444	STREAMLINE
23B	480	65.75	27.33	.767	.979	.856	.297	.444	STREAMLINE
24	486.2	56	24	.729	.982	.754	.357	.469	FAIR FORM
25	390	54	24.48	.772	.993	.863	.315	.367	FAIR FORM

TABLE IB

WAKE FRACTION, THRUST DEDUCTION, AND RELATIVE ROTATIVE EFFICIENCY FOR SINGLE-SCREW SHIPS

EXP. w	FORMULA w	Δw	EXP. t	FORMULA t	Δt	EXP. e_{TT}	MODEL NO.
.240	.205	.035	.240	.240	.000	1.006	1
.390	.444	−.054	.270	.331	−.061	1.039	2
.320	.241	.079	.325	.320	.005	1.028	3
.340	.371	−.031	.370	.340	.030	.937	4
.290	.318	−.028	.306	.290	.016	.975	5
.300	.245	.055	.165	.180	−.015	.984	6
.290	.228	.062	.300	.247	.053	1.030	7
.290	.271	.019	.290	.247	.043	.980	8
.324	.277	.047	.352	.324	.028	.985	9
.348	.386	−.038	.300	.296	.004	1.009	10
.336	.330	.006	.275	.336	−.061	.990	11
.290	.252	.038	.290	.290	.000	.982	12
.301	.313	−.012	.220	.196	.024	1.030	13
.305	.295	.010	.349	.259	.090	1.028	14
.345	.323	.022	.142	.207	−.065	1.031	15A
.322	.323	−.001	.160	.209	−.049	1.030	15B
.331	.314	.017	.155	.198	−.043	.969	16
.262	.268	−.006	.231	.223	.008	1.008	17
.415	.394	.021	.275	.353	−.078	.981	18
.339	.315	.024	.265	.339	−.074	.932	19
.271	.296	−.025	.220	.176	.044	1.080	20
.120	.209	−.089	.188	.102	.086	.991	21
.365	.386	−.021	.220	.219	.001	1.028	22
.352	.339	.013	.180	.229	−.049	1.051	23A
.336	.337	−.001	.160	.218	−.058	1.045	23B
.320	.367	−.047	.186	.272	−.086	1.003	24
.232	.263	−.031	.225	.197	.028	1.115	25

TABLE 1A
HULL CHARACTERISTICS OF SINGLE-SCREW SHIPS

MODEL NO.	L	B	H	b	m	p	D/B	E/H	TYPE OF RUDDER
26	390	54	24.5	.758	.993	.863	.315	.367	FAIR FORM
27	379	57.5	23.67	.650	.973	.732	.307	.433	STREAMLINE
28A	380	53	23.67	.769	.975	.854	.328	.433	FAIR FORM
28B	380	53	23.67	.769	.975	.854	.328	.433	STREAMLINE
29	455	63.5	22	.757	.984	.830	.280	.409	FAIR FORM
30	448	58	26.9	.642	.998	.751	.315	.372	FAIR FORM
31	390	54	24	.774	.998	.859	.283	.417	FAIR FORM
32	496.25	65.75	29	.749	.991	.833	.274	.397	CONTRA
33	290	48	16	.369	.666	.659	.286	.281	FAIR FORM
34A	480	72.83	27.67	.712	.896	.844	.264	.397	STREAMLINE
34B	480	72.83	27.67	.712	.896	.844	.264	.397	CONTRA
35	510	77	28.5	.706	.925	.819	.260	.421	CONTRA
36	450	60	24	.621	.976	.749	.286	.396	FAIR FORM
37	404	53.75	24	.746	.984	.825	.335	.427	CONTRA
38	435	66.5	27	.721	.928	.833	.259	.377	CONTRA
39	485.17	60	22	.715	.977	.788	.339	.512	STREAMLINE
40	500	70	29	.750	.990	.811	.278	.396	CONTRA
41	500	71	25	.651	.953	.759	.268	.400	STREAMLINE
42	425	64	27.5	.722	.930	.833	.278	.381	STREAMLINE
43	508	70	29.5	.797	.981	.867	.279	.407	CONTRA
44	470	62	25	.607	.981	.730	.323	.467	STREAMLINE
45	523	65.75	29.5	.758	.988	.842	.274	.390	STREAMLINE
46	470	66	25	.571	.920	.702	.303	.467	STREAMLINE
47	452	65	28.33	.718	.977	.816	.273	.397	STREAMLINE
48	452	65	28.33	.718	.942	.827	.273	.397	STREAMLINE
49	441	66.5	27.92	.716	.929	.829	.259	.370	CONTRA
50	479	65	27.75	.724	.989	.812	.277	.406	STREAMLINE

TABLE 1B

WAKE FRACTION, THRUST DEDUCTION, AND RELATIVE ROTATIVE EFFICIENCY FOR SINGLE-SCREW SHIPS

EXP. w	FORMULA w	Δw	EXP. t	FORMULA t	Δt	EXP. e_{rr}	MODEL NO.
.289	.277	.012	.245	.246	−.001	1.050	26
.270	.282	−.012	.200	.176	.024	.955	27
.400	.356	.044	.290	.340	−.050	1.033	28A
.310	.356	−.046	.180	.201	−.021	1.027	28B
.264	.339	−.075	.185	.224	−.039	1.013	29
.176	.205	−.029	.184	.150	.034	1.080	30
.324	.313	.011	.235	.275	−.040	1.025	31
.290	.306	−.016	.175	.174	.001	1.139	32
.154	.147	.007	.148	.131	.017	1.025	33
.312	.323	−.011	.255	.203	.052	1.125	34A
.346	.323	.023	.252	.208	.044	1.103	34B
.344	.340	.004	.163	.206	−.043	1.044	35
.222	.249	−.027	.160	.188	−.028	.983	36
.280	.310	−.030	.215	.168	.047	1.077	37
.358	.329	.029	.200	.215	−.015	1.068	38
.400	.349	.051	.198	.260	−.062	1.073	39
.310	.355	−.045	.165	.186	−.021	1.054	40
.266	.294	−.028	.166	.173	−.007	1.058	41
.350	.303	.047	.250	.228	.022	1.040	42
.371	.377	−.006	.180	.223	−.043	1.037	43
.243	.244	−.001	.180	.158	.022	1.002	44
.294	.299	−.005	.190	.191	−.001	1.034	45
.238	.254	−.016	.180	.155	.025	1.008	46
.328	.289	.039	.210	.213	−.003	1.005	47
.342	.291	.051	.205	.222	−.017	1.042	48
.377	.320	.057	.207	.226	−.019	1.028	49
.335	.298	.037	.240	.218	.022	1.120	50

TABLE 1A

HULL CHARACTERISTICS OF SINGLE-SCREW SHIPS

ODEL NO.	L	B	H	b	m	p	D/B	E/H	TYPE OF RUDDER
51	580	60	20	.854	.986	.893	.258	.466	CONTRA
52	190	37	14.5	.469	.824	.692	.366	.435	FAIR FORM
53	438.5	63	25.75	.678	.979	.756	.317	.427	CONTRA
54	531.25	70	30.08	.750	.982	.820	.282	.399	FAIR FORM
55	465	64	25.5	.645	.986	.748	.301	.431	CONTRA
56	470	62	25	.612	.980	.749	.323	.467	FAIR FORM
57	105.83	25	9.5	.437	.757	.726	.333	.395	FAIR FORM
58	465	64	25.5	.645	.986	.742	.292	.431	CONTRA
59	450	66	27	.629	.976	.730	.288	.395	CONTRA
60	195	38.5	14.15	.488	.858	.661	.346	.442	FAIR FORM
61	465	69.5	27.25	.654	.980	.779	.308	.422	CONTRA
62	465	65.5	25.5	.635	.987	.740	.294	.431	CONTRA

