ME - 5

TEACHING PROBLEM SOLVING IN MATHEMATICS LEARNING: REFLECTION FROM PISA AND TIMSS RESULTS OF THE STUDENTS OF INDONESIA

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Abstract

The quality of mathematics education in Indonesia is still low. This is at least acquired from the attainment of the students of Indonesia participating in international assessments, such as PISA and TIMSS. Some of the main causes are mathematics learning that is poorly related to the contexts of living that students experience. In addition, teacher lacks facilitating students in problem solving activities. Therefore, teacher needs to familiarize students with PISA and TIMSS like questions in mathematics learning and teach them the process and strategies of solving problem. Within this article, it is given an alternative problem solving process for PISA and TIMSS like questions with 4 (four) stages, along with 12 (twelve) core strategies of problem solving and other strategies that can also be taught to students.

Key Words: Problem Solving . Mathematics Learning . PISA . TIMSS

INTRODUCTION

A. Background

The quality of mathematics education in Indonesia is still low. This is in line with the statement by one of mathematics education scholars in Indonesia, Marpaung. This is acquired from the accomplishment of the students of Indonesia in international assessment. From the results of international assessments, such as PISA in 2000, 2003, 2006, 2009 and 2012, the students of Indonesia did not show valuable change in each participation (Ministry of National Education, 2011). In PISA 2012, Indonesia is in 64th level of 65 countries participating in the test (OECD, 2013). This is a worst attainment if it is compared to that in 2009 which places Indonesia at the 61st position of 65 participants. Meanwhile building on TIMSS 2007, the attainment of the students of Indonesia was also in apprehension due to the decreased of the average of our grade VIII students score, compared to that in 2003. In the TIMSS 2007, Indonesia was in the 36th position of 49 countries. In 2011, the achievement of the students of Indonesia is more deeply in apprehension, the score is decreased if it is compared to that in 2007, Indonesia was in the 38th level of 42 countries (Napitupulu, 2012).

There are lots of factors contributing to the low score results of TIMSS and PISA. Some of them are that students of Indonesia in general are lack trained in carrying out TIMSS and PISA like questions. Moreover, mathematics learning is lack connected to the contexts of living that students encounter and lack facilitating students in expressing their argumentation and thinking process. The fact is clearly not in line with the characteristics of questions or problems in TIMSS and PISA which are essentially contextual, require reasoning, argumentation and creativity in carrying them out (Ministry of National Education, 2011). Whereas the essence enables to reach students through problem solving activities that teacher can implement in mathematics learning.

Realizing the aforementioned facts, we should attempt to apply a variety of alternative and innovative methods in enhancing the mathematics ability of our students. One of key elements in the attempt is the improvement of the quality of mathematics teacher, as emphasized in various literatures and research results (Ministry of National Education, 2011). Therefore, it is important to discuss the processes and strategies of problem solving that is necessary for teacher to teach to students in mathematics learning involving TIMSS and PISA like questions.

B. Formulation of the Problem

Based on the background, the formulation of the problem is then what are the processes and strategies of problem solving that is necessary for mathematics teacher to teach to students in mathematics learning involving TIMSS and PISA like questions?

C. Goal of the Study

The goal of the study is to reveal the processes and strategies of problem solving that is necessary for mathematics teacher to teach to students in mathematics learning involving TIMSS and PISA like questions.

D. Benefit of the Study

On the basis of the goals, this study is then beneficial to:

1. Make mathematics teachers aware of the reality of the achievement of students of Indonesia in the subject of mathematics currently.
2. Provide information for mathematics teachers concerning the processes and strategies of problem solving that is necessary for mathematics teacher to teach to students in mathematics learning involving TIMSS and PISA like questions.
3. Promote reformation of teaching methods of mathematics teachers from teacher–oriented to student–oriented.

DISCUSSION

A. PISA and TIMSS at A Glance

First, *Programme for International Student Assessment* (PISA). PISA is a study on the program for assessing students in international level which is held by the *Organisation for Economic Cooperation and Development* (OECD). PISA aims at assess on how far as students in the last year of their primary education (students aged 15 years) have mastered the important knowledge and skills to be able to participate as the constructive and responsible citizen or the community element. The instances assessed in the study of PISA are account for mathematical, science and reading literacies. In 2012, it is added one more instance such *financial literacy* (OECD, 2010a).

The followings are the examples of mathematical questions in PISA *Released Items 2006.*

1. A result of global warming is that the ice of some glaciers is melting. Twelve years after the ice disappears, tiny plants, called lichen, start to grow on the rocks. Each lichen grows approximately in the shape of a circle. The relationship between the diameter of this circle and the age of the lichen can be approximated with the formula:

   $$d = 7.0 \times \sqrt{(t - 12)} \text{ for } t \geq 12$$

   where *d* represents the diameter of the lichen in millimetres, and *t* represents the number of years after the ice has disappeared. Using the formula, calculate the diameter of the lichen, 16 years after the ice disappeared. Show your calculation!

