

The mathematical model of the improved system of the seat with adjustable pressure profile

Tien Tran Xuan¹, Dong Nguyen Phu²

¹Department of Mechatronics, Faculty of Mechanical Engineering, Hung Yen University of Technology and Education, Hung Yen, Vietnam

²Department of Vehicles and Engines, Faculty of Mechanical Engineering, Technical University of Liberec, Liberec, Czech Republic

¹Corresponding author

E-mail: ¹tranxuantienc@utehy.edu.vn, ²dong.nguyen.phu@tul.cz

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Abstract. Following a patented solution, a seat which is possible to change its stiffness was created. The seat contains an actively controlled pneumatic spring element (the PSE). For the requirement of working faster and more precisely, an improvement was applied. This article deals with derivation of mathematical model of the improved PSE system used for subsequent analysis. The model is considered as a mixed model which is a combination of single-discipline subsystems as mechanical, electrical, fluid and control ones. The simulations are carried out for varied input parameters and both the system parameters and system characteristics are calculated. The results describe the behavior of the improved system in two modes of controller setup: constant pressure and constant stiffness under static and dynamic conditions

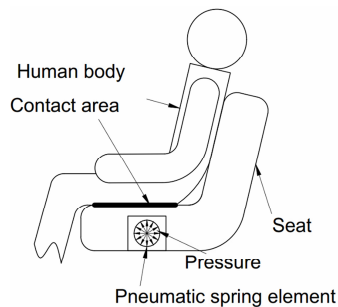
Keywords: pneumatic spring, mathematical model, improved system, transmission of acceleration.

1. Introduction

The seat with adjustable pressure profile is shown in Fig. 1(a). A PSE inserted inside the cushion allows changing the stiffness of cushion by a system of pneumatic actuator and a controller [1]. The PSE is inserted inside the car seat cushion (as shown in Fig. 1(b)). The mathematical model of the original system was presented in the article [2]. For the requirement of working faster and more precisely, the improved system is built. The detailed scheme of the system is in Fig. 2.



a) The seat in reality



b) A simplified scheme

Fig. 1. The seat with a PSE inserted inside

From the point of view that the PSE affects the human body locally (in contact with buttock), the model is built with the assumption that the mechanical subsystem represents a combination of foam block $100 \times 100 \times 50 \text{ mm}^3$ and a PSE (see Fig. 3). An additional latex tube is connected to the PSE (see Fig. 4). It can be considered that the volume change is not caused by the PSE but is

caused by the additional latex tube when compressed air is supplied. Therefore, the volume change in case of improved PSE is much larger than in case of the original PSE and the regulation of pressure can be enhanced.

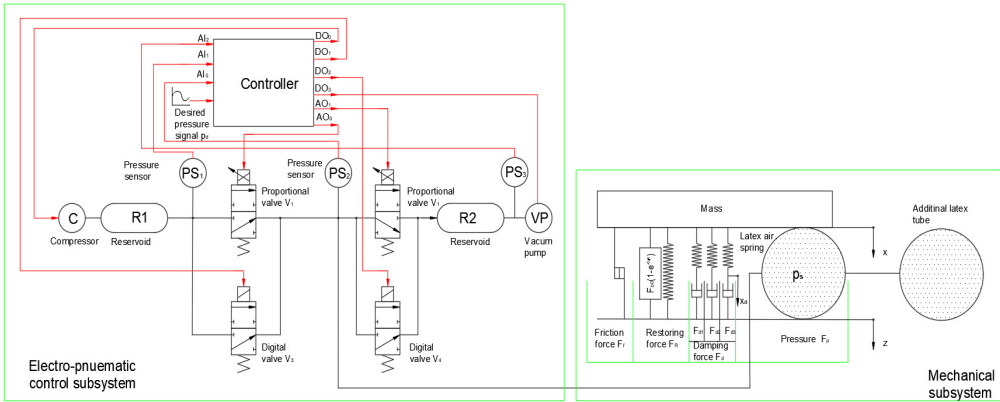


Fig. 2. Detailed scheme of the improved system

For convenience of modeling, the improved system is divided into two parts: electro-pneumatic control subsystem and mechanical subsystem.

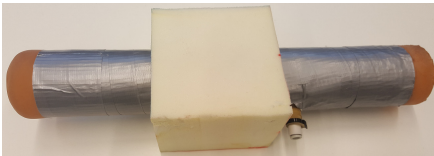


Fig. 3. The brick of foam with a PSE inserted inside

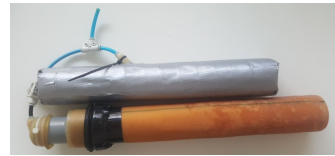


Fig. 4. The PSE connected to an additional latex tube

For the distribution of compressed air in and out of the PSE, the system of valves is divided into two pairs. Each pair (input and output) consisting of one proportional valve and one discrete valve. The first pair of input valves (V_1, V_3) is used to distribute the compressed air supplied from the reservoir R_1 into the PSE, the second pair of output valves (V_2, V_4) is to distribute the air from the PSE to the outlet. A vacuum pump generating output pressure (inside a reservoir R_2) less than 0 kPa is connected to the outlet valves. The proportional valves V_1, V_2 are intended for small airflow in case of small control error (the difference between desired and instant value) which is less than the threshold value set in the control software. The discrete valves V_3, V_4 for faster airflow are opened when the difference between the desired and the instant pressure value is bigger than the threshold value. Sensor PS₁ (SMC PSE540A-R06) measures pressure in the reservoir R_1 (denoted p_{cr}) and sensor PS₂ (PSE543A-R06) measures pressure inside the PSE (denoted p_s) and one more sensor PS₃ (PSE543A-R06) is added for measuring vacuum pressure inside the reservoir R_2 (denoted p_{vr}). The control software was created in Labview environment. It enables to display the courses of system values in time as desired pressure in PSE, measured unfiltered and filtered pressure in PSE, value of control error, etc. According to [3], the control software is also used for setting of system parameters as upper and lower limit value of pressure in the reservoir, constants of PID controller, selection of working mode (there are 2 modes: constant pressure and constant stiffness). The improved system can work with inlet air pressure up to 250 kPa and vacuum pressure at the output up to -60 kPa. The pressure inside the PSE can be changed in the range [0, 25] kPa.