TABLE 1B

WAKE FRACTION, THRUST DEDUCTION, AND RELATIVE ROTATIVE EFFICIENCY FOR SINGLE-SCREW SHIPS

EXP. w	FORMULA w	Δw	EXP. t	FORMULA t	Δt	EXP. e_{rr}	MODEL NO.
.408	.439	−.031	.260	.245	.015	1.078	51
.240	.234	.006	.214	.204	.010	1.027	52
.295	.315	−.020	.176	.177	−.001	1.065	53
.353	.322	−.031	.276	.300	−.024	1.052	54
.313	.269	.044	.220	.188	.032	1.077	55
.243	.240	.003	.200	.206	−.006	1.044	56
.198	.193	.005	.205	.169	.036	1.010	57
.275	.278	−.003	.198	.165	.033	1.025	58
.258	.260	−.002	.160	.155	.005	1.088	59
.249	.230	.019	.260	.212	.048	1.037	60
.273	.271	.002	.190	.164	.026	1.026	61
.270	.287	− 017	.185	.172	.013	1.000	62

TABLE 2A
HULL CHARACTERISTICS OF TWIN-SCREW SHIPS WITH BOSSINGS——PROPELLERS TURNING OUTBOARD

MODEL NO.	L	B	H	b	m	p	$\frac{D}{B}$	$\frac{E}{H}$
1	599.4	68.5	32.9	.754	.949	.851	.255	.300
2	577	83	26	.551	.977	.704	.229	.385
3	472.25	65	27	.784	.985	.869	.215	.358
4	469	57	31.85	.799	.973	.858	.243	.220
5	590	80	30	.697	.983	.774	.231	.338
6A	408	62	19	.568	.965	.724	.214	.386
6B	408	62	19	.568	.965	.724	.214	.386
7	260	41.5	16	.557	.939	.720	.261	.387
8	522	70	29.5	.780	.986	.869	.229	.362
9	310	48	16.75	.580	.950	.706	.229	.403
10	680	72	32.58	.642	.964	.764	.276	.374
11	625	77.5	26.5	.647	.988	.749	.232	.357
12	450	61.5	26	.727	.986	.821	.256	.354
13	607	80	32	.649	.981	.769	.225	.320
14	470	63	30	.684	.977	.784	.278	.318
15	630	81	30.25	.657	.986	.760	.228	.347
16	498.9	63.75	25.39	.632	.965	.764	.259	.368
17	517	65	28	.672	.974	.764	.258	.267
18	685	86	29	.641	.973	.744	.221	.362
19A	430	60	24.19	.612	.979	.735	.262	.393
19B	430	60	24.19	.612	.979	.735	.262	.393
19C	430	60	24.19	.612	.979	.735	.262	.393
20	500	70	25	.608	.977	.735	.236	.370
21	494.75	69.66	26	.615	.963	.717	.273	.369
22	480	63.75	24.81	.637	.965	.750	.239	.352
23A	459	61.5	26	.693	.966	.808	.244	.365
23B	459	61.5	26	.693	.966	.808	.244	.365

TABLE 2B. WAKE FRACTION, THRUST DEDUCTION, AND RELATIVE ROTATIVE EFFICIENCY FOR TWIN-SCREW SHIPS WITH BOSSINGS PROPELLERS TURNING OUTBOARD

BOSS ANGLE WITH HORI. IN DEGREES	EXP. w	FORMULA w	Δw	EXP. t	FORMULA t	Δt	EXP. e_{rr}	MODEL NO.
12.5	.300	.279	.021	.190	.215	−.025	.921	1
52.5	.050	.034	.016	.160	.153	.007	1.015	2
30	.220	.208	.012	.225	.195	.030	1.003	3
8	.280	.302	−.022	.212	.210	.002	1.010	4
30	.184	.180	.004	.216	.186	.030	1.006	5
50	.090	.044	.046	.220	.163	.057	1.018	6A
30	.130	.131	−.001	.165	.172	−.007	.950	6B
51	.080	.039	.041	.123	.160	−.037	.959	7
30	.188	.207	−.019	.244	.187	.057	.984	8
26.5	.110	.153	−.043	.162	.167	−.005	.959	9
30	.140	.158	−.018	.193	.175	.018	1.085	10
34	.127	.139	−.012	.170	.172	−.002	1.000	11
24	.259	.222	.037	.200	.205	−.005	.972	12
27.5	.133	.174	−.041	.155	.173	−.018	.985	13
28.5	.240	.183	.057	.200	.200	.000	1.026	14
35	.090	.138	−.048	.168	.162	.006	1.030	15
30	.144	.154	−.010	.185	.176	.009	.943	16
28.5	.220	.178	.042	.165	.195	−.030	.968	17
30	.152	.158	−.006	.170	.178	−.008	.940	18
30	.124	.147	−.023	.162	.171	−.009	.972	19A
37.5	.091	.109	−.018	.150	.163	−.013	.947	19B
45	.058	.076	−.018	.173	.155	.018	1.065	19C
62.5	.042	.046	−.004	.155	.151	.004	.951	20
37.5	.111	.110	.001	.193	.168	.025	.995	21
18.5	.189	.212	−.023	.189	.187	.002	1.008	22
24	.214	.209	.005	.200	.194	.006	1.016	23A
40	.176	.128	.048	.189	.184	.005	.983	23B

TABLE 2A

HULL CHARACTERISTICS OF TWIN-SCREW SHIPS WITH BOSSINGS —— PROPELLERS TURNING OUTBOARD

MODEL NO.	L	B	H	b	m	p	$\frac{D}{B}$	$\frac{E}{H}$
24A	519	73	29.8	.711	.992	.787	.247	.302
24B	519	73	29.8	.711	.992	.787	.247	.302
24C	519	73	29.8	.711	.992	.787	.247	.302
24D	519	73	29.8	.711	.992	.787	.247	.302
25	520	73	24.16	.646	.984	.744	.211	.352
26	520	74	30	.708	.991	.769	.244	.333
27	569.7	70	25.5	.685	.969	.798	.281	.392
28	486.5	64	26.29	.587	.968	.736	.285	.399
29	540	72	30.12	.685	.980	.772	.236	.398
30	486.5	64	26	.606	.961	.754	.273	.394
31	689	92	32.4	.587	.994	.711	.203	.345

TABLE 2B. WAKE FRACTION, THRUST DEDUCTION, AND RELATIVE ROTATIVE EFFICIENCY FOR TWIN-SCREW SHIPS WITH BOSSINGS PROPELLERS TURNING OUTBOARD

BOSS ANGLE WITH HORI. IN DEGREES	EXP. w	FORMULA w	Δw	EXP. t	FORMULA t	Δt	EXP. e_{rr}	MODEL NO.
13	.270	.263	.007	.200	.207	−.007	1.006	24A
19	.215	.239	−.024	.163	.194	−.031	.955	24B
26	.201	.206	−.005	.179	.190	−.011	.959	24C
32.5	.190	.172	.018	.190	.188	.002	.962	24D
33.5	.180	.142	.038	.142	.185	−.043	.919	25
32	.178	.174	.004	.157	.184	−.027	.976	26
24	.185	.206	−.021	.221	.186	.035	.993	27
16.25	.160	.204	−.044	.160	.180	−.020	.962	28
15	.219	.245	−.026	.210	.195	.015	1.035	29
32	.110	.134	−.024	.170	.168	.002	.980	30
37.5	.120	.100	.020	.152	.170	−.018	.989	31

