# MECHANIZED COMPUTATION OF SHIP PARAMETERS

by

Ralph C. Leibowitz
and
Robert L. Harder



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# NOTATION

| | |
|---|---|
| $A$ | Area |
| $A_{ij}$ | Symbol defined by a matrix (see Appendix A) |
| $ds$ | Differential distance along shear element |
| $dx, dy, dz$ | Differential distances along $x$-, $y$-, $z$-axes, respectively |
| $E$ | Local Young's modulus of elasticity |
| $EI$ | Flexural rigidity of beam |
| $E_o$ | Reference modulus of elasticity |
| $F_i(y,z)$ | Known stress functions ($i = 1, 2, 3$) |
| $G$ | Local modulus of elasticity in shear |
| $G_i(y,z)$ | Known stress functions which are linear homogeneous functions of $F_i(y,z)$ ($i = 1, 2, 3$) |
| $GJ_e$ | Torsional rigidity |
| $I_{ij}$ | Equals $\int_{\text{area}} \frac{G_i G_j}{E} dA$ |
| $\overline{I}_{ij}$ | Functions of $I_{ij}$ |
| $J_e$ | Areal polar moment of inertia of a cross section about its shear center |
| $KAG$ | Shear rigidity |
| $k$ | Effective area factor |
| $k'$ | $k \frac{E}{E_o}$ |
| $K_1, K_2, K_3$ | Temporary unknown constants used to derive equations |
| $M_x, M_y, M_z$ | Bending moments about the $x$-, $y$-, and $z$-axes, respectively; positive when the vector representing them is in the positive coordinate direction |
| $\overline{M}_y, \overline{M}_z$ | Bending moments about the $\overline{y}$-, $\overline{z}$-axes, respectively; positive when the vector representing them is in the positive coordinate direction |
| $\overline{\overline{M}}_x$ | Bending moments about a longitudinal axis passing through the center of shear; positive when the vector representing it is in the positive coordinate direction |
| $q$ | Shear flow |
| $s$ | Arc length along shear element |
| $t$ | Thickness |

| | |
|---|---|
| $U_x, U_y, U_z$ | Displacements of cross section of beam along $x$-, $y$-, and $z$-axes, respectively; positive when the vector representing them is in the positive coordinate direction |
| $\overline{U}_x, \overline{U}_y, \overline{U}_z$ | Displacements of cross section along $\overline{x}$-, $\overline{y}$-, and $\overline{z}$-axes, respectively; positive when the vector representing them is in the positive coordinate direction |
| $\overline{\overline{U}}_y, \overline{\overline{U}}_z$ | Displacements of cross section of beam along $\overline{\overline{y}}$-, $\overline{\overline{z}}$-axes, respectively; positive when the vector representing them is in the positive coordinate direction |
| $V_x, V_y, V_z$ | Forces acting on positive side of cross section of beam (i.e., portion of beam on $-x$ side of section) along $x$-, $y$-, and $z$-axes, respectively; positive when the vector representing them is in the positive coordinate direction |
| $\overline{V}_x$ | Force acting on positive side of cross section of beam (defined as for $V_x$) along $\overline{x}$-axis; positive when the vector representing it is in the positive coordinate direction |
| $\overline{\overline{V}}_y, \overline{\overline{V}}_z$ | Force acting on positive side of cross section of beam (defined as for $V_y$ and $V_z$, respectively) along $\overline{\overline{y}}$- and $\overline{\overline{z}}$-axes, respectively; positive when the vector representing them is in the positive coordinate direction |
| $W, \overline{W}$ | Strain energy and strain energy per unit length, respectively |
| $x, y, z$ or $\overline{x}, \overline{y}, \overline{z}$ | Coordinates of a right-hand rectangular Cartesian coordinate system (see Figure 3); $\overline{y}, \overline{z}$ are also the positions of the neutral axis in $x$-, $y$-, $z$-coordinate system |
| $\overline{\overline{x}}, \overline{\overline{y}}, \overline{\overline{z}}$ | Position of the shear center in $x$-, $y$-, $z$-coordinate system |
| $\Delta x, \Delta y, \Delta z$ | Length of segment along $x$-, $y$-, and $z$-axes, respectively |
| $\theta_x, \theta_y, \theta_z$ | Rotations of cross section of beam about $x$-, $y$-, $z$-axes, respectively |
| $\overline{\theta}_x, \overline{\theta}_y, \overline{\theta}_z$ | Rotations of cross section of beam about $\overline{x}$-, $\overline{y}$-, $\overline{z}$-axes, respectively |
| $\overline{\overline{\theta}}_x, \overline{\overline{\theta}}_y, \overline{\overline{\theta}}_z$ | Rotations of cross section of beam about $\overline{\overline{x}}$-, $\overline{\overline{y}}$-, $\overline{\overline{z}}$-axes, respectively |
| $\sigma_{ij}$ | Stress components in rectangular coordinate system defined as the force per unit area acting on a face perpendicular to the $i$-axis and in the $j$-direction ($i = x, y, z;\ j = x, y, z$) |
| $\mu$ | Mass per unit length of beam |
| $\tau$ | Equals $(\sigma_{xy}^2 + \sigma_{xz}^2)^{1/2}$ |

Subscript $i$ in test denotes node numbers unless otherwise indicated; thus $y_i$, $z_i$, $A_i$ are the coordinates and corresponding area for $i^{\text{th}}$ node.

## ABSTRACT

For the purpose of vibration and load analysis a ship hull is often regarded as a flexural beam. This report describes a method and numerical computer (digital) program to calculate ship section properties (i.e., equivalent beam parameters) needed for the beam vibration equations and its internal shear distribution, using data tabulations obtained from hull plans by a pre-established orderly procedure. The program has been written in FORTRAN and can be used on an IBM 650, 704, 709, or 7090. Comparison between digital computer and hand calculations for a sample problem shows excellent agreement.


## INTRODUCTION

For several years the David Taylor Model Basin has been concerned with the computation of the natural frequencies and mode shapes of a ship hull,[1] the whipping response of a ship subject to slamming loads,[2,3] and the flutter response of hull-appendage systems.[4] In solving these problems, the ship hull has been treated as a beam and the physical parameters (i.e., equivalent beam parameters) have been computed for a ship subdivided into $n$ sections of equal or unequal length (usually $n = 20$).[1] These parameters include the inertia properties (mass, location of center of gravity, and moments of inertia), bending (location of neutral axis and bending flexibilities), shear (location of shear center and shear flexibilities), and torsional flexibility parameters.

The accurate calculation of ship properties has been a laborious task because it requires a detailed examination of ship scantlings, a tabulation of pertinent basic data (such as location and cross-sectional areas of longitudinals), and the performance of routine but lengthy calculations. It is therefore of interest to develop a digital computer program for calculating the inertia-elastic parameters of a ship hull to materially reduce the time, labor, cost, complexity, and errors associated with the present method of hand calculation of these properties.

The objective of this report is to describe a method and numerical computer program for calculating the section properties (i.e., equivalent beam parameters) of the hull starting with information derived from drawings of the hull. These parameters are to be used in the finite-difference form of the beam vibration equations developed in Reference 1; these equations have been used in vibration, slamming, and hydroelasticity problem areas in which the hull is also treated as a beam.[1–4] The theory, program derivation, and operation associated with the determination of these parameters are presented.[5,6] This includes the mathematical development of the necessary equations and a description of the input and output statements

[1]References are listed on page 86.

of a digital computer routine which could be used to compute the parameters. The data to be furnished to the computer are discussed in detail. Parts of the task, the examination of ship scantlings and the tabulations of basic data, will remain manual operations not included in the program. The input forms for the digital computer program should also serve the auxiliary purpose of assisting in the orderly and efficient recording of the basic data. Data input is prepared on cards and the computer calculates parameters for one ship section at a time. The output of the program gives the internal shear flow (stress) distribution (in the hull, per unit beam shear or torque) in addition to the parameters needed in the beam equations. The program has been written in FORTRAN[5,6] and can be used on IBM 650, 704, 709, or 7090.

To test the program, a hand and digital computer calculation is compared for a sample problem.

The method has been developed for bodies with a plane of symmetry (typical of most ships) and also for the general case where there is no symmetry.

The report has been organized to meet the needs of the program user.

# DATA TO BE FURNISHED TO THE COMPUTER

## GENERAL

Based upon a theory presented in Appendix A, a digital computer program for calculating the section properties (i.e., equivalent beam parameters) of ship hulls, presented in Appendix B, has been devised. Computation of these parameters requires that certain data (geometry, areas and thicknesses, effectiveness, etc.) be furnished to the computer using input forms discussed in Appendix C; output forms are also discussed in that appendix. These data and the method for obtaining them are now discussed. In the next section of the text, a "hand" calculation of the beam parameter for a sample ship section shows how these data are used in making this calculation on the "digital" computer; this is true because a digital computer operates on these data in a similar fashion. A comparison of the results of hand and computer calculations is given.

## DATA

### Geometry

Consider the $y-z$ coordinate system of a ship cross section shown in Figure 1, where $y$ is taken in the plane of symmetry for a symmetrical cross section. Otherwise the origin is arbitrarily chosen. The geometry of every cross section obtained from ship plans is given by the $y-z$ coordinates of each node (Figure 1).

A numerical assignment of nodes is made (1) at every point where there is a longitudinal beam (2) at the junction of more than two plates* (e.g., junction of lower deck to hull), and

*"Plate" here designates segments of decks, hull, bulkhead, inner bottom, etc.

(3) if desired, at other points in the section; if there is a plane of symmetry, only one-half of the section need be used but a node is assigned where any number crosses this plane. Assuming linear plating between two nodes, extra nodes should be assigned along curved members. Moreover, subdivision of long straight sections by assignment of additional nodes along the length improves the accuracy of the results obtained. Each node, and plate which must lie between two nodes, is numbered with an integer which runs sequentially from 1 to 150; the positive direction of each plate is indicated by an arrow drawn beside the plate.

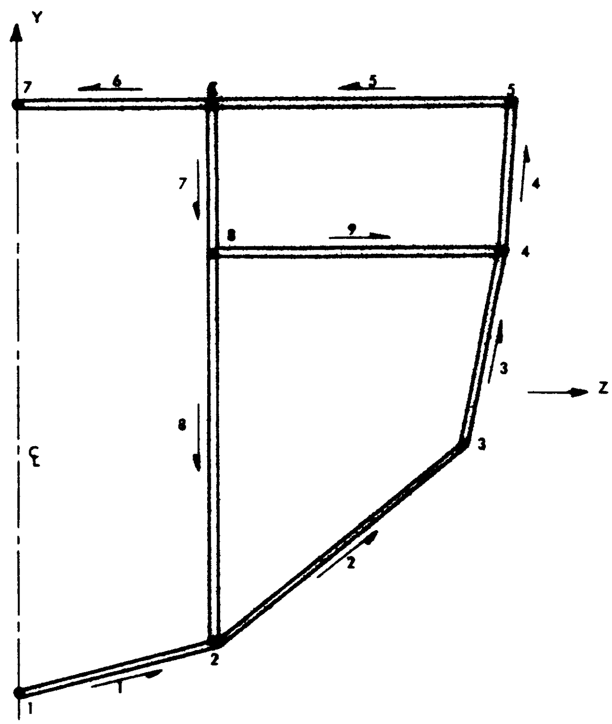



Figure 1 – Typical Symmetric Section Showing Node and Plate Numbering

## Areas* and Thicknesses

To calculate the elastic parameters, data are required on the areas (A) (see FORTRAN symbols defined in Table 1b). If a node represents a longitudinal beam, its area should be found; otherwise zero area is assigned to the node. The area of nearby plates is *not* assigned to the node because this is done by the computer program. The program also computes the length of each plate as the distance between the nodes it joins. The thickness (PT) of each plate is found and must not be zero. To maintain constant plate thickness, a node is assigned at each point where the thickness changes, thus subdividing the plate. If symmetry is used, nodes on the centerline and plates lying along the centerline are assigned only one-half the total area and thickness, respectively.

## Effectiveness (AK for Nodes, PK for Plates)

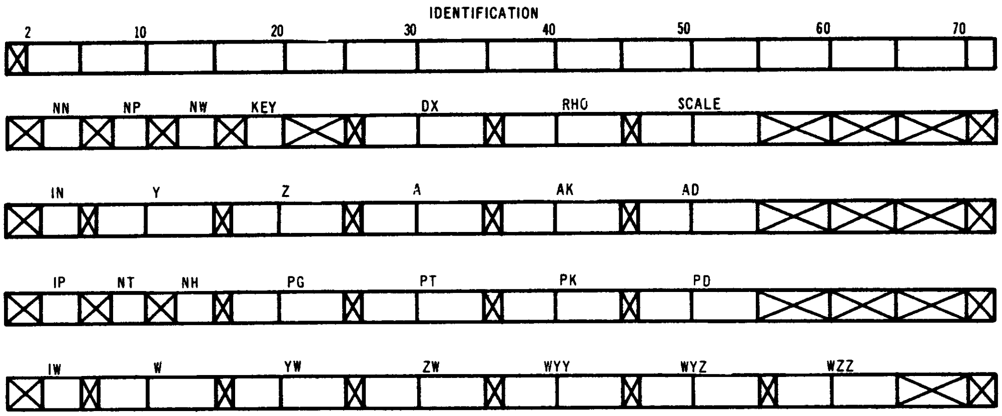
Longitudinal members which end a relatively short distance from the section to be analyzed will not be completely effective in carrying tension loads. An effectiveness value is assigned to each node area; 1.0 for completely effective members, 0.0 for members which

*It is convenient to replace the actual area distribution by a set of "concentrated areas" at a set of nodes. These nodes will be closely spaced and the area of any structure between nodes can be divided between the nodes at the ends of the segment. This idealization separates the problem so that the nodes (longitudinals) carry all the tension, and the panels between the nodes carry only shear.

# TABLE 1

## Input Forms

### TABLE 1a Column Headings for Input Forms




### TABLE 1b Definitions

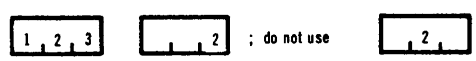
| | |
|---|---|
| IDENTIFICATION: | Any statement consisting of characters printable by the computer, a-numeric. This will not be used in the calculations, but will appear as a heading on the printout. |
| NN: | The number of nodes used, an integer. |
| NP: | The number of plates used, an integer. |
| NW: | The number of nonstructural weights used, an integer. |
| KEY: | A three digit integer used to control machine operation.<br>1st digit (1, 2) → (do not use symmetry, use symmetry).<br>2nd digit (1, 2, 3, 4) → (mass, bending, torsion and shear; bending, shear, torsion; bending only, torsion only).<br>3rd digit (1, 2) → (output beam parameters only; also output shear flows). |
| DX: | Length of hull section ($\Delta X$), floating point number. |
| RHO: | Density of structural material, floating point number. |
| SCALE: | Equals 1.0 unless mixed units are used. Node areas will be divided by $(SCALE)^2$ and plate thicknesses divided by (SCALE). |
| IN: | Number associated with a node, integer. |
| Y, Z: | Coordinates of the node, floating point numbers. |
| A | Area of the node, floating point number. |
| AK | Area effectiveness, floating point number. |
| AD | Area density ratio, floating point number. |
| IP | Number associated with a plate, integer. |
| NT: | Number associated with node at tail end of plate, integer. |
| NH: | Number associated with node at head end of plate, integer. |
| PG | Plate shear effectiveness, floating point number. |
| PT: | Plate thickness, floating point number. |
| PK: | Plate tension effectiveness, floating point number. |
| PD | Plate density ratio, floating point number. |
| IW | Number associated with mass item, integer. |
| W | Weight of item, floating point number. |
| YW, ZW | Coordinates of center of gravity, floating point numbers. |
| WYY, WYZ, WZZ | Moments of inertia about its c.g., floating point numbers. |

### TABLE 1c Format of Input Numbers

Integer.

Three spaces are provided for integers. All integers used by this routine are positive so no sign is needed. Leading zeroes may be omitted.

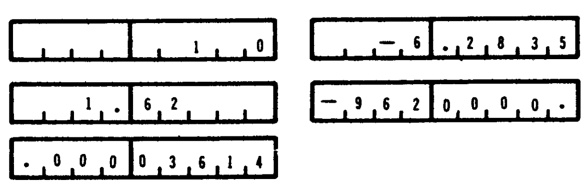
Samples:




Floating Point.

Nine spaces are provided for floating point numbers. One space must be used for the decimal point and one for the sign if negative.

Samples:



can carry no tension, and an intermediate value for partially effective members. A tension effectiveness is also assigned to the plates, which in addition may be used to account for cutouts such as hatches in the deck; see footnote on page 52.

"Effectiveness" can also be used if more than one material has been used in the construction. The effectiveness is taken as the product of the above number times the modulus ratio. The modulus ratio is the actual modulus of the material divided by a reference value of the modulus.

## Density

Structural mass is calculated as the product of the volume and density of the structural element. If more than one material is used, a density factor, which is the ratio of the actual density to the reference density, is associated with each element. For sections made of one material, all density factors will be 1.0.

## Plate Shear Factor (PG)

A plate shear factor (similar to the tension effectiveness) is needed for each plate. As with tension effectiveness, there are two factors, one due to inability to carry shear and the other due to shear modulus. Plates which end at a nearby cross section should have a low effectiveness, and plates with a modulus greater than the reference value should have an increased effectiveness. PG must *never* be 0. If the plate has no shear effectiveness, it is not considered as a structural element.

## Mass Items*

If mass calculations are to be made, additional information is needed for each nonstructural mass. Nonstructural mass includes machinery, cargo, fuel, virtual mass, etc. For each item, the weight (W), location of its center of gravity (YW, ZW), and moments of inertia about its own center of gravity (WYY, WYZ, WZZ) are required. Here WYY is the moment of inertia about an axis through the center of gravity of the item and parallel to the z-axis. The YY indicates that the integral which gives the moment of inertia has the factor $Y^2$.

## Other Numbers

A count of the total number of nodes (NN), plates (NP), and masses (NW) is needed. If masses are to be computed, the length of the section (DX) and the basic material density (RHO) are needed.

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*See pages 59 and 60.

## Key

Some control over which calculations should be made is provided by KEY. Each of the three digits which make up KEY (Table 1b) has a specific meaning and must be assigned one of the allowable values. The first digit is 1 if information is given about the complete section; 2 if symmetry is used and information given for half of the section. The second digit is 1 if all mass, bending, shear, and torsion parameters are to be calculated; it is 2 for only elastic parameters (bending, shear, torsion), 3 for only bending parameters, and 4 for only torsion parameters. The third digit controls only the output; if 1 it prints all section parameters specified by the second digit; if 2, it will in addition print the shear flows in the plates.

## Units

Any consistent set of units may be used. Masses may be computed in weight or mass units. The values of RHO, W, WYY, WYZ, and WZZ should be given in the same system. Provision is made to give lengths in mixed units if desired. If consistent units are used, then SCALE = 1.0. If lengths are given in feet (i.e., Y, Z, DX), areas in square inches, and plate thicknesses in inches, SCALE = 12.0 and RHO is in (–) per cubic foot.*

These data are collected and put on an input form such as Table 2, from which it is punched into cards for input to the computer. The input and output forms (Tables 2 and 3a, respectively) and the associated operation and rules of the computer program are discussed in Appendix C.

## COMPARISON BETWEEN COMPUTER AND MANUAL COMPUTATIONS FOR SAMPLE PROBLEM

To test the program, a sample hand calculation was made and compared with a solution of the same problem using the computer routine. Figure 1 shows the plates and nodes, and Table 2 indicates the data for the sample problem. The manual calculations are shown in Table 4, and are compared in Table 3b with the computer output shown in Table 3a. The shear flows (from the computer solution) are shown in Figure 2. The theoretical basis for these calculations is presented in Appendix A.

Node calculations are shown at the top of Sheet 1, Table 4; plate calculations at the bottom of Sheet 1; *shear flows due to y-shear* at the top of Sheet 2; *shear flows due to z-shear* at the bottom of Sheet 2; and *shear flows due to torque* on Sheet 3. On Sheet 1, data are given in Columns 1–5 for nodes and 1–5 for plates. Columns 6–9 for plates are found by Column 2 and node Columns 2 and 3. Plate Columns 10 and 11 are used to calculate 12, and thus also 13 and 14.

*(–) indicates the mass unit.

(Text continued on page 11.)

## Table 2

### Sample Input Sheets

**Table 2a — Sample Input Sheet Showing Data for Sample Problem**

IDENTIFICATION

| | | | | | |
|---|---|---|---|---|---|
| ST | TEST | CASE | 8/20/62 | STA 7 | APPROX |

| NN | NP | NW | KEY | DX | RHO | SCALE |
|---|---|---|---|---|---|---|
| 8 | 9 | 0 | 222 | 1.0 | 1.0 | 12.0 |

| IN | Y | Z | A | AK | AD |
|---|---|---|---|---|---|
| 1 | −22.05 | 0 | 17.6 | 1.0 | 1.0 |
| 2 | −20.45 | 15.15 | 23.5 | 1.0 | 1.0 |
| 3 | − 4.50 | 29.05 | 0 | 1.0 | 1.0 |
| 4 | 12.55 | 31.10 | 0 | 1.0 | 1.0 |
| 5 | 21.25 | 31.40 | 12.4 | 1.0 | 1.0 |
| 6 | 21.50 | 15.00 | 0 | 1.0 | 1.0 |
| 7 | 21.50 | 0 | 2.8 | 1.0 | 1.0 |
| 8 | 12.55 | 15.00 | 0 | 1.0 | 1.0 |

1 5 10 15 20 25 30 35 40 45 50 55 60 65 70 72

Table 2b — Sample Input Sheet Showing Data for Sample Problem

| IP | NT | NH | PG | PT | PK | PD |
|---|---|---|---|---|---|---|
| 1 | 1 | 2 | 1.0 | 1.478 | 1.474 | 1.0 |
| 2 | 2 | 3 | 1.0 | 0.957 | 1.399 | 1.0 |
| 3 | 3 | 4 | 1.0 | 0.750 | 1.223 | 1.0 |
| 4 | 4 | 5 | 1.0 | 0.983 | 1.110 | 1.0 |
| 5 | 5 | 6 | 1.0 | 0.947 | 1.030 | 1.0 |
| 6 | 6 | 7 | 1.0 | 0.675 | 0.927 | 1.0 |
| 7 | 6 | 8 | 1.0 | 0.625 | 0.358 | 1.0 |
| 8 | 8 | 2 | 1.0 | 0.625 | 0.707 | 1.0 |
| 9 | 8 | 4 | 1.0 | 0.459 | 1.098 | 1.0 |

1 5 10 15 20 25 30 35 40 45 50 55 60 65 70 72

**Table 2c — Sample Input Sheet**

IW W YW ZW WYY WYZ WZZ

1 5 10 15 20 25 30 35 40 45 50 55 60 65 70 72

# TABLE 3

## Sample Output and Comparison of Results of Sample Problem

| ST TEST CASE 8/20/62 STA 7 APPROX | |
|---|---|
| MASS = | 0.2136E 02 |
| Y − CG = | 0.5945E 00 |
| Z − CG = | 0. |
| I − YY = | 0.5486E 04 |
| I − YZ = | −0. |
| I − ZZ = | 0.8576E 04 |
| I − MX = | 0.1406E 05 |
| STRUCTURE AREA = | 0.2359E 02 |
| Y EL AXIS = | −0.1823E 01 |
| Z EL AXIS = | 0. |
| YY FLEXIBILITY = | 0.9469E-04 |
| YZ FLEXIBILITY = | 0. |
| ZZ FLEXIBILITY = | 0.1337E-03 |
| Y SHEAR CENTER = | −0.6138E 01 |
| Z SHEAR CENTER = | −0. |
| TORSION FLEXIBILITY = | 0.1083E-03 |
| YY SHEAR FLEXIBILITY = | 0.1090E-00 |
| YZ SHEAR FLEXIBILITY = | 0. |
| ZZ SHEAR FLEXIBILITY = | 0.1087E-00 |

(a) Sample Output (from Computer)

| Item | Sample Output (Program) | Table 4 (Hand Calculation) |
|---|---|---|
| $\bar{y}$ | −1.823 | −1.823 |
| $\bar{z}$ | 0 | 0 |
| YY-FLEX | 0.00009469 | 0.00009474 |
| YZ-FLEX | 0 | 0 |
| ZZ-FLEX | 0.0001337 | 0.0001337 |
| $\bar{\bar{y}}$ | −6.138 | −6.228 |
| $\bar{\bar{z}}$ | 0 | 0 |
| YY-SHEAR FLEX | 0.1090 | 0.1090 |
| YZ-SHEAR FLEX | 0 | 0 |
| ZZ-SHEAR FLEX | 0.1087 | 0.1085 |
| TORSION FLEX | 0.0001083 | 0.0001081 |

(b) Comparison of Results of Sample Problem

Column 13 under plates, which is defined as $1/2\,(12k\,t\Delta s) = 6.0$ 4 5 12, represents the division of the plate effective tension area, one-half being assigned to the node at each end. This division is effected in Columns 6 – 8 under nodes; then the net node area is computed in node Column 9.* Plate Columns 14 and 15 are used as Columns 7, 9, and 27 of Table 4, Sheet 2, in carrying out the calculations for the distribution of y- and z-shear. Compute $\bar{y}$ from $\Sigma$ 2 · 9 / $\Sigma$ 9. Because of symmetry, $\bar{z} = 0$.** Compute Columns 10 – 16 for nodes as indicated. Bending parameters are calculated on Sheet 1; the factor of 2 is for symmetry and 144 is to change from square inches to square feet.

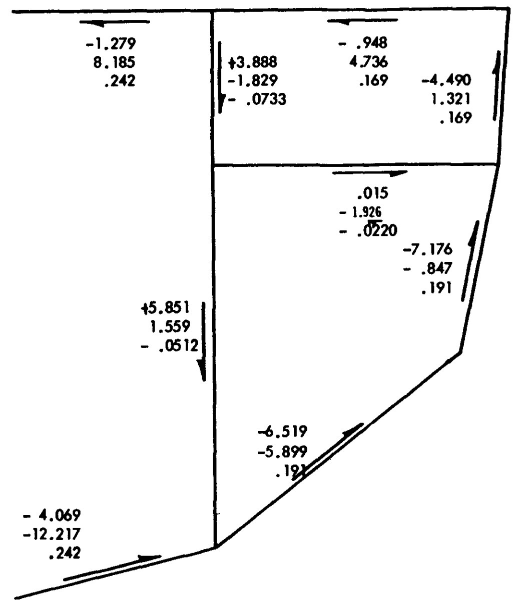


Figure 2 – Sample Problem

**Shear Flow × $10^3$ per Unit $V_y$ / 1/ft**

**Shear Flow × $10^3$ per Unit $V_z$ / 1/ft**

**Shear Flow × $10^4$ per Unit $M_x$ / 1/ft$^2$**

**The numbers in Figure 2 give the shear flows in the plates corresponding to the following conditions:**

$V_y = 1,\ V_z = 0,\ M_x = 0$ (top numbers)
$V_y = 0,\ V_z = 1,\ M_x = 0$ (middle numbers)
$V_y = 0,\ V_z = 0,\ M_x = 1$ (bottom numbers)

**The scales are indicated under the caption. The numbers come from Sheet 2, Col. 8; Sheet 2, Col. 28; and Sheet 3, Col. 8 of Table 4, respectively.**

To compute the shear parameters, first find a tree; that is, a set of plates so that one and only one path exists between every two nodes (see section Shear and Torsion in Appendix A.2). The tree chosen consists of all the plates except 5 and 9 (see Figure 1). For y-shear, Columns 2 and 3 of Sheet 2, Table 4, represent all nodes further from the root (node 1) than the plate in question. The $[T_{ji}]$ matrix discussed in Shear and Torsion of Appendix A.2 is applicable to these columns; in particular, see Equation [14]. Considering Plate 2, it is seen that shear flows from nodes 3, 4, and 5

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* 13 represents one-half the effective area of each plate employed in Columns 6, 7, and 8 at the top of the sheet in determining total node areas. Hence, the factor of 1/2 is introduced in calculating 13 (bottom). The assignment to proper nodes is carried out by entering values from 13 (bottom) in appropriate spaces under 6, 7, and 8 (top). For example, for Plate 7, which corrects nodes 6 and 8, one-half the effective area is 12.0 (Column 13, bottom). This value is entered as "Plate Area" once at node 6 (8 top) and once at node 8 (6 top). Whether a particular number is entered under Columns 6, 7, or 8 is of no significance. Effectiveness $k' = k\frac{E}{E_o}$ = PK, Column 5. Plate thickness (inches) t = PT, Column 4. Plate length (feet) $\Delta s$ = Column 12. $6 = 1/2 \times 12$ converts inch feet to square inches. Columns 4 and 5 were *arbitrarily* chosen as sample problem input data. They were used only as indicated in calculating the entries of Columns 13, 14.

