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# Pre-service education students' application of visualisation strategies to solve mathematical word-problems.

A thesis submitted in fulfilment of
the requirements for the degree of
Doctor of Philosophy



by

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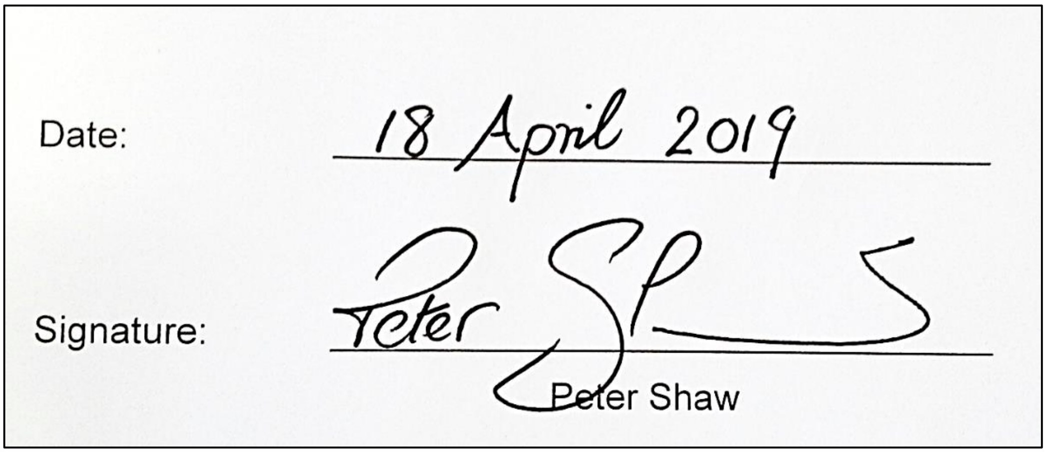
2018

DECLARATION

I, the undersigned, Peter Shaw, Student Number, 201415787, hereby declare that this dissertation is my own work and has not been submitted nor will be submitted to any other university for a similar or any other degree award.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## ACKNOWLEDGEMENTS

One does not write a thesis by ones-self. For the author, it is a focused, time-consuming and somewhat selfish undertaking; however, for those surrounding the writer, the dissertation can be very disruptive at home and at work. With this in mind, when I reflect upon the labour of this task, I am reminded of many sacrifices made by others who have assisted me to accomplish this goal.

I want to thank my family – my wife, Jean, and my sons, Alexander and Gabriel – for the countless hours of peace, support and tea that they have provided for me. As a happily married man and an incredibly proud father, I am all too keenly aware of the havoc my studies have imposed upon their lives. Without their steadfast presence in my life, I doubt that I would have completed this job. And I would particularly like to thank them for the wonderfully comfortable chair that they bought me four years ago: it has aided my writing immeasurably.

I would like to extend a special thanks to two special people in my life – Dr Roy and Prof Duku – from whom I have sought and found council. In their support of my work, they have moved heaven and earth to assist me to complete the thesis. Such encouragement and assistance goes beyond collegiality; it talks to deep friendship.



Classroom action research cannot occur in the absence of students who are willing to participate in the enterprise. I am deeply grateful to the 2016, IPS 413 E Mathematics participants for their willingness to share their mathematics stories with me. In so doing, you have helped me to become a better teacher.

Finally, I have to find a way to compliment my supervisor, Dr Melanie Drake, and co-supervisor, Dr Johannes Pylman, who are both aware of the many tribulations that accompanied this project. At times, when it was all getting too much, it was because of their resolve and resilience that any progress was made. In spite of many setbacks in this long and tempestuous expedition, never a cross word was uttered between us. I hold Dr Drake and Dr Pylman in very high regard and I thank them for their commitment to my thesis.

## ABSTRACT

This classroom-based action research dissertation examined visualisation strategies used by pre-service Intermediate Phase PGCE education students to solve mathematical word-problems. The setting was an Eastern Cape university. Previous literature indicated a positive correlation between the use of visual scaffolds and success in solving word problems. However, a gap was found insofar as little research had been published on the application of visualisation to word-problems by student teachers in South Africa. This thesis advances our understanding of the role visualisation may play in assisting student teachers to solve word-problems. The theoretic framework was informed by Bruner’s theory of learning. The research was grounded in the hermeneutic tradition. An interpretivist research paradigm was expedited by using an inductive, naturalistic perspective and relativist ontology. Thirty-eight student-teachers participated in the study. Parallel and convergent qualitative and quantitative data gathering instruments were used, thereby facilitating triangulation and examination for microgenesis. It was found that vestiges of past teaching practices initially limited the participants’ knowledge to a deeply-flawed, banking model of routines and an instrumental perception of mathematics. Disruptive calls for social justice impeded progress. Albeit visualisation strategies liberated understanding, many foundational concepts and skills had to be reconstructed. The confluence of time and rehearsal culminated in some measure of expertise. Sustained effort enabled new knowledge to be compressed and consigned to long-term memory. Salient visual representations assisted participants to conceptualise relational mathematical meta-concepts and reduced the cognitive demands imposed by word-problems but that achievement was a hard-won prize.



**Key Words: Mathematical; pre-service students; problem-solving; visualisation strategies; word-problems**

## LIST OF ABBREVIATIONS

| | |
|---|---|
| AIMSSEC | African Institute for Mathematical Sciences Schools Enrichment Centre |
| ANA | Annual National Assessments |
| B Ed | Bachelor of Education |
| BODMAS | Brackets, Of, Divide, Multiply, Add, Subtract |
| CAPS | Curriculum and Assessment Policy Statement |
| CDE | Centre for Development and Enterprise |
| CK | Content Knowledge |
| C-P-A | Concrete – Pictorial – Abstract |
| CRA | Concrete to Representational to Abstract |
| DoE | Department of Education |
| ESL | English Second Language |
| GET Band | General Education and Training Band |
| HG | Higher Grade |
| HOT | Higher Order Thinking |
| IFI | Interpreting Figural Information |
| IPS | Intermediate Phase Studies |
| LEP | Limited English Proficiency |
| LoLT | Language of Learning and Teaching |
| LOT | Lower Order Thinking |
| LMS | Learning Management System |
| MALATI | Mathematics Learning and Teaching Initiative |
| NGO | Non Govermental Organisation |
| NSC | National Senior Certificate |
| PCK | Pedagogical Content Knowledge |
| PGCE | Post Graduate Certificate in Education |
| SACMEQ | Southern and Eastern Africa Consortium for Monitoring Educational Quality |
| SRC | Students' Representative Council |
| TIMSS | Trends in International Mathematics and Science Studies |
| VP | Visual Processing |
| ZPD | Zone of Proximal Development |

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# CHAPTER 1 – INTRODUCTION

**Visualization** or **visualisation** is any technique for creating images, diagrams, or animations to communicate a message. Visualization through visual imagery has been an effective way to communicate both abstract and concrete ideas since the dawn of man.

(Rockswold, 2014)

## 1.0 PROLOGUE - MR GENDALL'S ENGLISH CLASS

When I reflect deeply about the life-journey that brought me to this place, this thesis, my earliest recollection of formally using visualisation goes back to around 1970/71. As best as I can remember, at that time I was in Standard 6 (Grade 8), and my first encounter with the use of visualisation, used as a deliberate cognising teaching and learning strategy, occurred in an English language lesson on parts of speech.

In those days English grammar formed a sizable part of language studies and was the stuff of children's nightmares for Standard 6 tests and examinations. Tenses, punctuation and parts of speech; past perfect tense; commas and colons; nouns, adverbs, conjunctions and prepositions – eight parts of speech in total. I remember wondering why on earth anyone would ever need to know these things, (naiveté is a very special gift bestowed upon the young), and I remember groaning, inwardly mostly, whenever we were asked to pull out *that* textbook.



In Standard 6, Mr Gendall became my English teacher. He was also my tennis coach. He was young, apparently very good-looking, and all the girls in my class thought that he was cool. And, like any other properly trained Christian National Education teacher would have known, Mr Gendall was aware of his curriculum obligations so there would be no opportunity to avoid the grammar lessons. But, it was *in how he taught* those lessons and particularly the parts of speech – it was in the *methodology* that he used – that he profoundly improved my understanding of that work.

At the beginning of our first term with him, Mr Gendall told us that just as all other Standard 6 children would be required to do, we were going to study parts of speech. Much groaning, not all inwards! Then he said that he was going to teach us a new way to learn this work, and he concluded with an instruction that we would need to use our colouring pencils – *our colouring pencils!* - and, “bring them tomorrow.”

Today, almost five decades later, I have to acknowledge that I have lost some of the fine detail of the methodology used by Mr Gendall, but its *visual principles* remain with me. And, as will be revealed later in the thesis, I think that is quite important. He used an approach in which, for example, a verb – a *doing* word, was conceived – visualised – as an action. Thus, said Mr Gendall, we were required to place a red-pencil, stick-figure, running person on top of any and all *doing* words. In the same way, above each noun, a *thing*, we would draw a green house. Adverbs are used to modify, to *paint* verbs, thus a red paintbrush was placed above the adverbs. Similarly, green paintbrushes were to be placed above adjectives; conjunctions were drawn in blue, as a circle with an arrow; the articles, *the, a* and *an*, had a yellow triangle and the prepositions, as I recall, received a purple hat, because one had to think about them. And so it continued, but the short sentence below should better demonstrate my memory of those experiences:


Mr Gendall was the first person to legitimise and liberate the colour-filled contents of my wooden pencil box. Before that, all I ever got to use was a Parker© pen and blotting paper. I had so much fun hunting down and inserting colourful sketches above the various words that I lost my loathing for parts of speech and quickly learned them all. In fact, as I recall, I started to describe the parts of speech as running man, or purple hat, etcetera, easily leaving the more mundane terms of *verb* and *preposition* behind. And, if my memory serves me well, I recall that my sense of elation was shared by all of my peers.

Those people who study semiotics would have a field day with Mr Gendall’s methodology. That a green pencil-sketch of a house, placed upon three abstract symbols, c – a – t, should simultaneously represent not a house, but both a noun and a cat, and not just a cat, but indeed, any other object, including houses, and that this all is acceptable and makes sense to us, is a wondrous thing. It goes to the unique cognitive capacity humans have to mentally leap from one context to another, to form – instantly – mental pathways that link and codify stimuli and ideas, construct schemas and reveal new understandings to ourselves. In this parts of speech example, the colourful icons act as visual scaffolds, as conduits to the concepts of prepositions, adjectives, etcetera.

To this day, I continue to have more than a passing understanding of verbs and nouns, but the path was made much easier for me by using Mr Gendall's visualising strategy. It has to be said that we never used vocabulary that contained words like *visualisation* or *methodology* in our classes. Nor did I, from Standard 6 through to matric, always draw colourful images on top of words. There was no need for this. As layers of understanding melded the iconic and symbolic representations of the parts of speech, I was able to consign that knowledge and those relationships to long-term memory. Subsequent to the assimilation and accommodation of the parts of speech visualising meta-concept, that is, its *compression* (Gray & Tall, 2007), I would only call up the visualising strategy to my working memory sporadically, so as to test a problematic word against my mental construct of those iconic images. Further, in as much as Mr Gendall's methodology enabled my grammar marks to lift quantitatively from say, $\frac{5}{10}$ to $\frac{9}{10}$ so too he qualitatively liberated a sense of being *able to do it* within me.



## 1.1 INTRODUCTION

I have worked as a mathematics teacher in higher education for thirty-two years. I have worked with thousands of students and, among other programmes of instruction, many cohorts of Post Graduate Certificate in Education (PGCE) students. In the PGCE programme, since 2012, I have been teaching the Intermediate Phase Studies, IPS 413 E Mathematics module and it is in this module that I conducted the action research. PGCE students are often in their late twenties or early thirties and, on occasion, older than that. For some of these students, any recollections they may hold of mathematics might be vague and peppered with misremembered facts and details, ill-formed procedures and flawed conceptual understanding. Although a pass in IPS 413 E module is a prerequisite for the awarding of the PGCE qualification, experience has taught me that some students may be hesitant, even reticent in coming into the mathematics class.

### 1.1.1 TEACHING AND LEARNING MATHEMATICS IN THE 21$^{ST}$ CENTURY

Albeit that PGCE students might be anxious about having to study mathematics, “the single most characteristic thing about human beings is that they learn. Learning is so deeply ingrained in man [sic] that it is almost involuntary, and thoughtful students of human behaviour have even speculated that our specialization as a species is a specialization for learning” (Bruner, 1966, p. 113). A modern view of mathematics is that it is a socio-cultural practice that can be undertaken by all members of a society (Siemon, Beswick, Brady, Clark, Faragher & Warren, 2012, p. 18). The capacities to think mathematically and solve mathematical problems are highly sought-after attributes; in the 21$^{st}$ century, a recognised qualification in mathematics has become a societal pre-requisite for many professions (Eastaway & Askew, 2010, p. 21).

Whether it be used for a simple financial transaction, to buy a loaf of bread, or for the purchase of a ticket to a far-away destination; whether it is used to read a stock-market share-price report in a newspaper, or it is used to establish a basic understanding of temperature, 32°C, and thereby informs the way one should dress for that day, increasingly, and on many vectors, all humans apply mathematical skills and knowledge in their daily lives. From this, it follows that there is a societal expectation for mathematics teachers, and by extension PGCE student teachers, to create learning environments and activities that facilitate deep learning of the subject.



Contemporary conceptions of our acquisition of mathematical knowledge suggests that the liberation of deep understanding occurs when teachers assist their pupils to actively construct and reconstruct their knowledge of mathematical concepts and skills (Bruner, 1966, p. 123 – 127; Hyde & Bizar, 1989, p. 89). Iterative reconstructions of mathematical knowledge and skills-sets should lead progressively towards a fuller understanding of mathematics. Thus, for PGCE student teachers to be able to thrive in the increasingly complex technological world, they themselves must hold requisite mathematical content, and this must be coupled with pedagogical and methodological knowledge. Only then, (Anghileri, 2007, p. 13) will they be able to assist their own pupils to successfully negotiate the numerous language, symbolic and operational complexities that are encountered when one *does* mathematics.

### 1.1.2 DOING MATHEMATICS AND PROBLEM-SOLVING

*Doing* mathematics is much more than simply working with numbers and writing lists of formulae, etcetera. If we return to the exemplar about temperature that was presented earlier, we find that the digits 3 and 2, abstract symbols themselves, linked to the symbol °, linked to a problem-question of what to wear on a particular day, in fact requires high levels of meta-cognitive processing. On one level, we need to understand that in this situation the abstract symbols 32° act as a "mediator" (Bruner, 1966, p. 18) which invokes a consideration that we are concerned with temperature. In a different context, 32° might imply an acute angle, or a bearing to be plotted on a map. Further, 32° on a Celsius scale implies a different temperature to the Fahrenheit or Kelvin scales and, if we successfully negotiate those hurdles, then we have to appreciate that, in different settings, 32°C evokes dissimilar human responses. If, while relaxing in a bistro, you are served a cup of 32°C coffee, it is highly probable that you will be very annoyed at receiving a *cold* cup of coffee. However, in the context of atmospheric temperatures, many humans would consider 32°C a *hot* day: armed with that conceptual understanding, most of us would dress in a way that enables us to remain as cool as possible. So, *doing maths* summons together many intersecting concepts, the cognitive synthesis of which enables us to construct cogent solutions to problems.



Thus, it comes as no surprise to find that our mathematics curriculum, the South African Curriculum and Assessment Policy Statement (CAPS), Intermediate Phase, Grades 4 – 6, Mathematics (Department of Basic Education, 2011), requires students to acquire concepts, routines and problem-solving skills that are integrated into subject content areas including numbers, patterns, geometry, measuring and working with data. Collectively, these content areas contribute to the construction of a wide base of mathematical skills and knowledge that in turn feeds into an even more important characteristic of mathematical endeavour, *vis-à-vis*, an ability to cogently combine relevant aspects of those knowledge structures to solve mathematical problems. Indeed, Siew Yin (2010) suggests that problem-solving lies at the heart of all mathematical endeavour; it is the principle reason for studying mathematics. Further, the inherent problem-solving capacities gifted to us through our "powers of ratiocination" (Bruner, 1966, p. 25), that is, our ability for reason and to form a train of thought, provides us with mathematical thinking capabilities which enable us to productively engage with the subject.

### 1.1.3 SOLVING WORD-PROBLEMS

As a sub-set of problem-solving, solving *word-problems* is a cognitively demanding aspect of schools-based problem-solving. Such problems require students to negotiate complex linguistic, semantic, schematic, strategic and procedural knowledge bases (Mayer, 1997, p. 458). However, “many learners dislike and even fear word problems” (Murray, 2012, p. 55). When students cannot cogently apply appropriate knowledge and skills bases to word-problems, they run the debilitating risks of becoming disenfranchised within the mathematics classroom (Breen, 2001, p. 42), of experiencing mathematical anxiety, and of limiting their future career paths. For many at-risk students (Belbase, 2013, p. 232), the mathematics they encounter happens to them and around them, but does not become a part of who they are and remains largely outside of their grasp.

The reasons for ill-success in mathematical problem-solving are many-fold (Gooding, 2009, p. 5). Thus, it follows that no single remedial strategy surmounts every hurdle; there is no silver bullet. However, visualising strategies that are rich in the deliberate application of enactive and iconic activities in the classroom (Naidoo, 2012, p. 1) appear to offer a methodology which seems to hold many benefits. Visual tools and techniques can assist students to construct their own cognitive frameworks of mathematical ideas; in turn, these frameworks may facilitate a better understanding of mathematical word-problems.



## 1.2 BACKGROUND TO THE STUDY

My PhD journey began seven or eight years ago and, over the course of time, it has become more focused. In June 2010, on a sunshine-filled, but icy-cold winter’s day, I had a chance meeting with a long-standing friend and academic colleague in Grahamstown. My vocalisation of my intention to do a PhD in mathematics education prompted my critical-friend to suggest that the last thing that was needed in South Africa would be yet another case study on factors that contribute to misunderstanding in mathematics. No, he said, the troubles are legion, and well-documented: rather, what is needed, are solutions. And that, singularly – *finding solutions* – became the catalyst of my own adventure.

It occurred to me that in the 21st century greater and greater emphasis is being placed upon teaching *and learning* mathematics in novel ways and in extending studies in mathematics beyond practice, rote and rehearsal-based exercises towards the integration of various sub-sets of mathematics for solving multi-step, non-routine, mathematical problems. Indeed, Henderson (2012, p. 46) suggests that modern mathematics curricula endeavour to replace more traditional operations, routines and procedural approaches with a more inquiry-based view of learning and *thinking* about mathematics, which is to say *working mathematically* to solve problems. As in other countries, this change of emphasis appears to be occurring in South Africa too. Because solving word-problems is a subset of problem-solving, it makes sense to briefly examine different conceptions of problem-solving as found in different parts of the world. I turn to this next.

### 1.2.1 PROBLEM-SOLVING IN MATHEMATICS CURRICULA



Hodgen, Marks and Pepper (2013, p. 31) make the point that, “fundamental differences in national education values and contexts mean that learning lessons from international comparisons is not straightforward, and we should be careful about ‘cherry-picking’ policies and approaches from overseas.” In any country, the curriculum is tightly located within the social norms, aspirations and cultural practices of that society. However, it is prudent to be aware of conceptions of problem-solving that are found in other countries.

Outside of African, in Singapore, problem-solving lies at the heart of mathematical teaching and learning (Ministry of Education, Singapore, 2012). In large measur,e visualisation strategies are used to make sense of problems (Naroth & Luneta, 2015). In Australia, problem solving requires learners to model ideas, learn to pose and solve problems, and perceive relationships between mathematics and other learning areas (Wernert & Thomson, 2016; Siemon, Beswick, Brady, Clark, Faragher & Warren, 2012). In England, problem solving is guided by a conception of “*working mathematically… develop fluency, reason mathematically* [and] *solve problems”* (Department of Education, 2013). Similar stem-phrases drive problem-solving in the

United States of America (Malley, Neidorf, Arora & Kroeger, 2016; National Governors Association Center for Best Practices, Council of Chief State School Officers, 2017b).

On the African continent, Namibian learners apply mathematics to their daily lives and are required to, “solve relevant problems in theoretical situations or in applications to everyday life” (Ministry of Education, 2010, p. 2). In Botswana, problem-solving is one of five mathematical content areas (Masole, Gabalebatse, Guga & Pharithi, 2016), and primary school learners are expected to use the four operations able to solve problems (Ministry of Education & Skills Development, 2002). Zimbabwe assigns 10% of assessment marks to problem-solving (Ministry of Education, 2006), but teachers confine problem solving to explicitly stated, drill-type calculations (Chauraya & Mhlolo, 2008, p. 75). In Zambia, problem solving is used to improve, “learners’ intellectual competence in logical reasoning, spatial visualization, analysis and abstract thought” (Ministry of Education, Science, Vocational Training and Early Education, 2013).

In South Africa, in the *Curriculum and Assessment Policy Statement* (CAPS) document, non-routine, problem-solving has become a facet of our conception of mathematics and is perceived “to develop mental processes that enhance logical and critical thinking, accuracy and problem-solving that will contribute in decision-making” (Department of Basic Education, 2011, p. 4). These opportunities are found in phrases such as, “solve or complete number sentences by inspection or trial and error; visualize, name, describe, sort and compare; solve problems related to time, distance, speed, length, perimeter, area, and volume; pose simple questions about the environment, school, and family” (Reddy, Arends, Juan, & Prinsloo, 2016, p. 4 – 6).



These exemplars demonstrate that across the world, societies are modernising their conception of mathematics, are moving away from rehearsal, rote and routines, and embracing problem-solving as a pivot for teaching and learning mathematics. The emerging trend is that teachers are required to integrate, “conceptual understanding and non-routine problem-solving” (Dalgarno & Colgan, 2007, p. 1054) into all aspects of mathematical. Teachers need to modify and/or reinvent their own mathematical skills-sets, content knowledge, pedagogical knowledge and curriculum knowledge to meet the requirements of new curricula. van der Sandt, (2007), suggests that steps must be taken to ensure a proper fit exists between teachers’ aspirations and the requirements of a curriculum and the society that it serves. South Africa, through its CAPS iteration of curriculum aspirations, is moving quite decisively towards an understanding based, activities-filled and problem-solving approach towards mathematics. This means that for PGCE students' mastery of mathematical content knowledge is not enough; that knowledge must be supported by subject content knowledge, pedagogy and methodological capacity.

### 1.2.2 CHARACTERISTICS OF WORD-PROBLEMS

Sepeng and Webb (2012, p. 1-2) describe word-problems as mathematical problems which are presented within a contextualizing, descriptive text. Mathematical word-problems should provide learners with opportunities to apply their mathematical knowledge to solve realistic mathematical problems. Such problems require learners to "comprehend the contextual situation described in the problem, keep track of the incoming information, embed the numerical values in the relational storyline structure and finally perform arithmetic calculations" (Novak & Tassell, 2017, p. 21). Indeed, the examples, exercises and mathematical problems that learners encounter in classrooms must be seen to be transferable to the wider world: such work must be seen to must serve a purpose other than a compliance, a test or an examination.



#### 1.2.2.1 INSTRUMENTAL AND RELATIONAL UNDERSTANDING

On a lower plane of mathematical capacity, instrumental understanding, "usually involves a multiplicity of rules rather than fewer principles of more general application" (Skemp, 2006, p. 90). As such, it is a way of knowing things by doing them (Bruner, 1966, p. 68). Relational understanding operates on a higher plane: in mathematics word-problems it is exemplified by an ability to use many different but intersecting complementary cognitive competences and skills-sets to solve the problem at hand.

Word-problems can elevate the study of mathematics from the level of knowing rules but not why or how they work, of knowing routines, procedures and holding only instrumental understanding into the realm of relational understanding (Siemon et al. 2012, p. 18; Reason, 2003, p. 6). That is, word-problems can improve mathematics competences to include, "knowing what to do and why" (Skemp, 1976, p. 25). Relational understanding prospers when students construct robust conceptual frameworks of underpinning mathematical principles and processes. With these in place, they can then attend to many different, but related mathematical problems (Skemp, 1976, p. 25).

However, Skemp (1976) cautions that exposure to the requisite cognitive skills-sets is not enough; sustained practice is needed - *practice makes perfect.* Further, a cognitive shift from instrumental to relational understanding is complicated by, “a tendency, among even bright undergraduates, to adopt an instrumental approach” (Anderson, 1996, p. 819). An adverse effect of the so-called tick-in-a-box compliance mentality and copious paper-trails that this record-keeping generates, is that it detracts time and energy away from the modern view of teaching and learning mathematics *for understanding*. Recording every discrete skill or knowledge byte reduces the measurement of understanding to an instrumental view of knowing (Pesek and Kirshner, 2000, p. 524 - 525). Such practices contradict educational theory which teaches us that the depth of understanding can be better evaluated in terms of the type and strength of concepts that are held by learners; it is these deep conceptions which foster metacognition and liberate relational understanding.



While relational understanding offers many cognitive benefits, “it is much more difficult to learn the fewer principles, and much easier to learn rules” (Reason, 2003, p. 6). However, the alternative to relational understanding, that is, learning by rigorously rehearsing and memorising routines and fixed procedural techniques, can predispose learners to resist venturing outside of their known instrumental skills and knowledge sets. Further, any fixation with learning routines, of being rule-bound, of being inflexible and attempting to consign all mathematical skills and knowledge to memory ultimately leads to a lack of enjoyment of the subject, and failure (Reason, 2003, p. 6). Indeed, Pesek and Kirshner, (2000, p. 525 – 527) caution that meticulously constructed layers of instrumental learning can interfere later in learning when pupils are required to apply relational understanding to problem-solving.

#### 1.2.2.2 CONSTRUCTION OF META-CONCEPTS

Word-problems draw upon many operational and procedural aspects of mathematics and require students to apply both deductive and inductive reasoning (Plotz, Froneman & Nieuwoudt, 2012, p. 75). It stands to reason, then, that an ability to navigate through the higher-order cognitive processes that are integral to solving word-problems also indicates an ability to work skillfully and *with understanding* with lower-order, procedural skills-sets such as multiplying and dividing, working with ratios, manipulating formulae and fractions, and so on. Gray and Tall (2007) and Chin (2013, p. 13-18) use the term *compression* to describe this mental agility. Stated differently, compression implies that when lower mathematics skills-sets become consolidated and automatic, cognitive space opens up for creative, higher order thinking to take place. A confluence of practice, integration, synthesis, meaning-making and time optimises compression. Deep understanding – compression – is a hard-won prize. It takes effort, attention to detail, agility of mind and much practice in both routine and non-routine problem-solving in order for one to consolidate and automate the numerous routines and skills-sets upon which compression and, by extension, problem-solving endeavours rest.

Unfortunately, many students find it difficult to construct the meta-concepts – the relational understanding – that assists us to solve different word-problems. This is because struggling students tend to rely on instrumental understanding and do not link new experiences easily to their existing but fragile knowledge frameworks. Thus, they do not forge mental links and do not compress interrelated aspects of mathematics. In other words, suggests Chin (2013, p. 29), many struggling students do not piece together the connections – the conceptual frameworks – that enable them to use supportive, previously encountered mathematical knowledge and experiences to assist them to solve new problems in new contexts.



#### 1.2.2.3 PROBLEMS ENCOUNTERED WHEN SOLVING MATHEMATICAL WORD PROBLEMS

Gooding, (2009, p. 5) suggests that word-problems confront students with five types of difficulties; all or some of these difficulties may affect different students by varying degrees. These difficulties include an inability to (1) read and understand word-problems, (2) form mental images of the contexts in which word-problems are set, (3) successfully construct number sentences or algorithms, (4) carry out mathematical calculations and, (5) judge the validity of the answers that they have calculated themselves. These difficulties are discussed below. Because language is a principal tool for teaching and learning mathematics, it plays a significant role in providing or restricting students' access to solving word-problems (Mahofa & Adendorff, 2014, p.

86). Even the most elementary word-problems require “translation” (Tobias, 2006, p. 2) into an appropriate mathematical code. When students read a word problem, they do not try to memorise the words; instead they try to use the cues in the sentence composition and words to construct meaning (Lieberman, 2004, p. 39). Where, for example, the language of instruction and word-problems are not presented in the mother tongue of the learners or, more precisely, where students cannot attach meaning to the words, syntax and phrasing that are found in word-problems, such learners are, “at a great disadvantage” (Murray, 2012, p. 55).

A modern view of *context*, apropos word-problems in mathematics, situates these problems in real-life extra-mathematical situations which are understood by the readers of such problems (Holtman, Julie, Mbekwa, Mtetwa & Ngcobo, 2011, p. 121). Often this appeal to use real-life contexts is injudiciously used by educators, such that said contexts may lie outside of the real-life experiences of the learner (Murray, 2012, p. 57). In such cases, the ability to understand and visualise the word problem scenario is obscured. When this happens, the context of a word problem is lost to the learner, who is thus precluded from *seeing* and fathoming its requirements. Indeed, it is conceivable that because of this hurdle, students, “are possibly being marginalised by the inclusion of word problems in the curriculum” (Tobias, 2006, p. 12).



However, even if pupils have negotiated the hurdles of language and context and can visualise a conception of the problem they are trying to solve, another barrier awaits them, that of, “problem execution” (Montague, 2012, p. 1). Problem execution, or the ability to *set-out* a mathematical strategy, is often multi-stepped and frequently requires the construction of a suitable algorithm. If children are mainly exposed to instrumental modes of learning (Skemp, 2016, p. 92), their lack of experience in relational understanding will impede success in establishing appropriate starting points to solve problems. In section 1.2.2.1, we have acknowledged that instrumental learning techniques are quite easy to apply; they rely mainly on rote learning and regurgitation, of knowing “rules without reasons” (Skemp, 2016, p. 92), and can achieve a single right answer quite easily. However, when such trained students are exposed to non-routine word-problems, they find problem execution, the construction of algorithms and the setting-out of solutions elusive.

An additional hurdle in solving word-problems lies in the fact that many students cannot easily and consistently *compute* foundational operations. In subtraction, for example, a group of practicing American teachers was found to hold misconceptions regarding borrowing and believed that, "you cannot subtract a larger number from a smaller one" (Ma, 2010, p. 3). This myth, held as it was by the teachers, cascaded into their teaching practices which embraced memorisation and rote recall of low-order, instrumental routines. Similar myths or misunderstandings of computations are found in the other operations, too. Because, "symbolic recording is very dense" (Siemon et al., 2012, p. 382), where teachers and, by implication, their students, do not deeply understand the applications of arithmetic operations, things can end badly.

A final stumbling block, as alluded to by Gooding (2009, p. 5), is that often students cannot perceive the *validity* of their answers. They lack the capacity to estimate what an acceptable answer might be. This mal-condition extends from the numbers found in an answer into the units which are attached to the said answers. Estimation is a process by which, by looking at the generalities of the numbers and operations found within a mathematical calculation, we are able to form an appropriate, if inexact, value-judgement on the size and shape of a result. However, "estimation is a complex skill that requires the integration of a number of ideas and strategies" (Siemon et al., 2012, p. 388). Estimation requires the application of relational-understanding teaching and learning methods rather than rote and rehearsal-based, instrumental-instruction.



From the above, it can be seen that word-problems impose heavy cognitive demands upon students. Word-problems require students to be able to critically call upon a repertoire of appropriate mathematical skills and tools; each problem will require its own particular combination of these skills and tools. To be successful, students need to be accomplished in the application of relational mathematics techniques (Skemp, 2016, p. 92). Relational techniques bring with them the benefits of being adaptable in new contexts; they are easier to remember (albeit harder to learn); they provide the reward of intrinsic motivation, and they assist learners to develop comprehensive schema – cognitive networks of ideas which are organic and flexible in nature (Skemp, 2016, p. 92 – 93).

#### 1.2.2.4 THE ROLE OF VISUALISATION IN PROBLEM-SOLVING

Visualisation plays an important role in assisting learners to contextualise word-problems. Manipulatives and pictorial stimuli – which Bruner (1966, p. 11) calls the enactive and iconic representations of knowing – act, "as the basic "building blocks" of mathematics" (Novak & Tassell, 2017, p. 21). Where the use of these stimuli are absent in helping children to contextualise word-problems, said children are impeded from forming mental images of mathematical ideas. In these situations, such children must rely on rote, recall and routines which, while cognitively demanding, do not provide much opportunity for mathematical development. Conversely, when teachers use visualising strategies as a methodological technique cognition is greatly enhanced (Siemon et al., 2012, p. 337 – 345).

In 1966, Bruner, reflecting upon the educational promise held by visualisation scaffolds, wrote, "I do not think that we have begun to scratch the surface of training in visualization – whether related to the arts, to science, or simply to the pleasures of viewing our environment more richly" (Bruner, 1966, p. 34). Five decades later, a modern-day view is that "visual literacy is crucially important in a contemporary society dominated by visual media" (Jordaan & Jordaan, 2013, p. 76). Good mathematics teachers make use of tactiles, visual representations, gestures and motions – embodied cognition – as visualising scaffolds that are used to assist their students to better understand mathematics problems (Boaler, Chen, Williams & Cordero, 2016).



Previously, visualising scaffolds were seen as offering at best, a series of transitory steps towards doing *real* mathematics. However, our contemporary understanding of how humans learn has overturned that perspective (Makina & Wessels, 2009, p. 58). Current research advises us that, "visual mathematics is an important part of mathematics for its own sake and new brain-research tells us that visual mathematics even helps students learn numerical mathematics… [in fact]… when we don't ask students to think visually, we miss incredible opportunities to increase students' understanding and to enable important brain crossing" (Boaler et al., 2016). Visual demonstrations of mathematical ideas provide a sound footing for conceptual development. However, real cognitive growth only occurs when students themselves are encouraged to use manipulatives or draw representational models of their thinking on paper or form mental images in their minds to assist in problem-solving. Indeed:

> a student's ability to develop and interpret various representations increases the ability to do and understand mathematics. When students gain access to mathematical representations and the ideas they represent, they have a set of tools that significantly expands their capacity to think mathematically. Representations extend a person's understanding of a concept, and shed light on an idea not fully understood in another form.
>
> Plotz, Froneman & Nieuwoudt, 2012, p. 76

In Plotz et al. (2012), the use of the term *representation* is used literally, and is used to imply the use of tactile or visual scaffolds whereas representation, for Bruner (1966), implies three ways – three conceptions – of knowing which are effected via enactive, iconic and symbolic techniques.

The use of visual tools invites the consideration of a constructivist epistemology: when students work with tactiles or develop visual ideas and solutions that are related to their mathematical word-problems, "they actively develop their own understanding of the world, rather than having such understanding delivered to them" (Mnguni, 2014, p. 2). These visualising *aides-mémoire* enable students to "tease out" (Fong & Lee, 2004, p. 108) strategies that assist them to find solutions. Even when such strategies fail in the first instance, visualising models often provide some embryonic clarification so that students can then begin to *problem-solve their problem*. In other words, students begin to use their visual models reflexively as part of a metacognitive-loop thought process.

Naidoo (2012) suggests that visual tools include concrete apparatus such as bottle-tops and pebbles and iconic stimuli such as diagrams, pictures and transparencies, and judicious use of colour. The models that the students make provide visual-mental hooks that assist deep thinking (Anghileri, 2007, p. 15). And, suggest Siemon et al. (2012), these visual hooks free up cognitive space that enables students to focus more cogently on the construction of viable solutions for word-problems.

Visualisation can play many scaffolding roles in assisting teachers and students to solve mathematics problems. Siew Yin (2010) suggests that these benefits include a better understanding of the problem; simplifying the problem; seeing connections to related problems; catering to individual learning styles; providing a substitute for computation; transforming the problem into a mathematical form; and a tool to check the solution. However, for students to benefit from the use of such scaffolding potentials, their teachers must hold sound mathematical subject knowledge and deep mathematical pedagogical content knowledge. Thus, when the benefits of compression (Gray & Tall, 2007; Chin, 2013), discussed earlier (1.2.2.2), are coupled with visualising techniques, it would seem that relational teaching and learning methods can free-up cognitive space which can then be committed to the focused task of solving word-problems.

However, it is prudent to end this section on a cautionary note. While visualisation offers – *potentially* – many opportunities to represent problems in interesting ways and thus – *potentially* – may improve mathematics understanding, "the generally accepted dictum that a picture is worth a thousand words masks the fact that visual images do not constitute a universally understandable means of communication" (Jordaan & Jordaan, 2013, p. 89). One has to acknowledge that life experience and enculturation within any society predisposes individuals to interpret tactile and visual and indeed symbolic representations of problems in different ways.



## 1.2.3 SUBJECT CONTENT KNOWLEDGE

Bloch (2009, p. 58) contends that, because South African schooling is a national disaster, our country fares badly in internationally-mediated mathematics tests. A principle reason for this disaster, suggests Bloch (2009, p. 83), is that most South African mathematics teachers hold low subject content knowledge of their subject. Thus, in effect, it is our teachers who disenfranchise their pupils. They do this by two means: many South African mathematics teachers do not understand the mathematics content they teach and, further, they use transmission modes of teaching and learning, which in turn encourages instrumental learning. However, as discussed earlier (1.2.2.1), instrumental learning has already been shown to severely limit deep mathematical conceptualisation (Skemp, 1976).

Ball (2015), , prospective teachers need to be pedagogically equipped to better do their work, to better teach mathematics: student-teachers need to be taught both *mathematical content knowledge* and *mathematical knowledge for teaching* or "pedagogical content knowledge" (Shulman, 1986, p. 13). If we superimpose Ball's

(2015) view upon my own work, my experiential learning suggests that, generally, PGCE student teachers have poor mathematical content knowledge and, further, because they typically enter the PGCE programme from undergraduate degrees which did not focus on methodological and pedagogical discussions, so here, too, their mathematical knowledge for teaching is of a very low standing.

This background (1.1 through 1.2.3) makes the point that an inability to solve mathematical word-problems negatively influences classroom performance and compromises future career-pathing. Many inter-woven skills, procedures and mathematical concepts need to be compressed before students become comfortable with this work. It is my experience that many of my PGCE student teachers have followed mathematical paths devoid of enactive and iconic representations of ways of knowing this subject. Further, instrumentalism, hallmarks of which include the use of one routine and rote-learned – or mis-learned – mathematical facts and ideas seems, *de facto,* to embrace their full experience of mathematics.

In the conception of mathematics word-problems, there exists a researchable gap. PGCE student teachers need to be given opportunities to progress beyond instrumental learning practices towards relational understanding. They need to be encouraged to compress smaller, instrumentally learned skills into larger, relationally-constructed meta-concepts; visualisation techniques offer much promise in assisting students to do this. I want to understand the experiences of PGCE student teachers who encounter a mathematics course in which relational understanding and visualisation informs the conception, design and delivery of the module.

## 1.3 STATEMENT OF THE RESEARCH PROBLEM

Problem-solving skills and an ability to solve word-problems are highly desirable in schooling and the workplace (Siemon et al., 2012; Eastaway & Askew, 2010). As a consequence of curriculum renewal, problem-solving now plays an expanded role in teaching and learning mathematics within the CAPS curriculum (Department of Basic Education, 2011). Thus, prospective teachers must be trained to meet the challenges of our evolving curriculum (van der Sandt, 2007). However, albeit that the cognitive demand requirements for Intermediate Phase mathematics word-problems are relatively low and can be solved by using arithmetic computations, many student teachers struggle to solve these problems. Partly, this is because, in South Africa, our traditional teaching methods have encouraged, “learner dependency and superficial understanding” (Malan, Ndlovu & Engelbrecht, 2014, p. 1). This inability to solve mathematical problems is not uniquely attributable to PGCE student teachers; world-wide, many experienced teachers may be fearful of and struggle with this work (Dalgarno & Colgan, 2007, p. 1051; Ma, 2010, p. xxiiv).

Improvements in teaching and learning mathematics are, “a function of appropriate levels of mathematics *and* pedagogic training” (Adler, 2017, p. 2). Student teachers may lack this necessary subject content knowledge and pedagogical content knowledge (Breen, 2001; Gooding, 2009). In cases where such students struggle to solve mathematical problems, underpinning mathematical skills sets may not have become compressed (Chin 2013). Visualisation should enable PGCE student teachers to conceive mathematical concepts in the abstract and relationally, should provide them with an innovative teaching methodology, and should assist them to develop pedagogical content knowledge, (Bruner, 1966; Skemp, 1976; Naidoo, 2012; Anghileri, 2007; Siemon et al., 2012).



## 1.4 MAIN RESEARCH QUESTION

*How will visualisation strategies assist student-teachers to better understand and solve mathematical word-problems?*

## 1.5 SUB-RESEARCH QUESTIONS

1. *What existing word problem, problem-solving strategies do the students hold?*
2. *What barriers to solving word problems do the students perceive that they hold?*
3. *How effective do the students perceive the visualisation strategies to be?*

## 1.6 OBJECTIVES OF THIS STUDY

- The first research objective is to examine the existing problem-solving strategies held by the pre-service students.

- A second objective is to try to provide redress for mathematical barriers that the students themselves may perceive that they hold.
- A third objective is to establish the students' views on the effectiveness of visualisation, as a teaching and learning strategy, for mathematical word-problem, problem-solving.

## 1.7 PURPOSE OF THIS STUDY

The purpose of this research study is to investigate the application of visualising strategies to solve word-problems by pre-service education students. Hopefully, the study will provide evidence to myself, the pre-service PGCE students and other interested persons that visualisation strategies can provide pre-service education students with a liberating philosophical and ontological learning experience, can provide novel and productive ways to relearn mathematics, can broaden pedagogical capacities, and can improve their capacity to solve word-problems.

## 1.8 SIGNIFICANCE OF THIS STUDY



Most of the PGCE student teachers who graduate will take up Intermediate Phase teaching posts; in these posts, typically, they will be required to teach mathematics. It has been argued that any improvement in school-based mathematics will rest heavily on the preparation of better-trained mathematics teachers who graduate out of South African teacher-education programmes (Adler, Pournara, Taylor, Thorne & Moletanse, 2009); further, they suggest that, "an *informed* research agenda that produces grounded but sound empirical and theoretical bases [should inform] advocacy in curriculum and teacher education." (Adler et al., 2009, p. 29). Such a research agenda, they suggest, needs to critically examine the relationships between content knowledge, curriculum and pedagogy, with research goals that include finding:

> - Productive ways for teachers to learn and/or re-learn subject content to prepare them for teaching;
> - Ways of offering and inducting teachers into new experiences of teaching to broaden their pedagogical imagination.
>
> Adler et al., 2009, p. 29

Internationally, a deep understanding of undergraduates' experiences of teaching and learning mathematics is a somewhat new research area (Brown & Murphy, 2000). Indeed, in South Africa, while universities have conducted some research into in-service mathematics-education, "the most obvious under-researched area is initial teacher preparation" (Adler et al., 2009, p. 36). They point out that there are, "notable absences in research on primary mathematics… teacher education" (Adler et al., 2009, p. 36). The academic significance of this thesis is that it might demonstrate that visualising scaffolds enable students to assimilate and accommodate salient instrumental and relational mathematical skills and concepts. If this occurs, that is, if visualisation facilitates compression, this will provide some evidence that visualisation, as a novel and productive methodology, can be used as a teaching and learning methodology to solve word-problems. Further, research coming out of this thesis might benefit other pre-service, primary-school-mathematics, teacher-education, university programmes in South Africa.

## 1.9 RATIONALE FOR THIS STUDY



In South Africa, teacher education programmes have moved from teacher training colleges into university settings. Spaces have opened up for professional qualifications that include innovative teacher-education programmes (Adler et al., 2009, p. 37). But this opportunity brings with it, its own tensions: in higher education settings, academics have limited contact time with students and must strike a balance in terms of the breadth and depth of the content knowledge that they cover in their lectures (Adler et al., 2009, p. 38). The *central task* (Gierdien, 2012, p. 2) of teacher preparation courses is mathematical knowledge for teaching. This requires a balance of content-knowledge with pedagogical and methodological knowledge and must assist student-teachers to use innovative methodologies that they may have never previously experienced.

Parker (2009, p. 19) suggests that mature graduates who are considering an alternative career-path should be flagged as potential PGCE student teachers. Such students, she suggests, would bring experience and maturity into the education sector, and could partially offset the annual short-production of new teachers who are entering the profession in South Africa. Parker (2009) concedes that these graduates require support in order to fulfil the goals of the *National Policy Framework for Teacher*

*Education and Development in South Africa* (Department of Education, 2006), which, under the banner "*more teachers, better teachers*" (Department of Education, 2006, p.1), has called for teachers who are better qualified in terms of their content knowledge and understanding of educational pedagogy.

Albeit they are already graduated persons, PGCE students are formally inducted into the world of education in a limited, intensive, single-year university-based qualification. In that short time-span, they have to transform themselves. This can prove to be a daunting task and, suggests Adler (2017), this short time-span may not enable students to be properly trained to teach mathematics. In this short time-span, PGCE student teachers need to develop an understanding of the synergies which play out between mathematical content knowledge, pedagogy and curriculum (van der Sandt, 2007). Indeed, Shulman (1986) suggests that neither content knowledge nor pedagogical knowledge alone can suffice for teaching and learning mathematics. It is only in their blending that opportunities for true knowledge growth are found.

Rather than trying to rediscover the underpinning features of our nation's poor performance in mathematics, *I am trying to seek a solution,* one of many that will be needed to assist in the turn-around of our "national disaster" (Bloch, 2009, p. 58). My research questions invoke the use of both quantitative and qualitative research orientations and instruments: by combining such instruments, I hope to understand a complex situation better. In as much I hope that visualisation enables their *mathematics scores to improve* so, too, I hope that the participants in my research also experience a *positive attitudinal shift* towards mathematics. I hope that the evidence which is revealed through the action research (Chapter 4) demonstrates that visualizing strategies provide productive experiences of teaching and learning mathematics and word-problems in a novel way and liberates deep insights and understandings of aspects of pedagogical content knowledge (Adler et al., 2009).



## 1.10 DELIMITATION OF THIS STUDY

This action research study is delimitated to 38 participants who studied IPS 413 E Intermediate Phase Studies – Mathematics in Semester 1 of 2016 in an Eastern Cape university in the Republic of South Africa. The mathematical content is delimitated to Grade 4 to Grade 7, numbers-based, word-problems and the invoked arithmetic skills. The methodologies and pedagogical content knowledge are delimitated to theories and skills-sets that influence teaching and learning when using visualization to solve Intermediate Phase mathematical word-problems.

## 1.11 DEFINITION OF OPERATIONALISED TERMS

*Problem-solving* – as a global or meta-concept talks to the predisposition that humans have, as specialists in learning (Bruner, 1966, p. 113), in attending to their continually developing understanding of their world. Human beings are driven by a deep curiosity of wanting to know the world and as, “a response to uncertainty and ambiguity” (Bruner, 1966, p. 43), where we perceive a hurdle or an alternative, we invoke the psychological problem-solving gifts bestowed upon us (Bruner, 1966, p. 24 – 25) to seek solutions to our human needs and wants. While, in the context of this thesis, problem-solving is confined to issues mathematical, in a wider sense, problem-solving embraces all of humankind’s attempts to improve, find alternatives, and seek answers.

In the context of this study, as an operationalised term, problem-solving moves beyond routines-based mathematical *exercises* (Mukwambo, Ngcoza, & Chikunda, 2015), and implies, “engaging in a task for which the solution is not known in advance,” (Akinsola, 2008, p. 80). Problems are solved by using a mixture of mathematical knowledge and skills bases, routines and techniques, modelling and intuitive guesses. “In problem-solving greater emphasis is placed on the process of arriving at the solution rather than on the solution itself” (Southwood & Spanneberg, 2000, p. 65). Schäfer (2010) suggests that problem-solving encourages people to talk mathematically, to make predictions and to form connections; she suggests that while the use of apparatus serves as a powerful scaffold – Bruner (1966) calls this enactive representation – problem-solving must also be extended to include meta-cognitive and abstract thinking.



*Word-problems* – A word-problem, also called a story or verbal problem (Sepeng, 2015), is, “a complex cognitive task” (Munez, Orrantia & Rosales, 2013, p. 337); are a sub-set of problem-solving. Word-problems provide short, descriptive narratives that are used to contextualise mathematical problems. Often, the contexts of word-problems are situated within real-world scenarios that are designed to provide realistic platforms for solving problems. This, in turn, assists students to reflexively consider how and when to apply their mathematical content knowledge and techniques to strategically solve these problems (Sepeng & Webb, 2012).

In the context of this study, as an operationalised term, word-problems can be understood to extend students beyond computational and algebraic performance, that is, beyond rote and routines based learning. Instead, word-problems can be understood to expose students to non-routine questions which may require multi-step solutions. In order to solve these problems, students must apply relational understanding to conceptualise and formulate appropriate solution strategies.

*Visualisation* – In the context of this research project, the global operationalisation of visualisation refers to the use of tactiles, drawings, computer-animation models and mental constructs as strategies for *seeing*, that is, understanding and revealing potential solutions to mathematical problems. Visualisation comprises especially two of Bruner's (1966) three parallel processing systems, that is, enactive and iconic forms of representation. As such, "visual stimuli act as tools to negotiate new ideas," (Mudaly, 2010, p. 66).



In this study, the operationalised use of visualisation is congruent with the global interpretation. When bottle-tops are used, to reveal the structure of a ratio, the construction is quickly accomplished and alerts the user to features of the problem. Where sketches are drawn to represent situations, the sketches are, "rapid and direct, and therefore cognitively economical, and provides instant feedback" (Goldschmidt, 2003, p. 81). Such practical and mental *tools* facilitate mathematical exploration and meaning-making (Rosken & Rolka, 2006).

*Compression* – Gray and Tall (2007) suggest that compression occurs when students gain the mental acuity to connect many mathematical concepts, facts and processes, to focus on salient features of these phenomena and draw from them those aspects which will enable them to cogently solve mathematical problems. Compression implies the *automation* of many mathematical knowledge and skills sets, and is a highly desirable outcome in teaching And learning mathematics. It is a particularly sought after attribute for solving mathematical problems.

In this study, as an operationalised term, when compression occurs, it liberates cognitive-space in the mind to attend to the precise details and requirements of the problem-at-hand. However, "if compression of the knowledge required for the next stage does not occur, then procedural learning becomes more likely" (Gray & Tall, 2007, p. 37). In other words, where underpinning routines and concepts are not properly consolidated and schematized, the cumulative effect of only using memorised, routines-based learning techniques – variously called *rote learning*, *procedural learning* and *instrumental learning* – precludes students from making meaningful links, conceptual understanding, relational learning and compression.

## 1.12 ASSUMPTIONS

Mouton (2003, p. 148) suggests that in qualitatively-oriented research it is acceptable to formulate, "guiding ideas or expectations." These ideas and expectations might also be called *assumptions*. Assumptions, suggests Hofstee (2006, p. 88) are, "things that you take to be true without checking whether or not they are true." I have made the following assumptions:



1. Many of the students may have experienced less-than-ideal mathematical stimulus in their formal schooling years;
2. Many students might be "at-risk" mathematically, and thus appropriate scaffolding exercises will have to be included into the IPS 413 E programme;
3. Many students will have difficulty in solving numbers-based word-problems;
4. Very few, if any of the students have encountered the deliberate use of visualisation as a strategy for solving numbers-based word-problems;
5. Some and possibly many students, because of inbuilt conservativism and previous instrumentalist learning approaches, may resist using the visualising strategies that are revealed to them.

## 1.13 SUMMARY OF CHAPTER 1 & OUTLINE FOR THE THESIS

Chapter 1: Introduction

Chapter 1 provides a context for the research. The chapter revealed that in South Africa, the mathematics curriculum is moving towards non-routine and relational, problem-solving endeavours. The chapter explained that many students struggle to solve word-problems and introduced visualisation as a scaffold to facilitate understanding. Chapter 1 disclosed the purpose of the thesis. A background was presented to frame the research questions, the purpose of the study, its rationale, and delimitations of the study. A list of defined terms and assumptions was also presented.

Chapter 2: Literature Review

The literature review contains three major components, namely (1) a socially-mediated conceptual framework for the thesis, (2) various empirical studies which inform my understanding of problem solving, mathematics teacher education and visualisation, and (3) a learning-theory theoretical framework which is used to inform my praxis.

Chapter 3: Methodology

In the methodology chapter, I demonstrate the strengths and limitations of action research and its praxis in problem-solving contexts. I unpack the research paradigm, the mixed methods research approach, research design and my conception of the action cycles. I discuss my data collection instruments, validity, understanding, trustworthiness, reflexivity and ethics in research. I also identify my sample-size and explain how I analyse the quantitative and qualitative data that is captured.on research.



Chapter 4: Data Presentation and Preliminary Analysis of Findings

This chapter provides an account of the fieldwork experiences of the research. I reveal trends from my analysis of a Thinkboard, discuss the data captured by the assessment tasks for Action Cycle 1 and Action Cycle 2, and reveal trends which were captured by my analysis of two questionnaires and discuss findings from the focus group interview. I also reveal opinions held by the participants after they completed the module.

Chapter 5: Summary of Findings, Conclusions and Recommendations

A restatement of the aspirations of the thesis is followed by a brief summary of the research findings. I provided answers to the sub and main research questions and investigate implications and limitations of the work. I discuss problems which were encountered and make recommendations. I give an account of contributions that this research has made and provide a short autobiographical reflection of the research journey and a conclusion.

# CHAPTER 2 – LITERATURE REVIEW

## 2.0 INTRODUCTION

My literature review is quite long. I wrote it in the knowledge that there can be, "no research without action and no action without research" (Cooke & Cox, 2005, p. xix), failing which, the cogence of the research must be compromised. Herr and Anderson (2005, p. 84) suggest that in action research, "the literature drawn for the study will develop as the researcher grows into deeper understanding of the issues under study." Cooke & Cox (2005, p. xx) point out that the researcher's voice is often a feature which is missing, but desired, in many action research reports. In part, I wrote some sections of the literature review during and after conducting the action research. Thus, the contents of the literature review were used as much to inform my action research *praxis* as to investigate the research questions.

Early in the action research, in February 2016, the participants were asked to complete a Thinkboard. Further, in the first few weeks of the action research, and sadly, throughout the research project period, numerous stay-aways and other interruptions served to continually disrupt teaching and learning. The convergence of the information captured from the Thinkboard activity coupled with the students' easy absconding from lessons heightened my awareness that many of my PGCE students carried battle-scars of previous encounters with mathematics and social injustices that had been served up to them in their lives.



I realised that, unless these issues were philosophically addressed, the purposes of the thesis would be thwarted. Indeed, suggests McAteer (2013, p. 102), in action research that is linked to teaching and learning, the literature review should consider matters of philosophical stance. Further, philosophical reflexivity, knowing thyself, and being aware of one's own "epistemic, metaphysical, and moral judgements, decisions and behaviors" (Gonnerman, O'Rourke, Crowley, & Hall, 2015, p. 673-674), are important in educational action research, and, to paraphrase Hughes (1990, p. 90), unless researchers appreciate the phenomena which have shaped people's lives, it is difficult to understand how people construct meaning. With this in mind, I reflected on some of the big ideas that have influenced my own thinking for the past two decades. In doing this and by writing about these perspectives in this literature review, I re-established my own view of knowledge – my epistemology. I called this first section of the literature review: *Two Affective Influences on my Personal Perspectives.* These affective influences guided my praxis and my daily interactions with the PGCE cohort for the duration of the research period.

Because classroom-based, teacher-led action research typically looks to alleviate real-world problematic situations (Herr & Anderson, 2005; Fraenkel & Wallen, 2009; Koshy, 2010; Newton Suter, 2012), it made sense that this literature review should demonstrate that such a real-world problem exists. Thus, in the second part of my literature review, I have demonstrated that different, independent data-sets reveal that mathematical performance in South African schools is in crisis. In other words, *there is a problem* (in fact, there are many problems) that needs to be fixed. I have called this: *Benchmarking South Africa's Mathematical Performance: International, Regional and Local Data-sets.*

I have also interrogated the South African conception of mathematics for Grades 0 – 9 so as to align my own work with that of our Department of Basic Education. This is not to suggest that I intended to slavishly apply the "*technicist* approach" (Schäfer, 2010, p. 34) that is currently espoused by the South African Department of Education. Indeed, I believe that; such schemes of work, etcetera, can stifle imagination and can, "potentially promote very rigid teaching approaches that are governed by teaching towards objectives and assessment criteria" (Ludhra, 2008, p. 60). Like Schäfer (2010), I am deeply concerned with the plethora of *teacher-proofing* text-books, lesson-plans, scheduling, worksheets which are now *de rigueur* our national mode of operationalising our mathematics curriculum. But, I believed that it was important to find some congruence between the ambitions of my courseware and the aspirations of the state. To offer anything less, I felt, was to short-change the participants. This third focus is called: *A 21st Century View of Mathematics: A South African Perspective.*



I felt that, as a fourth component of the literature review, it would be important to begin to focus my research lens more tightly on mathematical problem-solving in general and the solving of word-problems in particular. By doing this, I felt that I would be positioning myself to use that understanding to inform my own praxis. I called this aspect of the literature review: *Problem-Solving in Mathematics.*

In the first week of Action Cycle 1, I asked the participants to complete an arithmetic exercise. I used the knowledge that I gleaned from this task to create activities to offset holes in the students' knowledge of many low-level mathematical ideas. I then re-evaluated the students' in this work.  Unfortunately, there was little evidence of improvement. This was very disappointing and is discussed fully in Chapters 4 and 5. However, this set-back opened up space to look more deeply at mathematics teacher education, pedagogical content knowledge, difficulties that learners experience with multiplicative and fractional thinking, and principles which unpin effective mathematical teaching. This fifth part was called: *Mathematics Teacher Education.*

The sixth part of the literature review investigates the concept of visualisation. It flows from the five parts of the literature review that precede it and is a pivot for the thesis. My academic reading and many years of teaching experience suggest to me that the visualisation of mathematical word-problems, *as a learned problem-solving strategy*, can liberate many latent potentials in otherwise struggling students. I believe that when students visualise mathematical problems, the visualising scaffolds that they create can act as mediators which can assist them to identify probable solutions.  Visual scaffolds can mollify many of the misunderstandings which prevent students from starting and solving word-problems. This part of my literature review is called: *Visualisation as a Strategic Teaching and Learning Tool in Mathematics.*



Finally, in the seventh major part of this literature review, I present the theoretic framework for my thesis. It rests upon my understanding of Jerome Bruner's (1915-2016) theory of learning. This theory embraces the application of understanding and inquiry-based, cooperative and scaffolded teaching and learning perspectives and practices. I do not extend the theoretical framework to include social constructivism, *per se*. Albeit that in principle I subscribe to social constructivist practices, I am well aware of my own short-comings: too many years, working within a behaviourist-inspired, transmission-mode-dominated teaching and learning environment preclude me from comfortably wearing the mantle of a social constructivist and I will forever remain an emerging social constructivist. I have called this part of the literature review: *A Theoretical Framework for this Research.*

## 2.1 TWO AFFECTIVE INFLUENCES ON MY PERSONAL PERSPECTIVE

Below, in (2.1.1), I try to unpack my understanding of Plato's *allegory of the cave* and how that understanding has influenced my own philosophy and epistemology. The second influence upon my own work (2.1.2) is derived from a critical theorist, Paulo Freire (1921-1997), who exposed links between social class, poverty, hunger and educational performance on one hand, and redress on the other. Alongside Lewin, Freire, a liberationist, is considered to be one of the founders of action research (Reason & Bradbury, 2008). Collectively, these influences speak to the importance that all humans are capacitated to live liberated lives. By extension, this action research should offer a similar promise to its participants.

### 2.1.1 PLATO – THE ALLEGORY OF THE CAVE

The philosophical roots of my thesis can be traced back to Plato's *Allegory of the Cave.* The allegory is presented in Book VII of Plato's *Republic* and is written as a dialogue between Socrates (Plato's mentor) and Glaucon (Plato's brother). In the dialogue, Socrates directs increasingly sophisticated, philosophical questions to Glaucon. However, within the narrative of the allegory, Plato does not provide direct answers to the questions. Indeed, this is a feature found in much of his writing; typically, Plato "hides behind the narrative character" (Kohan, 2013, p. 315) of his propositions, thus we never have direct contact with Plato's opinions. We have to wrestle with his writing for ourselves. This gives rise to various interpretations of the figurative symbolism that is found within the allegory.