TABLE 3A
HULL CHARACTERISTICS OF TWIN-SCREW SHIPS WITH BOSSINGS —— PROPELLERS TURNING INBOARD

MODEL NO.	L	B	H	b	m	p	$\frac{D}{B}$	$\frac{E}{H}$
1	469	57	31.85	.799	.973	.858	.243	.220
2A	408	62	19	.568	.965	.724	.214	.386
2B	408	62	19	.568	.965	.724	.214	.386
3	500	70	25	.608	.977	.735	.236	.370
4	500	72	25.4	.614	.971	.742	.236	.387
5	500	72	25.2	.621	.979	.739	.243	.367
6	540	72	30.12	.685	.980	.772	.236	.398

TABLE 3B. WAKE FRACTION, THRUST DEDUCTION, AND RELATIVE ROTATIVE EFFICIENCY FOR TWIN—SCREW SHIPS WITH BOSSINGS PROPELLERS TURNING INBOARD

BOSS ANGLE WITH HORI. IN DEGREES	EXP. w	FORMULA w	Δw	EXP. t	FORMULA t	Δt	EXP. e_{TT}	MODEL NO.
8	.160	.210	−.050	.182	.180	.002	.976	1
50	.120	.121	−.001	.210	.170	.040	1.003	2A
30	.075	.071	.004	.150	.159	−.009	1.014	2B
62.5	.215	.198	.017	.140	.194	−.054	.932	3
58	.176	.176	.000	.183	.184	−.001	1.037	4
60	.192	.190	.002	.196	.188	.008	1.022	5
15	.135	.144	−.009	.222	.174	.048	1.005	6

TABLE 4A

HULL CHARACTERISTICS OF TWIN—SCREW SHIPS WITH SHAFTS AND STRUTS

MODEL NO.	L	B	H	b	m	p	D/B	E/H
1	510	85.5	27	.594	.976	.694	.214	.358
2	535	72.2	30	.628	.986	.737	.236	.295
3	496	67	28.21	.735	.981	.837	.240	.330
4	408	62	19	.568	.965	.724	.214	.386
5	575	108	28.5	.603			.168	.307
6	730	80	20.58	.459	.818	.668	.175	.377
7	730	80	20.58	.459	.818	.668	.175	.377
8	400.25	61	18	.541	.950	.701	.209	.593
9	274	47.25	21.25	.809	.972		.212	.251
10	308	41	12.4	.503	.890	.720	.215	.362
11	520	73	24.16	.646	.984	.744	.211	.352
12	519	73	29.8	.711	.992	.787	.247	.302
13	300	41.5	11.48	.552	.930	.726	.217	.479
14	300	41	11.5	.567	.969	.701	.220	.478
15	519	73	29.8	.709	.992	.783	.247	.302
16	695	90	29	.572	.966	.695	.216	.379

TABLE 4B
WAKE FRACTION, THRUST DEDUCTION, AND RELATIVE ROTATIVE EFFICIENCY FOR TWIN-SCREW SHIPS WITH SHAFTS AND STRUTS

EXP. w	FORMULA w	Δw	EXP. t	FORMULA t	Δt	EXP. e_{rr}	MODEL NO.
.125	.100	.025	.152	.148	.004	.999	1
.140	.113	.027	.160	.158	.002	1.040	2
.120	.153	−.033	.167	.144	.023	.925	3
.115	.091	.024	.120	.140	−.020	.922	4
.130	.103	.027	.145	.151	−.006	.967	5
.082	.062	.020	.088	.117	−.029	.922	6
.077	.062	.015	.100	.114	−.014	.984	7
.118	.083	.035	.143	.143	.000	.966	8
.148	.172	−.024	.180	.164	.016	.972	9
.030	.072	−.042	.075	.081	−.006	1.043	10
.100	.120	−.020	.120	.130	−.010	.975	11
.142	.145	−.003	.156	.159	−.003	.957	12
.051	.086	−.035	.122	.096	.026	.987	13
.084	.091	−.007	.140	.119	.021	1.006	14
.164	.144	.020	.143	.175	−.032	.938	15
.104	.092	.012	.083	.124	−.041	.932	16

The results of the analysis are presented in the form of empirical equations. In building up these equations the influence of a great many independent variables was investigated, but only those variables were incorporated in the final equations whose influence predominated and which could be presumed to be known in the early design stages of a ship. Furthermore, the formulas were so constructed that the dependent variables remain finite for the limiting values of the independent variables.

The formulas for wake fraction are expressed as functions of various hull and propeller coefficients; the thrust deduction coefficients, on the other hand, are expressed as simple functions of the wake fraction. This procedure is justified theoretically, as previously brought out, and works out well in practice. Separate formulas were worked out for single-screw and twin-screw ships and, in the case of the latter, for ships with inboard- and outboard-turning propellers and for ships equipped with bossings and struts. The formulas are as follows.

A. FOR THE WAKE FRACTION OF SINGLE-SCREW SHIPS

$$w = 0.1 + 4.5 \frac{\bar{v} l \; B/L}{(7 - 6\bar{v})(2.8 - 1.8 l)} + \frac{1}{2}\left(\frac{E}{H} - \frac{D}{B} - k'\eta\right)$$

A.D. = $\pm .027$

where: L is the length of ship (mean immersed)

B is the beam of ship

H is the draft of ship

E is the elevation of propeller shaft above the base-line

D is the propeller diameter

p is the coefficient of area of load water-line plane

b is the block coefficient

l is the b/m = Longitudinal prismatic coefficient

$\bar{v}$ is the b/p = Vertical prismatic coefficient

m is the coefficient of midship section area

η is the rake angle of propeller blades in radians

k' is .3 for a normal type stern and .5 to .6 for a stern with cut-away deadwood

A.D. is the average deviation from the mean for 66 spots.

B. FOR THE THRUST DEDUCTION COEFFICIENT OF SINGLE-SCREW SHIPS

$$t = kw$$

where t is the thrust deduction coefficient

w is the wake fraction

k is the coefficient varying between .50 and .70 for vessels with contra and streamline rudders; between .70 and .90 for vessels equipped with double-plate rudders carried behind posts with rectangular section; and between .90 and 1.05 for vessels equipped with orthodox single-plate rudders. (For ordinary practice the value .60 for contra rudders, .65 for streamline rudders, .85 for double-plate rudders, and 1.00 for single-plate rudders can be used).

C. FOR THE WAKE FRACTION OF TWIN-SCREW VESSELS

a. When equipped with bossings and outboard-turning propellers

$$w = 2b^5(1-b) + 0.2\cos^2\frac{3}{2}\psi - .02$$

A.D. = $\pm$.023 for 38 spots.

b. When equipped with bossings and inboard-turning propellers

$$w = 2b^5(1-b) + 0.2\cos^2\frac{3}{2}(90-\psi) + .02$$

A.D. = $\pm$ .012 for 7 spots.

c. When equipped with propeller struts

$$w = 2b^5(1-b) + .04$$

A.D. = $\pm$ .023 for 16 spots.

where b is the block coefficient.

ψ is the bossing angle with the horizontal in degrees.

D. FOR THE THRUST DEDUCTION COEFFICIENT OF TWIN-SCREW SHIPS

a. When equipped with bossings

$$t = .25w + .14$$

A.D. = $\pm$ .020 for 45 spots.

b. When equipped with propeller struts

$$t = .70w + .06$$

A.D. = $\pm$ .016 for 16 spots.