**See Figure 3.

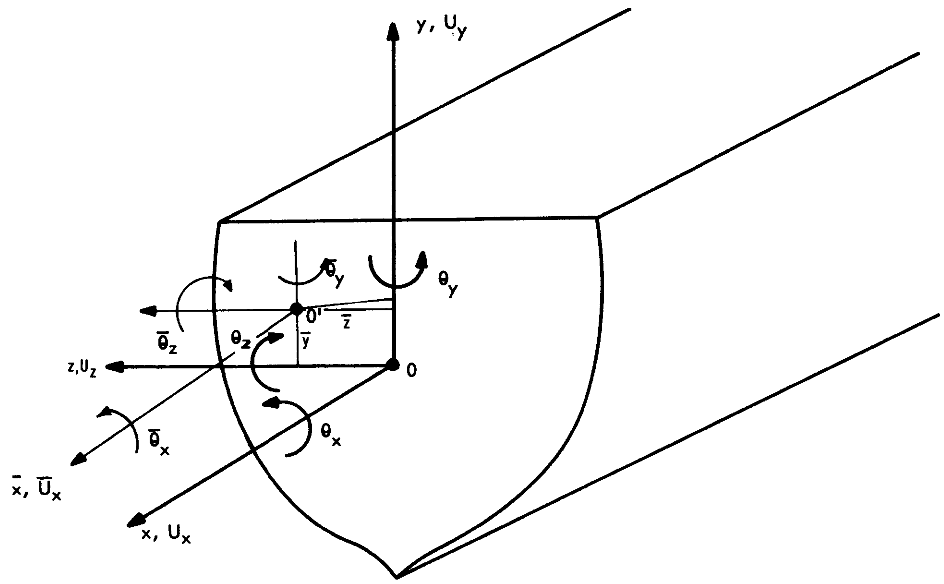



Figure 3 – Coordinate Systems

$U_x$, $U_y$, and $U_z$ are displacements; $\theta_x$, $\theta_y$, and $\theta_z$ are rotations. The forces $V_x$, $V_y$, and $V_z$ and moments $M_x$, $M_y$, and $M_z$ act upon the section shown

are effective, and that the sign of the summation is negative since the positive sense of Plate 2 is away from the root. Hence $q_{particular} = q_{part} = (-) \Sigma q_{out} = -q_{out\ 3} - q_{out\ 4} - q_{out\ 5}$.* $Q_{part}$ means $Q_{particular}$, or a particular solution of the shear flows out of the nodes, as discussed in Appendix A.2. However, since $Q_{part}$ is based on a tree which omits several plates or paths of flow (2 or 3 in the example) it is not completely general. Additional shear patterns (one for each plate omitted in the tree) are superposed. These are the $Q_{loop}$ terms. The *amount* of shear flow in each loop to be added to the particular solution is unknown *a priori*, and is indicated by the coefficients $K_1$, $K_2$, or $K_1$, $K_2$, $K_3$ (2 if symmetric and 3 if antisymmetric). For the method of solving for the K's, the matrix operations on Sheets 2 and 3 of Table 4 illustrate this for the sample problem and it is further discussed below. In general, the method of solution is outlined in section Shear and Torsion in Appendix A.2.

The solution of simultaneous equations for $K_1$, $K_2$, etc., is by matrix inversion and multiplication in the sample calculation, as indicated on Sheets 2 and 3 of Table 4. In the sample calculation, as indicated on Sheets 2 and 3 of Table 4. In the digital computer

---

*Columns 2 and 3 include the sign associated with each value of $q_{out\ i}$ and, therefore, represent $[T_{ji}]$ $[q_{out\ i}]$. Hence a separate computation for $[T_{ji}]$ is unnecessary. This is the reason Column 4 is multiplied by a factor of 1. It is possible, of course, to treat $q_{out\ i}$ (without regard to sign) and $T_{ji}$ separately as on page 65 (see Equation [14]). This is less convenient for calculation.

program, it is accomplished by the "Gram-Schmidt Reduction" indicated in Figure 4.* This is a standard technique in matrix algebra. Equations [25] and [32] of Appendix A.2 show the matrix derivation.

For reasons discussed below, Column 4 is formed by summing entries from 15, Sheet 1 (top) as indicated in 2, 3, of Sheet 2. For example, for Plate 2:

Column 4 = – (nodes 3 + 4 + 5) = – (–0.6572 + 2.6709 + 3.5423) = – 5.5560. Similarly, Column 23 is generated by the same combinations of nodes; however, in this case the shears are taken from the entries in Column 16, Sheet 1 (top).

That Columns 15, and 16 are proper expressions for $q_{out}$ from each node for the section sustaining y-shear and z-shear, respectively, is seen from the equations for $\Sigma q_i$ (out) in Appendix A.1, remembering that $I_{yz} = 0, \bar{z} = 0$ in the example. Columns 5 and 6 are loops (Column 5 is associated with Plate 5, 6 with plate 9 by random selection). They could as easily have been reversed. The selection of entries in 5 and 6 is based on the following statement in Appendix A.1, "For each plate which is not on the tree, there exists a closed loop through that plate and others in the tree." Reference to Figure 1 shows that the loop, including Plate 5 in the *positive sense* (but excluding Plate 9), includes Plates 2, 3, 4, 7, and 8 (all in the positive sense). Also, the loop involving Plate 9 in the *positive sense* (but excluding Plate 5) includes Plates 2, 3, and 8 (all in the negative sense). The entries in 5 and 6 reflect these statements.** See also Equations [13]–[15] for the $\{q_{loop\ j}\}$ and discussion of the $L_{j1}$ matrix in Appendix A.2. Column 7 of Sheet 2, Table 4, is plate Column 14 of Sheet 1, Table 4. Next, solve for the factors $K_1$ and $K_2$, which are the amount of shear flow in the loops. This is indicated as a matrix operation to the right on the calculation sheet (not the same method used in the program, but equivalent).† The following hand calculations and those used in the program are based on the set of Equations [32] of Appendix A.2. While these equations are for a general cross section, the equations of the sample problem are for cross sections with only two or three loops.

The y-shear calculation for the symmetric hull cross section involves the solution of two simultaneous equations for $K_1$ and $K_2$ (see Appendix A.2, Equation [32]). The numerical values for the elements in the matrix to the left of the $K_i$ matrix in Equation [32] are obtained as follows (see Table 4, Sheet 2):†† (Text continued on page 39)

---

*The digital computer program and Flow Charts (see Figures 4 and 5) are discussed in Appendix B.

**For z-shear the entries in Columns 24 and 25 are identical to those in 5 and 6, respectively. Since the force is antisymmetric (see Figure 6) a third loop consisting of Plates 6, 7, 8, and 1 must be considered. The entry in 26 reflects the shear flow in this loop.

†The solution of the matrix equation is performed differently in the sample problem and in the computer program because the sample calculation inverts a 2-by-2 or a 3-by-3 matrix by hand with a desk calculator, and the computer program permits inversion of an n-by-n matrix (where n may be any integer up to 30, the maximum number of loops) by high-speed digital computers (Gram-Schmidt Reduction). The optimum method is naturally different in the two instances.

††The rationale underlying the y- and z-shear calculations are similar. For the latter, a detailed calculation is given on page 39.

# TABLE 4
## Sample Calculations
### TABLE 4a Node Calculations and Plate Calculations

| | DATA | | | | PLATE AREAS | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| IN | Y | Z | A | AK | | | | A | $Y-\bar{Y}$ | $A(Y-\bar{Y})$ | $A(Y-\bar{Y})^2$ | AZ | $AZ^2$ | $\frac{A(Y-\bar{Y})}{I_{yy}} \times 10^3$ | $\frac{AZ}{I_{zz}} \times 10^3$ |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9* | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
| | | | | | | | | 4 + 6 + 7 + 8 | 2 + 1.823 | 9 × 10 | 10 × 11 | 3 × 9 | 3 × 13 | | |
| 1 | − 22.05 | 0 | 17 6 | 1 0 | 199 1 | 87 5 | | 216.7 | −20 227 | −4383.2 | 88659 | 0 | 0 | −4.0684 | 0 |
| 2 | − 20.45 | 15 15 | 23.5 | ↓ | 199 1 | 170 0 | 87.5 | 480 1 | −18.627 | −8942.8 | 166578 | 7273.5 | 110192 | −8.3006 | 4.759 |
| 3 | − 4 50 | 29 05 | 0 | | 170 0 | 94.5 | | 264 5 | − 2 677 | − 708 1 | 1895 | 7683.7 | 223211 | −0.6572 | 5.052 |
| 4 | 12 55 | 31 10 | 0 | | 94 5 | 57 0 | 48 7 | 200 2 | 14 373 | 2877 5 | 41358 | 6226.2 | 193635 | 2.6709 | 4.094 |
| 5 | 21.25 | 31.40 | 12 4 | | 57 0 | 96 0 | | 165 4 | 23 073 | 3816 3 | 88053 | 5193.6 | 163078 | 3.5423 | 3.415 |
| 6 | 21 50 | 15 00 | 0 | | 96 0 | 56 3 | 12 0 | 164.3 | 23 323 | 3832 0 | 89374 | 2464.5 | 36968 | 3.5568 | 1.620 |
| 7 | 21 50 | 0 | 2 8 | | 56 3 | | | 59 1 | 23 323 | 1378 4 | 32148 | 0 | 0 | 1.2794 | 0 |
| 8 | 12.55 | 15.00 | 0 | 1.0 | 12 0 | 87.5 | 48 7 | 148 2 | 14 373 | 2130 1 | 30616 | 2223.0 | 33345 | 1.9771 | 1.462 |
| | Σ 9 · 2 = −3096 3 | | | | | | | Σ 9 − 1698 5 | | | 538681 | | 760429 | | |
| | $\bar{Y}$ = 1 823 | $\bar{Z}$ = 0 | | | | | | | | $I_{yy}$ = 1,077,362 | | $I_{zz}$ = 1,520,858 | | | |

| IP | NT NH | PG | PT | PK | $Y_H$ | $Y_T$ | $Z_H$ | $Z_T$ | $Y_H - Y_T$ | $Z_H - Z_T$ | ΔS | ½(12k′tΔS) | ΔS/t | $Y_H Z_T - Y_T Z_H$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | 6 − 7 | 8 − 9 | $\sqrt{10^2 + 11^2}$ | 6.0 4 5 · 12 | 12 / 4 | 6 · 9 − 7 · 8 |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
| 1 | 1 2 | 1.0 | 1.478 | 1.474 | −20.45 | − 22.05 | 15.15 | 0 | 1 60 | 15 15 | 15.23 | 199 1 | 10 30 | 334 06 |
| 2 | 2 3 | ↓ | 0 957 | 1.399 | − 4.50 | − 20.45 | 29.05 | 15.15 | 15.95 | 13.90 | 21 16 | 170 0 | 22.11 | 525 90 |
| 3 | 3 4 | | 0.750 | 1 223 | 12.55 | − 4 50 | 31.10 | 29 05 | 17 05 | 2.05 | 17 17 | 94 5 | 22 89 | 504.53 |
| 4 | 4 5 | | 0.983 | 1.110 | 21.25 | 12.55 | 31 40 | 31.10 | 8 70 | 0.30 | 8.71 | 57.0 | 8.86 | 266 80 |
| 5 | 5 6 | | 0.947 | 1.030 | 21 50 | 21.25 | 15.00 | 31 40 | 0.25 | − 16.40 | 16.40 | 96 0 | 17 32 | 356.35 |
| 6 | 6 7 | | 0.675 | 0 927 | 21.50 | 21 50 | 0 | 15.00 | 0 | − 15.00 | 15 00 | 56 3 | 22 22 | 322.50 |
| 7 | 6 8 | | 0.625 | 0.358 | 12.55 | 21.50 | 15 00 | ↓ | − 8 95 | 0 | 8 95 | 12.0 | 14 32 | − 134 25 |
| 8 | 8 2 | | 0 625 | 0.707 | − 20 45 | 12 55 | 15.15 | | − 33.00 | 0.15 | 33 00 | 87.5 | 52 80 | −496.88 |
| 9 | 8 4 | 1.0 | 0.459 | 1 098 | 12 55 | 12.55 | 31.10 | 15.00 | 0 | 16.10 | 16.10 | 48 7 | 35.08 | −202.06 |

*If AK ≠ 1.0 then 9 = 4 5 + 6 + 7 + 8

**This holds by virtue of the equation for YY-FLEX in Appendix A, recognizing that for the symmetric hull of the sample problem, $I_{yz} = 0$. 144 is necessary because $I_{zz}$ here has the units ft² in.², and YY-FLEX. has units of ft⁻⁴. Similarly, ZZ-FLEX $= \frac{144}{I_{yy}}$

$$\text{AREA} = 2 \times \frac{1698\ 5}{144} = 23\ 59\ \text{FT}^2$$

$\bar{Y} = -1.823$ $\bar{Z} = 0$

$$\text{YY} = \text{FLEX.}^{**} = \frac{144}{I_{zz}} = 0.9474 \times 10^{-4}\ \frac{1}{\text{FT}^4}$$

$$\text{YZ} - \text{FLEX.} = 0$$

$$\text{ZZ} - \text{FLEX.}^{**} = \frac{144}{I_{yy}} = 1\ 337 \times 10^{-4}\ \frac{1}{\text{FT}^4}$$

# TABLE 4b Y-Shear and Z-Shear

| IP | $Q_{PART}$ | | Q LOOPS | | $\Delta s/t$ | $Q_{Vy} \times 10^3$ | $R\Delta s$ | ⑦ × ⑧ |
|---|---|---|---|---|---|---|---|---|
| ① | ② ③ | ④ | ⑤ | ⑥ | ⑦ | ⑧ | ⑨ | ⑩ |
| | $Q_{out}$ Y's | | $L_{j1}$ | | | | | |
| 1 | −(7+6+8+2+3+4+5) | −4.0687 | 0 | 0 | 10.30 | −4.0687 | 334.1 | 170.51 |
| 2 | −(3+4+5) | −5.5560 | 1 | −1 | 22.11 | −6.5187 | 525.9 | 939.53 |
| 3 | −(5+4) | −6.2132 | 1 | −1 | 22.89 | −7.1759 | 504.5 | 1178.69 |
| 4 | −(5) | −3.5423 | 1 | 0 | 8.86 | −4.4902 | 266.8 | 178.63 |
| 5 | −− | 0 | 1 | 0 | 17.32 | −0.9479 | 356.4 | 15.56 |
| 6 | −(7) | −1.2794 | 0 | 0 | 22.22 | −1.2794 | 322.5 | 36.37 |
| 7 | (6+7) | 4.8362 | 1 | 0 | 14.32 | 3.8883 | −134.2 | 216.50 |
| 8 | (6+7+8) | 6.8133 | 1 | −1 | 52.80 | 5.8506 | −946.9 | 1807.32 |
| 9 | −− | 0 | 0 | 1 | 35.08 | 0.0148 | −202.1 | 0.007 |
| | | {1.0000 | −0.9479 | +0.0148} | | | | $\Sigma$ = 4543.12 |
| | | FACTORS | | | | | | |

| IP | $Q_{PART}$ | | Q LOOPS | | | $\Delta s/t$ | $Q_{Vz} \times 10^3$ | | ㉗ × ㉘$^2$ |
|---|---|---|---|---|---|---|---|---|---|
| ㉑ | ㉒ | $Q_{out}$ Z's ㉓ | ㉔ | ㉕ | ㉖ | ㉗ | ㉘ | ㉙ | ㉚ |
| | | | $L_{j1}$ | | | | | | |
| 1 | | −20.402 | 0 | 0 | −1 | 10.30 | −12.217 | 334.1 | 1537.33 |
| 2 | | −12.561 | 1 | −1 | 0 | 22.11 | − 5.899 | 525.9 | 769.39 |
| 3 | | − 7.509 | 1 | −1 | 0 | 22.89 | − 0.847 | 504.5 | 16.42 |
| 4 | | − 3.415 | 1 | 0 | 0 | 8.86 | 1.321 | 226.8 | 15.46 |
| 5 | | 0 | 1 | 0 | 0 | 17.32 | 4.736 | 356.4 | 388.48 |
| 6 | | 0 | 0 | 0 | −1 | 22.22 | 8.185 | 322.5 | 1488.61 |
| 7 | | 1.620 | 1 | 0 | 1 | 14.32 | − 1.829 | −134.2 | 47.90 |
| 8 | | 3.077 | 1 | −1 | 1 | 52.80 | 1.559 | −946.9 | 128.33 |
| 9 | | 0 | 0 | 1 | 0 | 35.08 | − 1.926 | −202.1 | 130.13 |
| FACTORS | | 1.000 | 4.736 | −1.926 | −8.185 | | | | $\Sigma$ = 4522.05 |

$$\begin{pmatrix} K_1 \\ K_2 \end{pmatrix} = - \begin{bmatrix} 138.30 & -97.80 \\ -97.80 & 132.88 \end{bmatrix}^{-1} \begin{pmatrix} 132.55 \\ -94.68 \end{pmatrix} = \begin{pmatrix} -0.9479 \\ +0.0148 \end{pmatrix}$$

YY-FLEX. $= 2 \times 12 \times (4543.12 \times 10^{-6}) = 0.1090$

Z − SHEAR CENTER = 0

$$\begin{pmatrix} K_1 \\ K_2 \\ K_3 \end{pmatrix} = - \begin{bmatrix} 138.30 & -97.80 & 67.12 \\ -97.80 & 132.88 & -52.80 \\ 67.12 & -52.80 & 99.64 \end{bmatrix}^{-1} \begin{pmatrix} -294.20 \\ 287.14 \\ 395.80 \end{pmatrix} = \begin{pmatrix} +4.736 \\ -1.926 \\ -8.185 \end{pmatrix}$$

ZZ − FLEX. $= 2 \times 12 \times (4522.05 \times 10^{-6}) = 0.1085$

Y − SHEAR CENTER $= 2 \times \Sigma$ ㉘ $\times$ ㉙ $= 2(-3.114) = -6.228$

**NOTE:**

⑤, ㉔ Description of 1st loop

⑥, ㉕ Description of 2nd loop

㉖ Description of 3rd loop

$Q_{Vy}$ Shear flow in plates for $V_y = 1$, $V_z = 0$, $\overline{\overline{M}}_x = 0$ — see pages 72–75.

$Q_{Vz}$ Shear flow in plates for $V_y = 0$, $V_z = 1$, $\overline{\overline{M}}_x = 0$

⑨, ㉙ $R\Delta s$. Same as $Y_H Z_T - Y_T Z_H$ on Sheet 1, Column 15 (bottom). R is perpendicular distance from origin to plate. (See Figure 9 and page 72).

⑩, ㉚ Used in calculating $\sum_j \frac{\Delta s_j}{t_j} Q_{Vy,z,j}{}^2$ for determination of $\left(\frac{1}{KA_{yy}}\right), \left(\frac{1}{KA_{zz}}\right)$ (YY − FLEX. and ZZ − FLEX.)

(See Equations [46]–[48] of Appendix A.2 and pages 84–85).

TABLE 4c Torsion

| IP | $r\Delta s$ | $\Delta s/t$ | Q Loops $L_{ji}$ | | | $q_T^*$ Not Normal | $q_T \times 10^3$ $=0.888 \times$ (7) | (3) × (8)$^2$ |
|---|---|---|---|---|---|---|---|---|
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
| 1 | 334.06 | 10.30 | 0 | 0 | −1 | 0.273 | 0.242 | 0.6032 |
| 2 | 525.90 | 22.11 | 1 | −1 | 0 | 0.214 | 0.190 | 0.7982 |
| 3 | 504.53 | 22.89 | 1 | −1 | 0 | 0.214 | 0.190 | 0.8263 |
| 4 | 266.80 | 8.86 | 1 | 0 | 0 | 0.190 | 0.168 | 0.2500 |
| 5 | 356.35 | 17.32 | 1 | 0 | 0 | 0.190 | 0.168 | 0.4888 |
| 6 | 322.50 | 22.22 | 0 | 0 | −1 | 0.273 | 0.242 | 1.3012 |
| 7 | −134.25 | 14.32 | 1 | 0 | 1 | −0.083 | −0.074 | 0.0783 |
| 8 | −496.88 | 52.80 | 1 | −1 | 1 | −0.059 | −0.052 | 0.1426 |
| 9 | −202.06 | 35.08 | 0 | 1 | 0 | −0.024 | −0.021 | 0.0154 |
| | FACTORS | | 0.190 | −0.024 | −0.273 | | | $\Sigma = 4.504$ |

$$\begin{pmatrix} K_1 \\ K_2 \\ K_3 \end{pmatrix} = \begin{bmatrix} 138.30 & -97.80 & 67.12 \\ -97.80 & 132.88 & -52.80 \\ 67.12 & -52.80 & 99.64 \end{bmatrix}^{-1} \begin{pmatrix} 10.2 \\ -7.3 \\ -12.8 \end{pmatrix} = \begin{pmatrix} 0.190 \\ -0.024 \\ -0.273 \end{pmatrix}$$

$\Sigma$ (2) × (7) = 563

$$\text{FACTOR} = \frac{1}{2 \times 563} = 0.888 \times 10^{-3}$$

$$\text{TORSION FLEX.} = 2 \times 12 \times (4.504 \times 10^{-6}) = 0.1081 \times 10^{-3} \ 1/FT^4$$

NOTE: $r\Delta s$ same as $R\Delta s$, Cols. (9), (29) Sheet 2.

(4), (5), (6) Q Loops Specification of three loops.

$q_T^*$ (Not Normal) (7) = (0.190) (4) + (−0.024) (5) + (−0.273) (6)

These are shear flows for $V_y = V_z = 0$, $G \dfrac{d\theta_x}{dx} = 1$

(8) $q_T$. These are shear flows in plates for $V_y = V_z = 0$, $\bar{\bar{M}}_x = 1$.

(9) Calculation of $\sum_j \left(\frac{\Delta s}{t}\right)_j Q_{Tj}^2$ for torsion flexibility. See Appendix A.2, Equation [51] and page 84—85.

Figure 4 – FORTRAN Statements and Symbols

```
      DIMENSION A(150),AK(150),AD(150),C(30),LINK(150),IN(150),
     1NT(150),NH(150),NEXT(150),NRANK(150),NC(30),PK(150),IP(150),
     2PD(150),PG(150),PT(150),QOY(150),QOZ(150),Q(150,30),IW(100),
     3QY(150),QZ(150),QT(150),R(150),RDS(150),W(100),WYY(100),
     4WYZ(100),WZZ(100),Y(150),YW(100),Z(150),ZW(100),MTYPE(150)
   99 IF(SENSE SWITCH 5)524,523
  523 READ 1
      IF(SENSE SWITCH 6)500,501
  500 WRITE OUTPUT TAPE 6,1
      GO TO 502
  501 PRINT 1
  502 READ 2,NN,NP,NW,KEY,DX,RHO,SCALE
      READ3,(IN(I),Y(I),Z(I),A(I),AK(I),AD(I),I=1,NN)
      READ4,(IP(I),NT(I),NH(I),PG(I),PT(I),PK(I),PD(I),I=1,NP)
      IF (NW) 999,100,999
999   READ5,(IW(I),W(I),YW(I),ZW(I),WYY(I),WYZ(I),WZZ(I),I=1,NW)
      GO TO 100
    1 FORMAT(72HCOMPUTER ENGINEERING ASSOCIATES, A DIVISION OF SUSQUEHAN
     1A SCIENCES, INC.)
2     FORMAT (4I5,5X,3F10.4)
3     FORMAT (I5,5F10.4)
4     FORMAT (3I5,4F10.4)
    5 FORMAT(I5,6F10.4)
    6 FORMAT(10H MASS     = ,E11.4)
    7 FORMAT(10H Y - CG = ,E11.4)
    8 FORMAT(10H Z - CG = ,E11.4)
    9 FORMAT(10H I - YY = ,E11.4)
   10 FORMAT(10H I - YZ = ,E11.4)
   11 FORMAT(10H I - ZZ = ,E11.4)
   12 FORMAT(10H I - MX = ,E11.4)
   13 FORMAT(18H STRUCTURE AREA = ,E11.4)
   14 FORMAT(18H Y EL AXIS      = ,E11.4)
   15 FORMAT(18H Z EL AXIS      = ,E11.4)
   16 FORMAT(18H YY FLEXIBILITY = ,E11.4)
   17 FORMAT(18H YZ FLEXIBILITY = ,E11.4)
   18 FORMAT(18H ZZ FLEXIBILITY = ,E11.4)
   19 FORMAT(24H Y SHEAR CENTER       = ,E11.4)
   20 FORMAT(24H Z SHEAR CENTER       = ,E11.4)
   21 FORMAT(24H TORSION FLEXIBILITY  = ,E11.4)
```

INPUT

Figure 4a-1 – The FORTRAN Statements

```
 22 FORMAT(24H YY SHEAR FLEXIBILITY = ,E11.4)
 23 FORMAT(24H YZ SHEAR FLEXIBILITY = ,E11.4)
 24 FORMAT(24H ZZ SHEAR FLEXIBILITY = ,E11.4)
524 READ INPUT TAPE 5,1
    IF(SENSE SWITCH 6)525,526
525 WRITE OUTPUT TAPE 6,1
    GO TO 527
526 PRINT 1
527 READ INPUT TAPE 5,2,NN,NP,NW,KEY,DX,RHO,SCALE
    READ INPUT TAPE 5,3,(IN(I),Y(I),Z(I),A(I),AK(I),AD(I),I=1,NN)
    READ INPUT TAPE 5,4,(IP(I),NT(I),NH(I),PG(I),PT(I),PK(I),PD(I),I=1
   1,NP)
    IF (NW) 100,100,528
528 READ INPUT TAPE 5,5,(IW(I),W(I),YW(I),ZW(I),WYY(I),WYZ(I),WZZ(I),I
   1=1,NW)
100 DO 101 I=1,NN
101 A(I)=A(I)/(SCALE**2)
    DO 102 I=1,NP
102 PT(I)=PT(I)/SCALE
    KEYD=1
    DO 300 K=1,150
300 NRANK(K)=0
    DO 301 J=1,NN
    K=IN(J)
301 NRANK(K)=J
    DO 399 I=1,NP
    K=NT(I)
    IF(NRANK(K))303,303,304
304 NT(I)=NRANK(K)
    K=NH(I)
    IF(NRANK(K))305,305,302
302 NH(I)=NRANK(K)
    GO TO 399
398 FORMAT(7H PLATE ,I3,16H REFERS TO NODE ,I3,23H WHICH WAS NOT DEFIN
   1ED.)
303 IF(SENSE SWITCH 6)503,504
503 WRITE OUTPUT TAPE 6,398, IP(I),NT(I)
    GO TO 505
504 PRINT 398, IP(I),NT(I)
505 KEYD=2
```

INPUT

SCALE
RENUMBER

NUMBERING
ERROR
DIAGNOSTIC

Figure 4a-2

```
      GO TO 304
  305 IF(SENSE SWITCH 6)506,507
  506 WRITE OUTPUT TAPE 6,398, IP(I),NH(I)
      GO TO 508
  507 PRINT 398, IP(I),NH(I)
  508 KEYD=2
      GO TO 302
  399 CONTINUE
      GO TO(306,99),KEYD
  306 KEYA=(KEY/100)
      KEYB=(KEY/10)-10*(KEY/100)
      KEYC=KEY-10*(KEY/10)
      GO TO(103,106,106,108),KEYB
  103 SW=0.0
      SWY=0.0
      SWZ=0.0
      SWYY=0.0
      SWYZ=0.0
      SWZZ=0.0
      IF (NW) 997,997,998
 998  DO 104 I=1,NW
      SW=SW+W(I)
      SWY=SWY+W(I)*YW(I)
      SWZ=SWZ+W(I)*ZW(I)
      SWYY=SWYY+WYY(I)+W(I)*YW(I)*YW(I)
      SWYZ=SWYZ+WYZ(I)+W(I)*YW(I)*ZW(I)
  104 SWZZ=SWZZ+WZZ(I)+W(I)*ZW(I)*ZW(I)
 997  DO 105 I=1,NN
      G=A(I)*AD(I)*RHO*DX
      SW=SW+G
      SWY=SWY+G*Y(I)
      SWZ=SWZ+G*Z(I)
      SWYY=SWYY+G*Y(I)*Y(I)
      SWYZ=SWYZ+G*Y(I)*Z(I)
  105 SWZZ=SWZZ+G*Z(I)*Z(I)
  106 DO 107 I=1,NN
  107 A(I)=A(I)*AK(I)
  108 DO 110 J=1,NP
      JH=NH(J)
      JT=NT(J)
```

NUMBERING ERROR DIAGNOSTIC

SCALE RENUMBER

EXTRACT KEY

ADDITIONAL AND NODE MASS SUMS

EFFECTIVE TENSION AREA

PLATE LENGTH, rΔS ΔS/Gt, TENSION AREA

Figure 4a-3

```
      YH=Y(JH)
      YT=Y(JT)
      ZH=Z(JH)
      ZT=Z(JT)
      G=SQRTF((YH-YT)**2+(ZH-ZT)**2)
      RDS(J)=YH*ZT-YT*ZH
      R(J)=G/(PG(J)*PT(J))
      A(JH)=A(JH)+(0.5*G*PT(J)*PK(J))
      A(JT)=A(JT)+(0.5*G*PT(J)*PK(J))
      GO TO (109,110,110,110),KEYB
  109 G=G*PT(J)*PD(J)*RHO*DX
      SW=SW+G
      SWY=SWY+G*0.5*(YH+YT)
      SWZ=SWZ+G*0.5*(ZH+ZT)
      SWYY=SWYY+G*0.25*(YH+YT)*(YH+YT)
      SWYZ=SWYZ+G*0.25*(YH+YT)*(ZH+ZT)
      SWZZ=SWZZ+G*0.25*(ZH+ZT)*(ZH+ZT)
  110 CONTINUE
      GO TO(111,114,114,120),KEYB
  111 GO TO(113,112),KEYA
  112 SW=2.0*SW
      SWY=2.0*SWY
      SWZ=0.0
      SWYY=2.0*SWYY
      SWYZ=0.0
      SWZZ=2.0*SWZZ
  113 SWY=SWY/SW
      SWZ=SWZ/SW
      SWYY=SWYY-SW*SWY*SWY
      SWYZ=SWYZ-SW*SWY*SWZ
      SWZZ=SWZZ-SW*SWZ*SWZ
  114 SA=0.0
      SAY=0.0
      SAZ=0.0
      SAYY=0.0
      SAYZ=0.0
      SAZZ=0.0
      DO 115 I=1,NN
      SA=SA+A(I)
      SAY=SAY+A(I)*Y(I)
```