Plato uses the allegory as a symbol, a metaphor, to describe his belief in a two-level state of reality. He suggests that, in our day to day lives, humans function in the lower level, in the realm of things, sensations and naïve belief systems. Plato suggests that humans should use this lower realm as a springboard to seek the higher level, the realm of forms, in which abstract thought, truth and wisdom are found. It is in the higher realm, he believes, that humans find the full expression of their own emancipation.

Plato superimposes the allegory upon his views of education. As an early champion of liberal education (Losin, 1996, p. 49 – 50), Plato, "propounds the value of a *thinking* mind as greater than a *doing* person" (Nelsen, 2014, p. 102). He perceives, for example, a quest for vocational training as a servile goal and one that is incapable of expanding the human mind. It is only the liberally educated person who, "becomes capable of any desired intellectual skill because he or she learns how to think and to what end or purpose to think" (Nelsen, 2014, 102).

Plato suggests that there is an ideal state for everything, including for example, tables. When humans experience *tables* for the first time, their early experiences of tables is incomplete: their early understanding produces only a prototype of *tableness*. Knowledge, argues Plato, "is not knowledge of content, or possession, but knowledge of relationship, affection and passion" (Kohan, 2013, p. 315). Through a constant search to know tables better - some tables are made from reclaimed wood, some are large, most tables have four legs, etcetera, numerous experiences of different tables in various settings elevates the embryonic understanding of tables closer and closer to a complete knowledge of tables, that is, to *the form* of tables. Plato argued that the true knowledge – *understanding* – of tables, or mathematics, or anything, occurs only in the realm of forms (Nelsen, 2014, p. 104).



In the above, Plato's realm of forms finds some congruence with Maslow's (1943) hierarchy of needs, the highest levels of which are met when individuals obtain *esteem*, the capacity of be *self-actualizing*, and are free to pursue their desire to *know* (Maslow, 1943, p. 385). Indeed, it is only when we become self-actualising that we come to hold ideals such as truth and goodness in high regard. When self-actualisation is not achieved, such conceptions are mollified by the more pressing needs of safety and well-being (Biehler & Snowman, 1997, p. 408). From this we can extrapolate that the realm of sensations can be perceived to be analogous with instrumental learning while the realm of forms draws links to relational understanding and compression.

In Plato's allegory, prisoners are fettered, looking at the back wall of a cave. A fire burns behind them. It illuminates the cave such that the prisoners observe flickering, distorted shadows of themselves on the wall. Also behind them, the prisoners can hear dampened echoes of guards who are standing and talking beyond their scope of view. These guards move various objects about, creating moving shadows on the walls of the cave. However, because of their shackled circumstance, the prisoners cannot know that there are guards, a fire and moving objects in the cave.

The shadows and echoes are a metaphor that represents the full, but naïve, experience of these prisoners' lives; they have never known anything else. It represents their only way of knowing the world. But this, "knowledge is based on mere appearance, not on an understanding of the origins of the shadows, of the real things of which they are distortions" (Issitt, 2007, p. 392 – 393). The allegory talks to our distorted perception of the qualities of a good education (Losin, 1996, p. 50).

Eventually, one prisoner breaks his shackles, finds his own way out of the cave and discovers that all that he had previously held as true was actually an illusion. In literal and figurative terms, this prisoner has moved from the dark to the light. Dark to light is itself a metaphor for unknowing to knowing. Through this allegory, Plato presents his view that most of what passes for education is but an illusion of education.

Plato suggests that the purpose of a *good* education, of a good teacher, is to create enabling opportunities for students to emancipate themselves. This idea is best conveyed in Plato's own words:



> Education isn't what some people declare it to be, namely, putting knowledge into souls that lack it, like putting sight into blind eyes, [rather], education takes for granted that sight is there but that it isn't turned the right way or looking where it ought to look, and tries to redirect it appropriately.
>
> Plato, as cited in Curren, 2007, p. 21 - 22

In the allegory, the liberated prisoner, now acutely aware of the fullness of reality, of the world of forms, of his own journey towards wisdom, is confronted with a dilemma: does he continue into the world by himself, or does he return to the cave to liberate the other prisoners from their world of shadows. He realizes that going back carries with it two dangers. First, because his own new understanding of reality is incomplete, nascent, the other prisoners may perceive him to be mad and will simply ignore him; second, the prisoners, intellectually shackled by not knowing any better, may resist his offer to liberate them and might even attempt to kill him.

Plato suggests that any journey towards emancipation may well prove difficult – such a paradigm-shift is difficult to negotiate – but for those students (and teachers) who embrace new opportunity, the potential benefits might make the anticipated tensions worthwhile. Nor can one attend to this emancipatory journey towards knowing in a piecemeal manner. Plato makes the point that:

> The power to learn is present in everyone's soul and the instrument with which each learns is like an eye that cannot be turned around from darkness to light *without turning the whole body* [my emphasis]. This instrument cannot be turned around from that which is coming into being without turning the whole soul until it is able to study that which is and the brightest thing that is, namely, the one we call the good.
>
> Plato, as cited in Curren, 2007, p. 22

Plato suggests that the task of a good teacher is dual fold: through sustained effort teachers can awaken in their students a desire to emancipate themselves, to learn, and that burning desire to learn must be focused on that which is morally perceived to be good. A liberal education enables teachers to assist students to, "think and act for themselves" (Holmes, 1987, p. 16), but for this to happen, *the whole body has to be turned*, that is, there must be a constant striving to move beyond the realm of sensations into the realm of forms.



The allegory produces at least two important educational foci which speak directly to teaching and learning within the IPS 413 E Mathematics module, namely, the concept of two realms of reality, and the metaphor of the allegory itself.

#### 2.1.1.1 PLATO'S CONCEPT OF TWO REALMS OF REALITY

The realm of things, of sensations and untested belief systems, resonates with teaching practices in which the teacher is the sole dispenser of knowledge. In such settings, figures, facts and practiced routines represent the knowledge that is allotted to students. Data that is discussed in Chapters 4 and 5 suggests that, in mathematics at least, the 2016 cohort of PGCE students derived most of their mathematical knowledge in such environments. These transition-mode situations impose, "a powerfully positivist, and inherently limited and limiting regime of truth in education"

(Issitt, 2007, p. 383). Students are perceived of as holding no or little worthwhile knowledge – *tabula rasa* – and are required to become passive recipients of factoids and knowledge systems as fed to them by their *teachers*. In other words, they become a shadow of their full potential.

Plato argued in favour of – and modern societies need – students who have the capacity to think deeply, who are willing to take calculated risks and who are able to cognitively stretch their understanding of important issues. In effect, when students such as the 2016 IPS 413 E Mathematics cohort are encouraged to undertake creative, non-routine activities, they move from the realm of things into a higher, liberating plain, *vis-à-vis,* Plato's realm of forms.

#### 2.1.1.2 A METAPHOR FOR A GOOD EDUCATION

The metaphor of the allegory rings true in education settings where, albeit that liberating education practices are available, many teachers and students, knowing better, continue to cling to out-dated and often ill-working, educational practices. The task is to turn this resistance around, but liberating people who are shackled by their own conservativism, is problematic. Nor can effective change be done in half measures. For teachers such as myself, it means giving up some control; for students – the 2016 IPS 413 E cohort – it means embracing self-reliance and accountability.

Transmission mode teaching practices are enormously seductive for teachers. It gives them complete control over what is constituted as being worthwhile, of what needs to be known and of the timing and mode of transmission to effect that *transfer* of knowledge. For teachers, giving up that control exposes us to the discovery that we are not experts in our subject matter and, further, that loss of control challenges us to study our subjects more deeply.

For students, the removal of transmission mode practices requires them to shift towards embracing self-reliance; this implies that they take ownership of their lives, of liberating themselves and of striving to know more. Simultaneously, their liberalisation requires students to become accountable for their own actions; this may require them to confront their own ineptitude when they do not do well in their courses and, in turn, this might compel them to reflect upon their commitment to their studies.

### 2.1.2 FREIRE - THE OPPRESSION OF THE MANY BY THE FEW

In his seminal book, *Pedagogy of the Oppressed*, Paulo Freire (1997, p. 24 - 25), posits that the liberation of people lies in their life-long vocation for the humanization of themselves and others. However, he suggests that the historical evidence of civilizations and societies reveals that, "injustice, exploitation, oppression and… violence," (1997, p. 25) perpetrated by *the few* over *the many*, leads to the dehumanization of oppressors and the oppressed alike and creates in both illusions and distortions of what it is to be human. He stresses the point that the, "ontological and historical vocation," of oppressed men and women, world-wide, is to become, "more fully human" (Freire, 1997, p. 48).

Sadly, South Africa's history, like many other histories, is replete with many examples of *the oppression of the many by the few.* These examples have worked thorough our country at political, social and domestic levels. In education, oppression has been as covert as the *hidden curriculum* and as overt as *educational spend* on different population groups in South Africa (Christie, 1988), and now, in the second decade of the 21st century, the ramifications of this *violence* that was visited upon our society remain an unhealed wound.



Freire (1997, p. 37), writes, "Any situation in which "A" objectively exploits "B" or hinders his and her pursuit of self-affirmation as a responsible person is one of oppression." Violence, suggests Freire (1997, p. 37), is always *initiated* by the oppressor; it cannot be constituted in any other way. It is not possible for the oppressed to initiate violence, as their own violence is always a reply to violence already visited upon them by an oppressor. The oppressed may act violently, but always, it is a reaction to violence perpetrated by their oppressors. And, the oppressor, knowing that his or her interests are served best by maintaining the status-quo, will violently work to preserve the privileges of the few, that is, his or her constituency, at the expense of sharing with the many (Freire, 1997, p. 34).

The prestige of the oppressor is in large part manifested by the status that the oppressor imbues upon his or her accumulation of objects. The lens of the oppressor is one in which money, privilege, "more – always more – even at the cost of the oppressed having less or having nothing" (Freire, 1997, 40), commodifies humanity and is used to identify who is taken as worthy to be human. Indeed, Freire (1997) writes:

> For the oppressors, there exists only one right: their right to live in peace, over against the right, not always even recognised, but simply conceded, of the oppressed to survival.
>
> Freire, 1997, p. 39 - 40.

For the oppressed, their dehumanization by their oppressors is insidious, pervasive, complete. Typically, they will not even be aware that they have been subjugated and may not be aware of their own oppression. They will buy into a dominant fatalistic rhetoric that their own occupation (of the mind), their own demise, their low place in society, is of their own making. “So often do they hear that they are good for nothing, know nothing and are incapable of learning anything – that they are sick, lazy and unproductive – that in the end they become convinced of their own unfitness” (Freire, 1997, p. 45). A self-fulfilling prophesy, fulfilled…



Freire (in Curren, 2007) suggests that, because the ontological vocation of people is *humanization*, when oppressed people do get a sense of their oppression, when they get a sense that they have been duped and dehumanized, conflict and revolution will arise. When this happens, they will seek out ways to act, to liberate themselves and regain possession of their dispossessed humanness. And, Freire (1997) writes:

> It is only the oppressed who, by freeing themselves, can free their oppressors. The latter, as an oppressive class, can free neither others nor themselves.
>
> Freire, 1997, p. 38

However, Freire (1997, p. 30) suggests that for those who would liberate themselves, a duality of consciousness exists and niggles at the back of the mind: on one hand the oppressed realise that without freedom, they cannot fully be human; or the other hand, they are fearful of the implications and responsibilities that come with this freedom. Thus, the internalised struggle of their liberation is a hard-won, intra-mental battle.

Flowing from the above, Paulo Freire (2007), explains that in *banking* models of education, teachers *deposit* knowledge *into* students for them to, “receive, memorize, and repeat” (Freire, as cited in Curren, 2007, p. 68). However, ultimately, a banking model disenfranchises students. Freire suggests that banking is a deliberate and oppressive practice that encourages passive acceptance of the status-quo by the marginalised and oppressed, *vis-à-vis* students: such a model turns students, “into “containers,” into “receptacles” to be “filled” by the teacher” (Freire, 1997, p. 53).

Further, he suggests, “to alienate human beings from their own decision-making is to change them into objects” (Freire, 1997, p. 66). A banking model can only serve the interests of the dominant oppressor, that is, the teacher, the school, the education system. Often, teachers are unconscious of their role as oppressors, of their own subjugation within the system, and of the fact that they are perpetrating oppression upon others. The dire and often unconsciously achieved consequences of banking education are that the oppressors of marginalized students perceive them to be “incompetent and lazy” (Freire, 1997, p. 53). Indeed, in the course of time, students will begin to perceive themselves as such.



Banking models do not embrace the very creativity, risk-taking, inquiry-based exploration and transformational dialogue which are keystones of competence and deep understanding. In banking models, the scope of the students’ school experience is limited to, “receiving, filing, and storing the deposits” (Freire, 1997, p. 53) that are bestowed upon them by their benevolent, but oppressive, teachers.

Given that a feature of our humanness is our cognitive awareness that we are ever, “unfinished,” (Curren, 2007, p. 72), Freire calls for an overthrow of pervasive banking systems of education in favour of a problem-solving model of education that is philosophically inspired by the liberation of full human potential. Liberating education, Freire suggests, “consists of acts of cognition, not transferals of information” (Curren, 2007, p. 72). Through dialogue, students and teachers become co-constructors who meaningfully share their understanding of the world. The praxis of a problem-solving model of education demystifies reality, evokes critical reflection, and actions and unleashes the full creative cognitive capacities of teachers and students alike.

A problem-solving philosophy of education, “strives for the *emergence* of consciousness and *critical intervention* in reality” (Freire, as cited in Curren, 2007, p. 72). Conjoined as they are, students and teachers will critically reflect upon and constantly reconstruct their praxis. In doing this, rather than being perceived as *objects* who are *acted upon* in their education, Freire suggests that teachers and students become *subjects* who *mutually liberate each other* in their own education:

Currently, higher education in South Africa is characterised by mass stay-ways by students, violent disruptions of classes and severe damage to university infrastructures. Indeed, as is discussed in detail in Chapter 4 (4.1.1), for the duration of the action research many such interruptions were visited upon my own university. In large measure, agreements are reached and promises are made, but often these agreements are perceived to be unattended to or broken by one or other party and this has exacerbated simmering tensions between the *haves* and the *have nots* in education in South Africa. At least two important aspects are gleaned from Freire’s (1997) text. These include his view that (1) oppressed human beings are ontologically predisposed to seek becoming more human and that (2) banking models of education need to be reformed to liberate human potentials.



### 2.1.2.1 THE ONTOLOGICAL VOCATION OF OPPRESSED STUDENTS

In 2015 through 2017, much violent upheaval has occurred on university campuses in South Africa. Nationally, the wanton destruction of our campuses and the disruption of our classes has been roundly condemned. However, Freire’s (1997) commentary on oppression and the ontological drive to be more human provides a sounding board against which we can better understand the turmoil. Introspection suggests that decades of broken promises and of jobless, debt-ridden, graduated students hopelessly walking our streets have created a powder-keg of seething resentment and that these frustrations, in large measure, have caused the outpouring of hurt by our students. Further, many students have been victims of generations of political, social, economic and educational violence, but their ontological destiny – the destiny that they are now rightfully claiming for themselves – is simply a quest to be more human. We should hold their ontological aspirations very close to our own hearts.

#### 2.1.2.2 A BANKING VERSUS A LIBERATING MODEL OF EDUCATION

For university students to enjoy the full gifts of a liberating university education, both academics and students must abandon the pervasive banking model of teaching and learning that stubbornly persists in many classrooms. This paradigm shift, while difficult, has the potential, on one hand, to liberate creative endeavour and higher cognitive thinking and, on the other, should enable students to reap the rewards and esteem of being self-actualizing South African citizens.

Later, in Chapters 4 and 5, I will discuss the fact that, while this aspiration was constantly uppermost in my mind, the cohort of participants in the research and I only achieved this goal partially. In effect, in my role as an action researcher, I found that I was, “a living contradiction” (McNiff & Whitehead, 2006, p. 28) which was a difficult pill to swallow. I realised that while I espoused liberal approaches, all-too-easily I assumed a transmission mode teaching stance.

In the next section of this literature review, I will reveal the perilous state of our country’s mathematical performance, this measured against many sets of data. Measured against that data, it is prudent to anticipate that oppressed children who do make it into our university classes will have clawed their way into our universities. We can anticipate that many PGCE students will have gaps in their mathematical knowledge and skills, but, equally, we can anticipate that, in spite of all obstacles that are thrown at them, they are tenacious and have high ambitions to graduate.



## 2.2 BENCHMARKING SOUTH AFRICA’S MATHEMATICAL PERFORMANCE: INTERNATIONAL, REGIONAL AND LOCAL DATA-SETS

Research into South Africa’s mathematical performance, when measured against international and national benchmarks, suggests crisis (Bloch, 2009, p. 17). Indeed, this crisis can be exemplified by the fact that by the time that they have completed the first year of their Intermediate Phase studies, “grade 4 learners are already almost two grades behind grade level expectations” (Graven, 2015, p.1). Bloch (2009) continues, suggesting that many South African mathematics teachers hold low subject content knowledge and states that, “the basics of pedagogy are often absent” (Bloch, 2009, p.

102). Further, van der Berg and Spaull (2011, p. 1) write that, “few outside of policy circles are aware of the extent of the underperformance of South Africa’s education system.” However, suggests Bloch (2009, p. 68), unless we identify and acknowledge this crisis, we cannot begin to deal with it. Unfortunately, the exemplars below will reinforce these pessimistic views.

### 2.2.1 TRENDS IN INTERNATIONAL MATHEMATICS AND SCIENCE STUDIES REPORTS

At Grade 8 level, South Africa has participated in five Trends in International Mathematics and Science Studies (TIMSS) cycles. These participations occurred in 1994/5, 1998/9, 2003, 2011 and 2015.

In 1995, 40 countries participated in the TIMSS study, (Gonzales, Calsyn, Leslie Jocelyn, Mak, Kastberg, Arafeh, Williams, & Tsen, 2000). In that cycle, pupils in Singapore achieved the highest results in mathematics, obtaining a national average of 643 out of 800 points for the test; in the same study, South African children were the lowest scoring nation, scoring, as the national average, 354 out of 800 available points. In other words, while the national *average* for Singapore was ±80%, the South African national average was ±44%. What is salient here is not just the position occupied by South Africa, which was last, but more significant is the size of the gap between the averages for first place at 643/800 and South Africa at 354/800; that is a difference of 289 points in an 800 mark test.



Four years later, in 1999, with 38 participating countries in the Grade 8 mathematics test, once again Singapore’s pupils were first with 604 points as their national average out of 800 and, again, South Africa came last, this time with a national average of 275 points. These scores convert to national averages of ±76% and ±34% for Singapore and South Africa respectively. Howie (2011, p. 2) writes that, “The [South African] average score of 275 points out of 800 points is well below the international average of 487 points.” Indeed, our national average was 212 points *below* the international average and 329 points *below* the Singapore national average.

In 2003, 46 countries participated in the TIMSS study. In the 2003 cycle, the assessment of Grade 8 mathematical performance was measured against two cognitive domain league-tables – namely (1) a *content knowledge and technical procedure* table, and (2) an *application* table. In the content knowledge and procedure domain, Korea and Singapore achieved 592 and 591 out of 800 points, or 74% and 73,9%, respectively. South Africa, second but last in the content and technical procedure test, scored 261 out of 800 points or 32,6%. In the application domain test, Singapore was first with 611 out of 800 points, that is, 76,4%, and South Africa was last with 269 out of a possible 800 points, or 33,6% (Mullis, Martin, Gonzalez & Chrostowski, 2004).

South Africa did not take part in the 2007 TIMSS research.

However, four years later, it was suggested that, "in August 2011, the HSRC will administer the Trends in International Mathematics and Science Study to Grade 9 students and this will provide a trend analysis with TIMSS 2003 results and a comparison of our mathematics and science performance with 50 other countries" (Reddy, Kivilu, Cosser & Frempong, 2011, p. 1).



Indeed, this did happen. Note though, that at an international level, Grade 8 pupils were assessed, while in South Africa, it was *Grade 9 pupils* who wrote the test. Nonetheless, South Africa continued its own trend of being at the bottom of the class in this global research undertaking (Reddy, 2012).

Only 24% of our Grade 9 pupils managed to achieve the lowest benchmark in that test, that is, 400 out of 800 marks, and a scant 1% managed to score at the highest, or advanced benchmark, 625 out of 800, or higher. However, the report states that the average achievements for South African pupils was, "not reliably measured because the percentage of students with achievement *too low for estimation* [my emphasis] exceeds 25%" (Mullis, Martin, Foy & Arora, 2012, p. 114). That is, more than 25% of the South African Grade 9 pupils who wrote the 2011 TIMSS Grade 8 test, achieved scores too low to be estimated. In effect, they were outliers.

In the latest iteration of the TIMSS, conducted in 2015 and released on 29 November 2016, the trends presented above continue to manifest. In 2015, 39 countries took part in the Grade 8 mathematics TIMSS test (Mullis et al., 2016). In the 2015 cycle, as in the 2011 cycle preceding it, South Africa's Grade 9 children wrote the Grade 8 TIMSS mathematics test. In this cycle, as in the previous cycle, the data capturers had, "reservations about reliability because the percentage of students with achievement too low for estimation exceeds 25%" (Mullis et al., 2016).

Singapore, followed by Korea, came in first and second with respective national average scores of 621 and 606 achieved out of a possible 800 points. Indeed, East Asian countries took the top five berths in achievement, namely, Singapore (621); Korea (606); Chinese Taipei (599); Hong Kong (594) and Japan (586). This was followed by a 48 point gap with the sixth best performing country, that being the Russian Federation (538).  The middle mark for this international test was 500. South Africa, second but last, and Saudi Arabia, with scores of 372 and 368 respectively, were at the bottom of the pile (Mullis et al., 2016).

Four benchmarks with attendant descriptors of capability were set for the Grade 8 mathematics test:

| | | | |
|---|---|---|---|
| 1. | Advanced Benchmark: | 625 | Can apply and reason in a variety of problem situations, solve linear equations, and make generalizations. |
| 2. | High Benchmark: | 550 | Can apply understanding and knowledge in a variety of relatively complex situations. |
| 3. | Intermediate Benchmark: | 475 | Can apply basic mathematical knowledge in a variety of situations |
| 4. | Low Benchmark: | 400 | Have some knowledge of whole numbers and basic graphs |

Mullis et al., 2016.



It is worth noting the following comparisons of performance between Singapore and South Africa. In Singapore, 99% of the Grade 8 pupils achieved the low benchmark; 94% obtained the intermediate benchmark; 81% the high benchmark, and 54% scored above the advanced benchmark. In other words, 54% of Singapore's Grade 8 pupils scored 625 marks or more in the test (Mullis et al., 2016). In South Africa, only 34% of our Grade 9 pupils achieved the low benchmark; 13% achieved the intermediate benchmark; 3% obtained the high benchmark, and a scant 1% of South African pupils scored in the advanced benchmark (Mullis et al., 2016).

Collectively, this summary of South Africa's performance in various Grade 8 TIMSS's reports demonstrates that, at a Grade 8 level, when compared to their international peers in mathematics, South African Grade 9 pupils are hardly competitive. Now, it might be useful to refocus the lens more tightly to compare South African performance with those of its immediate African neighbours.

### 2.2.2 SOUTHERN AND EASTERN AFRICA CONSORTIUM FOR MONITORING EDUCATIONAL QUALITY REPORTS

The Southern and Eastern Africa Consortium for Monitoring Educational Quality (SACMEQ) was launched in 1995, and is now driven by 15 ministries of education in the southern African region. It has undertaken three regional surveys.



The first project, SACMEQ I, (UNESCO, 2010, p. 2) was conducted from 1995 to 1998. The research sample included 1 000 schools and 20 000 Grade 6 pupils from 7 African countries: South Africa did not participate in this project. Participating countries included Kenya, Malawi, Mauritius, Namibia, Tanzania (Zanzibar), Zambia and Zimbabwe.

The SACMEQ II research project (UNESCO, 2010, p. 2) took place from 1998 to 2004. The sample included 2 000 schools, 40 000 Grade 6 pupils, and 14 countries: South Africa participated in the project. The other participating countries included Botswana, Kenya, Lesotho, Malawi, Mauritius, Mozambique, Namibia, Seychelles, Swaziland, Tanzania (Mainland), Tanzania (Zanzibar), Uganda and Zambia.

In the SACMEC II project, 8 of the 14 countries that took part produced higher national averages than South Africa. Mauritian Grade 6 pupils achieved the highest national average at 585 out of 800 points, or ±73%. South Africa achieved a national average score of 486 out of 800 points, or ±61%. The worst performing country in the SACMEQ

II project was Namibia, which achieved a national average of 431 out of 800 points, or ±54% (Makuwa, 2010, p. 4).

The data for SACMEQ III, (UNESCO, 2010, p. 2), was captured in 2007. Approximately 61 000 pupils, 8 000 teachers, 2 800 schools and 15 countries were involved in the exercise. Participating countries included Botswana, Kenya, Lesotho, Malawi, Mauritius, Mozambique, Namibia, Seychelles, South Africa, Swaziland, Tanzania (Mainland), Tanzania (Zanzibar), Uganda, Zambia and Zimbabwe.

Seven of the participating countries obtained better results than South Africa. The results indicate that in 2007 Mauritius retained its position as best performer in the region: its Grade 6 pupils scored, as a national average, 623 out of 800 points, or ±78%. In that cycle, the South African national average was 495 out of 800, or ± 62%, and Zambia, last in the region, achieved 435 out of 800 points, or ±54%. (Makuwa, 2010, p. 4). Below, in Table 2.1, we can observe and compare regional performances in Grade 6 math in SACMEQ II (2000) and SACMEQ III (2007), respectively.



Table 2.1

*SACMEQ II & SACMEC III mathematical performance. Source: Makuwa, 2010, p. 4*

| | **Pupil Mathematics Score** | | |
|---|---|---|---|
| COUNTRY | **2000** | **2007** | |
| Botswana | **512.9** | **520.5** | ▶ |
| Kenya | **563.3** | **557.0** | ▶ |
| Lesotho | **447.2** | **476.9** | ▲ |
| Malawi | **432.9** | **447.0** | ▲ |
| Mauritius | **584.6** | **623.3** | ▲ |
| Mozambique | **530.0** | **483.8** | ▼ |
| Namibia | **430.9** | **471.0** | ▲ |
| Seychelles | **554.3** | **550.7** | ▶ |
| South Africa | **486.1** | **494.8** | ▶ |
| Swaziland | **516.5** | **540.8** | ▲ |
| Tanzania | **522.4** | **552.7** | ▲ |
| Uganda | **506.3** | **481.9** | ▼ |
| Zambia | **435.2** | **435.2** | ▶ |
| Zanzibar | **478.1** | **486.2** | ▶ |
| Zimbabwe | ×× | **519.8** | ×× |
| **SACMEQ** | **500.0** | **509.5** | ▶ |

For the comparison, a mean of 500 was established in the 2000 data, and 509,5 for the 2007 data-set. Scores written in green are more than 10 points above the mean, scores in black are within 10 points of the mean, and those in red are more than 10 points below the mean. A green arrow, pointing upwards, indicates a positive growth of at least 10 points between 2000 and 2007, a grey, horizontal arrow indicates sustained performance, and a red, downward pointing arrow indicates a drop of more than 10 points between 2000 and 2007. Zimbabwe did not take part in the 2000 research programme.

The data in Table 2.1 suggests that, when compared to our African neighbours, South Africa's Grade 6 pupils are not particularly competitive in mathematics. In both 2000 and 2007, South Africa lies below the regional mean scores.

Further, if one drills deeper into the 2007 data-set, (see Table 2.2 below), it reveals the low-standing performance of the Eastern Cape compared to the other South African provinces: only Limpopo fares worse.



Table 2.2

*SACMEQ III. South African provincial scores. Source: Hungi et al., 2010, p. 13*

| | Pupil Mathematics Score |
|---|---|
| PROVINCE | **2007** |
| Eastern Cape | **468.8** |
| Free State | **491.6** |
| Gauteng | **545.0** |
| KwaZulu Natal | **485.2** |
| Mpumalanga | **476.1** |
| Northern Cape | **498.7** |
| Limpopo | **446.7** |
| North West | **503.1** |
| Western Cape | **565.7** |

The empirical data shows that in comparison to international standards, the various TIMSS reports place South Africa *bottom of the class*; within the SACMEC 2007 programme, when compared with its fourteen regional neighbours, South Africa is in the *middle of the class*, and within South Africa itself, the Eastern Cape is *second but last* in mathematical performance.

SACMEQ IV was conducted in 2012/3. At the time of writing this text the regional data captured by that exercise was not available on the SACMEQ website. Indeed, it might have been embargoed. Spaull (2016) was appointed by the SACMEQ Secretariat as an analyst for SACMEQ IV, *but he subsequently resigned, stating matters of good conscience*. He suggests that the SACMEQ IV data and results are flawed on (1) methodological design and consistency, and (2) "particularly the fact that weaker students had been excluded from the final results," (Spaull, 2016). Further, he suggests, SACMEQ refused to investigate issues that he raised pertaining to the comparability and validity of the data (Spaull, 2016). This might explain why, *apparently*, the SACMEQ IV data for mathematics capability in South Africa has climbed from a national average of 495 in 2007 to 587 in 2013.

Spaull (2016) suggests that the data reveals to us that, "the fact that teacher test scores plummeted at the same time that student test scores soared should already make us very curious about the technical procedures that might lead to such a situation" (Spaull, 2016). And, he continues:



> Unfortunately, the contract that I have signed with SACMEQ prevents me from publishing any results that are based on that data until the international report has been publicly released, at which time I will provide a full account of my technical concerns and reasons for the non-comparability. I have also subsequently deleted the data on SACMEQ's request.
>
> Spaull, 2016.

### 2.2.3 THE PERCEPTION OF INFLATION OF MARKS

An additional and worrying trend in South Africa is a perception that we are artificially raising the matriculation examination mathematics marks. In part, this perception arose because, a decade ago, our national Department of Education (DoE) attempted to find parity between the now out-dated Higher Grade (HG) Mathematics matriculation examination and the now in vogue National Senior Certificate (NSC) for Mathematics. Indeed, a comparison between the results obtained in the 2007 Higher Grade (HG)

Mathematics and the 2008 National Senior Certificate (NSC) Mathematics matriculation examinations suggests that marks-inflation did occur.

In a report that was compiled by the Centre for Development and Enterprise (2010), *in principle* the South African National Department of Education equated a 40% HG examination result in 2007 with a 50% NSC result in 2008. *In practice*, the NSC mark was lifted to 54%. However, “the real leniency and grade inflation appears further up the scale” (Centre for Development and Enterprise, 2010, p. 9); a 64% HG matriculation result in 2007 generated an equivalency of 80% for a 2008 NSC counterpart. A comparative table, (Table 2.3), below, will demonstrate the full magnitude of this excess.

Table 2.3

*Comparisons between 2007 HG and 2008 NSC scores. Source: Centre for Development and Enterprise, 2010, p. 9*

| HG Mathematics 2007 | NSC Mathematics 2008 |
|---|---|
| 10 | 13 |
| 14 | 19 |
| 20 | 26 |
| 22 | 30 |
| 27 | 36 |
| 29 | 40 |
| 35 | 47 |
| 37 | 50 |
| 40 | 54 |
| 45 | 60 |
| 46 | 62 |
| 50 | 66 |
| 53 | 70 |
| 64 | 80 |



There has been widespread speculation that, after the 2007-2008 event, the DoE continued to inflate matric results. Indeed, the 2010 matric results gave so much rise to the speculation of inflation that, on 30 December 2010, Professor Sizwe Mabizela, the chairman of Umalusi offered a contradiction to this perception stating that, “We are satisfied that those examinations are credible and can be released,"with the rider that, “any errors had been "technical" in nature” (News24, 2010).

The TIMSS and SACMEQ reports, coupled with widely held views of inflation of matric maths marks, paint a bleak picture of our pupils’ mathematical competitiveness in international, regional and local contexts. However, the problem gets worse!

### 2.2.4 THE EARLY LOSS OF PUPILS FROM SOUTH AFRICAN CLASSROOMS

Bloch (2009, p. 67) writes the following, “It is quite clear that millions of children are not progressing through the school system. It is estimated that perhaps only 52 of every 100 who start Grade 1 make it to Grade 12.” By way of one example and confirmation of Bloch’s (2009) view, we can turn the clock back to 2000.

In that year, 1 055 397 children entered formal schooling in South Africa via the Grade 1 classroom (Department of Education, 2002, p. 8). As a cohort, those children would eventually matriculate in 2011. In 2011, 534 498 children were registered for Grade 12, which is approximately 51% of the 2000 registration class.



Further, at the end of 2011, the DoE announced that South Africa had achieved a matriculation pass-rate of 70,2% (Department of Education, 2013, p. 27). But, in making this announcement, the DoE used sleight-of-hand and possibly even a bit of deception.

As we know, in 2011, 534 498 children were registered for Grade 12. Of this number, 496 090 children sat for their matric exams and, of these children, 348 117 passed the matric exam (Department of Basic Education, 2013, p. 10). It is upon the fraction 348 117 / 496 090 that the Department of Education generated the 70,2% pass rate.

However, viewed from the perspective of those children who registered for Grade 1 in 2000, that is, 1 055 397, we see that *less than half* of the original class of 2000 actually made it into the matric examination room. When measured against the full registration class of 2000, the 70,2% pass-rate, actually drops to approximately 33%, which is 348 117 / 1 055 397.

In 2011, a mark of 30% was considered a pass in mathematics for the matriculation examination (Department of Education, 2013, p. 27). In that year, 224 635 pupils sat for the mathematics matric exam. Of these candidates, 104 033 pupils passed matric mathematics (Department of Basic Education, 2013, p. 27).

In other words, just less than half of the children (46%) who wrote mathematics managed to achieve a mark of 30% or more. Indeed, only 67 541 pupils managed to get a mark of 40% or more (Department of Basic Education, 2013, p. 27). Therefore, based on the original intake in 2000, only 1 in 10 pupils of the registration class of 2000 managed to write and pass the mathematics subject.

### 2.2.5 NATIONAL AVERAGE MARKS FOR ANNUAL NATIONAL ASSESSMENT IN MATHEMATICS

To monitor performance, progression and competitiveness across South Africa, the DoE has introduced compulsory annual national testing in mathematics. These benchmarking tests - the Annual National Assessments or ANAs - were first written in Grades 1 through 6 and Grade 9 in 2012. Every child in every state school in South Africa is required to write these tests.



A summary of the mathematical performance of South African pupils in these grades for 2012, 2013 and 2014 is presented in Table 2.4, below. The data-set in Table 2.4 is worrying: while the average marks obtained by pupils in the Foundation and Intermediate Phases show some positive improvement (albeit that they start, in 2012, off low bases), the mathematical performance at the end of the GET Band, in Grade 9, remains stubbornly, shockingly, low. Annually, by the end of the Foundation and Intermediate Phases, some marked improvement in the percentage of pupils who achieve 50% or more is noted. However, in parallel to the data revealed in Table 2.4, performance by Senior Phase Grade 9 pupils is shockingly poor. The delivery of ANAs in 2015 and 2016 was disrupted by education unions. Any limited amount of data that might have been captured is, to my best knowledge, unavailable

Table 2.4

*ANA results for mathematics. Source: Department of Basic Education, 2014, p. 41.*

| **National Average Marks for Annual National Assessment in Mathematics** | | | |
|---|---|---|---|
| **Grade** | **2012** | **2013** | **2014** |
| **1** | **68%** | **60%** | **68%** |
| **2** | **57%** | **59%** | **62%** |
| **3** | **41%** | **53%** | **56%** |
| **4** | **37%** | **37%** | **37%** |
| **5** | **30%** | **33%** | **37%** |
| **6** | **27%** | **39%** | **43%** |
| **9** | **13%** | **14%** | **11%** |

The data-set in Table 2.5 is even more forbidding. It reveals, for Grades 3, 6 and 9 respectively, the percentage of pupils who managed to achieve a mark of 50% or more when they wrote the ANA tests.

Table 2.5

*Pupils who achieved 50% or more in ANA for mathematics. Source: Department of Basic Education, 2014, p. 43.*



| **Pupils who Achieved 50% or more in the Annual National Assessment in Mathematics** | | | |
|---|---|---|---|
| **Grade** | **2012** | **2013** | **2014** |
| 3 | 36% | 59% | 65% |
| 6 | 11% | 27% | 35% |
| 9 | 2% | 2% | 3% |

The literature review has demonstrated that in mathematics, “South Africa comes at the bottom of the pile” (Bloch, 2009, p. 66). With this sober overview of poor mathematical performance in South African schools in mind, it is now important to become both introspective and forward-looking. Cognisant of the current crisis in mathematics in South Africa, we need to be looking for ways to improve the situation.

## 2.3 A 21st CENTURY VIEW OF MATHEMATICS: A SOUTH AFRICAN PERSPECTIVE

It is said that teaching only occurs when someone has learned something. If this idiom is applied as a pivot in mathematics, then what is it that is taught and what is it that students learn when teachers and students are engaged in mathematics? In asking this question, I am not seeking answers to the specific curriculum content that is encountered in the mathematics classroom. Rather, it is important to try to discover the principles, the philosophy, the critical driving forces upon which that curriculum is constructed because that must drive *what* we teach and *what* we learn.

### 2.3.1 THE CAPS-CURRICULUM DEFINITION OF MATHEMATICS

South Africa's Curriculum and Assessment Policy Statement (CAPS) document for mathematics defines mathematics in the following way:

> Mathematics is a language that makes use of symbols and notations to describe numerical, geometric and graphical relationships. It is a human activity that involves observing, representing and investigating patterns and quantitative relationships in physical and social phenomena and between mathematical objects themselves. It helps to develop mental processes that enhance logical and critical thinking, accuracy and problem-solving that will contribute in decision-making.
>
> Department of Basic Education, 2011, p. 4



The above-quoted definition, written in the present tense, contains only three sentences, but constitutes a deeply-conceptualised and tightly articulated definition of mathematics. Indeed, every word contained in the definition is saturated in meaning.

In the first sentence, the character of mathematics is identified by the abstract-noun, "language" – this word used instead of "mathematics is a science." Similarly, the use of the verb "describe" and abstract-noun "relationships" co-construct a focus on the core mathematical concepts which are number-systems, shapes and figures, and visual representation of data.

In the second sentence, the noun “activity” and the verbs “observing, representing and investigating” collectively suggest that *all* participants are actively engaging in their own construction of mathematical knowledge. I have italicised “*all”* because the phrase, “it is a human activity…” (Department of Basic Education, 2011, p. 4), suggests that the study and understanding of mathematics is a perfectly natural condition of human existence and, as such, is a subject which should be made available to all human beings.

The second sentence in the definition is important because it markedly shifts the philosophical stance of mathematics away from the *tabula-rasa* or empty-vessels view of learners that was an endemic feature of a behaviourist view of pupils and of transmission-mode teaching and learning for the greater part of the 20th century. Further, in the same sentence, the deliberate use of the nouns and abstract-nouns, “patterns, relationships, phenomena and objects” all point to a view of mathematics as being located, and thereby available to us, in the real world.

The third sentence describes an epistemological rationale for studying mathematics. “Develop[s]” is a doing word – a verb. Mathematics develops, “mental processes that enhance logical and critical thinking, accuracy and problem-solving, [and] decision-making.” (Department of Basic Education, 2011, p. 4). Collectively, these adjectives and nouns point to a view of mathematical knowledge that is far wider than, “there’s an example in the text-book, on page 24, and do not ask me again.” Further, as exemplified in Chapter 1, (1.2.1.1 – 1.2.1.9), the third sentence in the definition aligns the South African epistemology for mathematics with views held by other countries.



In addition to the above, an analysis of the Specific Aims contained within the CAPS (2011) policy document shows congruence between the specific aims and the definition. There are eight aims, and they contain many ***conscientizing* phrases. Such phrases include, “**critical awareness; confidence and competence; without being hindered by a fear of mathematics; spirit of curiosity and a love; beauty and elegance; creative part of human activity; deep conceptual understanding; application to physical, social and mathematical problems; the study of related subject matter” (Department of Basic Education, 2011, p. 4)

The CAPS definition of mathematics reveals a view of mathematics that is located in the real world, is socially mediated, actioned and problem-solving oriented. This being the case, and building upon the groundwork laid in Chapter 1, (1.1.2; 1.1.3; 1.2.2), the purpose and nature of mathematical problem-solving and word-problems must be explained more fully.

## 2.4 PROBLEM-SOLVING IN MATHEMATICS

Uniquely, human beings have the cognitive capacity to think about, strategize and solve complex problems (Bukatko & Daehler, 1995). From infancy through adulthood, the human capacity to solve problems becomes more sophisticated and multifaceted as layers of skills-sets, learning and life experience are brought to bear on problems at hand. Krulik and Rudnick (1980, p. 4) explain problem-solving as being, "the means by which an individual uses previously acquired knowledge, skills and understanding to satisfy the demands of an unfamiliar situation." Common features found in our distinctly human ability to solve problems include:

1. having an ability to represent problems in ways which enable investigation. These representations may take the form of tactiles, visual and/or mental images, words, symbols, maps and so on;
2. a mental flexibility to plan, to establish a sequential, route-marked path to follow to proceed through problems;
3. in the face of many alternatives, have an ability to strategically select an optimal path to follow, and,
4. when confronted with new problems, to be able to cognitively transfer and apply previously learned knowledge and skills into similar situations (Bukatko & Daehler, 1995)



These capacities resonate well with the CAPS definition of mathematics (2.3.1)

### 2.4.1 GOOD VERSUS POOR PROBLEM SOLVERS

Carson (2007, p. 9) suggests that a distinguishing feature between struggling versus successful problem solvers is that successful persons make use of comprehensive, cognitive knowledge bases. People who are good at solving problems draw on many rehearsed skills-sets and reflect upon experiences of previously encountered, similar problems. Indeed,

> …gifted mathematics students have a repertoire of ideas, strategies, and representations that seem to be organized into a highly sophisticated network of knowledge, equipping them with powerful ways to approach problem solving situations.
>
> English, Lesh, & Fennewald, 2008, p. 8

This *expert* knowledge is only acquired through experience and time: experts draw on domain specific knowledge, a vast content-specific vocabulary, factual, semantic, schematic and strategic knowledge (Mayer, 1997, p. 388 - 392). Further, Courtney-Clark and Wessels (2014), citing Kilpatrick, Swafford and Findell (2001), suggest that this proficiency is demonstrated through five interwoven strands, namely, conceptual understanding, procedural fluency, strategic competence, adaptive reasoning and productive disposition.



In effect, the critical thinking invoked by problem-solving requires us to delve into our cognitive toolboxes and to then extract the particular mix of content-specific skills, procedures and factual knowledge that will help us to successfully solve a particular problem (Carson, 2007, p. 10). Further, Carson (2007, p. 12), presents as an example a pair of chess players, one an expert, the other a novice, and makes the point that "an expert chess player is not a better problem solver, he or she just has a more extensive knowledge base than a novice player." Novice students have the facility to develop more comprehensive and better organised knowledge bases. When these expanding knowledge bases are coupled with *many* learning opportunities in which they practice *similar* problem-solving strategies, we can anticipate that *over time* these experiences will enable them to become expert problem solvers.

### 2.4.2 GALOTTI'S LIST OF PROBLEM-SOLVING TECHNIQUES

Galotti (2014), a psychologist, suggests that humans make use of a limited number of general problem-solving methods to solve most of the problems that we encounter. Techniques that humans use include methods such as:

1. *Generate and Test:* in which one generates an initial potential solution strategy, tests its efficacy and, upon reflection, may iteratively work through further cycles of generating and testing with the goal of refining towards an optimal solution (Galotti, 2014, p. 254).
2. *Means-ends Analysis:* in which one compares that which is available with that which is required. Although it is similar to the *generate and test* method listed above, it may not easily produce the most efficient and effective solutions. However, it does require the problem-solver to closely interrogate potential sub-sections within a problem (Galotti, 2014, p. 255).
3. *Working backwards:* in which one treats the desired endpoint as a frame of reference from which, working backwards, one perceives the various stages which one might need to negotiate to achieve that goal (Galotti, 2014, p. 255). Outcomes-based education provides a very powerful example of this problem-solving technique. When it was in vogue in South Africa in the late 1990s and first decade of the 21st century, teachers had to ascertain desired outcomes and then, *working backwards*, had to determine the various skills, knowledge-sets, values, etcetera, that needed to be built into their learning programmes to achieve those stated outcomes.



4. *Backtracking:* requires one to analyse and use provided data to make "provisional assumptions" (Galotti, 2014, p. 255), which thereby enable one to make a start to solve the problem. Often, false starts are generated, but if one keeps a record of these false starts, the errors lead to refinement as one carefully picks a path back to its source. A typical example of this is the situation where one has lost one's car-keys, and needs to reflect on possible places where the keys might have last been used/placed.
5. *Reasoning by analogy:* in which one reflects upon similar, past experiences to create viable analogies to solve the problem at hand (Galotti, 2014, p. 258 - 260). While the contexts and details may be different, the prepared brain can intuitively reveal to itself analogous characteristics between past and present problems which enable it to use those past experiences and problems to deal with current problems. In effect, the brain cognitively transfers past learning experiences into the conscious mind so as to use those experiences as mental scaffolds for solving the problem at hand. (Bukatko & Daehler, 1995, p. 348 - 349).

While any mathematical problem might draw from any or all five of Galotti's (2014) list of problem-solving techniques, *reasoning by analogy,* (later in this thesis, 2.6.1), referred to as *supportive met-befores*), plays a significant role in successfully solving mathematical problems.

### 2.4.3 MATHEMATICAL EXERCISES AND PROBLEMS

The term *problem* is used somewhat ubiquitously in mathematics to cover all manner of mathematical endeavour. Therefore, it is useful to distinguish mathematical exercises from mathematical problems. Mathematical exercises are typically presented as a series of low-order, repetitive, text-book based, practice *problems* that are presented to learners to complete: typically, they develop instrumental knowledge through the use of stylised routines and procedures to achieve predicable answers (Badger, Sanguin, Hawkes, Burn, Mason, & Pope, 2012, p. 10). While they are important, these routines-based, practice-makes-perfect, drill-and-rote mathematical exercises must be complimented by non-routine *problems* for which there are no immediate algorithmic solutions. Such problems require relational thinking and can elicit critical, creative, innovative and systematic thinking.



### 2.4.4 TYPES OF MATHEMATICAL PROBLEMS

Zanele (2015, p. 18 - 19) suggests that it is possible to conceive of all mathematical problems as existing on a three-tiered, hierarchical ladder. The least cognitively demanding of mathematical problems are said to be *highly structured problems*. Such problems might include mathematical exercises. These problems are solved through the consistent use of practiced, step-by-step solutions which generate predicable and convergent (the one correct answer) answers. Galotti (2014) uses a similar term, *well-defined problems*, to explain the nature of such problems, which:

> have a clear goal (you know immediately if you have reached the solution), present a small set of information to start from, and often (but not always) present a set of rules or guidelines to abide by while you are working towards a solution.
>
> Galotti, 2014, p. 252

On a slightly higher plane, we find *moderately structured problems* (Zanele, 2015, p. 18 - 19; Galotti, 2014, p. 252). When students encounter these problems, they are required to demonstrate flexibility and a capacity for adaptation, and to use different converging strategies to achieve an outcome.

On the highest plane of problem-solving, students work with *ill-structured problems* (Zanele, 2015, p. 18 - 19). These imprecise problems introduce a requirement to observe and understand the problem from multiple perspectives and to use past experiences to inform the solution strategies that are to be used by the problem-solver. From this work, many better or lesser satisfactory solution-strategies and answers may be generated and these, in turn, may give rise to further investigation.

### 2.4.5 BENEFITS OF ACQUIRING MATHEMATICAL PROBLEM-SOLVING SKILLS

Mathematics equips humans with skills that enable us to solve real-world problems (Siemon et al., 2012, p. 18). Although students may not perceive this as its end-goal, the processes of mathematics are driven by a desire to solve problems. Indeed, many mathematics teachers would support the view that, "solving problems is not only a goal of learning mathematics but also a major means of doing so" (Akinsola, 2008, p. 80). Still, it must be acknowledged that a requisite foundational-knowledge and skills-sets must be put in place: this cognitive toolbox of mathematical competences must be instantly available to the student and its construct must be informed by the labour of repetition and many *exercises*.



Mathematical problems bring the subject to life. They provide an affirming rationale for the requirements that students learn basic skills such as times-tables, BODMAS, measuring, converting, and so on. Problems enable students to transfer these previously-learned low-order skills into novel problem-solving situations. When solving problems, they may adopt and adapt numerous strategies to facilitate their finding solutions (Siew Yin, 2010). This, in itself, is quite a creative process. For those persons who are well-versed in the ways of mathematics, synergies in solving problems and interpreting visual stimuli, analysing patterns, constructing geometric figures, etcetera, are found in abundance.

## 2.4.6 TYPES OF UNDERSTANDING USED IN SOLVING MATHEMATICAL PROBLEMS

Siemon et al. (2012, p. 71) suggest that when students successfully integrate previously learned skills, concepts, routines and experiences, they demonstrate *instrumental* (or procedural) understanding. When intuitive judgement calls are added to the mix, in other words, when students successfully blend instrumental understanding with creative thinking and hunches, they demonstrate *relational* understanding. In turn, when mastery of content knowledge, skills and values is achieved, students demonstrate *compression* (Gray & Tall, 2007). Compression is a combining of instrumental and relational understanding. Put differently, knowing something deeply, "is not to know it as an entity having a life of its own, but it is to know it in relation to something else" (Mwakapenda, 2004, p. 35). The achievement of compression frees up cognitive space for problem-solving and focused logical and deductive thinking. In other words, compression – that is the skilful and automatic use of foundational mathematical skills and processes – is a prerequisite for effective problem-solving.



Gray and Tall (2007) found that when learners do not acquire sufficient levels of integration of mathematical processes, they cannot achieve compression of mathematical meta-concepts. They found that:

> in many cases the desired abstraction of thinkable concepts often does not occur as required, with many students remaining at a fragile procedural level of operation. This has long-term consequences for the successful teaching of mathematics at all levels.
>
> Gray & Tall, 2007, p. 6

The term *compression* has become a pivotal focus of my own meta-thinking. When I reflect deeply about the day-to-day engagements and meetings that I have with many of my mathematics students, I think that often one of the biggest hurdles that they face is their inability to join-the-dots. For many, compression has not occurred: for example:

1. Many students do not understand nor perceive any relationship between an image of a square, its constant side length and surface area with the symbols of $x^2$, $\sqrt{x}$ and *area*; they seem unable to link the image, symbols and concepts to each other;
2. nor do many students link the fraction $\frac{1}{3}$ to 0,333· to 33,333% and depressingly, nor can they link the concept of one third to a fair-proportioned, physical part of a tactile or a visual representation of one third as in, for example, a pie diagram.

In these sorts of cases, the consolidation of underpinning instrumental and relational sub-concepts and skills has not happened and compression has not been achieved.

### 2.4.7 PROBLEMS EXPERIENCED WITH WORD-PROBLEMS

The word-problems used in Intermediate Phase classrooms in South Africa are located in the real world. They are typically highly structured to moderately structured (Zanele, 2015), and are oriented towards instrumental rather than relational understanding (Siemon et al., 2012). However, many pupils, and indeed even university students, struggle to solve these problems. In large part, this is because many intersecting mathematical skills and knowledge sets have to be assimilated and accommodated *before* they can be applied seamlessly in novel, word-problem situations. Indeed, suggests Mayer (1997, p. 390 - 391), to become expert problem solvers, students must first acquire thousands of individual pieces of domain-specific knowledge. Often, these knowledge factoids and skills-sets are not in place. When any one or any combination of the bouquet of skills and knowledge is missing or misunderstood, where there are gaps in the foundational arithmetic concepts, then the successful solution of word-problems becomes elusive to learners.



In many countries across the world, pupils struggle to solve word-problems. However, albeit that, "an increase in globalisation is fostering the assumption of universalism in mathematics education" (Naroth & Luneta, 2015, p. 268), authors such as Leung, Graf, and Lopez-Real (2006), Hodgen, Marks, and Pepper (2013), and Naroth and Luneta (2015) caution that methodologies and practices that work in one country many not necessarily work in a second country. Indeed, as discussed later in this chapter (2.7.3), Bruner believes that, all other things being equal, our different cultures predispose us to interpret the same information in different ways.

Teachers and researchers are trying to unpack the causes of children's struggles with problem solving and are using that knowledge to seek remediation strategies to enable their learners to cope better with the rigour of word-problems. Two commonly encountered hurdles lie in (1) children's inability to read and understand the requirements of a mathematical problem; this frequently occurs when the language of teaching and learning is not in the learners' mother tongue (Kasule & Mapolelo, 2016, p. 266 - 267), and (2) in their inability to construct enabling representations for said problems. Two exemplars, below, are used to explore these hurdles.

#### 2.4.7.1 LANGUAGE AS A BARRIER TO UNDERSTANDING

In classrooms where mathematical problem solving rather than rote and routines-driven computations are used, good literacy skills are considered to be an indispensable requirement for mathematics problem solving (Evans, Ardito, & Kim, 2017). In such environments, where prior knowledge, skills and conceptual understanding are required, the sentence structures are designed to provide useful real-world contexts for learners. Learners who cannot fathom the asking requirements of word problems – in South Africa, this often occurs because the problems are stated in English while the learners' mother tongue is another language – are placed in a particularly difficult bind. They are precluded from success by a language barrier rather than or before any difficulty in mathematics can be entertained.



Impediments to understanding mathematical word-problems for *English second-language students* include, "the use of low-frequency terms (e.g. "spinner") as well as words that are ambiguous in meaning, such as "one" [won], …and change (a difference, or money received in a financial transaction)" (Barbu & Beal, 2010, p. 3). Long worded problems are awkward for non-mother tongue learners such that, "the impact of linguistic complexity on students' perceptions of the mathematics of the problem" (Barbu & Beal, 2010, p. 14) has a bearing on their ability to construct viable solutions. In effect, when students perceive the linguistic complexity of the mathematics problems to be of a high order, they also perceive that the construction of a solution strategy will prove to be equally difficult.

“Language provides the tools of thought, and carries the cultural inheritance of the communities (ethnic, gender, class, etc.) in which the individual grows up” (Lerman, 1996, p. 137). English, for example, is littered with homonyms and homophones that are easily understood – in context – by English speakers. However, for students who do not use English as a home language, such plays on words can become a nightmare. Thus, when creating word-problems in which the dominant language is intended to be read by non-mother tongue learners, it is prudent to reflect upon the multiple-meanings and nuances assigned to words and the cognitive-havoc that ambiguous words or turns of phrase might unleash. Indeed, one misunderstood word can alter the entire meaning of a mathematical sentence (Sherman & Gabriel, 2016, p. 473).

Evans, et al. (2017) suggest that support structures which may assist non-mother tongue learners include encouraging these learners to write up their mathematical thinking in reflective, peer-assessed journals; requesting these learners to work in collaborative groups – so-called *communities of inquiry* (Splitter, 1991) where bilingual learners may offer support as translators to other learners; getting the learners to apply themselves to real-world mathematics problem solving learning opportunities, and assisting learners to construct notes of problematic vocabulary, phrases and conventions.



In South Africa, Moloi (2015) used a participatory action research lens to investigate mathematical problem solving through play in rural regions of South Africa. Like Evans, et al. (2017), he found that (1) contextualised learning – that is, *contextualised for the rural setting* – enhanced learning, (2) problem solving should be learner centred, and (3) where “the teacher explains everything to learners their potentialities are oppressed and marginalised” (Moloi, 2015, p. 25). By interaction with others, the learners increased their social and linguistic capital, making breakthroughs in terms such as the comparative use of *big, bigger* and *biggest*.

Later in this thesis, in the data analysis chapter, I will recount how simple English phrases such as “twice as much” or “has one third more” or “double” triggers anxiety and misinterpretation for many PGCE mathematics students, so much so that theirs can become a test of vocabulary and literacy rather than of numeracy.

#### 2.4.7.2 POOR REPRESENTATIONAL SKILLS AS A BARRIER TO PROBLEM-SOLVING

Sajadi, Amiripour, and Rostamy-Malkhalifeh (2013, p. 4) suggest, "there is a significant correlation between using efficient representation and efficient word problem solving ability." They found that students who efficiently made use of tactile and iconic representations scored better marks in tests and examinations than those who did not, or could not, construct such representations.

A representation, they suggest, "is defined as any configuration of characters, images, concrete object, [sic] etc. [sic] that can symbolize or "represent" something else" (Sajadi et al., 2013, p. 4). Such representations enable students to visualise situations and to model potential strategies to solve mathematical problems (Barmby, Bolden, Raine & Thompson, 2013). However, carelessly constructed representations of word-problems can obfuscate the problem-at-hand and unwittingly oppress the demonstration of mathematical competence (Rellensmann, Schukajlow & Leopold, 2017).



My own views are congruent with Sajadi et al. (2013) and Rellensmann et al. (2017), above. Indeed, my own thesis is driven by a view that a capacity to construct visual representations of problems – visual scaffolds – potentially liberates problem solving endeavours. However, it is not all plain sailing. Later, in Chapters 4 and 5, I will reveal that (1) on occasion, participants resisted constructing any visual support, preferring to stick with symbolic representations, and (2) sometimes, especially when fundamental *drawing* skills were not practiced and compressed, the visual scaffolds become more of a hindrance than an aid.

Thus, it is fair to suggest that students who are able to use manipulatives or iconic representations to express their understanding of word-problems are potentially better positioned to be able to solve word-problems than students who cannot, but this is not always true (Siemens, 2006).

## 2.5 MATHEMATICS TEACHER EDUCATION

In this section, I examine aspects of mathematics teacher education, pedagogical content knowledge and some arithmetic concerns which have influenced this study.

Haylock and Mann (2014) suggest that in primary school education, one of the most important things that children should learn in mathematics is *how* to learn mathematics Because we can anticipate that in primary school education, most of the experiences of mathematics that learners encounter are orchestrated by their teachers, it is clear that teachers have a profound influence on children's understanding and disposition towards mathematics. Their experiences of mathematics might mostly develop a rote-driven learning mind set or, alternatively, might invoke meaningful-learning.

### 2.5.1 FACTORS WHICH INFORM OUR CONCEPTIONS OF MATHEMATICS

In classrooms where teachers present mathematics as a basket of discrete facts and formulae, rote learned operations and routines, this orientation to mathematics will typically become the view that their learners hold of the subject. Such a view might be said to be instrumentalist, that is, the teaching focus is content-driven and emphasises performance; for learners, the focus becomes mastery of disconnected skills, seeking the right answer and quiet acceptance of courseware (Beswick, 2005). In this sort of classroom, it is quite conceivable that learners may use memorization, mimicry and procedural rehearsal to produce *right* answers, but these answers might be done in the absence of any deep understanding of the completed assignment (Cornbleth, 1987; Tipps, Johnson & Kennedy, 2011). Such practices are unsustainable; learning without understanding, mindlessly trying to memorise list of maths-facts or formulae, and blindly rehearsing particular stratagems can only take one so far.



Similarly, when teachers portray mathematics as reflective, risk-taking and socially constructed endeavour, so too, their learners will usually adopt a similar stance – a so-called *meaningful-learning mind-set* (Haylock and Mann, 2014). Teachers who adopt these positions might be said to hold Platonist beliefs regarding the nature of mathematics: in their teaching they too would be focused on content but would emphasize understanding, and they would encourage their learners to actively construct their own understanding of mathematics (Beswick, 2005).

Thus, because student teachers will typically perceive their ways of knowing mathematics through the lens of their past (school-based) teacher-belief driven experiences of mathematics (Muir, 2008), it is vital that in their undergraduate studies of mathematics, student teachers are encouraged to consider and explore views and understandings of mathematics that might be at odds with their life-experience. Further, while a goal of modern education is to provide optimal teaching and learning environments for all learners, the reality is that many schools are not afforded that luxury (Berry, Bol & Mckinney, 2009). Thus, student teachers must be trained to deal teach in less than optimal working conditions.