In the course of the investigation an attempt also was made to correlate the relative rotative efficiency with known hull or propeller characteristics. This effort proved fruitless, however, since no factor or combination of factors tried gave a consistent variation. The averages of the experimental values for the models considered in the analysis worked out to be 1.02 for the single-screw models and .985 for the twin-screw models.

The accuracy of the wake and thrust-deduction formulas is shown graphically in Figs. 8 to 14. It is seen from these figures that the deviation of some individual spots is still quite large although the average deviation lies within reasonable limits. This suggests that the work be continued and the formulas be revised from time to time as more experimental material becomes available. Efforts along these lines will be made as opportunity presents itself.

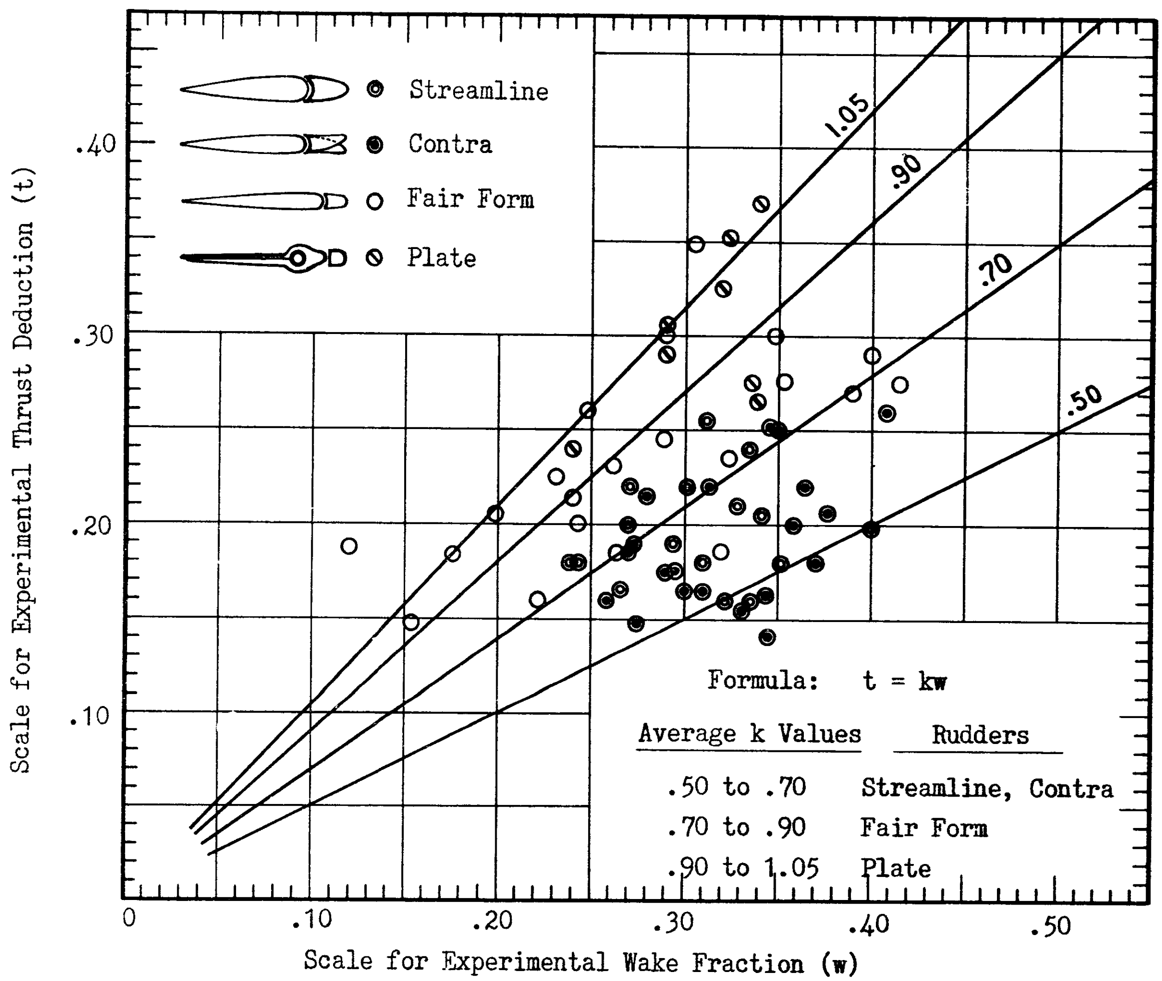

Figure 8 - Thrust-Deduction Coefficient for Single-screw Ships, Expressed as Function of Wake Fraction.

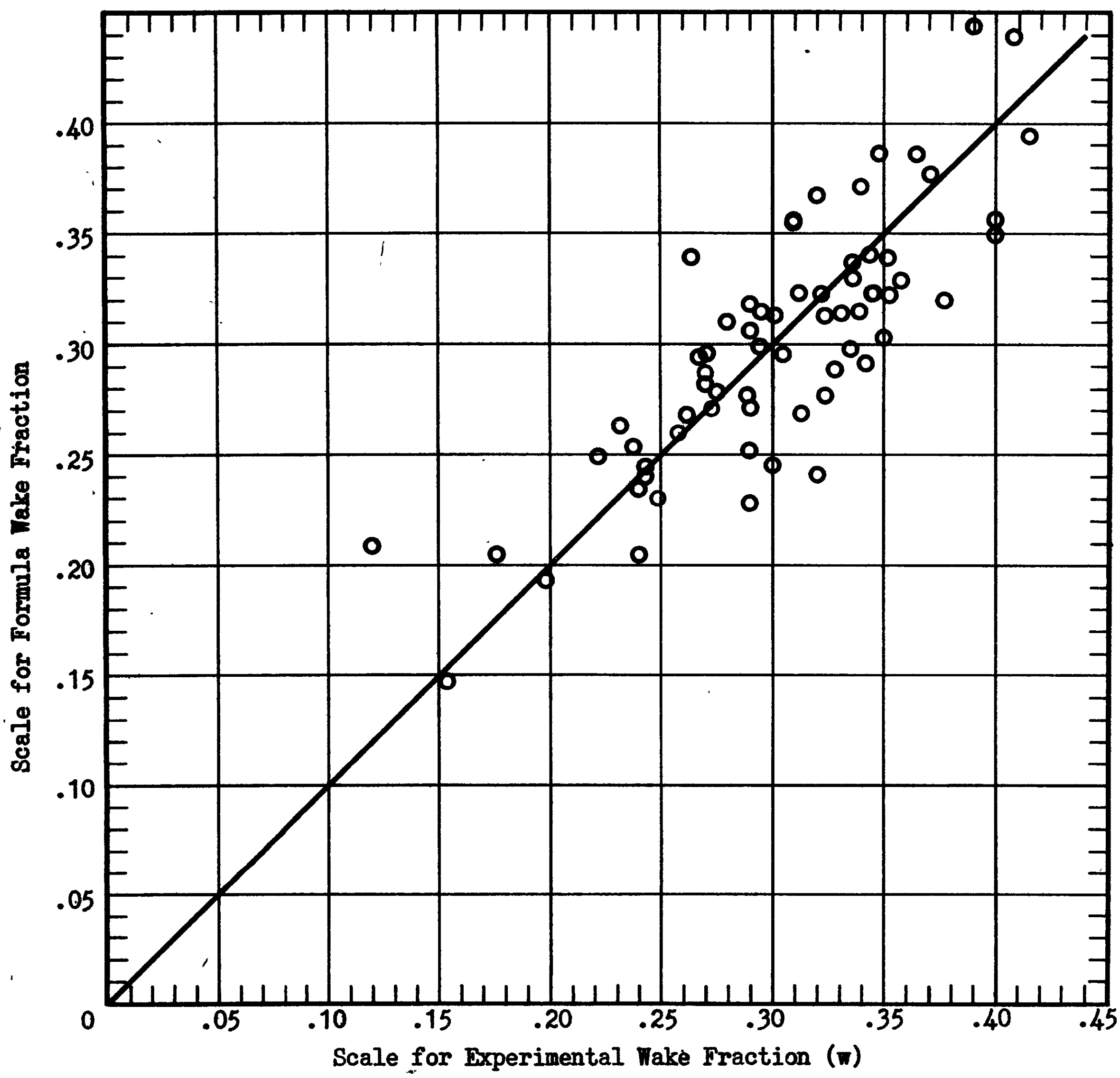

Figure 9 - Comparison of Experimental and Formula Wake Values for Single-screw Ships.

