Application of Convex Optimization Results of DE FINETTI's problem for Proportional Reinsurance (Study case CAARAMA insurance campany in Algiers)

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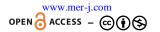✉Hanya.kherchi@gmail.com

Abstract :

The objective of this research is to find the optimal retention level for a proportional reinsurance treaty based on the results of the convex optimization developed in De Finetti's model. The latter makes it possible to determine the level of retention that achieves the expected profit by the insurer, while minimizing claims volatility.

The convex functions appear abundantly in economics and finance. They have remarkable specificities that allows actuaries to minimize financial risks to which some institutions are exposed, especially insurance companies. Therefore, the use of mathematical tools to manage the various risks is paramount.In order to remedy the optimization problem, we have combined the probability of failure method with the "De Finetti" model for proportional reinsurance, which proposed a retention optimization process that minimizes claim volatility for a fixed expected profit based on the results of the non-linear optimization.

Keywords: Convex function, Nonlinear optimization, Proportional reinsurance, Risk minimization.

JEL Codes: C02, C25, C61, G22.

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1. Introduction

The insurer is responsible for guaranteeing the various risks to which its customers are exposed, and for indemnifying claims that have occurred. However, the risks covered can have a strong influence on the security of the insurer and possibly on its result, which encourages it to seek cover. As a business, the insurance company applies to itself the techniques of risk management, namely reinsurance.As a matter of fact, reinsurance allows the insurer to transfer part of its risks for a premium. It is an essential operation which guarantees the insurability of these risks, and which thus contributes to the development of insurance both for damageand life insurance (Tosetti, Béhar, Fromenteau, & Ménart, 2011, p. 11).

Relations between insurers and reinsurers are based on a community of interest, each party seeking to determine the commitment that meets its own objectives, by setting the share of the risk to be retained. This part represents its own retention, whichis one of the most sensitive parameters in reinsurance(Blondeau & Partrat , 2003, pp. 14-83). As transactions between the two organizations are common, a poor appreciation of the level of retention directly impacts their profitability. To this end, each party must establish with precision a reinsurance program that optimizes its commitment.The search for an optimal reinsurance structure for the insurer is considered to be its major concern,therefore actuaries have used convex functions.In finance, optimization algorithms are essential to define models that minimize risks. For this purpose, convex functions are the main tool used in risk management (Charpentier, 2010). For example, De Finetti's work on nonlinear optimization has proved that the results of convex optimization allow to define a dynamic approach to programming in insurance when it comes to determine an optimal retention. Afterwards, both mathematicians Glineur and Walhin even extended De Finetti's resultson other reinsurance treaties.

This paper aims to define an optimal proportional reinsurance structure based on De Finetti's results on convex optimization in order to define an efficient reinsurance structure.To support this issue, we will develop a practical case devoted todetermination of an optimal retention level according to the De Finetti model using portfolio data from life insurance company, followed by acomparative analysis with the levelretention practiced by the managers of the company. The analysis of this problem will also allow us to measure the effects ofreassurance on the probability of ruin. The latter led us to define preciselyan efficient reinsurance structure for the combinations of profitability measures andrisk.

2. The reinsurance approach
2.1 Definition of reinsurance

Reinsurance can be defined as the technique by which an insurer transfers all or part of the risks it has taken out to another company. The idea that drives the reinsurance relationship is that of a sort of sharing of the ceding company by the reinsurer. The reinsurance transaction is based on the good faith of the parties and generally involves a lasting partnership over time (Blondeau & Partrat, 2003, p. 2).

2.2 Forms of reinsurance
2.2.1 Proportional reinsurance

We talk about proportional reinsurance when the reinsurer commits the same way to premiums and claims. There are again two types of treaties:

- **The quota share treaty:** This is the best-known treaty where the ceding company, which is the company that wants to reinsure, cedes to its reinsurer a percentage of the risk it has taken on as a direct insurer. This form of treaty makes it possible to transfer a share of the claims and the same share of the premiums. In addition, the treaty provides that the reinsurer also bears a share of the management and acquisition costs corresponding in principle to the proportion of its commitment to the portfolio. This treaty is characterized by equality between the proportion of premiums received by the reinsurer and the proportion of the cost of claims transferred to the reinsurer.