### 2.5.2 A NEED FOR SPECIALISED MATHEMATICS TEACHING KNOWLEDGE

In [2.3.1] above, I unpacked the current South African definition of mathematics. In support of this definition, the CAPS documentation provides specific aims that embrace views which encourage learners to develop a critical awareness of mathematical relationships, its relationship with society, our cultures and the environment, a curiosity towards and appreciation of the elegance of mathematics, a recognition that mathematics is a part of all human activity, and a desire to develop within all learners a deep conceptual understanding of mathematics (Department of Basic Education, 2011).



Clearly, this shifts us towards a Platonic view of the nature of mathematics. Analogous with the Platonic view, Tipps, Johnson and Kennedy (2011, p. 75), describe *directed teaching/thinking lessons* in mathematics not as transmission mode lessons but as "interactive and incorporates manipulative materials, visual aids, discussion and argument, and engaging tasks to encourage thinking." In such classrooms the teacher directs learning by introducing and scaffolding new knowledge but students are encouraged to construct their own understanding. This being the case, student teachers in South Africa must be exposed to mathematical ideas and materials that will contribute to these aims.

Indeed, student-teachers need to combine their ability to *do* mathematics – we call this content knowledge (CK) – with complimentary skills which enable them to *teach* mathematics – we call this pedagogical content knowledge (PCK) – which encompasses the unique skills, knowledge and understanding that mathematics teachers should hold in order to effectively teach this subject (Shulman, 1986).

Shulman (1987) identified seven aspects of PCK that student-teachers need to understand, namely (1) mathematical content knowledge; (2) a general understanding of pedagogy, management and organisation; (3) a knowledge of curriculum requirements, materials and programmes; (4) specialised pedagogical content knowledge and professional understanding; (5) a knowledge of their learners; (6) knowledge of the school and the community that it serves; and (7) an understanding of context, philosophical underpinning, drivers and values espoused by the curriculum.

### 2.5.3 MULTIPLICATIVE THINKING AS AN APPLICATION OF SHULMAN'S PCK



Multiplicative thinking is a big idea that enables human beings to work flexibly with (1) a range of numbers which might include whole numbers, rational numbers, fractions, decimals, ratios, proportions and percentages, (2) direct and indirect proportions, (3) enables us to communicate multiplicative thinking – multiplying and dividing – using models and diagrams, words and symbols, and written computations, and (4) enables us to accurately estimate the outcome of our computations (Siemon et al., 2012).

While repeated addition and skip-counting provides a useful access point, it is incorrect and inefficient to treat multiplicative thinking as simply repeatedly adding-on. Multiplicative thinking is a complex task which can take years for learners to properly understand. It incorporates proportional reasoning and the coordinated understanding of how multipliers (or multiplication operators) and multiplicands (the number of objects in each group) relate to each other (Tipps, Johnson & Kennedy, 2011). "The use of such composite units introduces the distinction between *how many* and h*ow much*" (Siemon et al., 2012, p. 354), and shapes our thinking processes. Early and informal introductions to multiplicative thinking exposes children to the concepts of fair sharing (simple proportion), and equal sharing and, at a later stage, enables them to comprehend complex relationships including one-to many, many-to-one, partitioning, unitising, equipartitioning and composite units (Siemon et al., 2012). Early school experiences of multiplicative thinking might be based on counting, which is additive in nature or splitting, which is oriented towards multiplicative operations.

Counting techniques might include counting large collections, repeated addition, skip counting, arrays//regions and repeated subtractions. Splitting might including equalling, doubling and halving (partitioning); and the use of arrays/regions, areas (often demonstrated as proportionally correct areas of rectangles or squares), and Cartesian products (often depicted on a tree diagram). Indeed, *arrays*, as rectangular arrangements of discrete objects, and *regions*, as a continuous rectangular (area based) representations provide particularly useful visual demonstrations of the multiplicative principles (Tipps, Johnson & Kennedy, 2011; Siemon, et al., 2012).

Multiplication and division facts such as the times-tables need to be prioritised, but this done such that these facts are not merely rote-learned but are laden with meaning. Arrays can be used to assist learners to appreciate the versatile and beneficial applications of the commutative property in multiplication (Siemon, et al., 2012). Over time, a more formal recording of multiplication and division and the use of algorithms is introduced into multiplicative thinking such that most children are able to use these techniques by the end of the middle years in primary school.



In South African schools, the mathematics content that is taught is largely driven by the CAPS curriculum policy documents. In turn, aspects of those content areas requirements inform the mathematical content that is offered to student teachers. Five content areas are identified in CAPS, namely (1) Numbers, Operations and Relationships; (2) Patterns, Functions and Algebra; (3) Space and Shape (Geometry); (4) Measurement; and (5) Data Handling (Department of Basic Education, 2011). Within the curriculum, “big ideas” provide a framework for progression. By way of example,

> …*multiplicative thinking* not only encompasses the various meanings and representations of multiplication and division,… but also supports connections between the operations and the base 10 system of numeration,

> the rational numbers, and generalisations associated with propositional reasoning.
>
> Siemon et al. 2013, p. 13.

In the Intermediate Phase (Grades 4 – 6), under the content area of Numbers, Operations and Relationships, Grade 4 learners must, for example, use number facts for units multiplied by multiples of 10 and 100, use skip-counting in 2, 3, 5, 10, 25, 50 and 100 between 0 and at least 10 000, must use estimation, doubling and halving, perceive multiplication and division as inverse operations, use input-output flow diagrams, solve problems of ratio and rate, and group and equally share with remainders (Department of Basic Education, 2011).

Thus, it is clear that it is not good enough that student-teachers simply have an ability to multiply and divide, as such limited subject knowledge would restrict their ability to teach conceptually (Ma, 2010). Indeed, “being able to do mathematics is simply the starting point for the journey into teaching mathematics” (Venkat, Mathews and du Plessis, 2012, p. 111). Instead, student teachers’ knowledge of these operations must extend beyond the instrumental skills of multiplication and division to a relational understanding of the operations. It flows from this that in order to then effectively *teach* multiplicative thinking, student-teachers must acquire the specialised PCK that will enable them to assist their future learners to make sense of this work.



Thus, on one hand, the student-teacher needs to hold a range of computational skills. This might start with perhaps the times-tables and would, over time, extend to long multiplication and multiplication with decimals and fractions. However, on the other hand, these techniques have to be consolidated with an understanding of nature of multiplication, of how the product is established through repeated aggregation – perhaps demonstrated in an array, and the scaling structure of multiplication – perhaps demonstrated on a number line.

While an expert teacher might easily turn to an empty number line, or might construct an array using bottle-tops or draw an area model of the multiplier and multiplicand or use a vertical layout for computation or ‘move the comma’ in the case of multiplications by 10, etcetera, this expert knowledge must be taught to student-teachers (Haylock, 2002). This tallies with a view that teaching is “an extremely complex task” (Siemon et al., 2013, p. 53), which, rather than relying on common sense and an innate ability to do it, in fact must be learned.

Where a Platonic understanding of the nature of mathematics is embraced, complimentary educational theorists voices should be introduced into the student-teachers’ university experience. Among many others, Ausubel’s discussion on prior knowledge and advance organisers (Ausubel, 1968); Piaget’s theory of cognitive development, the concepts of schema, adaptation via assimilation, accommodation, equilibration and stages of cognitive development (Piaget, 1986; Wadsworth, 1989; von Glasersfeld, 1996; Piaget, 2000); Vygotsky’s zone of proximal development (ZPD) (Bodrova & Leong, 1996); Bruner’s learning theory (Bruner, 1966); and Gardner’s theory of multiple intelligences (Gardner, 1993; Gardner, 1997), all provide useful educational theory footholds.

In addition to an ability to do mathematics and a knowledge of educational theoretical knowledge, student-teachers also need to know what covert hurdles might make mathematics difficult for learners. Ball (2009) suggests that in their university classes, student teachers should be required to continuously practice mathematics, represent mathematical ideas in different ways and seek different explanations for the types of mistakes that they and their future learners might make in mathematics.The specialised knowledge that effective teachers hold, Ball, Hill and Bass (2005), demonstrates such teachers not only have an ability to evaluate a correctly undertake multiplication calculation, but can also identify the nature of mistakes which learners make in achieving incorrect products, as seen, in *Figure 2.1*, below:

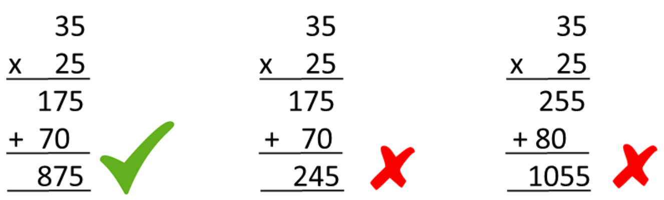


*Figure 2.1.* Example of PCK applied to multiplicative errors

While many individuals might compute the correct answer, it takes PCK to reveal the *nature* of the mistakes which might occur while working through such calculations. But more than that, having established the nature of the particular mistake that had occurred, competent teachers would then select an appropriate strategy to assist the learner to overcome the obstacle. The critical point to make here is that student-teachers must be taught to assess their learners' work not on the basis of marking it right or wrong, but with a view to understanding the nature of difficulties and finding appropriate mechanisms to support their learners. Such diagnostic assessment competence "is believed to be a prerequisite for teachers' abilities to promote specific skills in their students." (Friedrich, Jonkmann, Nagengast, Schmitz, & Trautwein, 2013, p. 26).

Gardner (1997), suggests that logjams in understanding can often be eliminated by reconceptualising problems in novel ways and suggests that:

> Reconceptualization is most-likely to come about if an individual has multiple representations of a problem – that is, if he or she can think about the problem in a number of different ways, particularly ways that have not previously been brought to bear on that problem. The more that an individual can make use of his unique strengths in attacking a problem, the more likely he will arrive at an approach that holds special, hitherto unanticipated promise for illuminating that problem.
>
> Gardner, 1997, p. 149



Thus, in overcoming an obstacle to find the product of 35 x 25, as in the example above, the expert teacher might encourage learners to adopt a strategy that might take the form of a visual representation – an array or an area diagram. It has to be acknowledged however, that different learners might be more or less inclined towards a visual representation (Nardi, 2014), in which case an alternative computational representation might be adopted. Indeed, the actual signs and symbols used in multiplications, the ways of presenting and recording information and multiplication with zeros, can all prove problematic for many learners (Siemon et al., 2012). Thus, strategies for providing remedial support need to be taught to student-teachers.

### 2.5.4 FRACTIONAL THINKING AS AN APPLICATION OF SHULMAN'S PCK

In South Africa, in the Intermediate Phase of schooling, that is, Grades 4 – 6, Grade 4 learners are exposed to intermeshing fractional thinking ideas. These include comparing and ordering halves, thirds, etcetera through eighths; describing and comparing common fractions in diagrams, adding common fractions with the same denominators and understanding equivalence. Learners are also expected to solve word problems containing fractions, grouping and sharing. By the end of the phase, when learners complete Grade 6, the sophistication of the asking requirements increases and includes addition and subtraction of mixed numbers, fractions of wholes, percentages, two digit denominators, equivalence between common, decimal fractions and percentages, and addition and subtraction of decimals to two decimal places (Department of Basic Education, 2011).

Clearly, even if student teachers understand all of these content aspects of fractions – often, however, they do not – then still, they need to assimilate requisite PCK skills so as to become effective teachers of fractions.



Many fractional facts and ideas are misunderstood and can hamper progression. Learners may not conceive the symbolic fractional numbers of $\frac{1}{2}$ , $0{,}5 \ and \ 50\%$ as representative of the same amount (Tipps, Johnson & Kennedy, 2011), or may not understand how to negotiate calculations with mixed numbers (Petit, Laird & Marsden, 2010).

Among conceptions of fractions that should inform student teachers' PCK understanding of fractions, they should be able to (1) demonstrate and use a variety of models of fractions to explain different perceptual features of fractions, (2) understand that learners may make use of flawed whole number reasoning to solve problems involving fractions, (3) use partitioning for understanding and explaining part-to-whole relationships, ordering and comparing; equivalence and operations with fractions, (4) use number lines to assist learners to understand the concepts of relative size, equivalence, the effects of adding and subtracting fractions, and the density of fractions, (5) employ procedural skills and understanding in formalised adding and subtracting, (6) understand that multiplying and dividing fractions are amongst the most complex tasks that are undertaken in mathematics in the primary school (Petit, Laird & Marsden, 2010). Siemon, et al. (2013, p. 569) suggest however, that because "fractions are inherently multiplicative, multiplying and dividing fractions is in many respects more straightforward than adding and subtracting fractions." Facets of working with fractions, and conceptual and skills aspects of applicable PCK are discussed below.

Haylock and Manning (2014) suggests that models offer visual and intuitive insights that symbolic representations might obscure. Where Haylock and Manning, 2014, and Petit, Laird and Marsden, 2010, use the term *models* other authors use *representations* (Siemon et al. 2013; Nardi, 2014). However, albeit that the mathematician's models provide a visible trace of the attributes of problems, the goal must be the development of an ability to mentally visualise scenarios and solutions: student teachers must understand that "modelling is a means to the mathematics, not the end" (Petit, Laird & Marsden, 2010, p. 1).



Visual models might include artefacts, number lines and number strips, arrays, shaded sections of rectangular and circular shapes, fraction charts, and base-ten materials. In cases where learners are at risk or have cognitive challenges, judiciously-used math cards which break down fractional computations can be used as scaffolds (Tipps, Johnson & Kennedy, 2011). Continuous fraction models – area diagrams and number lines – are based on conceptions of being infinitely divisible; discrete fraction models, as representations of a whole – a packet of sweets or a sack of potatoes – represent collections which are not intended for further partitioning (Siemon et al. 2012).

Each of these models offers particular benefits and limitations. Circles, for example, used as fractional representations (or pie charts), may display proportions, parts of whole and whole effectively, but cannot illustrate zero values. Similarly, the icons used in pictograms, when used as representations of larger groups, can be misinterpreted to imply singular items (Rickard, 2013).

Further, the homophones, *whole* and *hole*, and the colloquial use of whole, as in, *Trevor ate a whole half of a chocolate cake,* can create confusion. This language complication would be particularly true for non-mother tongue speakers. It is, however, quite acceptable to use informal language such as top number and bottom number alongside the formal vocabulary of fractions (Haylock, 2002). However, it must be noted that links between executive function, bilingualism and mathematical achievement have suggested that bilingualism can favourably enhance mathematical reasoning and problem-solving capacities (Hartanto, Yang, & Yang, 2018).

Other challenges which are linked to misunderstanding the concept of whole include learners' inability to perceive the whole when they are provided with a part of the whole – *If ¾ of the class is 24, how many in the class?;* not considering the original size of the whole when apportioning part-sizes to fractions – *If 0,5 of 120 sweets were eaten, how many sweets remain?*, and making incorrect comparisons with wholes that are not of the same magnitude – *Determine 40% of 50 and 100* (Petit, Laird & Marsden, 2010).

Fraction diagrams can be used to demonstrate the concept of partitioning, that is, the whole contains equally sized parts. In the case of common fractions, problems which are encountered include learners' misinterpretation of the role of the denominator or misinterpretation of a partitioned fraction diagram. Tipps, Johnson and Kennedy (2011) encourage teachers to allow learners to consider slices of bread, fruit and liquids for partitioning, and to then encourage learners to fold paper shapes such as squares, hexagons and circles into halves, thirds and so forth. Siemon et al. (2012), caution that when learners populate fraction strips that are pre-designed for their use – perhaps by their teacher or in s textbook – many misunderstandings can manifest; they suggest that deep understanding can only be achieved by encouraging learners to construct their own fraction diagrams and that then these diagrams are populated based on their own understanding of unit parts and partitioning.



Ratios, rates and percentages are built up as extensions of fractional understanding and equivalence. These concepts require proportional reasoning. Proportional reasoning requires learners to compare quantities, often as percentages, but this is a complex undertaking which develops over time. Indeed, Tipps, Johnson and Kennedy (2011), suggest that up to ninety percent of adults struggle with proportional reasoning. Typically such problems compare either two phenomena – for example a comparison of distances covered by two cars in different times to identify their average speeds, or problems where three out of four quantities are known, with the requirement that the fourth be established (Siemon, et al. 2012).

Flawed understanding of the effects of a percentage increase on an amount and subsequently a decrease on the revised amount can hamper understanding: for example, *raising 100 by 10% produces 110, but then subsequently decreasing 110 by 10% produces 99*. Clearly, for mathematicians, the starting points in the two calculations are different, but for many learners, the rationale seems counterintuitive (Haylock, 2002). For these sorts of stumble-stones, a visual representation in the form of a pair of partitioned arrays, might be used to clear up the misunderstanding.

Student teachers need to assist learners to comprehend relative size and number density, that is, the property that between any two fractions, common or decimal, there are an infinite number of other fractions.

Number lines are useful in assisting teachers to understand learners' spatial sense (Gardner, 1993), sense of relative size, and capacity to sort and order fractions, for example from smallest to biggest. Various combinations of plays on denominators and numerators, or one, two and three digit decimals can be used to explore learners' ordering skills (Tipps, Johnson & Kennedy, 2011).

Number lines are also particularly good at helping learners to make sense number density, and for this illumination, a magnifying visualisation of a section of a number line, displayed perhaps via a data-projector, might prove beneficial. Alternatively, teachers might use a *washing-line* activity in which the washing line becomes populated with fractions attached to pegs (Siemon et al., 2013). The point of this activity is to demonstrate that no matter how tightly two fractions might be packed alongside one another on the number line, there will always be a limitless number of other fractions between them (Petit, Laird & Marsden, 2010; Tipps, Johnson & Kennedy, 2011). Indeed, this zooming in, approaching zero,  is analogous with the first principles conceptions which underpin calculus.

In South Africa, by the end of Grade 6, learners are expected to demonstrate procedural fluency in adding and subtracting mixed numbers (Department of Basic Education, 2011). Petit, Laird and Marsden (2010), suggest that procedural fluency – that is, the application of formal computation techniques that are used to add and subtract mixed numbers – must be complimented with conceptual understanding and the use of appropriate models – visual representations – of fractions. They suggest that visual representations in the form of area models, partitions and number lines, can unlock meaning. Siemon et al. (2013), offer a similar view; they too suggest that mixed numbers are best understood when, in the early stages, visual stimuli – an array perhaps – are used in tandem with the more symbolic and somewhat ritualised computational techniques. Such models enable learners to visually link underpinning fractional concepts such as equivalence and common denominators and bring them to bear on the *steps* in procedural processes.

Decimals are handled in a somewhat similar manner. In South Africa, by the end of the Intermediate Phase, Grade 6 learners are expected to attend to "addition and subtraction of decimal fractions with at least two decimal places" (Department of Basic Education, 2011, p. 11). Here to, as above, student teachers should understand that visual representations – for example a number line or 10 x 10 grids (Siemon, et al., 2013) or Cuisenaire rods (Petit, Laird and Marsden, 2010) – used alongside the more formal columns-based setting-out techniques (Haylock & Manning, 2014), offer a conceptually powerful link for procedural fluency.



Multiplication and division of common fractions are not discussed in the Intermediate Phase in South Africa, but student teachers should understand the PCK skills, concepts and problems that learners need to negotiate when they encounter these types of calculations. Multiplication of decimal fractions is included in the Intermediate Phase (Department of Basic Education, 2011).

Multiplying and dividing common fractions requires a resetting of one's mind-space. Many learners carry over from their understanding of multiplication with whole numbers, the misconception that multiplication with common fractions will always result in a numerically larger product. Thus, while this conception is true for ¼ x 3, it is not true for ⅜ x ½. Petit, Laird and Marsden (2010), suggest that partitioned area models make the rationale of why these different outcomes are achieved, more accessible for learners. An example of the application of a visual strategy might be that the product of $\frac{1}{3} \times \frac{3}{4}$ can be found by folding a sheet of paper into thirds in one orientation and then folding it into quarters in the opposite orientation, thereby creating 12 congruent smaller areas. This folding and filling of spaces reveals the product to be $\frac{3}{12}$ or $\frac{1}{4}$ (Sowder et al., 1998). When such a strategy is used alongside the procedural routines of *multiply the top numbers, then multiply the bottom ones, then simplify if possible*, the conceptual meaning conferred by the visual scaffolds assists children to build up understanding (Tipps, Johnson & Kennedy, 2011).

Similarly, while division with whole numbers – for example, $34 \div 2 = 17$ typically generates a quotient that is smaller than the dividend, this is not necessarily true with common fractions. While, for example $\frac{1}{2} \div \frac{5}{2} = \frac{1}{5}$ and clearly, the quotient $\frac{1}{5}$ is smaller than the dividend $\frac{1}{2}$, in a separate example such as $\frac{1}{2} \div \frac{1}{4} = \frac{2}{1}$ the quotient $\frac{2}{1}$ $or$ $2$ is larger than the dividend $\frac{1}{2}$. PCK skills and knowledge are required to unpack this type of conundrum. While most learners are exposed to the phrase *invert and multiply* and successfully (procedurally) complete division computations, most do not perceive how it is that division with fractions supplies its surprising answers (Siemon et al, 2012).



If the approach that is adopted is quotitive division, the divisor is perceived as the size of a group, and the quotient tells us how many groups – of this size – can be obtained from the dividend. So, the answer tells us the number of parcels, *each the same size as the divisor*, that are available to us. Thus, in $\frac{3}{1} \div \frac{1}{2}$ the quotient 6 tells us that there are 6 groups of $\frac{1}{2}$ in 3. Similarly, we would find 8 groups of $\frac{1}{2}$ in 4; 10 groups of $\frac{1}{2}$ in 5, an so on. Although a bit obscured by the shapes of the fractions, the analogy extends to, for example $\frac{3}{5} \div \frac{2}{7}$; here, the quotient is found to be $\frac{21}{10}$ $or$ $2\frac{1}{10}$ In effect, this means that there are $2\frac{1}{10}$ groups of an amount $\frac{2}{7}$ in the dividend fraction $\frac{3}{5}$

Where a partitive approach is applied, the divisor is perceived as representing the total number of equal parts, and the quotient of division establishes the *size* of each part (Siemon et al. 2012).

### 2.5.5 PRINCIPLES WHICH UNPIN EFFECTIVE MATHEMATICS TEACHING PRACTICE

Muir (2008), identified six principles for effective teaching of numeracy, where numeracy implies extending mathematical competence to include contextualised applications in everyday life. These principles include attributes that should be inculcated into student teachers conceptions of mathematics and include (1) making connections between different areas of mathematics; (2) Challenging all learners in the class and holding high but realistic expectations of the learners; (3) teaching for conceptual understanding and linking concepts across the curriculum; (4) holding purposeful and challenging discussions on mathematical practices and reasoning with learners; (5) focusing on the big ideas in mathematics by engaging learners in the importance of being able to use a variety of methods to examine problems at hand; and (6) holding, displaying and encouraging positive attitudes towards mathematics as a worthwhile and enjoyable pursuit.

Similarly, Haylock and Mann (2014), suggest that meaningful-learning mind-sets are created when learners are encouraged to make cognitive connections between intersecting mathematical ideas. A *connections model* is proposed; in this model, deep mathematical understanding is presented as being able to connect vocabulary, use symbols, construct pictures and engage in real world experiences. Further, in the primary school, children should understand equivalence, transformation, conservation of number and classification as big mathematical ideas which underpin their understanding of mathematics. Teachers need to assist there learners to construct these "networks of connections" or schema (Haylock and Mann, 2014, p. 30).



Moschkovich (2013), suggests that classroom practices which support mathematical reasoning, use multiple resources and encourage all learners' participation create opportunities for all learners to develop mathematical thinking. Equity of opportunity for all learners is enhanced when teachers' practices are informed by an understanding of their students' history and life experiences and use this knowledge to create contextualised learning opportunities. Vocabulary, for non-mother tongue learners should extend beyond rote to understanding. Teachers should understand use curriculum materials. Moschkovich (2013, p. 48) suggests that mathematics teachers who make a positive difference in their learners accomplishment and their conceptual development, typically adhere to two principles, namely (1) in their classes, these teacher focus on concept development, not routines, and (2) they "give students the time to wrestle with important mathematics."

It is clear that in intent, there is close dove-tailing between the seven aspects of PCK (Shulman, 1987), the six principles for effective teaching of numeracy (Muir, 2008), Haylock and Mann (2014), and Moschkovich (2013), above.

Further, Muir (2008), demonstrated that the (1) relationship between teachers' belief systems of mathematics, their (2) knowledge of mathematics, and their (3) classroom practice, influenced what was taken as worthwhile. Here, Muir's (2008) views are analogous with Beswick (2005), who contrasted teaching practices in transmission-mode classrooms with Platonic classrooms.

Muir (2008) found that in primary schools, teachers often were able to list and use numerous facts and formulae but lacked conceptual understanding of the mathematics. Further, many teachers admitted that they were anxious, lacked confidence in and felt professionally unprepared to teach mathematics (Ma, 2008; Haylock & Manning, 2014). In turn, these factors contributed to their lesson-planning and the way they blended courseware, questions and assessment in their classes. Similarly, Ma (2010), found that in one group of average teachers in the United States of America, not one of the group displayed any deep understanding of elementary mathematics and concluded that any improvement in learners' mathematics scores would necessitate first, an improvement in their teachers' knowledge of school mathematics. And, to break the cycle of low-quality education and low quality teachers, she suggests, a refocusing on initial teacher preparation is required.



In their own research, Sowder, Philipp, Armstrong and Schappelle (1998), drew conclusions which were congruent with Muir (2008) and Ma (2010), above and found that many primary mathematics teachers held "weak and unconnected knowledge of a subject they taught daily … [and were]… struck by the teachers' lack of understanding of what their students knew and did not know" (Sowder, et al., 1998, p. 180). Similarly, Berry, Bol and McKinney (2009) found that pre-service teachers did not understand mathematical content, did not know how learners learn mathematics, lacked instructional and assessment knowledge, and did not know how to teach for understanding.

Muir (2008) established that teachers might be more or less categorized as being connectionist, transmission or discovery oriented. Teachers who used connectionist principles, that is, teachers who encouraged children to construct their own unique, deep and meaning-filled conceptions and representations of courseware, were found to provide the most beneficial learning opportunities for learners. Further, in her research, Muir (2008, p. 93), suggests that the "use of representations was found in this study to be particularly relevant to developing the principles of *conceptual understanding* and *making connections*." Nor are these representations limited to the use of manipulatives, but also include diagrams, words and symbols. However, the mere use of representations is not enough; it is *how* they are used that makes the difference.

Further, effective teachers provide instructional examples which are designed to provide conceptual and structural clarity for their learners. However, because one illuminating example cannot be assumed to provide a suitable scaffolding framework for all learners, numerous strategies must be employed to reveal the full scope of a mathematical idea or concept (Bruner, 1966; Gardner, 1997; Muir, 2008).



Further, the choice of tasks which are used in the classroom must be challenging for all learners and must bring with them varying aspects of cognitive demand and skills development (Bloom, Max, Furst, Hill & Krathwohl, 1956; Muir 2008). In this regard, the consideration of Vygotsky's Zone of Proximal Development, and his framework for understanding, namely that (1) learners need to construct their own knowledge, (2) development is linked to social circumstance, (3) learning can be in advance of development, and (4) language, or the lack thereof, plays a pivotal role in mental development, can assist student teachers to factor in appropriate engagement points and scaffolding assistances (Bodrova & Leong, 1996).

Gestures can be used by teachers and learners as a visual demonstration "that the body is involved in thinking and speaking about the ideas expressed in those gestures"

(Alibali & Nathan, 2011, p. 247). As such, *pointing*, *representational* and *metaphoric* gestures can represent embodiments of our perceptions of mathematics and thereby reveal our mathematical thinking to others. When gestures are used meaningfully in conjunction with other visual-verbal scaffolds, they assist learners to make sense of mathematical ideas. Gestures are often spontaneously produced and to serve as a universally understood form of communication that can express meanings and conceptions of mathematical ideas (Edwards, 2009).

Modelling is the practice where teachers demonstrate “what to do an / or how to do it” (Muir, 2008, p. 95), with learners. Modelling requires teachers to strategically seek inputs from learners and should not be confused with reinforcement-driven, behaviourist-inspired, transmission-mode practices where learners copy-down and mindlessly repeat routines (Wood, 1998). Instead, modelling provides opportunities to develop conceptual understanding while maintaining a focus on the mathematics at hand. In modelling, teachers and learners alike might include various gestures, demonstrations, representations and discussions that focus on interesting examples.



From all of the above, it is clear that albeit that student teachers need to hold many intermeshing skills and competences, a continuous thread demanding the use of models / representations / visual scaffolds has been revealed, their contribution being that said models can illuminate hidden mathematical concepts and can elevate rote-learned routines into deeply conceptualised mathematical computations. I turn next to the role visualisation can play in assisting learners to understand mathematical ideas.

## 2.6 VISUALISATION AS A STRATEGIC TEACHING AND LEARNING TOOL IN MATHEMATICS

Almost thirty years ago, Fosnot wrote, “The discipline to which the greatest disservice occurs in the schools is perhaps mathematics. Unfortunately, it is often taught solely as arithmetic computation, with little or no attempt made at facilitating reasoning or the development of logic” (1989, p. 71). Unfortunately, the data found in Chapter 4 will confirm that *thirty years later*, for many participants in this study, mathematics has been presented to them as a series of instrumentally learned, arithmetic routines and computations. However, as seen in [2.3, 2.4, and 2.5] above, South African learners are now required to deeply engage with mathematical ideas, are required to develop relational understanding and to use mathematical knowledge in creative and flexible ways to solve problems. Their teachers are encouraged to use PCK understanding and appropriate teaching techniques to assist learners to develop their own mathematical understanding. Teachers are encouraged to use apparatus and other visual representations of big mathematics ideas; these concrete and iconic representation can assist us to illuminate and understand concepts and characteristics that might be obscured by symbolic representations of mathematical ideas.

Because it is now recognised that mathematical understanding and reasoning can be negotiated and enhanced through the sensible use of manipulatives and sketches of ideas, in this section of the literature review I pay close attention to the role that visualising strategies can play in assisting students in their mathematics. However, it has to be acknowledged that visual strategies may not prove beneficial for all learners. This is discussed in [2.6.3.1], below.

### 2.6.1 THE ROLE OF SUPPORTIVE MET-BEFORES



Antonini, Presmeg, Mariotti, and Zaslavsky (2011, p. 191) make the point that mathematicians, epistemologists and mathematics teachers alike agree that mathematical thinking is enhanced when students are exposed to many different mathematical examples and exemplification. However, students have, “different abilities, interests, learning styles and cultural backgrounds” (Chamberlin and Powers, 2010, p. 114). Each student may present a different, “epistemological gap” (Carroll, 2017, p. 3), which occurs when, in past learning experiences, appropriate scaffolding was not introduced and the exemplifications that were used might have impeded rather than enhanced learning. It follows, then, that for *comprehensive development* to occur (Doyle, 2006, p. 31), all examples must be designed to enable previous learning to be linked to subsequent learning. This conduit, which Tall (2014, p. 223) calls, “supportive met-befores” assists students to construct new skills, procedures and concepts.

Supportive met-befores help students to form generalized conceptual frameworks of mathematical ideas. In this regard, supportive met-befores are analogous with the linking, integrative features of relational understanding.

### 2.6.2 COMPRESSION

As successive, positive experiences of mathematics are laid down, the coherence of these experiences leads to *compression* (Tall, 2014, p. 226) such that foundational overarching concepts – meta-concepts – are formed and become automatic, making future connections and success more easily attainable. In effect, success breeds success. However, when these successes are not achieved, this, "may lead to a negative feed-back loop in which the desire to avoid failure leads to less engagement with the mathematics" (Tall, 2014, p. 227). This avoidance of the subject leads to a downward spiral in mathematical confidence, and often causes at-risk students to adopt rote-learning approaches rather than attempt to understand their mathematics.

#### 2.6.2.1 EMBODIED COMPRESSION

Tall (2014, p. 226) suggests that when we are provided with opportunities to, for example, physically rearrange a fixed number of objects in different ways, we come to understand, to see, that the different arrangements do not change the total. Many such practical examples leads to embodied compression. This compression enjoins our eyes and hands; it is tangible: we can see it, it is self-evident, and we begin to perceive that a particular rule will hold true for all similarly defined cases. Tall (2014, p. 226) calls this *generic proof* – one case is seen to be a fair representation of a category of examples. These initial interactions with, in this example, a fixed group of objects help us to develop a generalizable understanding of, for example, number bonds.



#### 2.6.2.2 SYMBOLIC COMPRESSION

Symbolic compression (Tall, 2014, p. 226) occurs when, "embodied operations on objects such as counting, adding, sharing, is compressed into symbolic operations on whole numbers, fractions, signed numbers and so on." Symbolic compression occurs when numerous generic proofs are assimilated to develop a robust conceptual framework of a mathematical idea. The initial use of carefully selected examples, exemplified in practical and visible ways, can assist students to make meaning of their mathematical experiences. From this store of early physically and visually located understandings, that is, embodied compression, students should in time, link and compress numerous related memories into symbolic meta-concepts, *vis-à-vis*, symbolic compression.

### 2.6.3 A CASE FOR THE USE OF VISUALISATION TECHNIQUES IN MATHEMATICS

The use of visual representations has benefits for both teachers and their students. For teachers, visual representations, "play an important role in the explanation of mathematical ideas" (Barmby, Bolden, Raine & Thompson, 2013, p. 6), while, for students, a capacity to visually represent mathematical ideas fosters deep understanding of mathematical ideas. Indeed, "the ability to transform thoughts into images is often viewed as a test of true understanding" (Wolfe, 2004, p. xi). Indeed, albeit that the full scope of the benefits and limitations of visualisation in mathematics are yet to be revealed, there is now much interest in this field:

> Developers and practitioners are not waiting for educational theory and empirical study to guide them; they simply move forward based on their practical experience and informed inferences about what is useful and why.
>
> Macnab, Phillips & Norris, 2012, p. 103



As an application of the views held by Macnab, Phillips and Norris (2012), above, the visualising activities which are introduced to the participants in my research project have been developed and refined by myself. Hopefully, the data and findings of this thesis will add to a growing body of information about the role of visualisation in mathematics.

#### 2.6.3.1 VISUAL TECHNIQUES MAY NOT BE A BEST FIT FOR EVERYONE

Different individuals, based on their unique ways of thinking, may be drawn towards or reject the incorporation of visual solutions into their own schemata. Learners who prefer to make use of their kinaesthetic and/or spatial intelligences might be drawn towards visual constructions while learners who are more logically oriented might prefer the clarity brought to them by a more symbolic notation of mathematical ideas (Gardner, 1993).

Constructing visual representations can be problematic for students, because, "understanding a diagram as both a process and a product is critical if a student is to proficiently use a diagram to solve a word problem" (Poch, van Garderen, &

Scheuermann, 2015, p. 154). Thus, “teachers cannot assume that students recognise these representations in the manner expected” (Barmby et al., 2013, p. 7). Further, Presmeg (1986, p. 301) makes the point that where, in their past experiences of mathematics, the use of visualization was not a preferred methodological tool in the school curriculum, students may well not perceive any need to use concrete and visual methods. However, Presmeg (2009), Duval (2013), Clements (2014) and Wilkie & Clarke (2014) all support a view that there is, “compelling empirical evidence of the power of visualization in supporting sensemaking skills and growth in mathematical knowledge and understanding” (Rivera, Steinbring, & Aravi, 2014, p. 2).

Further, while Siemon et al. (2012), support the use of drawings of mathematical ideas that are constructed by learners, Rellensmann, Schukajlow and Leopold, (2017, p. 55), suggest that although drawing is widely perceived to be a useful aid in assisting learners to solve mathematical problems, “empirical studies on the benefits of *learner-generated drawing* [my emphasis] in mathematics have shown mixed results.”

Learner-generated drawings are intended to produce illustrations of learners' understanding of mathematical ideas and problems and to provide visual scaffolds to assist them to negotiate pathways. However, Rellensmann, Schukajlow and Leopold, (2017) found that in cases where learners feel they do not benefit from the use of such scaffolds, cognitive overload – caused by fulfilling the teacher's requirements – occurred, and thereby, reduced the learners' performance. This, they suggested, occurs because in such students, the cognitive requirements which are invoked to construct the sketches reduces the amount of mental space made available for information processing.



#### 2.6.3.2 PEDAGOGICAL ACCRETION

Mhlolo, Venkat and Schäfer (2012, p. 1), suggest that, “current reforms in mathematics education emphasize the need for pedagogy because it offers learners opportunities to develop their proficiency with complex high-level cognitive processes.” However, in my own experience, pedagogical accretion, in which teachers and children are encouraged to use novel approaches rather than routines and formulae, can be fraught with difficulty. On many occasions, in spite of using what I would have thought are innovative, interesting and conceptually accessible teaching strategies, I have found that my own students are hesitant and even resist the use of such approaches. Indeed, I reflect upon this issue in Chapter 4 and 5. It can be very discouraging.

Mhlolo et al. (2012, p. 1) state that when teachers in a Gauteng based research project were encouraged to present their mathematical ideas as visual representations, "the teachers' representations of mathematical connections were either faulty or superficial in most cases." This would be my experience as well. However, in spite of this sort of problem, Mhlolo et al. (2012) hold the view that:

> The ability to present a concept in several different ways shows a deep understanding of that concept, [and] recognising and producing alternate representations is a particularly fruitful way of conceptualising what a mathematical connection is.
>
> Mhlolo et al., 2012, p. 2

Thus, suggest Mhlolo et al. (2012, p. 8), in classrooms where meaningful and cogent representations of mathematical concepts are *not* made available to them, "most learners probably [lose] opportunities to develop a deep understanding of mathematical connections." Put differently, it is now understood that, "knowledge is amplified in the multiplicity of representation choices" (Siemens, 2006, p. 84).



#### 2.6.3.3 THE USE OF VISUAL TOOLS

Where Mhlolo et al. (2012) above use the term *representations*, Naidoo (2012) writes about visual tools. Naidoo (2012, p. 1) suggests that, "visual tools … make mathematical concepts easier to understand for learners. Visual tools include the use of concrete apparatus and iconic stimuli such as diagrams, pictures, transparencies and colour.

Master teachers deliberately incorporate visual tools into their explanations so as to enable their students to grasp abstract mathematical concepts (Naidoo, 2012, p. 2). Further, she contends that, "good teachers often use symbols, colour, diagrams and gestures in the classroom as an alternative to the routine approach of 'talk and chalk' teaching," and states that, "the use of visual tools assists in uncovering the role that visual reasoning plays in solving problems in mathematics" (Naidoo, 2012, p. 2). This view supports the position held by du Toit and Kruger (1993, p. 40), who wrote that, "the forming of visual images is indispensable for meaning attribution."

Rellensmann, Schukajlow and Leopold, (2017) found that strategic knowledge about drawing, the accuracy with which drawings are executed as fair representations of ideas, and types of drawings used – their fit-for-purpose – all contribute to the use of visual tools in problem solving. They suggest that a pictorial sketch, or *situational drawing*, which depicts a problem's surface structure might be useful in a low abstraction situation, for example, in a pictograph. However, a more succinct use of drawing – that is, *mathematical drawing* – is needed for tasks what require higher levels of abstractions, for example, an array or a tree-diagram. Mathematical drawings present an abstracted and idealised visual representation of a mathematical problem. Learners use these sketches to consider which strategies and computations to execute, and use the results to compare and validate against reality.

The pictorial orientation of situational drawings initially assists learners to makes sense of mathematical ideas, while the more abstracted mathematical drawing facilitates planning, computation and validation of findings. Where irrelevant information is presented on such sketches, they can serve as distractions from the purpose of the sketch (Rellensmann, Schukajlow & Leopold, 2017).



In concert with Rellensmann, Schukajlow and Leopold (2017), above, Kadunz and Sträßer (2004) suggest that as complimentary visualisation techniques, *images*, offer us analogous, heuristic representations of ideas while *diagrams* are used to represent our algorithmic thinking structures. Here, images and situational drawings imply much the same idea. Similarly, mathematical drawings and diagrams appear to be interchangeable in each case the former terms produce a more pictorial representation, while the latter pair, offers a more abstracted sketch of a mathematical idea or situation. Images may confer some ambiguity while diagrams "are created according to accepted rules of a system of representations and are used according to these rules" (Kadunz & Sträßer, 2004, p. 245). Images and diagrams are used to create and demonstrate meaning.

### 2.6.3.4 EMBODIED COGNITION

Naidoo's (2012) use of the term *gestures*, and Tall's (2014) explanation of *embodied compression* both resonate well with a conception of *embodied cognition.* Boaler et al. (2016), writing about embodied cognition, suggest that mathematics teachers and students should use their arms, legs and bodies as devices to mediate and explain mathematical ideas.

> We gesture because we see, experience and remember mathematics physically and visually, and greater emphasis on visual and physical mathematics will help students understand mathematics.
>
> Boaler et al., 2016, p. 5

In other words, "the ways we posture, gaze, point, and use tools when expressing ideas is evidence of our holding mathematical ideas in the motor and perceptual areas of the brain" (Boaler et al., 2016, p. 5). She suggests that mathematics teachers who use their bodies to explain ideas and concepts in physically demonstrative ways enable learners to perceive course-materials better. The visualisation of, for example, the sides of an equilateral triangle, or a large number versus a small number, by means of the movement of hands and arms – thereby embodying the idea which is under discussion – eases cognitive uptake of that idea.

Boaler et al. (2016) contend that because, "the body is an intrinsic part of cognition, the parts of our brain that control perception and movement of our bodies are also involved in knowledge representation" (Boaler et al., 2016, p. 5). It follows that students should be encouraged to use their bodies and gestures to ensure that they construct their own personalised muscle-memory banks of mathematical ideas. She suggests that such actions enable the mind to lay down mental pathways that provide links between, "visual and sensory motor memories" (Boaler et al., 2016, p. 5).

In summary, Naidoo (2012), du Toit and Kruger (1993) and Boaler et al. (2016), above, suggest that the deliberate use of visual stimuli (1) facilitates more comprehensive learning transfer and uptake, and (2) makes it possible for us to identify – teacher and pupil alike – the deductive processes employed by the problem-solver.

### 2.6.3.5 VISUALISATION AS A MEDIATOR FOR THE COMPLEXITY OF ORAL EXPLANATION

Mudaly (2010, p. 65) states that, generally speaking, mathematics teachers teach in, “a verbal way, where the teacher orally engages his or her learners with new or old concepts.” But, he continues, the, “abstract cues… [and] added language problems,” (Mudaly, 2010, p. 65) found in mathematical concepts and vocabulary may easily overwhelm learners. Contrasted with this verbal approach, Mudaly (2010, p. 65), cites examples where the use of visual information is used as a means of negotiating understanding in the world: babies visually discern their parents from other adults, teenagers use brand logos as status-symbols, and adults use road-signs to travel safely on our streets, etcetera. Mudaly (2010) makes the point that pictures make our ideas clearer to ourselves and to others (Muldaly, 2010, p. 65), and suggests that the use of well-conceptualised visual stimuli facilitates the linking of prior knowledge with new knowledge and, “act as tools to negotiate new ideas” (Mudaly, 2010, p. 66). Mudaly (2010), points out a subtle but important distinction between something that is visualised and visual literacy. The former, something visualised, (Mudaly, 2012, p. 67) refers to our ability to physically or mentally *see* an image; the latter, visual literacy, refers to a deeper construct, one in which such images can be mentally manipulated, reoriented and reimagined. Mudaly (2010, p. 67) holds the position that, “visual literacy is visualisation combined with logical thought,” and continues, stating that:

> …if visual literacy is successfully taught then there is a natural tendency for the mind to engage in logical and rational thought at the sight of a picture or diagram. Visual literacy … [refers] to the internal processes that the mind engages in after viewing an external picture or diagram or a mental image.
>
> Mudaly, 2010, p. 68



### 2.6.3.6 SEMIOTIC COMPLEXITY

Duval (2014) writes about the, “semiotic complexity” (2014, p. 159) or misunderstandings that visualization techniques might impose upon students. He cautions that imbuing artefacts such as bottle-tops or simple representative sketches with mathematical meaning can, for some students, prove to be counter-productive in conceptualising problems. It is quite possible that students may misunderstand or be unable to define the representational qualities of a visual scaffold. However, by assisting students to perceive diagrams as more than mere pictures can be a transformative process that opens up future mathematical development and understanding" (Poch, van Garderen, & Scheuermann, 2015). This benefit notwithstanding, as discussed in (2.6.3.1), above, we have to acknowledge and prepare for situations where students might find the use of visualisation obstructs their conceptualisation and learning.

### 2.6.3.7 INTERPRETING FIGURAL INFORMATION AND VISUAL PROCESSING

In the 1980s, Bishop (1983), having focused his eye on the problems that students had in interpreting and solving geometric problems, constructed a visualisation theory which connects spatial awareness and mental conjecture. Under the umbrella concept of *visualisation*, he conceived two separate but intersecting visual capacities that were required for problem-solving, (1) Interpreting Figural Information (IFI), and (2) Visual Processing (VP).

*Interpreting Figural Information,* or IFI, implies that we use our eyes, literally, to explore visual stimuli. If we use the example of a map, we find that we use IFI to identify features on that map to enable us to orientate ourselves and negotiate a path forward on that map. IFI speaks to our ability to look at external stimuli – a smiling face, body-language, a treasure map, a geometric shape – and, by mentally drawing upon memories of previous, similar experiences, use those memories to try to understand and interpret the new visual stimuli before us. The eyes serve as a conduit between the external world and our understanding of the world. Based on our interpretation of the visual data received by our eyes, IFI enables us to orient ourselves and to take action. IFI is akin to Mudaly's (2010) concept of visualising.



*Visual Processing*, or VP, occurs as a far more sophisticated undertaking. VP refers to our capacity to imagine and to mentally manipulate artefacts and situations. For example, when we close our eyes and mentally draw up a picture of a loved one, or we mentally rotate a geometric object, we are using visual processing. Visual processing is very powerful: it enables us to mentally remember images and manipulate things we have seen but, more importantly, it enables us to imagine objects, systems, etcetera, which we have never actually encountered. In this regard, Bishop's (1983) views of VP is congruent with Mudaly's (2010, p. 67) visual literacy.

#### 2.6.3.8 SPATIAL ORIENTATION AND SPATIAL VISUALISATION AND IMAGERY

Siemon et al. (2012, p. 223), suggest that, "when we are involved in visualisation, we are forming images, either mentally or externally through diagrams… [and that] being able to reason using visualisation is an important aspect of mathematics." Further, although we live in a three dimensional world, most visual representations of mathematics are presented as two dimensional drawings in text-books (Risma, Putri, & Hartono, 2013). Siemon et al. (2012, p. 224) suggest that, "Spatial sense is an important aspect of numeracy, being inherently mathematical and also essential for living and interacting in our world," and explain (2012, p. 223) that spatial sense comprises two main spatial abilities, namely, (1) Spatial orientation, and, (2) Spatial visualisation and imagery.

*Spatial orientation* finds a strong accord with Bishop's (1983) Interpreting Figural Information or IFI concept. Spatial orientation skills include being able to visualise dynamic changes in images; creating a mental warehouse of images of shapes and objects and being able to link "spatial knowledge to verbal/analytical knowledge" (Siemon et al., 2012, p. 223).



*Spatial visualisation or spatial imagery* is more than a simple remembered image of an event or object. Zimmerman and Cunningham (1991), in Siemon et al. (2012 p. 223), have defined it as, "the process of forming images (mentally, or with pencil and paper or with the aid of technology) and using such images effectively for mathematical discovery and understanding."

Risma, Putri, and Hartono (2013) suggest that when students are encouraged to make use of spatial visualisation, they begin to use spatial terms more confidently and develop spatial visualisation capacities – in other words, practice makes for conceptual and skills-based improvements. Further, the role of the teacher is important – over-teaching or providing too much support or too soon, reduces the efficacy of spatial visualisation as a learning and problem-solving scaffold. Further, where teachers themselves are insecure in using such techniques, little benefit accrues to learners.

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In this section, I have suggested that visual tools in the form of previous met-befores, tactiles, sketches, photographs and mental constructions can facilitate deep understanding of mathematical ideas and, in addition, when compression occurs, can assist students to solve word-problems. Throughout this chapter, many perspectives on mathematics have been presented, with each designed to build up a case to support the use of visual scaffolds in an IPS 413 E mathematics classroom as an appropriate, novel and beneficial teaching and learning methodology. It is now important to link all of these views into a suitable theoretical framework for the thesis. In this regard, the outstanding contribution made by Jerome Bruner (1915 – 2016) in assisting us to deeply understand *learning*, provides me with such a structure.

## 2.7 A THEORETICAL FRAMEWORK FOR THIS RESEARCH



My decision to write the theoretical framework at the end of this chapter stems from my view that, for action research, cogence in thinking about the issues which inform the research, should culminate in a theoretical framework that is consistent with the aspirations of the research.

In South Africa, many of us continue to teach by drilling, by rote and regurgitation, by using the one right method. In doing this, we reduce our subject to a 19th through middle 20th century transmission-mode view of mathematics. When we focus mostly on operations, processes, and convergence but do not provide our pupils with opportunities for risk-taking, experimentation and application, we short-change our pupils: we ignore a most essential epistemological aspect of 21st century mathematics, that is, mathematics as an understanding-based, problem-solving, human enterprise.

In contrast to the transmission-mode teaching and learning scenario outlined above, Jerome S. Bruner (1915 – 2016) posits a view that the accretion of relational knowledge and deep learning is driven by the active participation of intrinsically motivated, inquiry-oriented learners. My own epistemological, pedagogical and methodological positions rest heavily upon Bruner's conceptualisations of knowing and learning, therefore the theoretical framework of this research is guided by Bruner's theory of learning.

### 2.7.1 A RE-EMPHASIS OF THE ASPIRATIONS OF THE CAPS DEFINITION OF MATHEMATICS

If we return to the South African definition of mathematics for a moment (refer to 2.3.1), we are reminded that the CAPS (2011) view of mathematics is that it is, "a human activity that involves **observing** [my emphasis], **representing** [my emphasis] and **investigating** [my emphasis]" and that it, "helps to **develop mental processes** [my emphasis] that enhance logical and critical thinking, accuracy and problem-solving that will contribute in decision-making" (South Africa, 2011). The use of the verbs – observing, representing and investigating – as pivots, suggests that there are epistemological, methodological and pedagogical imperatives to provide visualising opportunities to students to enable them to really sink their teeth into these features of mathematics and thereby to engage with mathematics in ways which foster deep understanding. Indeed, it is clear that within the CAPS curriculum, rote-learned factoids withdraw – Bruner himself called these, "pellets of this or that" (Bruner, 1980, p. 408) – in favour of students actively learning for understanding.



Bush, Daddysman and Charnigo (2014) conceive three main types of learning experiences that students encounter – I would be more inclined to conceive a continuum – which includes *no learning*, *rote learning*, and *meaningful learning*. The first classification is self-explanatory; teaching might have taken place, but no learning uptake has been achieved. The second classification identifies an ability the students might develop to recall key facts, that is, rote learning, but does not extend to the capacity to do anything meaningful with that information. Meaningful learning, the third classification, occurs when students cogently, intuitively, creatively and strategically attend to non-routine problem-solving enterprises. Meaningful learning liberates the full knowing and understanding potentials in students.

Visual stimuli provide a gateway to understanding. Indeed, in this literature review, authors such as Siemon et al., (2012), Mudaly, (2010), Mholo et al., (2012), Naidoo, (2012), and Boaler et al. (2016), among others, promote a view that visualisation is an important mediating tool in teaching and learning mathematics. Visualising stimuli – visual scaffolds – can facilitate logical development, critical thinking and deeper understanding of mathematic ideas and can thereby assist students to solve mathematical problems. But visualising strategies, on their own, offer only a part solution for improvement in mathematical understanding. These strategies must be complemented by robust pedagogy and methodology, *a theory of learning*, one such as that espoused by Bruner. My own work and, by extension, the theoretical framework of the thesis are informed by Bruner's views on learning.

### 2.7.2 BRUNER – A BRIEF OVERVIEW

In the late 1940s, Bruner was a central figure in the cognitivist reform movement that rejected empiricist views of knowledge structures, rote-learned behaviours, and positivistic curricula. Instead, Bruner viewed humans as active and intelligent problem-solvers and suggested that, provided that the intellectual development of the learner was accounted for, "any subject can be taught effectively in some intellectually honest form to any child at any stage of development" (Bruner, 1960, p. 33). He suggested that learning involves three processes, all acting in unison, namely, *acquisition*, which adds to or overturns our previously held ideas and knowledge; *transformation,* which reorganises new knowledge to accommodate it within our existing knowledge structures and thereby enables us to use this new knowledge as a means of prediction, extrapolation, etcetera; and *evaluation,* which tests this newly acquired information to make sure that it *fits* with our world view (Bruner, 1960, p. 48 - 49).



Bruner (1986, p. 130), considered himself a constructivist and believed that, "what we call the world is a product of some mind whose symbolic procedures construct the world" (Bruner, 1986, p. 95). Continuing, he put forward an opinion that, "no one "world" is more "real" than all others, none is ontologically privileged as the unique real world" (Bruner, 1986, p. 96). Instead, every human being creates his or her own unique view and understanding of the world.

Bruner (1966) favoured an epistemology that encouraged pupils to construct their own knowledge (Bruner, 1966, p. 41 – 42), and recommended that schools should, "enlist the natural energies that sustain spontaneous learning – curiosity, a desire for competence, aspiration to emulate a model, and a deep-sensed commitment to the web of social reciprocity" (Bruner, 1966, p. 127). Bruner downplayed the effectiveness of tests and exams as a motivation for learning, suggesting that, "where grades are used as a substitute for the reward of understanding, it may well be that learning will cease as soon as grades are no longer given – at graduation" (Bruner, 1960, p. 51).

In his later years, Bruner amended his views of the cognitive perspectives of his earlier work, suggesting this needed to be supplanted by acknowledging that our culture, as a mediating and locating context, equips us with predispositions which shape our perception of ourselves and our world (Bruner, 1980, p. 102 – 103; Takaya, 2008, p. 2). When educational norms and practices are viewed from this position, it becomes apparent that epistemologically and culturally, there can be no neutrality in learning. Bruner unshackled himself from his earlier intrapsychic views of knowing and realigned himself with views which acknowledge the *social* construction of knowledge.

#### 2.7.2.1 FOUR CRITICAL IDEAS FROM BRUNER'S EARLY WORK

Four critical ideas manifested from his early work, (Bruner, 1960), those being that:



1. Many classrooms are set up for teaching, not for learning: such classrooms preclude learning of much more than the rote recall of facts and algorithms that transmission mode teaching habits endorse. However, deep learning is largely driven by curiosity (Bruner, 1966, p. 43) and success in future learning is dependent on the structuring of practical, understanding-based experiences in numerous earlier learning opportunities (Bruner, 1960, p. 11 - 15);
2. Often, teachers wait too long to introduce new or big ideas: they premise this delay on a view that such work lies outside of the comprehension of younger learners. However, through the mediation of a spiral curriculum, Bruner (1960, p. 33) contends that by accommodating the intellectual development of children and revisiting the same content over a number of years at increasing levels of sophistication, children can come to deeply understand that content;
3. Bruner posited that intuition and intuitive ways of knowing are a critical, but neglected, prerequisite for deep thinking. "Intuition implies the act of grasping the meaning or significance or structure of a problem without explicit reliance on the analytical apparatus of one's craft" (Bruner, 2007, p. 51). Experience-based intuition enables mathematicians, as an example, to have "a sense of

what combinations are likely to have predictive effectiveness and which are absurd” (Bruner, 1980, p. 211). Indeed, intuition comes before formal proof and the application of an algorithm;

4. Bruner argued that intrinsic rather than extrinsic motives for learning should drive the processes of teaching and learning. Extrinsic motives, *vis-à-vis* parental approval and avoiding failure in tests, conspires to encourage rote learning (Bruner, 1980, p. 406). However, the construction of knowledge, “is not a matter of getting him [sic] to commit results to mind. Rather, it is to teach him [sic] to participate in the process that makes possible the establishment of knowledge” (Bruner, 1966, p. 72). Bruner’s (1980) position aligns with Plato, (2.1.1), and Freire, (2.1.3), discussed earlier in this literature review.

## 2.7.3 TWO MAJOR THEMES FROM BRUNER’S WORK

The four ideas listed above consolidate into two major themes: Bruner posited that (1) the acquisition of knowledge requires active participation by the learner, and (2) we learn mostly by linking new information to our existing knowledge frameworks.



### 2.7.3.1 ACQUIRING KNOWLEDGE

Bruner (1980), as an advocate of inquiry-based, discovery-oriented, intrinsically-motivated and hypothetical modes of learning, suggested that we best acquire knowledge through participation in acts of problem-solving; he wrote:

> Mastery of the fundamental ideas of a field involves not only the grasping of its general principles, but also the development of an attitude towards learning and inquiry, towards guessing and hunches, towards the possibility of solving problems on one’s own.
>
> Bruner, 1960, p. 20

In the context of the above-written quote, *fundamental* implies a meaningful understanding of a foundational concept: for example, the conception of a square has powerful, deep and wide applicability in many directions. Accordingly, when teaching the fundamental concepts and big ideas in a subject, teachers must be employed to accomplish this knowledge construction (Bruner, 1960, p. 18 - 19). Further, he suggested that, “it is only through the exercise of problem solving and the effort of discovery that one learns the working heuristic of discovery" (Bruner, 1980, p. 410). In addition, "since learning and problem-solving depend upon the exploration of alternatives, instruction must facilitate and regulate the exploration of alternatives on the part of the learner" (Bruner, 1966, p. 43).

Bruner (1966) believed that with proper support from their teachers, students would learn to adopt problem-solving approaches that would predispose them to use novel and intuitive ideas to work through new, non-routine problem-situations and to discover optimal solutions *for themselves*. He suggested that, "discovery, like surprise, favors the well-prepared mind" (Bruner, 1980, p. 402). However, he acknowledged that the, "method of discovery would be too time-consuming for presenting all of what a student must cover in mathematics" (Bruner, 1960, p. 21). This sage advice compels teachers to expertly blend learning opportunities of discovery with moments of exemplification. Indeed, he conceded that, in mathematics at least, many exercises in, "computational practice may be a necessary step towards understanding conceptual ideas in mathematics" (Bruner, 1960, p. 29).



Bruner's (2007, p. 50) conception of discovery posited that it is only when we are actively engaging with solving problems that we are creating opportunities for discovery. Nor did he restrict his conception of discovery to something that was previously unknown to human-kind. Rather, he suggested, uppermost in his conception of discovery was the discovery of new ideas and knowledge *for oneself*. This enables one, "to be his [sic] own discoverer" (Bruner, 1980, p. 402). However, when teachers evoke a quest for discovery, within learners there will be an, "expectancy that there will be something to find" (Bruner, 1980, p. 404). In other words, the effort expended in taking risks and in thinking has to be matched with discovering something that is worthwhile.

Discovery-based-learning can be a time-consuming, risk-taking enterprise in which one stream of ideas may lead to a dead-end whereas a different stream of thought may lead to a suitable solution. While Bruner was aware of curriculum pressures that teachers face, nonetheless, he advocated that teachers should try to provide enough time for students to work through such problems and to achieve success. When this happens, “discovery, with the understanding and mastery it implies, becomes its own reward, a reward that is intrinsic to the activity of working” (Bruner, 2007, p. 51).

With reference to Piaget’s processes of assimilation and accommodation (Craig, 2016, p. 58), Bruner (2007) suggested that discovery requires a blending of (1) expository mode teaching, or *listening*, as a metaphor for accepting and unpacking new information, that is, the assimilation of information – and (2) hypothetical mode conceptualising, or *speaking,* as a metaphor for active problem-solving, that is, using the accommodation of information to best fit with the requirements of the new problem. When student and teacher cooperatively work in the hypothetical mode, that is, the world of ideas and theorising, the student ceases to be a recipient of information and knowledge and transforms, “to taking a part in the formulation and may at times even play the principal role in it” (Bruner, 1980, p. 403).

#### 2.7.3.2 LINKING PREVIOUS WITH NEW KNOWLEDGE

Bruner believed that, “the principal problem of human memory is not storage, but retrieval” (Bruner, 1980, p. 411). He suggested that we are biologically primed to store vast amounts of sensory and *learned* information. So, storage is not the issue; the problem, rather, is that often we appear to not have the facility to retrieve those memories. However, he suggested, where ideas – memories – are paired to additional stimuli, be it a linking picture, a paired word, or a mnemonic, for example, retrieval is simplified because of the better organisation of the information and the bonds it has with its linking devices.