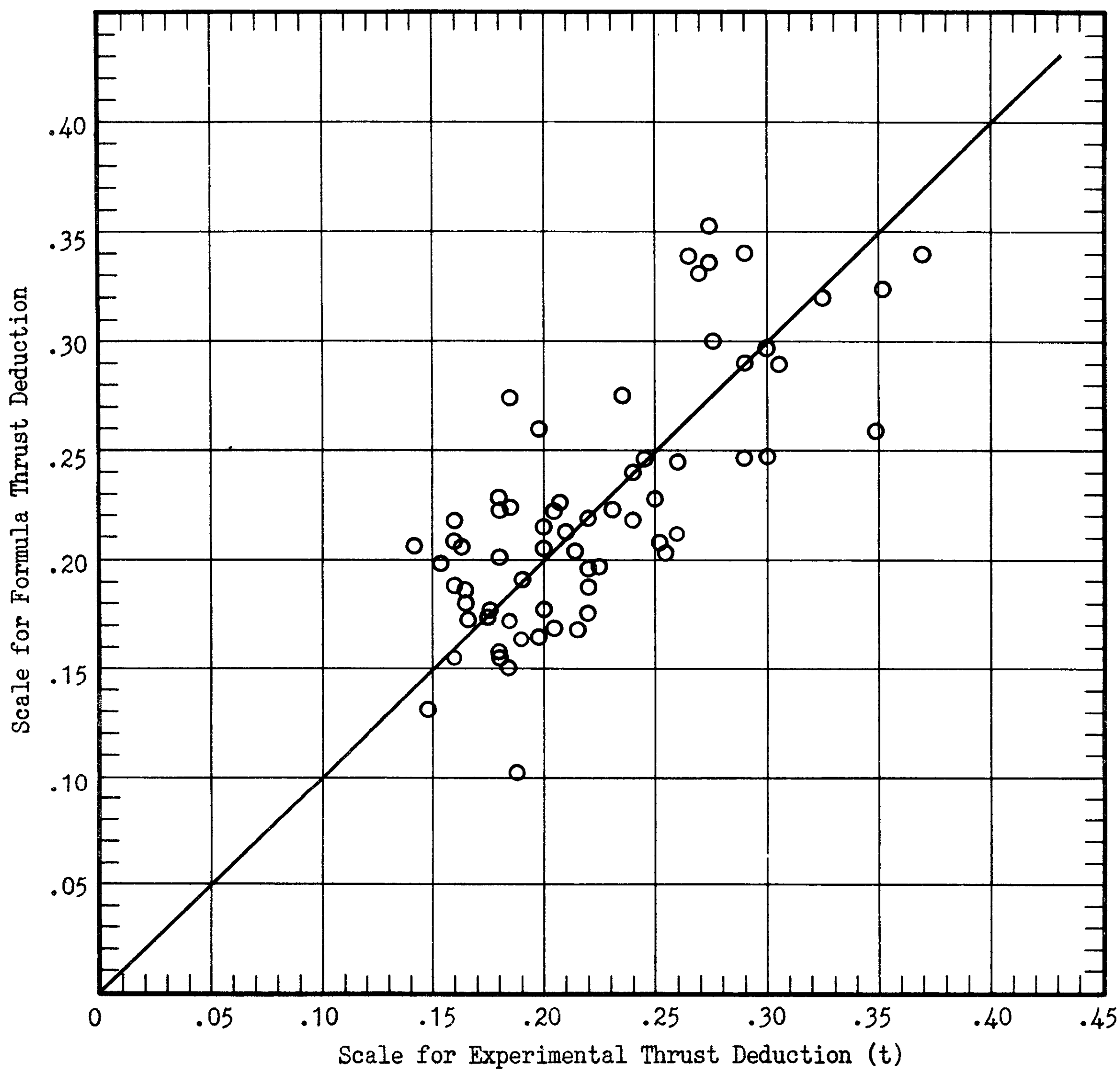

Figure 10 - Comparison of Experimental and Formula Thrust Deduction Values for Single-screw Ships.

