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GENERAL PRESSURE DISTRIBUTION FOR AN OSCILLATING SPHERE FLOATING IN A FLUID OF FINITE DEPTH

NAVAL SHIP RESEARCH AND DEVELOPMENT CENTER

Washington, D.C. 20007

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by

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NOTATION

A	$= \int_0^z (u^2 + R^2)^{-1/2} e^{Ku} du$
a	Radius of the sphere
C_n	$= PV \int_0^\infty \frac{u^{2n+2} e^{-u} du}{(Kh - u)(Kh \cosh u - u \sinh u)}$
D	Dipole weighting function
$Ei(x)$	Exponential integral
F	Force acting on the sphere
g	Acceleration of gravity
H_o	Struve function of order zero
h	Depth of the fluid
h_o	Heave amplitude
J_o	Bessel function of the first kind of order zero
K	Deep water wave number
K_o	Modified Bessel function of the second kind of order zero
k_o, k_n	Real and imaginary roots of equation $K = k \tanh kh$
P_n	Legendre function
$PV\int$	Principle value integral
p	Hydrodynamic pressure
p_a, p_v	Pressure components
$\tilde{p} = (p_a^2 + p_v^2)^{1/2}$	Pressure amplitude
$R = (x^2 + y^2)^{1/2}$	Horizontal distance from z-axis
r	Radial distance from origin
S	Monopole weighting function
t	Time
$U = \dot{z}$	Velocity of the sphere
Y_o	Bessel function of the second kind of order zero
xyz	Rectangular coordinate system

α	Added-mass coefficient
$\beta = Ka$	Dimensionless wave number
δ	Damping coefficient
θ	Polar angle between +z-axis and radius vector $\vec{r}$
λ	Wavelength
μ_o	Monopole strength
μ_1	Dipole strength
$\rho = \frac{r}{a}$	Dimensionless radial distance
ρ_o	Density of fluid
Φ	Velocity potential
Φ_a^d, Φ_v^d	Dipole potential functions
Φ_a^s, Φ_v^s	Monopole potential functions

ABSTRACT

The general pressure field caused by a heaving sphere floating half submerged in a fluid of finite depth is calculated. A sphere oscillating in an unbounded fluid develops a dipole pressure field. When oscillating on the free surface of a fluid, however, damping and buoyancy forces change the pressure field which now comes under monopole influence as well. The high- and low-frequency limits of singularities in an infinite fluid define the surface conditions for a bounded fluid. The general intermediate frequency case is considered, whereby the pressure field is related to the forces acting on the sphere.

ADMINISTRATIVE INFORMATION

This work was performed at the Naval Ship Research and Development Center under the General Hydrodynamics Research Program.

INTRODUCTION

The pressure field radiated from a sphere moving in an unbounded fluid is known to be equal to the pressure field radiated from a dipole placed in the center of the sphere. If, however, the fluid is bounded, and the motion occurs on or near the free surface, surface waves are generated which cause damping and, consequently, change the pressure field. The influence of surface waves due to a body heaving near the free surface on the radiated pressure field has often been subject to investigation in recent years, e.g., Ursell[1] and Porter[2] of the two-dimensional case of a heaving cylinder, and Havelock,[3] Barakat,[4] Franz,[5] and Wang[6] of the three-dimensional case of a heaving sphere. All investigations are based on the assumption of an ideal fluid, i.e., a fluid free of vortices; thus, a velocity potential must exist.

Thorne[7] (1953) treated the problem of generation of surface waves. His paper was fundamental in the development of the potential of two- and three-dimensional multipoles near a free surface. Havelock (1955) considered a heaving sphere half submerged in a fluid of infinite depth. Based

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[1]References are listed on page 32.

on the representation by Thorne, he calculated the respective added-mass and damping coefficients and the pressure on the sphere. Barakat (1962) studied the same problem, using a generalization of methods used by Ursell in his solutions for the two-dimensional case of a heaving cylinder. Wang (1966) attacked the problem of a heaving sphere in a fluid of finite depth. He gave values for the added-mass and damping coefficients. These become functions of the heave frequency and the depth of the fluid. He also calculated the pressure on the sphere and on the bottom of the fluid.

None of the mentioned papers discussed calculation of pressure anywhere in the fluid. An extension of the Wang presentation for this purpose is feasible; the performance, however, will be very cumbersome.

It is not intended to repeat all the steps which lead to an "exact" analytical solution, and reference is made to the authors cited, particularly to work by Wang. Instead, preference is given to evaluation of an idea that was evolved in 1965, making it possible to reach final results quicker. Franz found that the theory of classical dipole pressure of a body, such as a sphere oscillating in an infinite fluid, could be extended to include the presence of a free surface. When the sphere is floating half submerged in the fluid, not only does the damping caused by the created surface waves change the pressure field but the existing buoyancy force also requires additional consideration of a monopole pressure field. The low- and high-frequency limits of the fluctuating system characterize the conditions for the presence of a free or a rigid surface, respectively. Using these limiting cases, the general expression for the fluctuating pressure is then derived from the corresponding velocity potential. Franz described this pressure field for intermediate frequencies in terms of the classical high-frequency relation between the fluctuating forces and pressures on the sphere and certain weighting functions which become functions of the space coordinates and the heaving frequency. He calculated the pressure directly under the sphere when the fluid was of infinite depth. The forces exerted by the fluid on the sphere can be described by means of both added-mass and damping coefficients, which in the present report are assumed to be known. To relate the fluctuating pressure to the forces acting on the sphere is of practical significance insofar as measurements of the pressure in the fluid can lead to conclusions about the forces acting on the sphere and vice versa.

In the following, the Franz "force" method is used to investigate the fluctuating pressure anywhere in a fluid of finite depth with the intention of showing the usefulness of this method in obtaining pressure information, compared to that available from the more laborious, exact method presented by Wang.[6]

BASIC ASSUMPTIONS

Assume the origin of an xyz-system on the mean free surface of the fluid. The xy-plane coincides with the surface; the z-axis increases with increasing depth. The bottom of the fluid is at $z = h$.

The sphere with radius a, half submerged in the fluid, is performing simple harmonic oscillations with small amplitude h_o and angular frequency ω along the z-axis; thus, the position of its center at any time t is

$$z = h_o \cos \omega t \qquad [1]$$

The existing velocity potential Φ satisfies the Laplace equation

$$\nabla^2 \Phi = 0$$

anywhere in the fluid, and it is understood that the velocity components with respect to the three axes are given by

$$\dot{x} = v_x = -\frac{\partial \Phi}{\partial x} \quad \dot{y} = v_y = -\frac{\partial \Phi}{\partial y} \quad \dot{z} = v_z = -\frac{\partial \Phi}{\partial z}$$

In particular, the radial velocity becomes

$$\dot{r} = v_r = -\frac{\partial \Phi}{\partial r} \qquad [2]$$

The boundary condition on the free surface is known to be

$$K\Phi + \frac{\partial \Phi}{\partial z} = 0 \qquad [3]$$

where $K = \frac{\omega^2}{g}$ is the wave number in deep water, and g is the acceleration of gravity.

The boundary condition on the bottom $z = h$ is

$$\frac{\partial \Phi}{\partial z} = 0 \quad \text{on } z = h \qquad [4]$$

Further, the pressure is calculated from the linearized Bernoulli equation, namely,

$$p = \rho_o \frac{\partial \Phi}{\partial t} \qquad [5]$$

where ρ_o is the density of the fluid.

The length of the radius vector from center sphere to any point in the fluid is r, and the angle between radius vector and positive z-axis is θ.

FORCES ACTING ON THE SPHERE

To sustain steady-state oscillation (Equation [1]) the following forces are applied on the sphere:

1. The buoyant force of Archimedes F_b, which is a hydrostatic force in phase with the displacement. This force is caused by hydrostatic water pressures on the sphere and is equal to the weight of the displaced fluid. For small amplitudes, the volume of the displaced fluid can be taken as $a^2\pi z$. Therefore, the buoyant force F_b becomes

$$F_b = a^2\pi \; \rho_o gz = a^2\pi \; \rho_o gh_o \cos \omega t$$

All other forces are hydrodynamic in nature. They are

2. The inertial force F_i, which is the inertial resistance of the mass of the displaced hemisphere of fluid to its acceleration. This force is in phase with the acceleration of the sphere and is given by

$$F_i = \frac{2}{3} \pi a^3 \; \rho_o \ddot{z} = - \frac{2}{3} \pi a^3 \; \rho_o h_o \; \omega^2 \cos \omega t$$

3. The added-mass force F_α, which is the force on the hemisphere due to pressures on the sphere in phase with the acceleration and equals

$$F_\alpha = \alpha F_i$$

where α is the added-mass coefficient.