2. Mathematical model

2.1. Model of the electro-pneumatic control subsystem

The components of this part are proportional and discrete valves, compressor and PID controller.

2.1.1. Model of the valves

The proportional valves denoted V_1 and V_2 are of type SMC-PVQ13-6M-08-M5-A. They are characterized by dependence of flow rate q_{sj} ($j = 1, 2$) on pressure difference Δp ($\Delta p = p_{inlet} - p_{outlet}$) and coil current i_j (which is supplied to control the proportional valve j). The course of these characteristics is taken from the datasheet [3] and it is transformed by means of data-fit algorithm into a two-parametric Eq. (1):

$$q_{sj} = q_{pv}(i_j, \Delta p) = k_{00} + k_{10}i_j\Delta p + k_{01}\Delta p + k_{20}i_j^2 + k_{11}i_j\Delta p + k_{02}\Delta p^2 + k_{30}i_j^3 + k_{21}i_j^2\Delta p + k_{12}i_j\Delta p^2 + k_{03}\Delta p^3, \quad (1)$$

where: $p_{inlet} = p_{cr}$, $p_{outlet} = p_s$ for valve V_1 and $p_{inlet} = p_s$, $p_{outlet} = p_{vr}$ for valve V_2 .

For the purpose of controlling the system, we define a pressure error e as a difference between desired pressure p_d inside the PSE and instant internal pressure p_s Eq. (2):

$$e = p_s - p_d. \quad (2)$$

Beside that we define a threshold e_t which addresses an insensitivity of control system. We have:

$$q_{s1} = \begin{cases} q_{pv}(i_1, p_{cr} - p_s), & e < -e_t, \\ 0, & e \geq -e_t, \end{cases} \quad (3)$$

$$q_{s2} = \begin{cases} q_{pv}(i_2, p_s - p_{atm}), & e > e_t, \\ 0, & e \leq e_t. \end{cases} \quad (4)$$

The PID controller is used for control of the coil current which is supplied to proportional valves V_1, V_2 . Using the equation of PID controller the coil current is a function of the pressure error e and it is presented in the form:

$$i_j = K_p e + K_I \int_0^t e(\tau) d\tau + K_D \frac{de}{dt}, \quad (5)$$

where: $i_j(t)$ is the current supplied for controlling the proportional valve j ($j = 1, 2$).

For cases when we need to realize considerably higher flow rates to make system work faster we need to use the pair of discrete valves V_3, V_4 (in the type of SMC-S070B-6A) which increase the flow rate in step-change. According to [4], flow characteristics of the discrete valve include sonic conductance $C_{dv} = 0.083$ l/(s.bar) and critical pressure ratio $b_{dv} = 0.28$. The formula for the flow rate calculation of discrete valve is:

$$\frac{p_{downstream}}{p_{upstream}} \leq b_{dv} \rightarrow q_{sj} = q_{dv1}(p_{upstream}) = 600C_{dv}p_{upstream} \sqrt{\frac{293}{273 + T}} \quad \text{l/min}, \quad (6)$$

in case of choked flow, and:

$$\frac{p_{downstream}}{p_{upstream}} \geq b_{dv} \rightarrow q_{sj} = q_{dv2}(p_{downstream}, p_{upstream})$$

$$= 600C_{dv}p_{upstream} \sqrt{1 - \left(\frac{p_{downstream} - b_{dv}}{p_{upstream}}\right)^2} \sqrt{\frac{293}{273 + T}} \quad \text{l/min,} \quad (7)$$

in case of subsonic flow where $j = 3, 4$.

$p_{upstream}, p_{downstream}$ represents upstream pressure and downstream pressure of the valve respectively. In case of valve V_3 , $p_{upstream} = p_{cr}$ and $p_{downstream} = p_s$, in case of valve V_4 , $p_{upstream} = p_s$ and $p_{downstream} = p_{atm}$, and T is the temperature inside the PSE which is assumed to be constant (297 °K).

The discrete valves V_3, V_4 are used for increasing of airflow when control error is recognized as high which is beyond the sum of the threshold (e_t) and another parameter (denoted Δp_s) which is set in the control software and called the value of pressure sensitivity. We have:

$$q_{s3} = \begin{cases} q_{dv1}(p_{cr}), & \frac{p_{cr}}{p_s} \leq b_{dv}, \quad e < -e_t - \Delta p_s, \\ q_{dv2}(p_{cr}, p_s), & \frac{p_{cr}}{p_s} \geq b_{dv}, \quad e < -e_t - \Delta p_s, \\ 0, & e \geq -e_t - \Delta p_s, \end{cases} \quad (8)$$

$$q_{s4} = \begin{cases} q_{dv1}(p_s), & \frac{p_s}{p_{atm}} \leq b_{dv}, \quad e > e_t + \Delta p_s, \\ q_{dv2}(p_s), & \frac{p_s}{p_{atm}} \geq b_{dv}, \quad e > e_t + \Delta p_s, \\ 0, & e \leq e_t + \Delta p_s. \end{cases} \quad (9)$$

2.1.2. Mathematical model of the compressed air supply

The process of transporting air from the compressor to the reservoir and distributing compressed air from the reservoir through inlet valves (V_1 and V_3) to the PSE is shown in Fig. 5. The compressor characteristics in time and pressure domain are calculated on the experimental basis. Based on the scheme, the total air flowrate q_{cr} through the reservoir is then given by:

$$q_{cr} = q_c - q_{s1} - q_{s3}, \quad (10)$$

where: q_{s1}, q_{s3} are flow rates of both valves V_1, V_3 , q_c is supplied flow rate from the compressor.