**PLATE LENGTH, rΔS**
**ΔS/Gt, TENSION AREA**

**ADD PLATE MASS**
**TO MASS SUMS**

**MODIFY MASS SUMS**

**COMPUTE MASS PARAMETERS**

**COMPUTE AREAL MOMENTS**

Figure 4a-4

```
      SAZ=SAZ+A(I)*Z(I)
      SAYY=SAYY+A(I)*Y(I)*Y(I)                           COMPUTE AREAL MOMENTS
      SAYZ=SAYZ+A(I)*Y(I)*Z(I)
  115 SAZZ=SAZZ+A(I)*Z(I)*Z(I)
      GO TO(117,116),KEYA
  116 SA=2.0*SA
      SAY=2.0*SAY
      SAZ=0.0                                            MODIFY AREAL SUMS
      SAYY=2.0*SAYY
      SAYZ=0.0
      SAZZ=2.0*SAZZ
  117 SAY=SAY/SA
      SAZ=SAZ/SA
      SAYY=SAYY-SA*SAY*SAY
      SAYZ=SAYZ-SA*SAY*SAZ                               COMPUTE AREAL PARAMETERS
      SAZZ=SAZZ-SA*SAZ*SAZ
      G=1.0/(SAYY*SAZZ-SAYZ*SAYZ)
      SAYY=G*SAYY
      SAYZ=G*SAYZ
      SAZZ=G*SAZZ
      GO TO(118,118,509,120),KEYB
  118 DO 119 I=1,NN                                      COMPUTE Q OUT
      QOY(I)=A(I)*(SAZZ*(Y(I)-SAY)-SAYZ*(Z(I)-SAZ))
  119 QOZ(I)=A(I)*(SAYY*(Z(I)-SAZ)-SAYZ*(Y(I)-SAY))
  120 DO 121 I=1,NP
  121 MTYPE(I)=0
      DO 122 I=1,NN
      NEXT(I)=0                                          SETUP FOR TREE SEARCH
      LINK(I) =0
  122 NRANK(I)=0
      NRANK(1)=1
      GO TO(126,123),KEYA
  123 LC=0
      DO125 I=1,NN
      IF(Z(I))125,124,125
  124 LC=LC+1                                            MODIFY FOR SYMMETRY
      NC(LC)=I
  125 CONTINUE
      NRANK(1)=NC(1)
  126 K=1                                                TREE SEARCH
```

Figure 4a-5

```
127 DO 137 J=1,NN
    IF(NRANK(J))196,196,128
128 DO 136 I=1,NP
    IF(MTYPE(I))136,129,136
129 IF(NT(I)-NRANK(J))130,131,130
130 IF(NH(I)-NRANK(J))136,132,136
131 LA=NH(I)
    LB=-I
    GO TO 133
132 LA=NT(I)
    LB=I
133 IF(NEXT(LA))134,135,134
134 MTYPE(I)=-1
    GO TO 136
135 NEXT(LA)=NRANK(J)
    K=K+1
    NRANK(K)=LA
    MTYPE(I)=1
    LINK(LA)=LB
136 CONTINUE
137 CONTINUE
    DO 138 I=1,150
    DO 138 J=1,30
138 Q(I,J)=0.0
    L=0
    DO 149 I=1,NP
    IF(MTYPE(I))139,149,149
139 L=L+1
    Q(I,L)=1.0
    J=NH(I)
140 IF(LINK(J))141,144,142
141 K=-LINK(J)
    Q(K,L)=Q(K,L)-1.0
    GO TO 143
142 K=LINK(J)
    Q(K,L)=Q(K,L)+1.0
143 J=NEXT(J)
    GO TO 140
144 J=NT(I)
145 IF(LINK(J))146,149,147
```

TREE SEARCH

FIND TREE LOOPS

Figure 4a-6

```
146 K=-LINK(J)
    Q(K,L)=Q(K,L)+1.0
    GO TO 148
147 K=LINK(J)
    Q(K,L)=Q(K,L)-1.0
148 J=NEXT(J)
    GO TO 145
149 CONTINUE
    LA=L
    GO TO(156,996),KEYA
996 IF(LC-1)156,156,150
150 DO 155 I=2,LC
    LA=LA+1
    J=NC(I)
151 IF(LINK(J))152,155,153
152 K=-LINK(J)
    Q(K,LA)=-1.0
    GO TO 154
153 K=LINK(J)
    Q(K,LA)=1.0
154 J=NEXT(J)
    GO TO 151
155 CONTINUE
156 IF(LA) 196,196,310
310 DO 163 J=1,LA
    DO 162 K=J,LA
200 G=0.0
    DO 157 I=1,NP
157 G=G+R(I)*Q(I,J)*Q(I,K)
    IF(K-J)160,158,160
158 G=1.0/SQRTF(G)
    DO 159 I=1,NP
159 Q(I,J)=G*Q(I,J)
    GO TO 162
160 DO 161 I=1,NP
161 Q(I,K)=Q(I,K)-G*Q(I,J)
162 CONTINUE
163 CONTINUE
    GO TO(164,164,509,177),KEYB
164 DO 165 I=1,NP
```

FIND TREE LOOPS

GRAM SCHMIDT REDUCTION

FIND SHEAR FLOWS DUE TO Y-Z SHEARS

Figure 4a-7

```
      QY(I)=0.0
 165  QZ(I)=0.0
      K=NN
 166  I=NRANK(K)
      IF(LINK(I))167,170,168
 167  J=-LINK(I)
      QY(J)=-QOY(I)
      QZ(J)=-QOZ(I)
      GO TO 169
 168  J=LINK(I)
      QY(J)=QOY(I)
      QZ(J)=QOZ(I)
 169  J=NEXT(I)
      QOY(J)=QOY(J)+QOY(I)
      QOZ(J)=QOZ(J)+QOZ(I)
      K=K-1
      GO TO 166
170   IF(L) 312,312,311
311   DO 173 I=1,L
      G=0.0
      DO 171 J=1,NP
 171  G=G+R(J)*QY(J)*Q(J,I)
      DO 172 J=1,NP
 172  QY(J)=QY(J)-G*Q(J,I)
 173  CONTINUE
312   DO 176 I=1,LA
      G=0.0
      DO 174  J=1,NP
 174  G=G+R(J)*QZ(J)*Q(J,I)
      DO 175  J=1,NP
 175  QZ(J)=QZ(J)-G*Q(J,I)
 176  CONTINUE
 177  G=0.0
      DO 179 K=1,LA
      C(K)=0.0
      DO 178 I=1,NP
 178  C(K)=C(K)+Q(I,K)*RDS(I)
 179  G=G+C(K)*C(K)
 185  GO TO(510,186),KEYA
 186  G=2.0*G
```

FIND SHEAR FLOWS DUE TO Y-Z SHEARS

FIND SHEAR FLOWS DUE TO TORQUE

Figure 4a-8

```
510 G=1.0/G
    DO 180 I=1,NP
    QT(I)=0.
    DO 180 J=1,LA
180 QT(I)=QT(I)+G*C(J)*Q(I,J)
    GO TO(181,181,509,509),KEYB
181 UY=0.0
    UZ=0.0
    UYY=0.0
    UYZ=0.0
    UZZ=0.0
    DO 182 I=1,NP
    UY=UY+RDS(I)*QY(I)
    UZ=UZ+RDS(I)*QZ(I)
    UYY=UYY+R(I)*QY(I)*QY(I)
    UYZ=UYZ+R(I)*QY(I)*QZ(I)
182 UZZ=UZZ+R(I)*QZ(I)*QZ(I)
    GO TO(184,183),KEYA
183 UY=0.0
    UZ=2.0*UZ
    UYY=2.0*UYY
    UYZ=0.0
    UZZ=2.0*UZZ
184 UY=-UY
509 IF(SENSE SWITCH 6)511,187
187 GO TO(188,189,189,191),KEYB
188 PRINT 6,SW
    PRINT 7,SWY
    PRINT 8,SWZ
    PRINT 9,SWYY
    PRINT 10,SWYZ
    PRINT 11,SWZZ
    SWYY=SWYY+SWZZ
    PRINT 12,SWYY
189 PRINT 13,SA
    PRINT 14,SAY
    PRINT 15,SAZ
    PRINT 16,SAYY
    PRINT 17,SAYZ
    PRINT 18,SAZZ
```

FIND SHEAR FLOWS DUE TO TORQUE

COMPUTE Y-Z SHEAR PARAMETERS

OUTPUT

Figure 4a-9

```
      GO TO(190,190,193,193),KEYB
  190 PRINT 19,UZ
      PRINT 20,UY
      GO TO(191,191,193,193),KEYB
  191 PRINT 21,G
      GO TO(192,192,193,193),KEYB
  192 PRINT 22,UYY
      PRINT 23,UYZ
      PRINT 24,UZZ
  193 GO TO(99,194),KEYC
  194 GO TO (900,900,99,195),KEYB
  900 PRINT 25
      PRINT 26,(IP(I),QY(I),I=1,NP)
      PRINT 27
      PRINT 28,(IP(I),QZ(I),I=1,NP)
  195 PRINT 29
      PRINT 30 ,(IP(I),QT(I),I=1,NP)
      GO TO 99
  511 GO TO(512,513,513,515),KEYB
  512 WRITE OUTPUT TAPE 6,6,SW
      WRITE OUTPUT TAPE 6,7,SWY
      WRITE OUTPUT TAPE 6,8,SWZ
      WRITE OUTPUT TAPE 6,9,SWYY
      WRITE OUTPUT TAPE 6,10,SWYZ
      WRITE OUTPUT TAPE 6,11,SWZZ
      SWYY=SWYY+SWZZ
      WRITE OUTPUT TAPE 6,12,SWYY
  513 WRITE OUTPUT TAPE 6,13,SA
      WRITE OUTPUT TAPE 6,14,SAY
      WRITE OUTPUT TAPE 6,15,SAZ
      WRITE OUTPUT TAPE 6,16,SAYY
      WRITE OUTPUT TAPE 6,17,SAYZ
      WRITE OUTPUT TAPE 6,18,SAZZ
      GO TO(514,514,517,517),KEYB
  514 WRITE OUTPUT TAPE 6,19,UZ
      WRITE OUTPUT TAPE 6,20,UY
      GO TO(515,515,517,517),KEYB
  515 WRITE OUTPUT TAPE 6,21,G
      GO TO(516,516,517,517),KEYB
  516 WRITE OUTPUT TAPE 6,22,UYY
```

OUTPUT

Figure 4a-10

```
      WRITE OUTPUT TAPE 6,23,UYZ
      WRITE OUTPUT TAPE 6,24,UZZ
  517 GO TO(99,518),KEYC
  518 GO TO(519,519,99,520),KEYB
  519 WRITE OUTPUT TAPE 6,25
      WRITE OUTPUT TAPE 6,26,(IP(I),QY(I),I=1,NP)          OUTPUT
      WRITE OUTPUT TAPE 6,27
      WRITE OUTPUT TAPE 6,28,(IP(I),QZ(I),I=1,NP)
  520 WRITE OUTPUT TAPE 6,29
      WRITE OUTPUT TAPE 6,30,(IP(I),QT(I),I=1,NP)
      GO TO 99
 196  IF (SENSE SWITCH 6) 522,521
  521 PRINT 31
      GO TO 99                                              NOT PROPERLY CONNECTED
  522 WRITE OUTPUT TAPE 6,31
      GO TO 99
   25 FORMAT(29H SHEAR FLOWS PER UNIT Y SHEAR)
   26 FORMAT(I5,E15.4)
   27 FORMAT(29H SHEAR FLOWS PER UNIT Z SHEAR)
   28 FORMAT(I5,E15.4)
   29 FORMAT(28H SHEAR FLOWS PER UNIT TORQUE)
   30 FORMAT(I5,E15.4)
   31 FORMAT(40H THE STRUCTURE IS NOT PROPERLY CONNECTED)
```

Figure 4a-11

| | |
|---|---|
| A(150) | Area of node, input; ~~effective~~ tension area of node. |
| AK(150) | Tension effectiveness of node, input. |
| AD(150) | Density ratio of node, input. |
| C(30) | $Q^T R$ ($Q^T$ is the transpose of Q.) |
| DX | Length of hull segment, input. |
| G | Mass of node; length of plate; mass of plate; (used for many items in calculations), the last definition is 1/(effective polar moment for torque calculations) = $1/J_e$ . |
| I | Index used in many loops. |
| IN(150) | Node number, input ($\leq 150$). |
| IP(150) | Plate number, input ($\leq 150$). |
| IW(100) | Additional mass number (not used). |
| J | Index used in many loops. |
| JH | Node at head of plate. |
| JT | Node at tail of plate. |
| K | Index used in many loops. |
| KEY | Branching instruction, input. |
| KEYA | First digit of KEY, used to indicate symmetry. |
| KEYB | Second digit of KEY, used to choose which calculations to make. |
| KEYC | Third digit of KEY, used to output shear flows if desired. |
| KEYD | Used to indicate error in plate numbering. |
| L | Number of loop which is being found in matrix Q; number of loops. |
| LA | New node found on tree search; number of loops symmetric case. |
| LB | $\pm$ plate which makes connection in tree search. |

Figure 4b-1 — Index of FORTRAN Symbols and Switching
When one name is used for more than one variable, the definitions are separated by a semicolon.

| | |
|---|---|
| LC | Number of nodes on centerline. |
| LINK(150) | Plate which connects to next lower node on tree. |
| MTYPE(150) | Relation of plate to tree (+1 if it is on tree, -1 if it closes loop). |
| NC(30) | Numbers of nodes on centerline. |
| NEXT(150) | Next node down tree. |
| NH(150) | Number of node at head end of plate, input (≤150); changed to internal number. |
| NN | Number of nodes, input (≤150). |
| NP | Number of plates, input (≤150). |
| NRANK(150) | Internal node number for external node; order of searching nodes for tree. |
| NT(150) | Number of node at tail end of plate, input (≤150); changed to internal number. |
| NW | Number of additional mass items, input (≤100). |
| PD(150) | Density ratio of plate, input. |
| PG(150) | Shear modulus ratio of plate, input. |
| PK(150) | Plate effectiveness in tension, input. |
| PT(150) | Thickness of plate, input. |
| Q(150,30) | Matrix of tree loops; ortho-normal basis of loops. |
| QOY(150) | Shear flow leaving node per unit y-shear. |
| QOZ(150) | Shear flow leaving node per unit z-shear. |
| QT(150) | Shear flow due to torque. |
| QY(150) | Shear flow due to y-shear. |
| QZ(150) | Shear flow due to z-shear. |

Figure 4b-2

| | |
|---|---|
| R(150) | $\Delta s/Gt$ for plate. |
| RDS(150) | Twice the area of the triangle from origin to plate. |
| RHC | Density of material, input. |
| SA | Cumulative area. |
| SAY | Cumulative 1st moment of area; y coordinate of area centroid. |
| SAYY | Cumulative 2nd moment of area; moment about centroid. |
| SAYZ | Cumulative 2nd moment of area; moment about centroid. |
| SAZ | Cumulative 1st moment of area; z coordinate of area centroid. |
| SAZZ | Cumulative 2nd moment of area; moment about centroid. |
| SCALE | Scale factor to modify A(I) and PT(I), input. |
| SW | Cumulative mass. |
| SWY | Cumulative 1st moment of mass. |
| SWYY | Cumulative 2nd moment of mass. |
| SWYZ | Cumulative 2nd moment of mass. |
| SWZ | Cumulative 1st moment of mass. |
| SWZZ | Cumulative 2nd moment of mass. |
| UY | z-shear center |
| UYY | |
| UYZ | Shear parameters (last item in calculations). |
| UZ | |
| UZZ | y-shear center |
| W(100) | Weight of additional mass item, input. |
| WYY(100) | y inertia about center of gravity for mass item, input. |
| WYZ(100) | yz inertia about center of gravity for mass item, input. |
| WZZ(100) | z inertia about center of gravity for mass item, input. |

Figure 4b-3

| | |
|---|---|
| Y(150) | y coordinate of node, input. |
| YH | y coordinate of head of plate. |
| YT | y coordinate of tail of plate. |
| YW(100) | y coordinate of mass item, input. |
| Z(150) | z coordinate of node, input. |
| ZH | z coordinate of head of plate. |
| ZT | z coordinate of tail of plate. |
| ZW(100) | z coordinate of mass item, input. |

SENSE SWITCH 5 (Read Tape 5, Read Card)

SENSE SWITCH 6 (Write Tape 6, Print)

KEYA (General; Symmetric)

KEYB (Mass, Bending, Torsion, Shear; Bending, Shear, Torsion; Bending; Torsion)

KEYC (Beam Parameters Only; Output Shear Flow)

KEYD (Plate Numbering Okay; Error in Plate Numbering)

Figure 4b-4

Figure 5 – Flow Charts

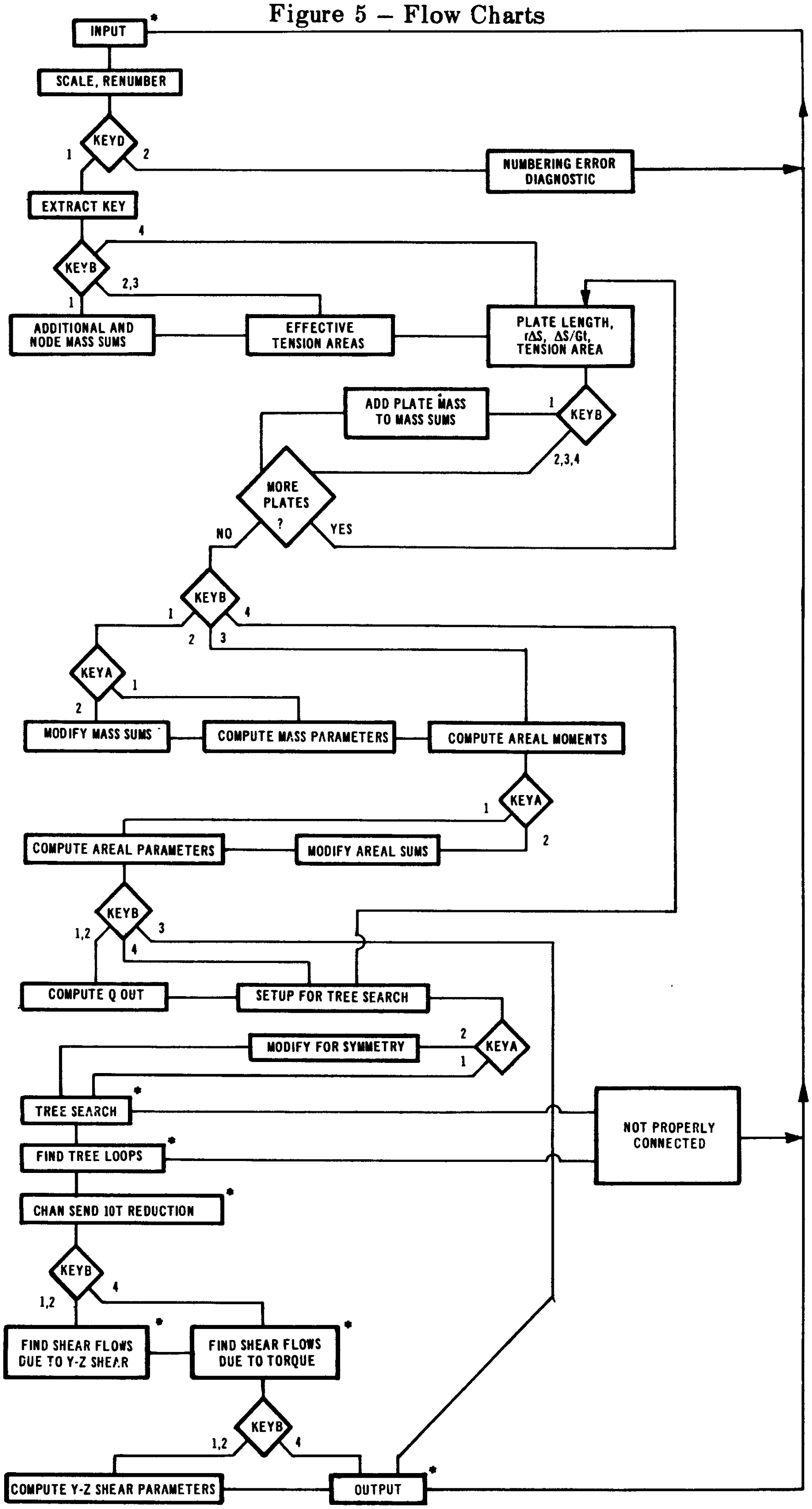
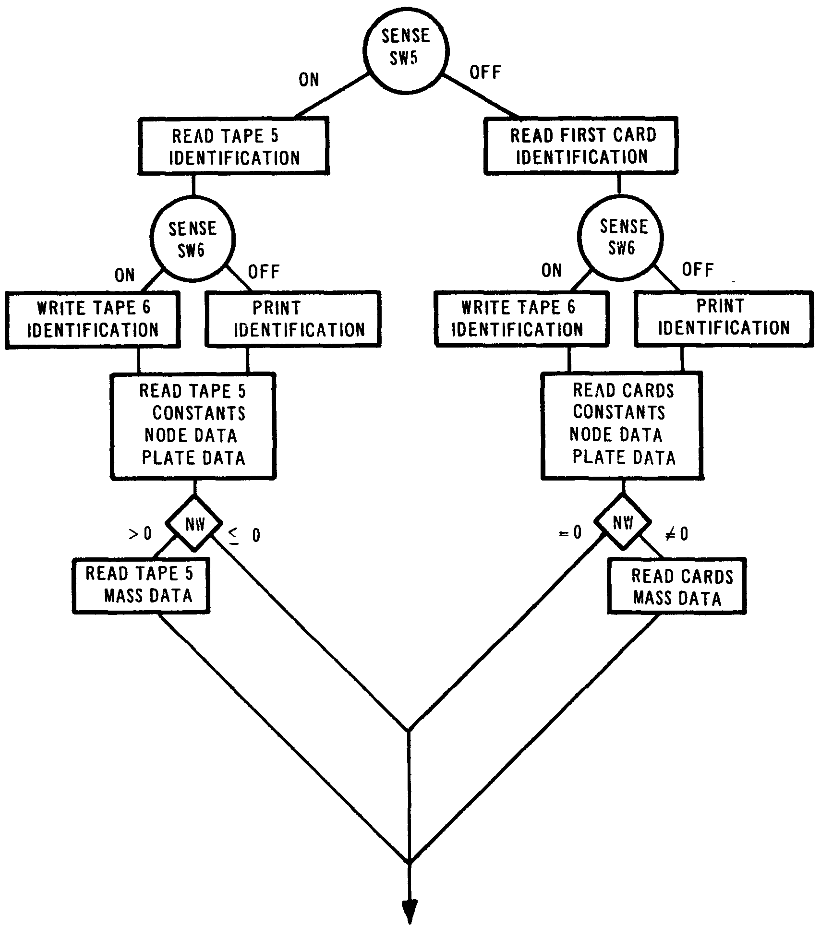



Figure 5a



SENSE SW5
ON
OFF
READ TAPE 5 IDENTIFICATION
READ FIRST CARD IDENTIFICATION
SENSE SW6
ON
OFF
WRITE TAPE 6 IDENTIFICATION
PRINT IDENTIFICATION
READ TAPE 5 CONSTANTS NODE DATA PLATE DATA
NW
> 0
≤ 0
READ TAPE 5 MASS DATA
SENSE SW6
ON
OFF
WRITE TAPE 6 IDENTIFICATION
PRINT IDENTIFICATION
READ CARDS CONSTANTS NODE DATA PLATE DATA
NW
= 0
≠ 0
READ CARDS MASS DATA


**Figure 5b**

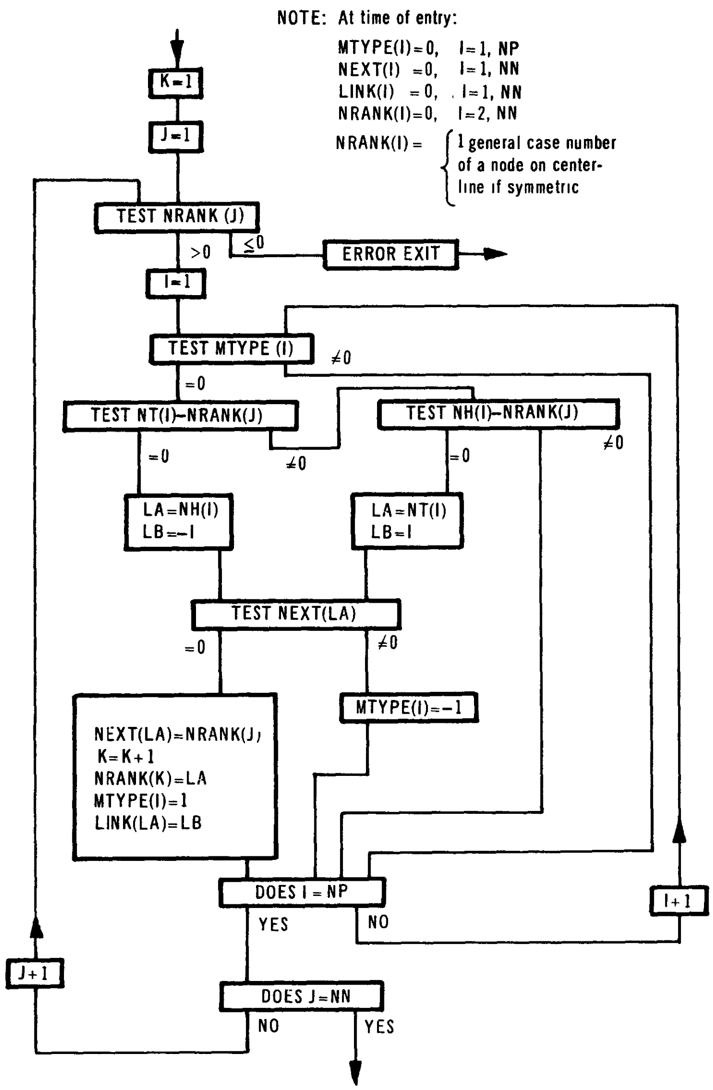

NOTE: At time of entry:
MTYPE(I)=0, I=1, NP
NEXT(I) =0, I=1, NN
LINK(I) =0, I=1, NN
NRANK(I)=0, I=2, NN
NRANK(I) = 1 general case number of a node on center-line if symmetric
K=1
J=1
TEST NRANK (J)
>0
≤0
ERROR EXIT
I=1
TEST MTYPE (I)
≠0
=0
TEST NT(I)–NRANK(J)
TEST NH(I)–NRANK(J)
=0
≠0
=0
≠0
LA=NH(I)
LB=–I
LA=NT(I)
LB=I
TEST NEXT(LA)
=0
≠0
NEXT(LA)=NRANK(J)
K=K+1
NRANK(K)=LA
MTYPE(I)=1
LINK(LA)=LB
MTYPE(I)=–1
DOES I = NP
YES
NO
I+1
J+1
DOES J=NN
NO
YES