4. The damping force F_δ, which is due to the pressures on the hemisphere in phase with the velocity of the sphere and is given in terms of the damping force coefficient δ by

$$F_\delta = \frac{2}{3} \pi a^3 \rho_o \omega \delta \dot{z} = - \frac{2}{3} \pi a^3 \rho_o \omega^2 \delta h_o \sin \omega t$$

Introducing $\beta = \frac{\omega^2 a}{g}$ and using Equation [1], the total fluctuating force F(t) is then

$$F(t) = a^2 \pi \rho_o g h_o \left\{ \left[1 - (1 + \alpha) \left(\frac{2\beta}{3} \right) \right] \cos \omega t - \delta \left(\frac{2\beta}{3} \right) \sin \omega t \right\} \qquad [6]$$

with its maximum value

$$\tilde{F} = a^2 \pi \rho_o g h_o \left\{ \left[1 - (1 + \alpha) \left(\frac{2\beta}{3} \right) \right]^2 + \left[\frac{2\beta\delta}{3} \right]^2 \right\}^{1/2} \qquad [6a]$$

The added-mass and damping coefficients vary with frequency and depth and are given for a certain range in Reference 6.

The oscillating hydrostatic buoyant force has a monopole character; the three hydrodynamic forces are of dipole nature.

LIMITING CASES OF THE SURFACE CONDITION $K\Phi + \frac{\partial \Phi}{\partial z} = 0$

This surface condition, valid for any frequency, includes two limiting cases.

For very low frequencies ($K \rightarrow 0$), the condition reduces to

$$\frac{\partial \Phi}{\partial z} = 0 \quad \text{on } z = 0$$

i.e., the vertical velocities vanish; consequently, the surface acts as a solid boundary.

For very high frequencies ($K \rightarrow \infty$), the condition is

$$\Phi = 0 \quad \text{on } z = 0$$

i.e., the surface becomes a free surface, neglecting gravity in comparison to the forces of inertia:

To avoid the inconvenience of a bounded fluid, the method of images is applied, yielding the following considerations:

The pressure field of a very low frequency monopole near the surface is equal to the pressure field of two monopoles of equal strength in an unbounded fluid. Likewise, a very high frequency monopole near a free surface can be replaced by a dipole in an unbounded fluid. These considerations hold for points of much greater distance from the singularities than the singularities are from the surface. In this case, the sphere can be treated as a mass point.

PRESSURE OF THE MONOPOLE NEAR A FREE SURFACE IN THE LIMIT OF VERY LOW FREQUENCY

The velocity potential Φ in an unbounded fluid caused by a simple source whose strength is given by the time rate of change of the volume $V = V_o \cos \omega t$ takes the form

$$\Phi = \frac{\dot{V}}{4\pi r} = - \frac{V_o \omega \sin \omega t}{4\pi r}$$

where the monopole strength μ_o is given by $V_o \omega / 4\pi$. According to Equation [5], the pressure becomes

$$p = \rho_o \frac{\ddot{V}}{4\pi r} = - \frac{\rho_o V_o \omega^2}{4\pi r} \cos \omega t$$

For a sphere oscillating in the free surface with an amplitude that is small compared to the radius of the sphere, the oscillating volume is $V_o = a^2 \pi z$, where $z = h_o \cos \omega t$. In the limit of very low frequency, the volume has to be doubled, so that

$$V = 2V_o = 2 \cdot a^2 \pi z = 2 \cdot a^2 \pi h_o \cos \omega t$$

and

$$\frac{\ddot{V}}{4\pi} = - \frac{a^2}{2} h_o \omega^2 \cos \omega t = - \frac{\omega^2 V_o}{2\pi}$$

Therefore, for low frequency oscillations the monopole pressure changes to

$$p = \rho_o \dot{\Phi} = \rho_o \frac{\ddot{V}}{4\pi r} = -\frac{a^2}{2r} \rho_o h_o \omega^2 \cos \omega t = -\rho_o \frac{\omega^2 V_o}{2\pi r}$$

With $\rho = \frac{r}{a}$ and $\beta = \frac{\omega^2 a}{g}$ the low frequency monopole pressure becomes

$$\frac{p^s}{\rho_o g h_o} = -\frac{\beta}{2\rho} \cos \omega t \qquad [7]$$

This pressure, caused by the buoyant force, can also be written as

$$\frac{p^s}{\rho_o g h_o} = -\frac{\beta}{2\rho} \frac{F_b}{a^2 \pi \rho_o g h_o} \qquad [7a]$$

PRESSURE OF THE MONOPOLE NEAR A FREE SURFACE IN THE LIMIT OF VERY HIGH FREQUENCY

Since a very high-frequency monopole near a free surface can be replaced by a dipole in an unbounded fluid, we consider the velocity potential of a dipole in the unbounded fluid

$$\Phi^d = \frac{a^3 U}{2r^2} \cos \theta \qquad [8]$$

where $U = \dot{z}$ is the velocity of the moving sphere (Reference 8, p. 123, Equation [2]). The pressure is

$$p^d = \rho_o \dot{\Phi} = \rho_o \frac{a^3 \ddot{z}}{2r^2} \cos \theta$$

or, written in dimensionless form

$$\frac{p^d}{\rho_o g h_o} = \frac{\beta}{2\rho^2} \cos \theta \cos \omega t$$

Now, for the hemisphere oscillating in the free surface in the limit of very high frequency, the pressure is determined by

$$p^d = \frac{F}{4\pi r^2} \cos \theta$$

This pressure representation holds in the near field, i.e., for points whose distances r from the sphere are less than the wavelength λ_a of the radiated sound.

F is the total hydrodynamic force acting on the full sphere in an unbounded fluid. This force has three components, namely, twice the hydrodynamic forces mentioned before, i.e.,

1. Inertia

$$F_i = m\ddot{z} = -\frac{2}{3}\pi a^3 \rho_o h_o \omega^2 \cos \omega t = -\frac{2\beta}{3}\pi a^2 \rho_o g h_o \cos \omega t$$

2. Added mass

$$F_\alpha = \alpha m\ddot{z} = \alpha F_i$$

3. Damping

$$F_\delta = \delta m\omega\dot{z} = -\frac{2}{3}\pi a^3 \rho_o h_o \omega^2 \delta \sin \omega t = -\frac{2\beta}{3}\pi a^2 \rho_o g h_o \delta \sin \omega t$$

Thus, in the limiting case of very high frequency, the total force to be applied is

$$F = 2(F_i + F_a + F_\delta) = 2F_o$$

and the dipole pressure, written in dimensionless form, becomes

$$\frac{p^d}{\rho_o g h_o} = -\frac{\beta}{3\rho^2}\cos\theta\left\{(1+\alpha)\cos\omega t + \delta\sin\omega t\right\} \qquad [9]$$

PRESSURE AT INTERMEDIATE FREQUENCIES AND THE WEIGHTING FUNCTIONS

At intermediate frequencies, the velocity potentials of both monopole and dipole must now satisfy the combined surface condition, Equation [3]. Further, considering a finite depth h of the fluid, the bottom condition (Equation [4]) imposes a relationship between frequency ω and depth h, expressed by the equation

$$K = \frac{\omega^2}{g} = k \tanh kh \qquad [10]$$

The respective potentials are given by Thorne.[7]

The theory shows that the source potential can be written as

$$\frac{\Phi^s}{\mu_o} = -\Phi_a^s \sin \omega t + \Phi_v^s \cos \omega t$$

Φ_a^s and Φ_v^s are components of the velocity potential, which become functions of frequency, space coordinates, and depth of the fluid. They are given later in detail.

Introducing the monopole weighting functions by

$$S_a = \frac{r}{2} \Phi_a^s \quad \text{and} \quad S_v = \frac{r}{2} \Phi_v^s \qquad [11]$$

and remembering that the source strength

$$\mu_o = \frac{V_o \omega}{4\pi} = \frac{a^2 h_o \omega}{4} \quad \text{with} \quad V_o = a^2 \pi h_o$$

the source potential for intermediate frequencies becomes

$$\Phi^s = \frac{a^2 h_o \omega}{2r} \left\{ -S_a \sin \omega t + S_v \cos \omega t \right\}$$

From this, the pressure is calculated as

$$p^s = \rho_o \dot{\Phi}^s = \frac{a^2 h_o \rho_o \omega^2}{2r} \left\{ -S_a \cos \omega t - S_v \sin \omega t \right\}$$

so that finally

$$\frac{p^s}{\rho_o g h_o} = - \frac{\beta}{2\rho} \left\{ S_a \cos \omega t + S_v \sin \omega t \right\} \qquad [12]$$

Again, from the Thorne presentation, the velocity potential of a dipole near the free surface can be written as

$$\frac{\Phi^d}{\mu_1} = -\Phi_a^d \sin \omega t + \Phi_v^d \cos \omega t$$

It will be shown later that the dipole potential functions Φ_a^d and Φ_v^d are related to the monopole potential functions Φ_a^s and Φ_v^s and also to the monopole weighting functions by

$$\Phi_a^d = -K\Phi_a^s = -\frac{2}{r} KS_a \qquad \Phi_v^d = -(k \tanh kh)\Phi_v^s = -K\Phi_v^s = -\frac{2}{r} KS_v$$