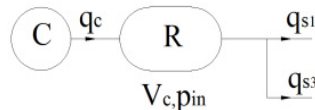


Fig. 5. The scheme of the process of transmitting compressed air to PSE

When the compressor is working, the flow rate q_c can be expressed in form of a linear function of the pressure inside the reservoir p_{cr} :

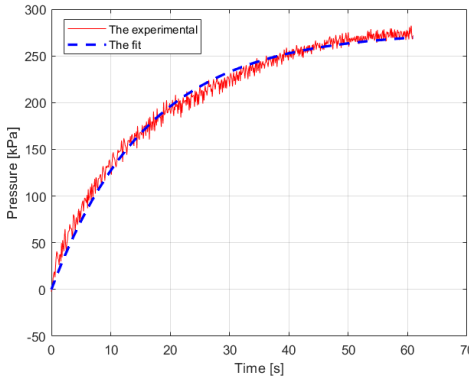
$$q_c = a_1 \left(\frac{p_{cr} - p_{atm}}{b_1} \right). \quad (11)$$

The constants $a_1 = 0.062$, $b_1 = 275.92$ are experimentally determined to correspond to the

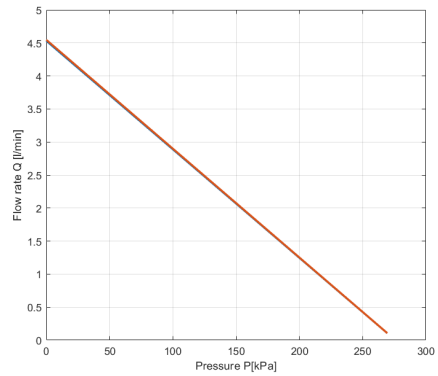
results in Fig. 6(b).

Pressure p_{cr} is measured by sensor PS_1 and it is kept in the interval $[p_{cr1}, p_{cr2}]$ by the control system. Values p_{cr1}, p_{cr2} are set in control software as user-defined system parameters. The compressor is switched on in case $p_{cr} < p_{cr1}$ (State = ON) and switched off in case of $p_{cr} > p_{cr2}$ (State = OFF). The “State” is an internal variable that is automatically set by the controller. Normally, the values of p_{cr1} and p_{cr2} are set to 190 kPa and 210 kPa respectively. The function describing the flow from the compressor to the reservoir is presented as follows:

$$\begin{cases} q_c = a_1 \left(\frac{p_{cr} - p_{atm}}{b_1} \right), & \text{State} = \text{ON}, & p_{cr} < p_{cr1}, \\ q_c = 0, & \text{State} = \text{OFF}, & p_{cr} > p_{cr2}, \\ q_c = a_1 \left(\frac{p_{cr} - p_{atm}}{b_1} \right), & \text{State} = \text{ON}, & p_{cr1} < p_r < p_{cr2}, \\ q_c = 0, & \text{State} = \text{OFF}, & p_{cr1} < p_{cr} < p_{cr2}. \end{cases} \quad (12)$$



a) Dependence of pressure of compressed air on time



b) Dependence of flowrate air on pressure

Fig. 6. The characteristics of the compressor



Fig. 7. The scheme of the process of releasing compressed air in the improved system

2.1.3. Mathematical model of the vacuum pump

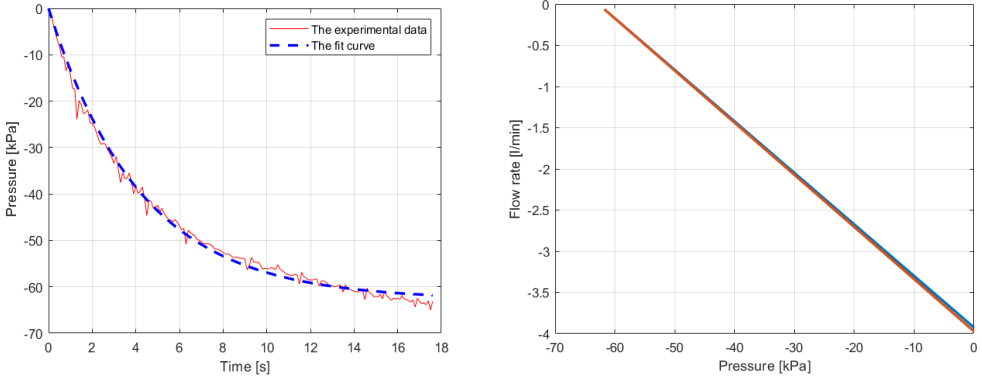
The mathematical model of the vacuum pump can be built similarly to the model of the air compressor because of the same operating principle. The total flow rate through the reservoir is then given by:

$$q_{vr} = -q_{vp} + q_{s2} + q_{s4}, \quad (13)$$

where q_{s2}, q_{s4} are flow rates of valves V_2, V_4 respectively. In this case q_{s2}, q_{s4} is calculated by Eqs. (6) or (7) with $p_{upstream} = p_s$ and $p_{downstream} = p_{vr}$.

When the vacuum pump works the flow rate q_{vp} can be expressed in the form of a linear function of the pressure p_{r2} inside the reservoir R_2 :

$$q_{vp} = a_2 \left(\frac{p_{vr} - p_{atm}}{b_2} \right). \quad (14)$$



a) Pressure of vacuum pump – time diagram b) Flow rate – pressure diagram
Fig. 8. The characteristics of the vacuum pump

The constants $a_2 = 0.23778$, $b_2 = -62.754$ are experimentally determined in correspondence with the results in Fig. 8(b).