2. You are asked to design a new set of coins. All coins will be circular and coloured silver, but of different diameters.

   Researchers have found out that an ideal coin system meets the following requirements:

- diameters of coins should not be smaller than 15 mm and not be larger than 45 mm.
- given a coin, the diameter of the next coin must be at least 30% larger.
- the minting machinery can only produce coins with diameters of a whole number of millimetres (e.g. 17 mm is allowed, 17.3 mm is not).

Design a set of coins that satisfy the above requirements. You should start with a 15 mm coin and your set should contain as many coins as possible.

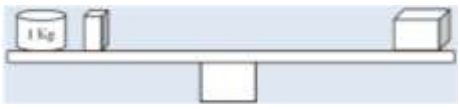
3. Which of the figures has the largest area? Show your reasoning!

Second, *Trends in International Mathematics and Science Study* (TIMSS). TIMSS is an international study about trend or direction or advance of mathematics and science. This study is held by the *International Association for the Evaluation of Educational Achievement* (IEA), namely an international association to assess students achievement in education. TIMSS is centered on Lynch *School of Education*, Boston College, USA.

TIMSS aims at investigating the improvement of mathematics and science learning. TIMSS is held once in 4 (four) years. This is for the first time held in 1995, and then respectively in 1999, 2003, 2007, and the fifth TIMSS, that is TIMSS 2011. One of TIMSS activities is to examine mathematics ability of grade IV students of elementary school and grade VIII students of junior high school. Grade VIII students of of junior high school in Indonesia have participated in TIMSS for 3 times, meanwhile those of elementary school have not.

The followings are the examples of TIMSS 2011 questions in *Assessment Framework*.

1. In a car rally two checkpoints are 160 km apart. Drivers must travel from one checkpoint to the other in exactly 2,5 hours to earn maximums points.
   a. What must the average speed be to travel the 160 km in this time?
   b. A driver took 1 hour to travel through a 40 km hilly section at the beginning of the course. What must be average speed in kilometres per hour, be for the remaining 120 km if the total time between checkpoints is to be 2,5 hours?
2. The objects on the scale make it balance exactly. On the left pan there is 1 kg weight (mass) and half a brick. On the right pan there is one brick. What is the weight (mass) of one brick?



   A. 0,5 kg B. 1 kg C. 2 kg D. 3 kg

Instructions:

Questions 3, 4, 5 are about Phone Plans. To answer these questions you may refer to any information shown on the pages in the Phone Plans section.

Betty, Frank and Darlene have just moved to Zedland. They each need to get phone service. They received the following information from the telephone company about the two different phone plans and it offers. They must pay each month and there are different rates for each minute they talk. These rates depend on the time of the day or night day they use the phone and on which payment plan they choose. Both plans include time for which phone calls are free. Details of the two plans are shown in the table below.

Plan		*Rate per minutes*		*Free minutes per months*
	Monthly fees	*Day (8am-6pm)*	*Night (6pm-8am)*	
Plan A	*20 zeds*	*3 zeds*	*1 zeds*	*180*
Plan B	*15 zeds*	*2 zeds*	*2 zeds*	*120*

3. Betty talks for less than 2 hours per month. Which plan would be less expensive for her? (Explain your answer in terms of both the monthly fee and free minutes)
4. Frank talk for 5 hours per month at the night rate. What would each plan cost him per month? Show your work!
5. Darlene signed up for the Plan B and the cost one month of service was 75 zeds. How many minutes did she talk that month? Show your work!

B. The Process of Solving PISA and TIMSS-Like-Mathematics Questions

Mathematics questions in PISA and TIMSS are dominated (if not all) with contextual problems. In terms of contextual or real-world problem, Tan (2003) states:

> In solving real-world problems, we need to realize that a whole range of cognitive processes and mental activities are involved. The mind has to go through cycles and iterations of systematic, systemic, generative, analytical and divergent thinking (p. 20).

Therefore, it can be asserted that problem solving is a complex activity that involves the confluence of both logical and extra-logical processes (Liljedahl, 2007). This means that problem solving requires a variety of skills including interpreting information, planning and working methodically, checking results and trying alternative strategies (Muir et.al., 2008).