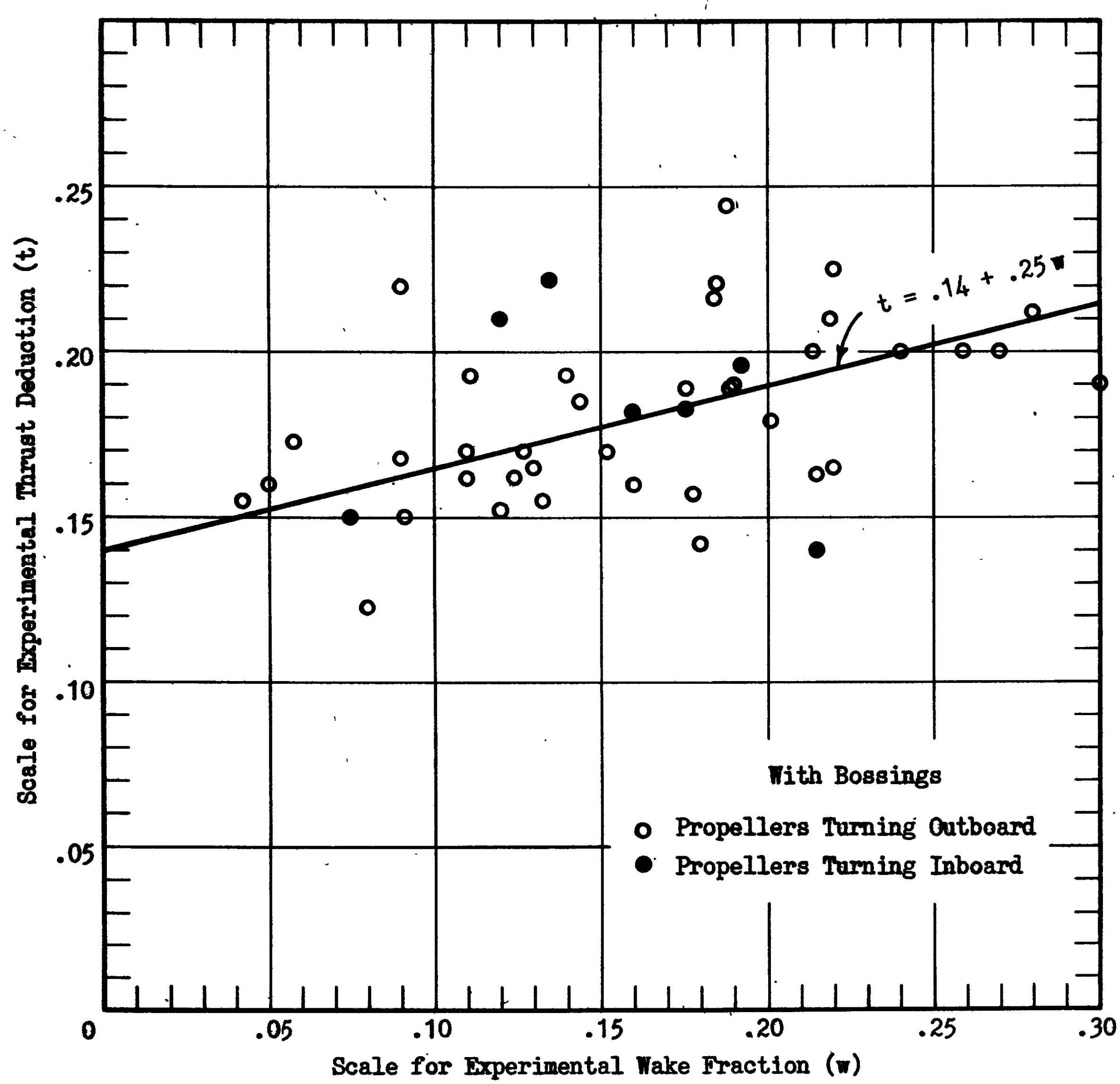

Figure 11 - Thrust Deduction Coefficient for Twin-screw Ships with Bossings, Expressed as Function of Wake Fraction.

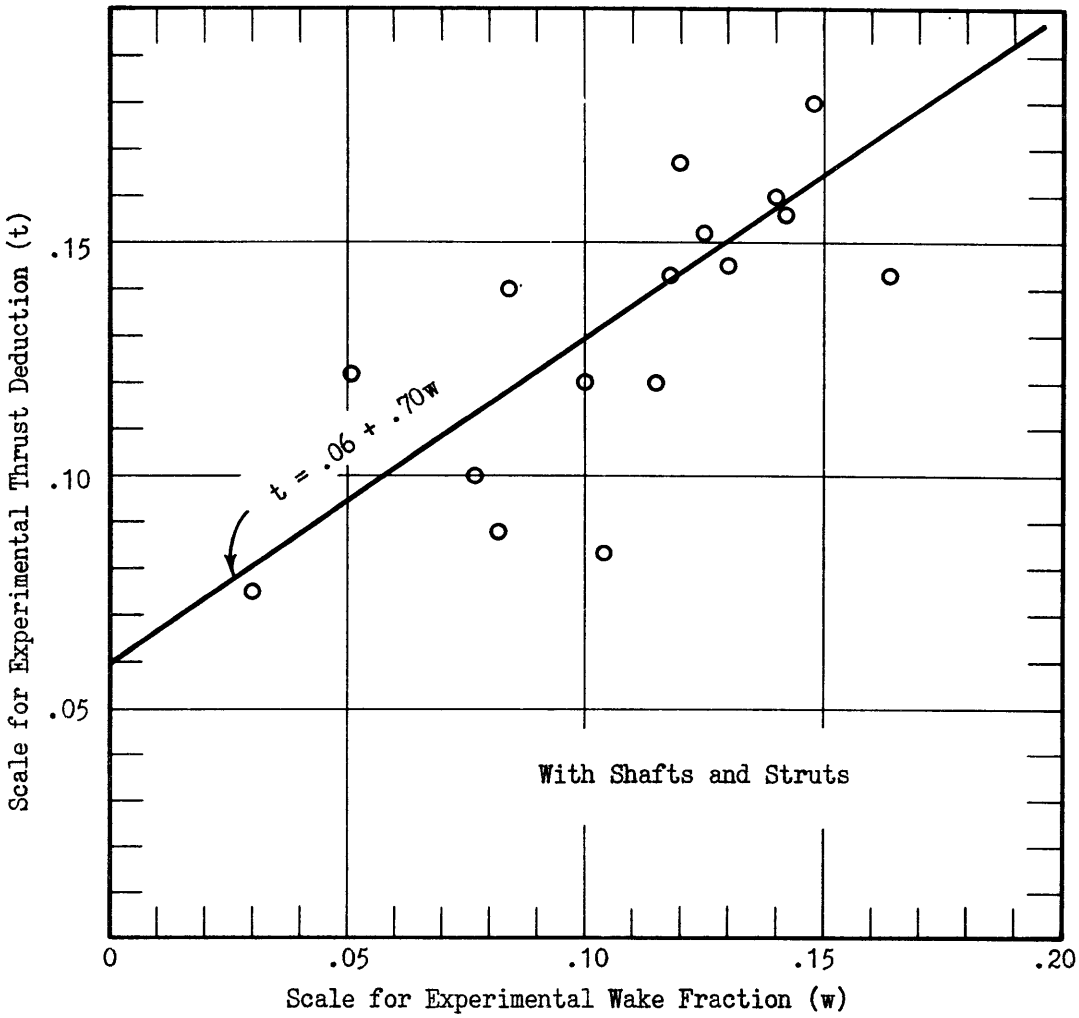

Figure 12 - Thrust Deduction Coefficient for Twin-screw Ships with Shafts and Struts Expressed as Function of Wake Fraction.