- **The treaty in excess of full, or treated in excess of capital (surplus):**To limit the drawbacks mentioned with the quota share treaty, we can introduce the full surplus treaty which seeks to homogenize the net portfolio by keeping all of the small risks and by limiting the custody on the most important risks to a level deemed acceptable: full.Once the full has been determined, all the risks with exposures lower than thefull are retained entirely by the ceding company and the risks exceeding the full are distributed between the ceding company and the reinsurer.

2.2.2 Non-proportional reinsurance

Unlike proportional reinsurance based on the original commitment and proportional cession, it is the amount of claims and the limited scope of coverage that predominate in non-proportional reinsurance.

In the context of non-proportional reinsurance coverage, the following treaties can be cited:

- **The excess of loss treaty:** Intuitively, we can estimate that the volatility of the loss experience is primarily influenced by the most

important losses. To reduce the volatility of claims, it is therefore preferable to play on a maximum claim limit retained by the ceding company. Thus no individual claim can by itself excessively unbalance the results.

The excess of claims contract specifies the reinsurer's commitment to each claim (per risk or per event). Most generally, this commitment is defined by an intervention limit (deductible) and the extent of the commitment (scope).

- **The annual stop loss treaty**: The annual excess loss allows you to work directly on the overall annual claims charges. As in the case of excess claims, the reinsurer makes a defined commitment beyond a single intervention (the deductible) and a commitment limit (the scope). The difference with excess claims is that the claim expense considered is the total annual claim expense.Another difference is that the limits are generally expressed as a percentage of the gross premiums written by the ceding company.

2. Convex function theory

Convex functions are one of the most basic types of functions.

A function f :$R_n \rightarrow$ R is convex if its domain is a convex set and for all x, y in its domain, and all $\lambda \in$ [0, 1], we have :

$$f(\lambda x + (1 - \lambda)y) \leqslant \lambda f(x) + (1 - \lambda)f(y) \qquad (1)$$

Convex functions has a lot of applications. It is used for proving some inequalities in easy manner. Also it has various applications in operation research, Quadratic and Geometric programming problems (Constantin & Persson, 2004, pp. 145-157).

Convex functions are particularly easy to minimize (for example, any minimum of a convex function is a global minimum). For this reason, there is a very rich theory for solving convex optimization problems that has many practical applications (for example, circuit design, controller design, modelling, etc.)

3. De Finetti'sresultats on non-linear problems

Historically, actuaries have focused on optimal reinsurance in a two-dimensional space. Their purpose is to compare and improve their strength and performance. It is obvious that the insurer is always looking to maximize its profit and minimize the standard deviation of this profit (Glineur & Walhin, 2006, pp. 2-8).

Consider a portfolio with n independent risksX_1, … ,X_n against a

premium $P_1, \ldots, P_n$. We will protect each risk by a proportional reinsurance with a percentage of transfer $(1 - \theta)$. The ceded premium is loaded by a technical load based on the expected value; it is written as follows:

$$P(\text{surrendered}) = (1 + \varepsilon_i^{re})(1 - \theta) E(X) \tag{2}$$

De Finetti suggested choosing a divested quantity $(1 - \theta_i)$ that minimizes the variance of the insurer's result and keeps its expectation value constant. The result of the insurer is given by:

$$Z(\theta) = \sum_{i=1}^{n}(P_i - (1 + \varepsilon_i^{re})(1 - \theta_i) E(X_1) - \theta_i X_i) \tag{3}$$

Θ is a vector of proportionally conserved quantities. $\Theta = (\theta_1, \ldots, \theta_n)$.