Pressure p_{vr} is measured by sensor PS_3 and is kept in the interval $[p_{vr1}, p_{vr2}]$ by the control system. Values p_{vr1} , p_{vr2} are set in control software as user-defined system parameters. The vacuum pump is switched on in case of $p_{vr} > p_{vr2}$ (State = ON) and switched off in case of $p_{vr} < p_{vr1}$ (State = OFF). The “State” is an internal variable that is automatically set by the controller. Normally the values of p_{vr1} and p_{vr2} are set to -60 kPa and -40 kPa respectively. The function describing the flow from the reservoir to the vacuum pump is expressed by:

$$\begin{cases} q_{vp} = a_2 \left(\frac{p_{vr} - p_{atm}}{b_2} \right), & \text{State} = \text{ON}, & p_{vr} > p_{vr2}, \\ q_{vp} = 0, & \text{State} = \text{OFF}, & p_{vr} < p_{vr1}, \\ q_{vp} = a_2 \left(\frac{p_{vr} - p_{atm}}{b_2} \right), & \text{State} = \text{ON}, & p_{vr1} < p_{vr} < p_{vr2}, \\ q_{vp} = 0, & \text{State} = \text{OFF}, & p_{vr1} < p_{vr} < p_{vr2}. \end{cases} \quad (15)$$

2.2. Model of the mechanical subsystem

Forces acting on the mass are illustrated in the free-body diagram (see Fig. 9) where x , \dot{x} , \ddot{x} represent the kinematic displacement excitation, velocity excitation and acceleration excitation, \vec{F}_{foam} , \vec{F}_p represent the contact forces between the foam block, the latex air spring and the mass sequentially. It is possible to set up the equation of motion of mass m in the form:

$$mg - F_{foam} - F_p = m\ddot{x}. \quad (16)$$

For further model derivation, we used this simplified approach.

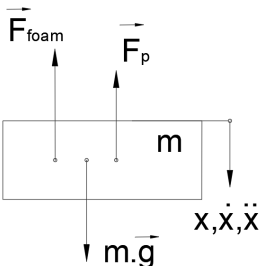


Fig. 9. Free-body diagram

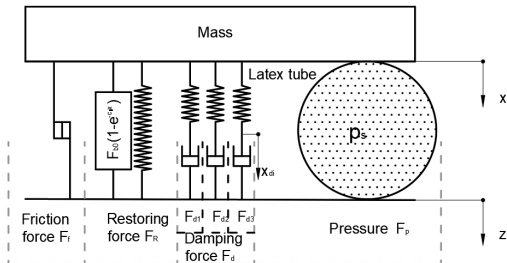


Fig. 10. Detailed scheme of the mechanical subsystem.

2.2.1. Model of polyurethane foam

Computational scheme of the rheological model the PU foam block is depicted in Fig. 10. It is the result of research taken from [6]. The concept of this lumped parameter model follows from the phenomenological approach. It comprises nonlinear restoring force F_R , damping force F_d and frictional force F_f in general force response F_{foam} . The model definition of PU foam is described in detail as below:

$$F_{foam} = F_{foam}(x, \dot{x}, x_{di}, \dot{x}_{di}) = F_R(x) + F_d(x, \dot{x}, x_{di}, \dot{x}_{di}) + F_f(x, \dot{x}, x_{di}, \dot{x}_{di}), \quad (17)$$

$$F_R = F_b + F_{dp}, \quad (18)$$

$$F_d = \sum_{i=1}^3 F_{di}, \quad (19)$$

$$F_f = f_f F_{Rd}, \quad (20)$$

$$F_b = F_{b0}(1 - e^{-c_b(x-z)}) \quad F_b = F_{b0} \cdot (1 - e^{-c_b(x-z)}), \quad (21)$$

$$F_{dp} = p_p S_p \left[\left(\frac{h_p}{h_p - (x-z)} \right)^{n_p} - \left(\frac{h_p}{h_p + (x-z)} \right)^{n_p} \right], \quad (22)$$

$$F_{di} = F_{dc}(x, x_{di}) = p_{0i} S_{0i} \left[\left(\frac{h_i}{h_i - (x - x_{di})} \right)^{n_{oi}} - \left(\frac{h_i}{h_i + (x - x_{di})} \right)^{n_{oi}} \right], \quad (23)$$

$$F_f = f_f F_{Rd}, \quad (24)$$

$$f_f = \frac{2f_{f0}}{\pi} \arctan(k_1 v) + k_2 |v|^{k_3} \text{sgn}(v). \quad (25)$$

The parameter of PU foam model is shown in Table 1.

Table 1. The parameter of PU foam model

Force component	Parameter	Physical unit	Value		
F_b	F_{b0}	[N]	80		
	c_b	[N/m]	600		
F_{dp}	S_p	[m ²]	0.0095		
	p_p	[Pa]	100		
	n_p	[1]	6.2		
	h_p	[m]	0.06		
F_{di}	S_{0i}	[m ²]	$i = 1$	$i = 2$	$i = 3$
	p_{0i}	[Pa]	1.2	0.03	0.8
	n_{oi}	[1]	100		
	h_i	[m]	3	3.5	2
	c_i	[-]	0.05	0.05	0.18
	n_i	[1]	50	300	300
	n_i	[1]	0.2	0.2	0.2
F_f	f_{f0}	[1]	0.05		
	k_1	[s/m]	5000		
	k_2	[-]	0.2		
	k_3	[1]	1		

2.2.2. Model of the combination of the latex air spring and the additional latex tube

The contact force F_p can be considered as a function depending on the deformation $(x-z)$ and pressure p_s . The change of pressure, deformation and change of contact area is a complex phenomenon, therefore, the dependence of contact force on the deformation and the pressure is determined experimentally. In this way, an experiment was performed on the Instron E3000

machine as shown in Fig. 11. The contact force F_p is possible to consider in the form of polynomial function:

$$F_p = F_p(x, p_s) = k_{00} + k_{10}(x - z) + k_{01}(p_s - p_{atm}) + k_{20}(x - z)^2 + k_{11}(x - z)(p_s - p_{atm}) + k_{02}(p_s - p_{atm})^2 + k_{30}(x - z)^3 + k_{21}(x - z)^2(p_s - p_{atm}) + k_{12}(x - z)(p_s - p_{atm})^2 + k_{03} \cdot (p_s - p_{atm})^3, \quad (26)$$

and the 3D graph of interaction force F_p is shown in Fig. 12.

