UNITED STATES EXPERIMENTAL MODEL BASIN

NAVY YARD, WASHINGTON, D.C.

THE TENSION IN A LOOP OF CABLE TOWED THROUGH A FLUID

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Introduction

This work was undertaken in an attempt to find what must be the minimum strength in a loop of wire cable when towed at a given speed by two towboats a given distance apart. To solve the problem it was necessary to determine the tension in the cable at the towboats. Mathematical expressions are derived from which the maximum tension may be computed for a prescribed loop of cable. The method developed is used to determine the tension in a particular case.

Analysis

In the following approximate solution, we suppose that the weight of the cable is negligible compared to its drag so that the cable may be treated as being in a horizontal plane. For the laws of force on the cable, we make the same physical assumptions as in Report 418 (Appendix I); i.e. that the force per unit length normal to the cable is given by $R \sin^2\phi$, where ϕ is the angle that the cable makes with the direction of motion, and R is the force per unit length of cable when normal to the stream, and that the force per unit length parallel to the cable is given by a constant F.

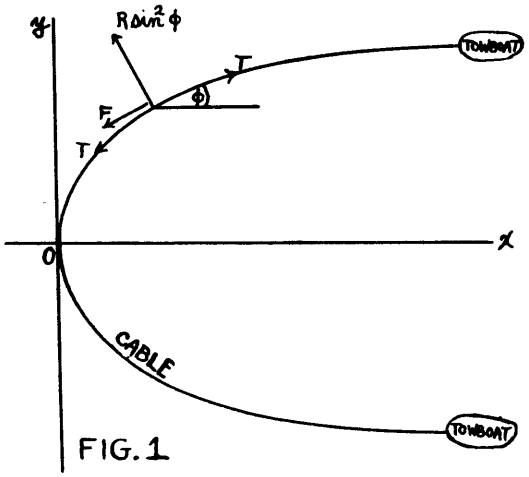


FIG. 1

Fig. 1 is a diagram showing the cable, the towboats and the forces acting. T is the tension in the cable at any point. O is taken at the point where the cable is normal to the direction of motion, ox. oy is normal to ox.

Let s = arc length along the cable measured from o. Then, relating y, s and ϕ we have the equation

$$dy/ds = \sin \phi \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

The element ds of the cable at P is in equilibrium under the action of the system of forces comprising the force $R \sin^2\phi$ ds normal to the cable, the force F ds along the cable, and the tensions T and T + dT. Resolving these forces along the cable we obtain the equation

$$dT/ds = F \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

which may be integrated to give

$$T - T_o = Fs \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

where T_o is the tension of the cable at the origin. Resolving also at right angles

to the cable we obtain

$$T\, d\phi/ds = -R \sin^2\phi \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

Eliminating ds from equations (2) and (4), we obtain

$$\frac{dT}{T\, d\phi} = -\frac{F}{R \sin^2\phi} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (5)$$

which may be integrated to give

$$\log T/T_o = \frac{F}{R} \cot \phi \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (6)$$

Also, eliminating ds between (1) and (2), we obtain

$$dy = \frac{1}{F} \sin \phi \, dT$$

or

$$y = \frac{1}{F} \int_{T_o}^{T} \sin \phi \, dT \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (7)$$

Now introduce the variable $t = T/T_o$. Then equation (3) may be written as

$$T - \frac{T}{t} = Fs$$

or

$$\frac{T}{Fs} = \frac{t}{t-1} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (8)$$

Also (6) becomes

$$\log t = \frac{F}{R} \cot \phi \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (9)$$

and changing the variable of integration from T to t in (7), it becomes

$$y = \frac{T_o}{F} \int_1^t \sin \phi \, dt = \frac{T}{Ft} \int_1^t \sin \phi \, dt = \frac{s}{t-1} \int_1^t \sin \phi \, dt, \text{ by (8)}$$

Hence

$$y/s = \frac{1}{t-1} \int_1^t \sin \phi \, dt \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (10)$$

Equations (8), (9) and (10) are the mathematical expressions from which T may be calculated in any given case. For a given value of R/F, $\sin \phi$ in (10) is given as a function of t from (9). The integral in (10) may then be evaluated numerically or graphically, giving y/s as a function of t. But by (8) T/Fs is also given as a function of t. Thus equations (8), (9) and (10) express in parametric form, with t as the parameter, that T/Fs (dimensionless tension) is a function of R/F and y/s; i.e. $T/Fs = f(R/F, y/s)$.