The determination of this vector will be a problem of minimizing the volatility of claims;

Under theconstraints

$$\text{Min Var } Z(\theta)$$

$$E(Z(\theta)) = K$$
$$\theta_i \geq 0, \quad i = 1, \ldots, n$$
$$\theta_i \leq 1, \quad i = 1, \ldots, n$$

Where K is the expectation value of the result chosen by the insurer.
To solve this problem, it is necessary to understand the variance of claims by policy.

$$\text{Var}(Z(\theta)) = \text{Var}\left[-\sum_{i=1}^{n}(\theta_i \text{Var}(X_i)\right] = \sum_{i=1}^{n}\theta_i^2 \text{Var}(X_i) \tag{4}$$

The expectation value of the result is given by $E(Z(\theta))$

$$\begin{aligned}E(Z(\theta)) &= \sum_{i=1}^{n}(P_i - (1 + \varepsilon_i^{re})(1 - \theta_i) E(X_1) - \theta_i E(X_i)) \\ &= \sum_{i=1}^{n} P_i - \varepsilon_i^{re})(1 - \theta_i) E(X_1) - \theta_i E(X_i))\end{aligned} \tag{5}$$

The problem to be solved is therefore;

$$\text{Min} \sum_{i=1}^{n}\theta_i^2 \text{Var}(X_i)$$

Under the constraints:

$$\sum_{i=1}^{n} \varepsilon_i^{re} (1 - \theta_i) \, \mathrm{E}(X_1) = -K + \sum_{i=1}^{n} E(X_i)$$

$$\theta_i \geq 0 \qquad , \quad i = 1,\ldots, n$$
$$\theta_i \leq 1 \qquad , i = 1,\ldots, n$$
$$-\theta_i \leq 0 \quad , i = 1,\ldots, n$$

The solution of this modeling is:

$$\theta_i = \min[1, \max(0, \, 1 - \frac{\lambda \mu \mathrm{E}(X_i)}{\mathrm{Var}(X_i)})] \tag{6}$$

λ is the constant of the Lagrange multiplier.

4.Optimization Steps
4.1 The probability of ruin

$$P(X > R+P) \leq \alpha \tag{7}$$

R: the amount of the reserves of the company
P: the amount of premiums
X: the overall burden of the claim
α :minimum level fixed a priori

The first goal of determining the probability of ruin is to logically evaluate the wealth of the company (Deelstra & Plantin , 2006), that is to say the probability that the scenario translating a failure is realized, and to estimate the initial level of reserves to make this probability sufficiently low.

The insurer may consider using reinsurance to minimize the likelihood of bankruptcy. In the case of a quota share, the minimization takes place as well (Azcue, P. &Muler, N. 2005);

We consider a homogeneous group of n risks. The random variable is the expenditure from risk no. I and the total amount of expenses of the group for the current year and therefore

$$X = Y_1 + Y_2 + \ldots + Y_n$$

It is assumed that the pure premium is charged proportionally to the expected value. For simplicity, it will be assumed that the technical loading

rate applied by the reinsurer to the pure reinsurance premium is also equal to λ, and that there is no commission payment by reinsurance.

By applying the following assumptions:

(H1): The random variables $Y_1, ... Y_n$ are independent and identically distributed.

Given the above ratings, the cumulative annual amount of claims rated X is:

$$X = Y_1 + Y_2 + ,...,+ Y_n$$

X follows the binomial law of parameters n and p, which is noted $X \sim \text{B}(n, p)$ from which one deduces the mathematical expectation value and the variance of the random variable X:

$$E(X) = n \text{ pet} Var(X) = n p q$$

Let us introduce the additional hypothesis (H2) below that will allow us to easily carry out mathematical calculations;

(H2): It is assumed that the number n of insureds is large enough for the law binomial B (n, p) can be approximated by the normal distribution:

$$N(m, \sigma 2) \text{ with } (m = n p \text{ and } \sigma = \sqrt{npq})$$

This approximation will be possible provided that the product npq is sufficiently large.

Approximately:

$$Xi \sim N(n p, n p q)$$

The random variable θX follows approximately the law N(θ E (X), V (X)).