In solving mathematics problem, heuristics are very important. There are many heuristics actually, 5 (five) of them are: (a) the problem-solving process involves four phases: understanding, planning, carrying out the plan and looking back (Polya, 1957); (b) learning is an active process where new knowledge is incorporated into what the learner already knows (Ausubel, 1968); (c) problem solving is a process that uses prior knowledge and produces new learning (Gagne, 1985); (d) a backwards approach is used to help set up a qualitative representation to solve the problem (Heyworth, 1999); (e) the process of problem solving goes beyond finding simple solutions; it encourages students to reflect on the solutions, make generalizations, and extend problems to include new possibilities for investigation (Frei, 2007).

The heuristics are intended to help students understand the phases involved in problem-solving (metacognitive tool) and to provide them with an organized approach to tackling problems in a systematic way, to increase their cognitive skills or procedural knowledge (cognitive tool) which then results in increased domain knowledge (Lorenzo, 2005). Nevertheless, the utilized heuristic within this discussion is Frei's heuristic consisting of 4 (four) stages. This is because it involves logical and systematic stages that enable teacher to teach mathematics problem solving, including solving PISA and TIMSS questions requiring communication skill. The most important reason is that Frei's heuristic basically constitutes the formulation of previous heuristics, such as Polya (1957), Ausuble (1968), Gagne (1985) and Heyworth (1999).

Through the four stages in Frei's heuristic, students will tackle mathematics questions structurally and meaningfully. However, it is important to note that teacher needs to plan time for teaching problem solving explicitly, modelling it, and providing guidance and individual practice to students. The four stages are as Frei (2007) states the following.

First, understanding the problem. Encouraging students to read problem carefully for some times until they really understand that is being asked. And then, students should highlight or underline the unfamiliar words and try to find the meaning of the words. Further, students pay attention to the likelihood of existing unnecessary information.

Second, planning and communicating solution. Students should decide on how they will solve problem that they face by thinking different strategies enabling to use. Sometimes it is necessary for students to use more than one strategy to solve problem. They may try to make prediction, or guess, about the encountered problem. The guesses frequently produce generalization which help them solve problem. They should not be encouraged to make arbitrary guesses, but encouraged to be responsible. They should always think about the relatedness between the encountered problem and other problems that they previously have solved.

Third, *reflecting and generalizing*. Solution and strategies of one problem frequently can assist students to know the way to solve other problems. Therefore, students need to learn the importance of reflecting what they have learned. Teacher needs to teach to students the process of critical reflection. This process should be modeled when teacher shows solution of the problem. Teacher can even display inappropriately problem solving to pass through the reflection process and "catch" the mistakes. Students need to decide whether their answer is meaningful or they have answer the questions asked. They should illustrate and write their thinking, estimation and approach. This provides them times to reflect their practices and develop the use of problem solving strategies. When they obtain an answer, they should explain the process of obtaining it to others.

Fourth, *extension*. This stage also needs to be taught explicitly and then modeled by teacher, because students need practice to internalize it. Student needs to learn the ways of asking questions to oneself such "what if" to connect the encountered problem to other problem. This process will make them explore problem until deep level and encourage them to use logical thinking process. Students should be able to think the possibility of existing more simply problem solving.

To run the Frei's heuristic, teacher may apply problem–based–learning (PBL) model with learning strategies by OECD. OECD (2010b) states three learning strategies. First, *memorization/rehearsal strategies*. Students use memorization strategies (e.g. learning facts or rehearsing examples) for many tasks; such strategies are appropriate when the learner needs to retrieve information, as presented, without any further elaboration or processing. Second, *elaboration strategies*. Elaboration is a measure of the extent to which students acquire understanding of new material by relating it to prior learning and knowledge. Elaboration strategies, unlike memorization strategies, can help to deepen students' understanding of the knowledge and skills in use. Third, *control strategies*. Students who control their learning ensure that they set clear goals for themselves and monitor their own progress in reaching them.

Frei's heuristic is actually relevant with the special features of the PBL model. Arends (2012) states that the features are driving question or problem, interdisciplinary focus, authentic investigation, production of artifacts and exhibits, dan collaboration. The syntax of this model is as follows.

Phase 1: Orient students to the problem. This means teacher goes over the objectives of the lesson, describes important logistical requirements, and motivates students to engage in problem-solving activity.

Phase 2: Organize students for study. This means teacher helps students define and organize study tasks related to the problem.

Phase 3: Assist independent and group investigation. This means teacher encourages students to gather appropriate information, conduct experiments, and search for explanations and solutions.

Phase 4: Develop and present artifacts and exhibits. This means teacher assists students in planning and preparing appropriate artifacts, such as reports, videos, and models, and helps them share their work with others.

Phase 5: Analyze and evaluate the problem-solving process. This means teacher helps students to reflect on their investigations and the processes they used.