If we introduce dipole weighting functions

$$D_a = -\frac{Kr}{\cos\theta} S_a \qquad D_v = -\frac{Kr}{\cos\theta} S_v \qquad [13]$$

and write the dipole strength from Equation [8] as

$$\mu_1 = \frac{a^3 \omega h_o}{2}$$

the dipole potential becomes

$$\Phi^d = -\frac{a^3 \omega h_o \cos\theta}{r^2} [D_a \sin \omega t - D_v \cos \omega t]$$

and the dipole pressure is

$$\frac{p^d}{\rho_o g h_o} = -\frac{\beta}{\rho^2} \cos\theta \, [D_a \cos \omega t + D_v \sin \omega t] \qquad [14]$$

or

$$\frac{p^d}{\rho_o g h_o} = -\frac{\beta}{\rho^2} \cos\theta \, [D_a \sin \omega t - D_v \cos \omega t] \qquad [15]$$

depending on whether the exciting forces are in phase with acceleration, such as F_i and F_a, or in phase with velocity, such as $F_v \cdot \Pi S_a$, S_v, and whether D_a, D_v are the time independent components of the weighting functions S and D of the source and dipole potential functions, respectively. When applied to the expressions for the low-frequency monopole (Equation [6]) and the high-frequency dipole (Equation [8]) the equation for the general pressure field anywhere in the fluid is obtained. With the help of Equations [10] through [14], the equation becomes

$$\frac{p}{\rho_o g h_o} = \left\{ -\frac{\beta}{2\rho} S_a - \frac{\beta \cos \theta}{3\rho^2} (1 + \alpha) D_a + \frac{\beta \cos \theta}{3\rho^2} \delta D_v \right\} \cos \omega t$$

$$+ \left\{ -\frac{\beta}{2\rho} S_v - \frac{\beta \cos \theta}{3\rho^2} (1 + \alpha) D_v - \frac{\beta \cos \theta}{3\rho^2} \delta D_a \right\} \sin \omega t \qquad [16]$$

or

$$\frac{p}{\rho_o g h_o} = -p_a \cos \omega t - p_v \sin \omega t$$

where p_a and p_v are the pressure components in phase with acceleration and velocity.

For the pressure amplitude, we find

$$\frac{\tilde{p}}{\rho_o g h_o} = (p_a^2 + p_v^2)^{1/2}$$

Introducing

$$S = (S_a^2 + S_v^2)^{1/2} \qquad D = (D_a^2 + D_v^2)^{1/2}$$

the formula for the pressure amplitude can be written in either form

$$\frac{\tilde{p}}{\rho_o g h_o} = \frac{\beta}{2\rho} S \left\{ \left[\frac{2\beta}{3} (1 + \alpha) - 1 \right]^2 + \left[\frac{2\beta}{3} \delta \right]^2 \right\}^{1/2}$$

or

$$\frac{\tilde{p}}{2\rho_o g h_o} \doteq \frac{\beta \cos \theta}{2\rho^2} D \left\{ \left[\frac{2\beta}{3} (1 + \alpha) - 1 \right]^2 + \left[\frac{2\beta}{3} \delta \right]^2 \right\}^{1/2}$$

Both formulas are convenient for computation; however, they no longer show the influence of the monopole and the dipole separately.

After the formula for the pressure is built up, the weighting functions are derived from the potential functions of concern.

SOURCE POTENTIAL Φ^s

When the source potential is written as

$$\frac{\Phi^s}{\mu_o} = -\Phi_a^s \sin \omega t + \Phi_v^s \cos \omega t$$

the components Φ_a^s and Φ_v^s as given by Reference 7 are as follows

$$\Phi_a^s = \frac{1}{r} + PV \int_o^\infty \frac{e^{-\nu h}[K \sinh \nu z - \nu \cosh \nu z] - (K + \nu) \cosh[\nu(h - z)]}{K \cosh \nu h - \nu \sinh \nu h} J_o(\nu R) \, d\nu$$

and

$$\Phi_v^s = 4\pi k_o \frac{\cosh(k_o h) \cosh[k_o(h - z)]}{2k_o h + \sinh(2k_o h)} J_o(k_o R)$$

Using the relation

$$\frac{1}{r} = PV \int_o^\infty e^{-\nu z} J_o(\nu R) \, d\nu$$

the expression for Φ_a^s is changed to

$$\Phi_a^s = 2\left\{\frac{1}{r} + PV \int_0^\infty \frac{e^{-\nu h}\left[K \sinh \nu z - \nu \cosh \nu z - K \cosh \left[\nu(h - z)\right]\right]}{K \cosh \nu h - \nu \sinh \nu h} J_o(\nu R)\, d\nu\right\}$$

$$= -2PV \int_0^\infty \frac{\nu \cosh\left[\nu(h - z)\right]}{K \cosh \nu h - \nu \sinh \nu h} J_o(\nu R)\, d\nu \qquad [17]$$

The principal value integral can be evaluated and is represented in two different forms that are useful for numerical calculations in the geometrical near and far field, respectively, namely[6]

$$\Phi^s_{a\,(near)} = 2\left[\frac{1}{r} + Ke^{-Kz}\left\{\frac{\pi}{2}\left[H_o(KR) + Y_o(KR)\right] + A\right\}\right]$$

$$+ 2 \sum_{n=0}^{\infty} \frac{Cn}{h^{2n+1}} \left[\frac{r^{2n}}{(2n)!} P_{2n}(\cos\theta) - \frac{Kr^{2n+1}}{(2n+1)!} P_{2n+1}(\cos\theta)\right]$$

$$\Phi^s_{a\,(far)} = -4\pi k_o \frac{\cosh(k_o h) \cosh\left[k_o(h - z)\right]}{2k_o h + \sinh(2k_o h)} Y_o(k_o R)$$

$$+ 2 \sum_{n=1}^{\infty} \frac{4k_n \cos(k_n h) \cos\left[k_n(h - z)\right]}{2k_n h - \sin(2k_n h)} K_o(k_n R)$$

where

$$A = \int_0^z (u^2 + R^2)^{-1/2} e^{Ku} du$$

$$C_n = PV \int_0^\infty \frac{u^{2n+2} e^{-u}}{(v - u)(v \cosh u - u \sinh u)} du \quad ; \quad v = Kh$$

k_o, k_n are the real (k_o) and imaginary (k_n) roots of the equation

$$K = k \tanh kh$$

so that

$$K = k_o \tanh k_o h$$

and

$$K = -k_n \tan k_n h \quad (n = 1, 2, \ldots, \infty)$$

Using the weighting functions (Equation [11]) the source potential is represented as

$$\frac{\Phi^s}{\mu_o} = \frac{2}{r} \left\{ -S_a \sin \omega t + S_v \cos \omega t \right\}$$

and

$$S_v = \frac{2\pi k_o r \cosh(k_o h) \cosh\left[k_o(h - z)\right]}{2k_o h + \sinh(2k_o h)} J_o(k_o R) \qquad [18]$$

$$S_{a(near)} = 1 - Kre^{-Kz} \left\{ \frac{\pi}{2} \left[H_o(KR) + Y_o(KR)\right] + A \right\} \qquad [19]$$

$$+ \sum_{n=0}^{\infty} \frac{C_n}{(2n)!} \left(\frac{r}{h}\right)^{2n+1} \left[P_{2n}(\cos \theta) - \frac{Kr}{2n + 1} P_{2n+1}(\cos \theta)\right]$$

$$S_{a(far)} = -2\pi k_o r \frac{\cosh(k_o h) \cosh\left[k_o(h - z)\right]}{2k_o h + \sinh(2k_o h)} Y_o(k_o R) \qquad [20]$$

$$+ \sum_{n=1}^{\infty} \frac{4rk_n \cos(k_n h) \cos\left[k_n(h - z)\right]}{2k_n h - \sin(2k_n h)} K_o(k_n R)$$

For points directly below the sphere (R = 0) the formulas for S_a as shown previously are not applicable; instead

$$S_a\big|_{\theta=0} = 1 - Kre^{-Kr} \, Ei(Kr) + \sum_{n=0}^{\infty} \frac{C_n}{(2n)!} \left(\frac{r}{h}\right)^{2n+1} \left[1 - \frac{Kr}{2n + 1}\right]$$

where Ei(Kr) is the exponential integral and

$$S_v\big|_{\theta=0} = \frac{2\pi k_o r \cosh(k_o h) \cosh\left[k_o(h - r)\right]}{2k_o h + \sinh(2k_o h)}$$

DIPOLE POTENTIAL Φ^d

The components of the dipole potential

$$\frac{\Phi^d}{\mu_1} = -\Phi_a^d \sin \omega t + \Phi_v^d \cos \omega t$$

are given by Reference 7 as

$$\Phi_v^d = -4\pi \frac{k_o^2 \cosh\left[k_o(h - z)\right] \sinh(k_o h)}{2k_o h + \sinh(2k_o h)} J_o(k_o R)$$

$$\Phi_a^d = \frac{\cos\theta}{r^2} + PV\int_o^\infty \frac{e^{-\nu h}\left[K \sinh \nu z - \nu \cosh \nu z\right] + (K + \nu) \cosh\left[\nu(h-z)\right]}{K \cosh \nu h - \nu \sinh \nu h} \nu J_o(\nu R) d\nu$$

Since (Reference 8, p. 431, Equation 17)

$$\frac{\cos\theta}{r^2} = PV\int_o^\infty \nu e^{-\nu z} J_o(\nu R) d\nu$$

we find

$$\Phi_a^d = 2K\ PV\int_o^\infty \frac{\nu \cosh\left[\nu(h - z)\right]}{K \cosh \nu h - \nu \sinh \nu h} J_o(\nu R) d\nu$$

Comparing with Equation [17] and applying Equations [11] and [13] shows that

$$\Phi_a^d = -K\Phi_a^s = -\frac{2K}{r} S_a = \frac{2 \cos \theta}{r^2} D_a$$

Further, it can be readily seen that

$$\Phi_v^d = -(k_o \tanh k_o h)\Phi_v^s$$

or

$$\Phi_v^d = -K\Phi_v^s = -\frac{2K}{r} S_v = \frac{2 \cos \theta}{r^2} D_v$$

Thus, the dipole potential Φ^d takes the final form

$$\frac{\Phi^d}{\mu_1} = \frac{2 \cos \theta}{r^2} \left[-D_a \sin \omega t + D_v \cos \omega t\right]$$

which is exactly the same representation as given in Reference 5. Of course, the weighting functions are different. They agree with the representations in Reference 5 for the special case $h \to \infty$, $\theta = 0$.