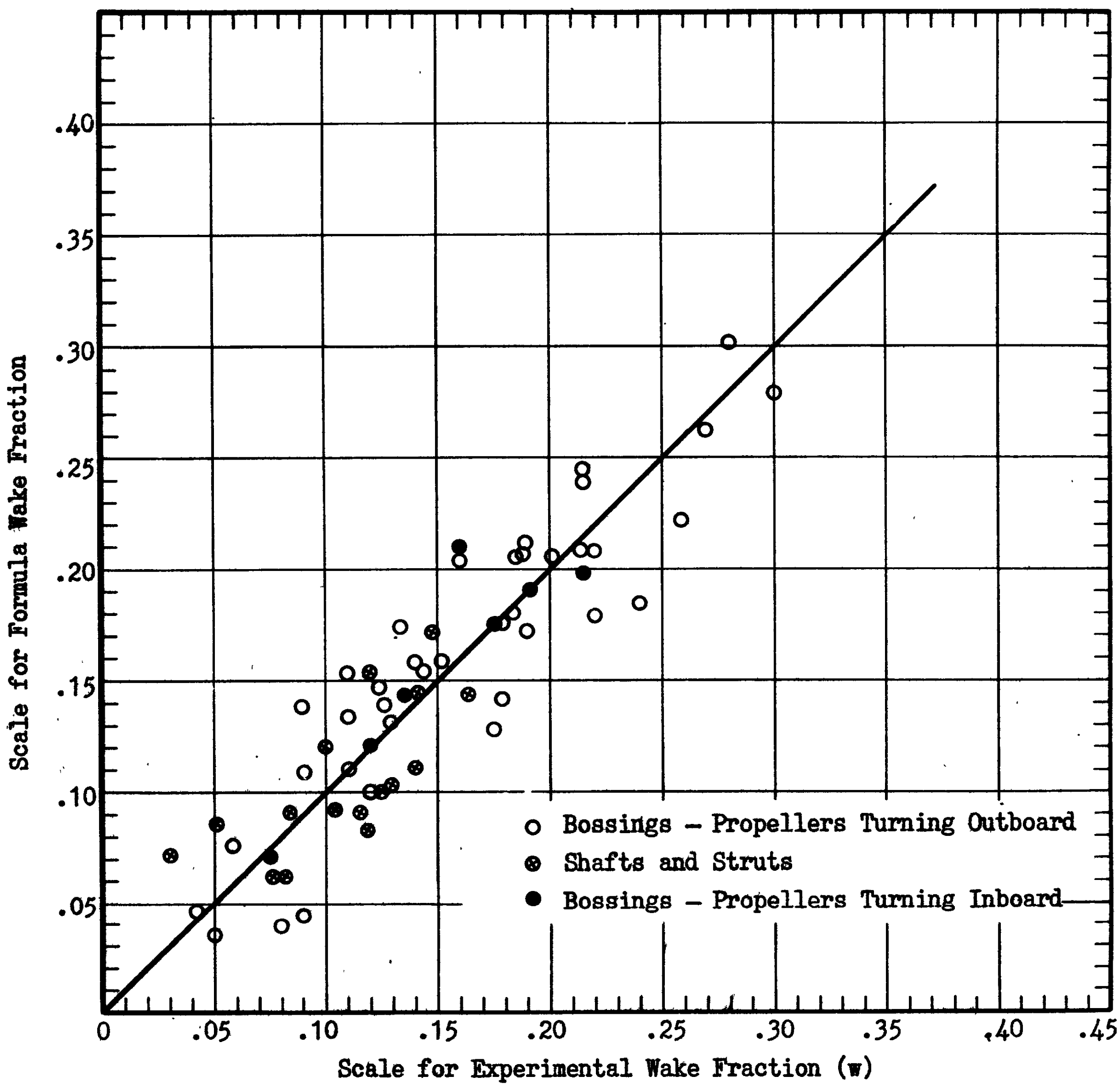

Figure 13 - Comparison of Experimental and Formula Wake Values for Twin-screw Ships.

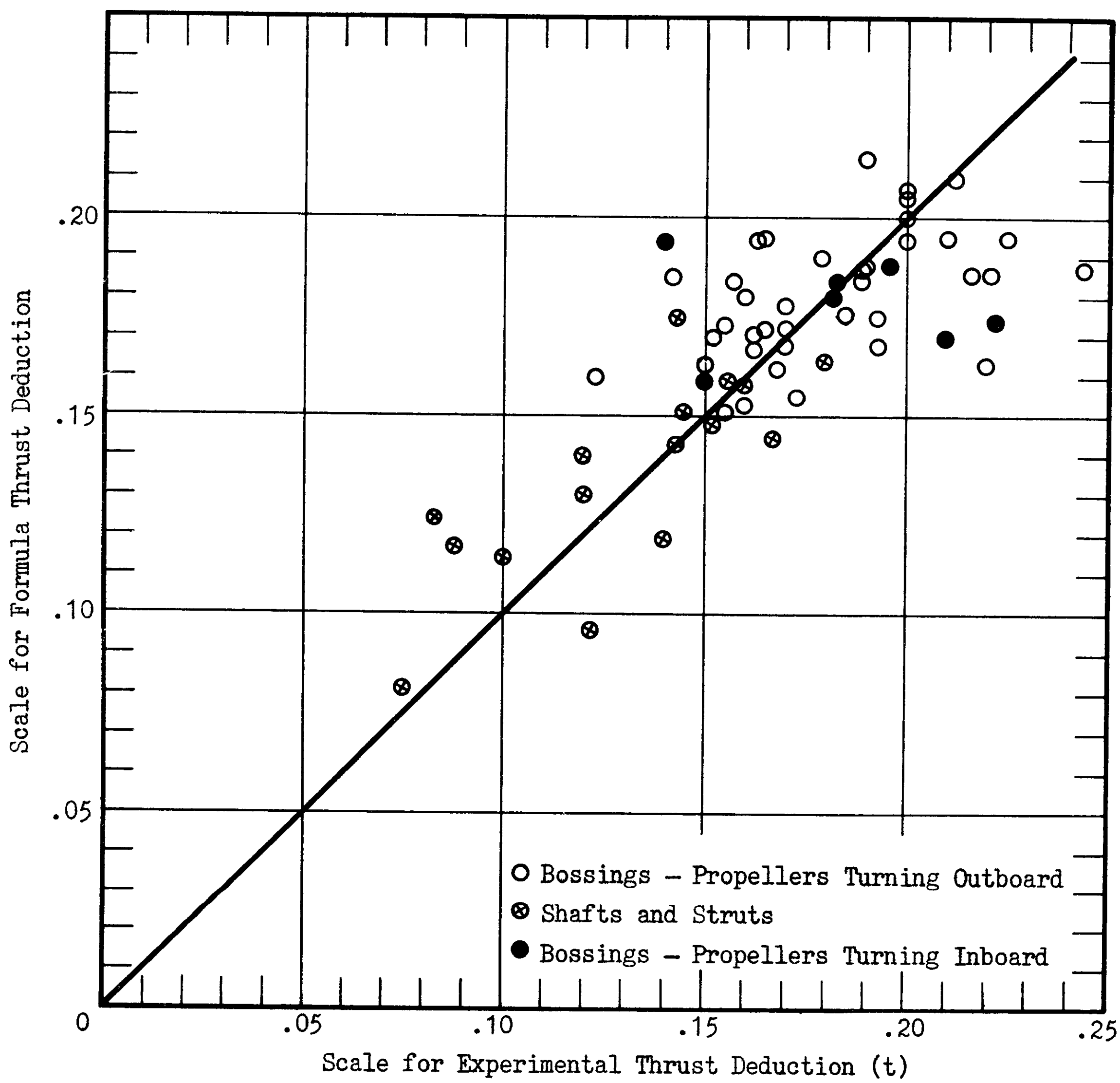

Figure 14 - Comparison of Experimental and Formula Thrust-Deduction Values for Twin-screw Ships.

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APPENDIX

PROHASKA AND VAN LAMMEREN'S ANALYSIS OF THE VANE-WHEEL WAKE METER

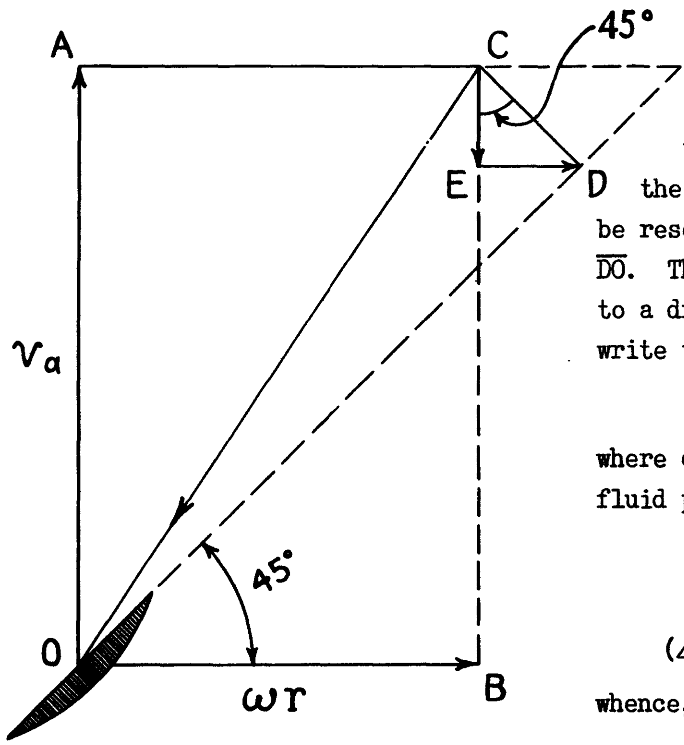

Figure 15

Referring to the sketch, Fig. 15, let v_a be the speed of advance and ωr be the circumferential velocity of the vane; $\overline{CO}$ is then the resultant inflow velocity. This can be resolved into the components $\overline{CE}$, $\overline{ED}$ and $\overline{DO}$. The component $\overline{ED}$ obviously gives rise to a driving force denoted by K. We can write therefore

$$K = c\, v_e \overline{ED} \tag{a}$$

where $c\, v_e$ is proportional to the mass of fluid passing through the meter per second.