The human mind constantly cycles through the three mental processes, namely acquisition, transformation, and evaluation. The act of learning, that is, our *acquisition* of new skills and knowledge, may contradict or overthrow an existing understanding that we hold. In trying to make sense of the new information, we *transform* it so as to cause the new knowledge to link with as many of our existing ideas as possible, and while doing this, we *evaluate* this information and try to judge whether we have made cogent sense of the new knowledge (Bruner, 1980, p. 421 - 422).

Because our skill-filled capacities are inherently the result of practice over time, Bruner suggested that provided, “one respects the ways of thought of the growing child”

(Bruner, 1960, p. 52), a spiral curriculum offers an optimal path for linking previously learned information to new information (Bruner, 2007, p. 53). Indeed, he suggested, “there are very few single or simple adult acts that cannot be performed by a young child” (Bruner, 1980, p. 327). Further, when complicated adult acts are reconstituted into simpler sub-components, even novices can achieve success. The difference between the expert and the novice lies largely in the, “orchestration of these components into an integrated sequence” (Bruner, 1980, p. 327). A spiral approach, in which sub-components are incrementally introduced to the orchestra offers learners opportunities to become skill-filled.

Where practitioners adopt a spiral approach in their teaching programmes, Bruner (2007) suggests that in planning their curriculum teachers need to consider (1) the problem of structure, (2) the problem of sequence, and (3) the problem of embodiment.

### 2.7.3.3 THE PROBLEM OF STRUCTURE

Bruner believed that teachers who avoid teaching *big ideas* to their students, and particularly their younger students, are often reluctant to entertain this work because of their awareness of their own lack of understanding of those ideas. However, as mentioned earlier, (2.7.3.2), Bruner (2007, p. 53) suggests that complex ideas can always be paraphrased in simpler, but nonetheless accurate restatements of general underlying principles and ideas.



Bruner (1960, p. 9) believed that teachers who structure their courseware do a great service to at-risk students who typically lose direction when courseware is loosely thrown together. However, structure should invoke more than the sequencing of facts and techniques. Structure should be conceptualised as operating at the heart of knowledge transfer (Bruner, 1960, p. 12). “To learn structure” suggested Bruner (1960, p. 7), “is to learn how things are related.” This, talks to the positive attributes of relational rather than instrumental learning, discussed in (1.2.2.1). Over time, simpler, generalizable ideas allow us to tease out deeper understandings of complex ideas.

#### 2.7.3.4 THE PROBLEM OF SEQUENCE

A second consideration requires teachers to sequence learning in ways which create supportive stepping stones. Bruner (2007, p. 53) suggested that to miss out a stepping stone, or to move to a new idea, premising the new knowledge upon a misunderstood previous idea, simply frustrates the enterprise of teaching and learning. This view finds congruence with the concept of supportive met-befores, discussed earlier in (2.6.1).

Sequencing can be linked to Bruner's (2007, p. 54) view on *readiness*, which proposes that, "any subject can be taught to anyone at any age in some form that is honest." Sequencing requires teachers translate ideas into a language and explanation which is age-appropriate. Furthermore, teachers should, "opt for depth and continuity in our teaching rather than coverage." (Bruner, 2007, p. 54). Failing this, teachers may limit the potential for children to learn deeply.

#### 2.7.3.5 THE PROBLEM OF EMBODIMENT

Embodiment addresses the need to establish ways for teachers to bridge, "the gap between the ideas in mathematics and the students' ways of understanding such ideas" (Bruner, 2007, p. 53). Big ideas, much like the idiom of *eating an elephant one bite at a time*, can be incrementally addressed, with successive layers of understanding building upon previous layers. Iterative experiences of a meta-concept can provide opportunities for embryonic ideas to become well-established within long term memory and, over time, leads to compression (2.6.2).



### 2.7.4 BRUNER'S THREE PHASES OF LEARNING

Bruner (1980) contended that humans, "can be described as a species that has become specialised by the use of technological implements" (Bruner, 1980, p. 327). He suggested that through the embodied actions of working with tools we have genetically wired ourselves to solve problems by using techniques which incorporate the use of tools. Indeed, the techniques and tools that we use impart a flavour to and influence our representations of the world and frame the ways in which we perceive the world. These learned techniques, "serve to amplify our motor acts, our perceptions, and our ratiocinative [common sense] activities" (Bruner, 1980, p. 327), so that we learn to react to external stimuli, to problems, in practiced ways.

> In short, the capacities that have been shaped by our evolution as tool users are the ones that we rely upon in the primary task of representation.
>
> Bruner, 1980, p. 327

Representations, broadly, imply the techniques which we use to understand and explain the world. The use of tools, as practiced techniques, must be stored, processed and encoded to become memories which must be made easily retrievable. Bruner (1980, p. 327) conceived three modes of representation, namely, (1) enactive representation, (2) iconic representation, and (3) symbolic representation. In *learning*, these representations are assimilated and accommodated such that enactive representations may be subsumed by iconic representation which, in turn, may be subsumed and supplanted by symbolic modes of representation. However, all three modes of representation remain available to the skilled mind.

#### 2.7.4.1 ENACTIVE REPRESENTATION

Enactive representation refers to our ability to physically negotiate our way through the world. "Enactive representation is based, it seems, upon a learning of responses and forms of habituation" (Bruner, 1966, p. 11). By practicing particular motor-skills sets, such as climbing a flight of stairs or touching biscuits as we count them out, we come to represent, "past events through appropriate motor response" (Bruner, 1980, p. 328).



Over time, "the nervous system converts a sequence of responses into an image or schema" (Bruner, 1966, p. 14). With proper attention to practice, these motor responses become automated, part of our *muscle-memory*, such that we do not expend undue cognitive effort attending to the acts of climbing stairs or counting-on.

#### 2.7.4.2 ICONIC REPRESENTATION

Iconic representation, "summarizes events by the selective organization of precepts and of images by the spatial, temporal, and qualitative structures of the perceptual field and their transformed images" (Bruner, 1980, p. 328). Further, "iconic organisation is principally governed by principles of perceptual organization" (Bruner, 1966, p. 11), and in this regard is linked to Gestalt theory and Gestalt cues (Appendix A), as it is "solidly based upon the analysis of naive phenomenology of experience and the manner in which perception and memory are linked by the rule of phenomenal similarity” (Bruner, 1966, p. 18).

By, for example, picking an apple off a tree, biting the fruit, and so on, our initial experiences of apples are stored in memory as an enactive representation. Over time, these experiences are subsumed by perceptions and mental images of apples that are arranged and stored in the mind. Consequently, when needed, a concept of apples can be conjured up in the mind’s eye as an iconic representation of apples. In this way, the enactive representation of apples is subsumed by an iconic representation.

It is important to note that humans are always free to return to the enactive stage: the subsumption of the enactive by the iconic mode of representation does not imply the extinction of the former in favour of the latter. We are free, as it were, to return to the kitchen table to eat another apple. And indeed, every time we do this, with each experience of apples, we further develop our enactive and iconic conception of apples.

#### 2.7.4.3 SYMBOLIC REPRESENTATION



Symbolic representation enables humans to assign meaning to symbols. The symbols themselves are an abstraction from the world of things and images of things; the symbolic world contains letters and words, musical notes and mathematical nomenclature. “Symbols (words) are arbitrary ... there is no analogy between the symbol and the thing” (Bruner, 1966, p. 11). In Bruner’s (1980, p. 328) own words, “a symbol system represents things by design features that include remoteness and arbitrariness. A word neither points directly to its referent here and now, nor does it resemble it as a picture.”

The elegance ascribed to symbolic representation is attributed to the fact that because symbolic notation, “remains invariant across transformations in imagery, the learner comes to grasp the formal or abstract properties of the things he is dealing with” (Bruner, 1966, p. 68). However, this achievement, properly discovered, requires us to first engage in the world, and thereby construct an enactive representation of it, following which we construct iconic representations of the world. Once these representations are stable, we are positioned to embrace the sophistication of abstract symbolic representation.

### 2.7.5 BRUNER'S THEORY OF LEARNING

In teaching and learning, it is possible to apply the three modalities of representation as a hierarchal methodology that can facilitate deep understanding. Bruner conceived a three-stage, constructivist-based learning theory. In it, Taber (2009, p. 149) posited that deep learning occurs when learners are given opportunities to sequentially work through tasks which are first saturated in the use of tactiles – manipulatives – with such learning experiences being gradually replaced by pictorial information as representations of ideas, and that, in turn, is eventually exchanged in favour of the abstraction of words and numbers (Naroth & Luneta, 2015, p. 269). Through the processes of (1) acting within the world, *vis-à-vis,* "instrumental activity" (Bruner, 1966, p. 68), (2) observing and constructing unique representational images of the world, and (3) acquiring the skills which enable one to craft symbolic descriptions and interpretations, "people convert reality into their own unique portrayal of reality" (Presno, 1997).

#### 2.7.5.1 APPLYING BRUNER'S THEORY OF LEARNING TO MATHEMATICS

Critically, in teaching and learning, the sequences and changes in methodological presentation from enactive through iconic through symbolic representational forms of knowing need to be scaffolded by knowledgeable teachers (Ndlovu, 2014, p. 3).



Bruner remained quite flexible on cycling back through the three forms, stating that in problem-solving he had noted that even though children might have reached the abstraction level for a particular aspect of mathematics, many preferred to return to the iconic representation as a means for making understanding (Presno, 1997).

#### 2.7.5.2 ENACTIVE REPRESENTATION IN MATHEMATICS

Witzel, Smith and Brownell (2001) applied Bruner's learning theory to mathematics. Writing about pedagogical *enactive* strategies that they used with students who were struggling in algebra, they suggest that teachers should:

> make instruction relevant and use explicit instruction to provide students with hands-on experiences. Hands-on experiences allow students to understand how numerical symbols and abstract equations are operating at a concrete level, making the information more accessible to all students.

Witzel, Smith & Brownell, 2001, p. 103

I think it important that we cherry-pick the terms, *relevant, explicit instruction, hands-on experiences* and *accessible* from the above quote. Collectively, they point to a view of teaching and learning mathematics which is explored in real-world contexts by teachers who directly manage their learning environments and who use tactiles and other apparatus to make the subject meaningful to all participants. The learning environment must be rich in its use of things that are found *in the world* as these provide useful tangible, visual scaffolds. It is important to note that Bruner himself believed that, "not teaching devices, but teachers were the principle agents of instruction" (Bruner, 1960, p. 15) and that it is through the conduit of *doing* that we begin to learn (Presno, 1997). These views support the CAPS definition of mathematics, as found in (2.3.1) in this chapter.

Witzel, Smith and Brownell (2001, p. 103), like Bruner, advocate a three-phase, "concrete to representational to abstract (CRA) sequence of instruction" teaching approach. Within the first phase or concrete level, teachers should make copious use of tactile stimuli: such models enable students to understand and self-correct deficiencies in their solutions (Kribs-Zaleta, 2008, p. 456). In the second phase, the iconic representational level, students should use a blend of tactile and iconic stimuli.



> The concrete phase involves manipulatives, such as toothpicks for counting. The representational phase uses pictures, such as tally marks. Pictorial representations relate directly to the manipulatives and set up the student to solve numeric problems without pictures. Matching pictorial representations to abstract problems helps students to understand. For basic algebra, it is important to include aids to represent arithmetic processes, as well as physical and pictorial materials to represent unknowns.
>
> Witzel, Smith & Brownell, 2001, p. 103

The above quote advances a view that teachers who set aside a great deal of teaching time for mathematical explorations in the concrete and representational worlds, in other words in the visualisation (2.6) of mathematical situations and problems, in fact set the scene for later success when symbolic abstraction is added to the mix.

### 2.7.5.3 ICONIC REPRESENTATION IN MATHEMATICS

Bruner's second stage – iconic representation – is based on Gestalt theory (Presno, 1997). Initially, images provide iconic summaries of our engagements with the world but, with experience, more sophisticated images are conjured up in the imagination. Scott Baumann et al. (1997) suggest that the iconic stage plays a vital role in linking the outside world – the world of things – to the mind of the cognising being. In the iconic stage the, "direct manipulation of objects is replaced by the beginning of more abstract thinking with visual or other perceptual imagery" (Özgün-Koco & Edwards, 2010, p. 51). We develop mental tools that enable us to look at iconic representations of objects and understand them. We also develop the capacities to rotate objects mentally and imagine objects, scenarios and phenomena that we have never seen.

### 2.7.5.4 SYMBOLIC REPRESENTATION IN MATHEMATICS

Abstract reasoning, the third level in Bruner's learning theory, introduces learners to the world of words and numbers and symbolic nomenclature. Success at this level is largely dependent on the quality and quantity of learning experiences in the concrete and representational levels of instruction. In the symbolic representation stage of conceptual development we, "grasp the formal or abstract properties" (Bruner, 1966, p. 68) of things and ideas with which we are dealing. This highest level of Bruner's three-stage learning theory draws parallels with Plato's higher realm of forms, discussed earlier (2.1.1). Through our deliberate and thought-filled, mark-making on paper and computer screens, for example, we demonstrate – for others to see – our particular understanding, our interpretation of the world.



Bruner's three stage learning theory finds cogence with dual-code theory which posits that, "students learn better when provided with visual and verbal representations rather than visual or verbal representations alone" (Moreno & Valdez, 2005, p. 43). Indeed, "visualizing what you read makes the text more meaningful and memorable" (Wilson, 2012, p. 189). Dual coding theory suggests that, "kinaesthetic and tactile experiences may be encoded not as verbal information but instead as a type of image" (Jones, Minogue, Tretter, Negishi & Taylor, 2006, p. 112), which reinforces the role that visual stimuli play in teaching and learning. Indeed, in the context of teaching word-problems that are written in English to students who are typically not English speakers, the use of suitable pictures, alongside the texts of word-problems, can play a significant role in supporting meaning-making of intent (Sadoski, 2005, p. 233 - 234).

### 2.7.6 IMPLICATIONS OF BRUNER'S THEORY OF LEARNING FOR THIS THESIS

Bruner's conception of learners is that they are intelligent, intuitive, problem-solving and action-oriented human beings. As such, they must be allowed to act in the roles of listener – as a metaphor for learning from others - and speaker - as a metaphor for constructing knowledge with others. While Bruner's conception of epistemology is firmly positioned in the construction of understanding – schema – the journey towards this goal is located first in learning experiences that occur in the real world. Bruner suggests that a structured, inquiry-based (discovery), spiral curriculum (embodiment), offers maximum educational beneficiation for learners. To this end, he envisages the use of a hierarchy for learning, moving learning from the world of enactive through iconic through symbolic modes of representation, and always presented with a view that *looping-back* is an inbuilt feature of the learning theory.



## 2.8 SUMMARY OF CHAPTER 2

Teachers hold philosophies, opinions and biases which are informed by their life experiences. In Plato's *Allegory of the Cave* I have suggested that deep learning is a continuous construction, is intrinsically motivated and requires effort and risk-taking. Freire posits that ontologically oppressed humans will seek redress and suggests that banking forms of education oppress learning. Bourdieu provides evidence of the devastating effects of marginalisation – oppression – of some human beings by others. In the second section of the review, I unpacked international, regional and local sets of data which confirmed that mathematics in South Africa is in poor health. In section three, I suggested that South Africa's curriculum policy encourages a liberalising mathematical pedagogy, an activities-based teaching and learning methodological approach and an understanding-based epistemological view of mathematics.

In the fourth section, I presented an overview of problem-solving in mathematics and a focused discussion on word-problems. I revealed barriers which preclude students from success and suggested that, where compression has not taken place, students muddle through problem-solving tasks. In section five, I discussed modern views on mathematics teacher education and pedagogy. In section six, I discussed the role that visualisation may play in assisting students to better understand and solve word-problems.

In the last part of the literature review, I revealed the theoretical framework for this thesis. Bruner's theory of learning draws together the pedagogical, methodological, ontological and epistemological aspirations of a *liberalising philosophy of education.* As such, Bruner's theory pulls together all of the viewpoints that precede it in the literature review and informs my praxis.

In Chapter 3 I will provide an account of the research methodology that was used in this research project. Its guiding principles, data collection instruments and ethical considerations were designed to answer the main and sub-research questions found in Chapter 1 of this thesis and to work harmoniously with the philosophical, academic and theoretical framework espoused in Chapter 2.

# CHAPTER 3 - METHODOLOGY

## 3.0 INTRODUCTION

Before I undertook this thesis, I vacillated over the research design for this research: the two contenders in my conceptualisation were case studies and action research. Although they are in some ways similar, it was the repeated suggestion that action research seeks to *solve problems* that led me to believe that a mixed methods approach and an action research design was the correct route for me to follow. Blaikie (2010, p. 50) suggests that, "few if any social research projects are exclusively concerned with advancing knowledge for its own sake." I did not want to simply write a report on a social circumstance; potential benefits were that the action research would expose the participants to a novel and meaning-filled experience of mathematics and would thereby, possibly emancipate them from imposed limitations on their understanding and conceptions of mathematics. I also felt this would enable me to reflexively improve my practice (Leitch & Day, 2000; Herr & Anderson, 2005).

## 3.1 FEATURES OF ACTION RESEARCH



Action research practitioners aspire to increase their understanding of problematic situations and thereby, to construct beneficial modifications to offset those challenging conditions (Blaikie, 2010, p. 73). It is suggested that, "action research resides mainly within the [theoretical] domain of social psychology and organizational development … the overriding philosophy is enhanced efficiency and effectiveness" (Ellis & Kiely, 2005, p. 94). Further, in action research, improving, being involved, supporting and constructing change is understood to be part of the research process, and indeed, action research can be seen to be "an embodiment of democratic principles in research" (Robson, 2002, p. 216).

In the context of this thesis, efficiency and effectiveness are derived from views that suggest that visual scaffolds can pull together and consolidate the important mathematical ideas, issues, requirements and features that are found in word-problems and the application of these visual scaffolds can be cast over a wide spectrum of mathematical problems (Fosnot, 1989; Siemon et al., 2012; Haylock & Manning, 2014; Rellensmann, Schukajlow & Leopold , 2017).

### 3.1.1 STRENGTHS OF ACTION RESEARCH

A main strength of action research is that, when it is successful, the emancipated participants embrace the experiences of the research which, in turn, "enhances chances of high construct validity, low refusal rates and 'ownership' of findings" (Mouton, 2003, p. 151). Indeed, "a major goal of action research is to generate local knowledge that is fed back into the setting" (Herr & Anderson, 2005, p. xv). In this regard, two settings which might potentially benefit from the research are my own practice in a university setting and, as a second setting, the classrooms in which, in the not too distant future, many of the participants will be teaching mathematics.

Bradbury (2015, p. 6 - 7) suggests that action research is underpinned by three principles, namely, that (1) *the self is relational* – we enjoy relationships with the earth and those upon it; that (2) *systems seek wholeness over time* – we strive towards collaboration among rather than domination over others and; (3) *the primacy of practical contribution* – we seek to balance science and art in achieving practical, emancipatory solutions. Such research is ongoing, thus the changes which might occur are not events, but rather part of ongoing processes (Robson, 2002). Bradbury writes, "Finding ourselves in relationship with complex, emergent systems, we seek to make a positive difference, to minimise suffering, to work towards justice, to muddle through" (2015, p. 7).



### 3.1.2 LIMITATIONS OF ACTION RESEARCH

The biggest limitations for action research are that the results might be written up to only reflect upon, "strong causal and structural explanations" (Mouton, 2003, p. 150); descriptive aspects of the research supplant the requirement to obtain hard data, and findings may be biased towards qualitative aspects of the research and, " lack of rigour in analysis" (Mouton, 2003, p. 150). Further, Altrichter, Posch and Somekh (1993, p. 191), suggest some researchers may, "offer very few analytical points and interpretations in order to reduce the risk of getting it wrong." In addition, the moral high ground that action researchers may claim for themselves, that they are "doing good" (Greenwood, 2005, p. 173) is no justification for sloppy research. Also, because the researcher has a stake in the work, the knowledge generated by action research cannot be said to be neutral (Herr & Anderson, 2005, p. 26).

### 3.1.3 DYNAMIC CONSERVATIVISM

Social institutions "are characterised by dynamic conservatism" (Herr & Anderson, 2005, p. 24). Often, members within a social organisation – *vis-à-vis*, for example, academics in a university faculty – prefer tradition, routines, processes and ways of knowing and doing to which they have become accustomed. "It is dynamic in that it constantly pulls practitioners back to a status quo that consists of norms, rules, skills, and values that become so omnipresent as to be taken for granted and go unchallenged… Action research can either reproduce those norms, rules, skills, and values or it can challenge them" (Herr & Anderson, 2005, p. 24). Often, faculty placed a strong emphasis, "on research as the action, and less emphasis on the effort to solve particular problems or induce particular changes in behaviour" (Sanford, 2005, p. 13). In my own faculty, our policies regarding thesis construction, research paradigm, approach, and design are well-worn and widely understood, and I have tried to work within the prescribed frameworks that are employed within the faculty.

### 3.1.4 EPISTEMOLOGICAL PERSPECTIVE



The epistemological view of my action research project lies within, "the *hermeneutic* tradition" (Hitchcock & Hughes, 2001, p. 227). "Hermeneutics focuses on recapturing the meanings of interacting with others, recovering and reconstructing the intentions of the other actors in a situation" (Cohen, Manion & Morrison, 2001, p. 29). A hermeneutic stance provides one with a, "powerful means of deepening self-understanding and building a conception of one's professional situation" (Brown & Heggs, 2011, p. 297). Such understanding, also called *verstehen* (Bryman, 2012, p. 29), explores peoples' perceptions of social phenomena. In my own research, I have used exchanges with the participants and collected data using different instruments to construct value-judgements regarding the efficacy of the activities in the action research and of the participants' opinions and experiences of the IPS 413 E Mathematics lessons. Having claimed a hermeneutic epistemological stance, it makes sense that my research lens is focused through an interpretivist paradigm.

## 3.2 RESEARCH PARADIGM – INTERPRETIVISM

Lincoln and Denzin (2006), in Mertens, Bledsoe, Sullivan and Wilson (2010, p. 195), state that paradigms encompass ontology and epistemology and suggest that, “paradigms are the overarching cosmological statements to which we subscribe when we engage in research. While mixed methods approaches are often associated with pragmatism (Teddlie & Tashakkori, 2009, p. 7), for the purposes of my thesis – which has an action research *design* – an interpretivist paradigm seemed better fulfil the philosophical, ontological and epistemological aspirations of the research. Interpretivism supports partnerships between the research and participants and thereby can focus on personal and professional development (Hammersley, 2012).

An interpretivist paradigm, “seeks to understand people’s lived experience” (Hennink, Hutter & Bailey, 2011, p. 14). Research that is driven by an interpretivist paradigm pursues social justice and an understanding of peoples’ perceptions of their lived experiences (Moyo, Modiba, & Simwa, 2015, p. 60). Such research is socially constructed. The researcher tries to empathetically interpret, understand and make sense of people’s subjective, but meaningful life experiences (Bryman, 2012, p. 28 - 30). Because this subjectivity impacts on research findings, interpretivist researchers have to problematize the notion of values-free research (Hennink, Hutter & Bailey, 2011, p. 14 - 15).



Reflexive analysis and interpretation of data enables the action researcher to discover problematic trends and thereby consciously try to improve the problematic phenomena. An interpretivist paradigm melds well with action research which is exploratory, descriptive, action-related inductive rather than deductive and places an emphasis on the well-being of its participants (Mouton, 2003, p. 151).

### 3.2.1 A NATURALISTIC PERSPECTIVE

The adoption of an interpretivist paradigm enables researchers to work alongside their research participants (Maxwell, 1996, p. 4), and thereby adopt a naturalistic perspective. This action research occurred within its most natural setting (Bryman, 2012, p, 622) – vis-à-vis the classroom.

Further, the research was (1) contextualized within the context of the PGCE IPS 413 E module; (2) as new evidence and information revealed itself, adaptations to the original planning were made; and (3) the emergent plan was informed by the interactions between researcher and participants.

By its very nature, such work required a fair degree of, "*inductive* theorizing, i.e. making sense of what you find after you've found it" (Gillham, 2000a, p. 7). Inductive reasoning, suggests Connole (1993, p. 10), "proceeds from specific observations (data) to general principles (laws)." That is, numerous specific observations enable the researcher to construct a composite situational understanding of the environment and activities that are being scrutinised.

### 3.2.2 A RELATIVIST ONTOLOGY

Ontologically, in action research, the reality of the researcher is understood to be socially mediated through experience, communication, interpretation and reflection. In action research, ontology is premised upon constructivist belief systems (Geelan, 2003, p. 13) and is said to be a relativist ontology. In other words, by constructing meaning and seeking consensus, a relativist ontology enables the researcher to arrive at a truth – *there may be other truths* – and any truth is but one of many (Bryman, 2012, 32-38). From this ontological perspective, it follows that although the researcher may be seeking specific and concrete understanding and resolution of a specific situation, the epistemological position is one in which knowledge remains uncertain and is subjectively constructed by the researcher (Koshy, 2010, p. 23 - 24). Fraenkel and Wallen (2009) suggest that action research practitioners ask questions such as:

> What kinds of methods, for example, work best for what kind of students? How can teachers encourage students to think about important issues? How can content, teaching strategies, and learning activities be varied to help students of differing ages, gender, ethnicity, and ability learn more effectively? How can subject matter be presented so as to maximise understanding?
>
> Fraenkel & Wallen, 2009, p. 589

Answers to such questions require analysis and interpretation of the on-the-ground experiences of the participants.

### 3.2.3 THE IMPOSITION OF PRE-SET THEORY IN TEACHING AND LEARNING

Mouton (2003, p. 151) suggests that in conceptualising action research projects, there is an, “emphasis on the participants and their world-views: [there is] a reluctance to impose any pre-set theory or explanation.” The requirement to establish the world-view positions held by the participants and to interact with them carries Platonic and Socratic overtones, and has been found to increase understanding of phenomena, situations and environments worldwide (Sepeng & Webb, 2012).

### 3.2.4 MICROGENESIS

Mouton (2003, p. 150-151) suggests that most types of action research, “have an explicit (political) commitment to the empowerment of participants and to changing the social conditions of the participants.” Action research can be used to demonstrate “microgenesis – that is, development within an observable time period” (Wertsch, in Cazden, 1997, p. 307). Such development can occur over relatively short time-spans (Birjandi & Ebadi, 2012).



## 3.3 RESEARCH APPROACH – MIXED METHODS

As an emerging approach, various authors apply different meaning to terms such as *paradigm*, *methodology* and *methods* and indeed many, “writings about MM blur the distinctions between paradigms, methodologies, and methods” (Teddlie & Tashakkori, 2009, p. 21). In this thesis, the terms research *paradigm*, research *approach*, and research *design* are formulated within the prescripts of my university.

Conceived variously as being the third path, the third research community, the third research paradigm or third methodological movement (Teddlie & Tashakkori, 2009, p. 3-4), mixed methods research approaches include, “qualitative and quantitative features in the design, data collection and analysis” (Mertens, 2005, p. 292), and thereby offer an alternative to the dichotomy of maintaining an only qualitatively oriented or only quantitatively oriented research stance. While some researchers suggest that, on the basis of epistemological position, it is not possible to combine qualitative and quantitative research, most researchers have adopted a technical approach which recognises that, “quantitative and qualitative research are each connected to distinctive epistemological and ontological assumptions, but the connections are not viewed as fixed and ineluctable" (Bryman, 2012, p. 631).

The rationale for using mixed methods approaches is informed by the fact that such an approach *mixes* qualitative and quantitative data rather than using them in tandem. In this thesis, I have conferred equal priority to the quantitative and quantitative aspects of my research and use both methods concurrently so as to comprehensively answer the research questions. Different tools are used to answer particular sub-questions more fully, to provide context for and illustration of the findings and to enhance the overall understanding of the findings. Such research lends itself to interpretivist explanations (Bryman, 2012, p. 628-634).

Because educational problems can be complex to resolve, mixed methods enable researchers to generate better understanding using the best-fit features of differently oriented qualitative and quantitative research instruments (Greene, Kreider, & Mayer, 2011, p. 260). Mixed methods approaches facilitate the purposes of action research that seeks the emancipation of its participants.



In action research, the researcher and participants co-construct an understanding of a problematic reality and then conceive of strategies for improving that reality. Action research is perceived as being less about using tried and trusted methods and more about being, "a work of art emerging in the doing of it" (Reason & Bradbury, 2008, p. 5). The action researcher, understanding the benefits and limitations of different quantitative and qualitative data gathering tools, uses a mixed methods approach so that, "their combined strength would result in improving the depth and accuracy of the findings" (Kumar, 2014, p. 20).

### 3.3.1. QUANTITATIVE FEATURES OF A MIXED METHODS APPROACH

In a mixed methods approach, the quantitative analyses provide a structured measurement of phenomena, produces an element of validity and reliability for the findings, and "communicates findings in an analytical and aggregate manner" (Kumar, 2014, p. 14). Measurement produces delineations, gauges differences consistently, and "provides the basis for more precise estimates of the degree of relationship" (Bryman, 2012, p. 164). As such, quantitative research tends to be linear, produce evidence of causality and generalization and can offer only a part answer to the emancipatory goals of action research.

### 3.3.2 QUALITATIVE FEATURES OF A MIXED METHODS APPROACH

Qualitative features of a mixed methods approach are that they are open and flexible, are unstructured, study multiplicities and "communicate findings in a descriptive and narrative rather than analytical manner (Kumar, 2014, p. 14). As such, qualitative research is inclined towards language, the well-being of its participants, contextualises conclusions and leans on experience (Henning, Stone, & Kelly, 2009, p. 4). In mixed methods research, these descriptive narratives are combined with quantitative findings to provide accurate answers to a multiplicity of research questions (Kumar, 2014, p. 25).

### 3.3.3 CONVERGENT PARALLEL MIXED METHODS



The use of many research tools is very time-consuming and generates copious amounts of data, but, suggest Fraenkel and Wallen (2009, p. 558), the payoff is that by integrating qualitative and quantitative data-sets, the different data will potentially help to reveal and explain patterns, trends, opinions and behaviours. This, in turn, provides cross-checks for the work, and enhances methodological triangulation (Creswell, 2014, p. 201) for the research.

In order to ensure methodological triangulation, a "convergent parallel mixed methods" strategy was used (Creswell, 2014, p. 15), that is, different qualitative and quantitative techniques were collected at about the same time. Later, in Chapters 4 and 5, the findings will show that in spite of the pre-planning, the execution of that planning did not go smoothly.

## 3.4 RESEARCH DESIGN – ACTION RESEARCH

Cohen, Manion and Morrison (2001) introduce their view of action research in the following way:

> One of the founding figures of action research, Kurt Lewin (1948) remarked that research which produced nothing but books is inadequate. The task, … is not merely to understand and interpret the world but to change it. Action research is a powerful tool for change and improvement at the local level.
>
> Cohen, Manion & Morrison, 2001, p. 226

In action research design, "the action aspect is central to the process and influences the methods" (Noffke & Somekh, 2011, p. 97). Action research treats theory and practice as intertwined, of equal value and, "integrates the development of practice within the construction of research knowledge in a cyclical process" (Noffke & Somekh, 2011, p. 94). Further, Cohen, Manion and Morrison (2001, p. 226) posit that action research can be used by individuals or groups who are seeking localised solutions to localised problems. As such, its practice can immediately impact on social settings and bringing about improvement (Noffke & Somekh, 2011, p. 97). In educational settings, for example, these problems can be found in wide-ranging areas that include the construction and exploration of new teaching methods, of trailing different learning strategies, experimenting with assessment, professional and attitudinal development programes and assessment.

### 3.4.1 TYPES OF ACTION RESEARCH

Fraenkel and Wallen (2009, p. 590-591), suggest that there are two main types of action research, namely (1) Practical Action Research, in which the, "primary purpose is to improve practice in the short term as well as to inform larger issues," and (2) Participatory Action Research, which, in addition to the above, also seeks to empower people and bring about beneficial social change. Henry and McTaggart (1996, p. 6), perceive a third research strand: Classroom Action Research, which provides opportunities for research which is conducted in classrooms:

> Classroom action research typically involves the use of interpretive modes of enquiry and data collection … with a view to teachers making judgments about how to improve their own practices …The emphasis is 'practical', that is, on the interpretations teachers and students are making in the situation.
>
> Henry & McTaggart, 1996, p. 6

For the purposes of this action research, the best fit alternative seemed to be classroom action research.

### 3.4.2 GENERAL ATTRIBUTES OF ACTION RESEARCH

Action research typically focuses on direct intrusions within the research setting; it is, "inquiry that is done *by* or *with* insiders of an organization or community, but never *to* or *on* them" (Herr & Anderson, 2005, p. 3). Often undertaken by teacher-researchers (Frankel & Wallen, 2009, p. 13), action research focuses on the improvement of existing situations and practices. Teachers who undertake action research reflexively consider the problematic situations and "taken-for-granted practices" (Newton Suter, 2012, p. 172) in their classrooms and attempt to improve problematic situations. This "leads not just to new practical knowledge, but to new abilities to create knowledge" (Reason & Bradbury, 2008, p. 5). Robinson (2005) writes that, "action researchers share a commitment to three main goals: the understanding and improvement of practice; the enhancement of the problem-solving capacities of the practitioners with whom they collaborate; and the advancement of knowledge about practice itself" (Robinson, 2005, p. 60). In this sense, action research can be perceived as a living entity, constantly adjusting and readjusting itself to the unfolding needs and aspirations of the participants that it would liberate.



### 3.4.3 PRAXIS

Praxis, suggests Woodwell (2014, p. 166), is a term that is synonymous with action research. Praxis describes the iterative, cyclical processes which are used by practitioners to reflect upon and learn from past experiences and improve their practice. Praxis is largely driven by intra-mental cognitive function and an intrinsic human motivation to do better. As such, it is informal and does not necessarily invoke the use of classification.

Reflection and intuitive thinking arouses "the intellectual technique of arriving at plausible but tentative formulations without going through the analytic steps by which such formulations would be found to be valid or invalid" (Bruner, 1960, p. 13). Bruner (1960, p. 13-14), suggested as well that, "the training of hunches is a much-neglected and essential feature of productive" (Bruner, 1960, p. 13-14). Indeed:

> those involved in action research want to solve some kind of day-to-day immediate problem, such as how to decrease absenteeism or incidents of vandalism among the student body, motivate apathetic students, figure out ways to use technology to improve the teaching of mathematics, or increase funding.
>
> Fraenkel & Wallen, 2009, p. 589

Thus, action research can be seen to not only report on an existing situation or phenomenon but also, through praxis – planning, acting, evaluating, refining and learning (Koshy, 2010, p. 9) – to also actively attempt to intervene, and when possible, to improve that situation or phenomenon.



### 3.4.4 CONCEPTUALISING A PLAN OF ACTION

Herr and Anderson (2005, p. 9) posit that some researchers, "see the goal of action research as *improving* practice or *developing* individuals, whereas others see its goal as *transforming* practice and participants." I tried to attend to both aspirations. In an attempt to solve a real world problem, *vis-à-vis* an inability to solve mathematical word-problems, I constructed a teaching and learning spiral that contained two action cycles.

I used mathematics content knowledge tasks to establish prior knowledge – that data was quantified. Interpretation of the data revealed gaps in content knowledge and thereby enabled me to create leaning opportunities and activities that might provide cognitive and skills support to the participants and to reflexively monitor the efficacy of these learning-oriented tasks (Henning, Stone, & Kelly, 2009, p. 128 - 129).

Sepeng and Webb (2012, p. 4), designed action cycles in which their prior-knowledge assignments were re-administered at the end of the action cycle. In this way, they were able to make a direct comparison of the two data-sets seamless. This technique enabled them to accurately quantify *the value-added* by the teaching and learning strategies that they had used. This, technique became a feature of my own work.

In order to establish and *understand* the participants' views on barriers that might preclude them from solving word-problems and their views upon the efficacy of the visualising strategies that were used in the IPS 413 E module, I used field-notes, a Thinkboard, questionnaires and a focus group interview to gather qualitative data.

### 3.4.5 PLANNING ACTION CYCLES

Koshy (2010) suggests that when they are presented on paper, action research cycles follow a somewhat linear path; conversely, in practice the various stages in action research tend to be overlapping and, "in reality the process is likely to be more fluid, open and responsive" Koshy (2010, p. 4). However, as one drills downwards towards a better understanding of a problem and its potential solutions, so consecutive, iterative activities should become better focused. In my planning, I envisaged working through two research action cycles.



In the first cycle, through a process of, "analysis, fact findings, planning, execution and evaluation" (Sanford, 2005, p. 13), I hoped to find starting points, and establish real and perceived ineptitudes and inadequacies that might be present within the cohort: some individuals might have previously developed a loathing/fear of mathematics while others might not fully understand basic arithmetic routines and conventions, etcetera. I planned to uncover those stories and mathematical gaps and use that information to try to alleviate fears and misconceptions and to assist the students to develop a more robust understanding of foundational mathematical concepts, practices and pedagogy.

To that end, reflexively, I conceived and constructed a series of education-theory lessons in which salient aspects of visualisation research was used to set the tone of the methodological and pedagogical aspirations of the module and this research. I coupled this work to CAPS (Department of Education, 2011) directives and various aspects of numbers-based, arithmetic content. I constructed numerous visual stimuli to embed patterning and fractions calculations. I was aware that all of these considerations would be subject to my interpretation of findings obtained in my initial lessons with the students.

The second cycle was conceived to build upon successes and failures that manifested in the first cycle. I used the stepping-stones and stumble-stones that were revealed in the first cycle to push towards developing a visual strategy for solving word-problems. I hoped that these evolutionary, iterative cycles of planning, acting and reflecting would produce improved ways of knowing and doing (Reason & Bradbury, 2008, p. 1 - 5).

### 3.4.6 FEATURES OF ACTION RESEARCH

Mouton (2001, p. 150-151), suggests that action researchers do this type of research "in order to gain understanding and insight into the life-worlds of research participants." Mouton (2001) also suggests that action research is empirically based; that is, the findings are derived from activities, experiments and observations rather than from theory. The data that informs the findings are a hybrid of existing data (as found in a literature review) and new data (as would be captured during the actual contact sessions with participants). Further, the research questions that drive action research are, "exploratory and descriptive or [have an] action-related focus. Conceptualisation is, "more inductive than deductive," in other words, one makes "sense of what you find after you've found it" (Gillham, 2000a, p. 7).



Mouton (2001) suggests that the most common modes of data collection include the observation of the actions of participants; semi-structured interviews; analysis of documents; and the construction of stories – a first person account of the trials and tribulations of the day-to-day experiences of the researcher and the participants. Both analytical and interpretive views can be woven into a research text to narrate this story (Gillham, 2000a, p. 22). The narrative should assist the reader to understand the lived experiences of the people involved in the action research, however, novice qualitative researchers may concentrate, "too much on description at the cost of analysis and interpretation" (Altrichter, Posch & Someka, 1993, p. 185), and may tend to, "offer very few analytical points and interpretations in order to reduce the risk of getting it wrong" (Altrichter et al., 1993, p. 191).

Gillham (2000a, p. 6) suggests that individuals and context have a direct bearing on the nature of activity and subsequently on the data that materialises. This gives rise to an *emergent* design: for all the planning that is done, nonetheless, the original grand research plan can be derailed by on-the-ground, lived experiences. Thus, while it is acceptable to conceive a hypothesis or hold a set of assumptions at the outset of the research, it is acknowledged that the actual research and the collected data might lie at a tangent to, or even contradict, the original proposition.

Gillham (2000a) writes, "working inductively from what's there in the research setting develops grounded theory: theory that is grounded in the evidence that is turned up" (Gillham, 2000a, p. 17). In other words, the evidence that is revealed through research – the data – enables one to conceptualise and reconceptualise a grounded theory: the richer the evidence, the richer the theory. Thus, in action research, it is a good practice to begin the research project by seeking out and gathering together useful information while adopting, as best possible, a neutral position and an unprejudiced mind (Gillham, 2000a, p. 18). Indeed:

> The American social anthropologist Clifford Geertz emphasizes the importance of beginning research into any culture by describing what you find in detail. He calls this thick description: a process which makes you pay attention to the fine grain of what you are observing, and reflecting on it.
>
> Gillham, 2000a, p. 19.



This open position, and the data that is assembled by the initial research, may force a, "*paradigm* shift – a complete change in the way we understand or theorize about what we are studying" (Gillham, 2000a, p. 19).

### 3.4.7 CONCEPTUALISING ACTION CYCLES FOR THIS THESIS

An action research is designed to contain a spiral of action cycles (Koshy, 2010, p. 4; Herr & Anderson, 2005, p. 5) such that data gathered in preceding cycles feeds into subsequent cycles. Analytic spirals (Hennink, Hutter, & Bailey, 2011, p. 237 - 238) within action cycles enable researchers to develop *thick descriptions* of data. "Each cycle increases the researchers' knowledge of the original question, puzzle, or problems and, it is hoped, leads to its solution" (Herr & Anderson, 2005, p. 5).

### 3.4.7.1 A MODEL OF AN ACTION RESEARCH PROJECT

An action research model, suggest Fraenkel and Wallen (2009, p. 592) and Koshy (2010 p. 4), includes:

- planning to implement a change;
- enacting the change, observing the effects of the process and the consequences of the change;
- reflecting on these processes and the consequences and after re-planning; then
- once again, acting and observing;
- reflecting and planning;
- and, iteratively, moving forward

Below, in *Figure 3.1*, I have constructed a simple flow-chart to demonstrate how the various aspects of action research flow from one to another.

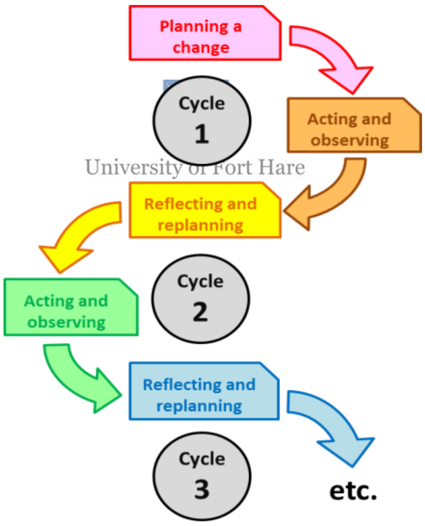




*Figure 3.1*. A flow-chart containing iterative cycles of action research

Various authors such as Cohen, Manion and Morrison (2001, p. 234 - 239), Mertler (2012, p. 14 - 22), and Rembe, Shumba, Maphosa and Musesengwe (2016, p. 38 - 93) provide slightly different models. However, when they are distilled to their essence, they all adopt a spiral methodology and they each employ a looping-back feature which is premised upon the view that, should a route prove fruitless or of little benefit, then a review of troublesome phenomenon will need to be identified and an alternative path will need to be put in place. In fact, action research provides teachers with a common-sense explanation of good teaching praxis.

### 3.4.7.2 A CONCEPTION OF A LAYOUT FOR MY ACTION CYCLES

Below, in *Figure 3.2*, I have constructed a broad-strokes representation of my general conception of the layout for an action cycle in my own research. The conception of this layout is based upon an action research design model: *Plan, Act, Observe, Reflect*, created by Kemmis and Wilkinson (1998).

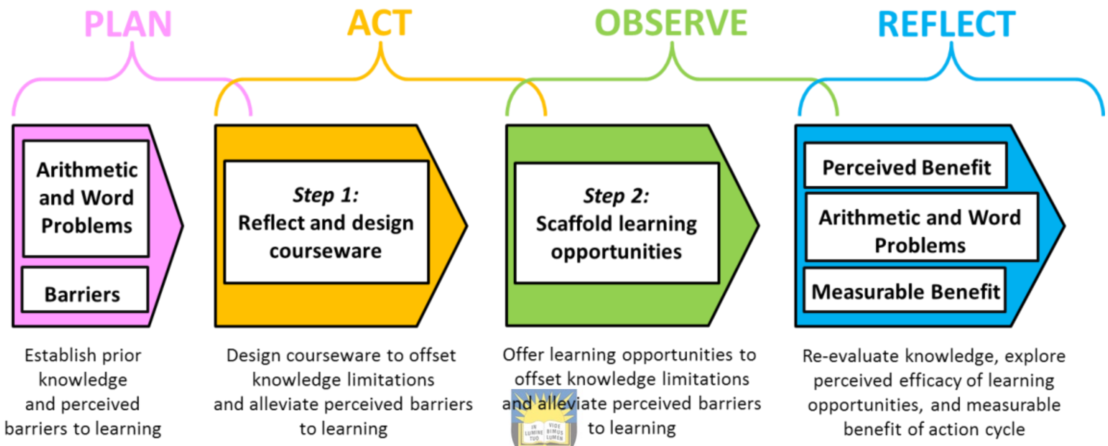





*Figure 3.2*. A flow-chart which demonstrates the general conception of a single action cycle

Initial planning was done prior to the implementation of the action research. In Action Cycle 1, two tasks were prepared for the participants one of which looked at existing foundational arithmetic knowledge and the second of which, looked at their existing ability to attend to and solve word-problems. The captured data provided information on the participants' prior knowledge and revealed gaps in their knowledge. The scrutinised data facilitated the construction of materials and activities that were conceived to offset some of the knowledge gaps. Subsequently, that courseware was offered to the participants. Their interaction with the courseware and classroom-based activities were observed and iteratively, on-the-ground experiences of each task informed the next. At the end of the action cycle, the participants were re-evaluated, and that data was used to reflect upon the efficacy of the action cycle.

In general conception, the main features of Action Cycle 2 mimicked Action Cycle 1.

### 3.4.8 CONCEPTION OF ACTION CYCLE 1 FOR THESIS

The Kemmis and Wilkinson (1998) model, above, was fine-tuned to define features of the conception of the first action cycle of the thesis. It is presented in *Figure 3.3*, below:

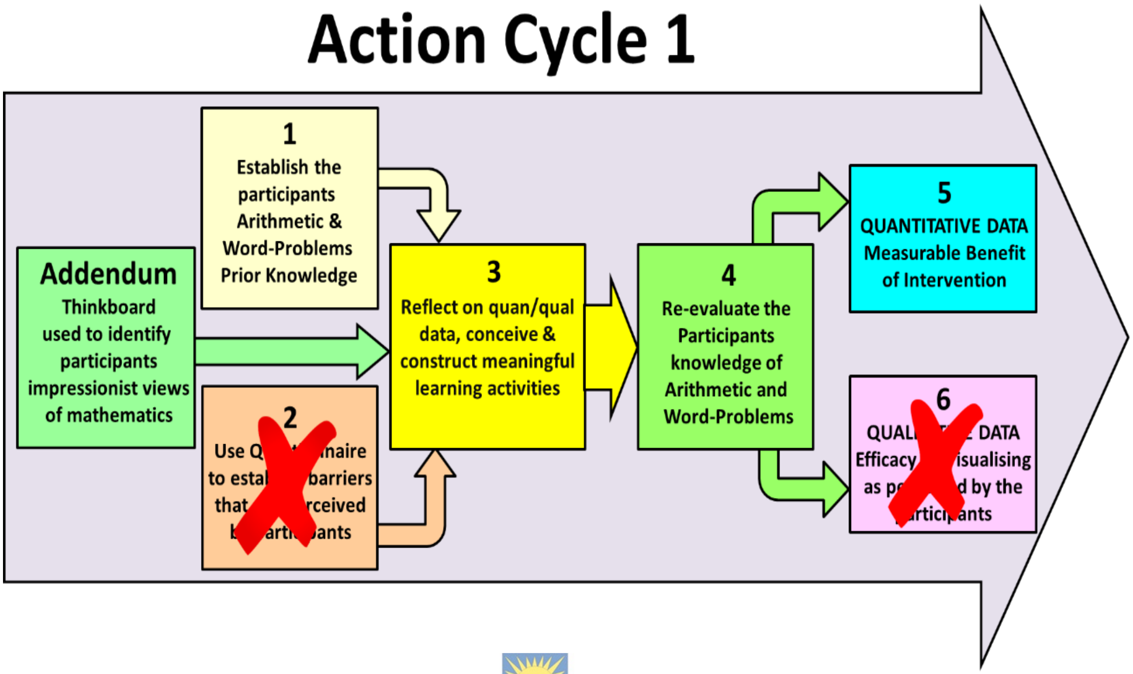



*Figure 3.3.* Conception of Action Cycle 1 for IPS 413 E Mathematics



#### 3.4.8.1 ACTION CYCLE 1: 1 – ESTABLISH PRIOR KNOWLEDGE

At the beginning of the first action cycle, the participants were asked to complete two assignments (Item 1 in Figure 3.3), one of which investigated Foundational Arithmetic Concepts while the other looked into Word-Problem Calculations. The quantitative data was captured and was then analysed. It was used to establish existing prior knowledge and knowledge gaps, and served as a baseline for the conception of and design of activities that might provide support to the participants.

#### 3.4.8.2 ACTION CYCLE 1: 2 – ADMINISTER QUESTIONNAIRE

Concurrent with the application of the prior knowledge tasks, I *intended* to ask the participants to complete a questionnaire (Item 2 in Figure 3.3). I intended to use that information to investigate the participants' perceptions of mathematics. *However, on-the-ground factors precluded the administration of the questionnaire during Action Cycle 1*. Because of different on-the-ground factors, a spontaneously-conceived

Thinkboard (Addendum in Figure 3.3) was presented to the participants. Later, this is fully discussed in (4.1).

### 3.4.8.3 ACTION CYCLE 1: 3 – REFLEXIVE SUPPORT

The quantitative data and qualitative data were considered and then, reflexively, were used to conceptualise and construct scaffolding mathematical activities (Item 3 in Figure 3.3) in the action cycle.

### 3.4.8.4 ACTION CYCLE 1: 4 – RE-EVALUATION OF MATHEMATICAL KNOWLEDGE

At the conclusion of Action Cycle 1, (Item 4 in Figure), the participants mathematical content knowledge was re-evaluated.

### 3.4.8.5 ACTION CYCLE 1: 5 – EXPLORE FOR MICROGENESIS

After marking and collating, the quantitative prior knowledge data was compared with the data captured by the re-evaluation tasks (Item 5 in Figure 3.3). The data-sets were used to ascertain whether *microgenesis*, (3.2.4), that is, improvement in performance over an observed period of time, had indeed occurred (Cazden, 1997).



### 3.4.8.6 ACTION CYCLE 1: 6 – PERCEPTIONS OF VISUALISING SCAFFOLDS

At the end of the first action cycle, I also *planned* to hold a semi-structured interview (Item 6 in Figure 3.3) with a purposefully selected sample of the participants. I planned to use this opportunity to try to establish their opinions regarding the mathematical content and the visual methodologies and scaffolds that were used in the first action cycle. As in (3.4.8.2), above, o*n the ground factors precluded the administration of the questionnaire during Action Cycle 1.* Later, this is discussed in (4.1).

In my conception of this action research project, the second action cycle began at the start of the second quarter of 2016. In it, I planned to use the thick research findings gleaned from the first action cycle as a baseline, a starting point, for the second cycle which, itself, was designed in much the same way as the first action cycle.

## 3.5 SAMPLE AND SAMPLING

The full population of the 2016 cohort of PGCE students for IPS 413 E Mathematics was 60 persons. Every student was invited to participate in the study. Thus, ideally, the entire population might have become part of the sample. Because I had direct access to the participants in the study, a "single-stage" (Creswell, 2014, p. 158) sample was invoked. Chin (2013, p. 64 - 65), working with a cohort of 24 PGCE students, achieved 8 participants for his PhD research and of these, selected 5 for data capturing purposes for his doctoral thesis. In this research, 38 out of 60 students became participants.

## 3.6 DATA COLLECTION INSTRUMENTS

In Chapter 1, I have suggested that my main research question states:

> *How will visualisation strategies assist student-teachers to better understand and solve mathematical word-problems?*

The main research question is framed by three sub-questions:



4. *What existing word-problem, problem-solving strategies do the students hold?*
5. *What barriers to solving word-problems do the students perceive that they hold?*
6. *How effective do the students perceive the visualisation strategies to be?*

In this thesis, Chapter 2 – the literature review – has revealed some of the conceptions and experiences of others, and has served to inform this research. However, in order to capture a more full understanding – and thereby answer the research questions, additional information has to be captured using the quantitative and qualitative data captured from the various assignments, questionnaires, classroom classroom-based activities, etcetera. Woven together, such data streams have a capacity to reveal a rich tapestry of information.

However, Herr and Anderson (2005) caution that:

> most insider action researchers are doing the inquiry while continuing to carry the rest of their workloads. … Because of this lived reality, the methodological approach to the data gathering need to be *researcher*

> *friendly*; by this we mean realistically doable, given the contexts and demands of our jobs.
>
> Herr & Anderson, 2005, p. 78

With this in mind, the research tools described below were chosen be appropriate for the purposes of action research and simultaneously, given that I was the only researcher involved this quest, the research instruments needed to be user-friendly and doable.

## 3.6.1 QUANTITATIVE ASSESSMENT TASKS

Like Sepeng & Webb (2012), (3.4.4), in each action cycle, an assessed assignment was offered at the beginning, and subsequently again, at the end of each action cycle. This strategy was appealing because it enabled me to quantitatively measure the *value* added by the teaching and learning strategies used *during* each action cycle.

In 2015, I conceptualised questions to insert into the action cycle 1 assignments. Because the B Ed Year 1, IPS 123 E Mathematics module had mathematical content that was similar to the work covered by the participants, in semester 2 of 2015, a pilot run was conducted in the B Ed classroom and information captured from that exercise was used to fine-tune the Action Cycle 1 tasks. A *Foundational Arithmetic Concepts* assignment was designed to examine arithmetic competencies (Siemon et al., 2012, p. 588 - 589); a second assignment, *Word-Problems Calculations* was used to establish how the participants coped with routines-based, well-structured, mathematical word-problems (Zanele, 2015, p. 18).



### 3.6.1.1 LOT AND HOT SKILLS

Bloom's taxonomy for learning, teaching and assessing (Bloom, Max, Furst, Hill, & Krathwohl, 1956), in its verb-driven, updated form (Krathwohl & Anderson, 2010) was used to support the construction on the questions in the assessment tasks. The revised taxonomy, "encourages learners to be actively engaged in classroom activities" (Maphalala, 2016, p. 105); it liberates both Low Order Thinking (LOT) and Higher Order Thinking (HOT) skills (Thompson, 2008, p. 97; Bennie, 2005, p. 82), and provides intrinsic motivation to learners to monitor their own cognitive development.

The hierarchical structure of the cognitive domain of Bloom's Revised Taxonomy enables teachers to use the questioning taxonomy to examine ideas in different ways.

The taxonomy provides modalities such as *question stems* which enable teachers to expand the scope of questions – and their imposed cognitive demands – progressively away from lower order thinking (LOT) skills towards higher order thinking (HOT) (Bezuidenhout & Alt, 2011, p. 1074; Mohammadi et al., 2015, p. 16). The view is that, over time, LOT skills are subsumed by HOT skills, schemata are constructed and compression occurs (2.6.2).

#### 3.6.1.2 DESCRIPTION OF PRIOR KNOWLEDGE ASSESSMENT TASKS

The prior knowledge assessment tasks were designed to provide answers to the first sub-research question, namely, *What existing word-problem, problem-solving strategies do the students hold?* The assessment tasks were provided to the participants in February 2016. Each assessment task was 40 minutes long.

The Foundational Arithmetic Concepts assessment task contained four main questions, with altogether twenty sub-questions. Each correct answer was awarded one mark. Questions 1 and 2 offered questions which tested *Remembering* skills while Questions 3 and 4 offered questions at a *Remembering* and *Understanding* level (Krathwohl & Anderson, 2010).



The Word-Problem Calculations assessment task contained ten word-problems, the first five of which were asked at an *Understanding* and *Applying* level on Bloom's Revised Taxonomy of Thinking Skills, and the latter five questions at an *Applying* and *Analysing* level (Krathwohl & Anderson, 2010).

### 3.6.2 QUESTIONNAIRES

Two questionnaires were constructed. The discussion below reveals the influences that affected the design and layout of the questionnaires.

#### 3.6.2.1 ACTION CYCLE 1 - QUESTIONNAIRE 1

Questionnaire 1 was constructed to elicit responses that reveal answers to the second sub-research question, namely, *What barriers to solving word-problems do the students perceive that they hold?* It contained two parts, namely Part A and Part B.

Part A had 3 sub-sections and 10 specific questions: it sought biographical and language information. Sub-section A1 explored gender, home language, age, and highest mathematical achievement in school. Sub-section A2 investigated attitudes towards mathematics: the questions were presented on a “Likert scales” type of grid, (Cohen, Manion & Morrison, 2001, p. 253). Sub-section A3 examined the participants’ personal views of their English language fluency in mathematical settings. It, too, used Likert scales questions.

Part B had 5 sub-sections and 10 specific questions. The questions in Part B, were used to establish the participants’ views on barriers that they perceived that they had when they try to solve mathematical word-problems. Part B in a novel way. To assist the participants to reflect deeply, I required them to read through a scaffold about word-problems *before* they attended to the answers for the questionnaire. I used Gooding’s (2009) five barriers that are linked to solving word problems as the pivot for the questions. Each of the five barriers was addressed through two Likert scales (Cohen, Manion & Morrison, 2000, p. 253), questions. Below each pair of questions, I inserted an area for short specified responses.



#### 3.6.2.2 ACTION CYCLE 2 – QUESTIONNAIRE 2

The second questionnaire was designed to be administered to the participants *after* they had worked through numerous visualising and activities-filled lessons. Its task, chiefly, was to provide insights and answers for the third research sub-question, namely, “*How effective do the students perceive the visualisation strategies to be?”*

The questionnaire focused on two central themes. Each theme has sub-sections. Each sub-section contained a Likert scales (Cohen, Manion & Morrison, 2001, p. 253) area for the participants to populate and an area in which, should they choose to do so, the participants might write a personal, reflective account of a point that they might wish to share.

Part 1 posed questions to establish whether, *from their personal perspective*, the participants felt any benefit was derived from the visual approach and visual scaffolds that they were exposed to in their IPS 413 E mathematics classes. The conception and layout of Part 1 of Questionnaire 2 was similar to that found in Questionnaire 1. As in the first questionnaire, the sub-questions explored the five categories of difficulties that students confront when they attend to word-problems (Gooding, 2009).

In Part 2 contained the same questions as Part 1. However, in Part 2, the participants were encouraged to imagine a future time, in 2017 and beyond, in which they might be teaching mathematics to Intermediate Phase children. In Part 2, the participants were encouraged to consider whether or not they believed that the visual scaffolds that were used in their IPS 413 E Mathematics classes would be applied into their own teaching practice.

#### 3.6.2.3 FACTORS THAT INFLUENCED THE USE OF QUESTIONNAIRES

Gillham (2000b, p. 1) suggests that, "good research cannot be built on poorly collected data; … poorly collected data will be wasteful of time and money, and discredit the name of 'research'." Further, in order to assure reliability (Newton Suter, 2012, p. 251), and validity (Shenton, 2004, p. 69); questionnaires must be treated as but one tool in a box filled with many research tools which are triangulated to form a comprehensive and coherent answer to a research question.



Methodological triangulation (Creswell, 2014, p. 201; Fraenkel & Wallen, 2009, p. 510) was achieved by complementing the findings from the questionnaires with the literature review, field notes, semi-structured interviews and the quantitative assessment tasks. Collectively, the interrogation of the multiple data sets that were gathered from the research tools were knitted together to form a cogent understanding and thereby offered deep insight – crystallization – of the research (Fraenkel & Wallen, 2009, p. 512).

#### 3.6.2.4 BENEFITS OF USING QUESTIONNAIRES

Gillham (2000b, p. 5-14) suggests that questionnaires have many benefits which include:

- Questionnaires are cheap;

- They are not time-consuming to administer;
- They can generate rich data quite easily and without too much difficulty;
- It is easy to collate True/False and Likert scales data;
- Participants can submit their completed questionnaires anonymously;
- All participants receive the same questionnaire and answer the same questions.
- Questionnaires can be used to test hypotheses.