According to [7], the internal pressure (p_s) depends on the volume (V_{ils}) and the flow rate of supplied air (q_s) and is expressed in the form of the differential equation:

$$\dot{p}_s = \frac{\kappa q_s R_{gas} T}{V_{ils}} - \kappa p_s \frac{\dot{V}_{ils}}{V_{ils}}, \quad (27)$$

where: V_{ils} is the total volume of the improved latex air spring, T is the temperature of the air inside the PSE which is assumed to be constant ($T = 297 \text{ }^\circ\text{K}$), $\kappa = 1.4$ is adiabatic exponent, $R_{gas} = 287 \text{ J kg}^{-1} \text{ K}^{-1}$ is the gas constant.

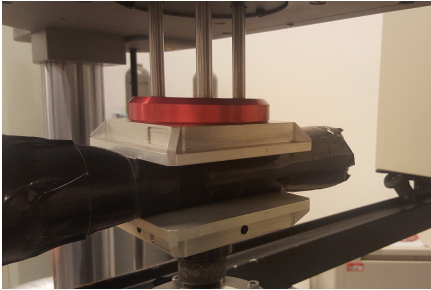


Fig. 11. Setup of the experiment

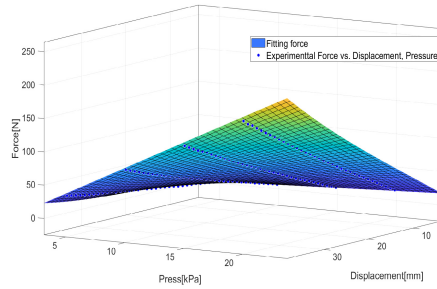


Fig. 12. Relationship between contact force, displacement and pressure

The additional latex tube is connected to the PSE as shown in Fig. 4. The PSE is totally covered with tape so it can be assumed that change of the PSE volume caused by internal pressure is negligible and the total change of volume is completely caused by the change of volume of the additional latex tube.

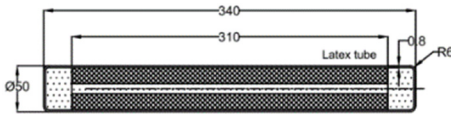


Fig. 13. Scheme of the latex tube with foam inserted inside

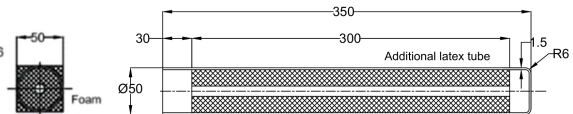


Fig. 14. The scheme of the additional latex tube

Firstly, we can use a simple calculation of volume of the PSE. The volume of the PSE is calculated in the way that is similar to the way and is given by:

$$V_{PSE} = \pi R^2 l - V_{foam}, \quad (28)$$

where: $R = 0.025 \text{ m}$ is the radius of the PSE, $l = 0.3 \text{ m}$ is the length of the PSE, $V_{foam} = 0.15l$ is the volume of foam (see Fig. 13).

Secondly, we consider the calculation of the volume of the additional latex tube. The additional latex tube is designed with the dimensions shown in Fig. 14. We can recognize that its structure is similar to the structure of the original PSE. The change of volume of additional latex tube

depends on pressure p_s and can be found experimentally. The setup of the experiment is shown in Fig. 15. At first the additional latex tube is filled with compressed air when the value of internal pressure meets the value of desired pressure. At second the additional latex tube is kept in a stable position and its surface is scanned by a 3D scanner machine.

The calculation is programmed in Matlab software. The result is a set of volume values of the additional latex tube in the dependence of pressure. So the volume (V_{add}) depending on internal pressure (p_s) can be given in form of a polynomial function Eq. (29):

$$V_{add} = V_{add}(p_s) = k_0 + k_1(p_s - p_{atm}) + k_2(p_s - p_{atm})^2 + k_3(p_s - p_{atm})^3 + k_4(p_s - p_{atm})^4 - V_{foam}. \quad (29)$$

By using Matlab a set of coefficients is obtained as shown below:

$$\begin{aligned} k_0 &= 8.512599267236148e - 04, & k_1 &= 6.586794838689470e - 09, \\ k_2 &= 4.111139460129753e - 12, & k_3 &= -4.030675121010709e - 16, \\ k_4 &= 1.163546770201630e - 20. \end{aligned}$$

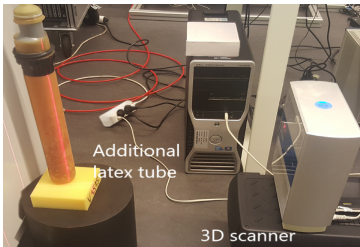


Fig. 15. The setup of the experiment of the scanning of the surface of the additional latex tube

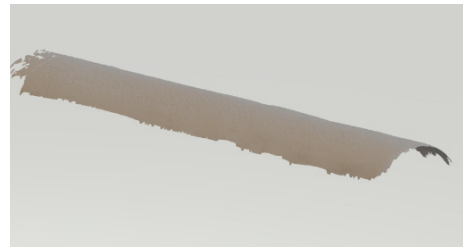


Fig. 16. Example of a point cloud of the surface of the additional latex tube

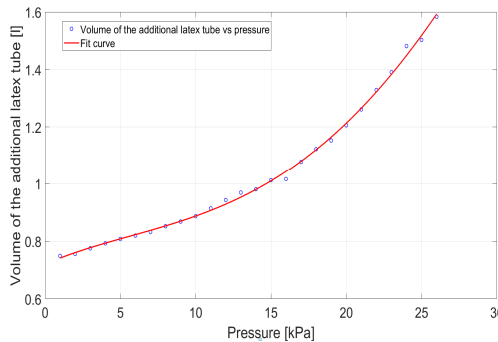


Fig. 17. Volume-pressure diagram and the fit function

Finally, the total volume of latex air spring in the case of the improved system can be given by:

$$V_{ils} = V_{ils}(p_s) = V_{PSE} + V_{add}(p_s). \quad (30)$$

Then the derivative of the total volume is:

$$\dot{V}_{ils} = \dot{V}_{ils}(p_s, \dot{p}_s) = \frac{dV_{add}}{dp_s} \dot{p}_s. \quad (31)$$