Figure 5c

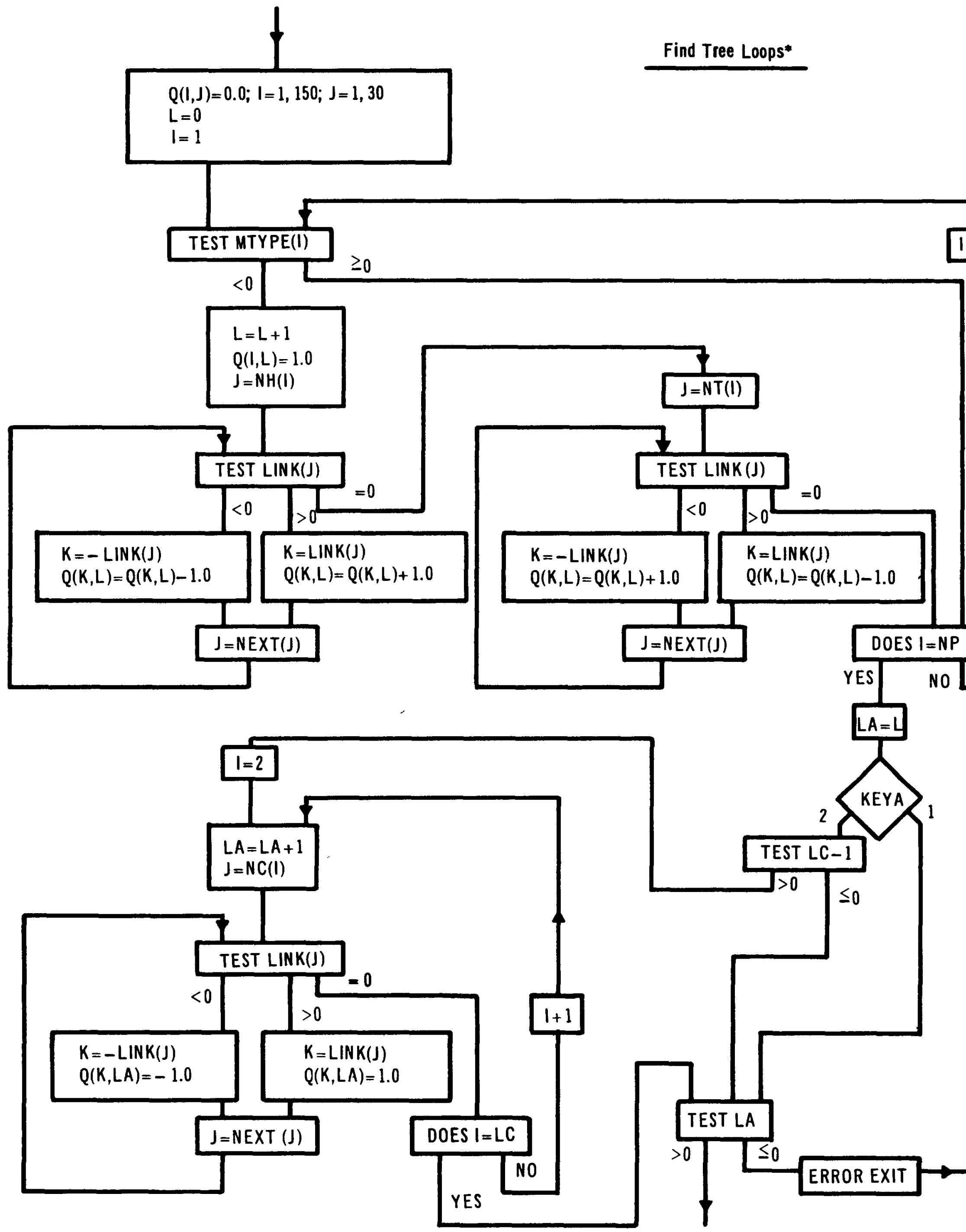
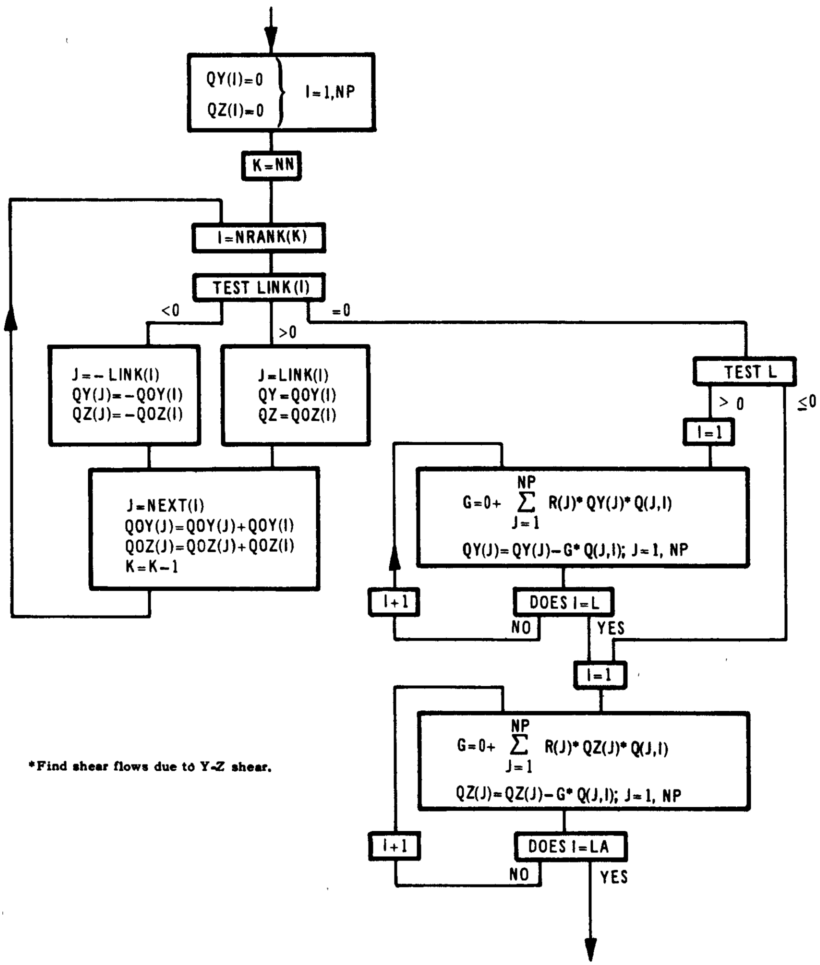
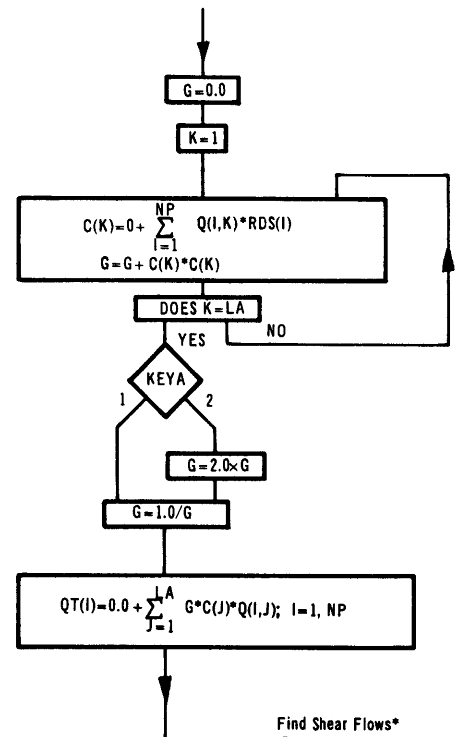

Find Tree Loops*
Q(I,J)=0.0; I=1, 150; J=1, 30
L=0
I=1
TEST MTYPE(I)
≥0
<0
I+1
L=L+1
Q(I,L)=1.0
J=NH(I)
J=NT(I)
TEST LINK(J)
=0
<0
>0
K=-LINK(J)
Q(K,L)=Q(K,L)-1.0
K=LINK(J)
Q(K,L)=Q(K,L)+1.0
J=NEXT(J)
TEST LINK(J)
=0
<0
>0
K=-LINK(J)
Q(K,L)=Q(K,L)+1.0
K=LINK(J)
Q(K,L)=Q(K,L)-1.0
J=NEXT(J)
DOES I=NP
YES
NO
LA=L
KEYA
2
1
I=2
LA=LA+1
J=NC(I)
TEST LC-1
>0
≤0
TEST LINK(J)
=0
<0
>0
K=-LINK(J)
Q(K,LA)=-1.0
K=LINK(J)
Q(K,LA)=1.0
I+1
J=NEXT (J)
DOES I=LC
NO
YES
TEST LA
>0
≤0
ERROR EXIT

Figure 5d




*Find shear flows due to Y-Z shear.

**Figure 5e**




Find Shear Flows*
Due to Torque

**Figure 5f**

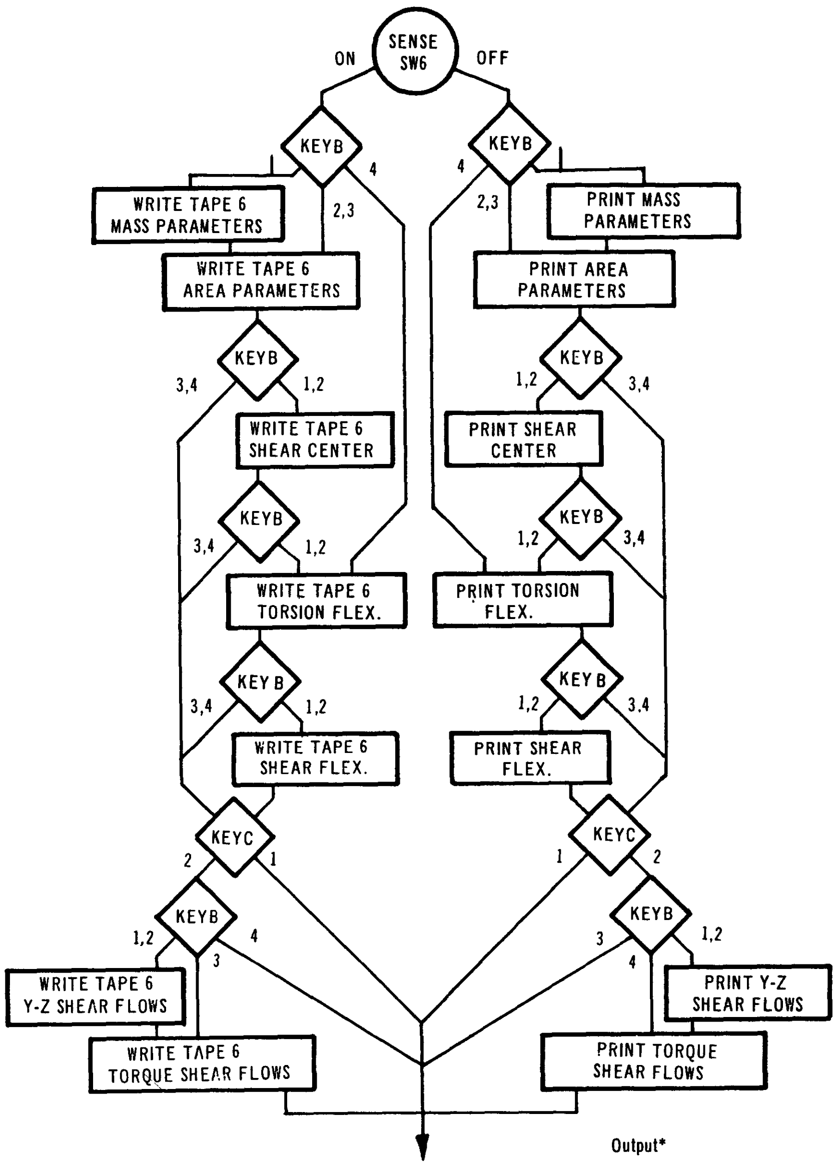

SENSE SW6
ON
OFF
KEYB
4
2,3
WRITE TAPE 6 MASS PARAMETERS
WRITE TAPE 6 AREA PARAMETERS
PRINT MASS PARAMETERS
PRINT AREA PARAMETERS
3,4
1,2
WRITE TAPE 6 SHEAR CENTER
PRINT SHEAR CENTER
WRITE TAPE 6 TORSION FLEX.
PRINT TORSION FLEX.
KEY B
WRITE TAPE 6 SHEAR FLEX.
PRINT SHEAR FLEX.
KEYC
1
2
3
WRITE TAPE 6 Y-Z SHEAR FLOWS
WRITE TAPE 6 TORQUE SHEAR FLOWS
PRINT Y-Z SHEAR FLOWS
PRINT TORQUE SHEAR FLOWS
Output*

Figure 5g

$$138.30 = \Sigma (7) \cdot (5)^2$$
$$-\ 97.80 = \Sigma (7) \cdot (5) \cdot (6)$$
$$132.88 = \Sigma (7) \cdot (6)^2$$

and the numerical value of the elements in the right-hand matrix of Equation [32] are

$$132.55 = \Sigma (7) \cdot (5) \cdot (4)$$
$$-\ 94.68 = \Sigma (7) \cdot (6) \cdot (4)$$

$\begin{Bmatrix} -0.9479 \\ +0.0148 \end{Bmatrix}$ = The solution $\begin{Bmatrix} K_1 \\ K_2 \end{Bmatrix}$ obtained by matrix inversion and multiplication, as indicated in Sheet 2 of Table 4.

In accordance with the statement following Equation [32], we substitute the $K_i$'s in Equations [11], [12], and [13] of Appendix A.2. This requires the use of Equations [14] and [15]. For the present problem, the procedure is then as follows:

Multiply Column (4) by 1.0 (see footnote on page 12), Column (5) by $(K_1) = (-0.9479)$ and Column (6) by $(K_2) = 0.0148$, and add to find Column (8), which is the shear flow distribution due to a unit y-shear.* The YY Flexibility is calculated at the right of Sheet 2 (see Equations [46]–[48] of Appendix A.2 and pages 84 and 85.) The z-shear center is obviously zero. In general, it is calculated by Equation [36] of Appendix A.2.

The z-shear calculation is almost the same, but now there is a third loop for antisymmetric forces, which is from nodes 1 to 6 (along centerline) and return via tree (see Column (26) on Sheet 2 and footnote on page 50).

Figure 6 and Sheet 2, Table 4 show how the sample hull cross section, which is symmetric and has five compartments, is treated with two loops for symmetric loading (y-shear), and with three loops for antisymmetric loading (z-shear). It is evident that the third loop cannot carry shear symmetrically. Column (26) defines the third loop. The z-shear solution now involves the solution of three simultaneous equations for $K_1$, $K_2$, and $K_3$ as follows (See Appendix A.2, Equation [32]).

$$\begin{bmatrix} 138.30 & -\ 97.80 & 67.12 \\ -97.80 & 132.88 & -52.80 \\ 67.12 & -\ 52.80 & 99.64 \end{bmatrix} = \begin{bmatrix} \Sigma (27)\ (24)^2 & \Sigma (27)\ (24)\ (25) & \Sigma (27)\ (24)\ (26) \\ \Sigma (27)\ (24)\ (25) & \Sigma (27)\ (25)^2 & \Sigma (27)\ (25)\ (26) \\ \Sigma (27)\ (24)\ (26) & \Sigma (27)\ (25)\ (26) & \Sigma (27)\ (26)^2 \end{bmatrix}$$

*The operation performed here is $(8) = (4) + (5)\,K_1 + (6)\,K_2$, which is equivalent to $\{q_j\} = \{q_{part\ j}\} + [L_{j1}]\{K_1\}$ (see Appendix A.2). Here, Column (4) is $\{q_{part\ j}\}$ by the indicated operation $\{q_{part\ j}\} = [T_{ji}]\{q_{out\ i}\}$ and Columns (5) and (6) constitute $[L_{j1}]$. Since this is done with $\bar{\bar{V}}_z = \bar{\bar{M}}_x = 0$ and $\bar{\bar{V}}_y = 1$, the values assumed by $\{q_j\}$ are those of $Q_{vy}$.

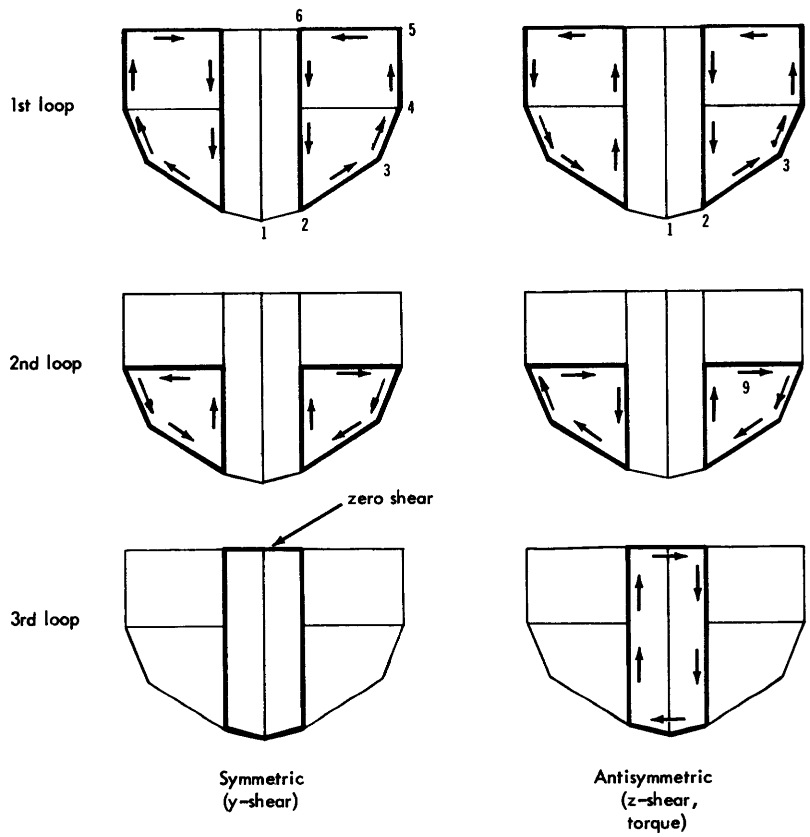



Figure 6 – Shear Flows in the Loops

$$\begin{bmatrix} \Sigma \frac{\Delta s_j}{t_j} L_{j1}^2 & \Sigma \frac{\Delta s_j}{t_j} L_{j1} L_{j2} & \Sigma \frac{\Delta s_j}{t_j} L_{j1} L_{j3} \\ \Sigma \frac{\Delta s_j}{t_j} L_{j2} L_{j1} & \Sigma \frac{\Delta s_j}{t_j} L_{j2}^2 & \Sigma \frac{\Delta s_j}{t_j} L_{j2} L_{j3} \\ \Sigma \frac{\Delta s_j}{t_j} L_{j3} L_{j1} & \Sigma \frac{\Delta s_j}{t_j} L_{j3} L_{j2} & \Sigma \frac{\Delta s_j}{t_j} L_{j3} \end{bmatrix}$$

$$\begin{Bmatrix} -294.20 \\ 287.14 \\ 395.80 \end{Bmatrix} = \begin{Bmatrix} \Sigma \; 27 \; 24 \; 23 \\ \Sigma \; 27 \; 25 \; 23 \\ \Sigma \; 27 \; 26 \; 23 \end{Bmatrix} = \begin{Bmatrix} \Sigma \frac{\Delta s_j}{t_j} L_{j1} \; q_{part\ j} \\ \Sigma \frac{\Delta s_j}{t_j} L_{j2} \; q_{part\ j} \\ \Sigma \frac{\Delta s_j}{t_j} L_{j3} \; q_{part\ j} \end{Bmatrix}$$

$\begin{Bmatrix} +4.736 \\ -1.926 \\ -8.185 \end{Bmatrix}$ = the solution $\begin{Bmatrix} K_1 \\ K_2 \\ K_3 \end{Bmatrix}$, obtained by matrix inversion and multiplication, as indicated in Sheet 2 of Table 4.

$Q_{vz}$ is obtained in a manner similar to that for finding $Q_{vy}$ as shown on Sheet 2 of Table 4. The ZZ Flexibility is calculated at the right of Sheet 2; see Equations [46]–[48] of Appendix A.2 and pages 84–85. The y-shear center is calculated by Equations [34] of Appendix A.2.

The solutions for torque involve the same loops as for z-shear; however, there is no particular solution due to $q_{out}$ because when $V_y = V_z = 0$, $q_{out} = 0$ at each node; see page 76 of Appendix A.2. The solution for $K_1$, $K_2$, and $K_3$ is now shown. (Refer also to Appendix A.2, Equation [37].)

$$\begin{bmatrix} 138.30 & -97.80 & 67.12 \\ -97.80 & 132.88 & -52.80 \\ 67.12 & -52.80 & 99.64 \end{bmatrix} = \begin{matrix} \Sigma \; 3 \; 4^2 & \Sigma \; 3 \; 4 \; 5 & \Sigma \; 3 \; 4 \; 6 \\ \Sigma \; 3 \; 4 \; 5 & \Sigma \; 3 \; 5^2 & \Sigma \; 3 \; 5 \; 6 \\ \Sigma \; 3 \; 4 \; 6 & \Sigma \; 3 \; 5 \; 6 & \Sigma \; 3 \; 6^2 \end{matrix}$$

$$\begin{Bmatrix} 10.2 \\ -7.3 \\ -12.8 \end{Bmatrix} = \begin{Bmatrix} \Sigma \; 2 \; 4 \\ \Sigma \; 2 \; 5 \\ \Sigma \; 2 \; 6 \end{Bmatrix} = \begin{Bmatrix} \Sigma \; (R\Delta s)_j \; L_{j1} \\ \Sigma \; (R\Delta s)_j \; L_{j2} \\ \Sigma \; (R\Delta s)_j \; L_{j3} \end{Bmatrix} = \begin{Bmatrix} \text{Area Loop 1} \\ \text{Area Loop 2} \\ \text{Area Loop 3} \end{Bmatrix}$$

(Entries are identical to those in the previous matrix) so that for $G \dfrac{d\theta}{dx} = 1$

$$\begin{Bmatrix} 0.190 \\ -0.024 \\ -0.273 \end{Bmatrix} = \text{the solution} \begin{Bmatrix} K_1 \\ K_2 \\ K_3 \end{Bmatrix}$$, obtained by matrix inversion and multiplication as indicated in Sheet 3. Columns (2) and (3) of Sheet 3 come from Columns (15) and (14) at the bottom of Sheet 1. Column (7) = (4) · $K_1$ + (5) · $K_2$ + (6) · $K_3$ , and is the shear flow per unit $G \dfrac{d\theta}{dx}$ ; see statement following Equation [37]. The net torque is (2) · (7) , and when Column (7) is divided by twice (for symmetry) this sum, then Column (8) is obtained.* The torsional flexibility is computed at the right of Sheet 3; see Equation [51] of Appendix A.2 and pages 84 and 85.

Comparison of these computed results shown on Sheets 1 through 3 can be made with Figure 2 and Table 3. The principal difference between these calculations and those used by the program are the units (in sample problem, scaling was done after computing, in program before computing) and the method of solving simultaneous equations. The weight calculation for the sample problem (discussed in section Inertial Parameters in Appendix A.2, which gives results agreeing with Table 3a) is presented in Table 4.

## CONCLUSIONS

A procedure has been developed for computing the inertia-elastic parameters of a ship hull in a mechanized manner by use of a digital computer. This procedure requires the routine tabulation of basic data systematically obtained in a prescribed fashion directly from ship plans for use as input to the digital computer. The computer then calculates the ship parameters as output to be used in the finite difference form of the beam vibration equations developed in Reference 1. Such mechanization fits the trend toward routinizing complex calculations leading to eventual design utility.

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*If $V_y = V_z = q_{out} = 0$, $\{K_i\}$ is found from Equation [37] of Appendix A.2 for $G \dfrac{d\theta}{dx} = 1$. Then from Equations [11], [13], and [38] $\{q_j\}_{G \frac{d\theta}{dx} = 1} = \{q_{loop}\}_{G \frac{d\theta}{dx} = 1} = [L_{ji}]\,\{K_i\}_{G \frac{d\theta}{dx} = 1} = \{Q_{\theta j}\}$ which is represented by Column (7). From Equation [26] the net torque is $\Sigma$ (2) · (7) where (7) and, therefore, the torque corresponds to a value of $G \dfrac{d\theta}{dx} = 1$. The shear flow for $V_y = V_z = 0$, $\overline{\overline{M}}_x = 1$ (but $G \dfrac{d\theta}{dx} \neq 1$) is obtained from Equation [40]. Thus $\{q_j\}_{V_y = V_z = 0} = \dfrac{(7)}{2\Sigma (7) \cdot (2)} = \dfrac{(7)}{2 \cdot 563} = 0.888 \cdot 10^{-3} \cdot (7)$ or $q_j \cdot 10^3 = 0.888 \cdot 10^7$, which is Column (8). The factor 2 in the denominator has been added for symmetry.

## RECOMMENDATIONS

1. Both manual and digital computer calculations of the section properties of ship hulls (i.e., equivalent beam parameters) should be made for a number of ships.

2. The digital computer method for obtaining these properties should be generally used if the comparison is favorable or if the comparison between theoretical results (e.g., frequencies, mode shapes, whipping response to slam, flutter response), based on the computer program, and experiment is at least as good as the comparison between theoretical results, based on hand computations, and experiment.

## ACKNOWLEDGMENTS

The author is indebted to William J. Dixon of Computer Engineering Associates for his contribution to the work presented in this report.

# APPENDIX A

The author recognizes that reader interest will vary widely on the theoretical aspect of the paper. A working knowledge of the physical meaning of the equations used in the coding may be required only. Or, interest may also be centered upon all of the fundamental ideas underlying the derivation and use of these equations. Therefore, this Appendix has been subdivided into two parts; Appendix A.1, which describes the method for evaluating the section properties of the ship (sufficient for understanding the general procedure), and Appendix A.2, which supplements this description with additional fundamental concepts and mathematical detail. Thus, the theory can be pursued to the degree desired.

## A.1 – METHOD FOR EVALUATING SECTION PROPERTIES

The theory of beams may be considered as the limiting case of the general theory of elasticity applied to slender objects. In the theory of elasticity, the displacements and stresses are unknown functions of position. Strain displacement, stress-strain, and equilibrium laws are available to solve for the unknowns. Most engineers consider the strain-displacement laws and the equilibrium laws as independent unrelated ideas; however, one is obtainable from the other by using the stress-strain law and a minimum principle (minimum potential energy theorem). In the theory of beams, instead of taking unknowns in three spatial dimensions, quantities are defined only along one line, the "axis" of the beam. The unknowns become six displacements (linear displacements in three directions and rotations about three axes) of the cross section, and six forces* (tension, two bending moments, two shears, and a torque). In the following, the elastic relations between the forces and displacements will be found for beams constructed of stringers and plates. The equilibrium laws which come from an application of Newton's Second Law (force = mass · acceleration) are not given, but they may easily be found to complete the beam theory.

Choose a rectangular cartesian coordinate system with the x-axis along the beam and the y- and z-axis such as to form a right-hand coordinate system; see Figure 3. The displacements of a cross section parallel and perpendicular to the x-axis will be given by $U_x$ and $U_y$, $U_z$, respectively. The rotations of the cross section about these axes are $\theta_x$, $\theta_y$, and $\theta_z$, where positive sense is given by the right-hand rule. The resultant force associated with these motions acting on the positive side of the cross section (acting upon the body which consists of those portions of the beam on the -x side of the section) will have three linear components $V_x$, $V_y$, and $V_z$ and three moments $M_x$, $M_y$, and $M_z$. All displacements, rotations, moments, and forces are positive if the vector which represents them is in the positive coordinate direction. In general, these 12 unknowns are functions of x (and possibly time). Six equations for the unknowns come from equilibrium; the other six from elasticity.

*Forces here is used in a generic sense in that it includes moments and torques.

In the following, we shall use the Theorem of Castigliano,[7] which is a corollary of the energy theorem. The theorem states that if the strain energy is written in terms of the applied forces, the displacements at the point of application of any force (in the direction of that force) is the partial derivative of the strain energy with respect to the force. Thus, if W is the strain energy in that part of the beam corresponding to a value of x, then

$$U_x = \frac{\partial W}{\partial V_x} \;; \qquad U_y = \frac{\partial W}{\partial V_y} \;; \qquad U_z = \frac{\partial W}{\partial V_z}$$

$$\theta_x = \frac{\partial W}{\partial M_x} \;; \qquad \theta_y = \frac{\partial W}{\partial M_y} \;; \qquad \theta_z = \frac{\partial W}{\partial M_z}$$

Consider a short segment going from x to x + Δx. The foregoing expressions will give the elastic deformations, to which we *add the rigid-body motions*, due to deflections at station x. Let the strain energy between x and x + Δx be $\Delta x\,\overline{W}$. Then the total deflections at x + Δx are given by

$$U_x\Big|_{x+\Delta x} = U_x\Big|_x + \Delta x \frac{\partial \overline{W}}{\partial V_x}$$

$$U_y\Big|_{x+\Delta x} = U_y\Big|_x + \Delta x\,\theta_z\Big|_x + \Delta x \frac{\partial \overline{W}}{\partial V_y}$$

$$U_z\Big|_{x+\Delta x} = U_z\Big|_x - \Delta x\,\theta_y\Big|_x + \Delta x \frac{\partial \overline{W}}{\partial V_z}$$

$$\theta_x\Big|_{x+\Delta x} = \theta_x\Big|_x + \Delta x \frac{\partial \overline{W}}{\partial M_x}$$

$$\theta_y\Big|_{x+\Delta x} = \theta_y\Big|_y + \Delta x \frac{\partial \overline{W}}{\partial M_y}$$

$$\theta_z\Big|_{x+\Delta x} = \theta_z\Big|_z + \Delta x \frac{\partial \overline{W}}{\partial M_z}$$

If Δx goes to zero, $\overline{W}$ becomes the strain energy per unit length. Then

$$\frac{\partial \overline{W}}{\partial V_x} = \frac{dU_x}{dx} \;; \qquad \frac{\partial \overline{W}}{\partial V_y} = \frac{dU_y}{dx} - \theta_z \;; \qquad \frac{\partial \overline{W}}{\partial V_z} = \frac{dU_z}{dx} + \theta_y$$

$$\frac{\partial \overline{W}}{\partial M_x} = \frac{d\theta_x}{dx} \;; \qquad \frac{\partial \overline{W}}{\partial M_y} = \frac{d\theta_y}{dx} \;; \qquad \frac{\partial \overline{W}}{\partial M_z} = \frac{d\theta_z}{dx}$$

If the strain energy per unit length $\overline{W}$ can be expressed in terms of $V_x$, $V_y$, $V_z$, $M_x$, $M_y$, and $M_z$, these six equations will give the desired elastic equations.