Application

Let us apply the above theory to the following problem: A wire cable 3600 ft. in length is towed through water at a speed of 20 knots by two towboats 1000 ft. apart. What is the tension in the cable at the towboats when the cable diameter is 1/2"; 5/8"; 3/4" ?

Assume $R^* = .27\ V^2 d$ lb./ft. . . . . (11)

$F^* = .006\ V^2 d$ lb./ft. . . . . (12)

where d = cable diameter in inches

V = speed of towboats in knots.

Then R/F = .27/.006 = 45 and y/s = 500/1800 = .278. Fig. 2 is a plot of Fs/T as computed from (8) against corresponding values of y/s as computed from (9) and (10). From Fig. 2, when y/s = .278, Fs/T = .210. Hence T = 1800 F/.210 = 8570 F. Taking V = 20 knots, and computing F from (12), we have the following table:

d	1/2"	5/8"	3/4"
F	1.2	1.5	1.8 lb./ft.
T	10300	12850	15400 lb.

* From early U.S.E.M.B. experiments.

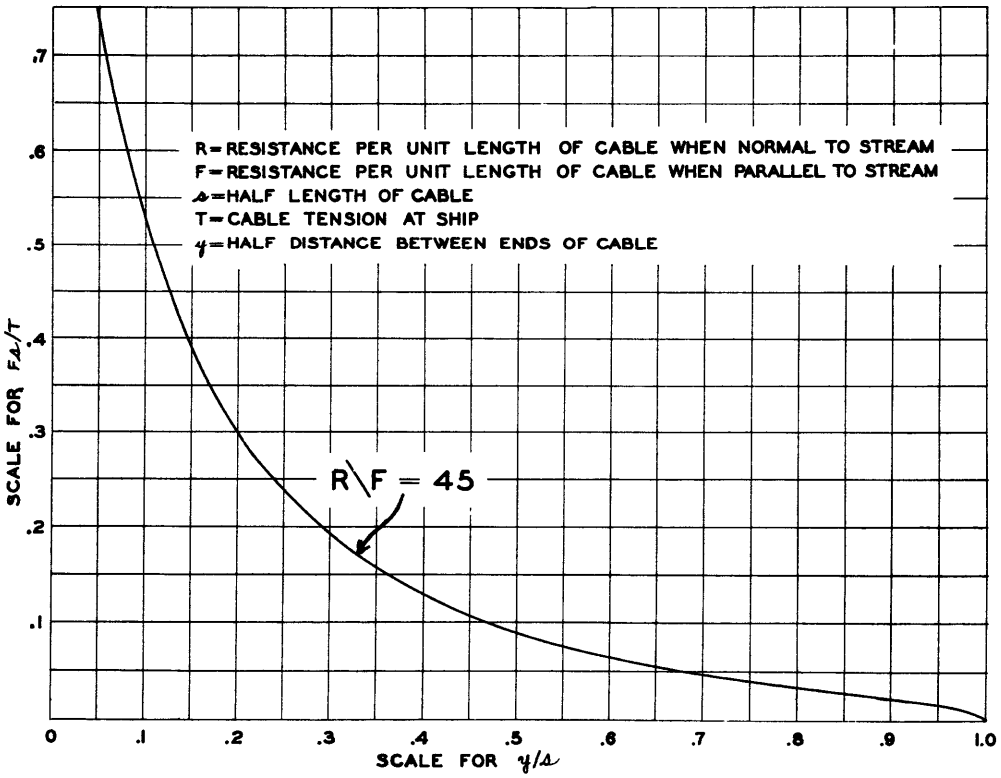


FIG. 2 CABLE TENSION AT SHIP

APPENDIX

ON THE LAW OF FORCE NORMAL TO A ROD AT VARIOUS ANGLES WITH A STREAM

In problems involving the resistance of cables towed through water, physical assumptions as to the law of force have been made. These assumptions, based on experiments of Relf and Powell, have been reviewed in Report No. 418, Appendix I. Since these experiments measured the forces on a cable in an air stream, it was thought desirable to secure new experimental data for the force normal to a rod at various angles to a stream of water.