In summary, the mathematical model describing the improved system is given by first-order differential equations. For the purpose of using Matlab software, it is useful to transform this

system of equations into matrix form:

$$\dot{\mathbf{X}} = \mathbf{F}(\mathbf{X}), \quad (32)$$

where:

$$\mathbf{X} = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \\ X_6 \end{bmatrix} = \begin{bmatrix} x \\ \dot{x} \\ x_{d1} \\ x_{d2} \\ x_{d3} \\ p_s \end{bmatrix}, \quad (33)$$

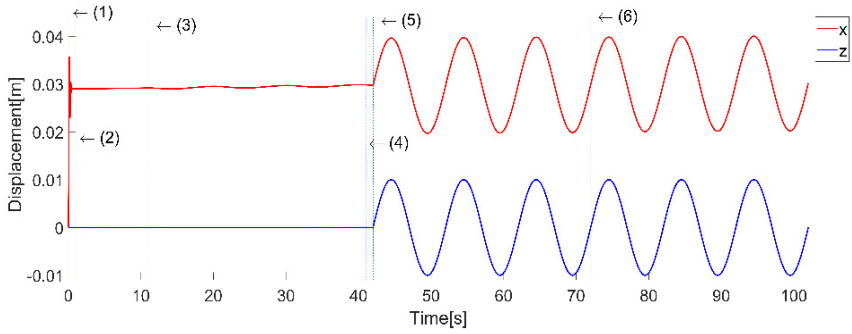
$$\mathbf{F}(\mathbf{X}) = \begin{bmatrix} F_1(\mathbf{X}) \\ F_2(\mathbf{X}) \\ F_3(\mathbf{X}) \\ F_4(\mathbf{X}) \\ F_5(\mathbf{X}) \\ F_6(\mathbf{X}) \end{bmatrix} = \begin{bmatrix} g - \frac{F_{foam}(X_1)}{m} - \frac{F_p(X_1, X_6)}{m} \\ \left(\frac{1}{c_1} F_{dc}(X_1, X_3)\right)^{\frac{1}{n_1}} \operatorname{sgn}(F_{dc}(X_1, X_3)) + \dot{z} \\ \left(\frac{1}{c_2} F_{dc}(X_1, X_4)\right)^{\frac{1}{n_2}} \operatorname{sgn}(F_{dc}(X_1, X_4)) + \dot{z} \\ \left(\frac{1}{c_3} F_{dc}(X_1, X_5)\right)^{\frac{1}{n_3}} \operatorname{sgn}(F_{dc}(X_1, X_5)) + \dot{z} \\ \frac{\kappa q_s R_{gas} T - \kappa X_6 \frac{dV_1(X_1)}{dX_1}(X_1) \cdot X_2}{V_{ls}(X_1, X_6) + \kappa X_6 \frac{dV_3(X_6)}{dX_6}} \end{bmatrix}. \quad (34)$$

The predefined functions of excitation z, \dot{z}, \ddot{z} are continuous in an interval of time. The variable q_s is calculated when the system of equations from Eq. (1) to Eq. (15) describing the function of the electro-pneumatic control subsystem is being solved. The solution of this system of equations gives as the results describing system response and system characteristics as kinematics of mass (x, \dot{x}, \ddot{x}), pressure inside the PSE (p_s) in case of mechanical subsystem, the flow rate ($q_s, q_r, q_c, q_{s1}, q_{s2}, q_{s3}, q_{s4}$), pressure (p_{cr}, e, \dots), supplied coil current of proportional valves i_j ($j = 1, 2$) in case of electro-pneumatic control subsystem and forces (F_{foam}, F_p, \dots), volume change of PSE (ΔV_b), etc.

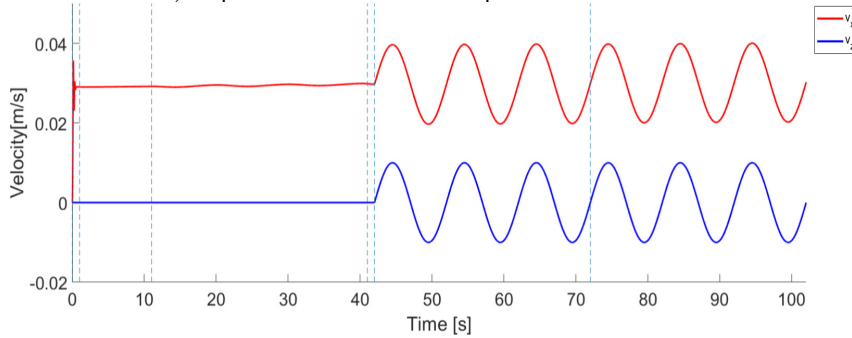
To solve the system of equations the initial and boundary conditions were provided. Initial conditions for the system of equations of the improved system are the values of the variables $x, x_{di}, \dot{x}, p_s, q_{si}, q_c, p_c, V_{ils}$ at time $t = 0$ and boundary conditions for the system equations are the values of the parameters $p_{cr1}, p_{cr2}, p_{vr1}, p_{vr2}, e_s, \Delta p_s$.