Beam theory assumes that the stresses $\sigma_{yy}$, $\sigma_{yz}$, and $\sigma_{zz}$ vanish ($\sigma_{ij}$ is defined to be the force per unit area acting on a face perpendicular to the i-axis and in the j-direction). Thus the strain energy per unit length is given by (see Chapter 6 of Reference 7):

$$\overline{W} = \frac{1}{2} \int_{\text{area}} \left[ \frac{\sigma_{xx}^2}{E} + \frac{\sigma_{xy}^2 + \sigma_{xz}^2}{G} \right] dA$$

The stresses $\sigma_{xx}$, $\sigma_{xy}$, and $\sigma_{xz}$ must be determined in terms of the beam forces $V_x$, . . . , $M_z$. Statics alone is not sufficient, and assumptions consistent with the theory of elasticity must be made to solve for the stresses. If the *distribution* (except for a constant factor) of the stresses is known, then statics can be used to find the stresses (find the factor). Assume that the $\sigma_{xx}$ stresses are due to $V_x$, $M_y$, and $M_z$; $\sigma_{xy}$ and $\sigma_{xz}$ stresses are due to $M_x$, $V_y$, and $V_z$.

For $\sigma_{xx}$ stresses, let $F_1(y, z)$, $F_2(y, z)$ be three given functions and $K_1$, $K_2$, and $K_3$ be three unknown constants. As a simple example, the functions might be selected (see Chapter 7 of Reference 8 or Chapter VI of Reference 9):

$$F_1 = 1; \qquad F_2 = y; \qquad F_3 = z$$

This would duplicate the stresses existing due to tension, moment about z-axis, and moment about y-axis if elementary beam theory is adopted, requiring tensile strain proportional to the distance from the elastic axis when bending moment is carried; i.e., the basic assumption of beam theory is that the longitudinal strain in the ship hull, deck, longitudinal members, etc., varies linearly with the coordinates of a cross section. Hence assume*

$$\sigma_{xx}(y, z) = K_1F_1(y, z) + K_2F_2(y, z) + K_3F_3(y, z)$$

Applying statics gives:

$$\begin{aligned}
V_x &= \int \sigma_{xx} dA &&= K_1 \int F_1 dA + K_2 \int F_2 dA + K_3 \int F_3 dA \\
M_y &= \int z\sigma_{xx} dA &&= K_1 \int zF_1 dA + K_2 \int zF_2 dA + K_3 \int zF_3 dA \\
M_z &= \int (-y)\sigma_{xx} dA &&= K_1 \int (-y)F_1 dA + K_2 \int (-y)F_2 dA + K_3 \int (-y)F_3 dA
\end{aligned}$$

Since $F_1$, $F_2$, and $F_3$ are assumed to be known functions, the above three equations can be solved for $K_1$, $K_2$, and $K_3$ as linear homogenous expressions in $V_x$, $M_y$, and $M_z$;

$$\text{i.e.,} \quad \begin{Bmatrix} K_1 \\ K_2 \\ K_3 \end{Bmatrix} = \begin{bmatrix} \int F_1 dA & \int F_2 dA & F_3 dA \\ \int zF_1 dA & \int zF_2 dA & zF_3 dA \\ -\int yF_1 dA & -\int yF_2 dA & -\,yF_3 dA \end{bmatrix}^{-1} \begin{Bmatrix} V_x \\ M_y \\ M_z \end{Bmatrix}$$

*This will be shown to give rise to the bending parameters.

Substituting these into the formula for $\sigma_{xx}(y, z)$, we get

$$\sigma_{xx}(y, z) = V_x G_1(y, z) + M_y G_2(y, z) + M_z G_3(y, z)$$

where $G_1$, $G_2$, and $G_3$ are known linear homogenous functions of $F_1$, $F_2$, and $F_3$. By substituting this expression for $\sigma_{xx}(y, z)$ into the above definitions of $V_x$, $M_y$, and $M_z$, we obtain:

$$V_x = V_x \int G_1 dA + M_y \int G_2 dA + M_z \int G_3 dA$$
$$M_y = V_x \int z G_1 dA + M_y \int z G_2 dA + M_z \int z G_3 dA$$
$$M_z = -V_x \int y G_1 dA - M_y \int y G_2 dA - M_z \int y G_3 dA$$

Then inserting the following conditions (one at a time)

$$V_x = 1; \quad M_y = 0; \quad M_z = 0$$
$$V_x = 0; \quad M_y = 1; \quad M_z = 0$$
$$V_x = 0; \quad M_y = 0; \quad M_z = 1$$

It is seen that the G functions must satisfy the following relations:

$$\int G_1 dA = 1; \quad \int G_2 dA = 0; \quad \int G_3 dA = 0$$
$$\int z G_1 dA = 0; \quad \int z G_2 dA = 1; \quad \int z G_3 dA = 0$$
$$\int y G_1 dA = 0; \quad \int y G_2 dA = 0; \quad \int y G_3 dA = 1$$

Then the terms in $\overline{W}$, depending upon $\sigma_{xx}$, become:

$$\overline{W}\bigg|_{\sigma_{xx}\ \text{terms}} = \frac{1}{2} \int_{\text{area}} \frac{\left[V_x G_1(y, z) + M_y G_2(y, z) + M_z G_3(y, z)\right]^2}{E} dA$$

Hence

$$\frac{dU_x}{dx} = \frac{\partial \overline{W}}{\partial V_x} = V_x \int_{\text{area}} \frac{G_1^2}{E} dA + M_y \int_{\text{area}} \frac{G_1 G_2}{E} dA + M_z \int_{\text{area}} \frac{G_1 G_3}{E} dA$$

$$\frac{\partial \theta_y}{\partial x} = \frac{\partial \overline{W}}{\partial M_y} = V_x \int_{\text{area}} \frac{G_1 G_2}{E} dA + M_y \int_{\text{area}} \frac{G_2^2}{E} dA + M_z \int_{\text{area}} \frac{G_2 G_3}{E} dA$$

$$\frac{\partial \theta_z}{\partial x} = \frac{\partial \overline{W}}{\partial M_z} = V_x \int_{\text{area}} \frac{G_3 G_1}{E} dA + M_y \int_{\text{area}} \frac{G_3 G_2}{E} dA + M_z \int_{\text{area}} \frac{G_3^2}{E} dA$$

These equations are three of the elastic equations for the beam. The other three can be derived from the $\sigma_{xy}$ and $\sigma_{xz}$ terms of the strain energy and will give expressions for

$\frac{\partial\theta_x}{\partial x}$, $\frac{\partial U_y}{\partial x} - \theta_z$, and $\frac{\partial U_z}{\partial x} + \theta_y$.* Thus, in order to find the beam parameters, all that is is needed from the theory of elasticity is the distribution of the stresses over the cross section!

These equations can be simplified somewhat by choosing a particular coordinate system. Let

$$I_{ij} \equiv \int_{\text{area}} \frac{G_i G_j}{E} dA$$

Then

$$I_{12} = I_{21}, \text{ etc.}$$

and

$$\frac{\partial U_x}{\partial x} = I_{11}V_x + I_{12}M_y + I_{13}M_z$$

$$\frac{\partial \theta_y}{\partial x} = I_{21}V_x + I_{22}M_y + I_{23}M_z$$

$$\frac{\partial \theta_z}{\partial x} = I_{31}V_x + I_{32}M_y + I_{33}M_z$$

Choose a new coordinate system whose origin is at $\bar{y}$, $\bar{z}$ in the original coordinate system. Let $\overline{U}_x$, $\overline{\theta}_y$, $\overline{\theta}_z$, $\overline{V}_x$, $\overline{M}_y$, and $\overline{M}_z$ be the unknowns in this new system (see Figure 3). Then

$$\overline{U}_x = U_x + \bar{z}\,\theta_y - \bar{y}\,\theta_z$$

$$\overline{\theta}_y = \theta_y ; \qquad \overline{\theta}_z = \theta_z$$

$$V_x = \overline{V}_x ; \qquad M_y = \overline{M}_y + \bar{z}\,\overline{V}_x ; \qquad M_z = \overline{M}_z - \bar{y}\,\overline{V}_x$$

---

*To similarly derive expressions for $\partial\theta_x/\partial x$, $(\partial U_y/\partial x) - \theta_z$, $(\partial U_z/\partial x) + \theta_y$ in terms of assumed distributions of $\sigma_{xy}$ and $\sigma_{xz}$ over the cross section we would repeat the development for $dU_x/dx$, $\partial\theta_y/\partial x$, $\partial\theta_z/\partial x$ almost identically. The importance of this derivation is not the integrals $(G_1G_2/E)dA$, etc., but the demonstration that the strain energy per unit length $\overline{\overline{W}}$ is a quadratic form in the terms $V_x$, $M_y$, $M_z$ (for terms dependent on $\sigma_{xx}$) and a quadratic form in the terms $V_y$, $V_z$, $M_x$ (for terms dependent on $\sigma_{xy}$ and $\sigma_{xz}$); see page 74.

The value of the expressions for $dU_x/dx$, etc., is to validate the use of Castigliano's Theorem in obtaining expressions for flexibility terms based on energy expressions.

As seen in the subsequent theory, the actual distribution chosen for $\sigma_{xy}$ and $\sigma_{xz}$ (i.e., the shear flows in the plates) is forced to be compatible with and dependent on the distribution chosen for $\sigma_{xx}$; namely, that indicated by $F_1 = 1$, $F_2 = y$, $F_3 = z$, and $\sigma_{xx}(y, z) = K_1F_1 + K_2F_2 + K_3F_3$, which is restated on page 52.

Then $$\bar{I}_{11} = I_{11} + 2\bar{z}I_{12} - 2\bar{y}I_{13} + \bar{z}^2I_{22} - 2\bar{y}\bar{z}I_{23} + \bar{y}^2I_{33}$$

$$\bar{I}_{12} = I_{12} + \bar{z}I_{22} - \bar{y}I_{32}$$

$$\bar{I}_{13} = I_{13} + \bar{z}\,I_{23} - \bar{y}\,I_{33}$$

$$\bar{I}_{22} = I_{22}; \quad \bar{I}_{23} = I_{23}; \quad \bar{I}_{33} = I_{33}$$

Since $I_{22}\,I_{33} - I_{23}^2$ will in general be nonzero, it is possible to solve for $\bar{y}$ and $\bar{z}$ such that $I_{12} = I_{13} = 0$ (i.e., select $\bar{y}$, $\bar{z}$ coordinate system such that $\bar{I}_{12} = \bar{I}_{13} = 0$). If this barred coordinate system is used, it is customary to call $\bar{I}_{11} = 1/EA$, $\bar{I}_{22} = I_{yy}/E(I_{yy}I_{zz} - I_{yz}{}^2)$, $\bar{I}_{23} = I_{yz}/E(I_{yy}I_{zz} - I_{yz}{}^2)$, and $\bar{I}_{33} = I_{zz}/E(I_{yy}I_{zz} - I_{yz}{}^2)$; Equation [10] of Appendix A.2 validates the expressions for $\bar{I}_{22}, \bar{I}_{23}, \bar{I}_{33}$. (These terms are defined conventionally either as geometric integrals, or by the geometric summations given below.) Thus*

$$\frac{\partial \bar{U}_x}{\partial x} = \frac{\bar{V}_x}{EA} \tag{1a}$$

$$\frac{\partial \bar{\theta}_y}{\partial x} = \frac{I_{yy}\bar{M}_y + I_{yz}\bar{M}_z}{E(I_{yy}I_{zz} - I_{yz}^2)} \tag{1b}$$

$$\frac{\partial \bar{\theta}_z}{\partial x} = \frac{I_{yz}\bar{M}_y + I_{zz}\bar{M}_z}{E(I_{yy}I_{zz} - I_{yz}^2)} \tag{1c}$$

Thus the choice of this coordinate system (elastic axis coordinates $\bar{y}, \bar{z}$) *uncouples* the tension and pure bending elastic equations. For the other three equations, a coordinate system (generally not the barred system for bending) $\bar{\bar{y}}$, $\bar{\bar{z}}$ can be found to uncouple the torsion from the shear. The center of this system is called the shear center.

$$\frac{\partial \bar{\bar{U}}_y}{\partial x} - \bar{\bar{\theta}}_z = \frac{1}{KA_{yy}G}\bar{\bar{V}}_y + \frac{1}{KA_{yz}G}\bar{\bar{V}}_z \tag{2a}$$

$$\frac{\partial \bar{\bar{U}}_z}{\partial x} + \bar{\bar{\theta}}_y = \frac{1}{KA_{yz}G}\bar{\bar{V}}_y + \frac{1}{KA_{zz}G}\bar{\bar{V}}_z \tag{2b}$$

$$\frac{\partial \bar{\bar{\theta}}_x}{\partial x} = \frac{1}{GJ_e}\bar{\bar{M}}_x \tag{2c}$$

The foregoing equations follow from the discussion in the footnote on page 48, that the shear and torsion deformations *can be* expressed in the *form:*

$$\frac{\partial U_y}{\partial x} - \theta_z = N_{11}V_y + N_{12}V_z + N_{13}M_x$$

*An alternative method of derivation is given in Appendix A.2.

$$\frac{\partial U_z}{\partial x} + \theta_y = N_{21}V_y + N_{22}V_z + N_{23}M_x$$

$$\frac{\partial \theta_x}{\partial x} = N_{31}V_y + N_{23}V_z + N_{33}M_x$$

where the N's are constants for a given section and $N_{12} = N_{21}$, etc.

Next, redefine quantities with respect to axes through a point s at $y = \bar{\bar{y}}$, $z = \bar{\bar{z}}$ by these equations:

$$V_y = \bar{\bar{V}}_y \qquad \bar{\bar{U}}_y = U_y - \bar{\bar{z}}\,\theta_x \qquad \bar{\bar{\theta}}_x = \theta_x$$

$$V_z = \bar{\bar{V}}_z \qquad \bar{\bar{U}}_z = U_z + \bar{\bar{y}}\,\theta_x \qquad \bar{\bar{\theta}}_y = \theta_y$$

$$M_x = \bar{\bar{M}}_x - \bar{\bar{z}}\,V_y + \bar{\bar{y}}\,V_z \qquad \bar{\bar{\theta}}_z = \theta_z$$

Substituting in the above equations for deformations gives:*

$$\begin{Bmatrix} \dfrac{\partial \bar{\bar{U}}_y}{\partial x} - \theta_z \\ \dfrac{\partial \bar{\bar{U}}_z}{\partial x} + \theta_y \\ \dfrac{\partial \bar{\bar{\theta}}_x}{\partial x} \end{Bmatrix} = \begin{bmatrix} 1 & 0 & -\bar{\bar{z}} \\ 0 & 1 & +\bar{\bar{y}} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} N_{11} & N_{12} & N_{13} \\ N_{12} & N_{22} & N_{23} \\ N_{13} & N_{23} & N_{33} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -\bar{\bar{z}} & +\bar{\bar{y}} & 1 \end{bmatrix} \begin{Bmatrix} \bar{\bar{V}}_y \\ \bar{\bar{V}}_z \\ \bar{\bar{M}}_x \end{Bmatrix}$$

$$= \begin{bmatrix} N_{11} - 2\bar{\bar{z}}N_{13} + \bar{\bar{z}}^2N_{33} & N_{12} + \bar{\bar{y}}N_{13} - \bar{\bar{z}}N_{23} - \bar{\bar{y}}\bar{\bar{z}}N_{33} & N_{13} - \bar{\bar{z}}N_{33} \\ N_{12} + \bar{\bar{y}}N_{13} - \bar{\bar{z}}N_{23} - \bar{\bar{y}}\bar{\bar{z}}N_{33} & N_{22} + 2\bar{\bar{y}}N_{23} + \bar{\bar{y}}^2N_{33} & N_{23} + \bar{\bar{y}}N_{33} \\ N_{13} - \bar{\bar{z}}N_{33} & N_{23} + \bar{\bar{y}}N_{33} & N_{33} \end{bmatrix} \begin{Bmatrix} \bar{\bar{V}}_y \\ \bar{\bar{V}}_z \\ \bar{\bar{M}}_x \end{Bmatrix}$$

Next choose $\bar{\bar{y}} = -\dfrac{N_{23}}{N_{33}}$ and $\bar{\bar{z}} = +\dfrac{N_{13}}{N_{33}}$ Now the above equations may be written:

---

*To transform the variables, let $\begin{Bmatrix} \bar{\bar{U}}_y \\ \bar{\bar{U}}_z \\ \bar{\bar{\theta}}_x \end{Bmatrix} = \begin{bmatrix} 1 & 0 & -\bar{\bar{z}} \\ 0 & 1 & \bar{\bar{y}} \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} U_y \\ U_z \\ \theta_x \end{Bmatrix}$ and $\begin{Bmatrix} V_y \\ V_z \\ M_x \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -\bar{\bar{z}} & \bar{\bar{y}} & 1 \end{bmatrix} \begin{Bmatrix} \bar{\bar{V}}_y \\ \bar{\bar{V}}_z \\ \bar{\bar{M}}_x \end{Bmatrix}$

Then $\{U\} = [N]\{V\}$, $\{\bar{\bar{U}}\} = [Q_1]\,[N]\,[Q_2]\,\{\bar{\bar{V}}\}$.

$$\begin{Bmatrix} \dfrac{\partial \bar{\bar{U}}_y}{\partial x} - \bar{\bar{\theta}}_z \\ \dfrac{\partial \bar{\bar{U}}_z}{\partial x} + \bar{\bar{\theta}}_y \\ \dfrac{\partial \bar{\bar{\theta}}_x}{\partial x} \end{Bmatrix} = \begin{bmatrix} \bar{\bar{N}}_{11} & \bar{\bar{N}}_{12} & 0 \\ \bar{\bar{N}}_{12} & \bar{\bar{N}}_{22} & 0 \\ 0 & 0 & \bar{\bar{N}}_{33} \end{bmatrix} \begin{Bmatrix} \bar{\bar{V}}_y \\ \bar{\bar{V}}_z \\ \bar{\bar{M}}_x \end{Bmatrix}$$

where the definitions of $\bar{\bar{N}}_{11}$, $\bar{\bar{N}}_{12}$, etc., are obvious. Such a choice of $\bar{\bar{y}}$ and $\bar{\bar{z}}$ uncouples shear deformation from torsion deformation and is said to locate point s at the "shear center" of the beam. The $\bar{\bar{N}}$ matrix may be written:*

$$\begin{bmatrix} \dfrac{1}{KA_{yy}G} & \dfrac{1}{KA_{yz}G} & 0 \\ \dfrac{1}{KA_{yz}G} & \dfrac{1}{KA_{zz}G} & 0 \\ 0 & 0 & \dfrac{1}{GJ_e} \end{bmatrix}$$

which yields Equations [2a, b, c]. The uncoupled form of this expression is validated by the above development. The symbols are arbitrary but are chosen to be written in conventional form. This expression (matrix) itself is a definition of the symbols $A_{ij}$, or $A_{yy}$, $A_{yz}$, $A_{zz}$, and $J_e$.

In the accompanying program, each of the above coefficients $\left(\dfrac{1}{KA_{ij}G}\right)$ appear as a single number. However, here they are written as products of several terms for comparison with the conventional shear and torsion flexibility coefficients $\left(\dfrac{1}{KAG}\right)$ and $\left(\dfrac{1}{GJ_e}\right)$.

Equations [1a, b, c] and [2a, b, c] are the *six elastic equations for beam theory*. For ship problems, Equation [1a] is usually not used. For motions symmetric with respect to the x-y plane, use Equations [1c] and [2a]. By rotating the y-z coordinates, it would be possible to completely uncouple the equations (i.e., choose principal axes so that $I_{yz} = 0$), but this has not been done here.** For symmetric sections typical of ship hulls, the axes chosen are principal axes.

---

*The $\bar{\bar{N}}$ matrix may be written as shown because the left-hand side of the matrix equation above is related to the shear and moment terms on the right side respectively, by constants which are called the shear and torsional flexibilities having the form $1/KAG$ and $1/GJ_e$, respectively.

**The terms $A_{ij}$ are defined by the above matrix. The program of this report is applicable to sections of any structures which are prismatic and may be treated as beams. These structures may be symmetric or unsymmetric. The sample problem chosen is symmetric (as are most ship hulls) and is, therefore, a special case of the general theory presented. The following terms exist in general, but are zero in the special case (symmetric with respect to the y-axis): $I_{yz}$, $\bar{z}$, $1/A_{yz}$, $\bar{\bar{z}}$. It is true that Figure 1 appears to have a symmetric outline, but it need not have.

For *cross sections consisting of stringers and plates,* we make the following assumptions in order to calculate the *tension stresses*:

1. All of the area has been concentrated into points which shall be called nodes. This is done by assigning the areas of the plates and stringers to the nearby nodes. By this means, the integrals on page 46 can be replaced by sums.

2. For effective members, the strain is a linear function of position $K_1 + K_2y + K_3z$. Some members may end near the cross section to be analyzed and, hence, their stress would be less than a completely effective member. For the nodes, an "effectiveness" is assigned which is 1.0 for effective members* and less for others. Thus the assumed form for stresses is (see page 156 of Reference 8 or page 209 of Reference 9):

$$\sigma_{xx} = K_1(kE) + K_2(kEy) + K_3(kEz)$$

where $K_1, K_2, K_3$ are unknown constants to be determined as shown above,
$k$ is effectiveness,
$E$ is Young's modulus, and
$y, z$ are coordinates.

Except for the addition of effectiveness and the possibility of having different moduli at the nodes, this is exactly the same as ordinary beam theory and would give the usual equations (this means that "ordinary beam theory" is based on a form of distribution of the tensile stress such as $\sigma_{xx} = K_1 + K_2y + K_3z$). Another factor $k' = k\dfrac{E}{E_o}$ ($E_o$ = reference value of modulus) is defined so that the tension at node i is given by:

$$(\sigma_{xx})_i = E_o\left[K_1(k'_i) + K_2(k'_iy_i) + K_3(k'_iz_i)\right]$$

(This expression for $\sigma_{xx}$ is the same as that previously used where $F_1 = 1$, $F_2 = y$, $F_3 = z$ except that a term k, providing for the effectiveness of the section, is included). The values of $\bar{y}$ and $\bar{z}$ are given by ($A_i$ is the node area and $y_i$, $z_i$ are the node coordinates):

$$\bar{y} = \frac{\sum_i k'_iy_iA_i}{\sum_i k'_iA_i} \quad ; \quad \bar{z} = \frac{\sum_i k'_iz_iA_i}{\sum_i k'_iA_i}$$

*Since the effect of cutouts (such as doors and hatches) and regions near the ends of members is to reduce the stress in that region, we introduce a tension effectiveness factor k, $0 \leq k \leq 1$. $k = 1$ if there are no cutouts or ends nearby in the axial direction. References 10 and 11 give rules for determining effectiveness.

The elastic constants to be used are:*

$$EA = E_o \sum_i k_i' A_i$$

$$EI_{yy} = E_o \sum_i k_i' (y_i - \overline{y})^2 A_i$$

$$EI_{yz} = E_o \sum_i k_i' (y_i - \overline{y})(z_i - \overline{z}) A_i$$

$$EI_{zz} = E_o \sum_i k_i' (z_i - \overline{z})^2 A_i$$

These numbers are calculated by the computer, and in the output statement:

$$\text{Structure Area} = \frac{EA}{E_o}$$

$$\text{Y EL Axis**} = \overline{y}$$

$$\text{Z EL Axis**} = \overline{z}$$

$$\text{YY Flexibility} = E_o I_{yy}/E(I_{yy}I_{zz} - I_{yz}{}^2)$$

$$\text{YZ Flexibility} = E_o I_{yz}/E(I_{yy}I_{zz} - I_{yx}{}^2)$$

$$\text{ZZ Flexibility} = E_o I_{zz}/E(I_{yy}I_{zz} - I_{yz}{}^2)$$

These latter equations are *not* independent of materials and effectiveness. The values of $I_{yy}$, $I_{yz}$, $I_{zz}$ are obtained from the equations for $EI_{yy}$, etc., given previously in which $k_i'$ accounts for effectiveness and modulus at each node.

For *cross sections consisting of stringers and plates,* assume the following in order to calculate *shear stresses* (see Chapter 6 of Reference 8):

1. All of the shear is carried in the plates (see Chapter 2 of Reference 7).† The plates are thin, and the component of shear perpendicular to the surface of a plate must vanish; hence, the shear stress $\tau$ has a direction along the plate direction (i.e., if the plate has a slope $\Delta z/\Delta y$, then the condition for zero component of shear perpendicular to the surface of the plate is $\sigma_{xz}/\sigma_{xy} = \Delta z/\Delta y$).†† Assume that the magnitude of the stress does not vary across the thickness and call this magnitude $\tau$.

$$(\tau^2 = \sigma_{xy}^2 + \sigma_{xz}^2)$$

*Note that A ($\neq \Sigma A_i$) is defined as part of the term EA which is defined as $E_o \Sigma k_i' A_i$. The term EA is defined as a single unit, and E and A are not employed separately. Similarly, for subsequent equations.

**Coordinates of the elastic axis.

†Shear stiffness of a rod is small compared to that of a plate and is assumed to vanish.

††The projection of $\tau$ along the y- and z-axis is equal to $\sigma_{xy}$ and $\sigma_{xz}$, respectively.

2. In order for the plate to be in equilibrium in the x-direction, the product of $\tau$ times the thickness must not vary along the plate [i.e., for a given plate, ( $\tau \cdot$ thickness) is independent of y and z]. This is explained as follows: Figure 7 shows the shear forces acting on a plate. For equilibrium in the x-direction, $F_1 = F_2$. But the thickness at end 1 could be different from the thickness at end 2: $t_1 \neq t_2$. Then defining the shear flow[8,9] q (force per unit length along the plate) by $q = \tau \cdot$ thickness:

$$F_1 = q_1 \; \Delta x = \tau_1 t_1 \Delta x$$

$$F_2 = q_2 \; \Delta x = \tau_2 t_2 \Delta x$$

Thus $\tau_1 \neq \tau_2$. But $F_1 = F_2$, and, therefore, $q_1 = q_2$. Also, by rotational equilibrium, $q_3 = q_1$ at corner 1 and $q_3 = q_2$ at corner 2.

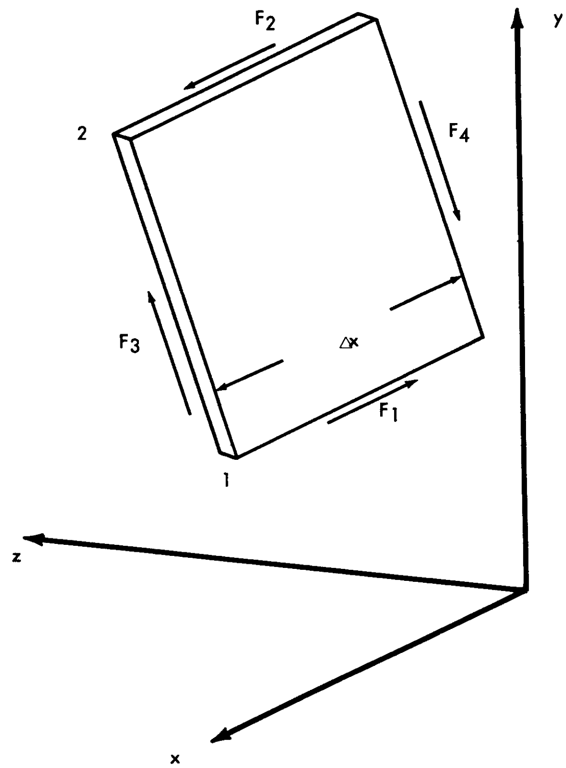


Figure 7 – Shear Forces Acting on a Plate

However, we could have selected slices 1 and 2 at any points on the plate, not just at the nodes at the end. Consequently, no matter where one looks along edge 3, $q_3 = q_1 = q_2$ is the same at any point on a single panel between nodes where tensile force (in the x-direction) acts on the plate from an external source; i.e., the shear flow is a constant for each plate. Thus the problem of finding the shear stresses has now been reduced to finding one unknown (shear flow) for each plate.

3. Each plate begins and ends at a node.* Also, each node has at least one plate attached to it. The shear stress in a plate exerts an axial force on the node. This force is q per unit length in the x-direction.[8,9] Assign a positive direction to shear flow. When looking at the cross section from the +x side, the shear stress acts upon the plate in one direction. This is the direction of the flow. If the shear flows into a node, the plate exerts a force on the node in the −x direction. Hence, the net force per unit length on a node by all the plates which join it is the sum of the shear flows out of the node. (Hence, the name "shear flow." For problems with no tension, the sum of the flows out of any node vanishes.) From this study of the nodes, the tensile stress in a node is given in terms of the forces $V_x$, $M_y$, and $M_z$ (see Equations [9] and [16] of Appendix A.2 and the preceding development):

---

*For additional detail on this section, see Figure 16 and the associated text in section Shear and Torsion in Appendix A.2; also see footnote on page 55.

$$(\sigma_{xx})_i = V_x \frac{k_i'}{A} - M_z k_i' \frac{I_{zz}(y_i - \bar{y}) - I_{yz}(z_i - \bar{z})}{I_{yy}I_{zz} - I_{yz}^2}$$
$$+ M_y k_i' \frac{I_{yy}(z_i - \bar{z}) - I_{yz}(y_i - \bar{y})}{I_{yy}I_{zz} - I_{yz}^2}$$

For ships with a plane of symmetry, $I_{yz} = 0$.