P (ruine) = P {θX > R + θΠ(X)}   (8)

Defining the reduced centered random variable U by:

$$U = \frac{\theta X - E(\theta X)}{\sigma(\theta(X))} \tag{9}$$

$$P \text{ (ruined)} = P \left\{ U > \frac{R + \theta(1+\lambda)E(X) - \theta E(X)}{\theta \sigma(X)} \right\} \tag{10}$$

$$P \text{ (ruined)} = P \left\{ U > \frac{R + \theta \lambda E(X)}{\theta \sigma(X)} \right\} \tag{11}$$

Or again;

$$P \text{ (ruined)} = 1 - F_0 \left(\frac{R}{\theta \sigma(X)} + \lambda \frac{E(X)}{\sigma(X)} \right) \tag{12}$$

Moreover, we have seen previously that the profit before reinsurance is equal to:

$$B= \Pi(X) - X \tag{13}$$

Its mathematical expectation value and its variance have for expression:

$$E(B) = \lambda\, E(X)\sigma(X) = \sigma(B)$$

If the factor of safety before reinsurance is greater than or equal to the value t desired by the insurer (which is judged 4), that is:

$$T= \frac{R+E(B)}{\sigma(X)} = \frac{X+\lambda E(X)}{\sigma(X)} \geq t \tag{14}$$

So, reinsurance is not necessary. Conversely, if the previous inequality is not verified, the insurer will have to resort to reinsurance.

We are now in this case and it is assumed that the insurer wishes to best adjust the full conservation for a given risk.

The insurer's annual profit after reinsurance, denoted B (θ), is given by:

$$B(\theta) = \Pi(X) - \theta X - (1 + \mu)(1 - \theta)E(X) \tag{15}$$

Or the technical bonus can also be written

$$\Pi(X) = (1 + \lambda)E(X) \tag{16}$$

Given the above assumptions, we deduce the expectation value and the variance:

$$E(B(\theta)) = (\lambda - \mu)E(X) + \mu\theta E(X) \tag{17}$$

And:

$$Var(B(\theta)) = \theta^2 Var(X) \tag{18}$$

Or the technical bonus can also be written $\Pi(X) = (1 + \lambda)E(X)$.
Given the above assumptions, we deduce the expectation and the variance:
$$0 \leqslant \theta \leqslant 1$$

$$T(\theta) = \frac{R+E(B(\theta))}{\sigma(B(\theta))} = \frac{R+(\lambda-\mu)E(X)+\mu\theta E(X)}{\theta\sigma(X)} \tag{19}$$

As we are in the context of proportional share reinsurance, the insurer commission = reinsurance commission, and we write:

$$T(\theta) = \frac{R+\theta\lambda E(X)}{\theta\sigma(X)} \tag{20}$$

Retention is sought from the retention coefficient $T(\theta)$:

$$T(\theta) \geq 4 \Rightarrow \frac{R+\theta\lambda E(X)}{\theta\sigma(X)} \geq 4$$

$$\theta \leqslant \frac{R}{4\sigma(X)-\lambda E(X))} \tag{21}$$

Therefore the probability of ruin will allow us to determine the insurer's retention interval. For this purpose, we had the idea of combining the probability of ruin theorem with convex optimization, and this to determine accurately the insurer's retention (Hess, 2000, pp. 16-22).

4.2 De Finetti's optimization on proportional reinsurance

After defining the retention interval from the probability of ruin, De Finetti's model developed (3) will allow us to accurately determine the percentage that represents the insurer's retention. To further explanation, we review De Finetti's results:

$$\theta i = \min \left[1, \max \left(0, 1-\frac{\lambda\mu E(X_i)}{Var(X_i)}\right) \right] \tag{22}$$

These results could be found based on results of nonlinear optimization that generalize the notion of Lagrange multipliers under inequality constraints.

5. Data application

For digital application, we used data from CAARAMA insurance campany in Algiers. Our work is presented as follow:

5.1 The probability of ruin calculation

Applying the probability of ruin to different reinsurance treaties would be a bit difficult, especially for non-proportional treaties that require more statistics (Hull, 2013). For our study, we will apply this theorem in a quota proportional treaty. This treaty is the most answered in Algeria given

the lack of experience and control of risk in personal insurance.
To be able to calculate the probability of ruin for this portfolio, it must be verified that:
- Claims are independent and identically distributed.
- The which represents the disaster attributed to each police, follow the normal law.