3. Calculation of the response of the improved system

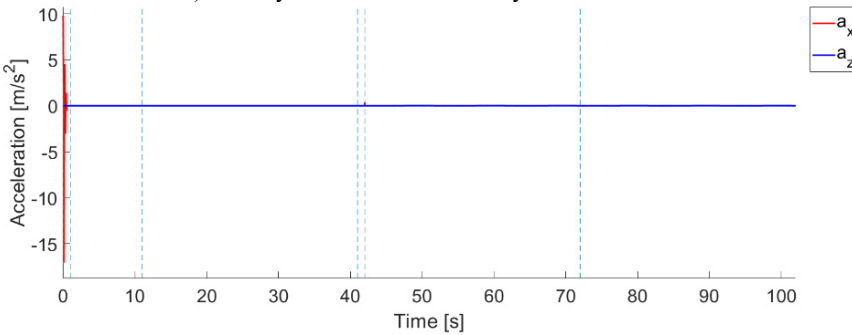
The examples below show the behavior of the original system when it works under static conditions (without excitation $z(t)$) and dynamic conditions (with excitation $z(t)$) for both modes of operation (constant stiffness and constant pressure) and the load $m = 10$ kg.



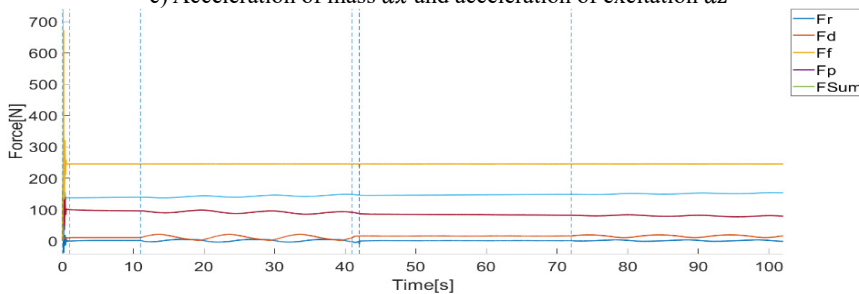
a) Displacement of mass x and displacement excitation z



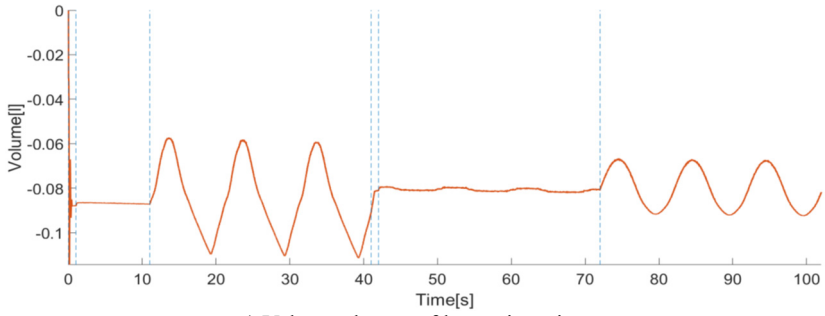
b) Velocity of mass v_x and velocity of excitation v_z



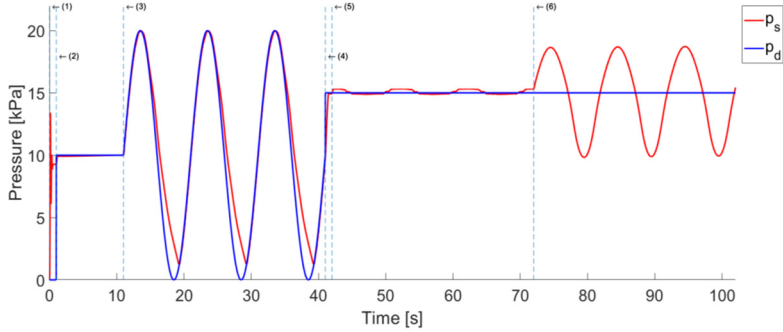
c) Acceleration of mass a_x and acceleration of excitation a_z



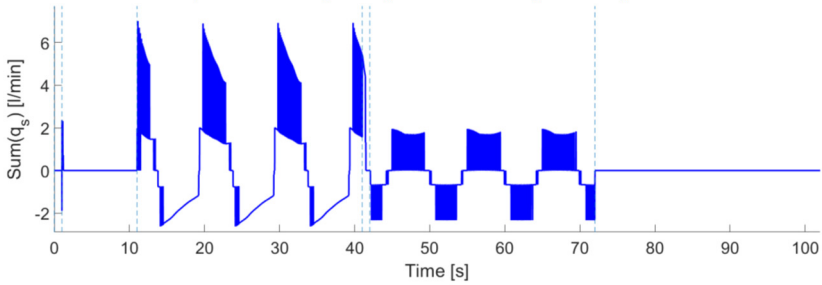
d) Forces



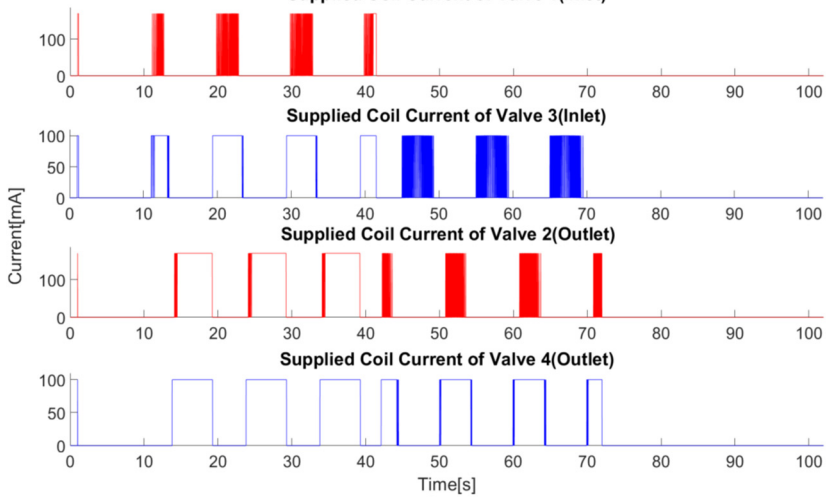
e) Volume change of latex air spring



f) Pressure response p_s and desired pressure p_d



g) The flow rate q_s through the PSE
 Supplied Coil Current of Valve 1(Inlet)