If this expression is differentiated with respect to x, assuming the node locations and areas do not depend upon x, then the rate of change of tension is the same expression except that $V_x$ is replaced by $\frac{dV_x}{dx}$, $M_y$ by $\frac{dM_y}{dx}$, and $M_z$ by $\frac{dM_z}{dx}$. Assume that $\frac{dV_x}{dx} = 0$, $\frac{dM_y}{dx} = V_z$, and $\frac{dM_z}{dx} = - V_y$ (equilibrium of beam). Then, since rate of change of tension in a node is the sum of the shear flows, it follows that (note that shear flow is out of node, hence, force on node is in +x direction; see Equations [16]–[20] of Appendix A.2):*

$$\Sigma\, q_i \text{ (out)} = - V_y k_i' A_i \frac{I_{zz}(y_i - \bar{y}) - I_{yz}(z_i - \bar{z})}{I_{yy}I_{zz} - I_{yz}^2}$$
$$- V_z k_i' A_i \frac{I_{yy}(z_i - \bar{z}) - I_{yz}(y_i - \bar{y})}{I_{yy}I_{zz} - I_{yz}^2}$$

This gives one equation involving the shear flows for each node. Usually there are more plates than nodes, so additional equations are needed to solve for the shear flows. Any set of shear flows which satisfies the above condition for the sum of q out of the nodes will automatically have the correct resultant $V_y$ and $V_z$, thus no additional information is gained by writing overall equilibrium equations; see page 70.**

---

*$\Sigma q_i$ (out) is better termed $(\Sigma q_{out})_i$ or merely $q_{out\ i}$, and is the algebraic sum of shear flows q on each plate connected to node i. Such a term q is positive if the force acting on the portion of the plate AA on the -x side of the section shown in Figure 16 (as viewed from the +x side) is away from the node; another plate or portion of a plate contiguous with side AA is assumed to exist to the left of AA. And $(\Sigma q_{out})_i$ is positive if the net shear flow in all connecting plates is outward.

**Overall equilibrium equates the total shear forces sustained by the section to the shear flows of the plates:

$$V_y = \sum_j q_j (y_{hj} - y_{tj}) \quad ; \qquad V_z = \sum_j q_j (z_{hj} - z_{tj})$$

Here, subscript j refers to all the plates, $q_j$ is the shear flow in a plate, and $y_{hj}$, $y_{tj}$, $z_{hj}$, and $z_{tj}$ locate the head (h) and tail (t) of the plate.

The point of the statement is that for shear flows $q_j$ based on a tree and values of $(\Sigma q_{out})_i$ at each node given by the preceding equation, the above equations are *automatically* satisfied (and for loop shear flows they give *zero* for $V_y$ and $V_z$); therefore, there is no point in invoking them.

4. By appealing to the equations of elasticity, and making assumptions about no change of shape of the cross section, it can be shown that the integral around any closed path per unit length through the section $\oint q \frac{ds}{Gt}$ equals twice the enclosed area times the rate of twist, or $\frac{d\theta_x}{dx} = \frac{1}{2A} \frac{\oint q ds}{Gt}$. (Rate of twist = $d\theta_x/dx$); see Chapters 16 and 17 of Reference 8, or Chapter VII of Reference 9; also, see pages 69–72 of Appendix A.2. By assuming that the plate segments are straight lines, it is possible to compute these areas by knowing about the connections made and the locations of the nodes. Since q is a constant along each path, the integral may be replaced by a sum $\sum (\pm q_i) \frac{\Delta s}{Gt}$, where $+q_i$ is used if the positive direction assigned to the unknown $q_i$ is in the same direction as the positive direction of the closed loop; see Equation [23] of Appendix A.2. The integral $\oint \frac{qds}{Gt}$ is evaluated around each of the loops. In the sample problem, remember that the tree is formed by omitting plates 5 and 9. For example, the loop formed by the tree and plate 9 is shown heavy in Figure 8, and the area associated with this loop is shaded. This area can be written (see page 72 of Appendix A.2):

$$A = \frac{1}{2} \sum_j \delta_j (R\Delta s)_j = \frac{1}{2} \sum_j \delta_j (y_h z_t - y_t z_h)_j$$

where the summation is over Plates 2, 3, 8, and 9 which make up the loop. Here R is the perpendicular distance from the origin to the plate and $\Delta s$ is the length of the plate. See Figure 9, in which the contribution of Plate 2 to this summation is shaded. The proper sign of $(R\Delta s)_j$ is assured by the head-tail polarity of the plate. The entry into the summation is multiplied by ($\delta_j = +1$) if the direction around the loop agrees with the sense of plate j and by ($\delta_j = -1$) if the direction around the loop is contrary to the polarity of plate j. In this example, $\delta_2$, $\delta_3$, $\delta_8$, and $\delta_9$ are $-1$, $-1$, $-1$, and 1, respectively. (See Column 5 , Table 4, Sheet 3, and the discussion of the $[L_{j1}]$ matrix in Appendix A.2, pages 68–75.) q is constant on any plate. Positive values of q in the plates are given by the arrows in Figure 8.

5. The resultant torque must be equal to the moment of the shear flows (see Chapter 6 and Figure 6.15 of Reference 8 or page 219 of Reference 9.) The resultant torque or twisting moment about the x-axis is $M_x = \sum_j q_j (y_{tj} z_{hj} - y_{hj} z_{tj})$ (see Equation [26], Appendix A.2). It represents moment about the x-axis. Its polarity is given by the $\theta_x$ arrow in Figure 3. Since the shear flow on a plate is a constant, the total force is just q times the length. The moment about the origin is the net force times the moment arm. Therefore, the net torque due to one plate about the origin is given simply by the shear flow times twice the area of the triangle which would be formed if the ends of the plate were joined to the origin by straight lines (see Equation [26] of Appendix A.2).

The above assumptions give exactly the correct amount of equations to yield a unique solution for the shear flows in the panels in terms of $M_x$, $V_y$ and $V_z$. Only a general discussion of the method of solution is given here. See pages 71–78 for a detailed discussion.

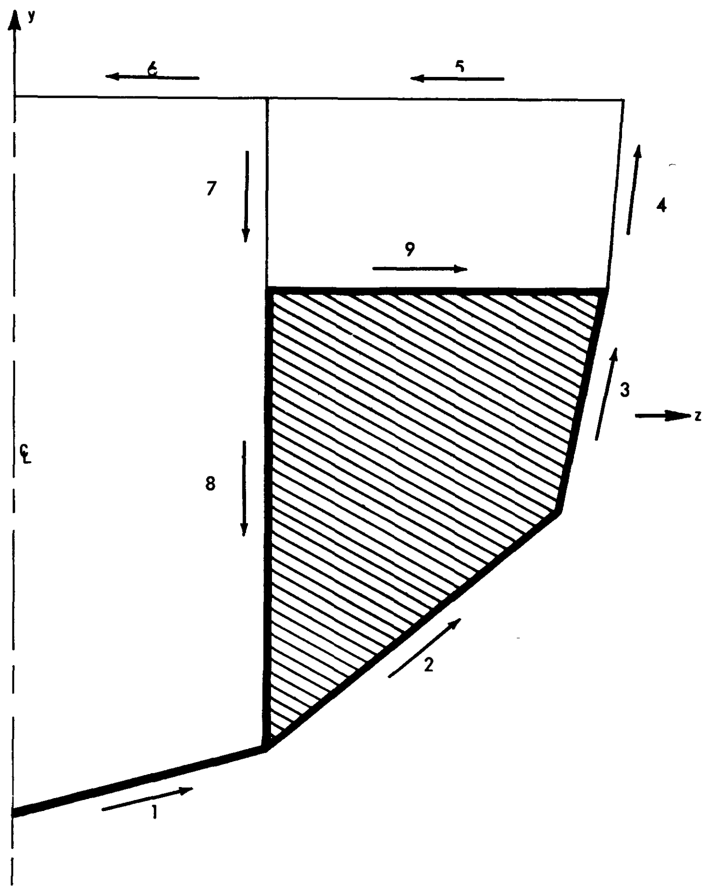


Figure 8 – Loop Formed by Tree and Plate

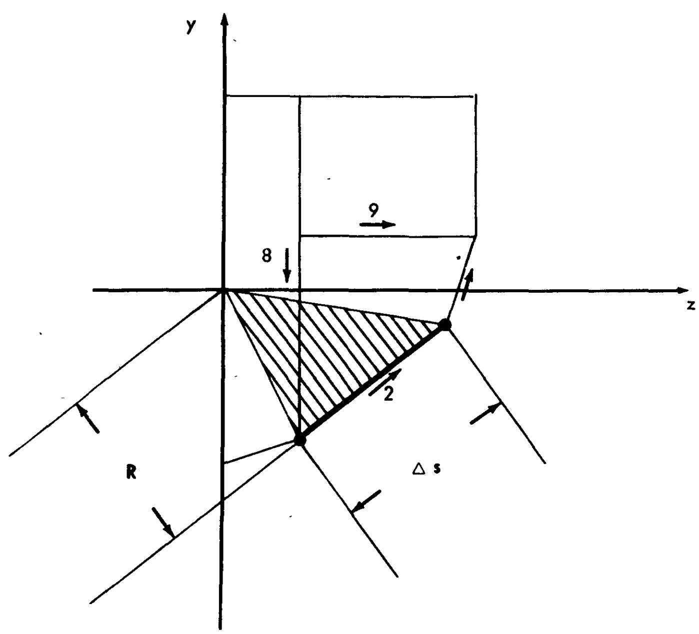


Figure 9 – Contribution of Plate 2 to Area

Three solutions for the q's are needed, one for $M_x = 1$, $V_y = V_z = 0$; one for $M_x = 0$, $V_y = 1$, $V_z = 0$; etc. $M_x$ is the twisting moment about the x-axis. There is no dependence of $(\sigma_{xx})_i$ on twisting moment. For that reason the solution for plate shear flows corresponding to $M_x = 1$, $V_y = V_z = 0$ has no component due to $q_{part}$, which is based on $(\Sigma q_{out})_i$ and, therefore, on $(\sigma_{xx})_i$; this solution arises entirely from $q_{loop}$ for the various loops (see Item 4 given previously). The reason for three solutions for $q_j$, the shear flows in the plates, being needed is covered extensively elsewhere in Appendix A.2. In particular, refer to the section Shear and Torsion, the section following Equation [26] through page 75.

The solution per unit torque $M_x$ is the solution for the shear flows in the plates $q_j$ existing when $V_y = V_z = 0$, $M_x = 1$. It is found as follows: First find the most general set of shear flows which satisfy the condition that the sum of the q's out of every node vanishes (see pages 54–55 and 75–76). This set can be found in terms of circulating flows. ("Circulating flows" refer to the plate shears in the loops, $q_{loop}$. The section Shear and Torsion in Appendix A.2 elaborates on the point.) In the computer program this was done by first finding a tree, that is, a *set of plates so that one and only one path exists between every two nodes.* For each plate not on the tree, there exists a closed loop through that plate and others in the tree. The most general set of shear flows that have zero net flow out of each node consists of a linear combination of flows in these loops. The new unknowns are the flows in the loops. By integrating around those loops (see Item 4, page 56), there will be sufficient equations to solve for these unknowns, but a new unknown $(d\theta_x/dx)$ is introduced (Equation [37] of Appendix A.2). The shear flows can now be written in terms of this one unknown, which can be found since the resultant torque is to be unity (Equations [38]–[41] of Appendix A.2).

The details of the method of solution for the plate shears $q_j$ existing for unit y-shear $V_y$ (or per unit z-shear $V_z$) are presented in the section Shear and Torsion in Appendix A.2. The location of the center of shear $\bar{\bar{y}}$, $\bar{\bar{z}}$ is also determined from $V_z$ shear and $V_y$ shear, respectively. In particular, see the paragraph following Equation [31].

The inertial parameters come from structural and nonstructural items. Structural items include ship hull, deck, longitudinal members, etc. Nonstructural items include machinery, cargo, superstructure, transverse bulkheads, the virtual mass of the water, etc. The inertial parameters which must be calculated are mass of a section $\Sigma M$, the position of the center of gravity Y-C.G., Z-C.G.; the rotary inertias I-YY, I-ZZ, I-MYZ; and the polar moment of interia I-MX. For symmetric motions of a symmetric ship, only $\Sigma M$ and I-ZZ are needed. For antisymmetric motion of a symmetric ship only $\Sigma M$, Z-C.G., I-YY, and I-MX are needed. For the general case, however, all parameters are needed. Y-C.G. and I-MYZ vanish in the case of symmetry. $\Sigma M$ is obtained by adding all mass items in a $\Delta x$ section. The Y-C.G. and Z-C.G. come from dividing the mass moments by the total mass. The rotary inertia will not be calculated by the equation on page 35 of Reference 1, since this can be properly evaluated from the input data (see section Inertial Parameters in Appendix A.2).

The values of the ship parameters should be plotted versus the axial coordinate x. Virtual mass and mass moments of inertia should be added to these curves (if not included in the input) and average values of the parameters over a $\Delta x$ section read from the curves (see Chapter 3 of Reference 1). The effects of other nonstructural items should be incorporated in accordance with this reference.

The determination of inertial parameters[1] (mass, $\Sigma M$; center of gravity Y-C.G., Z-C.G.; rotary inertias I-YY, I-YZ, I-ZZ; and polar moment of inertia I-MX) is reviewed separately in Appendix A.2. It is essentially identical to the determination of the terms A, $\bar{y}$, $\bar{z}$, $I_{yy}$, $I_{yz}$, $I_{zz}$, etc., for the elastic properties associated with tension and bending. The weight calculation for the sample problem which gives results agreeing with Table 3a is presented in Table 4.

## A.2 – ADDITIONAL THEORY USED IN EVALUATING SECTION PROPERTIES

### ASSUMPTIONS

Figure 10 illustrates the general class of structures to which this theory and digital program are applicable, and it shows the idealizations incorporated into the representation of the structure. In applying beam theory to a structure such as a beam hull, it is recognized that the section properties will vary with position along the beam; however, the calculation of the elastic parameters of the beam at a particular cross section is based on the assumption that the structure is prismatic; that is, all sections are identical, at least in the immediate vicinity of the section under consideration. Thus, Figure 10 shows the structure as a prism, with all tension and shear members parallel to the x-axis. It is assumed that the section lying in the plane $x = 0$ is the section to be analyzed. For the purpose of establishing the elastic properties of this section, the prismatic structure is assumed continuous, both in the $-x$ direction (shown) and in the $+x$ direction (not shown).

In Figure 10, the coordinate axis x, y, z locate points on the structure. The displacements of points from their basic positions as in Appendix A.1 is given by $U_x$, $U_y$, and $U_z$ in translation and by $\theta_x$, $\theta_y$, and $\theta_z$ in rotation, with positive directions in the same sense as the x-, y-, and z-axes (rotation established by the right-hand rule). Section forces are $V_x$, $V_y$, and $V_z$; section moments are $M_x$, $M_y$, and $M_z$. These forces and moments are positive if the force (or moment) exerted by the portion of the structure not shown ($x > 0$) upon the portion shown ($x \leqq 0$) is in the direction of positive displacement.

The figure shows that the structure has been idealized so that all the tensile stress is carried by a finite number of axial elements, each located at a distinct node of the section and having associated with it a finite area, $A_i$. The remainder of the structure carries only shear and consists of straight panels of constant thickness $t_j$ connecting pairs of nodes. For the sake of simplicity, it is assumed here (although not in Appendix A.1) (1) that each tensile element has full effectiveness and all are composed of the same material and (2) that each shear element has full effectiveness and all are composed of the same material. These assumptions do not really limit the generality of the theory.

In this report the following subscripts are used:

- i to indicate the various nodes of the cross section
- j to indicate the various plates of the cross section
- $\ell$ to indicate the independent loops formed by the plates

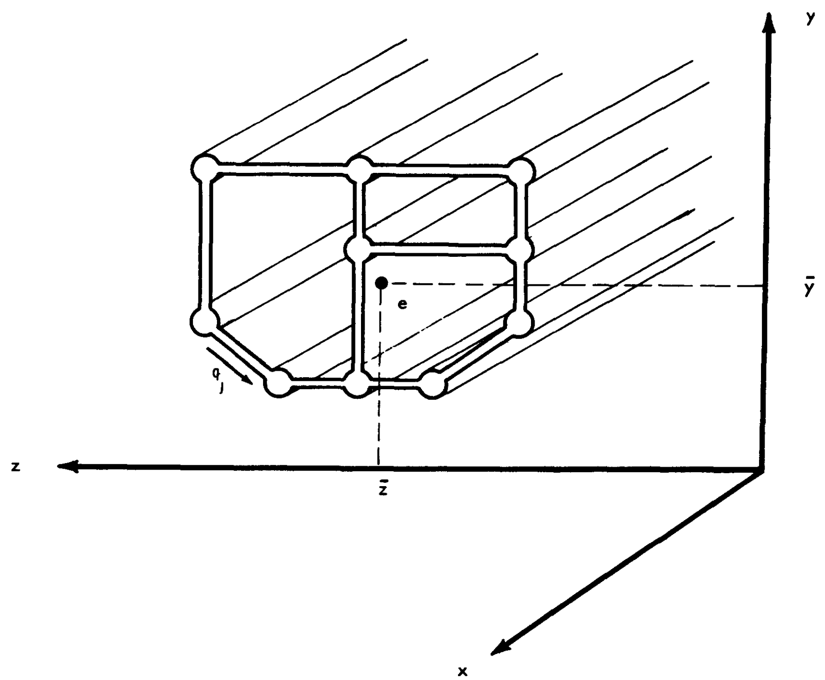


Figure 10 – Coordinate System for Idealized Prismatic Structure

## TENSION AND BENDING

As in Appendix A.1, it is assumed that the distribution of tensile strain over the cross section is linear in both y and z. Thus the axial stress in the tension-carrying material at node i, which is located at $y = y_i$, $z = z_i$, is

$$(\sigma_{xx})_i = B + C\,y_i + D\,z_i \qquad [1]$$

where the positive values denote tensile stress.

By summing over all the nodes of the section, we get the following expressions for tension and bending moments $V_x$, $M_y$, and $M_z$:

$$\left.\begin{aligned} V_x &= +\Sigma(\sigma_{xx})_i A_i = B\Sigma A_i + C\Sigma y_i A_i + D\Sigma z_i A_i \\ M_y &= +\Sigma(\sigma_{xx})_i z_i A_i = B\Sigma z_i A_i + C\Sigma y_i z_i A_i + D\Sigma z_i^2 A_i \\ M_z &= -\Sigma(\sigma_{xx})_i y_i A_i = -B\Sigma y_i A_i - C\Sigma y_i^2 A_i - D\Sigma y_i z_i A_i \end{aligned}\right\} \qquad [2]^*$$

*All summations given in section Tension and Bending are with respect to subscript i.

Now, transforming to forces, moments, and locations measured with respect to point e (see Figure 10) located at $y = \bar{y}$, $z = \bar{z}$, we define $\bar{V}_x$, $\bar{M}_y$, $\bar{M}_z$ to be forces and moments referred to axes at point e:

Also we define:

$$
\left.\begin{aligned}
\bar{V}_x &= V_x \\
\bar{M}_y &= M_y - \bar{z}\, V_x \\
\bar{M}_z &= M_z + \bar{y}\, V_x \\
y_i &= \bar{y} + y_{ei} \\
z_i &= \bar{z} + z_{ei}
\end{aligned}\right\} \quad [3]
$$

where $y_{ei}$, $z_{ei}$ locate node i relative to point e.

Substitution of the definitions of Equations [3] into Equations [2] leads to:

$$
\left.\begin{aligned}
\bar{V}_x &= (B + C\bar{y} + D\bar{z})\,\Sigma A_i && + C\,\Sigma y_{ei}A_i + D\,\Sigma z_{ei}A_i \\
\bar{M}_y &= (B + C\bar{y} + D\bar{z})\,\Sigma z_{ei}A_i && + C\,\Sigma y_{ei}z_{ei}A_i + D\,\Sigma z_{ei}^2 A_i \\
\bar{M}_z &= -(B + C\bar{y} + D\bar{z})\,\Sigma y_{ei}A_i && - C\,\Sigma y_{ei}^2 A_i - D\,\Sigma y_{ei}z_{ei}A_i
\end{aligned}\right\} \quad [4]
$$

Now choose $\bar{y}$ and $\bar{z}$ by the following:

$$
\bar{y} = \frac{\Sigma y_i A_i}{\Sigma A_i} \quad ; \qquad \bar{z} = \frac{\Sigma z_i A_i}{\Sigma A_i} \qquad [5]
$$

Then

$$
\Sigma y_{ei}A_i = \Sigma (y_i - \bar{y}) A_i = \Sigma y_i A_i - \bar{y}\,\Sigma A_i = 0
$$

$$
\Sigma z_{ei}A_i = \Sigma (z_i - \bar{z}) A_i = \Sigma z_i A_i - \bar{z}\,\Sigma A_i = 0
$$

and Equation [4] now becomes

$$
\left.\begin{aligned}
\bar{V}_x &= (B + C\bar{y} + D\bar{z})\,\Sigma A_i \\
\bar{M}_y &= \qquad C\,\Sigma y_{ei}z_{ei}A_i + D\,\Sigma z_{ei}^2 A_i \\
\bar{M}_z &= \qquad -C\,\Sigma y_{ei}^2 A_i - D\,\Sigma y_{ei}z_{ei}A_i
\end{aligned}\right\} \quad [6]
$$

Equations [6] indicate that this choice of $\bar{y}$ and $\bar{z}$ has uncoupled tensile force $\bar{V}_x$ from the bending moments $\bar{M}_y$ and $\bar{M}_z$. Point e, so located, is called the *elastic axis*, and is located at the center of effective tension-carrying area.

We make these further definitions:

$A = \Sigma A_i$ = total area of cross section.

$I_{yy} = \Sigma y_{ei}^2 A_i$ = area moment of inertia of cross section about axis through e and parallel to the z-axis.

$I_{yz} = \Sigma y_{ei} z_{ei} A_i$ = area product of inertia of cross section relative to axes through e.

$I_{zz} = \Sigma z_{ei}^2 A_i$ = area moment of inertia of cross section about axis through e and parallel to the y-axis.

Then Equations [6] may be written:

$$\left.\begin{aligned} \bar{V}_x &= (B + C\bar{y} + D\bar{z})A \\ \bar{M}_y &= I_{yz}C + I_{zz}D \\ \bar{M}_z &= -I_{yy}C - I_{yz}D \end{aligned}\right\} \quad [7]$$

We can solve Equations [7] for B, C, D in terms of $\bar{V}_x$, $\bar{M}_y$, $\bar{M}_z$, obtaining

$$\left.\begin{aligned} B &= \frac{\bar{V}_x}{A} - \frac{(\bar{z}I_{yy} - \bar{y}I_{yz})\bar{M}_y - (\bar{y}I_{zz} - \bar{z}I_{yz})\bar{M}_z}{I_{yy}I_{zz} - I_{yz}^2} \\ C &= \frac{-I_{yz}\bar{M}_y - I_{zz}\bar{M}_z}{I_{yy}I_{zz} - I_{yz}^2} \\ D &= \frac{I_{yy}\bar{M}_y + I_{yz}\bar{M}_z}{I_{yy}I_{zz} - I_{yz}^2} \end{aligned}\right\} \quad [8]$$

Substituting into Equation [1], the initial expression for $\sigma_{xx}$ at node i, gives

$$(\sigma_{xx})_i = B + Cy_i + Dz_i = (B + C\bar{y} + D\bar{z}) + Cy_{ei} + Dz_{ei}$$

$$(\sigma_{xx})_i = \frac{\bar{V}_x}{A} + \frac{(z_{ei}I_{yy} - y_{ei}I_{yz})\bar{M}_y - (y_{ei}I_{zz} - z_{ei}I_{yz})\bar{M}_z}{I_{yy}I_{zz} - I_{yz}^2} \quad [9]$$

The elastic parameters for bending and tension may be obtained from the associated *strains* as follows (see page 232 of Reference 7):

$$\left.\begin{aligned}
\frac{\partial \overline{U}_x}{\partial x} &= \epsilon_{xx}\Big|_{\substack{y=\bar{y}\\ z=\bar{z}}} = \epsilon_{xx}\Big|_{\substack{y_{ei}=0\\ z_{ei}=0}} = \frac{\overline{V}_x}{EA} \\
\frac{\partial \theta_y}{\partial x} &= +\frac{\partial \epsilon_{xx}}{\partial z} = +\frac{D}{E} = \frac{I_{yy}\overline{M}_y + I_{yz}\overline{M}_z}{E(I_{yy}I_{zz} - I_{yz}^2)} \\
\frac{\partial \theta_z}{\partial x} &= -\frac{\partial \epsilon_{xx}}{\partial_y} = -\frac{C}{E} = \frac{I_{yz}\overline{M}_y + I_{zz}\overline{M}_z}{E(I_{yy}I_{zz} - I_{yz}^2)}
\end{aligned}\right\} \quad [10]$$

Equations [10] summarize the elastic flexibility parameters of the beam, which describe bending and tensile deformations in terms of bending moments and tensile force.

## SHEAR AND TORSION

Next it is desired to determine the elastic flexibility parameters of the beam which relate shear and torsion deformations to beam shear forces and twisting moment. We will use Castigliano's Theorem, which requires that the total strain energy be expressed in terms of the beam shear forces and twisting moment $V_y$, $V_z$, and $M_x$. All the strain energy associated with these deformations is in the shear of the plates. By statics it is shown that any single plate sustains a shear flow q (force per unit length) which is the same at all points of the plate (see page 54). Thus the shear strain energy of any plate j depends only on the shear flow of that plate $q_j$. The first step is to express all the shear flows $q_j$ as functions of $V_y$, $V_z$, and $M_x$. The second step is to express strain energy per unit length of the beam $\overline{W}$ as a function of $V_y$, $V_z$, and $M_x$. The final step is to apply Castigliano's Theorem by differentiating this expression for $\overline{W}$ with respect to $V_y$, $V_z$, and $M_x$.

To find the panel shear flows $q_j$ as functions of $V_y$, $V_z$, and $M_x$, we will first compose $q_j$ of the sum of shears flowing in a tree $q_{part\ j}$, a particular solution, and shears flowing in loops $q_{loop\ j}$.

The tree is selected by omitting sufficient plates so that the remaining plates are simply connected. In a tree thus formed, all nodes of the section are part of the tree, and any two nodes are connected by one and only one path through the tree. One node is arbitrairly selected as the "root" of the tree.

Each loop is formed by taking one of the plates which was omitted in forming the tree and all of the plates which are part of the tree. One loop is formed by this procedure, and the plates of the tree which are part of this loop are retained in the loop, whereas extraneous plates of the tree (not needed in the loop) are omitted from a description of the loop.

The shear flows of the particular solution arise from the shear flowing out of the nodes $q_{out\ i}$. In a particular panel j, this shear flow $q_{part\ j}$ is given by the sum of $q_{out\ i}$ for all nodes farther from the tree root than panel j. The sign of this summation is plus (+) if the positive sense of panel j is toward the root, and minus (–) if the positive sense is away from the root.

The above relations and definitions may be summarized in matrix form by the following equations. In these equations the vector $\{q_{part\ j}\}$ represents shears in the plates due to $\{q_{out\ i}\}$ at the nodes, which is, in turn due to $V_y$ and $V_z$. Also the vector $\{q_{loop\ j}\}$ represents plate shears due to all the loop shears $\{K_\ell\}$. In Figures 13, 14, and 15 the light arrows associated with the plate numbers represent plate shears. The heavy arrows on the loops (heavy lines) represent loop shears.

$$\left\{q_j\right\} = \left\{q_{part\ j}\right\} + \left\{q_{loop\ j}\right\} \qquad [11]$$

$$\left\{q_{part\ j}\right\} = \left[T_{ji}\right] \left\{q_{out\ i}\right\} \qquad [12]$$

$$\left\{q_{loop\ j}\right\} = \left[L_{j\ell}\right] \left\{K_\ell\right\} \qquad [13]$$

The formation of the $[T_{ji}]$ and $[L_{j\ell}]$ matrices, which describe the tree and the loops, respectively, is illustrated for a simple example in Figures 11 to 15. Figure 11 shows a section composed of nodes numbered 1 to 7 and plates numbered 1 to 9. An arrow indicates the polarity of each plate. Figure 12 shows how a tree has been selected by omitting plates 6, 8, and 9. Node 1 is chosen as the root of the tree. The following equation gives the matrix $[T_{ji}]$, which describes the tree (also see page 11):

$$\underbrace{\begin{Bmatrix} q_{part\ 1} \\ q_{part\ 2} \\ q_{part\ 3} \\ q_{part\ 4} \\ q_{part\ 5} \\ q_{part\ 6} \\ q_{part\ 7} \\ q_{part\ 8} \\ q_{part\ 9} \end{Bmatrix}}_{\{q_{part\ j}\}} = T_{ji} \underbrace{\begin{Bmatrix} q_{out\ 1} \\ q_{out\ 2} \\ q_{out\ 3} \\ q_{out\ 4} \\ q_{out\ 5} \\ q_{out\ 6} \\ q_{out\ 7} \end{Bmatrix}}_{\{q_{out\ i}\}} = \underbrace{\begin{bmatrix} 0 & -1 & -1 & -1 & -1 & -1 & -1 \\ 0 & 0 & -1 & -1 & -1 & -1 & -1 \\ 0 & 0 & 0 & -1 & -1 & -1 & -1 \\ 0 & 0 & 0 & 0 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & +1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}}_{[T_{ji}]} \underbrace{\begin{Bmatrix} q_{out\ 1} \\ q_{out\ 2} \\ q_{out\ 3} \\ q_{out\ 4} \\ q_{out\ 5} \\ q_{out\ 6} \\ q_{out\ 7} \end{Bmatrix}}_{\{q_{out\ i}\}} \qquad [14]$$

Figures 13, 14, and 15 illustrate loops 1, 2, and 3, formed by adding to the tree plates 6, 8, and 9, respectively. In each case, a polarity of the loop has been chosen and indicated. The following equation shows how the matrix $[L_{j\ell}]$ is formed to relate the plate shear flows $\{q_{loop\ j}\}$ to the (as yet unknown) loop shear flows $\{K_\ell\}$ (see page 13 also):*

---

*In Equation [15] note that the loop including Plate 6 in the positive sense includes plates 1 through 6. Hence $L_{11}$, $L_{21}$, $L_{31}$, $L_{41}$, $L_{51}$, $L_{61}$ are denoted by +1, whereas $L_{71}$, $L_{81}$, and $L_{91}$ are designated zero. Similarly for elements in Columns 2 and 3.