LIMITING CASE OF INFINITE DEPTH $h \to \infty$

For infinite depth, the wave number changes to

$$k = K$$

as can be seen from Equation [10]. For the weighting functions, we obtain from Equation [18]

$$S_v = \pi K r e^{-Kz} J_o(KR)$$

from Equation [19]

$$S_{a_{(near)}} = 1 - Kre^{-Kz} \left\{\frac{\pi}{2}\left[H_o(KR) + Y_o(KR) + A\right]\right\}$$

and from Equation [20]

$$S_{a(far)} = -\pi K r e^{-Kz} Y_o(KR)$$

For the pressure directly below the sphere (R = 0) the weighting functions for $h \to \infty$ become

$$S_v = \pi K r e^{-Kr} \qquad D_v = -KrS_v$$

$$S_a = 1 - Kre^{-Kr}Ei(Kr) \qquad D_a = -KrS_a$$

as given in Reference 5.

For large argument x, the exponential integral is approximately

$$Ei(x) \sim \frac{e^x}{x}\left[1 + \frac{1}{x} + \frac{2!}{x^2} + \ldots\right], \qquad |x| >> 1$$

Therefore, the limiting cases for large values of K are

$$S_v \to 0 \qquad D_v \to 0$$

$$S_a \to -\left[\frac{1}{Kr} + \frac{2}{(Kr)^2} + \frac{3!}{(Kr)^3} \ldots\right] \qquad D_a \to 1 + \frac{2}{Kr} + \frac{3!}{(Kr)^2} \ldots$$

For K being very small, we obtain in the limit $K \to 0$

$$S_v \to 0 \qquad D_v \to 0$$

$$S_a \to 1 \qquad D_a \to 0$$

In the case of finite depth for very low frequencies $K \to 0$ for any angle θ, the limiting cases become

$$S_a \text{ undecided} \qquad D_a \to 0$$

$$S_v = \frac{\pi}{2}\frac{r}{h} \qquad D_v \to 0$$

and for very large frequencies $K \to \infty$

S_a and D_a approach zero asymptotically

S_v and $D_v \to 0$

RESULTS AND DISCUSSION

Before presenting any numerical results, some general remarks about the computations should be made.

The basic assumption that the fluid is of finite depth makes the numerical treatment of the problem much more difficult, compared to the calculations in the case of infinite depth. Further, the difficulties are increased, when computations are made for a point anywhere in the fluid and not necessarily on the bottom of the fluid or directly below the sphere. However, it pays to perform a series of such calculations insofar as some relations in the pressure distributions then become apparent, which are not quite obvious but can be helpful in pressure predictions. They are presented subsequently.

We do not intend to make quantitative statements about the accuracy of the obtained numerical values. They may be affected by truncation errors, by the particular choice of programming, or whatever might contribute when performing high-speed computations. These are well known facts, and often computations of this kind should merely be taken as approximations.

We are content with the fact that results obtained here are in satisfactory agreement with those presented in Reference 6, where the more exact method is applied.

In the following, we refer to the dimensionless pressure amplitude $\tilde{p}$ simply as pressure, hoping that no misunderstanding will occur.

CONCLUSION

The computed results obtained by applying the force method are in satisfactory agreement with those given from the exact method, as can be seen from the tables. Not too much emphasis should be given to the numerical data; it is more important that the general trend of the pressure curves obtained from both methods be similar and perhaps close to that in reality. Subsequent experimental work could confirm what at present can only be judged as supposition. This also applies to the pecularities mentioned in the figures.

The calculations performed with the help of the force method are quite simple, compared to those using the exact method. The theory on which the force method is based does not require the laborious calculations of an unlimited set of expansion coefficients as is required in using the exact method.

TABLES

Tables 1 and 2 show pressure values obtained from the force method, compared with those obtained from the exact method, for different angles and frequencies. The values of the latter are taken from Reference 6, where values are given for the pressures on the bottom for h = z = 4 and h = z = 2, respectively.

Table 3 gives the pressure directly below the sphere (θ=0) at z = 8 for fluids of different depths. It is recognized that h = 10 cannot be considered as a case of infinite depth. The last column of the table gives the calculated ratio of the pressure on the bottom to the pressure for the case of infinite depth, which should be 2:1.

Table 4 treats the same case as that previously mentioned for z = 4. Again, the pressure ratio is about 2:1.

TABLE 1

Pressure Values Obtained by Force and Exact Methods for h=z=4

θ Degrees	R/h	k=0.5	k=1	k=2	k=3	k=4	k=6
0	0	0.047	0.020	0.073	0.130	0.187	0.303
	0	0.052	0.032	0.061	0.119	0.176	0.290
5	0.087	0.047	0.019	0.071	0.128	0.185	0.300
5	0.085	0.051	0.031	0.067	0.117	0.175	0.285
10	0.176	0.046	0.019	0.067	0.122	0.177	0.289
10	0.175	0.051	0.030	0.057	0.113	0.168	0.275
20	0.364	0.043	0.016	0.053	0.100	0.148	0.244
	0.365	0.047	0.024	0.048	0.090	0.145	0.238
30	0.577	0.038	0.012	0.035	0.072	0.109	0.183
	0.575	0.041	0.015	0.036	0.074	0.113	0.184
40	0.84	0.032	0.007	0.021	0.044	0.069	0.117
	0.84	0.034	0.008	0.022	0.045	0.069	0.117
50	1.19	0.025	0.003	0.012	0.028	0.031	0.061
	1.19	0.027	0.005	0.010	0.023	0.035	0.061
60	1.732	0.019	0.003	0.003	0.006	0.011	0.020
63.5	2.00	0.020	0.006	0.002	0.005	0.008	0.014
70	2.747	0.015	0.002	0.001	0.001	0.002	0.003
71.5	3.00	0.016	0.004	0.000	0.001	0.001	0.003
80	5.671	0.010	0.002	0.000	0.000	0.000	0.000
none	none						

First row for each degree refers to force method.

Second row for each degree refers to exact method (Reference 6).

TABLE 2

Pressure Values Obtained by Force and Exact Methods for h=z=2

θ Degrees	R/h	k=0.5	k=1	k=2	k=3	k=4	k=6
0	0	0.113	0.080	0.446	0.767	0.985	1.41
	0	0.147	0.183	0.328	0.558	0.802	1.276
5	0.087	0.112	0.072	0.400	0.725	0.956	1.385
5	0.085	0.147	0.181	0.322	0.550	0.790	1.258
10	0.176	0.111	0.071	0.386	0.689	0.903	1.319
10	0.175	0.144	0.174	0.302	0.525	0.754	1.205
20	0.364	0.106	0.066	0.334	0.557	0.711	1.082
	0.365	0.132	0.149	0.241	0.433	0.629	1.014
30	0.577	0.097	0.059	0.252	0.347	0.466	0.783
	0.575	0.116	0.114	0.162	0.313	0.464	0.772
40	0.84	0.086	0.049	0.146	0.142	0.277	0.481
	0.84	0.099	0.082	0.081	0.189	0.291	0.489
50	1.19	0.061	0.038	0.060	0.081	0.168	0.300
	1.19	0.084	0.062	0.034	0.092	0.143	0.247
60	1.732	0.060	0.029	0.060	0.035	0.041	0.070
63.5	2.00	0.065	0.049	0.013	0.014	0.025	0.046
70	2.747	0.048	0.023	0.048	0.026	0.012	0.010
71.5	3.00	0.054	0.041	0.008	0.002	0.003	0.006
80	5.671	0.034	0.016	0.034	0.018	0.006	0.000
none	none						

First row for each degree refers to force method.

Second row for each degree refers to exact method.