It is already known that the claims are independent and identically distributed, remains to verify their normality :

Normality test results

data: Charge de sinistre
W = 0.64228, p-value < 0.22

Source :prepared by Authors with CARAMMA data elaborated with R3.5

If the p-value is below the fixed alpha level (often 0.05) then the null hypothesis is rejected and it is concluded that the disaster distribution is not normally distributed.
The p-value is equal to 0.22> 0.05, which implies the normality of the amounts of claims.
As a result, we continue:

$$E(X) = \frac{1}{n} \sum X_i = 3\ 363\ 088,08$$

$$\sigma(X) = (\frac{1}{n} \sum (X_i - E(x))^2)^{1/2} = 4\ 178\ 268,38$$

For a security load λ=15%

$$\Pi(X) = (1+\lambda)\ E(X) = 3\ 867\ 551,29$$
$$T = 2.51$$

We note that our safety factor is equal to 2.51 < 4 hence the need to resort to reinsurance. In this case, the retention interval is :

$$\theta \leqslant \frac{R}{4\sigma(X) - \lambda E(X))}$$

$$\theta \leqslant \frac{10000000}{4*4\ 178\ 268,38 - 0.15*3\ 363\ 088,08}$$
$$\theta \leqslant 61.7\%$$

And so the probability of ruin is :

$$P\ (ruined) = 1 - F_0\ (\frac{R + \theta \lambda E(X)}{\theta \sigma(X)})$$
$$= 1 - 0.999968$$

P (ruined) = 0.0032%

We can confirm that reinsurance in fact reduce the probability of ruin of the insurance.

After having determined the interval of the retention which corresponds to the commitments of the insurer, one tries to find the percentage which makes it possible to optimize this retention thanks to the modeling of "De Finetti".

5.2 Optimization by the De Finetti's modeling on a portfolio

De Finetti proposed to analyze proportional reinsurance structures by minimizing the variance of the insurer's gain under the constraint that the expected gain is fixed a priori.

To carry out this optimization process, we assume the following assumptions:

- The overall claim burden over a period of one year is: $X = \sum X_i$.
- Each Xi represents the victim attributed to each police. For i = 1, ..., n, the X_i are independent and identically distributed.
- The pure premium used to cover on average the loss ratio is worth P=E(X).
- The insurer applies an equal load to the identical pure premium for each policy. This shipment is supposed to represent the profit of the insurer. We do not include in this percentage management and acquisition fees, as well as any taxes.
- The technical premium that can be used to pay the claims is therefore.
- $P = (1 + \varepsilon_i^{re})E(X)$.
- Optimal retention refers to θ.

We chose to solve this problem through the R software.The latter allowed us to find an optimal value (or vector) (maximum or minimum) for a formula that is the objective to be achieved, under constraints or limits applied to the values or vectors sought, and this in order to solve linear or nonlinear problems, but it is limited to a reduced number of constraints. In the case of our problem, the size of our sample is very large, and as the number of constraints follows the number of variables, we face a large problem. But we can always find a workable solution for the reduced problem, while respecting the constraints.

The sub-portfolio we have selected includes the largest clients, representing 20% of the initial portfolio.

Table 1: Premium and claims data sub-portfolio (DZD)

insured	Pi	Xi
1	18 874 721,62	24 427 493,71
2	625 158,00	4 141 666,66
3	1 524 649,50	6 793 190,00
4	5 968 415,15	2 586 010,77
5	2 042 545,64	27 681 666,66
6	1 321 601,45	1 800 000,00
7	38 758 259,58	941 666,66
8	54 145 614,51	2 278 125,00
9	28 293 239,63	941 666,66
10	87 852,76	600 000,00
11	124 794,00	100 000,00
12	9 426 211,76	907 257,23
13	686 164,67	981 531,44
14	691 900,00	516 000,00
15	344 824,64	2 051 648,75
∑	162 5 952,92	76747 923,54

Source :insurance campany CAARAMA data prepared by Authorson R3.5

It should be noted that this sub-portfolio is rather homogeneous, insofar as premiums collected by the company sometimes cover the amounts of claims and make a profit, other times they take losses. But the turnover is still positive. However, the loss ratio is still important, and therefore will affect, possibly, the retention of the insurer.