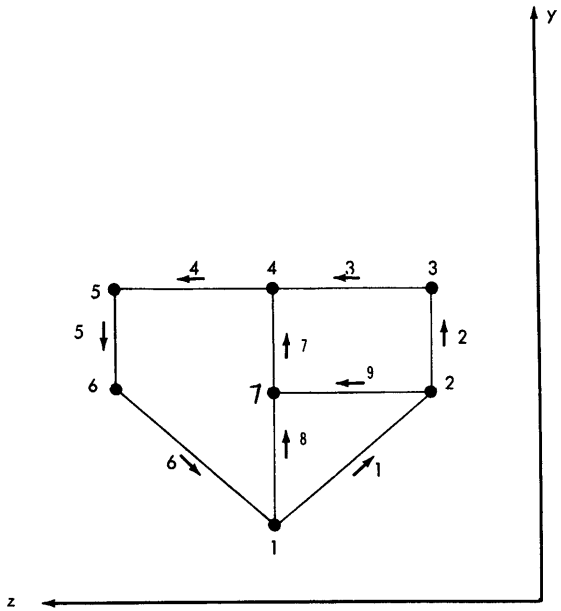


Figure 11 – Cross Section of Prismatic Structure Showing Nodes and Plates

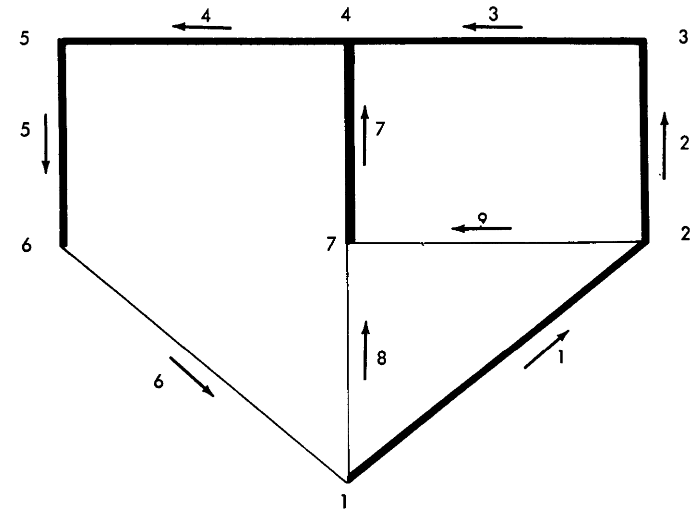


Figure 12 – Tree (Omits Plates 6, 8, and 9)

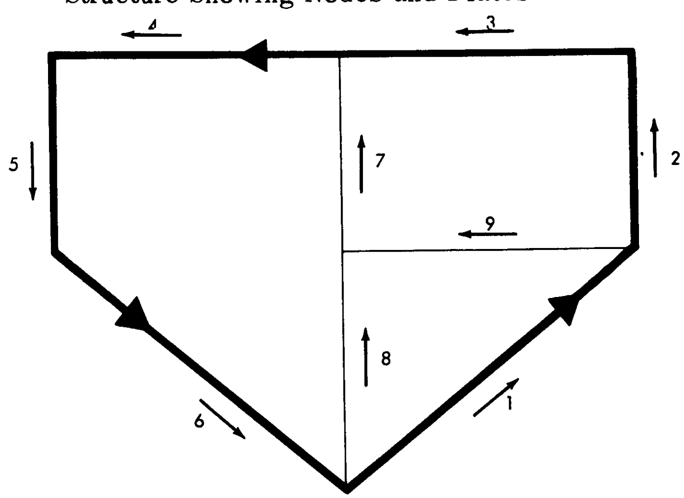


Figure 13 – Loop 1 (Shear Flow $K_1$)

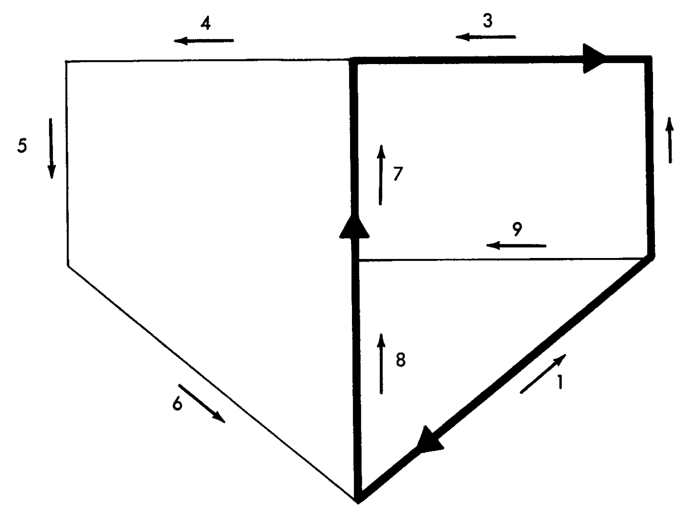


Figure 14 – Loop 2 (Shear Flow $K_2$)

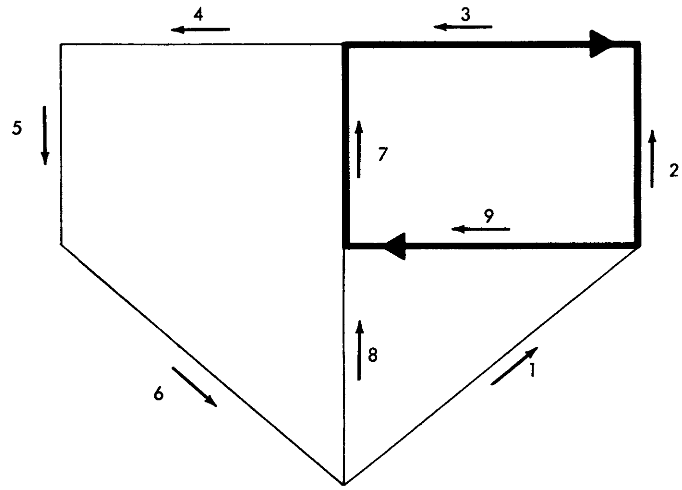


Figure 15 – Loop 3 (Shear Flow $K_3$)

$$\underset{\uparrow \atop \{q_{loop\ j}\}}{\begin{Bmatrix} q_{loop\ 1} \\ q_{loop\ 2} \\ q_{loop\ 3} \\ q_{loop\ 4} \\ q_{loop\ 5} \\ q_{loop\ 6} \\ q_{loop\ 7} \\ q_{loop\ 9} \end{Bmatrix}} = [L_{j\ell}] \underset{\uparrow \atop \{K_\ell\}}{\begin{Bmatrix} K_1 \\ K_2 \\ K_3 \end{Bmatrix}} = \underset{\uparrow \atop [L_{j\ell}]}{\begin{bmatrix} +1 & -1 & 0 \\ +1 & -1 & -1 \\ +1 & -1 & -1 \\ +1 & 0 & 0 \\ +1 & 0 & 0 \\ +1 & 0 & 0 \\ 0 & +1 & +1 \\ 0 & +1 & 0 \\ 0 & 0 & +1 \end{bmatrix}} \underset{\uparrow \atop \{K_\ell\}}{\begin{Bmatrix} K_1 \\ K_2 \\ K_3 \end{Bmatrix}} \qquad [15]$$

It has been shown that the particular solution $\{q_{part\ j}\}$ for shear flows in the plates arises from the shear flows out of the nodes $\{q_{out\ i}\}$. We will now derive an expression for the terms $q_{out\ i}$ in terms of $V_y$ and $V_z$ by differentiating Equation [9] with respect to x.

Figure 16 shows how the rate of change of tensile stress in a node is related to the shear flows out of that node via all connecting plates. The arrows indicate forces exerted on the members with which they are associated. The sum of $q_j$ of all the plates connected to the node (two plates are shown in Figure 16) equals $q_{out\ i}$ of the node. Equilibrium in the longitudinal direction of the small tensile element of cross section $A_i$ and length $\Delta x$ requires that (since, $\left(\sigma_{xxi} + \frac{\partial \sigma_{xxi}}{\partial x} \Delta x\right) A_i + q_{out\ i}\Delta x - \sigma_{xxi}A_i = 0$ :

$$\frac{(\partial \sigma_{xx})_i}{\partial x} A_i = -q_{out\ i} \qquad [16]$$

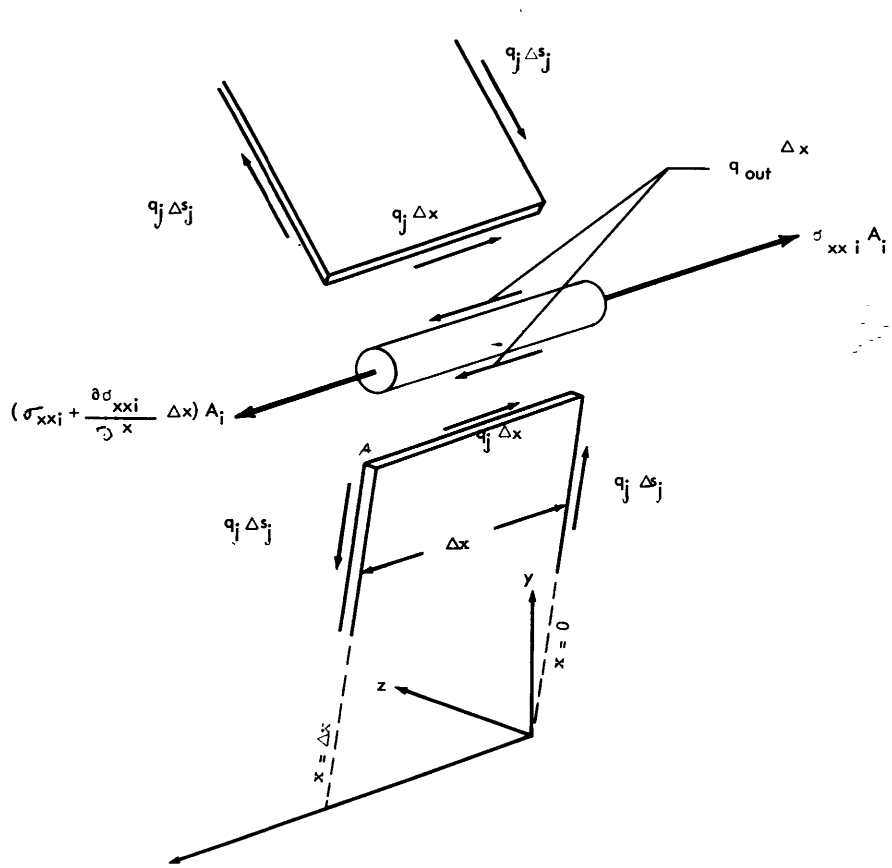


Figure 16 – Tensile Stress-Shear Flow Relationships at a Node

Lateral equilibrium of a small section of the entire structure requires that

$$\frac{\partial \overline{M}_y}{\partial x} = V_z \tag{17}$$

$$\frac{\partial \overline{M}_z}{\partial x} = -V_y \tag{18}$$

Longitudinal equilibrium of a small section of the entire structure requires that

$$\frac{\partial V_x}{\partial x} = 0 \tag{19}$$

Differentiating Equation [9] with respect to x and substituting Equations [16] to [19] gives

$$q_{out\ i} = -\frac{(y_{ei} I_{zz} - z_{ei} I_{yz})}{I_{yy} I_{zz} - I_{yz}^2} A_i V_y - \frac{(z_{ei} I_{yy} - y_{ei} I_{yz})}{I_{yy} I_{zz} - I_{yz}^2} A_i V_z \tag{20}$$

By means of Equations [11], [12], [13], [14], [15], and [20], we can express the plate shear flows $q_j$ in terms of $V_y$, $V_z$, $K_1$, $K_2$, . . ., $K_\ell$, . . ., $K_L$, where L is the number of loops and the K's are the unknown loop shear flows.

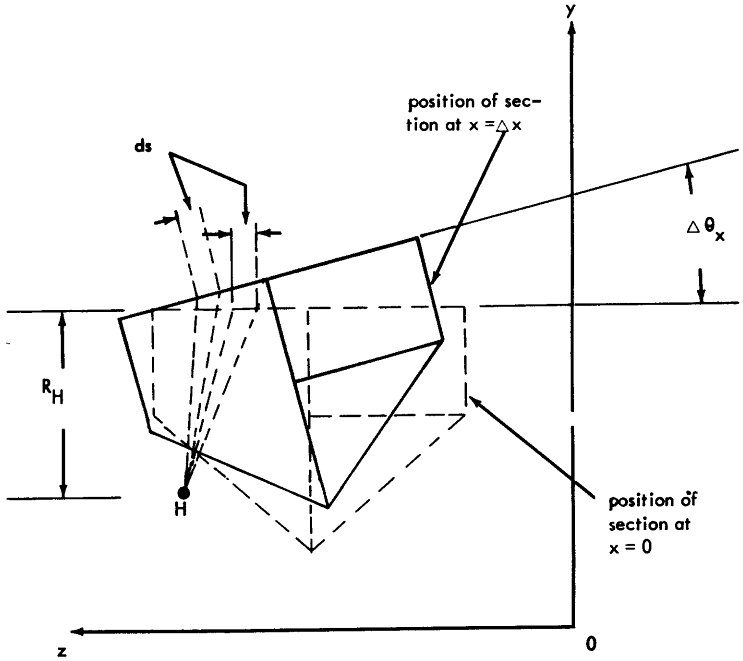


Figure 17 – Torsion in Two Sections of a Prismatic Structure

Now we can write L equations, introducing the unknown rate of twist $\frac{\partial\theta_x}{\partial x}$, by integrating around each of the loops an equation relating geometry to shear strain as shown in Figure 17. This figure shows two sections of a prismatic structure undergoing torsion, the section at x = 0 (dotted) and the section at x = Δx (solid). The solid section exhibits rotation of magnitude $\Delta\theta_x$ with respect to the dotted section. At some point H, not necessarily known, there is no relative translation between the two sections. As shown by the following detailed development, a segment of plate (shown), ds wide by Δx long by t thick, has a shear strain of $R_H \frac{\Delta\theta_x}{\Delta x}$, a shear stress* of $\tau = \frac{q}{t} = R_H G \frac{\Delta\theta_x}{\Delta x}$, where $R_H$ is the distance from H to ds, measured perpendicular to the direction of ds.

Figure 18 shows the shear strain in the plates comprising a loop. The definition of shear strain in the plates is conventional; assuming the shearing forces are applied in the x-direction (longitudinal) and the s-direction (circumferential), shear strain is

$$\epsilon_{xs} = \frac{du_x}{ds} + \frac{du_s}{dx}$$

Thus, if lines are inscribed on the undeformed plate which are parallel to the x- and s-axes, respectively, they form a right angle; after the plate undergoes shear deformation, the difference between the angle of intersection of these two lines and 90 deg is the shear strain.

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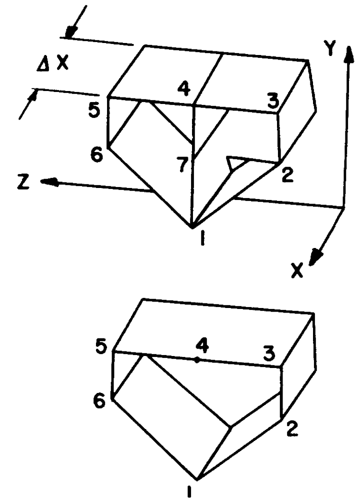
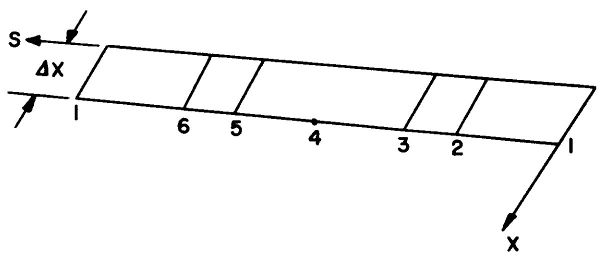
*See pages 53 and 54.



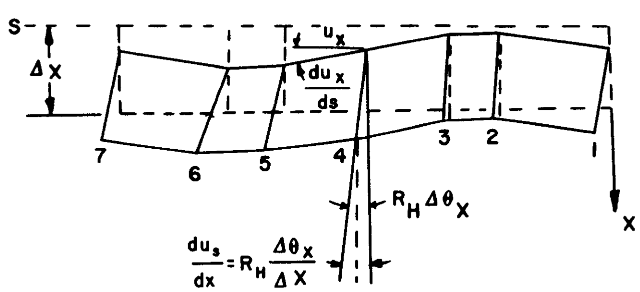

(a) SECTION OF THE HULL, $\Delta X$ IN LENGTH

(b) SECTION OF THE HULL, SHOWING ONLY THE PLATES COMPRISING ONE LOOP




(c) THE SECTION SHOWN IN (b) HAS BEEN UNFOLDED, OR DEVELOPED INTO A PLANE SHEET

(d) UNDEFORMED POSITION OF THE DEVELOPED SECTION (DOTTED LINES), AND DEFORMED POSITION (SOLID LINES)




(e) EQUATION FOR THE SHEAR STRAIN OF A PLATE OF FIGURE (d)

SHEAR STRAIN $\epsilon_{xs} = \frac{\tau}{G} = \frac{q}{Gt} = R_H \frac{\Delta\theta_x}{\Delta X} + \frac{du_x}{ds}$

$$\oint_\ell \epsilon_{xs}\, ds = \oint \frac{q\,ds}{Gt} = \oint_\ell R_H \left(\frac{d\theta_x}{dx}\right) ds + \oint_\ell \frac{du_x}{ds}\, ds = \frac{d\theta_x}{dx} \oint_\ell R_H\, ds = 2A_\ell \frac{d\theta}{dx}$$

(f) INTEGRATION OF EQUATION (e) AROUND THE LOOP

Figure 18 – Shear Strain in the Plates Comprising a Loop

In Figure 18, (a) shows a cross section of the hull, having a length $\Delta x$ in a direction parallel to the longitudinal axis of the hull; (b) shows only those plates which comprise the first loop ($\ell = 1$) of the section (refer also to Figure 13); and (c) shows the same plates as (b) but, for clarity in what follows, the structure has been unfolded, or developed, into the form of a plane sheet. In (c) the plates are undeformed by shear stresses. The same plates are shown in (d) but now they are deformed by shear stresses. (e) gives the equation for shear strain in a plate, making use of the definition of shear strain and the fact that $R_H \Delta\theta_x$ in the hull is the equivalent of $du_s$ in the definition, This is evident by inspection of the geometry of Figure 17 if we consider that a point on element ds on the cross section at $x = \Delta x$ is twisted through an angle $\Delta\theta_x$ relative to its original position on element ds (same position as that shown in Figure 17 for the section at $x = 0$). This point moves a distance $R_H \Delta \theta_x$ during the deformation. The point is also displaced a distance $\gamma\Delta x = \frac{du_s}{dx} \Delta x = du_s$ ($\gamma$ = angle of shearing strain and $dx \approx \Delta x$). Hence $R_H \Delta\theta_x = du_s$. In (f) this expression for shear strain is integrated around the loop $\ell$, resulting in the equivalent of Equation [21]. Equation [21], which gives a relation between the integral of shear strain around a loop and the rate of twist of the hull structure, is obtained by integrating $\tau$ around any loop $\ell$ as follows (letting $\frac{\Delta\theta_x}{\Delta x}$ take its limiting value, $\frac{\partial\theta_x}{\partial x}$):

$$\oint_\ell \tau ds = \oint_\ell \frac{q}{t} ds = G \frac{\partial\theta_x}{\partial x} \oint_\ell R_H \, ds = 2A_\ell \, G \frac{\partial\theta_x}{\partial x} \qquad [21]$$

The term $\frac{du_x}{ds}$ in the definition of shear strain will exist only if cross sections of the hull are permitted to warp out of their plane when the hull is deformed, as illustrated in (d) of Figure 18.

In Equation [21] the expression for shear strain neglects warping of the section and omits the term $du_x/ds$. However, the integration of the simpler expression for shear strain gives the correct result, as shown in (f), because the integral of the term $du_x/ds$ around the loop equals $u_x$ (end point) – $u_x$ (start point), which must equal zero if the loop is closed because the end point is the start point. Therefore, Equation [21] is valid whether or not warping is permitted.

Equation [21] shows another simplification compared with (f) of Figure 18 in that the shear modulus G is a constant outside the integral. The reason is that here the analysis is based on the simplification that all shear elements are composed of the same material. The last step in Equation [21] recognizes that $\oint_\ell R_H \, ds$ equals twice the area of the loop around which the integration is performed.* Note that this latter equality is independent of the location of H, the point from which $R_H$ is measured. Thus, although Equation [21] is derived based on rotation about H, it may be rewritten (for convenience in calculating), with $R_H$ replaced by R, the distance from the origin O to ds, measured perpendicular to the direction of ds. Thus

$$\oint_{\ell} \frac{q}{t} ds = (\oint_{\ell} R\, ds)\, G \frac{\partial \theta_x}{\partial x} = 2A_{\ell} G \frac{\partial \theta_x}{\partial x} \qquad [22]$$

For the idealized structure of this report, the loops are comprised of a finite number of plates, each of constant thickness $t_j$, so that the integrals may be replaced by summations:**

$$\ell \sum_j \frac{\Delta s_j}{t_j} q_j = (\ell \sum_j R_j \Delta s_j)\, G \frac{\partial \theta_x}{\partial x} = 2A_{\ell} G \frac{\partial \theta_x}{\partial x} \qquad [23]$$

Here $\ell \sum_j$ indicates a summation of only those plates j which comprise loop $\ell$, with signs altered, when necessary, to conform to the polarity of loop $\ell$; see page 56. This operation can be indicated by premultiplying the terms to be summed by the appropriate column of the $[L_{j\ell}]$ matrix:

$$\sum_j \frac{\Delta s_j}{t_j} L_{j\ell} q_j = (\sum_j L_{j\ell} R_j \Delta s_j)\, G \frac{\partial \theta_x}{\partial x} = 2A_{\ell} G \frac{\partial \theta_x}{\partial x} \qquad [24]$$

Now this summation is carried out over all plates j. For convenience in calculating, we can replace $R_j \Delta s_j$ by $y_{tj} z_{hj} - y_{hj} z_{tj}$, where $y_{hj}$, $z_{hj}$ locate the head end of plate j, and $y_{tj}$, $z_{tj}$ locate the tail end.† Also substituting for $q_j$ from Equation [11] and [13] yields:

$$\sum_j \frac{\Delta s_j}{t_j} L_{j\ell} (q_{part\ j} + \sum_j L_{j\ell} K_{\ell}) = \left[\sum_j L_{j\ell}(y_{tj} z_{hj} - y_{hj} z_{tj})\right] G \frac{\partial \theta_x}{\partial x}$$

---

*The loop $\ell$ about which we integrate to get Equation [21] is *any one* of the L loops, $\ell = 1, 2 \ldots\ldots, L$. For the example of Figure 17 in which L = 3, the integration would be done around loop 1 (Figure 13), then around loop 2 (Figure 14), and finally around loop 3 (Figure 15); $A_{\ell}$ is then the cross-sectional area enclosed by the loop around which the integration is performed, and it is that area which is indicated in Figures 13, 14, or 15, depending on whether $\ell = 1, 2$, or 3.

**The q of Equation [22] and the $q_j$ of Equation [23] and Equation [24] are the total plate shears $q_j$ of Equation [11]. In the steps from Equation [23] to Equation [25], the elements of the loop matrix $L_{j\ell}$ are introduced twice, once to ensure that the plate shears $q_j$ are correctly computed in terms of the loop shears $K_{\ell}$ and once to ensure that the summation of $(\Delta s_j / t_j) q_j$ over all plates j is restricted to those plates comprising the loop $\ell$ over which the summation (or integration) is to be performed.

†Twice the area of the triangle formed by the vector $\overline{\Delta s_j}$ and two position vectors A and B from the origin to the head and tail ends of the plate, respectively, are $|\overline{R}_j \times \overline{\Delta s}_j| = |\overline{A} \times \overline{B}| = |(i z_{hj} + j y_{hj}) \times (i z_{tj} + j y_{tj})|$ or $R_j \Delta s_j = y_{tj} z_{hj} - y_{hj} \cdot z_{tj}$.

giving the following L equations ($\ell = 1, 2, \ldots, L$):

$$\sum_j \frac{\Delta s_j}{t_j} L_{j\ell} q_{part\ j} + \left(\sum_j \frac{\Delta s_j}{t_j} L_{j\ell} L_{j1}\right) K_1 + \left(\sum_j \frac{\Delta s_j}{t_j} L_{j\ell} L_{j2}\right) K_2 + \ldots$$

$$\ldots + \left(\sum_j \frac{\Delta s_j}{t_j} L_{j\ell} L_{jL}\right) K_L = \left[\sum_j L_{j\ell}(y_{tj} z_{hj} - y_{hj} z_{tj})\right] G \frac{\partial \theta_x}{\partial x} \qquad [25]$$

In these equations, $q_{part\ j}$ is determined by $V_y$ and $V_z$ according to Equations [12] and [20].*

Another equation relates $M_x$, the twisting moment sustained by the entire section, to the plate shear flows (see Figure 9):

$$M_x = \sum_j q_j R_j \Delta s_j = \sum_j q_j (y_{tj} z_{hj} - y_{hj} z_{tj}) \qquad [26]$$

Here the summation is over all the plates.

To find $q_j$ in terms of $\bar{\bar{V}}_y$, $\bar{\bar{V}}_z$, $\bar{\bar{M}}_x$ (see Equation [30]), we solve Equations [25] for $K_1, K_2, \ldots, K_L$ and substitute into Equations [13], [12], and [11].** This is done three times:

once for $V_z = \bar{\bar{M}}_x = 0$, giving as solutions $q_j = Q_{Vyj} V_y$;

once for $V_y = \bar{\bar{M}}_x = 0$, giving as solutions $q_j = Q_{Vzj} V_z$;

and once for $V_y = V_z = 0$, giving as solutions $q_j = Q_{Tj} \bar{\bar{M}}_x$.

Since $V_y = \bar{\bar{V}}_y$ and $V_z = \bar{\bar{V}}_z$ (see below), we then have:

$$\{q_j\} = [Q] \begin{Bmatrix} \bar{\bar{V}}_y \\ \bar{\bar{V}}_z \\ \bar{\bar{M}}_x \end{Bmatrix} \qquad [27]$$

where the elements of Q are defined by the above solutions:

---

*The physical significance of Equation [25] is that it represents one expression of strain compatibility for each independent loop comprising the cross section of the structure. Basically, in a structure with a loop, strain compatibility ensures that when you go around the loop once you return to the starting point. Contrariwise, for a structure without a loop (for example, a deep channel or U-shaped section), there is no requirement that the adjacent, but unconnected, edges be aligned when torsion is carried.

**Equations [13], [12], and [11] may be combined to give:

$$\{q_j\} + [T_{ji}]\{q_{out\ i}\} + [L_{j\ell}]\{K_\ell\}$$

Solutions of Equation [25] (under the conditions listed) give $\{K_\ell\}$ in terms of $\bar{\bar{V}}_y$, $\bar{\bar{V}}_z$, and $\bar{\bar{M}}_x$. Also Equation [20] gives $\{q_{out\ i}\}$ in terms of $\bar{\bar{V}}_y$ and $\bar{\bar{V}}_z$.

Thus, substituting for $\{q_{out\ i}\}$ and $\{K_\ell\}$ in the above equation will give the plate shears $\{q_j\}$ in terms of $\bar{\bar{V}}_y$, $\bar{\bar{V}}_z$, and $\bar{\bar{M}}_x$.

$$[Q] = \begin{bmatrix} Q_{Vy1} & Q_{Vz1} & Q_{T1} \\ Q_{Vy2} & Q_{Vz2} & Q_{T2} \\ \cdots & \cdots & \cdots \\ Q_{Vyj} & Q_{Vzj} & Q_{Tj} \\ \cdots & \cdots & \cdots \end{bmatrix} \quad [28]$$

Because the strain energy per unit length $\overline{W}$ is quadratic in $q_j$ and, therefore, in $V_y$, $V_z$, $M_x$ (see page 48) we know that the *form* of the equations relating deformation to forces and torques will be:

$$\left.\begin{aligned} \frac{\partial U_y}{\partial x} - \theta_z &= N_{11} V_y + N_{12} V_z + N_{13} M_x \\ \frac{\partial U_z}{\partial x} + \theta_y &= N_{12} V_y + N_{22} V_z + N_{23} M_x \\ \frac{\partial \theta_x}{\partial x} \quad &= N_{13} V_y + N_{23} V_z + N_{33} M_x \end{aligned}\right\} \quad [29]$$

where $N_{ij}$ is constant and $N_{ij} = N_{ji}$.