TABLE 3

Pressure Directly below Sphere at z=8 for Different Frequencies and Depths

Ka	h=8	h=10	$h \to \infty$	Ratio
0.02	0.0028	0.0022	0.0014	2.0:1
0.04	0.0049	0.0039	0.0030	1.6:1
0.06	0.0068	0.0055	0.0041	1.7:1
0.08	0.0085	0.0069	0.0057	1.5:1
0.10	0.0101	0.0081	0.0067	1.5:1
0.2	0.0158	0.0126	0.0098	1.6:1
0.3	0.0176	0.0132	0.0100	1.8:1
0.4	0.0164	0.0117	0.0090	1.8:1
0.5	0.0139	0.0095	0.0074	1.9:1
0.6	0.0111	0.0076	0.0059	1.8:1
0.7	0.0085	0.0062	0.0045	1.9:1
0.8	0.0065	0.0043	0.0033	2.0:1
0.9	0.0046	0.0030	0.0026	1.8:1
1.0	0.0036	0.0025	0.0019	1.9:1
1.5	0.0075	0.0053	0.0039	1.9:1
2.0	0.0143	0.0103	0.0076	2.0:1
3.0	0.0286	0.0207	0.0151	1.9:1
4.0	0.0432	0.0312	0.0227	1.9:1
6.0	0.0714	0.0522	0.0392	1.8:1

TABLE 4

Pressure Distribution below Sphere at z=4 for Different Frequencies and Depths

Ka	h=4	h=8	$h \to \infty$	Ratio
0.02	0.0061	0.0035	0.0028	2.2:1
0.04	0.0108	0.0063	0.0058	1.9:1
0.06	0.0148	0.0089	0.0087	1.7:1
0.08	0.0185	0.0114	0.0115	1.6:1
0.10	0.0219	0.014	0.0140	1.6:1
0.20	0.0354	0.0232	0.0236	1.5:1
0.3	0.0437	0.0286	0.0282	1.5:1
0.4	0.0475	0.0300	0.0290	1.6:1
0.5	0.0472	0.0285	0.0278	1.7:1
0.6	0.0436	0.0254	0.0247	1.8:1
0.7	0.0377	0.0215	0.0209	1.8:1
0.8	0.0307	0.0173	0.0168	1.8:1
0.9	0.0242	0.0135	0.0130	1.9:1
1.0	0.0197	0.0110	0.0106	1.9:1
1.5	0.0414	0.0232	0.0223	1.9:1
2.0	0.0725	0.0411	0.0392	1.8:1
3.0	0.1298	0.0743	0.0706	1.8:1
6.0	0.303	0.174	0.165	1.8:1

FIGURES

In the figures, the points where the pressure is observed is fixed, and the sphere is moved on the free surface to different horizontal distances from the point of observation. Imagine that the sphere on the surface is approaching an observation point at a certain level (z) in the fluid. The angle θ becomes the azimuth angle between the upward vertical through point P of observation and the radius vector from point P to the position R of the sphere.

Figure 1 shows the pressure on the bottom of the fluid as a function of frequency for different positions of the sphere. Observe that

1. With respect to frequency, the pressure is fluctuating with a maximum value for $Ka<0.5$ and with a minimum value for about $Ka \sim 1$. For higher frequencies, the pressure increases almost linearly with increasing frequency. This behaviour is observed for any position of the sphere and leads to the remarkable conclusion that the sphere radiates the same pressure amplitude to a point of observation at three different frequencies for a given heaving amplitude.

2. With respect to distance, the pressure decreases with increasing distance of the sphere. The maximum occurs directly under the sphere ($\theta=0$).

Figure 2 shows the pressure at a fixed point for different locations of the sphere, oscillating at the frequency $Ka = 0.1$. The depth of the fluid is varied. Observe that the pressure decreases with increasing depth of the fluid.

Figure 3 shows the pressure for the same frequency, plotted against the azimuth angle θ. This presentation shows clearly the decrease of the pressure to zero, when the angle approaches 90 degrees, i.e., when the sphere is very far distant from the point of observation ($R \to \infty$).

Figure 4 is a particular case taken from Figure 2. The pressure directly under the sphere ($\theta=0$) is calculated as a function of frequency (1) on the bottom ($h = z = 8$), and (2) at the same level ($z = 8$), when the fluid is of infinite depth ($h \to \infty$). Observe that the pressure on the bottom is almost doubled, compared to the pressure at the same point in the case of infinite depth. This can be explained as the reflection effect from the bottom. The plot does not show exactly the ratio of 2 to 1. The doubled value is plotted as a broken line.

Figure 5 gives the pressure on the bottom ($z = h = 4$) as a function of the horizontal distance of the sphere for various values of the frequency parameters Ka. Intersections of curves indicate equal pressure for different frequencies at the same position of the sphere. Observe also the pressure fluctuations in the lower frequency range as described in Figure 1.

Figure 6 is a plot of the monopole weighting function S and its components S_a and S_v, versus frequency, for $h = z = 8$, $\theta=0$. Limiting values for S_v can be obtained from Equation [18] as

$$S_v = \frac{\pi}{2}\frac{z}{h} \text{ for } K = 0$$

and

$$S_v = 0 \quad \text{for } K \to \infty$$

Limiting values for S_a could not be obtained by analysis.

Figure 7 is the plot of the corresponding dipole weighting function and its components. The limiting values for $K = 0$ are zero.

Figure 8 shows for $h = z = 8$, $\theta = 0$ the pressure $\tilde{p}$ and the corresponding components in phase with acceleration (p_a) and velocity (p_v). For higher frequencies, the contribution of p_v decreases, whereas the influence of p_a increases. For lower frequencies, p_v dominates.

Figure 9 gives the pressure for $h = 10$ and $z = 4$ for different azimuth angles θ. The log-log representation permits plotting a wider range of frequencies.