3.6.2.5 PROBLEMS ENCOUNTERED WHEN USING QUESTIONNAIRES

Gillham (2000b, p. 9-14) raises the following problems with questionnaires:

- One needs a captive audience, otherwise the response returns tend to be low;
- Many researchers badly conceptualise and poorly design their questionnaires;
- Badly conceived questions result in badly answered questionnaires, *vis-a-vis*, they produce data which is of no value to the research enterprise;
- Questionnaires need to be short but impactful;
- They must written up using clear and unambiguous language;
- The words used in the questions will have a bearing on the quality of the answers;
- The literacy levels of participants and their mother-tongue versus the language in use impact on how the questions are understood and answered;
- For most people, oral forms of communication are better than written forms;
- The participants may not take the questionnaire seriously;
- Participants may be worrying about what is done with the data.



Most, of the participants were isiXhosa mother-tongue learners with mixed abilities when speaking, writing and thinking in English. Irwin (2002) suggests that, "local circumstances, conditions and values must be of paramount consideration if questionnaires are to be used effectively in southern Africa" (Irwin, 2002, p. 10). With that in mind, in 2015, two isiXhosa-speaking colleagues reviewed the questionnaires. Their advice was worked into the questionnaires. The words and sentences were reset to about a Grade 7 to Grade 8 level. While Gillham (2000b, p. 20) cautions that in all probability the views of these peers were, "impressionistic rather than specific," nonetheless their comments enabled me to coax out issues of ambiguity and to write up the questions in shorter sentences. Succinctly, Koshy (2010, p. 83), states, “Keep the questionnaire simple.” This advice, too, was applied to the questionnaires.

### 3.6.2.6 TYPES OF QUESTIONS USED IN QUESTIONNAIRES

Questionnaires can address three types of questions, namely (1) facts; (2) opinions, values and judgements; and (3) behaviours. Gillham (2000b, p. 28) and Fraenkel and Wallen (2009, p. 132) suggest *that selected response questions* are useful when trying to gather factually-based information that can be answered with True/False or Yes/No responses. These questions are also called closed-ended questions (Fraenkel & Wallen, 2009, p. 396) because the response is discrete. Selected response questions are easy to administer, require very little thinking on the part of the participants and are easy to collate.

Ranked response questions are similar in structure to selected response questions, but are used for questions that probe opinions and values. Ranking allows participants to allocate degrees of importance or, “relative degree of preference, priority, intensity, etc.” (Cohen, Manion & Morrison, 2001. p. 252). This allows participants to, “express preferential judgements” Gillham (2000b, p. 31). Likert scales offer an alternative type of ranked response technique. They contain “affective measures” (Newton Suter, 2012, p. 113), thus they have the ability to examine self- or other-directed perceptions, values and attitudes that participants might have towards mathematics, aspects of mathematics, or mathematics teachers, etcetera.



The difference between Likert scales and either selected responses or ranked order questions lies in the addition of a rating scale that is found in the Likert scales (Cohen, Manion & Morrison, 2001, p. 253). By introducing an interrogative adverb such as, *how*, into, for example, *how* seriously? or *how* often?, the answers open up possibilities for a continuum of degrees of acceptance or rejection of the position statement or question. In my own work, I placed statements rather than questions into my questionnaires. I adopted a five scale rating and provided spaces in a grid for the participants to populate with short textual responses.

*Specified response questions*, suggests Gillham (2000b, p. 30), are *open* questions that are used to probe people’s thoughts. They are often presented at the end of a questionnaire and ask participants to provide written responses and explanations of their opinions. Cohen, Manion and Morrison (2001) suggest that open-ended responses, "contain the 'gems' of information that otherwise might not have been caught," and continue, stating that, "an open-ended question can catch the authenticity, richness, depth of response, honesty and candour which… are hallmarks of qualitative research" (Cohen, Manion & Morrison, 2001, p. 255).

#### 3.6.2.7 FACTORS THAT INFLUENCE THE READABILITY OF QUESTIONNAIRES

Gillham (2000b, p. 38) suggests that the ordering of the questions in the questionnaire should follow a logical pattern, leading the participant from one question to the next. Further, he suggests, "it is extremely boring… to answer a series of scaled-response questions; and people stop thinking about what they are doing" (Gillham, 2000b, p. 39). To offset that negative potential, a mixture of selected response, ranked response and specified response questions were blended into the questionnaire.

Gillham (2000b, p. 37) suggests that a well-designed questionnaire should *look* and *work* well. His view talks to a hallmark of my own work: I have always believed that the *good-design* of a paper, a slide-show, a text or an explanation provides a professional and critical – but often overlooked – contribution to the successful outcome of learning endeavour. From an aesthetic point of view, Gillham (2000b, p. 37) suggests that one should aim for an uncluttered layout and the use of easily-read fonts such as Arial, Comic Sans MS or (more formally) Times New Roman. These fonts present well on paper. Further, a well-designed questionnaire should have a cover page and title and should contain pertinent information. He suggests that questionnaires should be, at most, five to six pages long, with each page containing at most five to six questions.



In this research, Questionnaire 1 was six pages long. Two of these pages were reserved for the cover page and an information page, while the four other pages contained the questions. There were two main themes which together contain six main areas of interest, with a combined 20 sub-questions in total. In Questionnaire 2 too, there were six pages, two of which were for the cover page and information, while the other pages contained the questions. In this questionnaire, there were two main themes, six main areas of interest, and 15 sub-questions.

### 3.6.3 FIELD NOTES

Altrichter et al. (1996, p. 68-118) suggest when one writes field notes, it is important for the researcher to try to adopt the lens of a neutral observer. However, despite this desire to embrace a neutral position, Cohen, Manion and Morrison (2001, p. 145) suggest that this type of data gathering can bring many tensions into a research setting: it can be difficult to establish suitable codes of conduct and communicative techniques that are agreeable to all parties, and it can be tricky to try to handle issues which have emotional content attached to them. Fraenkel and Wallen (2009, p. 506) suggest field notes, "are the researchers' written account of what they hear, see, experience, and think in the course of collecting and reflecting on their data." But, writes Silverman (2002, p. 64), unwittingly, researchers, may try to report on everything such that they do not see the trees for the woods.

#### 3.6.3.1 BENEFITS OF USING FIELD NOTES

Field notes, "can be written *in situ* and away from the situation" (Cohen, Manion & Morrison, 2001, p. 146), and they enable the researcher to construct a qualitative, unstructured narrative (Cohen, Manion & Morrison, 2001, p. 190). Further, for educators, they enable the researcher to reveal the context and constraints of teaching and learning and help to flesh-out the detail of what actually occurs in the classroom.



#### 3.6.3.2 RECORDING OBSERVATIONS

The recording of observations can be attended to in different ways. These may include:

- quick, fragmentary jottings of key words/symbols;
- transcriptions and detailed observations written out fully;
- descriptions that, when assembled and written out, form a comprehensive and comprehensible account of what has happened;
- pen portraits of participants;
- reconstructions of conversations;
- descriptions of the physical settings of events;
- descriptions of events, behaviour and activities;

Cohen, Manion & Morrison, 2001, p. 311

### 3.6.4 FOCUS GROUP INTERVIEWS

Interviews, or, "vocal questionnaires" (Newton Suter, 2012, p. 114) enable researchers to better understand the mathematical knowledge, skills, reasoning, solution strategies and conceptual understanding of their participants (Siemon et al., 2012, p. 132). Benefits include understanding the needs of the participants better; finding out how they think and what they think; developing a sense of their beliefs and motivation pertaining to mathematics and informing and optimising future lessons.

#### 3.6.4.1 PURPOSIVE SAMPLING

A "purposive sample" (Fraenkel & Wallen, 2009, p. 99) was selected from the group of participants for the focus group interview. Purposive sampling requires researchers to think critically about the features of a research project that they want to examine and to then use that knowledge to choose participants whom they judge will be best suited to assist them in achieving that goal (Silverman, 2002, p. 250). Such samples are selected by researchers who, "use their judgement to select a sample that they believe, based on prior information, will provide the data they need" (Fraenkel & Wallen, 2009, p. 99).



Even so, it is important to note that because, "the selection of a focus group does not follow strict methodological dictates, … it cannot be considered a valid representation of larger public opinion" (Woodwell, 2014, p. 29). A focus group should ideally comprise six to eight selected participants who, with the support of a moderator, reflect upon a selected group of issues for about sixty to ninety minutes (Hennink, Hutter & Bailey, 2011, p. 136). In this research, the data that was captured from the various quantitative assessments, questionnaires and field notes were used to identify a purposive sample of the participants for the focus group interview.

#### 3.6.4.2 BENEFITS OF USING FOCUS GROUP INTERVIEWS

Focus group interviews are a primary source of data collection (Mouton, 2003, p. 69; Roth & Bradbury, 2013, p. 354). Because the participants are in a group, they get to hear each-other's opinions. Cohen, Manion and Morrison (2001, p. 267) suggest that focus group interviews are exchanges of thoughts and ideas which are shared between people on matters of common interest. These opportunities validate and assist all participants to better understand each other's perspectives. Prompted by the viewpoints of others, participants may be encouraged to share additional information (Bantwini & Arowolo, 2015, p. 505). This enables the researcher to, "get close to the social actors' meanings and their interpretations, to their accounts of the social interaction in which they have been involved" (Blaikie, 2010, p. 207). Focus group interviews, like other qualitative research tools, "can be used for exploratory, explanatory or evaluative research" (Hennink, Hutter & Bailey, 2011, p. 136 - 137), but it is important to acknowledge that because the information is captured within the framework of a *group*, the group dynamic may distort the data. In addition, Fraenkel and Wallen, (2009, p. 452) make the point that a focus group interview is not a group discussion, and caution the moderator against allowing the participants to go off topic.

3.6.4.3 UNAVOIDABLE FEATURES OF FOCUS GROUP INTERVIEWS

Cicourel (1964), in Cohen, Manion and Morrison (2000, p. 267), describes, "five unavoidable features of the interview situation."

1. There are many factors which inevitably differ from one interview to another, such as mutual trust, social distance and the interviewer's control.
2. The respondent may well feel uneasy and adopt avoidance tactics if the questioning is too deep.
3. Both interviewer and respondent are bound to hold back part of what it is in their power to state.
4. Many of the meanings which are clear to one will be relatively opaque to the other, even when the intention is genuine communication.
5. It is impossible, just as in everyday life, to bring every aspect of the encounter within rational control.

Cohen, Manion & Morrison, 2001, p. 267 - 268



Despite the fact that the participants were mature students, in interview situations, the interviewer is typically perceived to be the one who is in power. That, I knew, was a tension that could not be totally removed from any interview situation. Thus, with reference to (1) above, I was aware that it might prove difficult to win over the trust of the participants.

Item (4) above was equally problematic. Most of the participants were isiXhosa mother-tongue speakers who have to negotiate the hurdles of language opacity and demonstrate their understanding of mathematical concepts in *English*.

To offset the power-relations, trust and language barriers, I asked an expert mathematician – a person who was also an isiXhosa mother-tongue colleague – to conduct the focus group interview.

#### 3.6.4.4 INTERVIEW QUESTIONS

The design of interview questions, much like the construction of the questions within questionnaires, depends on what it is that the researcher wants to know. Issues that impact on the design of an interview include:

- the nature of the subject matter;
- whether the interviewer is dealing with facts, opinions or attitudes;
- whether specificity or depth is sought;
- the respondent's level of education;
- the kind of information she can be expected to have;
- whether or not her thought needs to be structured;
- some assessment of her motivational level;
- the extent of the interviewer's own insight into the respondent's situation; and
- the kind of relationship the interviewer can expect to develop with the respondent.

Cohen, Manion & Morrison, 2001, p. 274



An advantage of focused group interviews is that the investigator has situational awareness and can, "recognize symbolic or functional silences, 'distortions', avoidances, or blockings" (Merton and Kendall, in Cohen, Manion and Morrison, 2001, p. 290), which can then be flagged to be explored more deeply, thereby facilitating naturalistic, emergent design and inductive theorising.

## 3.7 UNDERSTANDING, TRUSTWORTHINESS, REFLEXIVITY AND RELIABILITY

### 3.7.1 UNDERSTANDING

Because the issues of subjectivity, opinion, attitude and world-view conspire have the potential to create bias Cohen, Manion and Morrison (2001), to make sense of the action research findings, *understanding* has been applied to the research. While acknowledging the potential for bias, the descriptive explanations and deep understanding of on-the-ground experiences (Abma & Schwandt, 2011, p. 103) work well with the pedagogical aspirations of action research. Understanding in research is "addressed through the honesty, depth, richness and scope of the data achieved, the participants approached, the extent of triangulation and the disinterestedness or objectivity of the researcher" (Cohen, Manion & Morrison, 2001, p. 105).

### 3.7.2 TRUSTWORTHINESS

Often, in action research, researchers use qualitative and quantitatively oriented research instruments. Where this is the case, trustworthiness (Herr & Anderson, 2005; Feza, 2015) is used as a "demonstration that the researcher's interpretations of the data are credible, or "ring true," to those who provided the data" (Herr & Anderson, 2005, p. 50). Trustworthiness is perceived to provide a fitting lens for assessing action research.



### 3.7.3 REFLEXIVITY

Researchers, unwittingly or otherwise, may influence the research field in which they conduct their work; individual ontologies may impact on research decisions which govern what is chosen for observation or rejection and what it is that is deemed as important or not (Calas & Smircich, 1999, p. 664). As this can be the undoing of research programmes, cautious researchers apply reflexivity to their work.

> Reflexivity is, in basic terms, thinking critically about what you are doing and why, confronting and often challenging your own assumptions, and recognising the extent to which your thoughts, actions and decision-making processes shape what you see and how you research.
>
> Drake, 2015, p. 103

Reflexivity requires the researcher to deliberately and deeply consider both the field that is under investigation and also the personal, internal habits of mind that may impact upon the researcher's understanding of that field (Watt, 2007, p. 82). Thus, through the analysis of multiple sources of data, triangulation (Koshy, 2010, p. 98; Silverman, 2002, p. 234) is used to construct comprehensive understandings of the findings of the research; this, in turn, leads to, "richer, 'thicker' descriptions" (Geertz, in Geelan, 2003, p. 11).

However, triangulation can imply that a single finding or result is implicit in the analysis of the data. But for action research, this is not necessarily correct. More to the point, the multiple-data sets that are obtained during the research can generate different perspectives and understandings; these data-sets can be used to lead to *a truth* or crystallisation which will provide an account of the interpretations of the data by the researcher. But, quite conceivable, there will be other truths.

### 3.7.4 RELIABILITY

Reliability refers to the *consistency* of an outcome (Koshy, 2010, p. 98; Newton Suter, 2012, p. 251). Put another way, reliability implies that whether you attempt to prove or disprove a certain point, the research conclusion will yield a similar result (Hofstee, 2006, 24). However, where researchers use, "judgement calls" (Woodwell, 2014, p. 104) with data that cannot be replicated by others, reliability is compromised. Examined through the lens of quantitative research endeavour, qualitative methods are often criticized as being, "impressionistic, biased, commonplace, insignificant, ungeneralizable, idiosyncratic, subjective and short-sighted" (Cohen, Manion & Morrison, 2001, p. 120), but that tension occurs because the reliabilities of the quantitatively versus qualitatively inspired research lie on different planes.



Qualitative research is conducted in the real world and addresses issues of context, situation, authenticity, detail and meaningfulness. Thus, indeed there are aspects of being impressionistic, etcetera, but this flows from the philosophical, ontological and epistemological standpoints inherent in this type of research. Accordingly, "in qualitative research, reliability can be regarded as a fit between what researchers record as data and what actually occurs in the natural setting that is being researched" (Cohen, Manion & Morrison, 2001, p. 119).

Because action researchers use a mixture of quantitative and qualitative research instruments and triangulation, these in-built features of action research tend to balance out the impressionistic aspects of the study. Further, in order to enhance the reliability of their work, researchers can use “low-inference descriptors” (Silverman, 2002, p. 226 - 227), that is, they can use precise *verbatim accounts* to complement their impressionistic interpretations of situations. Using a process called *member checking* (Fraenkel and Wallen, 2009, p. 504; Creswell, 2014, p. 201), participants can review the truthfulness and accuracy of findings of the research.

## 3.8 DATA ANALYSIS

Newton Suter (2012, p. 173) suggests that, “data analysis and interpretation involve the discovery of trends or patterns in the data and the conclusions (if any) possible from the analysis.” In this thesis, various qualitatively and quantitatively oriented data-gathering research instruments were combined such that, hopefully, the whole was “more than the sum of its parts” (Bryman, 2012, p. 699). While quantitatively captured data can be quite easy to enumerate, qualitative data requires the use of descriptive, coherent, credible and accurate narratives Koshy (2010, p. 101).



Action research findings are stylistically less formal than other research and are often presented in a, “quasi-story form, and as such, are more personal” (Newton Suter, 2012, p. 173). “Practitioners tend to use narrative and story as a way to communicate professional knowledge, which makes it particularly appropriate for action research” (Herr & Anderson, 2005, p. 34). In addition, “since good stories are memorable, a story format can be an effective way to share findings and ideas” (Newton Suter, 2012, p. 173 - 174).

### 3.8.1 ANALYSIS TECHNIQUES

The application of both quantitative and qualitative research gathering techniques enable researchers to explore more deeply the, “complexity of human phenomena” (Sandelowski, 2000, p. 246). Descriptive narratives and numerical data can be combined and analysed to enhance the richness of the research. Qualitative data can be sorted and coded while quantitative data can be examined for trends and percentages (Christ, 2010, p. 647). For the purposes of action research, software such as Microsoft Word© and Microsoft Excel© are appropriate for capturing, analysing, numerating and codifying data (Newton Suter, 2012, p. 383).

In order to establish whether the different data-sets converge, researchers use a "side-by-side" (Creswell, 2014, p. 222) technique to compare trends in the various data-sets. Using an additional research technique called "data transformation" (Creswell, 2014, p. 223), quantitative data can also be re-examined through a qualitative lens and, similarly, qualitative data can be analysed and reduced to numerical data.

Sandelowski (2000, p. 253) proposes that q*uantitizing* occurs when qualitative data is interpreted by using quantitative tools, while *qualitizing* implies that quantitative data sets are analysed through the use of qualitative tools. Using numbers to explain verbal and written data, or alternatively, using a narrative to describe numerical data can reveal multiple layers of understanding of the research project.

#### 3.8.1.1 QUANTITATIVE DATA ANALYSIS

In this research, numerical data was captured for each action cycle from the prior knowledge assessment tasks and the re-assessment tasks. The primary data was consolidated into Microsoft Excel© spreadsheets. The formula-generating capacity of Microsoft Excel© was used to establish the mean, median, mode and range of each data-set. Summaries of these data-sets were visualised in Box and Whisker plots. The numerical data was qualitized to provide a detailed explanation of interesting aspects that were observed in the data. The questionnaires contained a number of attitudinal Likert scale questions (Fraenkel & Wallen, 2009, p. 124 - 126). That data was quantitatively analysed in Microsoft Excel© and was then qualitized.



#### 3.8.1.2 QUALITATIVE DATA ANALYSIS

Because qualitative data can generate copious and clumsy amounts of data (Bryman, 2012, p. 565), researchers need to avoid offering superficial analyses of such research data. Newton Suter (2012, p. 351 - 352) suggests that, "conclusions in qualitative research are typically derived from identified patterns and uncovered conceptual, not statistical, relationships." Inductively, these patterns lead from loosely-connected generalizations to specific positions of understanding which in turn reveal information that is hidden in the data.

A Thinkboard was used to establish the PGCE students' impressionistic views of mathematics. Three other questionnaires sought perspectives on barriers they perceived they held, and their sense of the efficacy of the visualisation strategies that were used in the classroom. Transcribed verbal responses from the focus-group interview were analysed for, "frequencies" (Fraenkel & Wallen, 2009, p. 453). Through iterative levels of coding and abstraction, as clarity of the frequencies invoked better coding, the data was thematically contextualised (Newton Suter, 2012, p. 355).

Collectively, the various quantitatively and qualitatively oriented data-sets provided a holistic view of the participants' experiences of the visually mediated learning experiences and revealed trends of acceptance or rejection, embracing of the new techniques or loathing of them, etcetera. In turn, the findings enabled me to answer the sub and main research question of the thesis.

## 3.9 ETHICAL CONSIDERATIONS

Ethics, or moral philosophy (Coetzee, le Roux & Mohangi, 2016), functioning as a singular noun, is "the philosophical study of the moral value of human conduct and of the rules and principles that ought to govern it" ("Ethics", 1999, p. 488). When treated as a plural noun, ethics is understood as, "a code of behaviour considered correct, especially that of a particular group, profession, or individual, the moral fitness of a decision, course of action, etc." ("Ethics", 1999, p. 488).



When researchers are awarded the privilege of working with other human beings, it is important that they are scrupulously honest and do their research with a commitment to preserving the dignity and privacy of participants who, by the very act of participating in the research, unwittingly may put themselves at risk. Ethics helps to foreground the, "principles and values researchers build into their work so that research integrity can be maintained" (Sotuku & Duku, 2015, p. 113). As such, ethics transcends the vagaries found in different cultural settings and their practices, and rather, embraces principles which are held to be truthful, respectful, fair and honest (Coetzee, le Roux & Mohangi, 2016). In reporting, where participants' opinions or quotes are transcribed, such material must not identify the participant (Booth, Osman & Venkat, 2016).

Because real world research can inadvertently uncover illegal activities, it is understood that in those situations, the legal requirement to report such activities to the authorities must override the confidentiality agreements that are made between the research and participants (Robson, 2002).

However, in real world research, for the most part it is the participants themselves who may potentially occupy a vulnerable position that expose themselves to exploitation. Unacceptable practices include (1) involving people without their knowledge, (2) coercion, (3) withholding information about the true nature of the work, (4) deception, (5) diminishing the social standing of participants, (6) imposing the researcher's will on the participants, (7) exposing participants to mental distress of physical harm, (8) invasion of privacy, (9) selectively withholding benefit, and (10) treating participants with disrespect (Robson, 2002).

Thus, research participants must be fully aware of the nature, size and shape of the research project in which they might become involved. If and when they are suitably convinced that they might wish to contribute to the research, participants are required to give their informed consent to allow the research to move forward. Cohen, Manion and Morrison, 2001 (p. 50 - 51), explain *informed consent* as the process by which potential participants think about and then use free-will to decide whether or not to participate in a research activity. Indeed, Mouton (2003, p. 243) advises that people have, "the right to refuse to participate in research, [and have a] right to anonymity and confidentiality."



Further, it is unacceptable to impose the researcher's own values and expectations upon the participants in a research project, the "ethical course is to try to seek an understanding of what is going on by telling it as it is" (Robson, 2002, p. 71)

In my university, ethical considerations are attended to in a professional manner. We are aware of considerations of cultural differences, power-relations, vested interests, etcetera, and how, unchecked, these factors can influence and possibly contaminate research. Numerous stepping stones need to be carefully negotiated before ethical clearance is awarded to the researcher. These steps include the requirements to complete and have approved a Protocol Synopsis of the research, a General Ethics form for Academics and Students, an Ethics Application Form for Humans, an Ethics Checklist, an Ethics Investigator's Declaration – Conflict of Interest, and an Ethics Confidentiality Consent Form.

It is only after the above requirements have been properly administered that the university provides an Ethical Clearance Certificate.

## 3.10 LIMITATIONS OF THE STUDY

*Limitation 1:* The field data for this research project was collected in Semester 1 of 2016. Quantitative and qualitative data was captured from 38 Intermediate Phase PGCE participants in the IPS 413 E – Mathematics class. Thus, the data captured and conclusions drawn from the action research were bounded by that time-span.

*Limitation 2:* I was keenly aware that the English language and mathematical vocabulary might act as barriers-to-learning in the class. My empirical experience has shown that many English-explained concepts are difficult for non-mother tongue students to grasp. Indeed, I was aware that the students' test results might be skewed downwards, not because of mathematical incompetence *per se*, but because they were disenfranchised, excluded from success, by language-limitations. Thus, I have had to concede that the generalisability of this work is somewhat restricted.



*Limitation 3:* Although the generalisability of this action research findings may be limited, Herr and Anderson (2005, p. 6) make the point that, while the results of an *individual* action research may not offer a universal truth, each narrative becomes a reflective study of both the process and the product of a unique set of contextualised experiences. Collectively, these narratives add immeasurably to practitioners' understanding of similar problems solved in different contexts.

## 3.11 SUMMARY

In any thesis, the methodological chapter is tasked with providing a roadmap of the rationale, features and procedural details of the study that was undertaken. This chapter began with an explanation of the features of action research projects and specifically made mention of their intrinsic motivation to *solve real-world problems.* I discussed strengths, limitation, potential bias and dynamic conservativism.

This thesis is positioned within the hermeneutic tradition, thus an interpretivist research paradigm was adopted. This gave rise to a naturalistic perspective, a relativist ontology and an acknowledgement that the findings would offer only one of many accounts which might be drawn from the experience. Because of the interdisciplinary, inductive and emergent nature of action research, quantitative and qualitative data collection instruments were used. Microgenesis was used to detect improvements. The research unearthed unexpected problems which, in turn, needed to be addressed. The various research tools were designed to converge, thereby facilitating methodological triangulation.



In each of two action cycles, prior knowledge assessment tasks were provided to the participants are the beginning of each cycle, and thereby provided baseline data, and then, these tasks served as an assessment tool for reassessment at the end of each action cycle. This enabled me to construct a quantitative comparison of mathematical competence between each pair of assessments. Bloom's Revised Taxonomy was applied to the assessment questions.

Qualitative data was used to capture the participants' views about mathematics, the IPS 413 E Mathematics module and the visualising methodology that was used within the module. Qualitative data was captured using a Thinkboard activity, three questionnaires, field notes and a focused group interview. Issues of understanding, trustworthiness of the findings, reflexivity, samples and sample sizes, reliability, data analysis techniques and ethical considerations are also supplied in this chapter.

Next, in Chapter 4, the data findings are discussed. The chapter will first give an account of important fieldwork experiences; it will then discuss the Thinkboard activity, the assessment tasks for both Action Cycles; the findings from Questionnaires 1 and

2; the focus group interview; and the qualitative data that was obtained from a questionnaire that was provided to participants after the completion of the action research.

# CHAPTER 4 – DATA PRESENTATION AND PRELIMINARY ANALYSIS OF FINDINGS

## 4.0 INTRODUCTION

This chapter was originally written in the form of a journal as an account of the day to day experiences and observations which occurred during the research period with the participants. However, the chapter became very cumbersome and long. Thus, after extensive consultation with my supervisor, the chapter was reconstructed into its current size and shape. And, rather than try to account for every interaction, it was agreed that instead, I would use exemplars of factors that worked through the action research period would be introduced into this chapter. Because of my stance as a practitioner who is working within his own setting, it is acknowledged that I am working as "an insider committed to the success of the actions under study" (Herr & Anderson, 2005, p. 33). The data collection chapter now has five distinct sub-sections which discuss the Fieldwork Experiences, a Thinkboard Activity, Action Cycle 1, Action Cycle 2, and the participants' reflective research perspectives.



In writing-up this chapter, I have been guided by Herr and Anderson (2005) who suggest that action research is written in the first person and can contain, "elements of humour and irony and a narrative hook that leads the reader into wanting to read more" (Herr & Anderson, 2005 p. 34). This writing technique is ideally suited for the purposes of conveying the muddiness and the on-the-ground realities that action researchers might try to describe to their readers. Because action researchers deal with real and complex problems, their work can result in unanticipated outcomes. In these fluid situations, creativity and flexibility are highly desirable attributes and the informal voice that is used serves to make complicated situations more easily understood by the reader. Further, action researchers, "often punctuate their reports in a provocative way, maybe by asking challenging questions or upsetting our traditional thinking about something" (Newton Suter, 2012, p. 373).

## 4.1 FIELDWORK EXPERIENCES

In this part of the chapter, I will recount some of the important lows and *aha* moments which occurred in the field. I will provide an account of the unfolding landscape as I understood it and I will attempt to provide an honest, sober and accurate record of my research experiences.

### 4.1.1 DISRUPTION – THE FRAGMENTATION OF A UNIVERSITY MODULE

In my discussion on Freire (2.1.2), I have written about the negative implications of ontological oppression and societal neglect. Here, I present a localised manifestation of such oppression – my IPS 413 E classroom – and of the disrupted teaching and learning experiences that the participants had to endure in the first semester of 2016.

In the past few years, intermittent, unanticipated disruptions have thwarted teaching and learning processes at my university. Recently, these disruptions have increased in frequency to the point where they threaten the viability of our programmes. While I have always been aware of these interruptions to learning, chronicling my fieldwork experiences has focused my attention on the fragmentation of our students' education into disjointed periods of calm, alternating with brooding tension, boycotts and violence. I did not set out to give a report on *absenteeism* in this thesis, however, its prevalence – and there are many reasons for its manifestation – is an important *unexpected* finding and needs to be introduced into the thesis. Below, I give an account of attendance and absenteeism which played out in the IPS 413 E Mathematics module and which, thereby, had a direct bearing on my action research.



On Wednesday, 10 February 2016, I met with the IPS 413 E – Mathematics cohort for the first time. I arrived at my classroom 40 minutes early, but the PGCE cohort were late for my lesson - they had been kept back in their earlier lecture. Further, even though some of these students were up to 15 to 20 minutes late for the lesson, they seemed to drag their feet into my classroom. Indeed, albeit that in the course of time I would come to learn that the cohort contained a very affable group of mature learners, this first lesson also heralded a pattern of late arrivals and foot dragging. Forty-four students attended the first lesson, but, correctly, I anticipated that the class-size would grow in the following weeks. This too – late registrations for university programmes – has become a trend which conspires against inspiring efficient and effective introductions to academic modules.

Higher Education in South Africa is in a state of flux: starting in 2015, under banners such as *@nofeeincrease* and *@feesmustfall*, large swathes of university students have, on occasion, vacated their classrooms in favour of seeking social justice. And, indeed, on Monday 15th February, in only the second teaching week in 2016 and after I had only had one lesson with the PGCE cohort, the mass-mobilisation of students manifested yet again. Because of the volatility of the on-the-ground situation, classes were suspended. At that time, the principle reasons for the mass-action were unclear and no one had any idea about when and how the strike action would be resolved. For my part, having achieved a small measure of goodwill in my first lesson, I was troubled that that goodwill might be lost.

Four days later, on the 19th February, university management met with various student factions. Collectively, they resolved to call for a respite from the stay-away and our students agreed to resume attending classes. With that in mind, I began preparations to meet with the PGCE class, *for only the second time*, on Monday 22nd February and indeed, that lesson did take place.



However, on the 22nd February, yet another problem manifested: it was announced that changing, on the ground circumstances now required all of the B Ed and PGCE students to undertake a two-week school-experience practicum in the first semester of 2016. This requirement caught staff and students off guard.

The first semester of study contains about 12 to 15 contact weeks which typically creates about 24 to 30 lessons for IPS 413 E Mathematics module. Thus, because of the stay-away, I entered the third week of the semester with two lessons already lost to strike action, and the spectre of losing four more lessons during the newly scheduled semester 1 practicums. This impacted on my pre-planning for the action-cycles and data-collection phases of my thesis. But more trouble was to come…

Two days later, on the 24th February, my lesson with the PGCE cohort got off to a rocky start. In that week we were living through an appalling heat wave and were experiencing unbearably high humidity percentages, too. On that day, when I entered my classroom, I was hit by a blast-furnace-hot wall of thick, super-saturated air. The air-conditioning had been set to its maximum 31°C setting and left running. I turned down the temperature to 16°C but that did not offer any reprieve: while the heating coil in the air-conditioner worked properly, the condenser had no refrigerant gas, so all that I really managed to do was circulate the hot air. To add to my miseries, the PGCE students, who now numbered 60 in total, arrived very late for my class. As had happened previously, they explained that the lecturer in their previous lesson typically always arrived late for lessons and then worked through into the next (my) lesson. I realised that I would have to address this matter.

On Friday, 26th February, our students boycotted their university classes and after this mass action, the following week, on Monday 29th February, only 38 of my students attended my class; on Wednesday 2nd March, there were 44 students, and on Monday 7 March, only 37 out of 60 registered students attended the IPS 413 E Mathematics lesson.

Human Rights Day, a public holiday in South Africa, was celebrated on 21 March 2016. This celebration did mean, however, that I lost a lesson with my PGCE cohort. Similarly, Easter Monday, also called Family Day in South Africa, is a public holiday and was celebrated on 28 March 2016 so I lost a lesson with the PGCE cohort. Further, immediately after the Easter weekend and a short university break, the PGCE students were away for two weeks, doing the aforementioned unplanned practicums in Eastern Cape local schools; the practicums cost me 4 lessons.



After the practicums had been completed, only 33 of the PGCE students returned to the university on the 18th April. However, on that day, while I was negotiating the way forward with these students, members of the Students' Representative Council (SRC), entered my classroom and, for reasons unknown to me, closed down all teaching and learning for the day.

On the 19th April, Mr Muisi Maimane of the Democratic Alliance political party paid a visit to our university and that visit brought the university to a stand-still. And, as if that was not enough, the greater part of East London was without water and we learned that for the following three or four days, while emergency repairs were effected to our major water-reticulation pipeline, no running water would be available on our campus.

Because of this water crisis, at 12h00 on 19$^{th}$ April, the vice chancellor closed the university.

In South Africa, 27$^{th}$ April is celebrated as national Freedom Day. Thus, because this public holiday occurred on a Wednesday, I lost a lesson with the PGCE cohort. Further, in our country, as in many other countries, 1$^{st}$ May of each year is celebrated as Workers' Day. Because the 1$^{st}$ May fell on a Sunday, Monday 2$^{nd}$ May was celebrated as a public holiday and so another lesson with my PGCE cohort was lost.

On Wednesday 11$^{th}$ May, fearful as I was that my students were not properly prepared for their June examination, I negotiated an extra lesson for the PGCE cohort. On Friday 13$^{th}$ May, most, but not all of the class attended the extra lesson. However, many of them seemed to be in a bad mood. During the lesson, I did not interrogate this *heaviness* with them, but certainly, it was not a pleasant lesson. And so, rather than being a, "yes, an extra lesson…" it was more like pulling teeth.

At the end of the lesson, I asked a few students to explain the brooding atmosphere. They suggested that, (1) because our university administrators required them to sign off on financial matters, many students queued for long and frustrating hours at the Finances Offices:  some of the students encamped in front of the offices as early as 04h00 and, I was told, were only attended to well after midday. Also, (2), on the 13$^{th}$ May, some academics had cancelled their lessons with the cohort, thus many of the students felt it was *unfair* of me to call them in for this extra lesson.



I squeezed in a second make-up lesson on the17$^{th}$ May. Many students attended the lesson and it was conducted in a far better atmosphere than the first extra lesson. Twelve to fifteen students stayed on *after* the make-up lesson to ask me to help them with various mathematical hiccups. This smaller group worked together for an additional hour and a half and appeared to make some profound strides.

On the 18th May, only about one third of the 180 students my B Ed Year 2 class attended their lesson with me. The rest were absent and were either at home or were milling in the street immediately outside my classroom. They were protesting in support of their peers who, having been arrested for the destruction of university property in a previous violent mass-action incident, were appearing in court on that day. In my university, unless we are told to do otherwise by senior management, university staff are instructed to carry on with their lessons, thus I began my work with the students who had indeed come to the lesson.


*Figure 4.1.* A typical boycott poster

However, early in the lesson, a member of the Students Representative Council (SRC) made her way into our classroom and requested all of the students to leave the class; the SRC, see *Figure 4.1*, above, had called for a general stay-away. I pointed out to the SRC student that as *democratically* elected representatives of the students, the SRC would appreciate the sanctity of the South African Constitution and rights such as freedom of movement and freedom of association. I pointed out to her that freely, students were engaging in protest action, *outside the classroom*, and freely, other students, those in my class, had made a choice to study. The logic of my comments was lost on her and did not deflect her from her purpose: she insisted that the students *must leave* and *must join the strike-action*. She stated that should we remain in class, *we would be violently removed from the classroom.* When I indicated to her that she should choose her words more cautiously – that she was in fact threatening us – that too went right over her head. Prudently, I dismissed the class. Thus, later in the day, another lesson with my PGCE students would be lost.



Two days later, on the 20th May, the university held a centenary celebration. Presidents Jacob Zuma and Robert Mugabe were in attendance (*Figure 4.2*, below). On that day, in spite of ongoing protest-action, I managed to work in a third extra lesson for the PGCE cohort. Almost every single student arrived for the lesson.


*Figure 4.2.* Presidents Zuma and Mugabe attend centenary celebrations

The extra lesson was three hours long. At its conclusion, I administered a make-up task for students who needed an extra opportunity to "boost" (the students use this term) their marks up to our minimum, duly-performed, semester mark of 40%. This meant that some students were in my class for about four and a half hours, but all in all, the lesson and boost-task were attended to in high spirits.



Three days later, on the 23rd May, our faculty received an instruction from the Vice Chancellor's office which announced an amendment for the *Duly Performed* requirement for the June 2016 exam period: any student holding a Duly Performed mark lower than 40% was to be automatically awarded 40%. This consideration, taken to alleviate continuing simmering tensions, was a part of a larger negotiated agreement between the management of the university and the SRC.

The following week, from 30th May through 3rd June, the students went into *Study Week.* They used this time to prepare themselves for their examinations which commenced on 6th June 2016 and on the 9th June, at 13h00, the PGCE students sat for their IPS 413E Mathematics exam.

Thus, in summary, 29 lessons were potentially available to the module, but because of the stay-ways, very late arrival for classes, the co-incidence of public holidays falling on my teaching days and the unscheduled school experience practicums, 16 teaching days were either totally or partly lost. The module retained only 13 unimpeded lessons and 3 extra lessons were offered to the students. The various upheavals made it difficult to sustain any traction. After each interruption, it became necessary to invest time and energy into renewing relationships, refreshing and consolidating previous learning before any headway might be made. From the beginning of the semester through to the examination, progression was repeatedly marred by disruption. Such disturbances have become an endemic feature of teaching and learning, so much so that the abnormal is now normal. And, the lens of my research has foregrounded the magnitude of this aberration.

### 4.1.2 DESPAIR – A QUEST FOR AN ETHICAL CLEARANCE CERTIFICATE

Correctly so, all research that is undertaken in my university must receive an ethical clearance certificate and research number. It gives one a licence to conduct research. Obtaining an ethical clearance certificate is a technical and tedious task, but it is an important undertaking. The ethical clearance certificate carries with it a clearance number which, for research purposes, has to be attached to consent forms, questionnaires and any other research documentation.



On 20th November 2015, I submitted the requisite forms to the faculty for scrutineering so as to demonstrate that, ethically, my proposed PhD study would be in good standing. All that day the documentation was inspected by senior members of staff, returned to me to effect minor adjustments and was re-submitted for final approval, a clearance certificate and a research number. Typically, it then takes about two to three weeks to obtain approval and the all-important clearance number. I anticipated receipt of the approval between 1st and 15th December 2016. However, by the end of January 2016, I had received neither feedback nor the clearances. This saga continued to drag itself out for the month of February 2016.

In February, too, I conducted the first pair of prior-knowledge tasks for Action Cycle 1. These diagnostic tasks were used to ascertain the participants' base-line capabilities (Department of Basic Education, 2011). My supervisor and I agreed that, although the clearances had not been issued, such testing is stock-in-trade for reflexive teaching, thus we felt this action was sound. I subsequently marked, collated and analysed the two data-sets and then used the information to create concept-building learning opportunities for the cohort, which again, is stock-in-trade for my work.

On the 8th March, I learned that my ethical clearance documents had never left our premises. In error, they had sat unattended to on a university desk for three months. Thus, although they were long overdue on my planning agenda, I decided that I could not administer the questionnaires, nor conduct any small-group interviews. These tasks were scheduled as activities that would be undertaken by my research *participants.* I realised that there would be little chance of recording any evidence by way of interviews or questionnaires for the first action-cycle of the action research.

After the administration error was uncovered, the due processes were hastily actioned and I finally received my ethical clearance certificate (Appendix B). The number is: DRA011SSHA01 / 6408640984 and it was issued on 22 March 2016. It had taken four months to complete this task, but at least I now had it to hand. Armed with these numbers, I was able to insert the information onto my questionnaires and was free to move forward with the research.



On the 18th April, after completing their school practicums, my students returned to the university. My research supervisor and I used that opportunity to formally ask the PGCE cohort to consider becoming part of my research programme. In possibly too much detail, we explained everything to them and discussed how very important it was that they not feel coerced into participation: each student was also provided with a full, written account of the purposes of the thesis and asked to carefully read the documents so as to fully understand the ethical dimensions of the research.

None the less, although only 33 students attended the lesson, when asked whether they would like to become formal participants in the research, every single person signed-up on the consent forms. And then, on the 25th April, an additional 5 students approached me and volunteered to become participants in the research, thus bringing the total to 38 participants.

### 4.1.3 DISTRESS – A THREAT TO THE VIABILITY OF THE PGCE QUALIFICATION

On 26th February, a few of our ex-PGCE students – students who had graduated and were working – returned to the university, deeply distressed and seeking advice. The King Williams' Town, Department of Education, District Office had employed them as teachers; they had been awarded full-time teaching posts and were at work, teaching. However, while sifting through the reams of paper-work that formalized their employment, these ex-students had had to deal with department officials in East London. And, in those offices, an official had flatly rejected their qualifications and told them very bluntly to get out of their state-run schools. The official suggested that their PGCE qualifications did not serve as an entry point for teaching in the GET Band and so they would all lose their jobs.



From these students, I also learned that qualified PGCE graduates from other universities in the Eastern Cape were experiencing the same frustrations. After some initial parries and ripostes, as a collective, the universities elected to engage with the Department on this matter. Our academic and administrative position was that the DoE officials had misinterpreted their own guiding framework and instructions. However, no resolution was found. Further, the tension between the DoE and the universities spilled over into my own interactions with the 2016 PGCE cohort who became very fearful that their time spent in the programme might come to nought.

By the 8th March, the deadlock had taken on an ominous lustre. A growing number of ex-PGCE students had been affected by the Department of Education (Eastern Cape) view that their PGCE qualification did not provide an entry point to teach in Eastern Cape public schools. And, despite ongoing meetings between the universities and the DoE, a stalemate prevailed. In my university, because an unsatisfactory outcome would prove disastrous for them, the ex-PGCE students petitioned the Vice Chancellor of the university to assist them. Many had already been working without payment for months and had been told by the DoE that they had to vacate their teaching posts.

Further, and understandably so, the ex-students felt that if they did lose their posts, the chances of their being paid for the work they had already done, was very low.

It was clear that the ramifications of a negative outcome for the *existing* cohort of PGCE students was that it would nullify their ability to obtain gainful employment with the DoE. Also, because of the ongoing volatility in Higher Education, our faculty was aware that any perception of poor treatment of students, past or present, had the potential to ripple through the campus and might-well set off even more stay-away action. Fortunately, time would show that this did not happen but, sadly, during the course of writing this thesis, the impasse remained unresolved and I think might have contributed to the high levels of absenteeism in my IPS 413 E Mathematics classes.

### 4.1.4 DISCUSSION – SCAFFOLDING MIS-UNDERSTOOD FOUNDATIONAL ARITHMETIC CONCEPTS

By way of example, below I discuss how I used the primary data that was captured from the Foundational Arithmetic Concepts, Action Cycle 1, prior knowledge task to try to provide conceptual support for participants who had struggled to correctly answer the questions in the task. In much the same way, I used the primary data captured from the other prior knowledge tasks to identify problems and to then conceive strategies that might provide support to the participants.



The baseline setting Foundational Arithmetic Concepts, Action Cycle 1, task was administered on the 10th February 2016. I marked the assignments and then, in MS Excel©, I collated the primary data and constructed summaries and graphs of the trends that I found. Using intuition and insights that I had gained from the primary data, I created feedback and scaffolding lessons for the PGCE cohort, as outlined below"

*1.* The quantitative primary data enabled me to ascertain and rank performance for each question. This, in turn, enabled me to introduce my first design feature into my teaching and learning strategy:
   *In Microsoft Word©, I reordered the questions in the Foundational Arithmetic Concepts assignment so that, in descending order, the question that had achieved the highest success rate appeared first, followed consecutively*

*through to the worst performing question. On the right hand side of each question I inserted, as a percentage, the class success rate for that question.*

2. Next, I introduced a second design feature:

   *Based on the skills, routines and procedural mis-conceptions that precluded the students from success, I constructed new sets of questions that required the students to solve numerous problems that were similar to the original questions. Bloom's Revised Taxonomy* (Krathwohl & Anderson, 2010) *was incorporated into the design of these questions.*

Thus, my over-arching plan was that each type of mathematical content question and its attendant mathematical skills, routines and concepts would be visited at least four times prior to the June 2016 examination. Those times were occasioned by (1) the prior-knowledge task; (2) scaffolding of problematic questions via the re-ordered issue and investigation of the task, (3) practicing mathematical ideas in numerous activities; and, (4) the re-evaluation task.



3. Having gained permission from the PGCE cohort, I applied a third design feature:

   *At a meta-cognitive and ethical level, I had promised myself that I would share all of my data with the participants and that this sharing would happen spontaneously and sporadically, as and when new data was revealed to me*.

Thus, in class, I used a data-projector to display the trends that I found in the data. We scrutinized the data sets, often finding trends that demonstrated that many participants were struggling with similar mathematical procedural and conceptual problems. These expositions introduced a sense of camaraderie and optimism among the participants. They found the shared information captivating and, for many, it dispelled any sense of isolation, of being the only one who *didn't get it*.

Mathematical content knowledge and the construction of a connections model – linking pictures, symbols, language and real-world scenarios (Haylock & Mann, 2014) – were major foci for the semester. Further, into many lessons, I also introduced aspects of, for example, Ausubel's concept of prior learning (Ausubel, 1968), or important issues pertaining to CAPS (Department of Basic Education, 2011), or social constructivism, etcetera. I did this to integrate mathematical content knowledge (Muir, 2008; Tipps, Johnson & Kennedy, 201), subject content knowledge (Sowder, Philipp, Armstrong & Schappelle, 1998), and pedagogical content knowledge (Shulman, 1986) and used these cross-linking opportunities to establish pivots for our shared exploration of mathematics.

By way of example, on 24th February I cross-linked the conception of compression (Tall, 2014), Blooms' Revised Taxonomy, the BODMAS acronym and mathematical operations and routines. I used this linking technique to demonstrate how evidence captured from a test in which a student might have correctly *remembered* a low-order fact such as the BODMAS acronym, but subsequently might not have solved a low-level *understanding* task such as 19 + 16 x 17 – 5 = ?, would signal that the compression of the relevant unpinning facts, routines and ideas might not have occurred and that a robust schema (Piaget, 1986), might not have been constructed.

New or problematic mathematical content was scaffolded and many tactile and visual stimuli were brought to bear upon the teaching and learning processes (Bruner, 1966). Such stimuli included the use of found resources such as bottle-tops or pebbles, biscuits and Smarities© which are useful in sharing situations, arrays, formulating fractions; ratios, building number-lines, directed numbers and number patterns. In the South African context, I believe that my use of found resources is appropriate; models and posters of fractions or number-lines and Cuisenaire© rods, etcetera, largely, are not available in South African schools. In class I introduced the practice of using ribbons (made either from card or drawn in fair proportion on paper), which facilitate the construction of visible interpretation of ideas as diverse as fractions and word-problems. I also made use of a document camera and data-projector to create *directed teaching/thinking lessons* (Tipps, Johnson & Kennedy, 2011, p. 75), which were developed to assist the participants to improve their mathematical knowledge. And, in support of all of the above, I constructed numerous MS PowerPoint© slideshows.

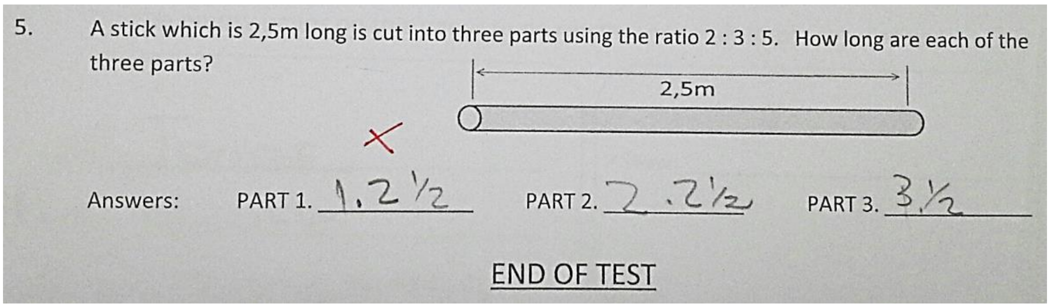
### 4.1.5 DISCUSSION – REFLECTIONS ON A TYPICAL LESSON – 7 MARCH 2016

Within the constraints of this thesis, it is not possible to reflect upon every lesson which occurred within the semester. However, I would like to offer, as one example, my reflections on a lesson I had with the participants on 7th March 2016.

Although only 37 out of 60 registered students attended that IPS 413 E Mathematics lesson, I used the lesson to begin to unpack the various mathematical skills-sets, vocabulary and concepts which had proved to be outside of their understanding in the Action Cycle 1, Foundational Arithmetic Concepts, prior-knowledge task. On an ad-hoc and informal basis, I incorporated numerous tactile and iconic scaffolds into our discussions

Q 4.5 is presented in *Figure 4.3,* below. It is taken from the Action Cycle 1, Foundational Arithmetic Concepts prior knowledge assessment tasks. Because only 1/44 of the students who wrote the test managed to correctly answer this ratios-based question, it was reviewed in class on 7th March. This question and its answer enable me to discuss the methodology that was applied in my lesson.




5. A stick which is 2,5m long is cut into three parts using the ratio 2 : 3 : 5. How long are each of the three parts?

2,5m

Answers: PART 1. 1.2½ PART 2. 2.2½ PART 3. 3½

END OF TEST

*Figure 4.3* Question 4.5 as found in Action Cycle 1, Foundational Arithmetic Concepts, Prior Knowledge task.

In my reflexive planning, I had realised that Q 4.5 had proved to be massively problematic for the entire cohort. Thus, in order to provide a robust conceptual scaffold for this poorly understood problem, I created a four-page ratios activity for the students (Appendix C). This document was provided to the participants in the form of a photocopied handout and was used in conjunction with colourful found resources. My plan was for the students to work through the document by themselves, in pairs or small groups. I hoped that, *without my assistance*, they would be able to use a combination of the information that was provided to them in the worksheets, their own intuition and their past learning experiences to reconstruct and liberate lost memories of important features of ratios. However, in this conception of discovery and social construction of knowledge, I was only mildly successful. Many students found it difficult to more forward by themselves or with the assistance of their peers and so, eventually, I reverted to a transmission teaching and learning mode. Indeed, the passage of time would show that this was a trap I fell into easily.

I gathered the class around me – we formed a large, loose semi-circle – and then, using biscuits, I demonstrated the sharing of biscuits between a father and two sons, such that, on the basis of *one for me, one for you*, the father enjoys twice the number of biscuits as that boys. This amusing modelling was used to then discuss different types of sharing possibilities and different meanings attached to fractions notations, and was linked our use of vocabulary and language, and the manipulatives and images that were being used in the lesson.

The larger exercise was designed to help the participants to visualise ratios in terms of concepts such as the size of the whole, the sum of the parts of a whole and why it is that we might divide the whole by the sum of its parts (so as to establish a unit size), etcetera. The explanation, combined with the vocabulary, real-world scenarios, visual and activities-driven parts of the work, and worksheets assisted the students to make better sense of ratios, but the rationale of a *unit size* proved evasive for some of the students and, intuitively, I believed that most of the students did not construct a meta-concept of ratios. That is, compression did not occur. Thus, I realised that, in subsequent lessons, that meta-concept would need to be reviewed and carefully attended to, failing which, all the fractions and ratios concepts would flounder.

In *Figure 4.4*, below, taken from the worksheets, a ratio of 3 : 7 is presented. On the top line a rectangular representation of a *whole* has been sub-divided into 10 equal unit parts. The sum of the parts in this ratio is 10 (3 + 7 = 10). The participants were required to use the symbolic (3 : 7) and iconic (rectangular bar) cues to colour the unit cells in fair proportion so as to properly represent 3 : 7. There are many possible, correct answers. A correct solution, and I think, the most obvious one, is presented in the lower sketch. All the students easily negotiated the requirement.

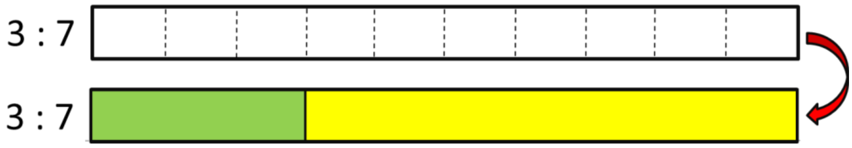



*Figure 4.4.* Direct correspondence between ratio and shading in of parts

However, when the questions on the worksheets progressed to the order found in the Figure 4.5, below, some of the students stumbled:

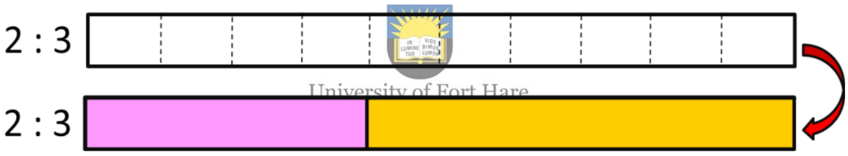





*Figure 4.5.* Indirect correspondence between ratio and shading in of parts

I think that because rectangular bars had been sub-divided into 10 units, inadvertently, I complicated the situation. The question raised two sticking points:

1. there were 10 divisions in the whole but only 5 parts in the ratio; and
2. there were too many 2s in the solution: the sum of the parts 2 + 3 was 5; 5 divided into 10 is 2, that is, the unit size and coincidentally 2 was also a number in the ratio.

In my planning, I did not anticipate this problem. So, assisting the students to conceptualise an answer to this question became a bit of a mess. I reverted to a chalkboard sketch of the bar, first sub-divided into 10ths and then repeated in 5ths. This comparison model – in effect a fractions wall and equivalent fractions – seemed to reduce the misunderstanding, but again reinforced my understanding that this type of work would require much more support.

Fortunately, towards the end of the exercise, when confronted with questions of the order below, *Figure 4.6*, (which did not contain any printed sub-divisions of the whole), most of the participants finally began to put the various sub-tasks of this procedural work together.

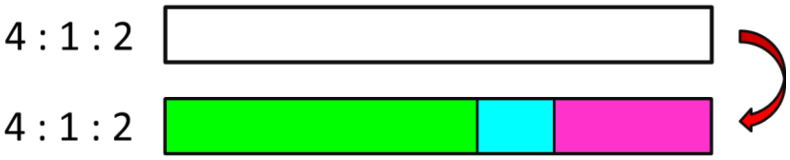



*Figure 4.6.* Representation of ratio in a rectangular bar by means of calculation and measurement

By rote mostly, the students easily added the parts in the ratio, measured the full length of the rectangular bar, divided that length by the sum of parts, and used that quotient to construct, in fair proportion, the visual representation of the ratio.



The three examples presented above were used to consolidate many interrelated skills and content knowledge that are applied to ratios. Following the use of these preliminary visualising activities, the work was formalised and ratios were inserted into contextualising word-problems, as in *Figure 4.7*, presented below.

**2. Thando, Sipokazi and Jane eat 240 sweets in the ratio of 3:4:5. How many sweets does each child eat?**

**3. Lulu owns pens, rulers and erasers in the proportion 4:3:5. She owns 16 pens. How many rulers and erasers does she have?**

*Figure 4.7.* Routine application of ratios in Intermediate Phase word-problems

While the students easily worked through question 2, above, question 3 required additional scaffolding. Question 3 did not fit into the *routine* set up by the questions which preceded it. In question 2, for example, the total of 240 sweets is shared. In question 3, the total is not supplied; instead, what we get is a relationship between pens as parts (4 parts) and a number of pens (16 pens).

In order to bring some sense of the asking requirements of the question and its solution to bear on our discussions, I returned the group to our informal semi-circle, and using Smarties©, I placed 16 red smarties into a 4 x 4 discrete representation of an array, and used that array as a link to the ratio (4 : 3 : 5), in equal groups to the 4 in the ratio, and from there, to the 3 parts (each a group of 4), and 5 parts, respectively.

While the difficulty in Q3 was unanticipated, the discovery of this conceptual glitch was important. With the ritual of the first routine, as set up in Q2 disrupted, many participants had become disoriented. This demonstrated, yet again, that even when the students appeared to be *getting it right,* their understanding of the concepts and processes was often tenuous and that compression was elusive.



Towards the end of the lesson, I presented a blended-learning, asynchronous activity in the form of a photocopied handout to the students, (Appendix D). In it, the students were directed to use an internet website – one which is saturated in visual explanations, examples and video clips – to learn for themselves many foundational arithmetic ideas and vocabulary. My thinking was that by putting the students in charge of their own learning, I was in fact offering them an opportunity to liberate their own dormant potentials.

### 4.1.6 DISCUSSION – MULTIPLICATIVE REASONING

On the 10th February 2016, I met the IPS 413 E Mathematics cohort for the first time. I used that lesson to introduce myself and the module to the participants and I also used the first period to give the participants an arithmetic competence diagnostic assessment task. By carefully marking and reflecting on the answers that were provided to me in their answer books, I was alerted to problematic arithmetic concepts and practices that were found in the scripts, and was thus able to consider appropriate starting points with the participants. The data revealed many problems, of which multiplication and multiplicative reasoning was but one.

Twelve of the twenty questions in the task used, in different ways, multiplicative facts, skills and reasoning. These questions included mental multiplication, ascertaining perfect squares and cubes, multiplication in mixed operations calculations, *long* multiplication, multiplication by zero, factors and, multiplication and division as part of a ratio calculation. The numbers used in the questions and their answers, (with the exception of the ratio question), were restricted to whole numbers. All of the solutions were obtained without the aid of a digital calculator.



With the exception of Q1.2 which required a True/False response to the statement: **In the pattern: 1; 4; 9; 16; 25; ____; ____; the next two numbers are 34 and 43,** and which garnered 25/34 correct responses, the other eleven multiplication/division-based questions enjoyed very low returns. The unifying fact that emerged from the data was that many students simply did not know their times-tables, and thus, any conception build upon that missing pivotal link, was bound to fail.

I believe, albeit they are learned number facts, and mostly rote-learned at that, pupils and research participants alike, must know the times-tables. Haylock (2002), Siemon, et al. (2012), and Rickard (2013), agree. Indeed, Rickard (2013, p. 91) suggests that, “armed with predominately instant recall of multiplication facts, learners can then employ these quickly and easily,” in a range of multiplicative ways. These ways include multiplying digits in mental and long multiplication, calculating related facts for, for example multiples of 10, application to decimal multiplication, doubling and halving, etcetera.

However, in concert with Haylock (2002), Siemon, et al. (2012), and Rickard (2013), I also agree that the times-tables should be contextualised and understood. Haylock (2002), suggests that while an early understanding of multiplication is built upon repeated adding, a second facet – scaling up and down – as found in ratios, is an important multiplication concept with which learners must grapple. In this regard, he suggests (Haylock, 2013), arrays, as found in the blocks of chocolate in chocolate bars or constructed with arrangements of bottle tops, are ideal for demonstrating the commutative and distributive laws, scaling, repeated addition, etcetera.

Building on these starter activities, it is then possible to use learners' early understanding of multiplication to introduce visually rich, area models (locally, South Africans also call this the Grid Method), an understanding of which, in the course of time, can be brought to bear on algebraic computations (Siemon et al., 2012).

Alongside this work, appropriate language needs to be introduced into teaching and learning. Vocabulary should be introduced incrementally and should start with informal terminology which builds up to a more precise mathematical language. Thus, one might start multiplicative investigations with everyday language and real-life contexts such as *two boxes of biscuits and nine biscuits in each*, which then becomes *two nines*. The formal language acquisition requirements then progress to the use of *times*, as in 3 x 4 = 12 and hence, to simple number sentences such as, *six multiplied by three* and so forth (Rickard, 2002).