From the figure it is seen that

$$\omega r + \overline{ED} = v_a - \overline{ED}$$

($\triangle$ CED being an equilateral triangle)

whence,

$$\overline{ED} = \frac{v_a - \omega r}{2}$$

so that equation (a) becomes

$$K = c_1 v_a (v_a - \omega r) \tag{b}$$

Calibration curves show that in a uniform flow the circumferential speed of the meter is very nearly proportional to the forward speed, that is,

$$\omega r = q\, v_a \tag{c}$$

so that

$$K_0 = c_1 (1 - q) v_a^2 \tag{d}$$

In a non-uniform flow we can now analyze the performance of a vane-wheel from two points of view. First, that we have a single-vane meter which is so light that the fluctuations in wake produce instantaneous and proportional changes in the rotative speed of the meter; second, that the meter is of the multi-vane type and has sufficient inertia to turn at a constant speed, the fluctuations in wake producing fluctuations in slip angle.

In the first case we can write, since by assumption Eq. (c) holds for any part of a revolution,

$$K_m = c_2 (\overline{1 - w_m})^2 v^2 = \frac{c_2 v^2}{2\pi} \int_0^{2\pi} (1 - w)^2 \, d\theta \tag{e}$$

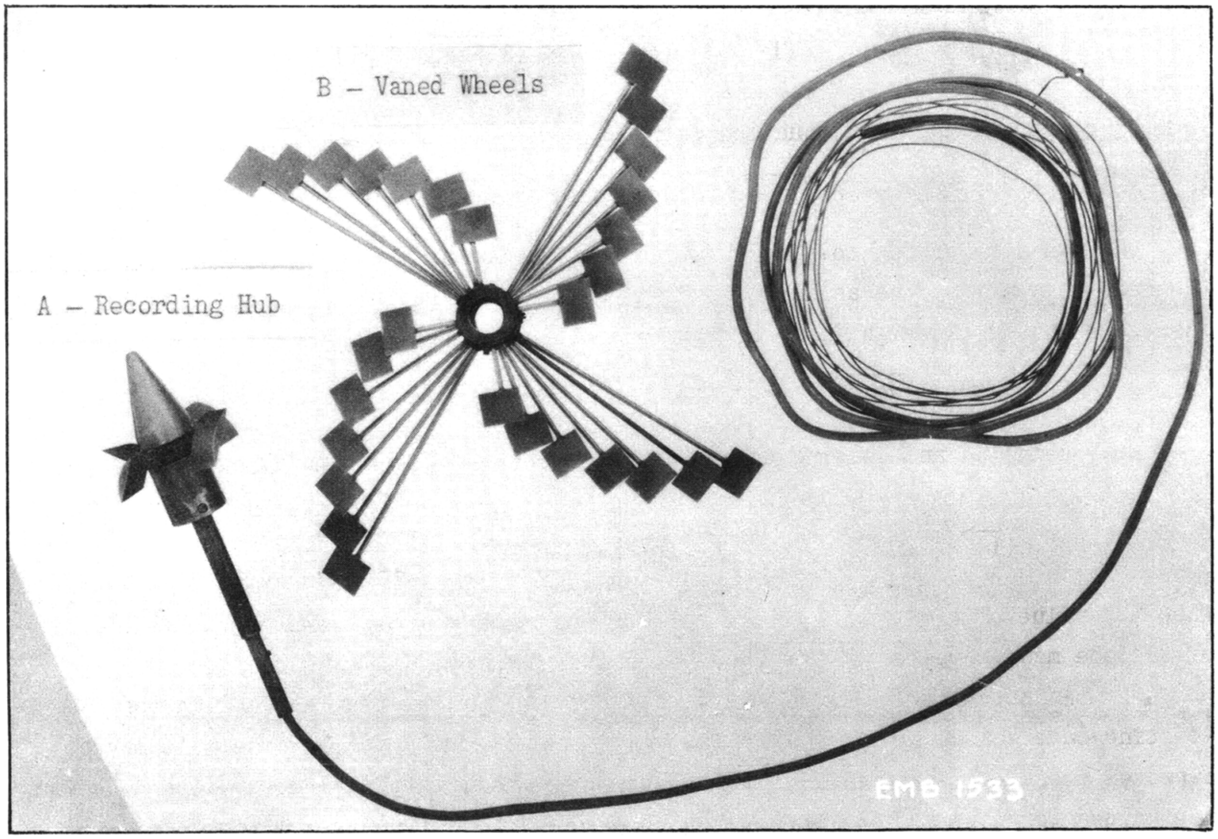

Figure 16

where w_m is the mean wake fraction, w is the instantaneous value of wake fraction and v the body speed.

Comparing Eq. (e) with Eq. (26) and (27), it follows that

$$(\overline{1-w_m}) = \sqrt{(\overline{1-w_v})(\overline{1-w_i})} \tag{f}$$

where w_v and w_i have the previously defined meaning.

The second case is somewhat more complicated. Assuming that Eq. (c) holds, we obviously have

$$\omega r = q(\overline{1-w_m})v \tag{g}$$

where w_m is again the mean wake fraction.

Introducing this equation into Eq.(b) and integrating between the limits zero and 2π, we get

$$K_m = \frac{c_1}{2\pi}\int_0^{2\pi} v_a\left[v_a - q(\overline{1-w_m})v\right]d\theta$$

$$= \frac{c_1}{2\pi}\int_0^{2\pi} v_a^2\,d\theta - \frac{c_1 q}{2\pi}(1-w_m)v\int_0^{2\pi} v_a\,d\theta$$

or, replacing v_a by its equivalent $v(1 - w)$, this is equal to

$$K_m = \frac{c_1 v^2}{2\pi} \int_0^{2\pi} (1-w)^2 \, d\theta - \frac{c_1 q v^2}{2\pi} \overline{(1-w_m)} \int_0^{2\pi} (1-w) \, d\theta \tag{h}$$

Then by comparing the last equation with Eq. (26) and (27), we get

$$K_m = c_1 v^2 \overline{(1-w_i)}\,\overline{(1-w_v)} - c_1 q v^2 \overline{(1-w_m)}\,\overline{(1-w_v)} \tag{i}$$

When referring to a calibration plot to obtain the mean forward speed from the mean revolutions, we assume that K_m is equal to K_o given by Eq. (d). We can therefore equate the right-hand members of these expressions, so that we have

$$c_1(1-q)\overline{(1-w_m)}^2 v^2 = c_1 v^2 \overline{(1-w_i)}\,\overline{(1-w_v)} - c_1 q v^2 \overline{(1-w_m)}\,\overline{(1-w_v)}$$

Reducing and solving this equation for $(1 - w_m)$ we finally obtain

$$\overline{(1-w_m)} = \frac{q}{2(1-q)} \overline{(1-w_v)} \left[-1 + \sqrt{1 + \frac{4(1-q)}{q^2} \frac{\overline{(1-w_i)}}{\overline{(1-w_v)}}} \right] \tag{k}$$

When the value of q is known from a calibration curve and w_i and w_v obtained by pitot-tube measurements, $\overline{(1 - w_m)}$ can be calculated.

An actual meter is of course neither as light as that assumed in the first limiting case nor as heavy as that assumed in the second limiting case. It may be expected therefore that the $\overline{(1 - w_m)}$ value obtained by an actual meter lies between the values found by Eq. (f) and (k) which is in general found to be the case.