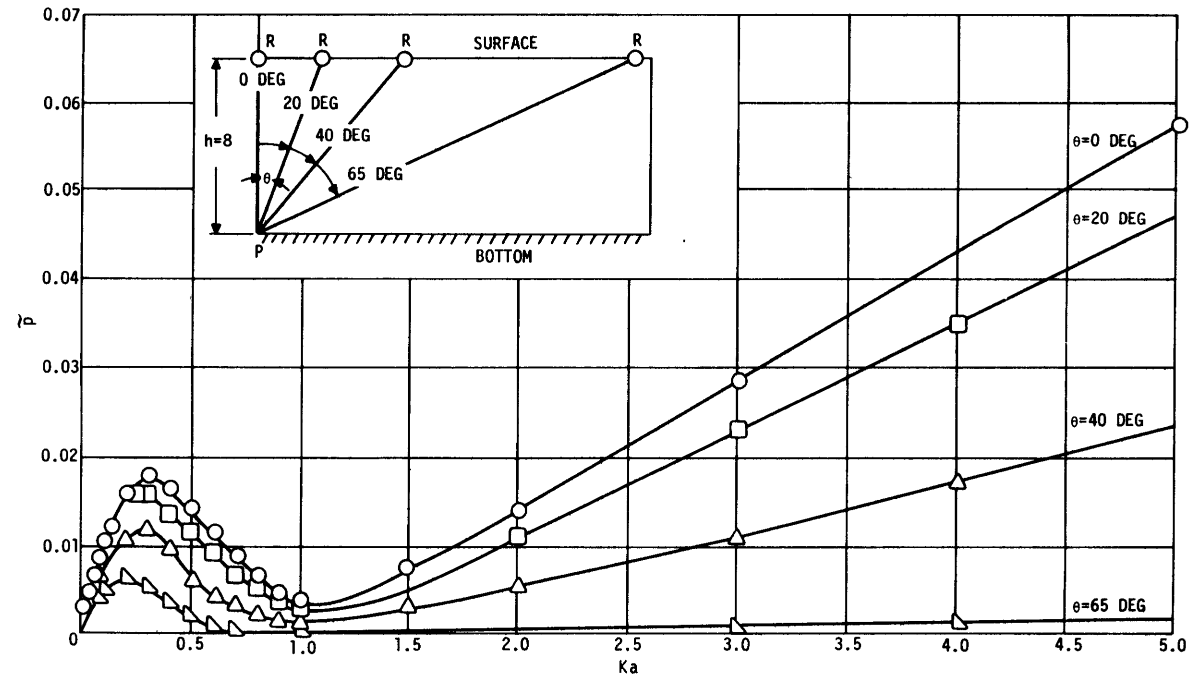

Figure 1 - Pressure on the Bottom as a Function of Frequency

The sphere is at different horizontal distances (R) from
tan θ = R/h

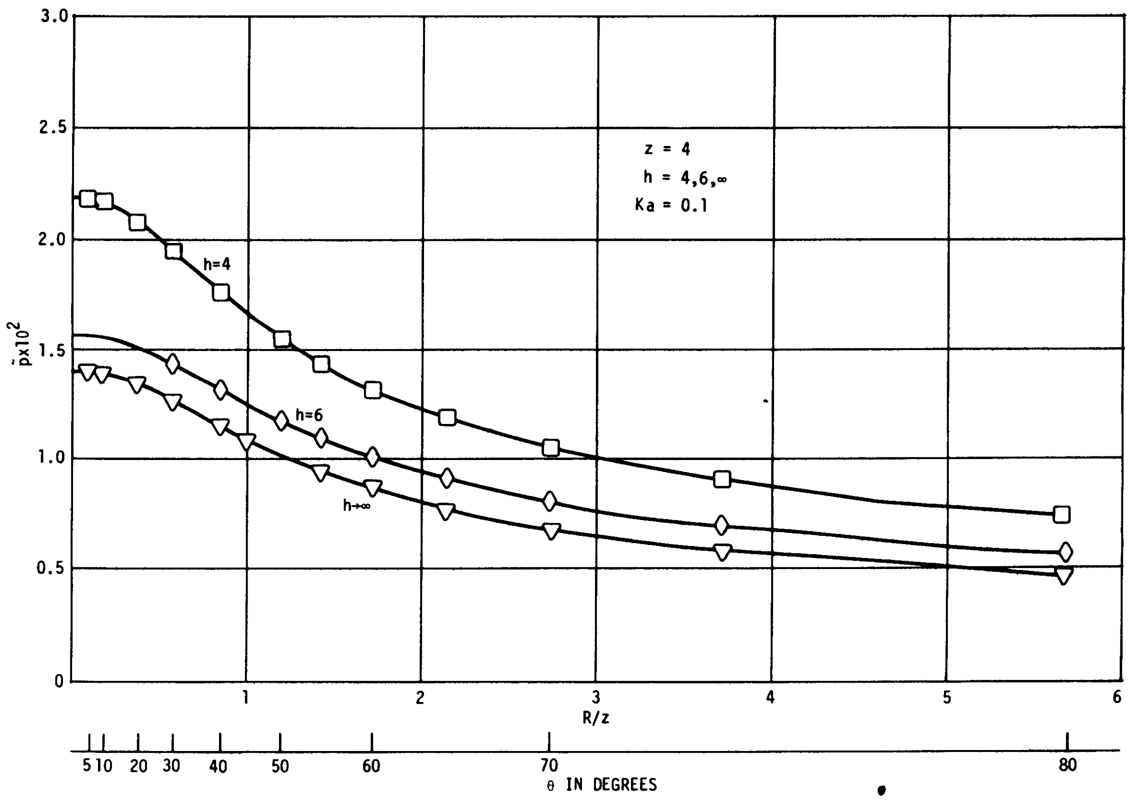

Figure 2 - Pressure as a Function of Distance R for Different Depths and Equal Frequency