h) Supplied coil current of the valves V_1, V_2, V_3, V_4

Fig. 18. The calculated response of the improved system

4. Calculation of transmission of acceleration

The transmission of acceleration is an important property of efficiency of vibration isolation systems. This is a way how to evaluate the influence of the PSE under dynamic conditions. In this article, we use the signal with continual change of frequency what is advantageous for investigation of nonlinear systems. For such cases so-called sweep sine functions are usually used. According to [8] the harmonic excitation function can be replaced by a sweep sine with variable frequency. Numerous human vibration studies conducted over the past several decades have shown that the human body is sensitive to low-frequency vibration occurring below 10 Hz [9]. Based on this the range was set to 1-11 Hz. The amplitude of acceleration was set to constant value of 0.1 g. The excitation function used for experiments under dynamic conditions is expressed in the form:

$$z(t) = Z_a(t + t_0)^{2(1-n)}\sin(c(t + t_0)^n), \quad (35)$$

where: n is the exponent of excitation function ($n = 2$), f_{start} is start frequency, f_{stop} is the stop frequency, T is the total time of the excitation signal, g is the gravitational acceleration and the other parameters are calculated as follows:

$$c = \pi \frac{f_{stop} - f_{start}}{T} \quad 1/s^2, \quad (36)$$

$$t_0 = \left(\frac{2\pi f_0}{nc} \right)^{\frac{1}{n-1}} \quad s, \quad (37)$$

$$Z_a = \frac{0.1g}{n^2 c^2} \quad m/s^2. \quad (38)$$

Fig. 19 and Fig. 20 show calculated results of the transmission of acceleration in case of using constant pressure mode and constant stiffness mode of control operation. The transmission curves are shown in cases of desired pressure $p_d \in \{0, 5, 10, 15, 20, 25\}$ kPa in dependence on the frequency of excitation in the range [1, 11] Hz. With a load of 10 kg, peaks of the transmission curves are positioned in the range of [6, 8] Hz and peak values are in range [2.5, 4].

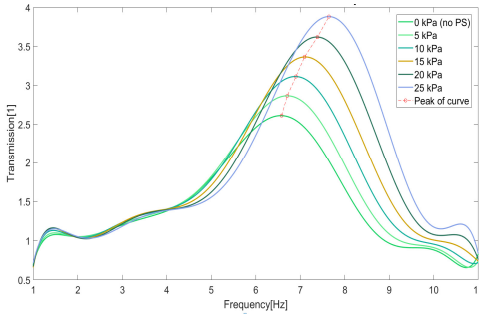


Fig. 19. Transmission of acceleration – constant pressure mode

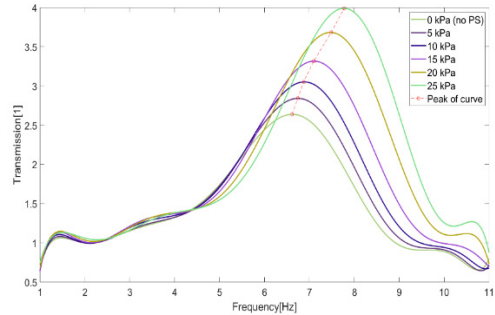


Fig. 20. Transmission of acceleration – constant stiffness mode

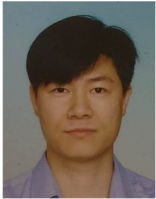
5. Conclusions

The model of the original system consists of the model of foam block which is based on the article [6] and the model of a latex air spring and the model of electro-pneumatic control which are found experimentally. This model allows to investigate the response of the system to kinematic excitation or the change of the desired pressure inside pneumatic spring. Calculated results describe the behavior of characteristics of the improved system in detail when the improved

system works in control modes (constant pressure and constant stiffness) and under static and dynamic conditions. The model is also used for evaluation of vibration isolation effect of the system which is possible to represent in form of the curves of transmission of acceleration. The transmission of acceleration shows the same tendency to shift the peak position toward the higher frequency when the desired pressure increases. This frequency shift is followed by simultaneous increase of peak value.

References

- [1] **Cirkl D.** Seat. Patent No. 303163, 2012.
- [2] **Cirkl D., Tran Xuan T.** Modelling of dynamical behavior of pneumatic spring-mass system. *Engineering Mechanics Proceedings*, Vol. 24, 2018, p. 865-868.
- [3] **Cirkl D., Tran Xuan T.** Simulation model of seat with implemented pneumatic spring. *Vibroengineering Procedia*, Vol. 7, 2016, p. 154-159.
- [4] Compact Proportional Solenoid Valve. Series PVQ, SMC catalog.
- [5] Port Solenoid Valve. Series S070, SMC catalog.
- [6] **Cirkl D., Hrus T.** Simulation model of polyurethane foam for uniaxial dynamical compression. *Vibroengineering Procedia*, Vol. 1, 2013, p. 53-58.
- [7] **Cirkl D.** Mechanical Properties of Polyurethane Foam. Ph.D. Thesis, Technical University of Liberec, Liberec, 2005.
- [8] **Sivčák M.** Dynamics of the Vibration Isolation System with More Degrees of Freedom. Ph.D. Thesis, Technical University of Liberec, Liberec, 2009.
- [9] **Griffin M. J.** Handbook of Human Vibration. Academic Press, London, 1990.



Tien Tran Xuan is a lecturer at HungYen University of Technology and Education, VietNam at the Department of Mechatronics. Now he is an Ph.D. student at Department of Applied Mechanics, Faculty of Mechanical Engineering, Technical University of Liberec.



Nguyen Phu Dong is a lecturer at Nha Trang University, Vietnam at Faculty of Transportation Engineering. Now he is an Ph.D. student at Department of Vehicles and Engines, Faculty of Mechanical, Technical University of Liberec.