Using this PBL model with learning strategies by OECD (2010) accompanied with problem solving or open–ended approaches (Shimada and Becker, 2005) and guided discovery method, teacher can apply the process (heuristic) and strategies for solving PISA and TIMSS–like–mathematics questions.

C. Strategies for Solving PISA and TIMSS–Like–Mathematics Questions

When teacher has acquired and taught heuristic of problem solving, he or she then should also equip them with strategies of problem solving in order to be able to undertake the heuristic well. The strategies for solving problem that may be taught to them are many. Some of them are as Frei (2007) states as follows:

1. Drawing a diagram

This strategy often reveals aspects of the problem that may not be apparent at first. A diagram that uses simple symbols or pictures may enable students to see the situation more easily and can help them keep track of the stages of a problem in which there are many steps. Students need to develop the skills and understanding to use diagrams effectively.

2. Drawing a table

This strategy helps students to organize information so that it can be easily understood and relationships between one set of numbers and another become clear. A table makes it easy to see the known and unknown information. The information often shows a pattern or part of a solution, which can then be completed. It also can help reduce the possibility of mistakes or repetitions. Students need to develop the skills and understanding to create and use tables effectively.

3. Acting it out or using concrete materials

This strategy uses objects or materials to represent people or things in the problem. This helps students visualize the problem in a concrete way and find the solution.

4. Guessing and checking

This strategy allows a student to make an educated guess and check the guess against the problem. If it is not a correct solution, the student revises the guess and checks until a correct solution is found.

5. Creating an organized list

This strategy is used instead of a table when a greater amount of information is available. Students need to follow a procedure or sequence to find the solution to the problem and make sure no information is left out or repeated.

6. Looking for a pattern

This strategy is an extension of drawing a table and creating an organized list. It is often used because mathematical patterns can be found everywhere. There are many ways to check for a pattern.

7. Creating a tree diagram

This strategy uses a diagram with different branches to represent relationships between different factors in a problem. A tree diagram enables the students to visualize the different factors in the problem and ensures that no factors are repeated or missed.

8. Working backwards

This is used to solve problems that include a number of linked factors or events, where some of the information has not been provided. The object is to determine the unknown information. The events occur one after the other and each stage or piece of information is affected by what comes next. To solve the problem, begin with the ending information and work backwards until the problem is solved. It is important to remember that mathematical operations will have to be reversed.

9. Using simpler numbers

This strategy can be used to solve a difficult or complicated problem in order to simplify the numbers.

10. Open-ended problem solving

This strategy is used to explore problems that might be answered in a number of ways. Although finding the correct answer is important, teachers should value the student's process for solving the problem and gain insight into the student's developmental understanding.

11. Analyzing and investigating

When using this strategy, analyze what is known and what needs to be known. Use the known information to investigate the problem and collect data.

12. Using logical reasoning

This strategy is used when the problem gives information as pieces of a puzzle. Each piece of information is important to solve the problem. Process of elimination is one approach in this strategy, where each piece of information builds to the solution. Drawing a grid to organize given information is also an approach that can be used.

Despite the twelve main strategies, there are still other strategies, namely breaking down ideas into smaller pieces, writing a number sentence, writing down ideas as work progresses so students, do not forget how the problem was approached, approaching the problem systematically, rereading the problem in order to rethink strategies if the student becomes 'stuck', orally demonstrating and explaining how an answer was reached.

CONCLUSION AND SUGGESTIONS

On the basis of the outline above, it is concluded that when teaching problem solving, teacher needs to teach the process of problem solving consisting of 4 (four) stages, that is: (1) understanding the problem, (2) planning and communicating solution, (3) reflecting and generalizing, (4) and extension to the higher level of problems. To underpin the success of problem solving, teacher also needs to teach the 12 (twelve) main strategies of problem solving to students, that is (1) drawing a diagram, (2) drawing a table, (3) Acting it out or using concrete materials, (4) guessing and checking, (5) creating an organized list, (6) looking for a pattern, (7) creating a tree diagram, (8) working backwards, (9) using simpler numbers, (10) open-ended problem solving, (11) analyzing and investigating, (12) using logical reasoning. Further, there are some additional strategies, that is breaking down ideas into smaller pieces, writing a number sentence, writing down ideas as work progresses so students, do not forget how the problem was approached, approaching the problem systematically, rereading the problem in order to rethink strategies if the student becomes 'stuck', orally demonstrating and explaining how an answer was reached.

Therefore, it is suggested to teachers in order to continuously improve the learning process in mathematics classroom, particularly the use of context and the increment of reasoning, problem solving, arguing and communicating portions. Further, they need to provide practice for students in conducting reasoning and problem solving. Furthermore, a variety of learning methods and approaches need to be implemented by teachers in order for students have abilities to make argumentation and communication.

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