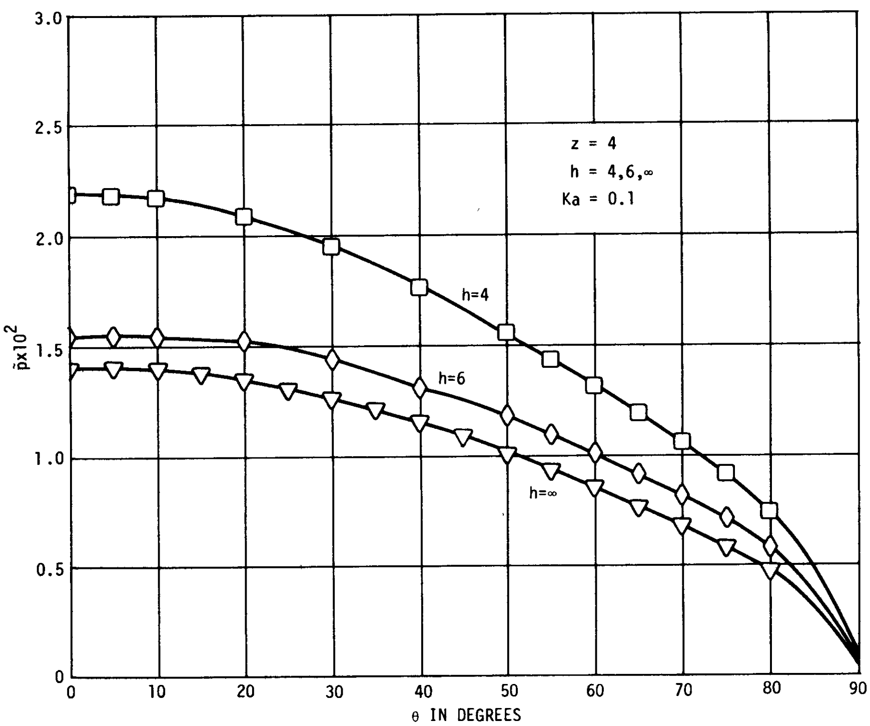

Figure 3 - Pressure as a Function of Angle θ

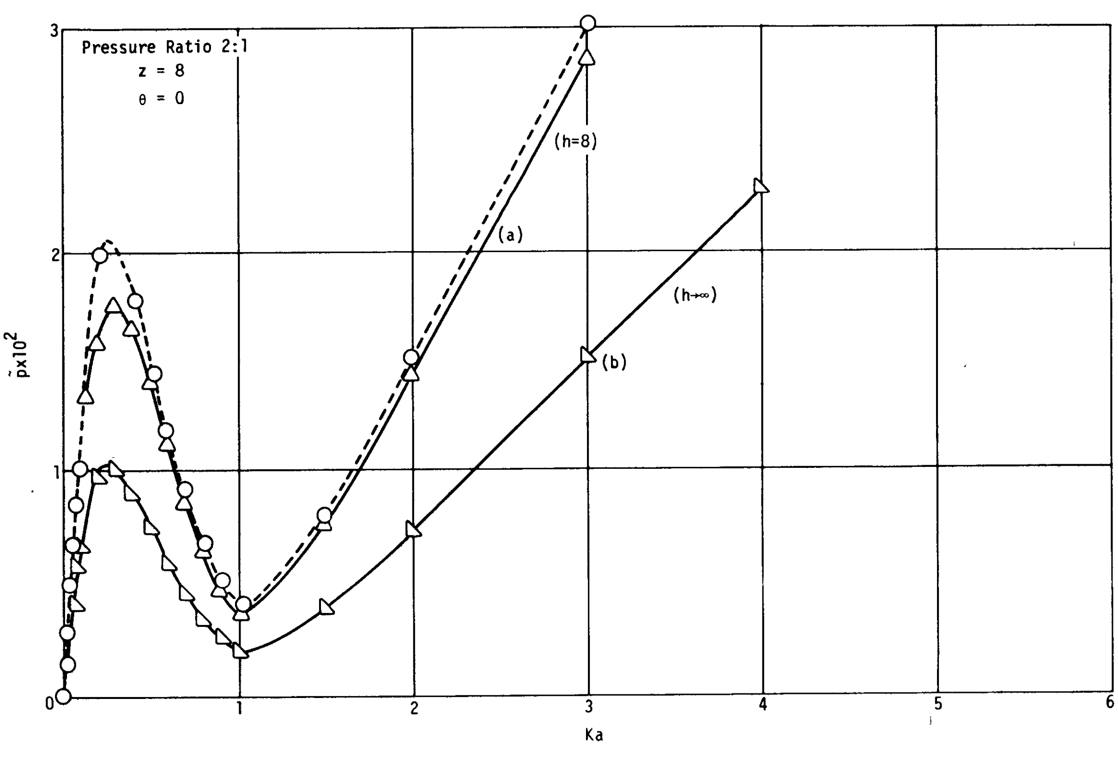

Figure 4 - Comparison of Pressure on the Bottom with Pressure for Infinite Depth

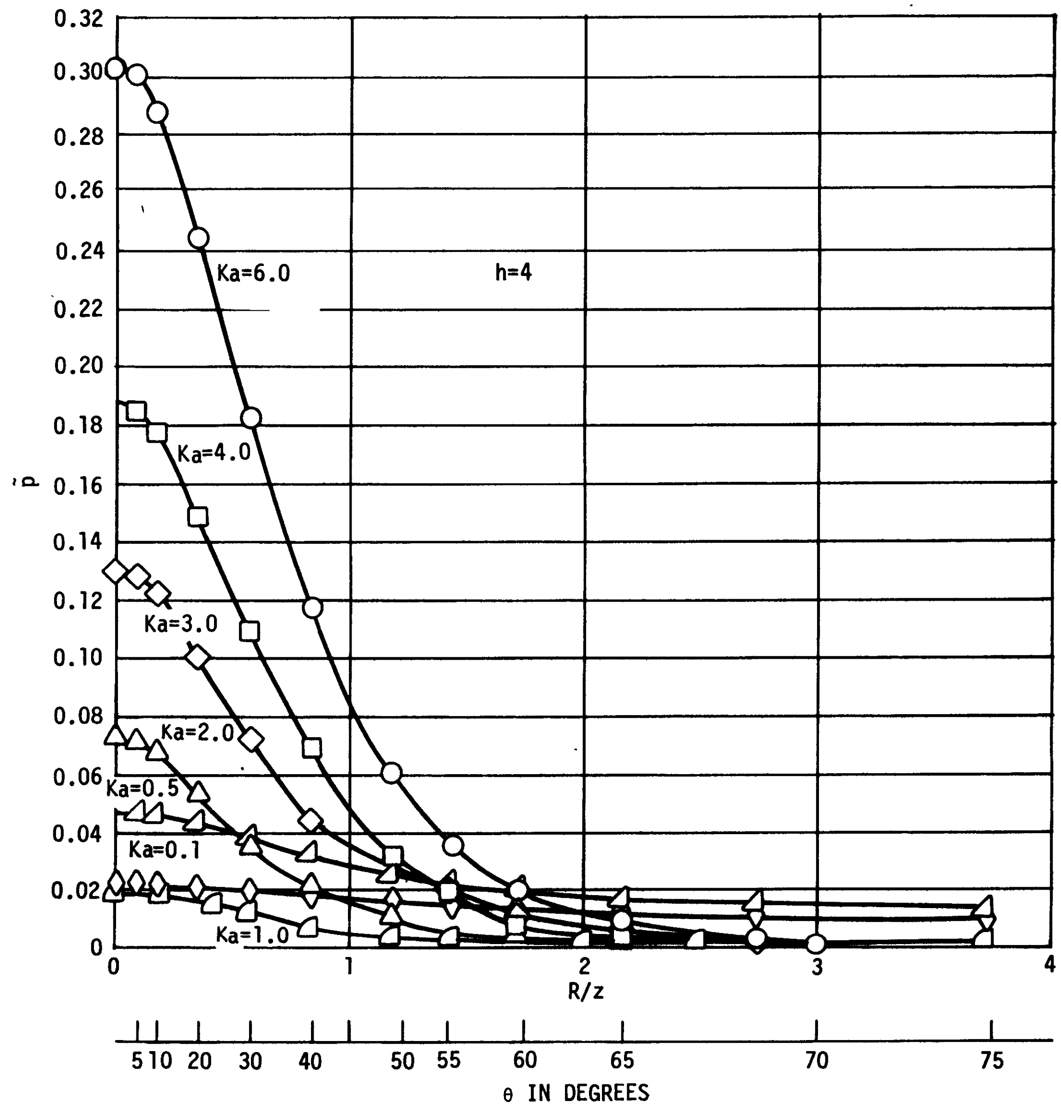

Figure 5 - Pressure on the Bottom as a Function of Distance for Various Frequencies

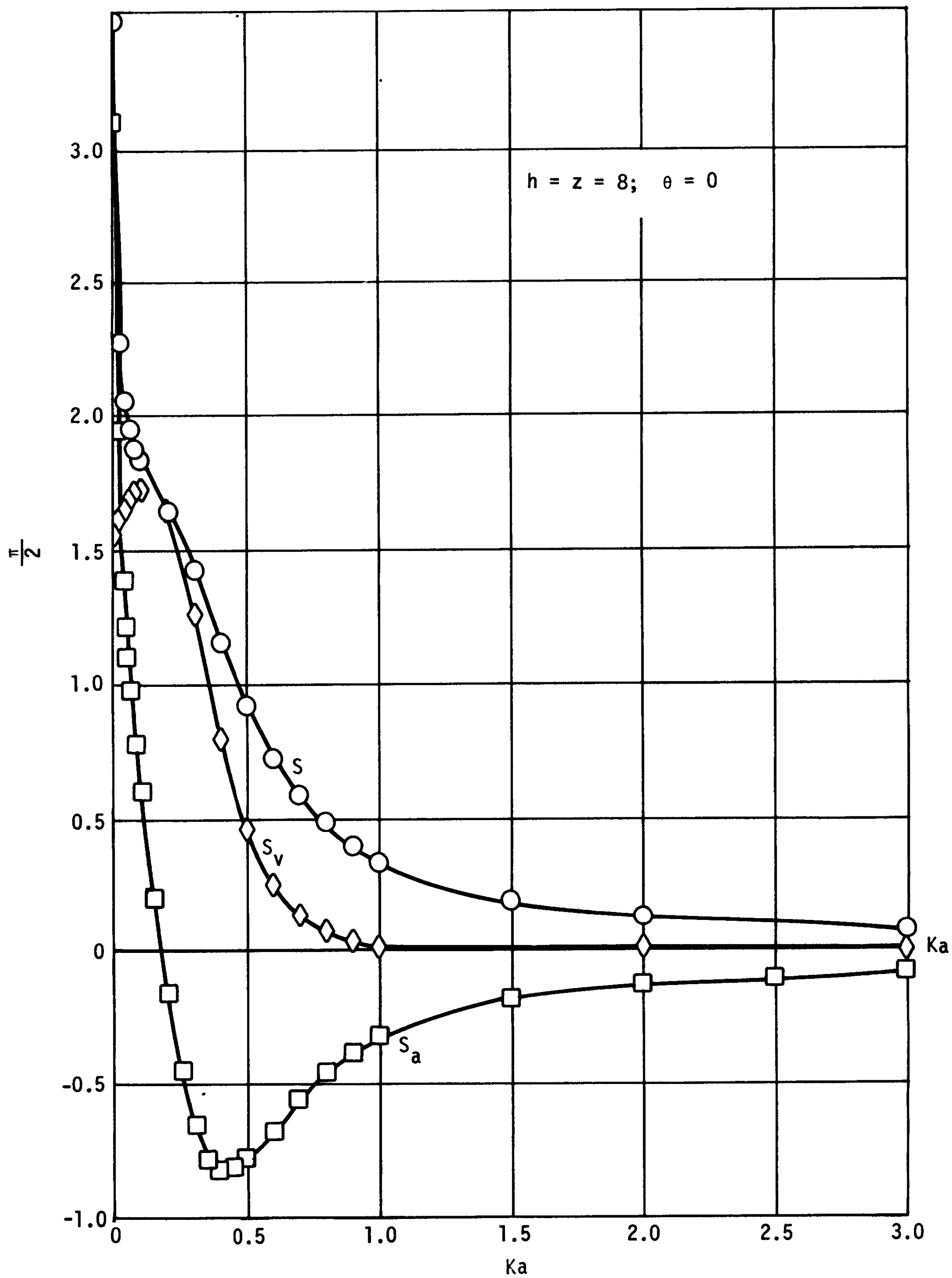

Figure 6 - Monopole Weighting Function S and its Components S_a and S_v on the Bottom Directly below the Sphere as a Function of Frequency

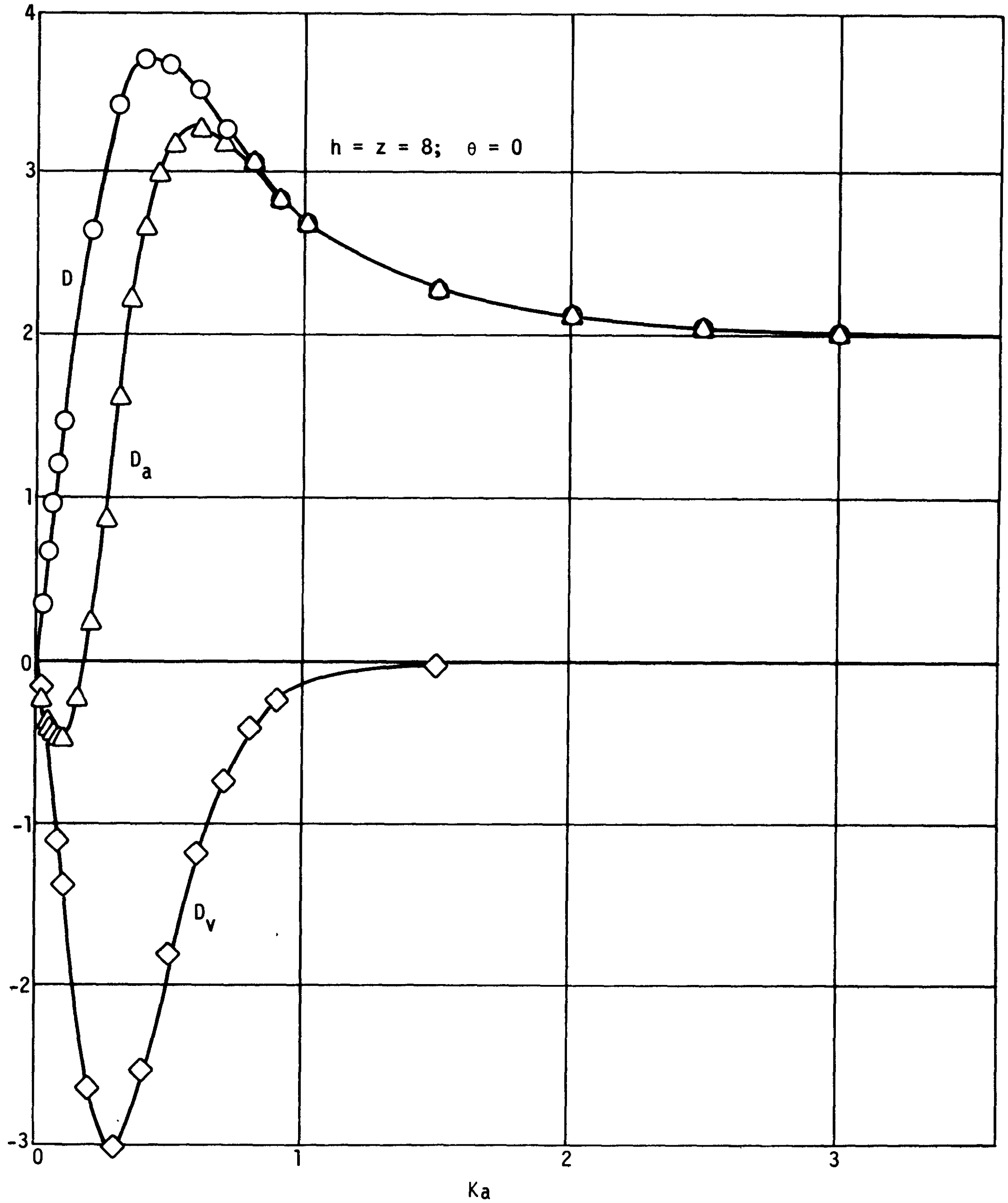

Figure 7 - Dipole Weighting Function D and its Components D_a and D_v on the Bottom Directly below the Sphere as a Function of Frequency