Aligned with this introduction to language and symbols, Van Der Walle, Karp and Bay-Williams (2010), suggest that an understanding of multiplicative symbolism can be negotiated via the use of simple story problems and the use of smallish numbers so as to allow learners to focus on appropriate use of symbols, context and concepts rather than the burden of onerous computations. Remainders too, should be introduced early. Further, it is suggested that the use of models – typically, arrays, ribbons and number lines – facilitates cognitive uptake of multiplicative thinking and brings some clarity to the counter-intuitive view that, for example 3 x 6 = 6 x 3 = 2 x 9, etcetera. Van Der Walle, Karp and Bay-Williams (2010), also remind us that zero and one, as factors, present many learners with cognitive challenges.

Armed with this knowledge, and with Plato's allegory of the cave (2.1.1), Freire's liberating conception of education (2.1.2) and Bruner's social constructivist learning theory (2.7) offering me a philosophical and epistemological bed-rock, I set about trying to address the many interrelated multiplicative thinking problems that manifested in the participants' workings. I set about trying to create learning opportunities that would enable the participants to develop multiplicative reasoning and would encourage them to become more self-reliant. I created a review exercise, discussed in (4.1.7), in which all of the questions in the prior knowledge task were rewritten with similar, but not identical questions. The questions in the review document were reordered; the best answered questions in the prior knowledge task were listed first, and then, in descending order of achievement, I listed all the other questions.

In our classroom, while reviewing their arithmetic assignments, the first hurdle that I had to overcome was a view that the participants believed that, had they had access to their calculators, they would have fared better in the arithmetic task. While we agreed that in all probability this most most-likely true, our discussion gave me an opportunity to discuss teaching and learning practices and requirements in South African primary schools (for example, see Table 4.7) and the attendant, requisite skills that the participants, as teachers, must hold in order to fulfil those roles.



The participants readily acknowledged the importance that primary school mathematics teachers attach to multiplicative thinking and the times-tables – indeed, in their thinkboards (4.2) which they had completed before doing the arithmetic task, the participants made it clear that they were keen to become really good teachers. In the course of the lesson, this first fruitful discussion worked itself into three short, interlinked discussions.

On the data projector, I first presented a traditional list of the times-tables and used that to introduce the concept of transmission mode, rote learning, vis-à-vis, Freire's banking model (2.1.2.3). The slide elicited memories of chanting the time-tables.

Second, I introduced a slide which contained an array-display of the times-tables and used that to unpack multiplicative number facts in a visual way. I used this slide to demonstrate that, for example 4 x 5 = 5 x 4. I also show the participants how the array revealed bounded areas, for example 4 x 5 = 20, and I highlighted how the perfect squares were presented, to the seeing mind, on a diagonal. It can be suggested that, via this mechanism, I was attempting to move beyond rote learning into a mode which liberated understanding, vis-à-vis, I was applying Plato's metaphor for a good education (2.1.1.3), Freire's liberating model of education (2.1.2.3) and Bruner's iconic and symbolic modes of representation (2.7.4.2 and 2.7.4.3).

During this lesson, I switched between slideshow and chalkboard, expository teaching and whole class discussion. Indeed, this was to become a feature of all lessons, but often, more-times than I had hoped for, the direct instruction component of our lessons became a/the pivot for progression. Killen (2015, p. 137), suggests that direct instruction "remains one of the most effective ways of promoting student learning and it has a long history of strong research support," but it should also give rise to guided practice and independent practice. This too, guided and independent practice, became a feature of our lessons.

The third component of our discussions focused on a vocabulary list of multiplicative (and other) facts that I issued to the participants. In class, I used my computer to link us to a web-site: https://www.mathsisfun.com/ . I demonstrated how the homepage in the web-site was put together and I paid particular attention to the dictionary feature in the homepage. I encouraged the participants to put aside a few hours to use the vocabulary list with the web-site dictionary to study, by themselves, some of the important ideas which they needed to understand. It is important to note that the dictionary feature offers much more than a short description of mathematical vocabulary; it offers full explanations in a user-friendly format that blends text with pictures, examples with short exercises, and provides links to adjacent ideas.

In that lesson, I believed that the above explanations and demonstrations would provide a suitable refresher for the participants and that, really, all that was needed was a brushing away of cobwebs to remind the participants of competences that they most-likely already held. However, the following weeks, two things quickly became clear to me in class. First, albeit that they were a lovely group of people to work with, it rapidly became obvious that the participants were continuously distracted from their university work and thus did not seem to apply themselves to their studies. This is discussed in (4.1.1) and (4.1.3). Second, I realised that the multiplicative problems that the participants held went beyond simply brushing aside cobwebs. I came to find that many of the participants held very little cogent understanding of the operations and arithmetic conventions which, as seen in (4.3 – 4.10). Nor did we ever fully repair this aberration, thus an inability to recall multiplicative facts and apply multiplicative reasoning limited progression throughout the semester.

In South Africa, long multiplication can be attended to by partitioning or using the standard form, with columns for units, tens, hundreds, etcetera (Department of Basic Education, 2011). The participants called this the *house* method.

In a lesson, where the *house* technique was discussed in class, some of the participants were perplexed and derailed by such matters as which number to write on top, which side of the columns to affix the multiplication and addition signs, and in which order – units, tens, hundreds or alternatively hundreds, then tens and units – to conduct the multiplication computations. These factors became sticking points, and trivial as it might seem, some of the participants attached a great deal of importance to the side on which to place the operations signs. Similarly, in spite of the evidence presented to them on the chalkboard, some participants seemed inflexible in perceiving that different routines, properly executed, would achieve the same answer.

In unpacking this work, I reminded the participants that, in an earlier lesson, we had acknowledged that 3 x 5 = 5 x 3 (the commutative property of multiplication), thus we could anticipate that, for example, 423 x 512 = 512 x 423 and that therefore which number *goes on top*, was not really problematic. For convenience, however, we eventually agree that the first-listed number would go on top.

Later, after the lesson, I realised that, in trying to discuss the commutative property in the context of a long multiplication calculation, and given that the participants were strongly predisposed to applying (mis-understood) rituals and different rituals at that, I may have cognitively stretched some participants too far. The solving of, for example 125 x 364, alternately, 364 x 125 and multiplying, for example, first the units and then tens, and hundreds or alternatively, hundreds, tens then units, etcetera, with the accompanying zeros as placeholders, might have been a bit too complex. However, in that lesson, I did not realise that some participants remained unsteady.

What I did notice though, was that when it came to the multiplication itself, alongside the columns, some participants constructed lists of the times tables – 1 x 4 = 4; 2 x 4 = 8; 3 x 4 = 12… or lists of repeated additions – 4 + 4 + 4… Not only was this an onerous and ill-efficient strategy, but on top of this, the participants were prone to making mistakes in their listings. And, it indicated that they had still not revised their times tables. Indeed, this missing skill-set – an inability to instantly recall multiplication facts – was carried through, through the full period of research. Its manifestation in assessment tasks is discussed further in (4.5.9.1) in this chapter.

Linked to their difficulties with long multiplication, I realised that, in setting out their work, some students did not appear to grasp the concept of the columns. Two errors were noted, namely that generally, (1) many students were very untidy – sloppy even – in setting out their long multiplications, and, (2) many did not understand the idea that the units, the tens, and hundreds, were attached vertically to columns. With either of these situations in play, and often both, errors were bound to occur, and did. Evidence of this too, is found in this chapter (4.3.1 and 4.4.1).

In class, my expository work is always neat, colour-coded, and properly set-out. This is true whether I construct a slideshow or work on a chalkboard. I used this fact to discuss, at length, a framework for setting-out work that would find favour in schools. I also drew a distinction between the setting-out conventions we use in exercise books and so-called rough-work. Indeed, because I believe deeply in trialling mathematical ideas, my participants were encouraged to keep a scribble-pad close to hand to give effect to their thinking.

After the class, I conceptualised and constructed a letter about setting out which I subsequently provided to the participants. The rationale which informed the development of the letter was a view that I hold that, particularly students who are at risk, benefit enormously when they learn to neatly structure their mathematical solutions. This allows them to visualise patterning in their steps and to follow paths towards a final answer. Thus, quick ideas aside, even in their scribble-pads, I believe that a *neatish* approach can benefit students who are struggling with mathematical ideas. By extension, I have also found that, especially students who are struggling, get lost in a sea of symbols and numbers and jottings when they do not have the wherewithal to cogently structure their thoughts. These facts notwithstanding, in my own private calculations, I often *scribble*, so I am aware of the contradiction in my writing. However, as in all things, a balance must be struck, and for at risk students – and many of the participants appeared to be struggling – I thought that the letter would prove to be of benefit.

Also, as a reflective activity, partly because the discussion about long multiplication had not gone to plan, I asked the participants to each bring to class about twenty to thirty bottle-tops or buttons or pebbles, etcetera. Such artefacts can be used to construct arrays to visualise the commutative, associative and distributive properties of multiplication. I thought that, rather than confine such work to a teacher-led demonstration, it might be useful for the participants to use these manipulatives themselves to assist their mathematical thinking. I also felt that such participant-involved investigations would serve the purposes of my thesis.

With the benefit of hindsight, I continue to believe that this was a constructive and beneficial way to try to assist the participants to make sense of multiplicative and other arithmetic facts, but two niggles persisted throughout the research period, namely, (1) many participants never brought such resources to class, and (2) of those who did – about a half of the participants – many seemed loathe to use them. In quiet discussion with some of them, in much the same way that (incorrectly) many teachers in their past had frowned upon the use of fingers in thinking and counting, so too, they quietly felt that the use of bottle-tops was for *babies*. My counter-position that modern perspectives of mathematics teacher education (2.5), Shulman's (1987) conception of complimentary content knowledge (CK) and pedagogical content knowledge (PCK) (2.5.2), and effective mathematics teaching practice (2.5.5) all contradict these myths, did not really bear any fruit.

I did, however, often use these manipulatives on number lines and arrays to explain multiplicative facts and ideas. They facilitated explorations into factors, factor pairs, remainders, primes, squares and square roots, and also provided me with an entrance point to area models of multiplication. However, typically, because so many participants would not take a first step, it would often be me, sitting cross-legged on the floor, moving small, square wall tiles or pebbles into different configurations with the participants observing rather than doing. It seemed as if, while I was doing it, it was fine (and they appreciated the exposition), but they could not/would not enact the same practices (2.7.4.1). Further, when, through exposition, they realised that a number and its square root could be visually linked to a square and its side length, they appreciated that new understanding, but seemingly, few returned to their desks to make notes of these discoveries.

In the second quarter of 2016, as a visual application of the distributive property of multiplication, I introduced the participants to a paper and pencil-based model – the grid method – which in fact is simply an areas-based representation of multiplication. I did this reflexively, partly to relook at long multiplication from a new, visual, perspective, and partly to improve the participants PCK. I used numbers such as 34 x 26 to achieve areas – for example four areas from (30 + 4) x (20 + 6) – the sums of which were added together to achieve the final products.

In the context of this class, the participants perceived the technique as novel and interesting. However, although some students quickly understood the multiplicative links between the iconic and symbolic representations of long multiplication, others did not grasp the intersection. To counter this, on my chalkboard, I reverted to much smaller numbers, 8 x 7, reconstructed an area model (5 + 3) x (5 + 2), then carefully drew a rectangular representation of 8 x 7, reconfigured it to (5 + 3) x (5 + 2), drew in horizontal and vertical lines to represent unit squares, and counted out the answer, 56. With the exposition completed, the participants used the grid technique as an alternative to long division and were encouraged to try different distributive combinations to test the concept. All seemed to go well, and in subsequent lessons, the technique was used again. However, when they were re-evaluated on this work, only 3/38 participants correctly completed a grid/areas based multiplication task (4.9.8). Seemingly, compression of this technique did not take place.

### 4.1.7 DISCUSSION – FRACTIONAL REASONING

In the arithmetic task that was completed by the participants on the 10th February 2016, only one question was directly linked to an understanding of fractions. In it, the participants were required to shade $\frac{5}{8}$ of a large square which had been sub-divided into sixteen smaller squares. Only 8/34 participants, or 24% managed this task.

In a word-problems task, completed on 22nd February 2016, five of the ten word problems that were contained in the assignment required the direct use of either fractions or ratios in their solutions. These questions, 2, 3, 4, 6 and 7, returned success rates of 17%, 37%, 37%, 8% and 16% respectively. A full discussion about the participants' solutions to these questions is provided in (4.4) of this chapter.

Because the arithmetic and word problems tasks were presented to the participants at the beginning of the semester, careful analysis of the answer sheets enabled me to get some sense of misunderstandings and to thereby consider activities that might help the participants to better understand fractions. However, to give some sense of the magnitude of the difficulties that the participants encountered, Q2 in the word problems task might prove enlightening. The question, written in full, stated: **Altogether Shirley and Jean have 120 teaspoons. Shirley owns $\frac{3}{8}$ of the teaspoons. How many teaspoons does Shirley have? How many teaspoons does Jean have?**

Only 6/37 participants (17%) correctly answered this question. With the exception of one other student, who was awarded one mark out of three for this question, all the other participants got it completely wrong. While a full description of the errors is provided in (4.4.3), it is worth looking at the conceptions that need to be in place – *but were missing* – to answer the question.

As a minimum, readers of the question would have to perceive that the sharing of the teaspoons is between two people and that, based on the $\frac{3}{8}$ fraction, the teaspoons are distributed in a fixed manner, in effect, a 3 parts : 5 parts ratio. Furthermore, the reader would have to perceive that the 120 count represents the entirety of the teaspoons that are shared – that is, the 120 does not represent one person's stock – rather the 120 is distributed between the two people. Further, one has to understand the roles of the numerator and denominator in the $\frac{3}{8}$ fraction.

For the prepared mind in which a fractions meta-concept or schema is in place, the relationship between the denominator (8) and the whole (120), invites division of 120 by 8, thereby generating a unit-size 15. This quotient is then multiplied by 3 and 5 respectively to achieve 45 and 75, the sum of which, in a quick check, returns us to 120. However, 31 of the 37 participants could not do this work.

While this discussion will focus on some reflexive activities that were used to try to improve the participants understanding of fractions, it has to be acknowledged that success in fractional thinking depends on, and in turn, symbiotically contributes to, success in algebraic thinking, ratios, decimals, percentages, measurement, estimation and proportional reasoning. When these relationships are perceived and understood, compression is effected; that compression provides problem-solvers with an ability to fluidly integrate many intermeshed underpinning skills and ideas.

Siemon et al. (2012) suggest that our fractional understanding develops best when our early experiences of fractions are informal, draw from our life-experience, and include visual stimuli which allow us to perceive parts, (fair shares); comparisons (bigger, smaller, half, etcetera); perceive conceptual differences between how many and how much, enable us to construct, partition and understand fraction diagrams (paper ribbons, area representations, etcetera), before we introduce fractional symbols and formal computations. Indeed, such informal use of models "should permeate instruction, not just be an incidental experience, but a way of thinking, solving problems, and developing fractions" (Petit, Laird & Marsden, 2010, p. 1).

It is clear then, that if we introduce the formal vocabulary and formal computational aspects of fractional work into learning programmes at too early a stage, we may inhibit rather than contribute to fractional understanding. Also, when banking models are used, such as when, in the multiplication of one common fraction with another, we reduce computation to the routine that *we multiply the top numbers and then multiply the bottom numbers*, any sight deviation from a fixed path – for example, the inclusion of mixed numbers or mixed operations – precludes success. Further, Petit, Laird and Marsden (2010), caution that, unwittingly, teachers may use models in a rote learning manner – for example, always reverting to the use of fraction circles – and suggest that to reduce the likelihood of this happening, teachers should multiple models (manipulatives, number lines, partitioned rectangular and circular areas, arrays, etcetera), which cause learners to continuously reapply their minds to the perceptual features found in the different models.

So, it is important to begin fractional work with (1) simple contextualised tasks which encourage informal use of fractional thinking – *John has twice as many sweets as…; Harry has half the number of CD's as…;* (2) link fractional thinking and whole number thinking; (3) use estimation and (4) construct models of fractional ideas In particular, learners should be encouraged to find and invent their own informal models and approaches to make sense of and anticipate what the outcome of a particular fractional computation might be: fraction circles, fractional strips, area models and number lines are commonly used to visualise fractional thinking. (Van de Walle, Karp and Bay-Williams, 2010). In this way, understanding is built into the learners' early experiences of fractional thinking and computation.

All of the above considerations informed my reflexive planning. In the context of this research, it is fair to conjecture that having matriculated from school and having completed a first degree in a university environment, all of the participants would have encountered fractional work and thinking in previous learning experiences. It is also fair to suggest that my own mathematics teaching knowledge, coupled with the data captured from the participants Thinkboards (4.2), pointed to less than satisfactory perceptions and understandings in mathematics. My difficulty in negotiating starting points was that the prior knowledge assignments uncovered many misunderstandings – many of these at a foundational level – so, finding the most beneficial ways to support the participants, in the limited time that was available to me, was, I realised, going to be challenging.

As happened with the arithmetic task and the discussion on multiplicative thinking discussed in (4.1.6) above, when I scrutinised the word problems assignments, I was alerted to problematic fractional mistakes that were made by the participants. However, it was only in early March 2016, when we reviewed their assignments together, that I began to realise how profoundly low the levels of misunderstanding of fractional work really were. As was found in (4.1.6), I realised that my initial belief that the participants simply needed to brush aside cobwebs was incorrect. Although I had planned to work through the review of the word problems with the participants in only one lesson, instead, the review spanned three lessons. During these lessons, it became clear that many intersecting fractional facts, ideas and vocabulary would have to be relearned. The review of the fractional work was part of a bigger enterprise, namely, a review of the entire word problems assignment.

For purposes of the review, the marked prior knowledge tasks were returned to the participants and I issued a new photocopied, word problems review assignment to the students. Notable features of the review assignment were that (1) it contained questions that were similar to the original tasks, and that (2) the order of the questions was reassigned to reflect the most-commonly correctly answered question first, followed, in descending order to the most problematic cases. Alongside each question, I introduced as a percentage, the success rates that had been found in the original assignments.

The rationale for the reordering of the questions in the review papers was that I felt that, by first working through questions that mostly, were well-answered, I would be able to iron out small problems and use their discussions and explanations as stepping stones towards forming connections with the more troublesome questions.

Thus, for example, because Q1 enjoyed the highest success rate (64%) in the prior knowledge task, it remained the first question in the review assignment. However, Q6, with an 8% success rate, was the worst answered question in the prior knowledge task, so it became Q10 in the review assignment. Both variants of Q6 – that is Q6 and Q10 – are transcribed below.

In the prior knowledge assignment, Q6 read: **Altogether, Lulu and Irene have sixty new books. Irene has half the number of books that Lulu has. How many books does each lady have?**

In the review assignment, Q10 read: **Altogether, Lulu and Irene have ninety-six keys. Irene has one third the number of keys that Lulu has. How many keys does each lady have? [8%]**

If we remain with Q6 and Q10, it can be used as an exemplar of the teaching and learning which occurred during the three days review of the word problems prior knowledge assessment task. Albeit that the review occurred in early March 2016, at that time, because of unfolding and stubbornly difficult working circumstances (4.1.1; 44.1.2 and 4.1.3), I had hardly begun to interact with and *know* the participants, thus, much of what was revealed through the review required spontaneous reactive support and guidance.

A feature of the review of was that, in the review document, was we moved from Q1 to Q2, etcetera, we would first look at its companion in the prior knowledge task. I would try to discover the various mis-understandings that had resulted in the mistakes in the questions – these were jotted down on the chalkboard – and then, on the fly, I would work through and try to massage a cogent reply.

Typically, for each question, such negotiations of understanding would enjoy some debate, some directed teaching and thinking, some visual modelling of ideas with manipulatives, iconic and symbolic representations, some development of understanding of problematic vocabulary and turns of phrase and some consolidation of ideas and skills. Such interactions, done to try to develop content knowledge (CK), pedagogical content knowledge (PCK), conceptual understanding and compression would find favour with authors such as Bruner (1966), Ausubel (1968), Shulman (1987), Gardner (1993), Bedrova and Leong (1996), Gray and Tall (2007), Ball (2009), Presmeg (2009), Ma (2010), Department of Basic Education (2011), Tipps, Johnson and Kennedy (2011), Siemon et al. (2012), Friedrick et al. (2013); Clements (2014), Haylock and Manning (2014), Gonnerman, O'Rourke, Crowley, & Hall (2015), Sherman & Gabriel (2016), Rellensmann, Schukajlow and Leopold, (2017), Hartanto, Yang and Yang (2018), and others. I list this long string of authors to demonstrate that there is a long history of academics drawing together various facets of fractional and mathematical thinking so as to encourage meaning-making and understanding rather than rote learning.

In Q6: **Altogether, Lulu and Irene have sixty new books. Irene has half the number of books that Lulu has. How many books does each lady have?** it rapidly became apparent that the sticking points were *altogethe*r and *has half the number of.* Many participants either misunderstood one or both of these turns of phrase. The context – two ladies sharing books and the number, 60 books, was not problematic – it was the relationship of how the books and the ladies were understood to have these books that was problematic.

Of the two sticking points, altogether was dispensed with quite easily. I used bottle-tops to convey understanding. The bottle-tops were placed on a demonstration table in two uncounted piles, and then, literally, pulled into one larger grouping. Similarly, different, smaller groupings were combined to form the larger group. Simultaneously, I explained how the smaller groups contributed to make the larger whole - the full amount – and how the smaller groups contained parts which, when combined together – altogether – implied the total number of available bottle-tops, and by extension, books.

At the back of their books, or on a notepad, the participants were encouraged to construct a personal dictionary of problematic vocabulary and turns of phrase, to provide explanations of these terms for themselves.

Further, as mentioned earlier in (4.1.7), I also encouraged the participants to make use of a user-friendly web-site, https://www.mathsisfun.com/ to make use its dictionary and explanations, to use other online resources such as Google© and Youtube©, and our university online management system, Blackboard©.

In Blackboard©, I had previously constructed, and consequently continuously updated, many support documents that I inserted into numerous folders into the IPS 413 E module contents page. These folders included a (1) *Learning Guide* folder; a (2) *Letters from Peter* folder, (3) *CAPS documentation*, (4) *Reading Articles*, (5) *Slideshows*, (6) *Mathematical Infograp*hics, (7) *Pencasts*, (8) *Past Tests and Exam Papers,* (9) *Test and Exam Preparation*, (10) *Year marks*, and (11) an *Announcements* page.

My Blackboard© repository was developed to provide asynchronous support to my participants. In it, for example, within the *Letters from Peter* folder, is the file: *Letter 2 – Setting out Rules for Math* that was discussed in (4.1.6) above; in the *Articles* folder, the participants had access to numerous user-friendly readings on fractions, word-problems, blended learning, learning theory, math anxiety and so forth, and in the *Slideshow*s folder, comprehensive slideshows on setting out, number patterns, teaching mathematics, and fractions among others. Each slideshow might contain 100 to 200 slides. The *Pencasts* folder contained numerous audio-visual micro-lessons that I created using an Echo Smartpen☺ and Livescribe☺ software. Thus, while some of the resources were developed for generic use, others, such as the Slideshow on fractions were reconfigured to suit the purposes of the IPS 413 E participants' problems with fractions, and yet other resources, such as the setting out document and slideshow, were constructed to specifically address problems that were uncovered by the action research data.

The second sticking point in Q6, *has half the number of,* proved move vexing to solve. Although it received careful attention, the data captured at the end on the first quarter in 2016 demonstrated that the re-evaluation task showed only modest improvement, (8% rises to 22%), and this, in a question – the actual question - that had been seen numerous times in the period leading up to the re-evaluation.

It has to be acknowledged that the sentence, *Irene has half the number of books that Lulu has*, can be written in different ways. More explicitly, one might write, *For every two books that Lulu has, Irene has only one.* Alternatively, one might write, *Lulu has twice the number of books as Irene*. Or, perhaps, *Lulu has double the number that Irene has*, or *For every one book Irene has, Lulu has two*, and so forth. And indeed, in class, as a first explanation, the question was repackaged into these shapes. Nonetheless, I was left with a distinct feeling that many participants continued to hold onto an incorrect conception of the explanation, that the participants held to a view that the sharing was half and half.

In order to get over this sticking point, as a second step, I rewrote Q6 on the chalkboard, but reduced the total from 60 books to 18 books. The smaller number, I felt, would enable me to use visual scaffolding demonstration that would focus on the unpinning concepts in play rather than the numbers. Indeed, Van de Walle, Karp and Bay-Williams (2010), support such actions, stating that these smaller quantities make it easier for learner's to understand, analyse and come to grips with their problems, and often lead to insights which can then be transferred back into their original calculations.

Next, at a demonstration desk, I counted out 18 bottle-tops, and then, with an onus on informal explanation, I tried to convey through sharing, that every time Irene was given one bottle-top, Lulu was given two. In other words $\frac{1}{3}$ 's not $\frac{1}{2}$ 's were in play, or alternatively the ratio of parts was 1 : 2. However, I did not introduce common fractions nor a formal explanation of ratios into the discussion. In effect, the demonstration could be aligned with Piaget's concrete operations stage, in which hands-on experience heightens understanding (Ojose, 2008), and further, the demonstration could be understood to be an exhibition of embodied cognition (Boaler, et al., 2016) which are both concepts and skills which the participants were encouraged to assimilate.

After we completed the chalkboard-based activity, the participants returned to Q6 and seemed to easily solve it. When we then focused on Q10 in the review task, my immediate sense was that the participants easily perceived the sharing in terms of 1 for me and 3 for you. Notwithstanding the fact that some participants struggled to divide 96 by 4 – missing multiplicative facts – most of the students seemed to ease with the problem.

A confluence of other mathematical considerations, problematic on-the-ground developments, and the imminent end of term 1 of 2016 precluded any further in-class discussions on this type of problem in the first quarter.

### 4.1.8 DISCUSSION – IFI, VP AND VISUALISING REASONING

Because this part of the chapter reflects upon fieldwork experiences, and in light of the importance I have attached to the use of manipulatives and iconic stimuli, it is well-worth reflecting on some of their attributes before I present two exemplars of the Interpreting Figural Information (IFI) and Visual Processing (VP) techniques that were used within the IPS 413 E Mathematics module. After that, I present exemplars of how IFI and VP were applied in the research. Siew Yin (2010) has identified seven roles that visualisation plays in assisting math students to solve problems:

1. To **understand** the problem. By representing the problem visually, students can understand how the elements in the problem relate to each other.
2. To **simplify** the problem. Visualization allows students to identify a simpler version of the problem, solving the problem and then formalizing the understanding of the given problem and identifying a method that works for all such problems.
3. To see **connections** to a related problem. This involves relating the given problem to previous problem-solving experiences.
4. To cater to individual **learning styles**. Each student has his or her own preference when it comes to the use of visual representations when solving problems.
5. As a **substitute** for computation. The answer to the problem can be obtained directly from the visual representation itself, without the need for computation.
6. As a tool to **check** the solution. The visual representation may be used to check for the reasonableness of the answer obtained.
7. To **transform** the problem into a mathematical form. Mathematical forms may be obtained from the visual representation to solve the problem.

Siew Yin (2010)



Further, Bishop's (1983) explanation of Interpreting Figural Information (IFI) and Visual Processing (VP), (2.6.3.7), and similarly, the explanation Siemon et al. (2012), present on spatial orientation and spatial imagery (2.6.3.8), demonstrates that visual skills and scaffolds represent more than a remembered image of an event or object. Zimmerman and Cunningham (1991), in Siemon et al. (2012 p. 223), have defined visualisation as, “the process of forming images (mentally, or with pencil and paper or with the aid of technology) and using such images effectively for mathematical discovery and understanding.” But, in practice, what does all of the above mean?

Below, in *Figures 4.8* and *4.9*, I have presented examples of problems that were presented in my lessons; these and other exemplars were used to connect mathematical ideas (Haylock and Mann, 2014), and were linked to mathematical teaching practice (Muir, 2008). In the first example, I will unpack an IFI solution; following that, I will present a problem which will be solved by using VP strategies. In each example, Siew Yin’s (2010) visualisation roles will be linked to the solutions.

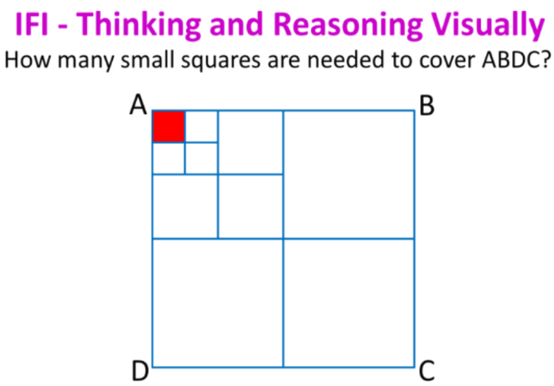


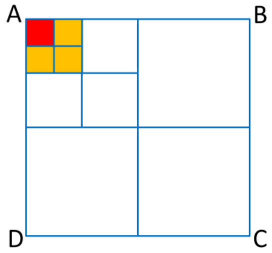

*Figure 4.8.* An application of a visually mediated IFI solution to a problem

Above, our task is to establish the number of the smallest square that are needed to completely cover – to tile – the surface of ABCD. In this question, the answer is quite simple to achieve, but it also enables me to explain the role that IFI plays in achieving the answer. The properly prepared mind will look carefully at the visual information that is presented in the sketch. That mind will look for patterns in the sketch – gestalts – and having established those patterns, will use that understanding to solve the problem.

Below, I will use text to support my explanation of a visual solution, but for many students, the visual presentation in the sketches that I draw rapidly become internal constructs of the mind.

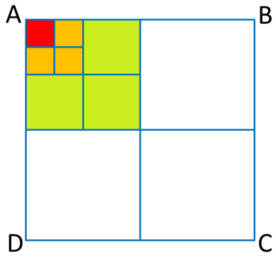
STEP 1

Because the original sketch is drawn neatly and in fair-proportion, it is relatively easy to deduce that there are four congruent squares enclosed in the first level of visual stimuli – in this example, I have presented the original square in red and its three congruent siblings in orange.




STEP 2

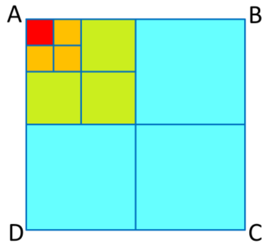
Additional squares are revealed to us and our IFI capacity enables us to see that there are four *larger* congruent squares. This is, in-effect, a visual stepping-stone. Already, the brain is constructing connections between previous and subsequent steps in the solution. The computation 1 x 4 x 4 = 16 occurs visually.




STEP 3

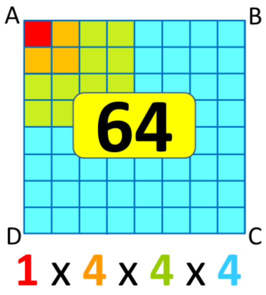
The pattern logically repeats itself. As in previous steps, the visual clues that enter our eyes enable us to see that there are, yet again, four larger congruent squares. Different people will achieve the final answer in different ways, but will generally detect an answer of 64 squares.






SUMMARY

Siew Yin (2010) suggests that one of the roles of visualisation is that it enables students to transform visual stimuli into mathematical equations and to generate an algorithm to solve the problem. In this case, the algorithm has revealed itself as 1 x 4 x 4 x 4, or $4^0 \times 4^1 \times 4^2 \times 4^3$ and has also provided a visual representation of consecutive powers of four or $4^3$.




A second problem is presented below. It is a bit more challenging and is an exemplar of the application of Visual Processing. Although primary school children in other countries routinely solve this sort of problem, it stumped many of my participants who could find neither a symbolic nor visual technique to achieve the answer.

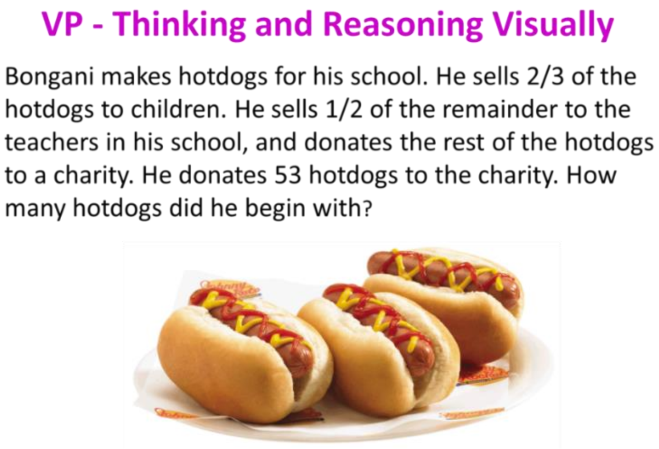


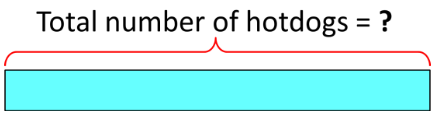

*Figure 4.9.* An application of a visually mediated VP solution to a word problem

The *intent* of sentence-structure is to enable students to succeed. *But many don't!* Often, especially second-language English speakers are excluded from success because they do not understand the story. In this example, we will invoke Bishop's (1983) concept of Visual Processing (VP). We want to know: *How many hotdogs did Bongani bring to school*?



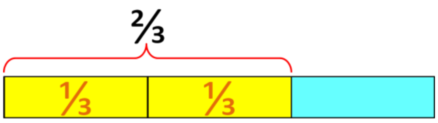
STEP 1

We do not know how many hotdogs Bongani brought to school, but previous experiences with the concepts of parts and whole enable us to imagine that the blue bar represents all of his hotdogs.




STEP 2

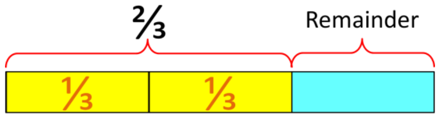
The text tells us that Bongani sold 2/3 of his hotdogs to the children in his school, so we can imagine and then sub-divide the whole into three equal pieces.




In this sketch, above, the hotdogs that were sold to the children are presented as two fairly-proportioned yellow rectangles.

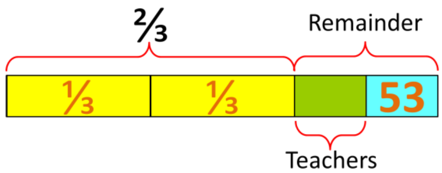
STEP 3

After selling his wares to the children, Bongani still has some hotdogs. The blue bar on the right represents the third, ⅓, that is, the remainder.




STEP 4

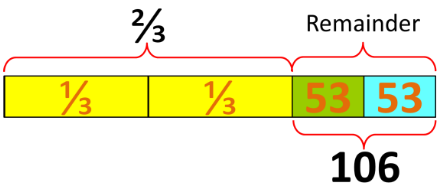
½ of the remainder was sold to the teachers in the school, thus the remainder is sub-divided into two equal parts.




The green rectangle, above, represents the hotdog sold to the teachers. The other ½ of the remainder, that is, 53 hotdogs, were given to a charity.

STEP 5

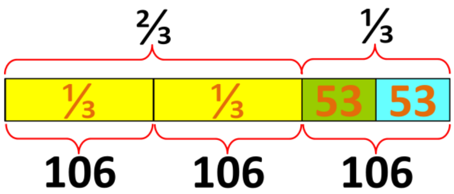
If 53 hotdogs were given to a charity, that means that similarly, 53 hotdogs were sold to the teachers. We can deduce the remainder is 53 + 53 = 106.





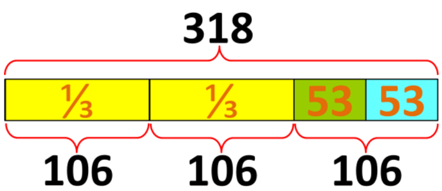

STEP 6

The remaining hotdogs, that is 53 + 53 = 106, also represents 1/3 of the total number of hotdogs. Our understanding of fractions and fraction walls reminds us that each ⅓ must represent a constant unit size, thus each ⅓ must have a value of 106.




STEP 7

If the remainder 1/3 is worth 106 hotdogs, each 1/3 is also worth 106 hotdogs. From this realisation, we can deduce that Bongani brought 318 hotdogs to school.




In this example, links are forged between a visual methodology and Siew Yin's conception of the roles that visualisation plays in supporting the construction of its solution. The logical progression of the steps (1) help us to *understand* the problem;

(2) enables us to *simplify* the problem; (3) allows us to see *connections* – in this case, between the parts of this problem and the whole; and, (4) allows us to *substitute* a visual model for an algorithm (Siew Yin, 2010). No visually-solved mathematical word-problem is necessarily exclusively an IFI or VP problem, nor will every visualised-problem invoke all of Siew Yin's (2010) visualising roles. However, visualisation and visualisation strategies enable us to free up cognitive space so that we can attend to procedural and intellectual aspects of mathematical problems (Siemon et al. 2012).

Action Cycle 1 had revealed many misunderstandings and it is acknowledged that many extraneous factors prevented our contact classes from attending to much more than trying to make sense of the mistakes that were found in the first pair of prior knowledge tasks. Action Cycle 2, building on experiences in Action Cycle 1, attempted to assist the participants to construct more cogent conceptions of mathematics. In both action cycles, the re-evaluation tasks did show some improvement, see Table 4.1 below, but generally the rates of return continued to be low; these data are discussed fully in (4.3 – 4.10) in this chapter.



Table 4.1

*Summary of quantitative results for Action Cycle 1 and Action Cycle 2*

| | | | | |
|---|---|---|---|---|
| Action Cycle 1 | | | | |
| Foundational Artithemtic Concepts | | | | |
| | Q1 | Q2 | Q3 | Q4 |
| Piror Knowledge | 63% | 36% | 38% | 12% |
| Re-evaluation | 88% | 63% | 52% | 33% |
| Word Problems | | | | |
| | Q1 | Q2 | | |
| Piror Knowledge | 45% | 38% | | |
| Re-evaluation | 56% | 44% | | |
| | | | | |
| Action Cycle 2 | | | | |
| Fractions and Ratios Concpets | | | | |
| | Q1 | Q2 | Q3 | |
| Piror Knowledge | 53% | 24% | 30% | |
| Re-evaluation | 60% | 42% | 66% | |
| Word Problems | | | | |
| | Q1 | | | |
| Piror Knowledge | 16% | | | |
| Re-evaluation | 71% | | | |

Action Cycle 2 was handled in a different manner to Action Cycle 1. The prior knowledge assessments in Action Cycle 1 had demonstrated many misunderstandings of foundational arithmetic competences. The data captured in the re-evaluation tasks demonstrated that overall, the activities in Action Cycle 1 did not lead to any great improvements. Thus, in Action Cycle 2, I concentrated my efforts into conceptualising and constructing a teaching and learning programme that would assist the participants to develop a deeper understanding of the mathematics they were doing.

Beginning with the prologue to this thesis, at many places in chapters 1 and 2 and in preceding parts of this chapter, I have identified how rote learning stifles cognitive development whereas activities-based, liberal teaching and learning practices promote relational understanding, the formation of robust schema, and the compression of big mathematical ideas. I will not repeat all of these considerations here, but I knew that I wanted these more dynamic perspectives to wash through the participants experiences of mathematics. Thus, in fashioning a plan of action for Action Cycle 2, the experiences of Action Cycle 1 and all of the benefits and limitations of these ideas were used to inform the thrust of the work.

For Action Cycle 2, two prior knowledge assessment tasks were prepared for the participants (Appendix L). Careful scrutiny of these tasks will show that the whole gamut of stratums that have been discussed in this thesis and that are considered to facilitate deep mathematical thinking, representation and understanding were introduced into the questions. Within the tasks, I have placed a strong emphasis upon visualisation and informal, intuitive problem-solving, these as applications of IFI and VP, (Bishop, 1983), as discussed earlier in this section. There is also some carry over to more formalised mathematical computations. In part, the questions were constructed as an addendum to the findings in Action Cycle 1, in part, they offer evidence of my perception of the skills-sets and ideas that might prove most beneficial for the participants cognitive development. A full description of the data for these Action Cycle 2, prior knowledge tasks is found in (4.7) and (4.8).



Within the tasks, one will find questions that encourage the participants to examine part-shaded-in areas and alternatively, shapes to be shaded, as visual representations of common fractions, decimal fractions and ratios. Number lines, fraction circles, area rectangles and squares, arrays, fraction ribbons, comparative iconic images and contextualised word problems are found in abundance. The visual stimuli are presented to assist the participants to estimate, to model, and to conceive solution strategies and answers. The use of these techniques finds favour with authors such as Bruner (1966), Shulman (1987), Gray and Tall (2007), Ball (2009), Presmeg (2009), Ma (2010), Petit, Laird and Marsden (2010), Tipps, Johnson and Kennedy (2011), Siemon et al. (2012), Clements (2014), Haylock and Manning (2014), Boaler et al. (2016), and others.

In the context of this thesis, it is not possible to discuss all of the activities that occurred during the research period, but, Q2.8, by way of example, can be used to demonstrate the manifold hurdles that the participants encountered. In that question, the participants had to determine the product of $\frac{4}{5} \times 2$, and, having achieved that answer, plot its position onto a number line – the number-line was presented with units partitioned into fifths. For participants in which appropriate compression might have occurred, an intuitive solution would have been to refer to the sketch of the number line, count out four spaces, and then, four spaces again, to achieve a touchdown point on $1\frac{3}{5}$. However, this was not the norm; in the prior knowledge and re-evaluation tasks, Q2.8 enjoyed success rates of 14% and 18% respectively. The 18% score in the re-evaluation assignment, was the second but lowest return.



Q2.8, and others like it, was discussed, unpacked and reformulated in a number of ways. First, as an application of multiplicative and fractional thinking, the participants were encouraged to estimate the size of the product – did they perceive that the answer would be bigger or smaller than four-fifths; bigger or smaller than one; what did they believe happens when a common fraction such as $\frac{4}{5}$ is multiplied by a whole number such as 1, 2, 3, 4; does multiplication always make a bigger number, etcetera?

Second, fractional circles and rectangular areas were brought to bear on the problem. The fractional circles were used as a visual representation of repeated addition, that is $\frac{4}{5}$ of a whole in one circle, plus $\frac{4}{5}$ of a whole in a second circle, describes 8 equal sectors in 10. A rectangular area-model was used to demonstrate a similar outcome, but this time by partitioning first the larger rectangle into two congruent rectangular parts (the two wholes), and then by partitioning the wholes into smaller, congruent fifths. Each whole was then shaded to cover four of five parts.

While structured and unstructured number lines seem well-positioned to assist in exemplification of many mathematical ideas, a basic understanding of number lines and place value must first be in place in order to derive that benefit. In the early stages of the research and continuing throughout the research period, it was evident that many participants, at some or other stage in their mathematical development, had either misunderstood or adopted an incorrect understanding of number lines. Indeed, this persistent mal-understanding extended to their use of the ruler, where often, participants might incorrectly construct a measured dimension, say 9cm, but draw it at 8cm, in other words, 1cm less than the desired outcome.

When using a ruler, the participants often began measurements from the 1cm mark, thereby ignoring the space between 0cm and 1cm. On the number line, perceiving fractional positions in the *empty* spaces between the units, was found to be problematic.



I used a *clothes-line* activity to try to bring some clarity to the participants understanding of number lines. On a three to four meter long string, I used pegs, pen and paper to identify 0, 2, 4, 6. These positions were set out in fair proportion on the clothes line. As a short, first task, the participants were invited to populate the clothes-line with 1, 3 and 5. They did so, but invariably did not concentrate much attention on the positionality of the numbers – thus instead of placing, for example 3, equidistant from 2 and 4, it might have been skewed left or right. I used these misrepresentations to explain the *obvious* IFI nature of the mistakes. However, throughout the duration of this activity, to the point where we were placing thirds or fifths or mixed numbers onto the clothesline, the perception of fair proportion remained elusive for some of the participants.

Thus, although we did establish the correct touchdown point on the number line, it was clear to me that some participants simply waited for me to complete the task and then copied my solution into their exercise books.

With a score of 0% in the Action Cycle 2 prior knowledge assessment and 80% in the re-evaluation, Q4.4 returned the most spectacular result difference in the two assignments. However, to be fair, Q4.4 was a carryover from Action Cycle 1, that is, it was very similar to Q6 in the prior knowledge and re-evaluation tasks, where it scored 8% and 22% respectively in the two tasks. So, while the 0% score in the prior knowledge task was disconcerting, the continued attention to this type of question no doubt influenced the positive outcome in the Action Cycle 2 re-evaluation.

Q4.4 in Action Cycle 2 read: **Altogether, Noludwe and Karin have 96 books. Noludwe has one third the number of books that Karin has. How many books does each lady have?** Clearly, this was similar to Q6, as discussed earlier in (4.1.7). What changed however, between the first action cycle and the second, and between the prior knowledge task and re-evaluation task in the second action cycle was that during Action Cycle 2, I actioned a deliberate IFI and VP methodology into the teaching and learning programme.

In Action Cycle 2, the participants were encouraged to construct visual scaffolds to assist them to make sense of the word problems. But, in order for this to be effected, I had to first create a framework to which they might connect their ideas. This is outlined below.



In our class, I spent some time negotiating an understanding of fraction walls and I used that opportunity to point out fractional facts such as the uniform size/value of parts of fractions holding the same denominator, the equivalence of some fractions with others holding different denominators, vis-à-vis, for example $\frac{2}{6} = \frac{1}{3} = \frac{3}{9}$ , and the roles played by numerators and denominators in fractions, etcetera.

I also gave the participants many congruent paper strips so as to enable them, by folding in half and half again, to construct ribbons of halves, quarters, eighths, or alternatively, by folding in thirds and then in halves, to produces strips of thirds and sixths, etcetera. These strips were used to create mixed numbers and to add together various couplings of the strips such as $\frac{1}{2} + \frac{1}{4}$ or $\frac{3}{4} + \frac{1}{2}$ etcetera. Incrementally, we adjusted these couplings to include slightly move advanced pairings such as $\frac{1}{2} + \frac{2}{3}$ and so on.

With the experience of the tactile models in place, I then encouraged the participants to use the blue lines in their exercise books to construct representative ribbons of their paper models. The sketches had to be drawn as fair-proportional representations of common and decimal fractions which were then used to visually add various combinations of fractions together.

Finally, the IFI and VP directed activity was tied into an introduction to the more routines that are associated with the formal addition of common fractions, which is, seeking a common denominator, establishing equivalent fractions, adding the numerators and simplifying.

Similar, compatible techniques were used to unpack fractional understanding with the other operations.



Paper ribbons, arrays and sketches were also used to introduce ratios to the participants. The visual models produced a big breakthrough when the participants made the connection that in a fractions driven word problem, when for example $\frac{3}{5}$ is in play (and thereby $\frac{2}{5}$ as well), the ratio 3 : 2 is derived from the numerators, not, as was often thought 3 : 5 or alternatively 2 : 5.

The background discussed above, contributed to success with ratios questions such as Q4.4. When the participants folded or drew a representative strip in three parts and made the link that for every one book Noludwe received, Karin got three, this set up the opportunity to construct a fair ratio (1 : 3), which logically produced 4 parts in play or $\frac{1}{4}$ and $\frac{3}{4}$ .

Once that was understood, division, in this case of 96 by 4, produced a quotient of 24, this being the unit size that populated each smaller, congruently drawn rectangle in the ribbon and thereby led to answers of 24 and 72, the sum of which was 96.

### 4.1.9 SUMMARY OF FIELDWORK

The fieldwork experiences have revealed three problems, (1) the fragmentation of the module by disruptive circumstances, (2) the difficulty obtaining an ethical clearance certificate, and (3) a threat to invalidate the PGCE qualification, which, taken together, have demonstrated that life is messy and one has to deal with it.

On a more positive note, explanations of actions that I took to improve the participants' mathematical understanding, that is, the (1) discussions regarding the planning, design and application of the activities, the (2) reflections on a typical lesson, and (3) the exemplars on the development of multiplicative and fractional conceptions and construction of visual scaffolds, all attest to a module that was cogent, comprehensive and liberating in design.

## 4.2 TRENDS FOUND IN THE THINKBOARDS

On Wednesday, 10 February 2016, I met with the IPS 413 E – Mathematics cohort for the first time. After a brief welcome and a short introduction to the IPS 413 E – Mathematics course, I asked the participants to complete a four-field Thinkboard.



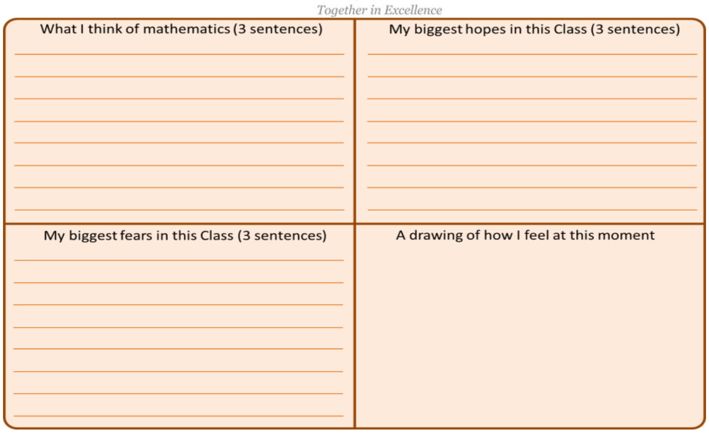



*Figure 4.10.* An unpopulated, four-field Thinkboard, as used in the IPS 413 E - Mathematics class

The Thinkboard, Figure 4.10 above, demonstrated to the participants that they could anticipate a great deal of interaction, sharing and participation in my classes. Further, although it was a novel activity to them, to a person they responded enthusiastically to the task.

As *Figure 4.11* below demonstrates, Thinkboards can explore various perspectives on many matters. Depending on what is sought, a page is organised into fields with each field being allocated a title, and possibly a descriptor. Three or four fields work best.

While the fields can be populated in different ways, my experience has taught me that my students are most comfortable writing, but one might also use symbols, patterns, images, etcetera. Typically, to get the right-brain (R-mode) working (Edwards, 2008), I always incorporate at least one drawing field into the Thinkboard.

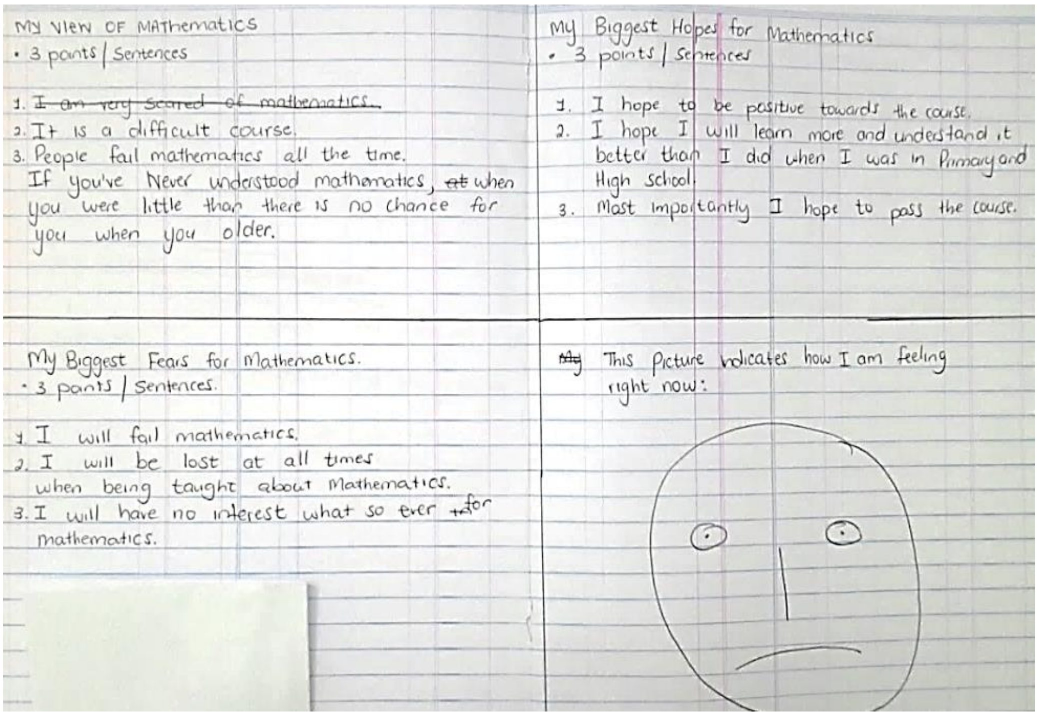



*Figure 4.11.* An example of a Thinkboard as constructed by a participant in the IPS 413 E Mathematics module on 10 February 2016

In part, the findings have assisted me to construct an answer for the second sub-research question, namely: *What barriers to solving word-problems do the students perceive that they hold?*

The qualitative data that was contained in the Thinkboards was recorded into MS Excel©. Each field was analysed separately. As various themes within each field revealed themselves, I colour-coded the individual cells so as to visually explore commonly reported opinions and their quantitative frequency.

In Table 4.2 below, I have produced a representative part of the spreadsheet. Because the spreadsheet contains primary data, I have masked the students' names and student numbers. The representation is of three of the fields; the field containing the students' drawings is not presented. The various colours contained in the cells pertain to a qualitative theme which was revealed through the analysis of the raw data. The complete spreadsheet is found in Appendix E.

Table 4.2

*A representative part of the Thinkboard primary data*

**THINKBOARD - TEXTUAL RESPONSES**

| | | | Field 1 | Field 2 | Field 3 |
|---|---|---|---|---|---|
| | **Student Number** | **Student Name** | **My view of Mathematics** | **My Biggest hopes for Mathematics** | **My Biggest fears for Mathematics** |
| 1 | | | it's a great course to do | to be easy to understand | I fear the fact that technology has made me not to use my mind often and especially when calculating |
| | | | it's practical and improves memory | maths must not be feared subject | I want to be able to transfer positive attitude towards math |
| | | | maths is part of our daily lives | todays's learners to be eager to do it | to fail |
| | | | | | |
| 2 | | | during my time at high school I did not have much problem because I was young and active in class,asked question if I don't understand | My biggest hope is to not disappoint myself | my biggest fears is the changing of curriculum system |
| | | | | to be able to cope | its more than 10 years now of being not involved in mathematics |
| | | | | to pass | another fear is the pace, too much to cover in a short space of time |
| | | | | | |
| 3 | | | Intricate and often seen as difficult | younger generation to get over ther fear of the level of difficulty | More professionals in this field imported |
| | | | needs regular practice to master | teachers who inspire and motivate | dying out of pure maths and more students opting for math literacy |
| | | | is a big part of our daily lives | willing or positive attitude from learners | students or applicants unable to pursue a course of their choice due to not having maths |



Each student was encouraged to supply three responses per text field. Thus, if we focus on Field 1, we see Students 1 and 3 supplied three inputs for each field: these are entered into individual cells. Student 2 supplied 1 response for Field 1, hence two cells remain unpopulated. Below, I will reflect upon the trends that have been uncovered in the data.

### 4.2.1 THINKBOARD – FIELD 1: MY VIEW OF MATHEMATICS

Field 1 was given the title, '**My view of mathematics.**' Deliberately short, the title was designed to be as open-ended as possible. 33 participants applied their minds to this field, and in total it garnered 97/99 possible responses or a 98% return. As seen in Table 4.3 below, Field 1 of the Thinkboard produced eight major trends.

Table 4.3

*Summary of Thinkboard data: Field 1*

| **THINKBOARD ACTIVITY - 10 FEBRUARY 2016** | | |
|---|---|---|
| | **Field 1: My View of Mathematics** | |
| | Number of Participants | 33 |
| | Number of Responses | 97/99 |
| | Return Rate | 98% |
| | **Major trends** | **Number of submissions** |
| 1 | Maths is integrated into daily life | 17 |
| 2 | Practice makes perfect | 15 |
| 3 | It is interesting but challenging | 13 |
| 4 | Maths promotes brain activity | 12 |
| 5 | Maths is perceived as just calculations | 11 |
| 6 | A tricky, complicated and confusing subject | 9 |
| 7 | Maths prepares one for success in life | 8 |
| 8 | Keep away from this subject | 7 |



#### 4.2.1.1 POSITIVE TRENDS

Five major trends reveal a neutral to optimistic disposition towards mathematics. 17 submissions suggested that mathematics is integrated into their daily lives and perceive that mathematics has wide applications in the working world. However, one participant wrote, "even though I don't see it being applied in real situations, but I think it is important." 15 submissions suggested that the students are aware that mathematics needs much practice and they acknowledge the importance of working continuously at the subject. 13 submissions suggest that mathematics is an interesting, if challenging, and enjoyable subject. Comments such as, "it's a great course to do," and "Mathematics is a very interesting subject," exemplify this trend. However, one student describes an interesting paradox, "it is very much enjoyable when understood and devastating when not." 12 submissions believe mathematics encourages brain activity, broadens the mind and causes one to think a lot. One student wrote, "I love it because you become a fast thinker and can think out of the box." Another suggested, "I view mathematics as a very important subject or learning area that keeps the whole head move around.” 8 submissions suggest that, in the modern world, it is very important to be numerate and that a good understanding of mathematics facilitates future success in life. The following comment demonstrates how deeply that submission was considered:

> If I understand mathematics, I will be able to explain to kids how important to have mathematics so that even in high school they can choose to shape their careers in future since there is a lot of opportunities in mathematics and in South Africa we have a shortage of students who have mathematics as a subject.

#### 4.2.1.2 NEGATIVE TRENDS

Three other trends, all less optimistic, were also revealed. 11 submissions suggest mathematics is driven by calculations: what is not clear is whether the participants perceive this type of activity positively or negatively. 9 submissions make the point that mathematics is difficult, tricky, complicated and confusing. 7 submissions suggested that mathematics is not an interesting subject and is best left to bright people only. The comments carry an undertone of rejecting the subject. Below, I include all 7 of the comments:



1. “Many people are not interested in it anymore”;
2. “Most people don't like it”;
3. “I don't feel like”;
4. “people fail mathematics all the time”;
5. “if you never understood mathematics when you were little than there is no chance for you when you older”;
6. “one is either good at it or bad, there's no in between”; and
7. “requires really smart people to pass above average.”

#### 4.2.1.3 COMMENTARY ON TRENDS FOUND IN FIELD 1

Although this data was captured early in this first action cycle of the action research, the major trends reveal that largely the participants are neutral to optimistic about the roles of mathematics in the classroom and outside world. Given the perilous nature of mathematical performance in South Africa, this is a pleasing outcome. However, the data also reveals undercurrents of fear and resistance. These negative dispositions seem to talk to matters of previous encounters that were saturated with symbolic representations of mathematics (Bruner – 2.7.5.4) and traditional, banking models of education (Freire – 2.1.2.3)

### 4.2.2 THINKBOARD – FIELD 2: MY BIGGEST HOPES FOR MATHEMATICS

The second field in the Thinkboard activity was given the heading, '**My biggest hopes for mathematics**.' The students supplied 97/99 or 98% of all the available spaces for submissions. In Table, 4.4 below, 4 major trends were revealed.

Table 4.4

*Summary of Thinkboard data: Field 2*

| THINKBOARD ACTIVITY - 10 FEBRUARY 2016 | | |
|---|---|---|
| | **Field 2: My Biggest Hopes for Mathematics** | |
| | Number of Participants | 33 |
| | Number of Responses | 97/99 |
| | Return Rate | 98% |
| | **Major trends** | **Number of submissions** |
| 1 | A desire to be a great mathematics teacher | 23 |
| 2 | A need to understand mathematics | 27 |
| 3 | Committed to overcoming a fear of maths | 22 |
| 4 | I just want to pass | 19 |



#### 4.2.2.1 POSITIVE TRENDS

Two trends are optimistic. 23 submissions indicated that in their future careers as teachers the participants want to be able to teach mathematics well to South African pupils. Comments such as, "to be able to teach it in an excellent manner," and, "making it fun for myself and others," and, "I want my students to be very good at maths because of me," serve to indicate to me that this cohort of PGCE participants are keen to improve mathematics teaching and learning in our country.

20 of the submissions make the point that the students are keen to *understand* their mathematics. The participants suggest that they want, "to acquire more understanding about it," and "I hope as a person I can know more in mathematics," and, "is to understand it and at least know it better." The nature of the writing indicates two strands

of thought. One strand reveals a fear of not coping with the mathematics, the other is a desire to understand the module course-materials.

#### 4.2.2.2 NEGATIVE TRENDS

Two trends are pessimistic. 19 submissions write directly about the fears that participants hold of the subject and of their determination to get past that fear. One student wrote, “I hope that mathematics could be taught in such a way that it creates excitement within me and not to instil fear,” while another student stated, “to have a great attitude towards it.”