To transform these equations into a more meaningful form, we can redefine quantities with respect to a point s at $y = \bar{\bar{y}}$, $z = \bar{\bar{z}}$ instead of at the origin. This redefinition of quantities is given by:

$$\left.\begin{aligned} V_y &= \bar{\bar{V}}_y & \bar{\bar{U}}_y &= U_y - \bar{\bar{z}}\,\theta_x \\ V_z &= \bar{\bar{V}}_z & \bar{\bar{U}}_z &= U_z + \bar{\bar{y}}\,\theta_x \\ M_x &= \bar{\bar{M}}_x - \bar{\bar{z}}\,V_y + \bar{\bar{y}}\,V_z & \bar{\bar{\theta}}_x &= \theta_x \\ & & \bar{\bar{\theta}}_y &= \theta_y \\ & & \bar{\bar{\theta}}_z &= \theta_z \end{aligned}\right\} \quad [30]$$

It can be shown (see Equations [2a, b, c] and associated material in Appendix A.1) that if we take $\bar{\bar{y}} = -N_{23}/N_{33}$, $\bar{\bar{z}} = +N_{13}/N_{33}$, the transformed equations are of the form:

$$\begin{Bmatrix} \dfrac{\partial \bar{\bar{U}}_y}{\partial x} - \bar{\bar{\theta}}_z \\ \dfrac{\partial \bar{\bar{U}}_z}{\partial x} + \bar{\bar{\theta}}_y \\ \dfrac{\partial \bar{\bar{\theta}}_x}{\partial x} \end{Bmatrix} = \begin{bmatrix} \bar{\bar{N}}_{11} & \bar{\bar{N}}_{12} & 0 \\ \bar{\bar{N}}_{12} & \bar{\bar{N}}_{22} & 0 \\ 0 & 0 & \bar{\bar{N}}_{33} \end{bmatrix} \begin{Bmatrix} \bar{\bar{V}}_y \\ \bar{\bar{V}}_z \\ \bar{\bar{M}}_x \end{Bmatrix} \quad [31]$$

decoupling the shear and torsion terms, so that this location of point s is called the shear center of the beam.

The first application of Equations [25] sets $V_z = \bar{\bar{M}}_x = 0$. By Equations [30] and [31], this implies that $\frac{\partial\bar{\bar{\theta}}_x}{\partial x} = \frac{\partial\theta_x}{\partial x} = 0$ also. Using these relations and the fact that the terms $q_{part\ j}$ are known multiples of $V_y$ (by Equations [12] and [20]), Equations [25] become:

$$\begin{bmatrix} \sum \frac{\Delta s_j}{t_j} L_{j1} L_{j1} & \sum \frac{\Delta s_j}{t_j} L_{j1} L_{j2} & \cdots & \sum \frac{\Delta s_j}{t_j} L_{j1} L_{jL} \\ \sum \frac{\Delta s_j}{t_j} L_{j2} L_{j1} & \sum \frac{\Delta s_j}{t_j} L_{j2} L_{j2} & \cdots & \sum \frac{\Delta s_j}{t_j} L_{j2} L_{jL} \\ \cdots & \cdots & \cdots & \cdots \\ \sum \frac{\Delta s_j}{t_j} L_{jL} L_{j1} & \sum \frac{\Delta s_j}{t_j} L_{jL} L_{j2} & \cdots & \sum \frac{\Delta s_j}{t_j} L_{jL} L_{jL} \end{bmatrix} \begin{Bmatrix} K_1 \\ K_2 \\ \cdot\cdot \\ K_L \end{Bmatrix} = - \begin{Bmatrix} \sum \frac{\Delta s_j}{t_j} L_{j1} q_{part\ j} \\ \sum \frac{\Delta s_j}{t_j} L_{j2} q_{part\ j} \\ \cdots \\ \sum \frac{\Delta s_j}{t_j} L_{jL} q_{part\ j} \end{Bmatrix} \qquad [32]$$

These equations are solved for $K_1, K_2, \ldots, K_L$. Then, substituting into Equations [11], [12], and [13], we write $q_j$ in terms of $\bar{\bar{V}}_y = V_y$, giving $Q_{Vy1}, Q_{Vy2}, Q_{Vy3}, \ldots$, the solution of plate shear flows per unit $V_y$. Having these, we substitute into Equation [26] to get:

$$M_x = \left[\sum_j Q_{Vyj} (Y_{tj} z_{hj} - y_{hj} z_{tj})\right] V_y \qquad [33]$$

The third equation of Equations [30] then gives the z location of the shear center:

$$\bar{\bar{z}} = \frac{\bar{\bar{M}}_x - M_x + \bar{\bar{y}} V_z}{V_y} = - \frac{M_x}{V_y} = - \sum_j Q_{Vyj} (y_{tj} z_{hj} - y_{hj} z_{tj}) \qquad [34]$$

The second application of Equations [25] sets $V_y = \bar{\bar{M}}_x = \frac{\partial\bar{\bar{\theta}}_x}{\partial x} = \frac{\partial\theta_x}{\partial x} = 0$, leading to equations identical to Equations [32] except that the terms on the right side are now all proportional to $V_z$. The solution gives $K_1, K_2, \ldots, K_L$ in terms of $V_z$ and , by Equations [11], [12], and [13], $q_j$ are written in terms of $\bar{\bar{V}}_z = V_z$, giving the terms $Q_{Vzj}$ of matrix Equations [27] and [28]. Again we substitute into Equation [26] to get:

$$M_x = \left[\sum_j Q_{Vzj} (y_{tj} z_{hj} - y_{hj} z_{tj})\right] V_z \qquad [35]$$

The third equation of Equation [30] now gives the y location of the shear center:

$$\bar{\bar{y}} = \frac{M_x - \bar{\bar{M}}_x + \bar{\bar{z}} V_y}{V_z} = + \frac{M_x}{V_z} = + \sum_j Q_{Vzj} (y_{tj} z_{hj} - y_{hj} z_{tj}) \qquad [36]$$

The third application of Equations [25] sets $V_z = \bar{\bar{V}}_z = V_y = \bar{\bar{V}}_y = 0$. By Equations [12] and [20], this means that $\{q_{part\ j}\} = \{q_{out\ i}\} = 0$ also. Equations [25] now become:

$$\begin{bmatrix} \text{(L by L coefficient} \\ \text{matrix is identical} \\ \text{to the coefficient} \\ \text{matrix of Equation} \\ \text{[32].)} \end{bmatrix} \begin{Bmatrix} K_1 \\ K_2 \\ \cdot\cdot \\ K_L \end{Bmatrix} = \begin{Bmatrix} \sum_j L_{j1} (y_{tj} z_{hj} - y_{hj} z_{tj}) \\ \sum_j L_{j2} (y_{tj} z_{hj} - y_{hj} z_{tj}) \\ \cdots\cdots\cdots \\ \sum_j L_{jL} (y_{tj} z_{hj} - y_{hj} z_{tj}) \end{Bmatrix} \cdot G \frac{\partial \theta_x}{\partial x} \qquad [37]$$

Equations [37] are solved, giving $K_1, K_2, \ldots, K_L$ in terms of $G \frac{\partial \theta_x}{\partial x}$. Again, substituting into Equations [13] and [11] gives $\{q_{loop\ j}\} = \{q_j\}$ in terms of $G\ \partial\theta_x/\partial x$. Express the latter as

$$\{q_j\} = \{Q_{\theta j}\}\ G\ \frac{\partial \theta_x}{\partial x} \qquad [38]$$

Now substituting into Equation [26], using $M_x = \bar{\bar{M}}_x$ from Equation [30] gives:

$$M_x = G\ \frac{\partial \theta_x}{\partial x} \sum_j Q_{\theta j} (y_{tj} z_{hj} - y_{hj} z_{tj}) = \bar{\bar{M}}_x \qquad [39]$$

Solving Equation [39] for $G \frac{\partial \theta_x}{\partial x}$ and substituting into Equation [38] gives:

$$\{q_j\} = \frac{1}{\sum_j Q_{\theta j} (y_{tj} z_{hj} - y_{hj} z_{tj})} \{Q_{\theta j}\}\ \bar{\bar{M}}_x \qquad [40]$$

whereupon, by definition of $\{Q_{Tj}\}$, we have

$$\{Q_{Tj}\} = \frac{1}{\sum_j Q_{\theta j} (y_{tj} z_{hj} - y_{hj} z_{tj})} \{Q_{\theta j}\} \qquad [41]$$

Thus, by three applications of Equations [25] we have determined $\bar{\bar{y}}$ and $\bar{\bar{z}}$ (the shear center location) and the entire matrix [Q] in

$$\{q_j\} = [Q] \begin{Bmatrix} \overline{\overline{V}}_y \\ \overline{\overline{V}}_z \\ \overline{\overline{M}}_x \end{Bmatrix} \begin{bmatrix} Q_{Vy1} & Q_{Vz1} & Q_{T1} \\ Q_{vy2} & Q_{Vz2} & Q_{T2} \\ \cdots & \cdots & \cdots \\ Q_{Vyj} & Q_{vzj} & Q_{Tj} \\ \cdots & \cdots & \cdots \end{bmatrix} \begin{Bmatrix} \overline{\overline{V}}_y \\ \overline{\overline{V}}_z \\ \overline{\overline{M}}_x \end{Bmatrix} \qquad [27],\ [28]$$

We have also determined (from Equation [39]) the term

$$\left(\frac{1}{GJ_e}\right) \equiv \overline{\overline{N}}_{33} \equiv \frac{\frac{\partial \overline{\overline{\theta}}_x}{\partial x}}{\overline{\overline{M}}_x} = \frac{1}{G\sum_j Q_{\theta j}\,(y_{tj}\,z_{hj} - y_{hj}\,z_{tj})} \qquad [42]$$

Now, to apply Castigliano's Theorem, note that the shear strain energy per unit length in all the plates is (see pages 45, 53, 54 and 69):

$$\overline{W} = \frac{W}{\Delta x} = \frac{1}{\Delta x}\sum_j \frac{\tau_j^{\,2}}{2G}\ (\text{Vol.})_j = \frac{1}{\Delta x}\sum_j \frac{1}{2G}\left(\frac{q_j}{t_j}\right)^2 (t_j\,\Delta s_j \Delta x)$$

or

$$\overline{W} = \frac{1}{2G}\sum_j \frac{\Delta s_j}{t_j}\ q_j^{\,2} \qquad [43]$$

Substituting from Equations [27] and [28] yields:

$$\overline{W} = \frac{1}{2G}\sum_j \frac{\Delta s_j}{t_j}\ (Q_{Vyj}^{\,2}\,\overline{\overline{V}}_y^{\,2} + 2Q_{Vyj}\,Q_{Vzj}\,\overline{\overline{V}}_y\,\overline{\overline{V}}_z + 2Q_{Vyj}\,Q_{Tj}\,\overline{\overline{V}}_y\,\overline{\overline{M}}_x$$

$$+ Q_{Vzj}^{\,2}\,\overline{\overline{V}}_z^{\,2} + 2\,Q_{Vzj}\,Q_{Tj}\,\overline{\overline{V}}_z\,\overline{\overline{M}}_x + Q_{Tj}^{\,2}\,\overline{\overline{M}}_x^{\,2}) \qquad [44]$$

Application of Castigliano's Theorem gives:

$$\frac{\partial \overline{\overline{U}}_y}{\partial x} - \overline{\overline{\theta}}_z = \frac{\partial \overline{W}}{\partial \overline{\overline{V}}_y} = \frac{1}{G}\left[\ \overline{\overline{V}}_y \sum_j \frac{\Delta s_j}{t_j}\ Q_{Vyj}^{\,2} + \overline{\overline{V}}_z \sum_j \frac{\Delta s_j}{t_j}\ Q_{Vyj} Q_{Vzj}\right.$$

$$\left. + \overline{\overline{M}}_x \sum_j \frac{\Delta s_j}{t_j}\ Q_{Vyj}\,Q_{Tj}\right] \qquad [45]$$

$$\frac{\partial \overline{\overline{U}}_z}{\partial x} + \overline{\overline{\theta}}_y = \frac{\partial \overline{W}}{\partial \overline{\overline{V}}_z} = \frac{1}{G}\left[\overline{\overline{V}}_y \sum_j \frac{\Delta s_j}{t_j} Q_{Vyj} Q_{Vzj} + \overline{\overline{V}}_z \sum_j \frac{\Delta s_j}{t_j} Q_{Vzj}{}^2 + \overline{\overline{M}}_x \sum_j \frac{\Delta s_j}{t_j} Q_{Vzj} Q_{Tj}\right]$$

$$\frac{\partial \overline{\overline{\theta}}_x}{\partial x} = \frac{\partial \overline{W}}{\partial \overline{\overline{M}}_x} = \frac{1}{G}\left[\overline{\overline{V}}_y \sum_j \frac{\Delta s_j}{t_j} Q_{Vyj} Q_{Tj} + \overline{\overline{V}}_z \sum_j \frac{\Delta s_j}{t_j} Q_{Vzj} Q_{Tj} + \overline{\overline{M}}_x \sum_j \frac{\Delta s_j}{t_j} Q_{Tj}{}^2\right]$$

From Equations [45] and Equations [2a, b, c] of Appendix A.1 (see also pages 51 and 74) we can write the shear flexibility terms of the beam:

$$\left(\frac{1}{K A_{yy} G}\right) \cdot \equiv \cdot \overline{\overline{N_{11}}} \equiv \frac{1}{G} \sum_j \frac{\Delta s_j}{t_j} Q_{Vyj}{}^2 \qquad [46]$$

$$\left(\frac{1}{K A_{yz} G}\right) \cdot \equiv \cdot \overline{\overline{N_{12}}} = \frac{1}{G} \sum_j \frac{\Delta s_j}{t_j} Q_{Vyj} Q_{Vzj} \qquad [47]$$

$$\left(\frac{1}{K A_{zz} G}\right) \cdot \equiv \cdot \overline{\overline{N_{22}}} = \frac{1}{G} \sum_j \frac{\Delta s_j}{t_j} Q_{Vzj}{}^2 \qquad [48]$$

To verify results previously determined or defined, the following relations will always hold:

$$G\,\overline{\overline{N_{13}}} = \sum_j \frac{\Delta s_j}{t_j} Q_{Vyj} Q_{Tj} = 0 \qquad [49]$$

$$G\,\overline{\overline{N_{23}}} = \sum_j \frac{\Delta s_j}{t_j} Q_{Vzj} Q_{Tj} = 0 \qquad [50]$$

$$\left(\frac{1}{G J_e}\right) \cdot \equiv \cdot \overline{\overline{N_{33}}} = \frac{1}{G} \sum_j \frac{\Delta s_j}{t_j} Q_{Tj}{}^2 = \frac{1}{G} \; \frac{1}{\Sigma Q_{\theta j} (y_{tj} z_{hj} - y_{hj} z_{tj})} \qquad [51]$$

## INERTIAL PARAMETERS

In this section we discuss the method for computing the inertial parameters by the digital program, the data required as input for the inertial calculations, the form of input and output data, and the weight calculation for the sample problem.

In determining the inertial parameters, the weight and first and second moments are calculated first by the following equations. The operations performed by the computer are those indicated by these equations:

| Item | Due to Additional Masses | Due to Longitudinal Members at Nodes | Due to Plates |
|---|---|---|---|
| $\Sigma M$ | $= \Sigma M_m$ | $+ \rho \, \Delta x \, \Sigma \, \zeta_i \, A_i$ | $+ \rho \, \Delta x \, \Sigma \, \zeta_i \, t_j \ell_j$ |
| $\Sigma MY$ | $= \Sigma M_m y_m$ | $+ \rho \, \Delta x \, \Sigma \, \zeta_i \, A_i y_i$ | $+ \rho \, \Delta x \, \Sigma \, \zeta_j \, t_j \ell_j \, \overline{y}_j$ |
| $\Sigma MZ$ | $= \Sigma M_m \, z_m$ | $+ \rho \, \Delta x \, \Sigma \, \zeta_i \, A_i \, z_i$ | $+ \rho \, \Delta x \, \Sigma \, \zeta_j \, t_j \ell_j \, \overline{z}_j$ |
| $\Sigma MY^2$ | $= \Sigma(M_m y_m{}^2 + I_{yym})$ | $+ \rho \, \Delta x \, \Sigma \, \zeta_i \, A_i \, y_i{}^2$ | $+ \rho \, \Delta x \, \Sigma \, \zeta_j \, t_j \ell_j \, \overline{y}_j{}^2$ |
| $\Sigma MYZ$ | $= \Sigma(M_m y_m z_m + I_{yzm})$ | $+ \rho \, \Delta x \, \Sigma \, \zeta_i \, A_i \, y_i z_i$ | $+ \rho \, \Delta x \, \Sigma \, \zeta_j \, t_j \ell_j \, \overline{y}_j \, \overline{z}_j$ |
| $\Sigma MZ^2$ | $= \Sigma(M_m z_m{}^2 + I_{zzm})$ | $+ \rho \, \Delta x \, \Sigma \, \zeta_i \, A_i \, z_i$ | $+ \rho \, \Delta x \, \Sigma \, \zeta_j \, t_j \ell_j \, \overline{z}_j{}^2$ |

In the above, the subscript m refers to the additional (nonstructural) mass items, for which

$M_m$ = weight of the item,

$y_m z_m$ = coordinates of the center of gravity of the item,

$I_{yym}$ = weight moment of inertia of the item about an axis through its center of gravity and parallel to the z-axis,

$I_{yzm}$ = weight product of inertia of the item with respect to axes through its center of gravity, and

$I_{zzm}$ = weight moment of inertia of the item about an axis through the center of gravity of the item and parallel to the y-axis.

The subscript i refers to the longitudinal structural members associated with node i and the subscript j refers to the plates. Here:

$\rho$ = density of structural material (basic),

$\Delta x$ = length of hull section for which weights are calculated,

$\zeta_i$ = density ratio, for material at node i, relative to $\rho$,

$A_i$ = cross-sectional area of longitudinal structural members associated with node i,

$y_i, z_i$ = coordinates of node i

$\zeta_j$ = density ratio, for material of plate j, relative to $\rho$,

$t_j$ = thickness of plate j,

$\ell_j = \sqrt{(y_{tj} - y_{nj})^2 + (z_{tj} - z_{hj})^2}$ = length of plate j, and

$$\left.\begin{aligned} \overline{y}_j &= \frac{1}{2}(y_{hj} + y_{tj}) \\ \overline{z}_j &= \frac{1}{2}(z_{hj} + z_{tj}) \end{aligned}\right\} \text{ \{these are the coordinates of the midpoints of the plate j.}$$

Note that, contrary to the method employed in the calculation of elastic parameters of the cross section of the hull, the weight of the plate j is not combined with the weights associated with the nodes at either end of the plate; instead, the entire mass of each plate is accounted for as a lumped mass located at a point midway between the ends of the plate.

To take advantage of symmetry (if it exists) of the cross section about the y-axis, the next step is to double the terms:

$$\Sigma\, M; \quad \Sigma\, MY; \quad \Sigma\, MY^2; \quad \Sigma\, MZ^2$$

and to set to zero the terms:

$$\Sigma\, MZ; \quad \Sigma\, MYZ$$

In either case (symmetry or nonsymmetry), the final step is the use of the following equations to determine the mass, the location of the center of gravity, and the moments of inertia about the center of gravity:

$$\begin{aligned}
\text{MASS} &= \Sigma M \\
\text{Y-CG} &= \Sigma MY \div \Sigma M \\
\text{Z-CG} &= \Sigma MZ \div \Sigma M \\
\text{I-YY} &= \Sigma MY^2 - (\text{Y-CG})^2\, \Sigma M \\
\text{I-YZ} &= \Sigma MYZ - (\text{Y-CG})\,(\text{Z-CG})\, \Sigma M \\
\text{I-ZZ} &= \Sigma MZ^2 - (\text{Z-CG})^2\, \Sigma M \\
\text{I-MX} &= (\text{I-YY}) + (\text{I-ZZ})
\end{aligned}$$

The final term represents the polar moment of inertia of the weight of the section about a longitudinal axis.

Data required as input for the inertia calculations include items describing the portions due to structural items (these are necessary for the flexibility calculations and need not be duplicated) and the items describing the portions due to nonstructural members. The following FORTRAN symbols are used for these latter terms:

| FORTRAN Symbol | Symbol in above equations | FORTRAN Symbol | Symbol in above equations |
|---|---|---|---|
| IW | m | WYZ | $I_{yzm}$ |
| NW | max. value of m | WZZ | $I_{zzm}$ |
| W | $M_m$ | RHO | $\rho$ |
| YW,ZW | $y_m$, $z_m$ | AD | $\zeta_i$ |
| WYY | $I_{yym}$ | DX | $\Delta x$ |

For other FORTRAN symbols refer to Table 1b and to Figure 4b.

It is, of course, necessary that these inputs be in consistent units. For example, if SCALE = 1.0 (no mixed units), the following may be used:

| | |
|---|---|
| $y_i$, $z_i$, $y_j$, $z_j$, $y_m$, $z_m$, $t_j$ | in. |
| $A_i$ | in.$^2$ |
| $M_m$ | lb |
| $\rho$ | lb-in.$^{-3}$ |
| $\zeta_i$ | (dimensionless) |
| $I_{yym}$, $I_{yzm}$, $I_{zzm}$ | lb-in.$^2$ |
| $\Delta x$ | in. |

In a second example, let SCALE = 12.0. Then the following may be used:

| | |
|---|---|
| $y_i$, $z_i$, etc. | ft |
| $t_j$ | in. |
| $A_i$ | in.$^2$ |
| $M_m$ | lb |
| $\rho$ | lb-ft$^{-3}$ |
| $I_{yym}$, etc. | lb-ft$^2$ |
| $\Delta x$ | ft |

For a description of the form of the input data, refer to Tables 1a and 1c (for format), Tables 2a and 2b (for structural items, sample problem) and Table 2c (for nonstructural items, of which there are no entries in the sample problem).

For the form of the output data (sample problem), see Table 3a.

The weight calculation for the sample problem, which gives results agreeing with Table 3a, is presented in Table 4.

# APPENDIX B

## DESCRIPTION OF DIGITAL COMPUTER PROGRAM AND FLOW CHARTS

A digital computer program has been devised to solve for the problem just discussed. The coding was done in FORTRAN. An index of FORTRAN symbols used (Figure 4b), a listing of the FORTRAN statements (Figure 4a), and flow charts (Figure 5) are provided to explain the program. Items will be discussed in the order they appear in the listing.

### INPUT

Provision has been made for both card or tape input, under control of sense switch 5. The input tape has been designated 5. The designation is immediately read out after it is read; see Figure 5b.

### SCALE RENUMBER

SCALE refers to modifying input data to allow the machine to work with numbers of the same units if the user wants to use a certain mixed set of units for the data. Every node and plate is assigned a number by the person who prepares the input cards. It is not required that they be in sequential order. (This is useful because if one wants to see the effect of removing a plate, the card in question can be removed, and the others do not have to be renumbered; the count NP must be changed of course.) A dictionary (called NRANK) is made such that if NRANK(5) = 3, then the node that the user calls 5 is in the third location. The nodes will be referred to (in the program) by the order in which they appear in memory (the order of input), so that references (NH and NT) of the plates to nodes must be changed. Each NH and NT is found in the dictionary and replaced by that number. If a number is specified on one of the plate cards for NH or NT and no node of that number had been given, an output statement to that effect will be made, and after checking the rest of the cards to see if any more errors were made, the computer will immediately return to try the set of data for the next section, if any.

### EXTRACT KEY

No test was made here to see that the value is allowable. The first and third digits must be 1 or 2, the second digit may be 1, 2, 3, or 4. Other values will probably result in the computer becoming lost. See Figure 4b for the meaning of these symbols.

The arithmetic statements are quite straightforward and should cause no difficulty. No provision is made to branch an overflow.

## TREE SEARCH

The tree search (Figure 5c) is given a node to start. It looks through all plate items (both NH and NT) which have not yet been used to see if one joins the node. If not, it is by-passed. If so, it looks at the node at the other end and determines if that node has been joined to the tree. If not, it puts the new node on the tree by giving the value of NEXT for the new node, the present node, and puts the new node on the bottom of the list of nodes to be searched, NRANK. A value for LINK is given each node as it is entered on the tree, which tells which plate goes back to NEXT. The sign of LINK specifies the direction of the plate. The branch is put on the tree by making MTYPE = 1. If, when the other end of a branch is being examined to see if it should be added to the tree, it is found that it is already on the tree (NEXT $\neq$ 0), then the branch is one that will close a loop and MTYPE = - 1. After all branches have been examined to see if they touch the first node, the computer will take the next item on NRANK, and if it is on the tree (NRANK $\neq$ 0), it will look through the remaining branches, etc. There are two possible exits. Normal exit occurs if all nodes are found (it finds NN of them). If all nodes are not connected, then, at some time all nodes which were connected to the first node have been searched, but no more nodes are in NRANK to use. The computer prints a statement if nodes are not properly connected, and starts to the next case.

## FIND TREE LOOPS

To find the loops (Figure 5d), take each plate which is not on the tree (MTYPE = −1). Go back down the tree from both ends to the origin to close loop. For symmetric sections, shear flow can cross the centerline for z-shear and torsion but not for y-shear. More loops are generated by going directly from the first node to the other nodes on the centerline and then returning via the tree. L = number of loops for y-shear, LA = number of loops for z-shear and torsion.

The word NEXT was used in connection with going back down the tree to the first node.

## OUTPUT

Output may be either on-line (printer) or off-line (tape) by proper choice of sense switch 5 (see output flow chart, Figure 3b).

# APPENDIX C

# OPERATION AND RULES OF THE COMPUTER PROGRAM

## INPUT FORMS

All data are to be collected and put on an input form from which it will be punched into cards for input to the computer. Table 2 is an example of one such input form and includes data for the sample problem treated on page 6. Only one "identification" (see Tables 1a and b) is used per section. This first card is used to identify the deck of cards which is punched and will also appear as a heading on the output (Table 3). A second card will contain the number of cards which follow in each of the three categories (nodes, plates, and masses); operating instructions; and some constants. The data cards follow these first two cards. All of the node cards must come next, and there must be more than one such card. Next come all of the plate cards; there must be at least one of these. The mass cards, if any, follow next. The cards must be stacked in the order indicated, and there must be exactly as many of each type as indicated on the second card; however, the cards within any of the three categories may be in any order. If more than one section is to be analyzed, the cards for each section may be stacked together. Each section must begin with its identifier. After the calculation for one section is completed, the computer will automatically begin the next section.

## OUTPUT FORMS

The output data will be identified by headings (see Table 3). The units are consistent, the length always being the same as Y and Z. (If areas are given in square inches and Y in feet, SCALE = 12.0, the output will all be in feet units.) Mass output includes mass, location of the center of gravity, and the moments of inertia about the center of gravity. Areal output includes the total tension area (not used in beam analysis unless there is an axial load), location of the elastic (neutral) axis, and bending flexibilities. If $I_{yy}$ is the moment of inertia about an axis parallel to the Z-axis through the elastic axis ($I_{yy} = \Sigma AY^2$), etc., and $\Delta = I_{yy} I_{zz} - I_{yz}^2$, then*

$$\text{YY Flexibility} = I_{yy}/\Delta, \ (=1/I_{zz} \text{ for symmetric section})$$

$$\text{YZ Flexibility} = I_{yz}/\Delta\ , \ (= 0 \text{ for symmetric section})$$

$$\text{ZZ Flexibility} = I_{zz}/\Delta, \ (=1/I_{yy} \text{ for symmetric section})$$

*See page 53 and Sheet 1 of Table 4.

These flexibilities should be divided by the reference value of E to be used in the beam equations. Shear output includes the location of the shear center and flexibilities (torsion and shear).

$$\text{Torsion Flexibility} = 1/J_e, \ (GJ_e \text{ is torsional rigidity})$$

$$\text{YY Flexibility} = \frac{1}{K_{yy} A}$$

$$\text{YZ Flexibility} = \frac{1}{K_{yz} A}.$$

$$\text{ZZ Flexibility} = \frac{1}{K_{zz} A}$$

These should be divided by the reference value of G for use in the beam equations.

## TROUBLE SHOOTING

If the machine stops on an arithmetic overflow, the operator should make sure that:

1. SCALE $> 0$.
2. All PT$> 0$.
3. All PG $> 0$.
4. There is some mass if second digit of KEY = 1.
5. There is some area, and it does not all lie upon a straight line if the second digit of KEY = 1, 2, 3.

The machine may print THE STRUCTURE IS NOT PROPERLY CONNECTED. This may be due to one of the following two causes: All nodes *must* be joined by at least one path through the plates and there *must* be at least one closed loop. (For the case of symmetry when only half of the structure is drawn, it is sufficient to have at least one closed loop in the complete section.)

Limits

$2 \leq$ NN (number of nodes) $\leq 150$

$2 \leq$ NP (number of plates) $\leq 150$

$0 \leq$ NW (number of masses) $\leq 100$

## FLOW CHART

A copy of the flow chart of the computer program (Figures 5a–h) is included for reference.

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For the purpose of vibration and load analysis a ship hull is often regarded as a flexural beam. This report describes a method and numerical computer (digital) program to calculate ship section properties (i.e., equivalent beam parameters) needed for the beam vibration equations and its internal shear distribution, using data tabulations obtained from hull plans by a pre-established orderly procedure. The program has been written in FORTRAN and can be used on an IBM 650, 704, 709, or 7090. Comparison between digital computer and hand calculations for a sample problem shows excellent agreement.

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