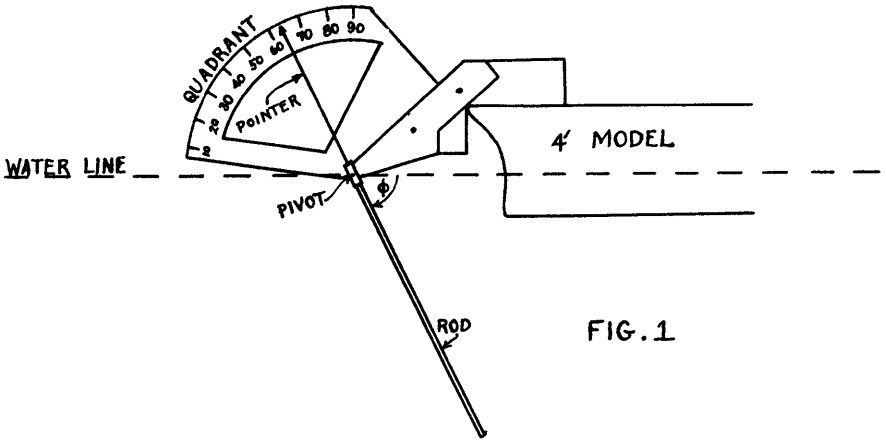


FIG. 1

A diagram of the apparatus used is shown in Fig. 1. The rod was free to swing in a vertical plane about a pivot on the water surface, attached about 5" forward of the bow of a 4-foot model. The pointer and quadrant shown are rigidly connected to the rod and model respectively.

The test was conducted in the eighty-foot model basin. The experimental procedure consisted of reading the angle assumed by the rod (as indicated by the position of the pointer along the quadrant) and measuring the model's speed through the water by means of a chronograph. The rods tested were of 1/4" diameter and ranged from 6.1" to 48.7" in length.

The readings obtained are shown plotted as angle against speed in Fig. 2 for a series of steel rods, and in Fig. 3 for a brass rod 4 ft. in length. The curves were computed for a rod of infinite length on the assumption that the normal force per unit length is given by $R \sin^2\phi$, where R is the force per unit length of rod when normal to the stream. Let w be the weight per unit length of the rod when submerged in water. Then the force per unit length normal to the rod is in equilibrium with the component $w \cos \phi$ of the weight of the rod. Hence the computed curves are determined by the equation