To determine the average profit, we used the result found by the ruin probability method, that is, calculate E (Z (61.7%)).

$$E(Z(61.7\%))= 162\ 915\ 952.92-(0.15*0.617*10\ 603\ 839.13)-10\ 603\ 839.13$$

$$E(Z(61.7\%))= 151\ 702\ 923,2\ DA$$

We can conclude that this amount represents the maximum expected profit that an insurer could expect. Given its reserves which have limited the level of retention to 61.7%, the insurer is not aiming for more than its expected profit, although it can increase its real profit if it keeps more than 61.7%, but it does not have the necessary reserves which would allow it, in the event of the occurrence of disasters, to assume its retention.

We can also calculate the minimum expected profit to make it vary between the two terminals:

$$E(Z(0\%))= 152\ 368\ 128,8\ DA$$

On R 3.5 we have varied the amount of profit expected to observe, for each amount, the level of retention that minimizes the volatility of claims using the LP function. Our results are as follows:

Table 2:Rentetion results

The expected gain	The reinsurerloading	The retention	Min σ(X)
153 217 000,00	15%	61,70%	2 643 006,26
153 150 000,00	15%	56.83%	2 242 253,10
153 050 000.00	15%	49.56%	1 705 372,60
152 950 000,00	15%	42.29%	1 241 847,81
152 890 000,00	15%	37.93%	998 944,21
152 800 000,00	15%	31.39%	648 105,44

Source :Prepared by Authors using CAARAMA's data on R3.5

For each expected return value, R application has found a level of retention considered optimal because it minimizes the loss volatility.

The more the expected profit increases, the more the retention of the insurance will be important, but this is not reassuring because the volatility of the accident increases with the expected return value. Indeed, a company wishing to improve the profitability of its portfolio must accept to bear more risks.

The volatility of this portfolio is not really significant compared to the expected profit. Here it is a portfolio that contains good risks. However, it is important to consider the level of retention that minimizes the variance in earnings.

The volatility of the claim provides a good indication of the risk of loss. Indeed, when a company decides to increase its level of retention to aim for a more attractive profit, it must expect a greater loss volatility that could hit its portfolio and fluctuate its results.

The company CAARAMA insurance has opted for a retention of 50% applied to this portfolio. Given the lack of data, we can assume two cases:
- The expected profit was 153,050,000 DA;
- Or the negotiations with its reinsurers ended up agreeing on 50% as retention.

This level of retention can be considered optimal because it reduces the volatility of the claim, but only if the expected profit is equal to 153,050,000 DA.

Determining the optimal level of retention for this portfolio depends on the policy of the insurer and its attitude to risk (which is measured by the

volatility of the claim), the insurance company must target a goal rather realistic that corresponds to its target of the year and optimize it (Deelstra, & Plantin, 2006).

The level of optimal retention will not necessarily be put into practice in the portfolio concerned, it will also depend on negotiations with the reinsurer, which in turn optimizes its own retention.

5.Conclusion

Nonlinear optimization provides a key decision-making tool in quantitative finance, and our work is a critical illustration of the importance of convex function studies in decision-making.

As for the insurer, the principle of risk sharing between him and reinsurer is based on actuarial methods that determine the commitment that corresponds to each party. For the retention to be optimal, it must correspond to the strategies predefined by the insurer according to the expected return torque, loss volatility.

The optimization process can be achieved by combining two methods that aim respectively at determining the retention interval and optimizing it for a fixed expected benefit.

- The theory of ruin, helped us to:
- Find the interval of our retention, which should be less than 61.7%;
- To note the positive effect of reinsurance on the insurer's security by minimizing its probability of ruin;
- To calculate the maximum expected profit of the insurer, which was subsequently used in the "De Finetti" method.

Lastly, the application of the "De Finetti" method allowed us to determine the exact value of the retention, which contributes to the profit desired by the insurer, and for which the volatility of the claims is minimal. But this value is not necessarily put into practice, everything depends on the negotiations with the reinsurer, and the attitude of the insurer against the risk.

We can propose as extension for our study a realistic modeling of the expected profit value.

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