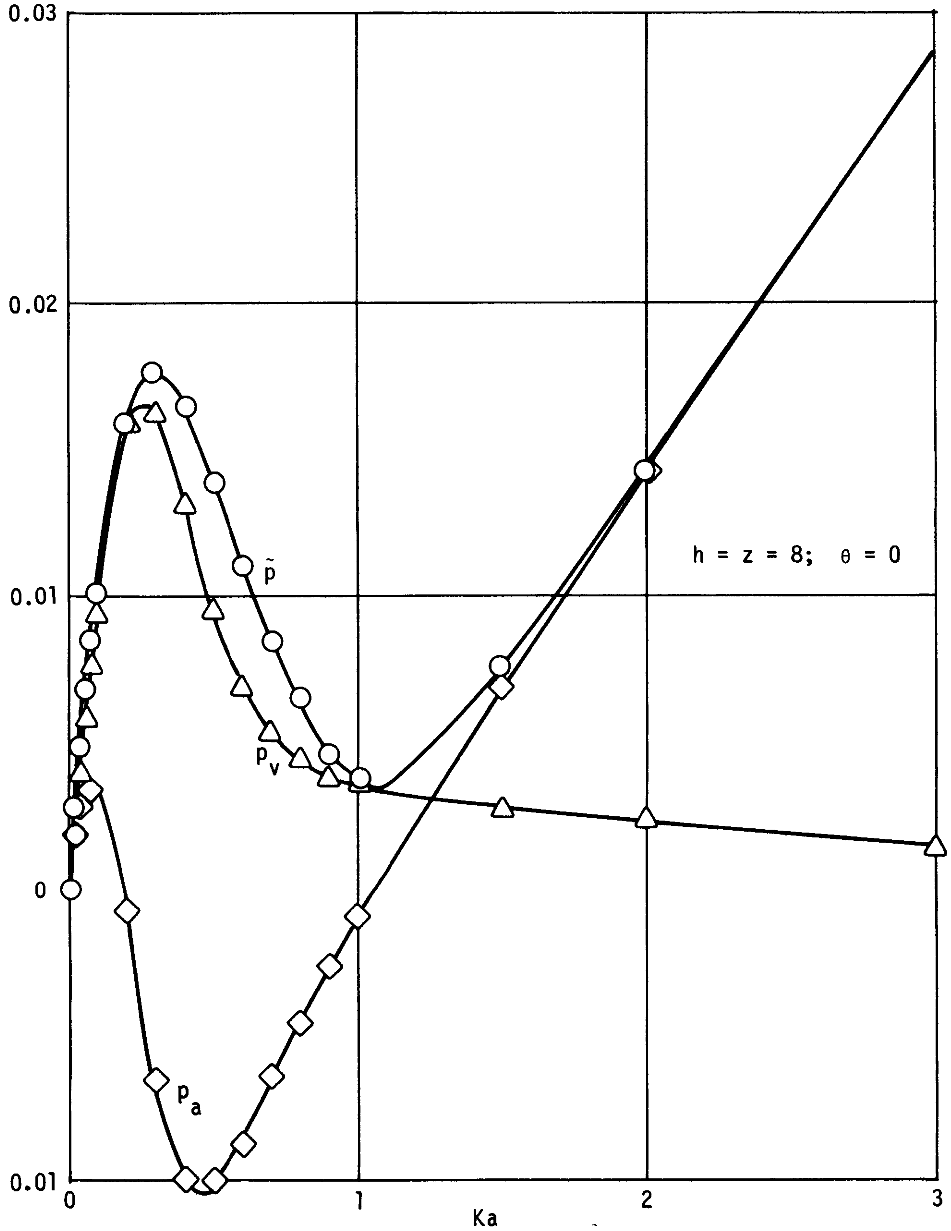

Figure 8 - Pressure $\tilde{p}$ and its Components p_a and p_v on the Bottom

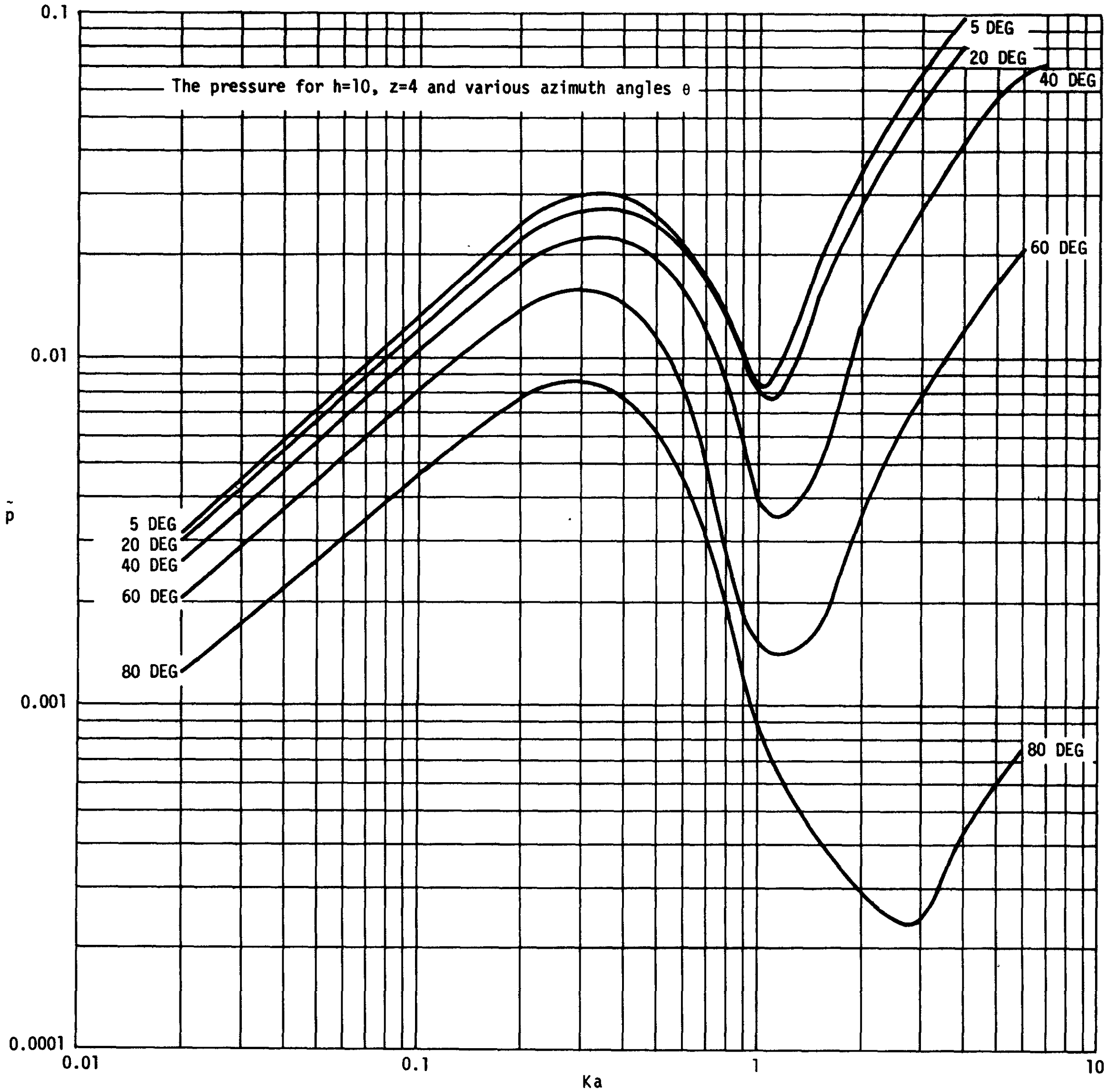

Figure 9 - Log-Log Representation of Pressure, Plotted against Frequency for Various Angles

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13. ABSTRACT

The general pressure field caused by a heaving sphere floating half submerged in a fluid of finite depth is calculated. A sphere oscillating in an unbounded fluid develops a dipole pressure field. When oscillating on the free surface of a fluid, however, damping and buoyancy forces change the pressure field which now comes under monopole influence as well. The high- and low-frequency limits of singularities in an infinite fluid define the surface conditions for a bounded fluid. The general intermediate frequency case is considered, whereby the pressure field is related to the forces acting on the sphere.

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