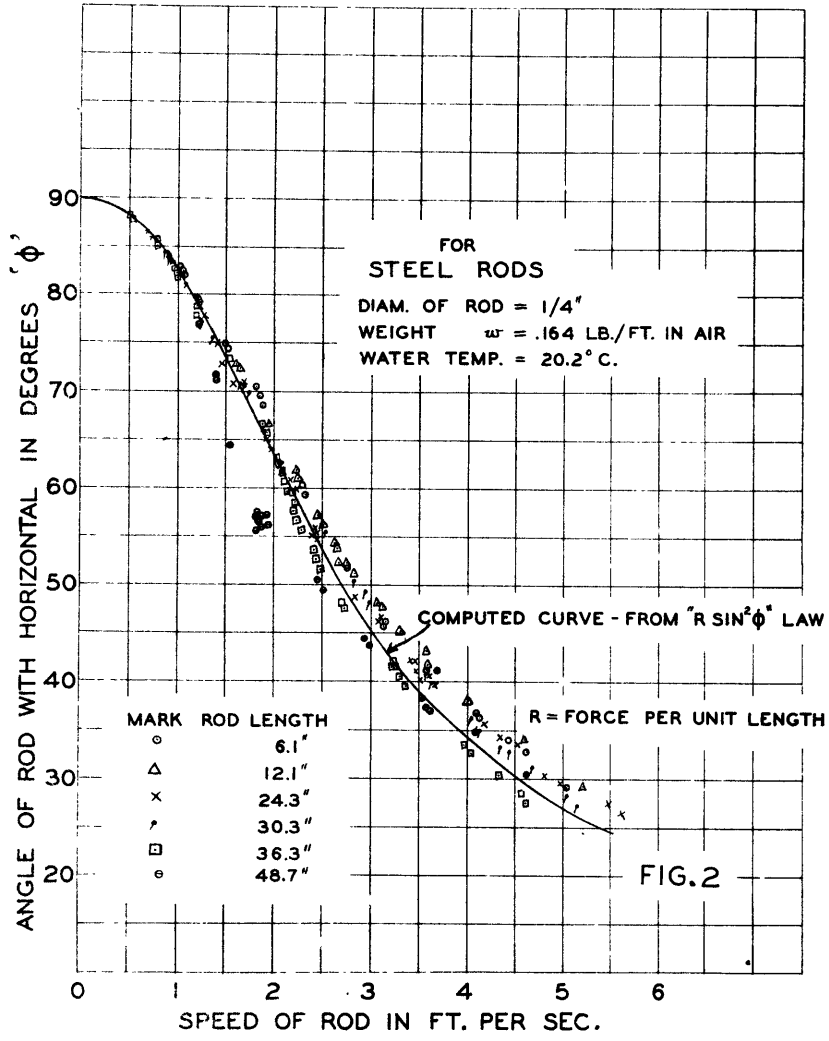
$$w \cos \phi = R \sin^2\phi$$

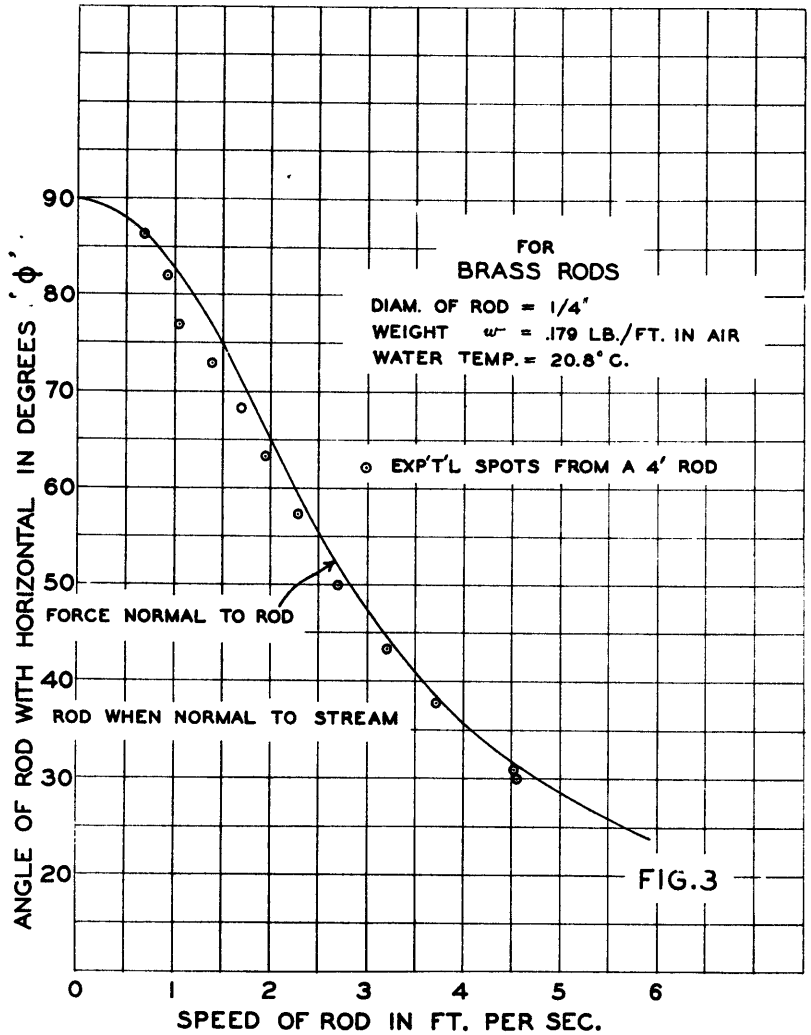
R is given as a function of the Reynolds' number vD/ν from the data of Wieselsberger and Relf. v is the speed, D the diameter of the rod, and ν the kinematic viscosity.

From Fig. 2 it is seen that the experimental spots approach the computed curve with increasing rod length. The hump determined by the spots for the four foot rod over the range 1.5 to 2.5 ft./sec. was correlated with a maximum lateral vibration of the rod. A less pronounced hump for the three foot rod over the range 2.2 to 3.2 ft. per sec. is of the same origin. These facts imply that the stream impresses upon the rod an oscillating force whose frequency increases with the speed. For the shorter rod, which had a higher natural frequency, reached a condition of maximum vibration (resonance) at a higher speed. For further verification a four foot brass rod (Fig. 3) was tested. Since the elastic modulus for brass is 1.3×10^7, and that for steel is 3.0×10^7, while the masses are in the ratio of 9 to 8, the ratio of the resonance frequency of brass to that of steel is $\sqrt{\frac{1.3}{3.0} \times \frac{8}{9}} = 0.62$. Consequently the hump should occur at a lower speed for brass, as is observed to be the case. Some residual vibration was observed to the highest speeds. To this may be attributed the small but uniform deviation of the spots from the computed curve in Fig. 3.

Conclusion:

In agreement with the results of Relf and Powell, the function $R \sin^2\phi$ furnishes a good approximation to the law of force normal to an infinite rod at an angle ϕ with a stream.





SPEED - ANGLE CURVES