Thus, it would appear that, while the submissions acknowledge fear of the subject, the participants are committed to working through and overcoming those fears. 13 submissions speak honestly about a desire to simply pass the module with students writing comments such as, “to pass,” or, “I hope to know and pass mathematics.” Contained within that trend are some students who write a bit more optimistically, and make statements of the order, “I want to pass the maths in distinction” and “to pass it at least by 70% and above” and “I will enjoy and pass it.” Collectively, the comments read more optimistically rather than pessimistically.



#### 4.2.2.3 COMMENTARY ON TRENDS FOUND IN FIELD 2

Given that Field 2 had as its heading, *My biggest hopes for mathematics*, trends 1 and 2 suggest that many participants carry a positive disposition towards this subject and a desire to teach the subject in ways that will inspire their pupils to do well in mathematics.

However, trends 3 and 4, almost as a counterpoint, suggest that students hold deep-seated fears of this subject and would be happy to simply pass it. This negative circumstance is often found in PGCE students. From the perspective of this action research, educational activities and a teaching and learning spirit which is inspired by Plato’s two realms of knowing (2.1.1.2) and the allegory of the cave (2.1.1.3) may offset these negative sentiments.

## 4.2.3 THINKBOARD – FIELD 3: MY BIGGEST FEARS FOR MATHEMATICS

The third field, **“My biggest fears for mathematics,”** garnered 93/99 or 94% submissions made by the participants. As demonstrated in Table 4.5, below, 5 major trends were revealed.

Table 4.5

*Summary of Thinkboard data: Field 3*

| THINKBOARD ACTIVITY - 10 FEBRUARY 2016 | | |
|---|---|---|
| | **Field 3: My Biggest Fears for Mathematics** | |
| 1 | Number of Participants | 33 |
| 2 | Number of Responses | 93/99 |
| 3 | Return Rate | 94% |
| | **Major trends** | **Number of submissions** |
| 1 | A deep-seated fear of calculations | 28 |
| 2 | A fear of not teaching mathematics well | 23 |
| 3 | Memories of past failures in mathematics | 15 |
| 4 | A fear of failing the IPS 413 E module | 11 |
| 5 | A fear of giving up on the subject | 9 |



### 4.2.3.1 NEGATIVE TRENDS

The data revealed five negative trends. 28 submissions write of a deep fear of mathematics. Participants worry about the calculations, the formulae and the types of mathematical problems that they will encounter in the IPS 413 E module. They ask of themselves whether they will be up to the task. Some of their comments include, “You have to know and use correct methods in order to have right answer when calculating,” and, “being lost in the steps, of never find the solution in a problem given,” and, “all those calculations that we are going to do in tests and assignments.”

23 submissions suggest the participants worry that they will not be able to teach mathematics in effective ways to South African children. Although they voice it as a concern, this trend can be treated as a compliment to the participants. None of them has any formal teaching experience and yet, looking to the future, they (1) anticipate that they will teach mathematics and (2) hope to teach it well.

The following comment captures the spirit of many views, “My biggest fear is that I am wondering if I can be able to explain it better or to view it on a way that kids can understand.” A second participant suggested, “I fear that there are many maths teachers who are negligent to the students that are struggling to grasp a lesson as quick as others.” While this comment is ambiguous, in that it might be directed either at university staff or teachers in schools, it demonstrates that the students have applied deep insight and reflective thinking in their writing.

15 submissions write to a fear that the students have they will fail this course. Comments such as, “that I'll never be good at it” and, “when I was in high school I was not good in it, so I fear that what if I can be like that again” and, “that I'll probably fail every time, like I did in high school” get to the very essence of the problem of the fear that confronts so many students. It is as if a millstone has been placed over their necks and so, in attending the IPS 413 E – Mathematics class, yet again this debilitating load will be set upon them.

11 submissions get directly to the point: “to fail” or, “I am scared of failing because I hate failing,” or, “to fail it,” or, “to fail mathematics,” and, “My biggest fear is to fail maths,” and 9 submissions stated that because of their own prejudices, boredom, not being interested, or a history of giving up too easily, they might not receive the full benefit of this module. They wrote comments such as, “I have a tendency to ‘cut off’ when I see numbers,” and, “I have no interest what so ever for mathematics.”



#### 4.2.3.2 COMMENTARY ON TRENDS FOUND IN FIELD 3

While somewhat anticipated, nonetheless, many of the comments located in Field 3 give cause for concern. That competent and able university students should hold these fears is an indictment on all of us who teach mathematics.

In large part, these trends talk to a mismatch between the philosophical, ontological, pedagogical and epistemological aspirations of this thesis and validate the liberating positions that were adopted throughout the literature review.

### 4.2.4 THINKBOARD – FIELD 4: HOW I FEEL AT THIS MOMENT

This field garnered 44/44 or 100% submissions, made by the participants. As seen in Table 4.6, below, 3 major trends were revealed.

Table 4.6

*Summary of Thinkboard data: Field 4*

| | THINKBOARD ACTIVITY - 10 FEBRUARY 2016 | |
|---|---|---|
| | **Field 4: A Drawing of how I Feel at this Moment** | |
| 1 | Number of Participants | 33 |
| 2 | Number of Responses | 33 |
| 3 | Return Rate | 100% |
| | **Major trends** | **Number of submissions** |
| 1 | So many faces | 33 |
| 2 | Happy/ambivalent/anxious/shocked/ crying/angry | 20/5/3/3/1/1 |
| 3 | Simplistic representations | mostly |

#### 4.2.4.1 TRENDS IN THE FACES

Every student drew a face to express their emotions. 14 attached a body to the face, while the rest, simply drew a head. The facial features were used to express emotion. I interpreted 20 of the faces as being drawn to express happiness and joy. Of the rest, my interpretation was that 5 students were ambivalent and/or resigned, 3 students were anxious, 3 shocked, and 1 each were either crying or angry.



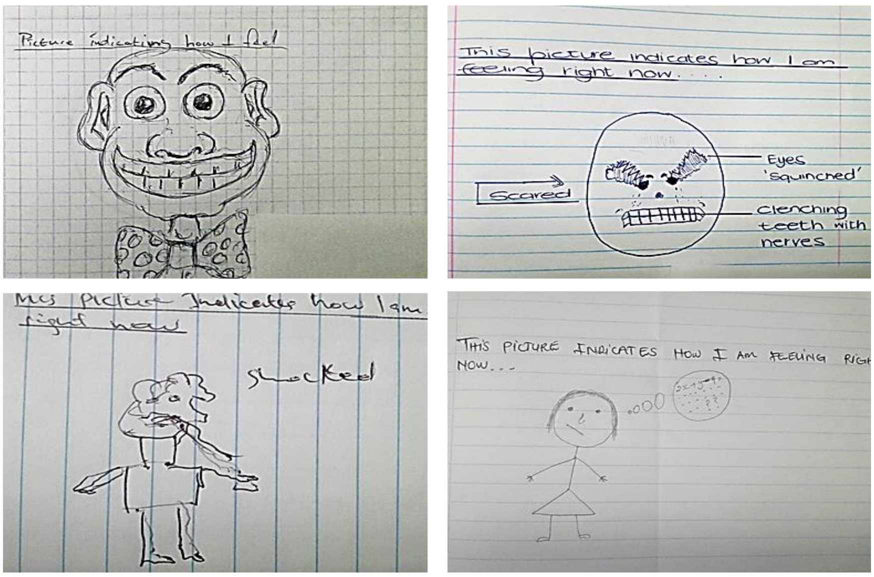



*Figure 4.12.* Sketches drawn by students in Field 4 of Thinkboard

Above, in *Figure 4.12*, I have included some of the pictures that were drawn by the students. They are indicative of the range of feelings that were coursing through the students while they were doing this Thinkboard activity.

#### 4.2.4.2 COMMENTARY ON TRENDS FOUND IN FIELD 4

I acknowledge there can be some contestation of my interpretations of the sketches. Most of the drawings are cartoon-like, simplistic and often drawn to a very low order. Largely, the sketches were unsophisticated and ill-proportioned. In my classroom observations, my sense was that very little attention to detail was considered in the compositions and that many of the participantss completed this aspects of the thinkboard just to get it done.

While about three quarters of the participants present images which suggest they are comfortable being in the IPS 413 E – Mathematics class, the remainder appear to be ill-at-ease and may feel somewhat threatened by the subject.

### 4.2.5 SUMMARY OF THINKBOARD ACTIVITY



The Thinkboard activity revealed two major themes in the participants' conceptions of mathematics and studying it in the IPS 413 E Mathematics module.

On a positive footing, the participants repeatedly reflect upon their desire to do well in this subject, to become great ambassadors of the subject, and to positively contribute to improving school pupils' success in this subject.

On a negative footing, many participants have suggested that they are fearful of mathematics. They fear failure and, in their writing, they imply that mathematics has damaged them psychologically. Some comment that they do not know how to proceed through mathematical problems and that, in as much as they feel that the subject abandoned them in favour of smarter people. They suggest that having been abandoned by the subject, they would be happy to either dispense with mathematics altogether or, hopefully, simply pass it.

## 4.3 FOUNDATIONAL ARITHMETIC CONCEPTS, ACTION CYCLE 1, PRIOR KNOWLEDGE ASSESSMENT

Between the 10th and 14th February, I marked the Foundational Arithmetic Concepts, Action Cycle 1, prior knowledge tasks, then transcribed the results into MSExcel©, and identified trends in the primary data. The task is appended (Appendix F). Four main questions were created for the evaluation. Each main question had five sub-questions. I will refer to the various main and sub-questions in the following manner, for example: Q4.2 means main question 4, sub-question 2. One mark was awarded to each sub-question. I have supplied a representative part of the primary data in Table 4.7, below. The full spreadsheet is appended (Appendix G).

Table 4.7

*Illustrative example of raw data as captured in the Foundational Arithmetic Concepts, Action Cycle 1, prior knowledge assessment tasks*

**FOUNDATIONAL ARITHMETIC CONCEPTS - ACTION CYCLE 1 - PRIOR KNOWLEDGE ASSESSMENT (10/2/2016)**

**UPDATED TO REFLECT ONLY DATA CAPTURED FROM THE PARTICIPANTS WHO WERE PRESENT FOR THE ASSESSMENT**

| | Question 1 - REMEMBERING | | | | | Question 2 - REMEMBERING | | | | | Question 3 - REMEMBERING & UNDERSTANDING | | | | | Question 4 - REMEMBERING & UNDERSTANDING | | | | | **TOTALS** |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 | **20** |
| 1 | 1 | 1 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | **7** |
| 2 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | **4** |
| 3 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | **8** |
| 4 | 1 | 1 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | **11** |
| 5 | 1 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | **6** |
| 6 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | **15** |



In each cell, 1 means that a correct answer was supplied; 0 means it was incorrect. The participants' marks are presented horizontally in rows. Each main question and its sub-questions are presented left to right, with totals per student on the far right.

I applied the revised, verb-driven, Bloom's taxonomy to the Foundational Arithmetic Concepts task. The questions were based on content-knowledge requirements for the Intermediate Phase. The questions were designed to ascertain, at the lowest level of the taxonomy, *Remembering* and one stage up, *Understanding*. Questions 1 and 2 elicit *Remembering* and Questions 3 and 4 form a bridge that links *Remembering* and *Understanding*. Questions 1 and 3 require True/False and Multiple Choice responses respectively. Questions 2 and 4 require students to show their working. Below, I will recount 10 major trends that this first prior knowledge assessment tasks has revealed to me.

### 4.3.1 TREND 1: POOR HANDWRITING AND SETTING OUT

Many students have appalling handwriting and set out worked solutions very poorly.

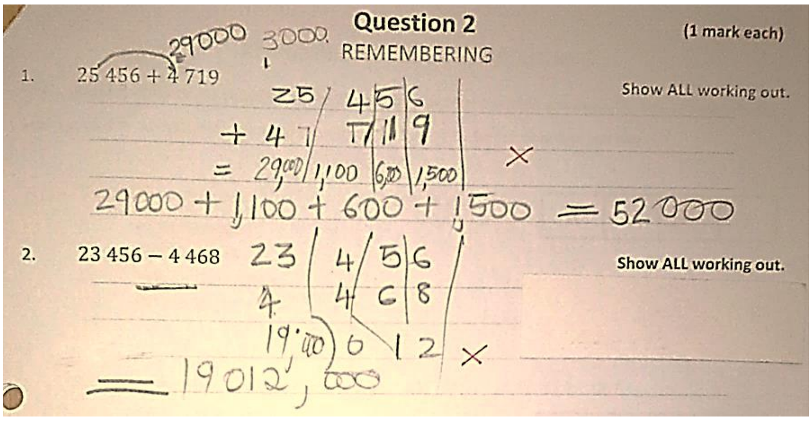



*Figure 4.13.* An example of poor setting out in the Foundational Arithmetic Concepts, Action Cycle 1, prior knowledge assessment tasks



*Figure 4.13,* demonstrates how poor writing and setting out exacerbates mathematical difficulties for struggling participants. If one discards computational ineptitude, it is clear that the poor presentation of the written solutions creates obfuscation and additional cognitive burdens for the student. Because of the visual mess, the student never gets to see the unfolding of a solution. Further, because of the poor setting out, the student does not create an opportunity to judge the feasibility of the answer.

### 4.3.2 TREND 2: LOW PROCEDURAL KNOWLEDGE

Generally, each student did best in Questions 1 and 3, with adjacent drops in Questions 2 and 4 respectively. In the sample piece of the spreadsheet provided above, (Table 4.2), participant 4 demonstrates this trend, having scored 4/5 and 4/5 for Questions 1 and 3 respectively, and then scored 2/5 and 1/5 for Questions 2 and 4, respectively. In Questions 1 and 3, the subtle cues found in true/false and multiple-choice questions guide students towards best guesses. This can skew marks upwards. However, this benefit is not available in Questions 2 and 4.

Many students performed poorly in *single-operations* calculations in Main Question 2:

1. ***Q 2.1: 25 456 + 4 719*** Only 27/34 or (79%) of the students correctly added a five digit and four digit number together.
2. ***Q 2.2: 23 456 – 4468*** Only 14/34 or (41%) correctly subtracted a four digit number from a five digit number.
3. ***Q 2.3: 2 658 × 52*** Only 8/34 or (24%) successfully multiplied a four digit by a two digit number
4. **Q 2.4: 5 392 ÷ 8** Only 3/34 or (9%) of the students achieved the correct quotient.

Although the data above shows poor returns, Table 4.8, below, demonstrates that the operations questions found in Main Question 2 are typical of exercise-calculations offered to Grade 4, 5 and 6 pupils and are well within the prescribed minimum requirements for the Intermediate Phase. Thus, it is of great concern that the participants fared badly in these routines-based tasks.



Table 4.8

*Content Knowledge requirements for Intermediate Phase. Source: Department of Basic Education, 2011, p. 9*

| **Numbers, Operations and Relationships**<br>**Topic 1.1 – Whole Numbers** | | |
|---|---|---|
| Grade 4 | Grade 5 | Grade 6 |
| Number Range for Calculations | Number Range for Calculations | Number Range for Calculations |
| Addition and subtraction of whole numbers of at least 4 digits | Addition and subtraction of whole numbers of at least 5 digits | Addition and subtraction of whole numbers of at least 6 digits |
| Multiplication of at least whole 2-digit by 2-digit numbers | Multiplication of at least whole 3-digit by 2-digit numbers | Multiplication of at least whole 4-digit by 3-digit numbers |
| Division of at least a whole 3-digit by 1 digit number | Division of at least a whole 3-digit by 2 digit number | Division of at least a whole 4-digit by 3 digit number |
| | | Multiple operations on whole numbers with or without brackets |

## 4.3.3 TREND 3: MAINLY REMEMBERING SKILLS

Most students' scores dropped as they delved deeper into the task. The trend showed strong *Remembering* competence contrasted with low *Understanding*. Table 4.9, below, will demonstrate this correlation vividly.

Table 4.9

*Summary of data captured in the Foundational Arithmetic Concepts, Action Cycle 1, prior knowledge assessment tasks*

| ACTION CYCLE 1 - PRIOR KNOWLEDGE 34 PARTICIPANTS | Question 1 - REMEMBERING | | | | | Question 2 - REMEMBERING | | | | | Question 3 - REMEMBERING & UNDERSTANDING | | | | | Question 4 - REMEMBERING & UNDERSTANDING | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ARITHMETIC CONCEPTS | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 |
| Correct responses per Sub- Question | 32 | 25 | 16 | 14 | 20 | 27 | 14 | 8 | 3 | 9 | 3 | 29 | 6 | 11 | 16 | 8 | 3 | 1 | 8 | 1 |
| Sub-question Averages (%) | 94 | 74 | 47 | 41 | 59 | 79 | 41 | 24 | 9 | 26 | 9 | 85 | 18 | 32 | 47 | 24 | 9 | 3 | 24 | 3 |
| Correct responses per Main Question | 107 | | | | | 61 | | | | | 65 | | | | | 21 | | | | |
| Main Question Averages (%) | 63 | | | | | 36 | | | | | 38 | | | | | 12 | | | | |

In the 'Correct responses per Sub-Question' row, I have provided totals of the correct responses per sub-question. Thus, for example, for **Question 1 – Remembering**, the sub-question scores are 32/34; 25/34; 16/34; 14/34 and 20/44 respectively. Below these raw scores, I have calculated their respective percentage values.



Further, referring to **Question 1 – Remembering**, the students supplied 107/170 correct responses for the five sub-questions: this translates into a 63% correct response rate. Reading left-to-right across the table, by the time we get to **Question 4 – Remembering and Understanding**, the correct responses for the entire class crashes to 21/170 or 12%.

## 4.3.4 TREND 4: LACK OF COMPRESSION

Sub-question Q1.1 required participants to identify the acronym BODMAS as being a mnemonic device that is used to sequence solutions of multi-operations calculations. 32/34 participants or 94% of the participants achieved the correct answer. Two questions later, in Q1.3, they were required to provide a True/False response to the question:

> **Q1.3:** ***In the calculation: 19 + 16 x 17 – 15, the first step would be to add the 19 and 16 together.***

Only 16/34 or 47% of the participants provided the correct answer (False). Thus, comparing data from Q1.1 and Q1.3, we learn that albeit that the BODMAS mnemonic is used to solve Q1.3, 23 of the students who correctly answered Q1.1, also *incorrectly* answered Q1.3. Further, 2 students incorrectly answered Q1.1, but *correctly* answered Q1.3. The inconsistency between Q 1.1 and Q1.3 suggests that many students hold fragile and discontinuous mental constructs of the BODMAS mnemonic. If we treat Q1.1 and Q1.3 as a truth-test pairing, we must accept that participants lack compression of BODMAS and may have guessed answers.

### 4.3.5 TREND 5: LACK OF FOUNDATIONAL KNOWLEDGE OF BASES AND POWERS

Sub-questions Q1.2, Q1.4, Q3.1 and Q4.3 test the students' recall and understanding of bases and powers. The students demonstrated low recall and understanding of this foundational mathematical prerequisite.

1. **Q1.2:** ***In the pattern, 1; 4; 9; 16; 25, ____; ____; etc., the next two numbers are 34 and 43*** **(T/F).**

   25/34 or 74% of the students answered the question correctly.

2. **Q1.4:** ***$4^3 = 12$*** **(T/F)**

   14/34 or 41% of the students answered the question correctly.

3. **Q3.1:** ***The number sequence: 1, 3, 9, 27, 81, etc., is a list of:***

   ***A. Prime Numbers;***

   ***B. Perfect Cubes;***

   ***C. Powers of a number;***

   ***D. Perfect Squares.***

   3/34 or 9% of the students answered the question correctly

4. **Q4.3:** ***If the area of a square is 169cm$^2$, what is the perimeter of the square?***

   1/34 or 3% of the students answered the question correctly.

### 4.3.6 TREND 6: A FRACTIONS META-CONCEPT IS MISSING

In main question 3, Q 3.4 required students to select one of four options:

**Q 3.4:** ***As a number value, $2\frac{1}{4}$ is:***

***A. smaller than 220%;***

***B. equal to $\frac{9}{4}$;***

***C. greater than 2,3;***

***D. none of the above answers is correct.***

The correct response is B. Only 11/34 or 32% of the students answered the question correctly. This raises at least two red flags. Firstly, had nomenclature such as $<$; $=$; $>$ , been introduced, the results might have been shifted even more downwards. Secondly, meta-concepts hold together the fabric of many smaller, intersecting concepts. A meta-concept of fractions would contain visualizations of fractions, equivalence, vocabulary, procedures, etcetera. The incorrect answers for Q3.4 suggest that compression and the construction of a robust fractions meta-concept are not in place.



### 4.3.7 TREND 7: AN INABILITY TO VISUALLY REPRESENT A FRACTION

In Q 4.4 only 8/34 or 24% of the students could correctly shade in $\frac{5}{8}$ of a large square which contained 16 smaller, congruent squares. It is difficult at this early stage of the research to provide more than a hunch regarding the answers submitted for this 'seemingly easy' question. Where compression has not taken place, configuration of the fraction $\frac{5}{8}$ might lead students to shade in 5 squares, (based on the value of the numerator), and indeed, six students did do just that.

Also, in the absence of compression, an 'educated' guess might suggest that one should fill in thirteen squares – thereby leaving 3 open squares, (that is, the missing 3 parts in $\frac{5}{8}$) (maybe) and indeed, five students did so.

Many of the answers demonstrate no application of understanding of fractions and their visual representations. Three examples of the submissions are presented in *Figure 4.14*, below: of these, possibly the top-right image is most interesting. It appears that the student identified 5 parts (of 8) in black, three parts (of 8) in white, and then in grey, 8 parts of 8. In effect, the student might be seeing the large square as representing two wholes.

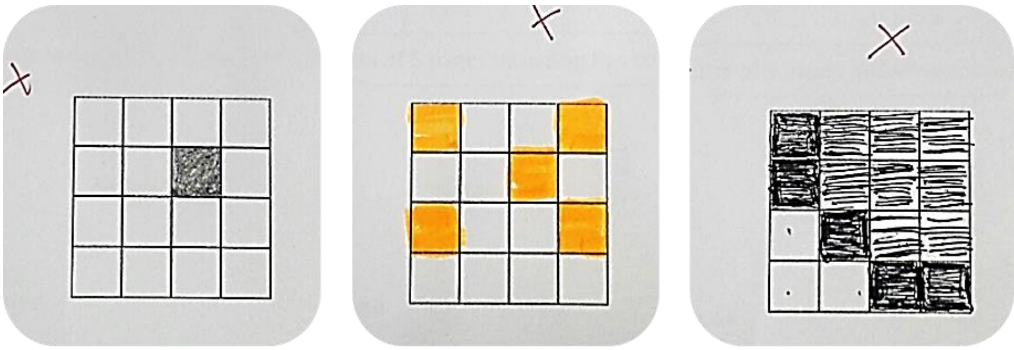


*Figure 4.14.* Students' attempts to answer Q4.4 in the Foundational Arithmetic Concepts, Action Cycle 1, prior knowledge assessment task



But many of the other answers, where the shading of squares included, for example 1 or 14 squares shaded, etcetera, suggest a much bigger misunderstanding of symbolic value of $\frac{5}{8}$ and visual representations.

## 4.3.8 TREND 8: A RATIOS META-CONCEPT IS MISSING

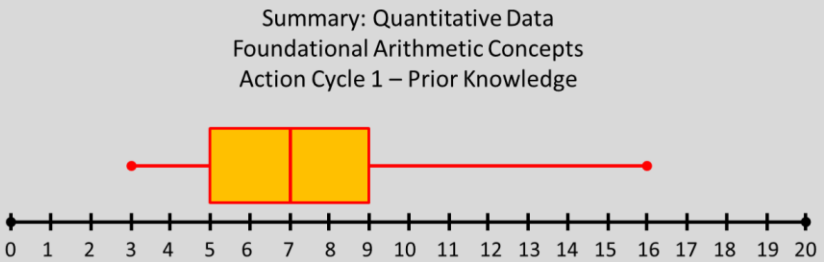
Q 4.5 is a ratios-based question. The question itself reads:

> ***Q 4.5: A stick which is 2,5m long is cut into three parts using the ratio 2:3:5. How long are each of the three parts?***

Only 1/34 or 3% of the participants managed to correctly answer this question. To be fair, two students correctly achieved the *numerical* answers of 0,5; 0,75 and 1,25 but these students left out the units, m. I felt that at university level, the insertion of units should be a prerequisite so I marked the answers as incorrect. This *understanding* question is quite easy to solve if the compression of fractions and ratios concepts has occurred. Clearly, that has not happened. However, the poor response rate amplifies the problem with fractions, outlined in TREND 6, above.

## 4.3.9 SUMMARY OF QUANTITATIVE DATA FOUND IN FOUNDATIONAL ARITHMETIC CONCEPTS, ACTION CYCLE 1, PRIOR KNOWLEDGE ASSESSMENT

Many students may not have studied mathematics for a while; this might explain why many performed poorly in the task. The results suggest that much procedural and skills-based remedial work will have to be undertaken. Foundational concepts and skills-sets need to be put in place so as to generate understanding-filled intersections between visual and symbolic representations. Below, in *Figure 4.15*, I reveal the overall picture of the test. I have used a box and whisker plot because I believe that (1) it is a visually interesting graph; (2) it is rich in data, and (3) it is easy to read.




| Quantitative Data | |
|---|---|
| Date of Test | 10/2/2016 |
| Number of submitted Scripts | 34 |
| Number who did not write | 4 |
| TOTAL | 38 |
| Mean | 7 |
| Mode (6 occurrences) | 4 |
| Median | 7 |
| Range (16-3) | 13 |
| Quartiles | 5; 7; 9 |
| Interquartile Range (9-5) | 4 |

*Figure 4.15.* Summary of data capture in Foundational Arithmetic Concepts, Action Cycle 1, prior knowledge assessment tasks.

The box-and-whisker plot and data-table reveal that one quarter of the students achieved scores of 3/20 through 5/20, that is, the red whisker on the left side of the plot. Half of the students, those found between quartiles 1 and 3, scored at between 5/20 and 9/20, that is, the two boxes. The top quarter of the students achieved marks of 9/20 through 16/20, that is, the right whisker. There were 34 participants; the mean was 7/20; the mode was 4; the median was 7; the range of marks was between 3 and 16 (range 13); the quartile indicators were found at 5, 7, and 9 respectively; and the interquartile range ran between 5 and 9, that is, an interquartile range of 4.

Thus, as a part answer to the first sub-research question: *What existing word-problem, problem-solving strategies do the students hold?* it is clear that many of the foundational knowledge and skills sets which are used by students to solve word-problems are not in place. On Bloom's Revised Taxonomy, at an understanding level, the students falter badly. Indeed, already, it is clear that meta-concepts of fractions (4.3.4 and 4.3.6), of bases and powers (4.3.5) and ratios (4.3.8) are not in place. In other words, compression of these mathematical ideas has not occurred.

## 4.4 WORD-PROBLEM CALCULATIONS, ACTION CYCLE 1, PRIOR KNOWLEDGE ASSESSMENT TASK

On 22 and 23 February, I marked, captured and collated the marks for the Word-Problem Calculations, Action Cycle 1, prior knowledge assessment tasks (Appendix H). Below, I disclose important trends that were found in the tasks.

### 4.4.1 TREND 1: POOR HANDWRITING AND SETTING OUT

Congruent with the reflections for the Foundational Arithmetic Concepts, Action Cycle 1 prior knowledge assessments, poor handwriting and untidy setting-out seem to conspire against success for many students.



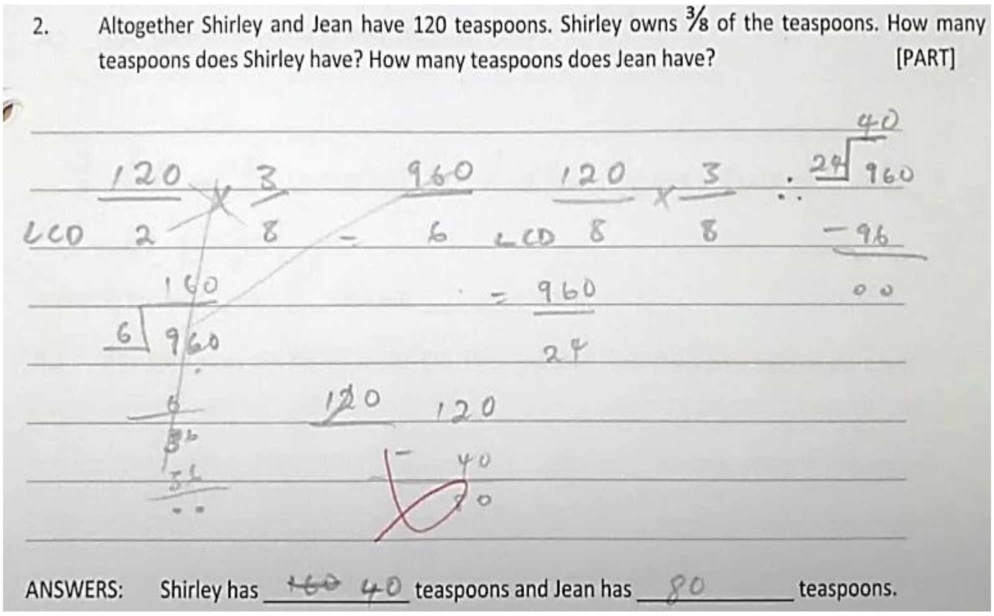



*Figure 4.16.* An unsuccessful attempt to solve Question 2 in the Word-Problem Calculations, Action Cycle 1, prior knowledge assessment tasks

A Question 2 solution, as presented in *Figure 4.16,* above serves as an exemplar of the phenomenon. The messiness in Figure 4.16 indicates more than simply poor setting out capability; it points to a lack of cogence, conceptualisation and visual clarity. The presentation indicates that mal-formed concepts and misunderstandings underpin the construction of the incorrect solution for this low-order word-problem.

The student is unable to engage confidently with the problem. The initial starting point of 120 multiplied by $\frac{3}{8}$ looks favourable, but then, mysteriously, the 120 acquires a denominator of 2 with the abbreviation LCD (lowest common denominator?) written to the far left. I write *mysteriously* because often one is left wondering from where students conjure up the information they put on paper. The 8 (not the 3) is then multiplied with the 120, generating 960, and the 960 is divided by 6, which is the product of 2 and 3. And so it continues… Arithmetically, 8 x 120 = 960, and 2 x 3 = 6. However, procedurally, *the computation is wrong.* I get a sense that the underpinning long-term memories of the routines that are required to solve this arithmetic problem are not in place. I also have a hunch that, intuitively, the student perceived that the correct answers would be 40 and 80 respectively, and then tried to construct a solution strategy that would fit that pairing of incorrect answers.



Indeed, many other students have supplied poorly constructed and ill-conceptualised solutions. As suggested by Bukatko and Daehler (1995, p. 348-9), they too seem unable to represent their problems in ways that enable them to plan and implement a successful path to follow.

The solutions for all the questions in the Word-Problem Calculations prior knowledge assessment tasks sit comfortably within Zanele's (2015), conception of highly structured problems. Such problems are said to invoke low cognitive demands, follow standardised routines, and generate single, right answers. They are *standard fare* exercise questions. Largely, is seems that the mistakes can be attributed to a lack of compression (Gray and Tall, 2007), The poor quality of the setting-out and construction of solutions and answers – as manifestations of their thinking – support Gray and Tall's (2007) view that many students failed to construct *thinking* meta-concepts of the problems they encountered. Rather, they seem to have pulled up into working memory fragmented and disjointed sprinklings of ill-formed long-term memories of mathematical processes and procedures. It appears that they then tried to weave all of this together – they fudged it together – in the vain hope the central mass of their creation would hold. But, of course, it did not.

### 4.4.2 TREND 2: LOW LEVELS OF BLOOM'S UNDERSTANDING, APPLYING AND ANALYSING CAPABILITIES

Bloom's Revised Taxonomy was used to construct the questions in the Word-Problem Calculations prior knowledge assessment tasks. Questions 1 through 5 were designed to elicit *Understanding and Applying.* Questions 6 through 10 were constructed to investigate *Applying and Analysing*.

Table 4.10

*Summary of data captured in the Word Problems Calculations, Action Cycle 1, prior knowledge assessment tasks*

| ACTION CYCLE 1 - PRIOR KNOWLEDGE<br>37 PARTICIPANTS<br>WORD PROBLEM CALCULATIONS | UNDERSTANDING AND APPLYING | | | | | | APPLYING AND ANALYSING | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | | 6 | 7 | 8 | 9 | 10 |
| **Correct responses per Sub Question** | **71** | **19** | **41** | **52** | **66** | | **9** | **18** | **64** | **64** | **56** |
| **Sub-question Averages (%)** | **64** | **17** | **37** | **47** | **59** | | **8** | **16** | **58** | **58** | **50** |
| **Correct responses per Main Question** | **249** | | | | | | **211** | | | | |
| **Main Question Averages (%)** | **45** | | | | | | **38** | | | | |



The data-table, Table 4.10 above, reveals that 37 participants completed the test (Appendix I). The class achieved 249/555 correct responses or 45% for the first five questions. The return for Questions 6 - 10 was lower, at 211/555 or 38%. Hindsight shows that Questions 8, 9 and 10 really belonged in the first category in the test, namely Understanding and Applying. Thus, Table 4.10, shows that my questions actually skewed the marks in questions 6 – 10 *upwards* quite a bit.

Overall, all the word problems achieved relatively low returns. The high point was a 64% return for Sub-Question 1, but even that must be reflected upon in the light that all of the questions were low-level Intermediate Phase arithmetic word problems. Table 4.9 also reveals the skewing effect in Questions 8, 9 and 10. Further, the table reveals that Questions 2, 6 and 7, achieved particularly low returns; next, I will specifically turn to these problematic questions.

## 4.4.3 TREND 3: INTERESTING RESPONSES TO QUESTION 2

Question 2, is transcribed below:

> **Q 2. Altogether Shirley and Jean have 120 teaspoons. Shirley owns ⅜ of the teaspoons. How many teaspoons does Shirley have? How many teaspoons does Jean have? (3 marks)**

Mathematically speaking, the participants had to establish the numerical values of $\frac{3}{8}$ and $\frac{5}{8}$ of 120, namely 45 and 75 respectively. 30/37 or 81% of the respondents scored 0 for this 3 mark question. 1/37 or 3% of the class scored 1 mark. The remaining 6/37 or 16% of the students scored the full three marks. In other words, the students either *got it*, or they did not. Among those students who did not achieve three marks for Question 2, three categories of answers appeared. Next, I will unpack general features that I found within these categories of incorrect answers.



### 4.4.3.1 CATEGORY 1: MESSAGES

11 students did not provide a calculated answer for Question 2. 8 of these students did write short messages in the answer area for Question 2. As per the exemplar, *Figure 4.17,* below, such students suggested that they struggled to do calculations that incorporated operations and fractions.

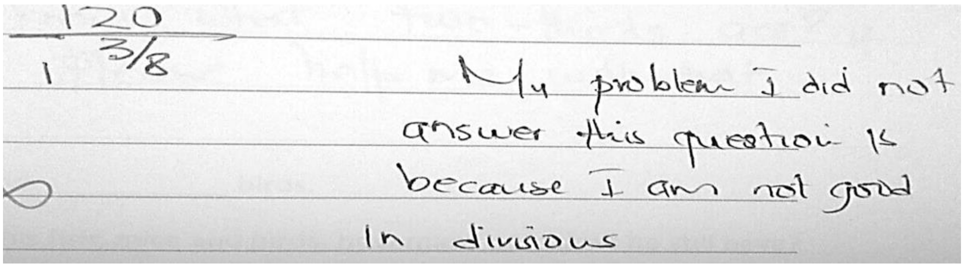



*Figure 4.17.* A participant’s response to Question 2 in the Word Problems Calculations, Action Cycle 1, prior knowledge assessment task

### 4.4.3.2 CATEGORY 2: PROCEDURAL INEPTITUDE

Many participants produced answers that, while incorrect, at least generated a plausible answer in that the two parts of their answers added up to 120 teaspoons. Some suggested the teaspoons were divided equally, that is 60 and 60 to each lady, while others offered 40 and 80 as the split – in effect $\frac{1}{3}$ and $\frac{2}{3}$ respectively. Yet other participants offered 30 and 90 or $\frac{1}{4}$ and $\frac{3}{4}$ respectively, and other answers included 100 and 20; 5 and 115, 65 and 55, and 49 and 71.

*Figure 4.18*, below, is interesting because if the student had constructed groups of 8 rather than 5, the visual solution had the potential to work. The group-bundles of 8 might then be redistributed into new, larger sets of 3 and 5 that would be tallied up to achieve the correct answers.

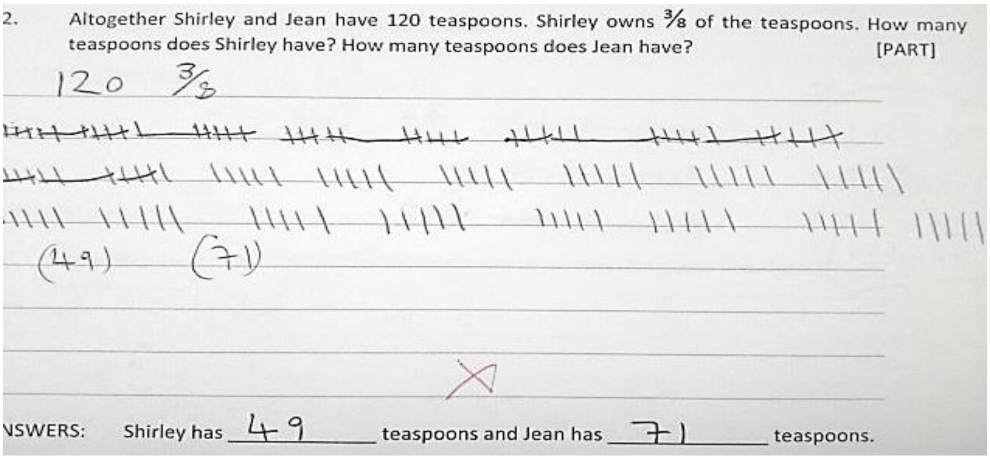



*Figure 4.18.* A student's unsuccessful visual solution to Question 2 in the Word Problems Calculations, Action Cycle 1, prior knowledge assessment

Albeit these participants displayed a sense of what to do, procedurally and/or conceptually, they could not create appropriate algorithms to achieve correct answers. Question 2 required the students to understand and apply previously learned skills, concepts and routines, learned while in school; in effect, they were required to display aspects of instrumental and relational understanding (Siemon et al., 2012, p. 71), but clearly they struggled to do so.

When held up for examination, answers of the order of 5 and 115 cannot be conceived to be representative of $\frac{3}{8}$ and $\frac{5}{8}$ of 120: such answers suggest that many participants did not perceive the *equivalence* of a fractional unit for a given whole, a given denominator and its proportional size.

On many occasions I have come across a misunderstanding of the roles of numerators and denominators in common fractions. The misunderstandings play out clearly in activities where students are required to construct fractions walls. On occasion, students will construct, for example, an *eighths* fraction-ribbon, but subdivide it into seven parts. On other occasions, students will create eight sub-divisions but the cells will be unevenly distributed by size. Nor are these mistakes, in my experience, simply a result of mere carelessness and/or untidiness. At a very foundational level, a very basic conceptual framework of common fractions, and the *equivalence of congruent parts* is not in place.

### 4.4.3.3 CATEGORY 3: IMPLAUSIBLE ANSWERS

Some participants provided implausible answers such as 60 and 40. Here, two problems manifest. First, the sum of 60 + 40 ≠ 120. Second, in terms of the sharing, Shirley - the first-mentioned name - owns fewer teaspoons than Jean. Also, one student supplied 94 and 60, and another suggested the split was 40 and 40. Albeit that the salient mathematical competences were unavailable to these participants, they did perceive *sharing* occurred between the two ladies. However, in the absence of understanding and compression, they did not comprehend that the sum of the two parts had to add up to the original total of 120 teaspoons.



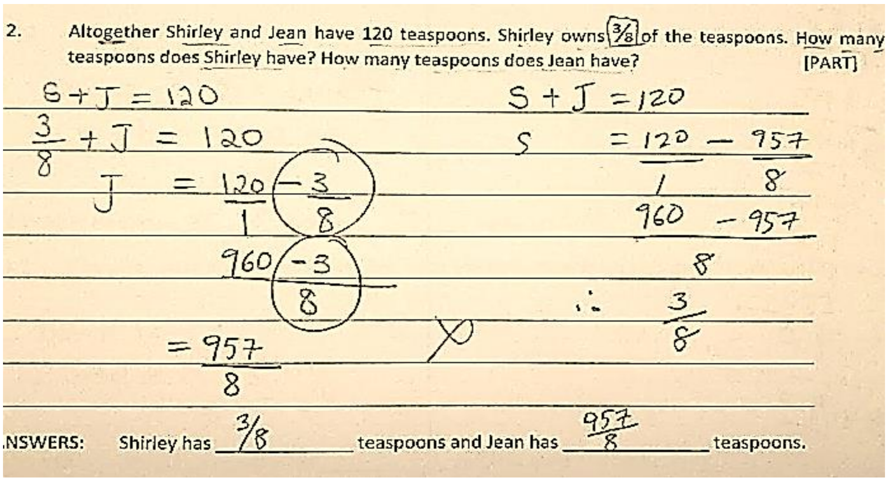



*Figure 4.19.* An incorrect solution to Question 2 in the Word Problems Calculations, Action Cycle 1, prior knowledge assessment task

The remaining students in this category demonstrated a deep misunderstanding of the task at hand. The solution in *Figure 4.19*, above, demonstrates that, in this problem at least, the participant operated off a deeply flawed misunderstanding of mathematical processes and procedures. Six students correctly answered Question 2, and, as seen in *Figure 4.20,* below, three of them used three used visual tools. Each of these solutions demonstrates a deep grasp of the concept of fractions as *equal* parts of a whole. It is worth mentioning that, in other answers to other questions, some students used a visual scaffold to support their thinking, but this practice was not commonly found; indeed, the overall use of visual strategizing is sporadic.

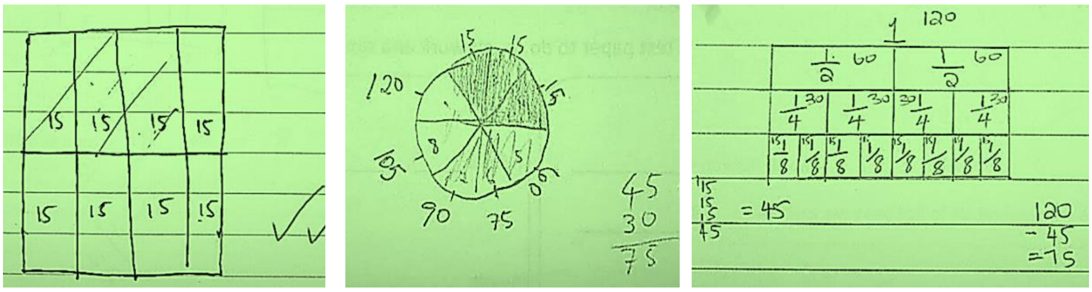



*Figure 4.20.* Three visual scaffolds, as constructed by the PGCE students, when answering Question 2 in the Word Problems Calculations, Action Cycle 1, prior knowledge assessment task



### 4.4.4 TREND 4: LANGUAGE AS A BARRIER

Question 6 reads:

> ***Q 6. Altogether, Lulu and Irene have sixty new books. Irene has half the number of books that Lulu has. How many books does each lady have?***

Only 3/37 or 8% of the students got the correct answer. All the others got it completely wrong. Previous experiences with this sort of question have taught me that often the poor performance lies with the wording of the question. Many students are confounded by the subtleties and turns of phrase found in the English language.

Thus, it became quite clear that these words and their impact on our thinking would have to be reviewed. Words such as *altogether* and the implied meaning in the phrases such as, *has half the number of*, would have to be carefully unpacked. Nor, with skilful adjustment, might/should one avoid the use of these terms – they are everyday, operationalised terms which are used in countless applications in mathematics. However, once explained and consolidated into their schema of this type of question, I anticipated that a positive turnaround would be effected for the research participants.

Drilling into the answers to the question, most of the cohort provided 30 and 30 as the two answers, the sum of these numbers returning us to 60 books. In these cases, it is conceivable that text was incorrectly understood to suggest the *Irene had half of the total amount*. Other participants provided 45 and 15 as their answers. Yet other students provided answers which, like in my exposition for Question 2 (4.4.3.3), suggested deeper problems were in play. Some participants provided 60 and 30, and other answers included 30 and 90; 30 and 15; 3 and 3; 60 and $\frac{1}{2}$; and 90 and 80.

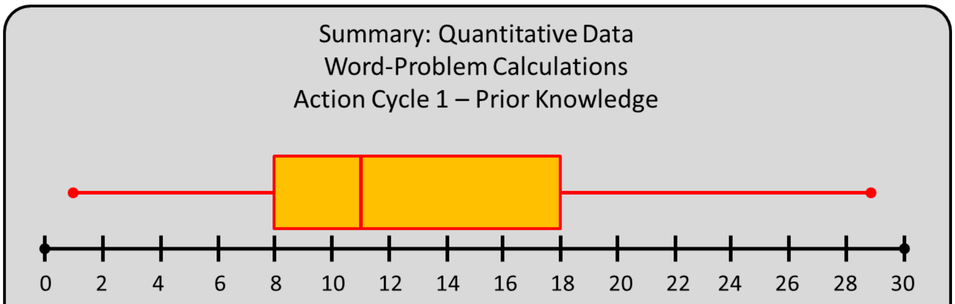
### 4.4.5 SUMMARY OF QUANTITATIVE DATA FOUND IN WORD-PROBLEM CALCULATIONS, ACTION CYCLE 1, PRIOR KNOWLEDGE ASSESSMENT TASKS



Four major trends were discovered. Collectively, inability to set out word problems (4.4.1); limited ability to solve *understanding* based word problems and low ability to solve *applying* and *analysing* problems (4.4.2); manifestations of Gooding's (2009) barriers as found in Question 2 (1.2.2.3 and 4.4.3); and language as a barrier (4.4.4) suggest that adjoined to the findings for the Foundational Arithmetic Concepts, Action Cycle 1, prior knowledge assessment tasks, (4.3), a full answer emerges to the sub-research question: *What existing word-problem, problem-solving strategies do the students hold*?

It seems that, albeit many dormant skills may simply require a re-awakening, at the beginning of the IPS 413 E module in 2016 most members of the cohort held only scatterings of remembered foundational mathematical routines and, typically, were unable to apply these skills to LOT, highly structured word problems. With few exceptions, no evidence of cogent strategizing was presented.

The Box and Whisker plot, *Figure 4.21*, below, demonstrates a large variance between lowest performers (1/30) and highest performers (29/30) in the test.




| Quantitative Data | |
|---|---|
| Date of Test | 22/2/2016 |
| Number of submitted Scripts | 37 |
| Number who did not write | 1 |
| TOTAL | 38 |
| Mean | 12 |
| Tri-Modal (3 occurrences each) | 8; 11; 19 |
| Median | 11 |
| Range (29-1) | 28 |
| Quartiles | 8; 11; 18 |
| Interquartile Range (18-8) | 10 |

*Figure 4.21.* Summary of data capture in Word-Problem Calculations, Action Cycle 1, prior knowledge assessment tasks



The lowest performing quarter of the cohort scored 1/30 through 8/30. Half of the class scored 8/30 through 18/30, that is, between quartiles 1 and 3. The top quarter of the class achieved marks between 18/30 and 29/30. The class mean was 12 and the median was 11.

Next, I will discuss the data that was captured in the Foundational Arithmetic Concepts, Action Cycle 1, re-evaluation tasks.

## 4.5 FOUNDATIONAL ARITHMETIC CONCEPTS, ACTION CYCLE 1, RE-EVALUATION TASKS

### 4.5.1 TREND 1: SMALL IMPROVEMENTS

Below, in Tables 4.11 and 4.12 respectively, I have supplied summaries of quantitative data for the Action Cycle 1, Foundational Arithmetic Concepts, Prior knowledge and re-evaluation tasks. The full data-set is appended (Appendix J)

Table 4.11

*Summary of data captured in the Foundational Arithmetic Concepts, Action Cycle 1, prior knowledge assessments*

| ACTION CYCLE 1 - PRIOR KNOWLEDGE 34 PARTICIPANTS ARITHMETIC CONCEPTS | Question 1 - REMEMBERING | | | | | Question 2 - REMEMBERING | | | | | Question 3 - REMEMBERING & UNDERSTANDING | | | | | Question 4 - REMEMBERING & UNDERSTANDING | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 |
| Correct responses per Sub- Question | 32 | 25 | 16 | 14 | 20 | 27 | 14 | 8 | 3 | 9 | 3 | 29 | 6 | 11 | 16 | 8 | 3 | 1 | 8 | 1 |
| Sub-question Averages (%) | 94 | 74 | 47 | 41 | 59 | 79 | 41 | 24 | 9 | 26 | 9 | 85 | 18 | 32 | 47 | 24 | 9 | 3 | 24 | 3 |
| Correct responses per Main Question | 107 | | | | | 61 | | | | | 65 | | | | | 21 | | | | |
| Main Question Averages (%) | 63 | | | | | 36 | | | | | 38 | | | | | 12 | | | | |

Table 4.12

*Summary of data captured in the Foundational Arithmetic Concepts, Action Cycle 1, re-evaluation assessments*

| ACTION CYCLE 1 - RE-EVALUATION 38 PARTICIPANTS ARITHMETIC CONCEPTS | Question 1 - REMEMBERING | | | | | Question 2 - REMEMBERING | | | | | Question 3 - REMEMBERING & UNDERSTANDING | | | | | Question 4 - REMEMBERING & UNDERSTANDING | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 |
| Correct responses per Sub- Question | 36 | 31 | 37 | 33 | 31 | 33 | 31 | 21 | 15 | 19 | 3 | 25 | 22 | 22 | 27 | 11 | 14 | 12 | 23 | 3 |
| Sub-question Averages (%) | 95 | 82 | 97 | 87 | 82 | 87 | 82 | 55 | 39 | 50 | 8 | 66 | 58 | 58 | 71 | 29 | 37 | 32 | 61 | 8 |
| Correct responses per Main Question | 168 | | | | | 119 | | | | | 99 | | | | | 63 | | | | |
| Main Question Averages (%) | 88 | | | | | 63 | | | | | 52 | | | | | 33 | | | | |

Table 4.11 and Table 4.12, above, demonstrate that quantitatively, microgenisis has occurred and that the impact of the teaching and learning activities in Action Cycle 1 were quite successful. With the exceptions of Q3.1 and Q3.2 which achieved reduced success rates in the re-evaluation, all the other main and sub-questions enjoyed positive growth. In Main Question 1, the overall class average moved from 63% to 88%, a jump of 25% or, in effect, a 40% improvement on the original result. Main Question 2, with 36% as a class average in the prior knowledge tasks, the average moved to 63%, a jump of 27%, or a 75% improvement in the re-evaluation. For Main Question 3, with 38% achieved in the prior knowledge assessment, the re-evaluation moved to 52%. This was a jump of 14%, or an overall improvement of 37%. For Main Question 4, the prior knowledge came in at a class average of 12%, and in the re-evalaution it climbed to 33%, a positive jump of 21%, or 275% improvement.



In the re-evaluations, moving from Main Question 1 through 4, the overall percentages dropped from 88% to 63% to 52% to 33%. This was consistent with the increasing *asking* complexity of the questions. Main Question 1, with success returns for the five sub-questions of 95%, 82% 97%, 87% and 82% seemed well understood. However, in the other main questions, some arithmetic skills continued to cause trouble.

## 4.5.2 TREND 2: LOW PROCEDURAL KNOWLEDGE

Many participants are precluded from success because they cannot compute foundational arithmetic operations tasks; they lack the procedural knowledge for this work. The underlying problems include a lack of knowledge of the times-tables, of how to set out solutions and an inability to easily apply often-used routines: evidence of this is presented in *Figure 4.22*, below.

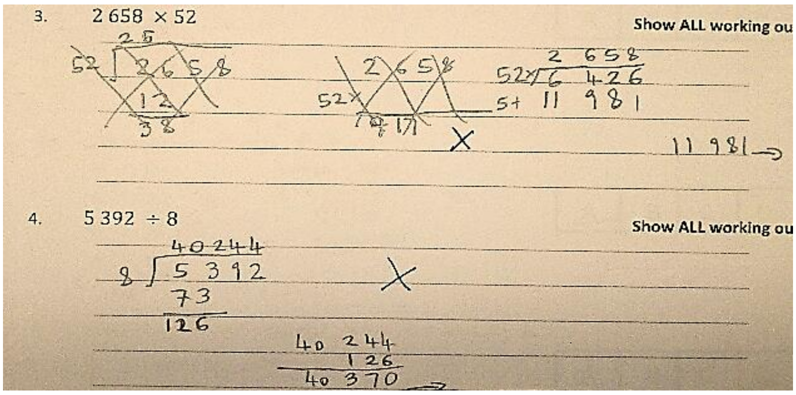



*Figure 4.22.* Evidence of poor setting out and misapplication of arithmetic operations routines



Q2.3 and Q2.4 were low-order, routines-based calculations. For Q2.3, of which *Figure 4.22,* above, is an exemplar, the prior knowledge tasks returned 8/34 or 24% while the pre-evaluation returned 21/37 or 55% success. Further, in the prior knowledge task, only 3/34 or 9% of the cohort managed to complete Q 2.4 correctly. The re-evaluation return was 15/37 or 39% which was a significant jump, but at 39% was still a low performing question.

## 4.5.3 TREND 3: INABILITY TO WORK WITH MIXED OPERATIONS

In the prior knowledge assessment, the participants returned a 9/34 or 26% success rate while, in the re-evaluation, the same question returned 19/38 or 50%. The re-evaluation result, applied as it was after a great deal of teaching and learning, was somewhat disappointing. Q 2.5 was a routines-based calculation.

However, largely, the participants failed to conceptualise the order of operations nor could they apply the BODMAS routine to the calculation.

Q 2.5, is transcribed: ***Q 2.5: Simplify: 7(16 + 9 – 4 (3 + 2))***

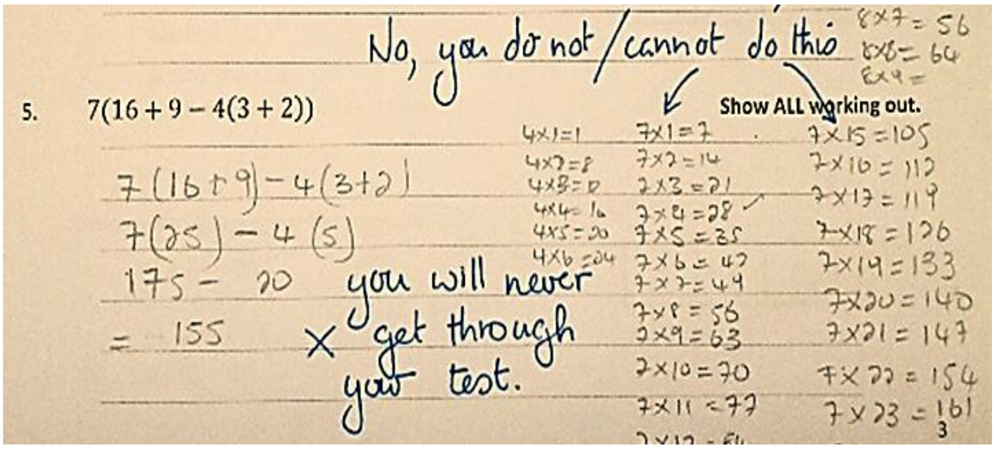



*Figure 4.23.* Evidence of lack of compression

As per the exemplar, *Figure 4.23*, above, many participants constructed the times-tables on paper, as a strategy for dealing with multiplication. The technique is ill-effective and time-consuming. It exhausts cognitive space and reduces the students' capacity to focus on a strategy for simplifying the arithmetic expression. In *Figure 4.23*, subsumption of low-order but important routines and skills did not take place, compression did not occur, and the participant quickly made computational errors.



## 4.5.4 TREND 4: LACK OF COMPRESSION OF POWERS AND BASES

Like the other four sub-questions in Main Question 3, Q 3.1 provided a statement with a choice of four possible solutions attached to it. Q 3.1 is transcribed below:

***Q 3.1 The number sequence: 1, 3, 9, 27, 81, etcetera, is a list of:***

***A. Prime Numbers***
***B. Perfect Cubes***
***C. Powers of a Number***
***D. Perfect Squares***

| | | | |
|---|---|---|---|
| A | B | C | D |

In the first quarter of 2016, that is when the first action cycle was in play, the lessons presented to the participants were saturated in activities, explanations and discussions which were aimed at assisting the students to assimilate basic number concepts and number patterns, but, clearly, they offered little benefit. In the prior knowledge assessment, the class scored 3/34 or 9%; in the re-evaluation, 3/38 or 8%. Albeit marginal, *the students did worse after instruction than before it*.

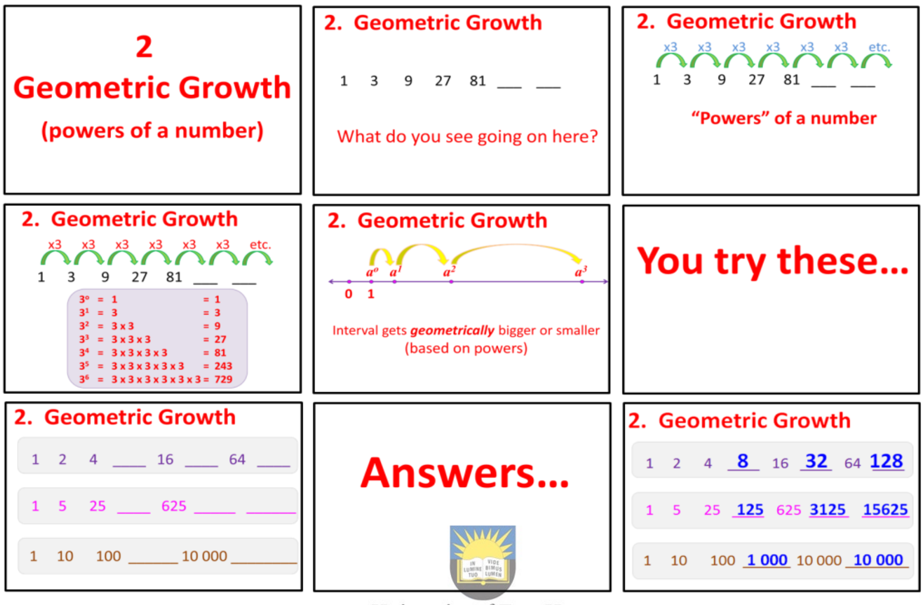


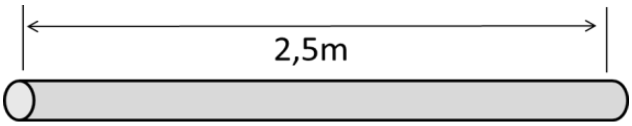



*Figure 4.24.* Part of a slideshow that was designed for and presented to the 2016 – IPS 413 E Mathematics cohort.

## 4.5.5 TREND 5: LACK OF COMPRESSION OF RATIOS

Q4.5 is transcribed below:

**Q 4.5 A stick which is 2,5m long is cut into three parts using the ratio 2 : 3 : 5. How long are each of the three parts?**




Generally, the participants did not do well with any of the sub-questions in Main Question 4. In particular, Q 4.5, presented the cohort with many difficulties. In the prior knowledge assessment the return was 1/34 or 3% of the students got it correct, and in the re-evaluation, the score rose to only 3/38 or 8%.

## 4.5.6 TREND 6: INABILITY TO CONCEPTUALISE LARGE NUMBERS

Q 4.1 returned 8/34 (24%) in the prior knowledge assessment and 11/38 (29%) in the re-evaluation. The sub-question required the students to write a numerical equivalent for a textually composed number. As seen in the exemplars in *Figure 4.25*, below, many students struggled to change text into digits.

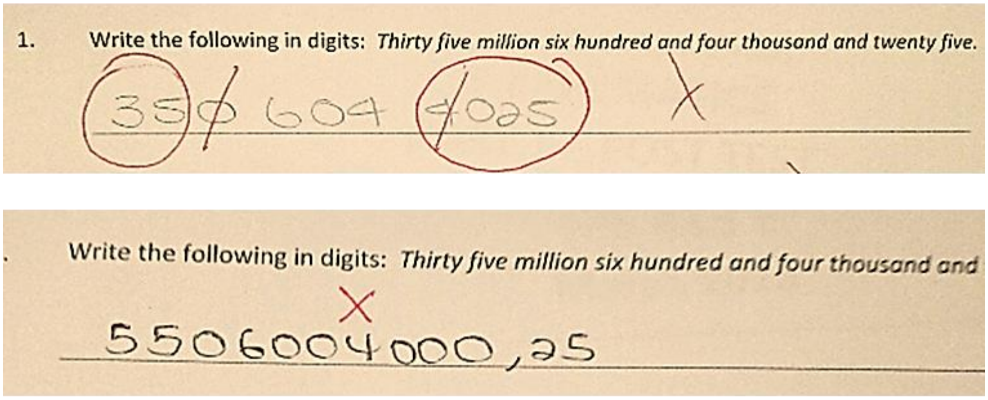



*Figure 4.25*. Examples of misconstruction of numerical representation of large numbers.

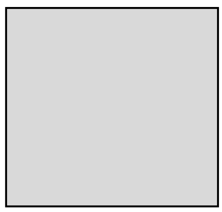


While language barriers may have played a role in the ill-successes with this question, the *shapes* of the answers often demonstrated a complete absence of number understanding, grouping, decimals, etcetera.

## 4.5.7 TREND 7: LACK OF COMPRESSION OF CONCEPTS LINKED TO SQUARES

Below, Q 4.3 is transcribed in full:

***Q 4.3 If the area of a square is 169cm$^2$, what is the perimeter of the square?***



In the prior knowledge assessment, participants achieved 1/34 (3%) success; in the re-evaluation, the return was 12/38 (32%). A far higher return was anticipated for the re-evaluation. The concepts of length, perimeter and area, squares and square-roots, as a two dimensional shape and in the form of $x^2$ and $\sqrt{x^2}$, were key foundational arithmetic concepts that students needed to assimilate and accommodate during the course of Action Cycle 1. A great deal of time and energy was expended on that work. The participants were given opportunities to construct sketches, physically measure squares, discuss and construct their own understanding of this work.

These methodologies are indicative of a visualising, constructive and liberalising pedagogy. Without such knowledge, very little progress can be made. That notwithstanding, the exemplar in *Figure 4.26,* below, is indicative of the wide range of mis-concepts and arithmetic ineptitude that prevailed after the completion of the first action cycle. It is taken from an answer which a participant produced in the re-evaluation task.

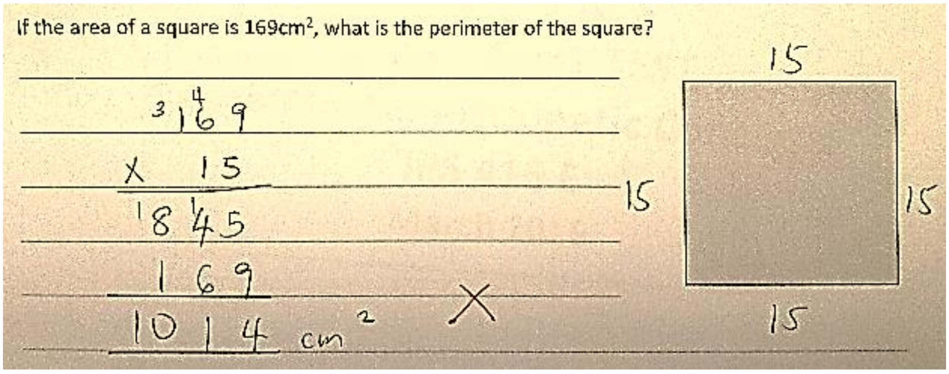



*Figure 4.26*. An exemplar which demonstrates a lack of compression of the meta-concept of perfect squares and square roots, area and perimeter

## 4.5.8 TREND 8: LACK OF COMPRESSION OF FACTORISATION

Q4.2 required participants to compile a list of the factors of 28. These are: 1; 2; 4; 7; 14 and 28, that is, the natural numbers that *exactly* divide into 28. In the prior knowledge assessment, the students achieved a 3/34 or 9% success rate while in the re-evaluation, success climbed to only 14/38 or 37%.

Further, deriving from their study of factors in the IPS 413 E class, the participants learned about factor pairs: 1 x 28; 2 x 14, etcetera; prime factors of numbers which, in the case of 28 are 2 and 7; and the prime factor form of numbers, which for 28 is 28 = $2^2$ x 7. Thus, it can be seen that the factors of 28 constitute a low level LOT arithmetic skill, but much is built upon it. Thus, poor performance in establishing factors of natural numbers implies that the abilities to construct factor pairs and find prime factors is also compromised.

### 4.5.9 TREND 9: POOR COMPUTATION AND SETTING OUT SKILLS

In my teaching, I encourage students to use a scribble pad. It provides a wonderful space in which to trial ideas, quickly set out a solution strategy, test a solution. The scribble pad can also provide visual evidence of mathematical ineptitudes.

Below, in two sets of photographs, in *Figure 4.27* and *Figure 4.28* respectively, I present scribblings that were found in the re-evaluation scripts. I think that these exemplars offer much evidence of the perilous state of foundational mental arithmetic structures that were held by students in the PGCE cohort

#### 4.5.9.1 CATEGORY 1: AN INABILITY TO MULTIPLY



The images presented in *Figure 4.27*, below, attest to students' weak capacity to perform basic multiplication calculations. The methods used by the students to circumvent their shortfall of knowledge of the times-tables and the procedural routines used in multiplication are seen to be laborious, time-consuming and ill-effective.

Within these exemplars, we find that incorrectly, 3 x 3 = 6; 3 x 12 = 32; and 3 x 13 = 41. Indeed, when constructing a list of multiples of 3, one student even listed 3 x 10 = 31, which demonstrates a complete misunderstanding of the concept of multiplying, and by extension, dividing by 10

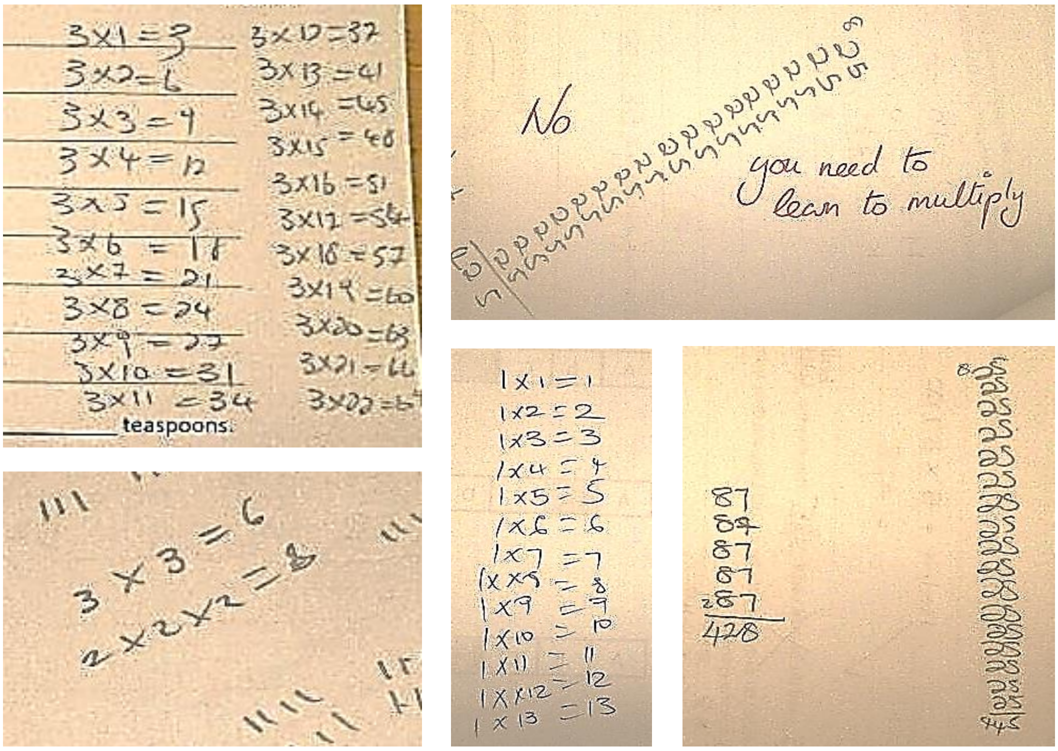



*Figure 4.27.* Exemplars of rough work carried out by participants in the Foundational Arithmetic Concepts, Action Cycle 1, re-evaluation task



In *Figure 4.27*, the two images on the right provide an example in which, on top, instead of multiplying 17 x 25, a participant correctly adds 25 to itself 17 times to achieve 425, then, on the bottom, checks the addition and achieves an incorrect answer of 445. Albeit multiples of 25 go up in 25, 50, 75, etcetera, the student then accepts 445 as the correct answer. Further, bottom-middle, a student wrote 1 x 1 = 1; 1 x 2 = 2, etcetera. This was an absolutely fruitless enterprise: a best-case scenario might be that the student was doodling. Taken with the other exemplars presented in *Figure 4.27*, it would seem that in terms of their ability to multiply, many students were operating off a very low multiplication facts and skills knowledge base.

#### 4.5.9.2 CATEGORY 2: POOR COMPUTATIONAL AND SETTING OUT SKILLS

The second set of photographs in *Figure 4.28*, below, highlights a second endemic feature, one of poor computation and poor setting out skills which, taken together, collude and conspire against success in simple arithmetic tasks.

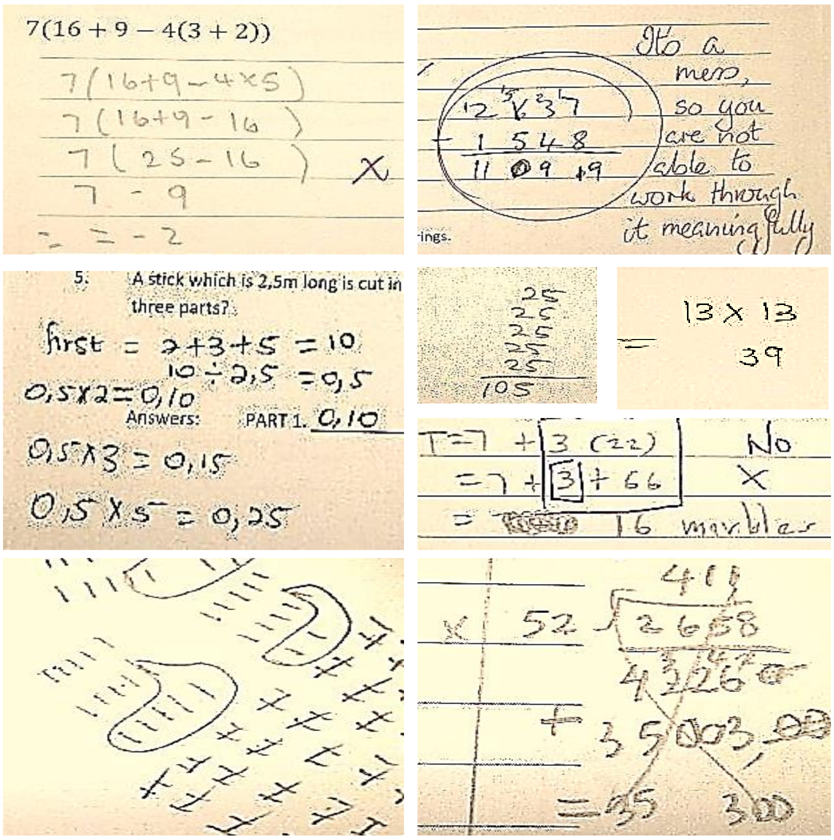


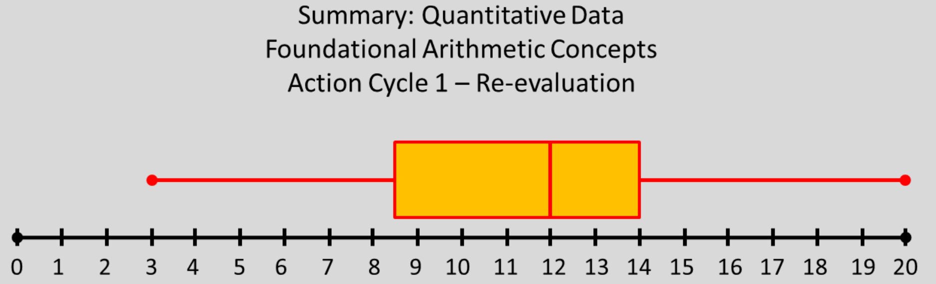



*Figure 4.28*. Exemplars of poor computation and setting out, in the Foundational Arithmetic Concepts, Action Cycle 1, re-evaluation tasks

In their classwork and tests the students were encouraged to set out their solutions neatly. This position was informed by a view that the neat patterning of the routines used in foundational operations-related calculations could act as visual scaffolds that might assist students to work cogently through their calculations.

### 4.5.10 SUMMARY OF QUANTITATIVE DATA FOUND IN FOUNDATIONAL ARITHMETIC CONCEPTS, ACTION CYCLE 1, RE-EVALUATION TASK

Below, in *Figure 4.29,* I have submitted a summary of the quantitative data that was collated from the Action Cycle 1 – Foundational Arithmetic Concepts – re-evaluation task. The graph shows that the scores in the test range from 3/20 through 20/20. Students with final scores of 3 through 8,5 marks reside to the left of quartile 1; half of the students, which is those found between quartiles 1 and 3, scored between 8,5 and 14 marks; and the top quarter of students earned marks between 14 and 20. The mean was 12, the mode (there were 5 occurrences) was 14, and the interquartile range was between 8,5 and 14, that is, 5,5.




| Quantitative Data | |
|---|---|
| Date of Test | 16/3/2016 |
| Number of submitted Scripts | 38 |
| Number who did not write | 0 |
| TOTAL | 38 |
| Mean | 12 |
| Mode (5 occurrences) | 14 |
| Median | 12 |
| Range (20-3) | 17 |
| Quartiles | 8,5; 12; 14 |
| Interquartile Range (14-8,5) | 5,5 |

*Figure 4.29.* Summary of data capture in Foundational Arithmetic Concepts, Action Cycle 1, re-evaluation tasks



Largely, the data findings are disappointing. Informed as it was by Bruner's three-phase theory of learning (2.7.4), the liberating-oriented, visually-rich, activities-based lessons had brought scant reward for the participants. Many of the tasks had been designed to offset misunderstandings of LOT routines and low order procedural knowledge. With few exceptions, compression of mixed operations, powers and bases, ratios, number sense, squares, and factorisation has proved elusive for many of the participants.

## 4.6 WORD-PROBLEM CALCULATIONS, ACTION CYCLE 1 – RE-EVALUATION TASKS

### 4.6.1 TREND 1: SMALL IMPROVEMENTS

Summarised data for the prior knowledge and re-evaluation of the Word Problem Calculations in Action Cycle 1 are presented in Table 4.13 and Table 4.14 respectively, below. Further, the full data-set is appended (Appendix K).

Table 4.13

*Summary of data captured in Word Problem Calculations, Action Cycle 1, prior knowledge assessment*

| ACTION CYCLE 1 - PRIOR KNOWLEDGE<br>37 PARTICIPANTS<br>WORD PROBLEM CALCULATIONS | UNDERSTANDING AND APPLYING | | | | | APPLYING AND ANALYSING | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| Correct responses per Sub Question | 71 | 19 | 41 | 52 | 66 | 9 | 18 | 64 | 64 | 56 |
| Sub-question Averages (%) | 64 | 17 | 37 | 47 | 59 | 8 | 16 | 58 | 58 | 50 |
| Correct responses per Main Question | 249 | | | | | 211 | | | | |
| Main Question Averages (%) | 45 | | | | | 38 | | | | |



Table 4.14

*Summary of data captured in Word Problem Calculations, Action Cycle 1, re-evaluation task*

| ACTION CYCLE 1 - RE-EVALUATION<br>37 PARTICIPANTS<br>WORD PROBLEM CALCULATIONS | UNDERSTANDING AND APPLYING | | | | | APPLYING AND ANALYSING | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| Correct responses per Sub Question | 76 | 40 | 65 | 56 | 73 | 24 | 47 | 59 | 58 | 57 |
| Sub-question Averages (%) | 68 | 36 | 59 | 50 | 66 | 22 | 42 | 53 | 52 | 51 |
| Correct responses per Main Question | 310 | | | | | 245 | | | | |
| Main Question Averages (%) | 56 | | | | | 44 | | | | |

A comparison between the assessments shows that the overall results in the re-evaluation indicate some improvement. In *Understanding* and *Applying* questions, the class average climbed from 45% to 56%, a numerical jump of 11% or, as an improvement on the prior knowledge result, 24%. The *Applying* and *Analysing* questions fared less well, starting at 38%, and moved to 44%, a jump of 6%, or a 16% improvement in the re-evaluation compared to the prior knowledge assessment.

### 4.6.2 TREND 2: LACK OF COMPRESSION OF FRACTIONS

In the prior knowledge assessment task, the participants returned a 17% success rate for Q.2; in the re-evaluation, the class-score rose to 36%, which was a 19% numerical rise and a 212% improvement on the first attempt. But, realistically, a 36% return was not good. My efforts to improve the low success rate in Q.2 did not result in the compression of the requisite concepts and skills. The question read:

> **Q 2:** ***Altogether Shirley and Jean have 120 teaspoons. Shirley owns $\frac{3}{8}$ of the teaspoons. How many teaspoons does Shirley have? How many teaspoons does Jean have?***

Q.2 was discussed previously (4.4.3): because of that finding, scaffolding was introduced to ameliorate the difficulties that the participants had with the problem. Teaching methods included the use of tactile, iconic and symbolic modes of representation and discussed the big ideas around which such problems are solved. An example of a visually-mediated, paper and pencil solution is presented in *Figure 4.30*, below.



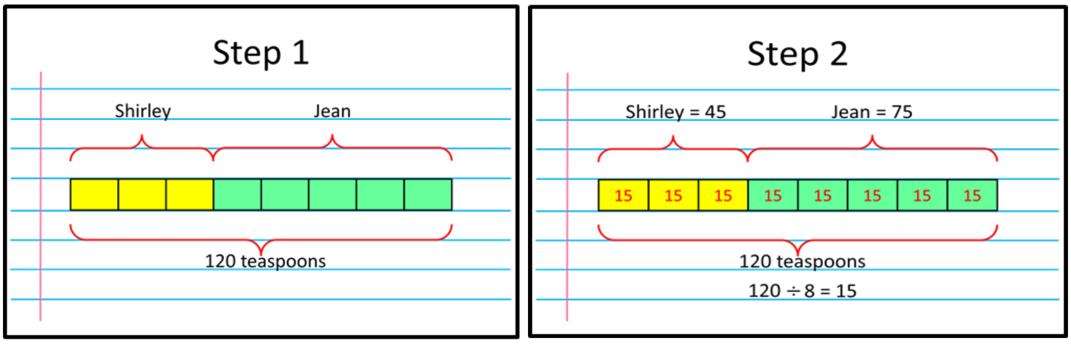



*Figure 4.30.* A visual solution for Q 2 in Word-Problems Calculations, Action Cycle 1, re-evaluation task

In the first step, the participants were taught to construct a ribbon – some call it a box, others a model – which was sub-divided in to 8 equal parts. The full length of the rectangle represented the *whole* while the 8 parts represented the total number of parts in play – that is, the denominator in the fractions. In effect, all that the participants were really doing was constructing an eighths-based fraction wall. The methodology required them to glean information from the word-problem and to use that information to populate and give meaning to the ribbon drawing. In the second step, students should have used the visual cues provided by the 120 teaspoons and 8 equal parts in the ribbon to deduce that, by division, they could establish the size of each of the equal parts, namely 15. From that, they should have been able to populate the parts and thereby establish the number of spoons that were owned by each of the two ladies. Whether by enactive or iconic or symbolic means, many students could not solve this *Understanding* and *Applying* task.

### 4.6.3 TREND 3: LANGUAGE AS A BARRIER

In the Word-Problems Calculations prior knowledge assessment, Q.6 delivered an 8% success rate. In the re-evaluation, this figure rose to 22%. Thus, the participants scored better by 14%, with a (somewhat misleading) improvement in the re-evaluation over prior knowledge task of 275%. Q.6 question reads as follows:



> ***Q 2. Altogether, Lulu and Irene have sixty new books. Irene has half the number of books that Lulu has. How many books does each lady have?***

This question, like Q.2 in 4.6.2 above, was discussed in my analysis of the prior knowledge assessment tasks data for Action Cycle 1 (4.4.4.). However, subsequent to writing the first test and in spite of the in-class activities, very little changed.

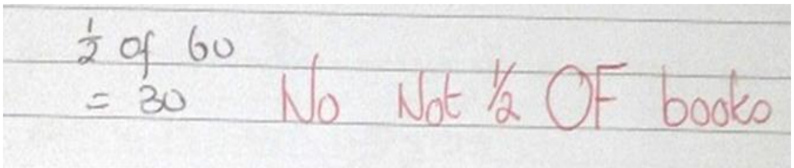



*Figure 4.31.* Example of an incorrect solution to Q.6 in Word-Problems Calculations, Action Cycle 1, re-evaluation

As seen in *Figure 4.31,* above, the sentence structure of the word-problem might have tripped up the participants, who may have interpreted this question to read that each girl has one half of the total of 60, that is, 30 books each. Turns for phrase such as *half* *the number of* are stock-in-trade in mathematics, thus, rather than avoid their use, it is appropriate that the participants learn to use them meaningfully.

## 4.6.4 TREND 4: LACK OF COMPRESSION OF RATIOS

Q.7 was a ratios-based question. The prior knowledge tasks returned 16% success; the re-evaluation, 42%. That was a growth of 26% or, compared to the prior knowledge assessment tasks, a 263% increase. Using visualising strategies, I had hoped that even students who might not be able to attend to the solution symbolically, would easily cope with the requirements of the question. However, I was incorrect. The question is transcribed below, in *Figure 4.32*, and as an indication of one methodology that was used, I have also provided a paper and pencil, visual solution for the problem.

***Q 7. Shirley collects red, blue and yellow coffee mugs in the ratio 5:3:4. She owns 12 yellow coffee mugs. Use this information to answer the following questions.***

***7.1 How many red mugs does Shirley have?***
***7.2 How many more red mugs than blue mugs does Shirley have?***
***7.3 Altogether, how many red, blue and yellow mugs does she have?***



In Step 1, the ratio – the parts – are all drawn in fair proportion and the representation of 12 yellow mugs is inserted on the sketch.

In Step 2, we use the visual cues to deduct that if the 12 mugs span 4 yellow cells, each yellow cell must have a unit value of 3.

In Step 3, we populate the cells which must have equivalent unit value; we use the visual information to answer the various questions in Q.7.

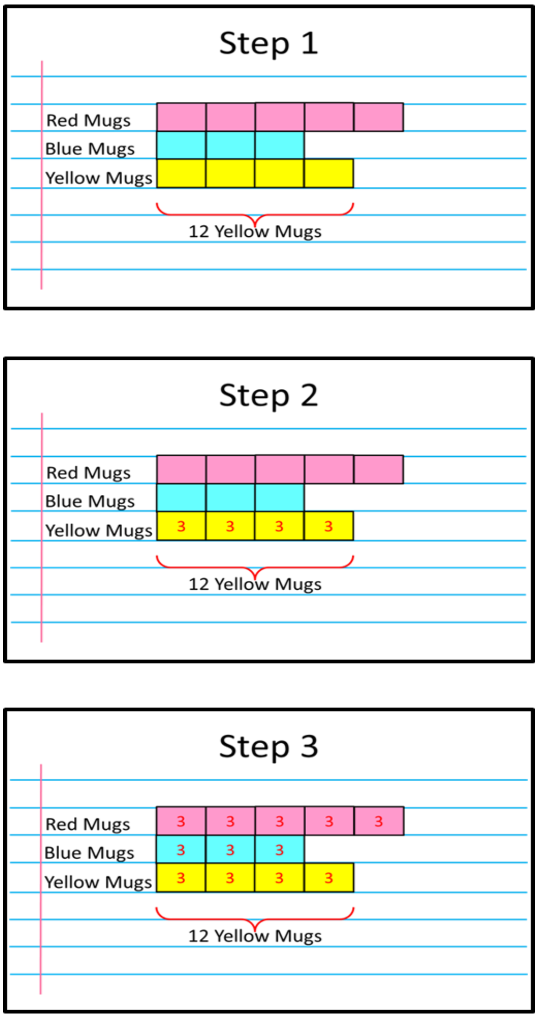
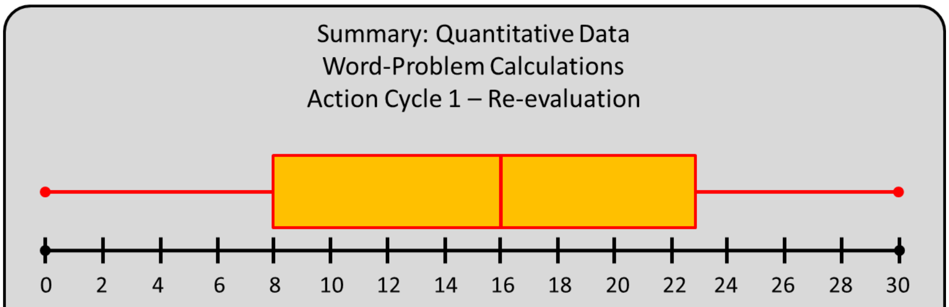



*Figure 4.32.* A visual solution for Q 7, Word-Problems Calculations, Action Cycle 1, re-evaluation

## 4.6.5 SUMMARY OF QUANTITATIVE DATA FOUND IN WORD-PROBLEMS CALCULATIONS, ACTION CYCLE 1, RE-EVALUATION TASKS

The box and whisker plot, *Figure 4.33*, below, summarises the data captured from the Word-Problems Calculations, Action Cycle 1, re-evaluation tasks.




| Quantitative Data | |
|---|---|
| Date of Test | 16/3/2016 |
| Number of submitted Scripts | 37 |
| Number who did not write | 1 |
| TOTAL | 38 |
| Mean | 15 |
| Bi-Modal (4 occurrences each) | 11; 25 |
| Median | 16 |
| Range (30-0) | 30 |
| Quartiles | 8; 16; 23 |
| Interquartile Range (23-8) | 15 |



*Figure 4.33.* Summary of quantitative data captured in Word-Problem Calculations, Action Cycle 1, re-evaluation tasks.

37 participants wrote the test. The lowest and highest scores achieved were 0/30, (2 students), and 30/30, (1 student), respectively. The class average was 15/30. A bi-modal condition (4 each) of 11 and 25 occured and the median was 16. The range was 30. Students below quartile 1 scored between 0 and 8; half of the students achieved scores between 8 and 223, and the top performing quarter of the class scored between 23 and 30. The interquartile range placed 50% of the students within a range of 15 marks, between 8 and 23.Compared to the prior knowledge assessment, the overall range in the re-evaluation expanded rather than contracted and the boxes shifted marginally to the right.

Overall, the first action cycle did not yield much benefit to the PGCE cohort. In the re-evaluation, at least four of Gooding's (2009) barriers occurred frequently. The participants have shown that they (1) struggled to read and understand word-problems

(4.6.3), and (2) struggled to form mental (nor physical) images of the contexts of word problems (4.6.2). Further, they (3) found it difficult to construct number sentences or algorithms (4.6.2 and 4.6.4), nor could they (4) carry out mathematical calculations (4.6.2 and 4.6.4).

## 4.7 VISUAL FRACTIONS AND RATIO CONCEPTS, ACTION CYCLE 2, PRIOR KNOWLEDGE ASSESSMENT

On April 20, 2016, the participants wrote the Action Cycle 2 *Visual Fractions and Ratios Concepts* prior knowledge assessment tasks. 30 marks were allocated to the arithmetic work (Appendix L). There were 36 participants. Below, I discuss trends found in the Visual Fractions and Ratios Concepts tasks.

### 4.7.1 TREND 1: LOW SCORES IN PRIOR KNOWLEDGE ASSESSMENTS

Table 4.15, below, shows that as the cognitive demand became more complex from Main Question 1 – *Remembering Fractions and Ratios* to Main Question 2 – *Remembering and Understanding Fractions*, so too, correct responses declined from 53% to 24%. Main Question 3 – *Understanding and Applying*, achieved a 30% success rate. The primary data is appended (Appendix M).



Table 4.15

*Summary of data captured in the Visual Fractions and Ratios Concepts, Action Cycle 2, prior knowledge assessment tasks*

| ACTION CYCLE 2 - PRIOR KNOWLEDGE<br>36 PARTICIPANTS<br>FRACTIONS AND RATIOS | REMEMBERING FRACTIONS AND RATIOS | | | | | | | | | | | REMEMBERING & UNDERSTANDING | | | | | | | | | | | UNDERSTANDING AND APPLYING | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| Correct responses per Sub Question | 36 | 29 | 26 | 18 | 18 | 24 | 14 | 11 | 12 | 1 | | 13 | 8 | 7 | 8 | 13 | 3 | 18 | 5 | 6 | 6 | | 25 | 2 | 0 | 12 | 22 | 13 | 2 | 8 | 8 | 15 |
| Sub-question Averages (%) | 100 | 81 | 72 | 50 | 50 | 67 | 39 | 31 | 33 | 3 | | 36 | 22 | 19 | 22 | 36 | 8 | 50 | 14 | 17 | 17 | | 69 | 6 | 0 | 33 | 61 | 36 | 6 | 22 | 22 | 42 |
| Correct responses per Main Question | 189 | | | | | | | | | | | 87 | | | | | | | | | | | 107 | | | | | | | | | |
| Main Question Averages (%) | 53 | | | | | | | | | | | 24 | | | | | | | | | | | 30 | | | | | | | | | |

In Main Question 1, the participants struggled particularly with sub-questions Q 1.10; Q 1.8 and Q 1.9. In Main Question 2, the worst-answered questions were Q 2.6; Q and 2.8.In Main Question 3, the biggest problems lay in Q 3.3; Q 3.2 and Q 3.7 respectively.

### 4.7.2 TREND 2: LACK OF RELATIONAL UNDERSTANDING OF FRACTIONS

Only one student in the cohort (1/36 or 3%) managed to answer sub-question Q 1.10 correctly. Most answers were similar to those found in *Figure 4.34*, below.

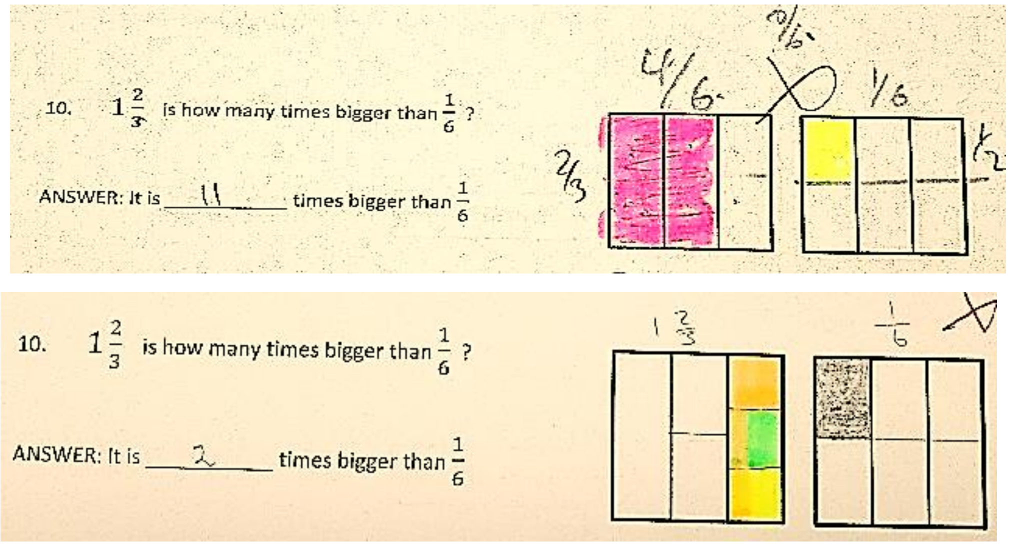



*Figure 4.34.* Two incorrect interpretations and solutions to Q.10 in the Visual Fractions and Ratios Concepts, Action Cycle 2 prior knowledge task



As demonstrated in *Figure 4.34*, above, 2 participants interpreted the two boxes as being linked separately to $1\frac{2}{3}$ and $\frac{1}{6}$ respectively. Many of the supplied answers followed no discernible pattern, and included 1's, 2's, 3's, etcetera, and many variations on different fraction combinations $\frac{2}{6}$; $\frac{9}{6}$; $1\frac{3}{9}$, etcetera.

Sometimes it is difficult to interpret what students might be thinking, but, clearly, they found this task vexing. Although the two squares were sub-divided into thirds, that clue offered no useful visual pivot. The students could not make any sense of their possible contribution to a viable solution for this question.

Further, in many cases the answers demonstrated that albeit that deep thinking about the problem did occur, no cognitive linking up of any relational understanding of fractions and the equivalence of fractions was in place.

### 4.7.3 TREND 3: INABILITY TO PLOT A MIXED NUMBER ON A NUMBER LINE

In their answers for Q 1.8, as seen in *Figure 4.35*, below, many participants were unable to precisely plot $2\frac{3}{8}$ on a number line. Only 11/36 or 31% of the students managed to achieve the answer. Similarly, in Q 1.7, the students had to plot $1\frac{7}{10}$ on a number line: there, they achieved only marginally better results, with 14/36 or 39% of the students being able to correctly identify that point.

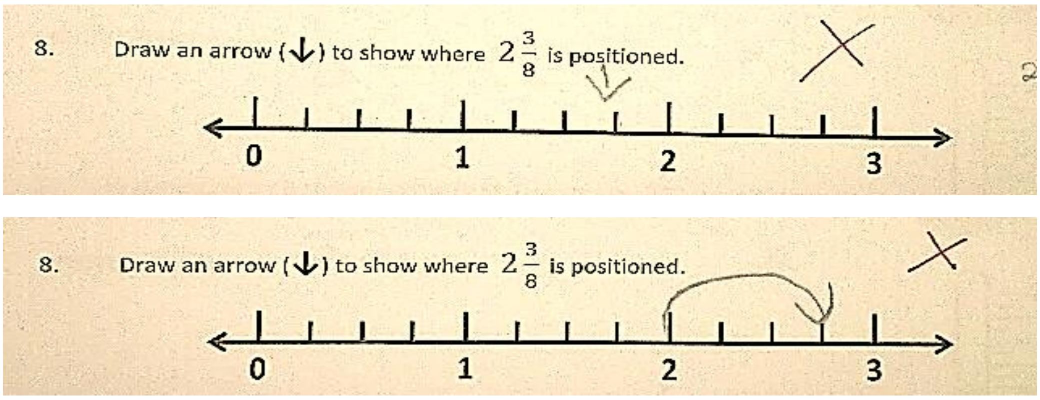



*Figure 4.35.* Two incorrect solutions for Q 1.8 in the Visual Fractions and Ratios Concepts, Action Cycle 2, prior knowledge task



The two exemplars above display ill-coherence in transferring a mixed-number, in this case, $2\frac{3}{8}$ , onto a number line. While many different misunderstandings may contribute to this situation, my experience suggests two probable reasons:

1. Often, students cannot conceptualize that in a fraction such as $\frac{3}{8}$, the denominator implies the fracturing of a whole into 8 equal parts. Indeed, the conception that the parts are *equal* proves somewhat evasive.
2. Many students seem to have little previous experience of number lines, have difficulty with the concept of 1, struggle with understanding how to use the linear space between 0 and 1 and, overall, have poor measurement skills. These missing skills-sets manifest in many ways. Many students will consistently measure dimensions at 1cm less than the true length: they ignore the linear distance between 0 and 1 on their rulers or steadfastly measure off from the 1cm graduation mark on the ruler.

### 4.7.4 TREND 4: INABILITY TO VISUALISE A FRACTION

Only 12/36 or 33% of the participants managed to correctly answer Q 1.9. The exemplars found in *Figure 4.36*, below, demonstrate that some students (1) intuitively knew the answer, and (2) others were able to construct visual representations that assisted them to answer the question.

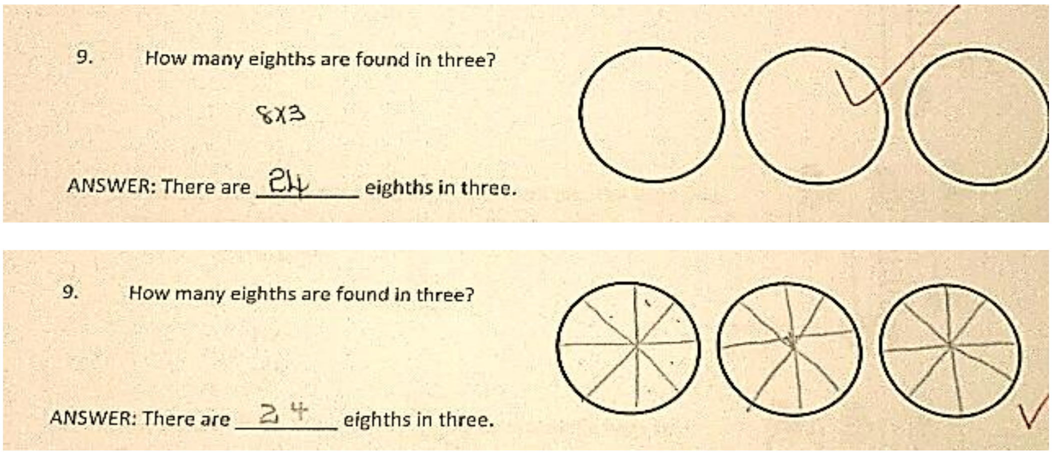



*Figure 4.36*. Two correct solutions for Q 1.9 in the Visual Fractions and Ratios Concepts, Action Cycle 2 prior knowledge task



However, for the most-part, as seen in the exemplars found in *Figure 4.37*, below, participants were unable to apply prior knowledge of fractions nor relational understanding nor use the visual scaffolds to achieve the correct answer. In the 1st quarter, our conversations about fractions had included the sharing of biscuits and pizza, and links to pie charts, thus this low return gives pause for thought.

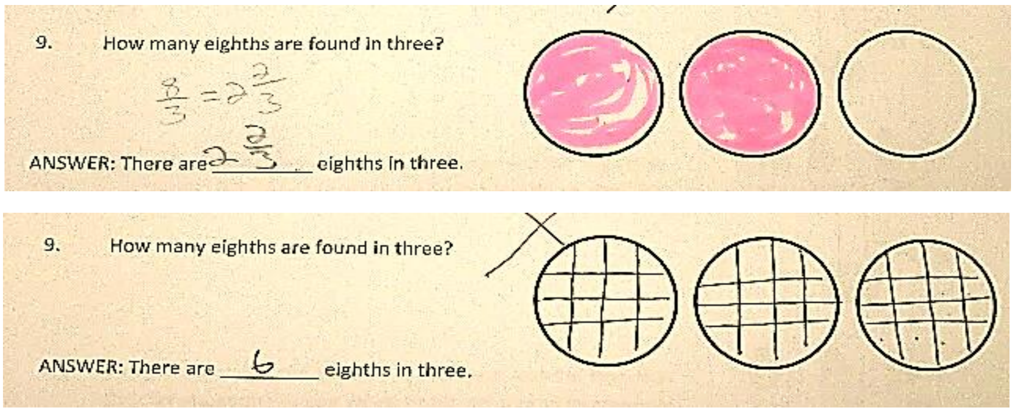



*Figure 4.37.* Two incorrect solutions for Q 1.9 in the Visual Fractions and Ratios Concepts, Action Cycle 2 prior knowledge task

## 4.7.5 TREND 5: INABILITY TO INTERPRET FIGURAL INFORMATION (IFI) IN A VISUAL REPRESENTATION OF A COMMON FRACTION

Main Question 2, Q 2.6 required the participants to identify shapes in which $\frac{2}{5}$ of those shapes where shaded in, in grey. Only 3/36 or 8% of the students correctly answered the question. Among the other respondents, different incorrect answers were presented. In many cases, it seemed that the students simply *took a stab* at the answers. *Figure 4.38*, below, shows two interesting examples of incorrect solutions.

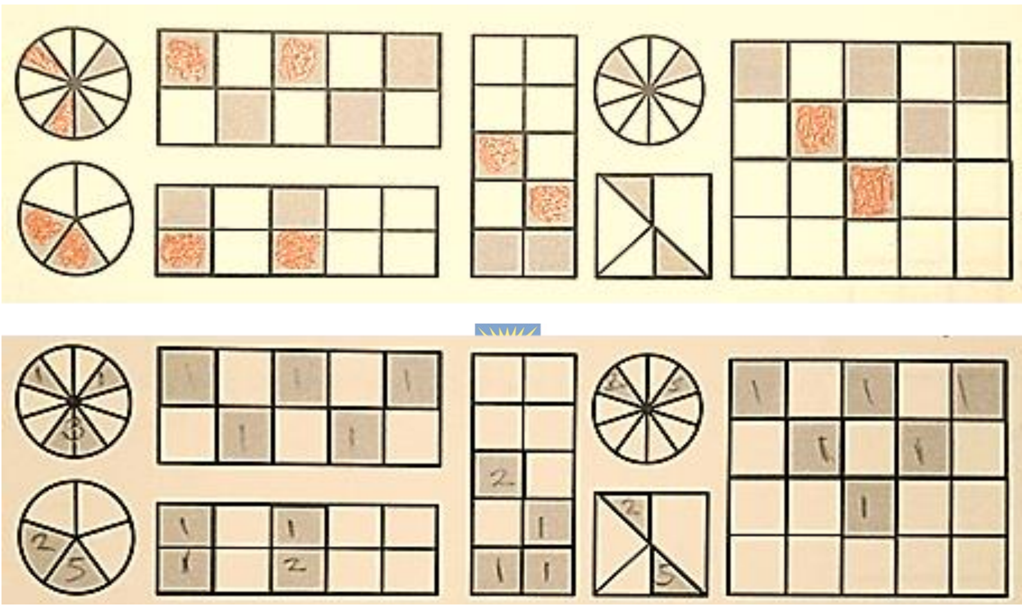


*Figure 4.38*. Two students' misinterpretations of $\frac{2}{5}$ of a whole as found in Q.9 in the Visual Fractions and Ratios Concepts, Action Cycle 2, prior knowledge task

Most of the participants incorrectly selected the triangles within the square as a visual representation of $\frac{2}{5}$ of a whole. It would seem that they perceived that because the square contained five parts with two parts shaded in grey; such a configuration would imply $\frac{2}{5}$ of a whole. However, selecting the square in this task indicated that compression of the concept of fractions had not occurred and suggested that the foundational notion that the visual representation of fifths, as combinations of spaces of congruent area, had not been accommodated.

## 4.7.6 TREND 6: FURTHER EVIDENCE OF AN INABILITY TO PLOT A NUMBER ON A NUMBER LINE

Q 2.8 proved problematic. In similar fashion to Q 1.7 and Q 1.8 before it, this sub-question required the participants to plot: $\frac{4}{5} \times 2$, that is, $1\frac{3}{5}$, onto a number line. Only 5/36 or 14% of the students correctly answered the question. As per *Figure 4.39*, below, many participants either wrote words to the effect that they did not know what to do, or simply did not attempt to produce an answer.

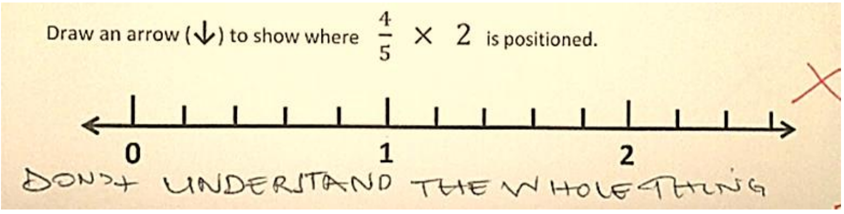



*Figure 4.39.* A textual response as found in in Q 2.8 in the Visual Fractions and Ratios Concepts, Action Cycle 2. prior knowledge assessment tasks



Others produced incorrect solutions in which they (1) could not plot the correct position or they (2) carried out their computations incorrectly. In *Figure 4.40*, below it appears that the student treated the incremental notch above the zero as *one*, and then counted-on until 8, thereby ending up at $1\frac{2}{5}$ .

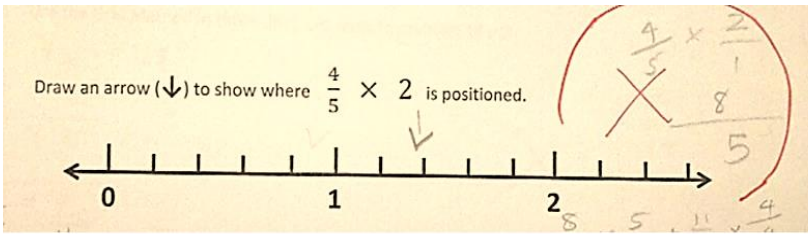



*Figure 4.40.* A response as found in Q 2.8 in the Visual Fractions and Ratios Concepts, Action Cycle 2 prior knowledge task

Others unsuccessfully plotted a point on the number line. As seen in the exemplar in *Figure 4.41*, below, it was hard to interpret if they thought that they had registered the correct point on the number line, or if they were simply guessing.

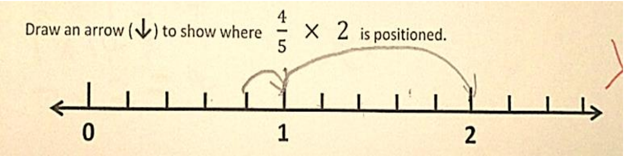



*Figure 4.41.* An incorrectly plotted response as found in Q 2.8 in the Visual Fractions and Ratios Concepts, Action Cycle 2 prior knowledge task

## 4.7.7 TREND 7: INABILITY TO ADD MIXED NUMBERS

In Q 2.9, only 6/36 or 17% of the participants identified that: $1\frac{1}{4} + 1\frac{3}{8} = 2\frac{5}{8}$. Visual scaffolds were inserted into the question. Of the 6 students who correctly answered the question, only 2 made use of the visual scaffold. The other 4 correct answers were completed without referring to the sketch. An exemplar of a visually mediated solution is provided in *Figure 4.42*, below.

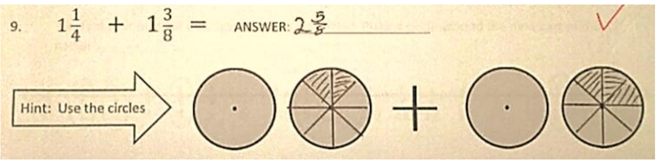



*Figure 4.42.* A visually mediated solution for Q 2.9 in the Visual Fractions and Ratios Concepts, Action Cycle 2. prior knowledge assessment tasks

Most participants were unable to add the two mixed numbers together by either visual or computational means. Indeed, 4 students did not attempt an answer, while 8 others, see *Figure 4.43*, used the visual scaffold, but interpreted its use incorrectly. Every other incorrect answer – 18 in total – either misapplied arithmetic techniques or simply supplied a fraction or mixed number.

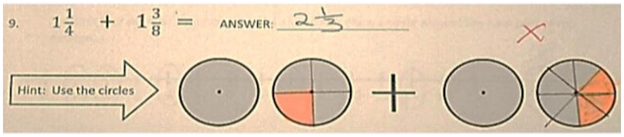



*Figure 4.43.* An incorrect, visually mediated solution for Q 2.9, found in the Visual Fractions and Ratios Concepts, Action Cycle 2 prior knowledge task

### 4.7.8 TREND 8: INABILITY TO APPLY THE GRID METHOD

In Main Question 3, sub-question Q 3.3, the participants were required use the *Grid Method* to multiply 14 x 7. The cohort scored a 0/36 or 0% success rate. This did not alarm me: I was aware that prior to the task, the students had probably not used the technique or might be unfamiliar with the term *grid-method*. I knew this problem would be resolved in the course of Action Cycle 2 . The Grid Method is derived from the distributive law: $a(b + c) = ab + ac$. The multiplicands are explored and reconstructed as suitable groupings of addends, so as to make multiplying easier. The purpose of this technique is not to speed up calculating, but rather to assist in predicting and judging the correctness of the product. 14 could be reconstructed as 10 + 4 or 5 + 5 + 4, etcetera, and 7 might become 4 + 3 or 5 + 2 or simply remain as 7. Below, in *Figure 4.44,* I have constructed some possible answers.

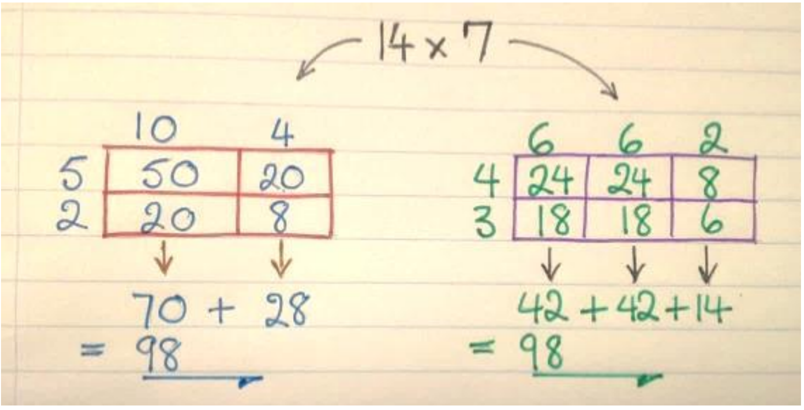



*Figure 4.44.* Two different solutions for Q 3.3 as found in the Visual Fractions and Ratios Concepts, Action Cycle 2 prior knowledge task

Past cohorts of students have found this technique very user-friendly It made it easier for them to multiply – many were notoriously bad at multiplying – and I predicted a similar outcome for the participants.

### 4.7.9 TREND 9: LACK OF COMPRESSION OF PRIME NUMBERS

Q 3.2 required participants to plot the prime numbers found between 0 and 18 onto a number line. This sub-question delivered a result of 2/36 or 6%. However, unlike the Grid Method multiplications, above (4.7.8), prime numbers and factors were extensively covered within Action Cycle 1. So, that left me feeling a bit perturbed.

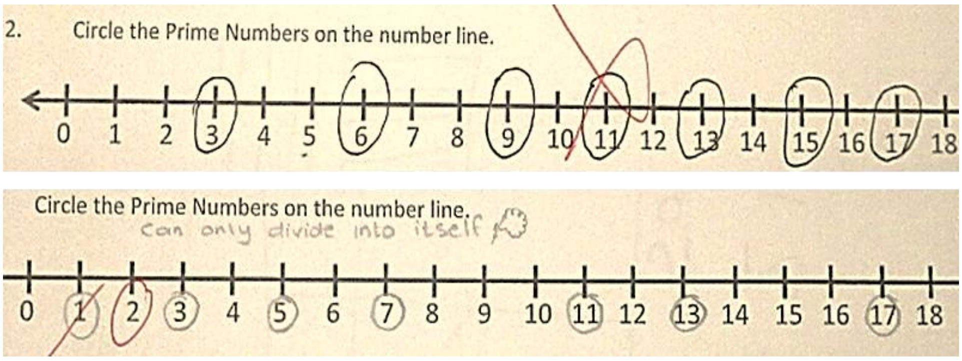



*Figure 4.45.* Two different incorrect solutions for Q 3.2 as found in the Visual Fractions and Ratios Concepts, Action Cycle 2 prior knowledge task

The two exemplars in *Figure 4.45,* above, were indicative of the answers that were received for Q 3.2. They demonstrate that largely, there was a complete absence of conception of prime numbers. In their answers, the participants presented all the usual culprits of misunderstanding; they circled the 0, and the 1, but not the 2; they filled in multiples of 3, and filled in all the odd numbers. The only constant was that they were all wrong. It was all quite disheartening!



At the time, I could not say where the problem lay; it might have been in my explanation, or the students might be distracted, or something else. But I also knew that the conceptual up-take and recall of prime numbers, *after* full explanations, scaffolding and accompanying activities exercises, should not have generated only 2/36 correct replies for this L-mode, LOT, low cognitive demand requirement.

### 4.7.10 TREND 10: LACK OF COMPRESSION OF ARRAYS

The results for Q 3.7 were particularly worrying. The question required participants to construct a visual solution, *an array*, to demonstrate that 3 x 5 = 5 x 3. In effect, I anticipated that the students would either draw appropriate arrangements of circles, as a discrete representation, or similarly, would employ square grids, as a continuous representation (Siemon et al., 2012), and that they would *all* get this right. However, only 2/36 or 6% of the students managed to correctly answer this question.

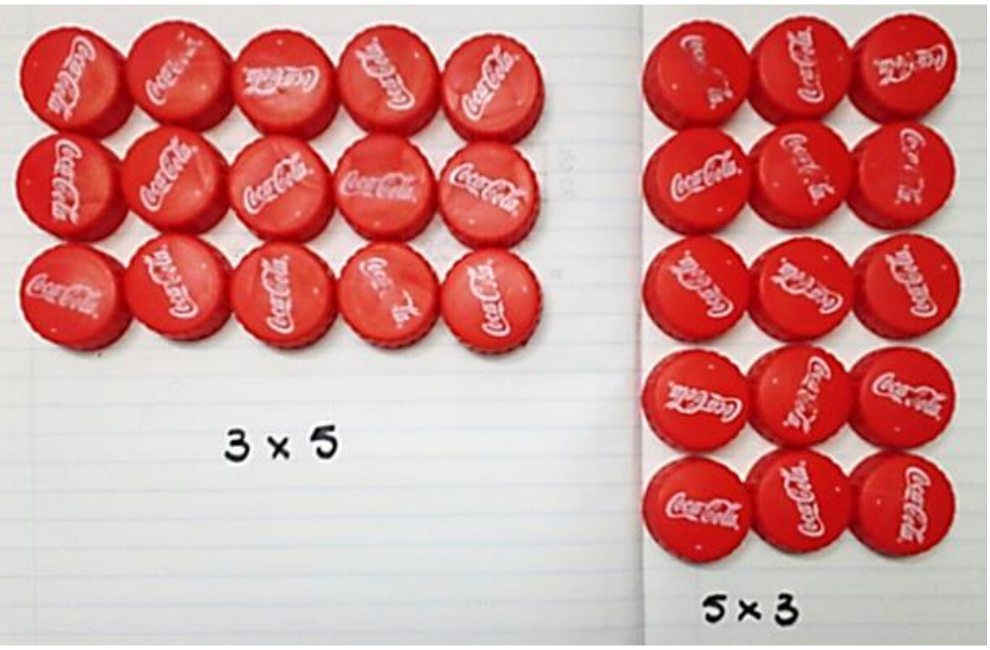


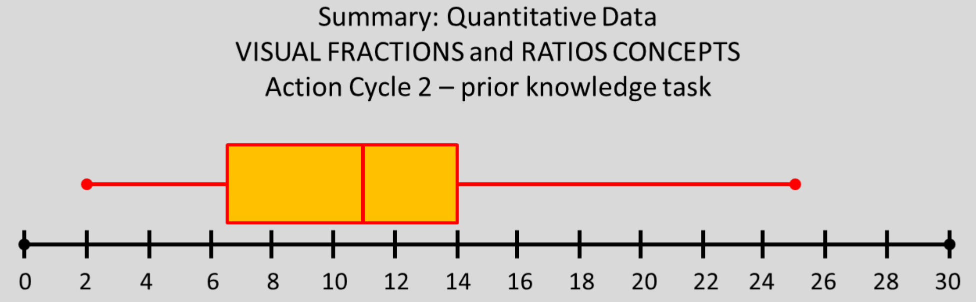

*Figure 4.46.* A reconstruction of the use of bottle tops to construct an array and thereby visualise the commutative law

The poor result was very worrying, given that within Action Cycle 1 classes and as presented in *Figure 4.46, above,* in almost every lesson and in different ways, I had made use of bottle-tops, buttons, biscuits, etcetera, as tactile and visual apparatus to represent arrays, the four operations, fractions and ratios. Indeed, within the confines of the IPS 413 E module, the participants had completed many activities that were conceived to assist them to construct robust memories of big mathematical ideas like arrays. The data captured from Q 3.7, suggested that, in large part, the activities with arrays had not proved to be of much educational benefit to the students and suggested that the students had not constructed any meaning-filled, long-term memories of our classroom-based activities with arrays. This was very disheartening.



### 4.7.11 SUMMARY OF QUANTITATIVE DATA FOUND IN VISUAL FRACTIONS AND RATIO CONCEPTS, ACTION CYCLE 2, PRIOR KNOWLEDGE TASK

A summary of the quantitative data that was captured by the Visual Fractions and Ratios Concepts, Action Cycle 2 prior knowledge tasks is provided by *Figure 4.47*, below.




| Quantitative Data | |
|---|---|
| Date of Test | 20/4/2016 |
| Number of submitted Scripts | 36 |
| Number who did not write | 2 |
| TOTAL | 38 |
| Mean | 11 |
| Bi-Modal (5 occurrences each) | 6; 9 |
| Median | 10 |
| Range (25-2) | 23 |
| Quartiles | 6,5; 11; 14 |
| Interquartile Range (14-6,5) | 7,5 |

*Figure 4.47.* Summary of quantitative data captured in the Visual Fractions and Ratios Concepts, Action Cycle 2 prior knowledge tasks



36 students completed the test. One participant each got the lowest mark, 2/30, and highest mark, 25/30, respectively. The left whisker, representing the lowest performing quarter of the students, was narrow, between 2 and 6,5; half of the students, that is, scores between quartiles 1 and 3, scored between 6,5 and 14 marks, and the top quarter, in the right whisker, scored between 14 and 25. The mean was 11, the median was 10 and a bi-modal situation saw occurrences each of students scoring 6 and 9. The range was 23, (25 – 2), and the interquartile range was quite narrow at 7,5 marks (14 – 6,5). This meant that 75% of the cohort achieved marks of 14 or less out of 30.

Action cycle 1 had been saturated in *doing* and *seeing* activities, and thereby, should have at least partly prepared the students for the action cycle 2, prior knowledge tasks, but largely, this did not happen. The scripts produced little evidence of memories of mathematical ideas that were applied in earlier lessons being transferred into the task. These discouraging results reinforced a view that I was developing that the aspirations of the action research might need to be subjugated by a far more pressing requirement *for the students* in that they needed to acquire a robust foundation of arithmetic conventions, skills, concepts, and vocabulary.

## 4.8 WORD-PROBLEM CALCULATIONS, ACTION CYCLE 2, PRIOR KNOWLEDGE ASSESSMENT

In the Action Cycle 2 prior knowledge assessment task, the word problems were set at A*pplying* and *Analysing* on Bloom's Revised Taxonomy. I created problems that used low level arithmetic facts and skills in novel situations, hence the *applying* aspect. Further, by setting the questions within stories, the students were required to *analyse* textual information.

### 4.8.1 TREND 1: LOW LEVELS OF BLOOM'S APPLYING AND ANALYSING CAPABILITIES

Overall, the Word Problems Calculations, Action Cycle 2, prior knowledge assessment task delivered a poor set of returns. The questions fell within the scope of arithmetic word-problems for the Intermediate Phase; however, many participants struggled to make sense of any of the questions. The main question contained 5 sub-questions and each sub-question was worth 4 marks: thus, with a gross score of 118/720, the overall average was 16%. A summary of the quantitative data is presented below, in Table 4.16. The full data-set is appended (Appendix N).



Table 4.16

*Summary of data captured in the Word Problem Calculations, Action Cycle 2, prior knowledge assessment task*

| ACTION CYCLE 2 PRIOR KNOWLEDGE 36 PARTICIPANTS WORD PROBLEM CALCULATIONS | APPLYING AND ANALYSING | | | | |
|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 |
| Correct responses per Sub Question | 42 | 27 | 24 | 4 | 21 |
| Sub-question Averages (%) | 29 | 19 | 17 | 3 | 15 |
| Correct responses per Main Question | 118 | | | | |
| Main Question Averages (%) | 16 | | | | |

Next, I will discuss the low level of visual problem-solving ability as exemplified in the tests and after that I will discuss the three worst performing sub-questions in the Word Problems Calculations, Action Cycle 2, prior knowledge assessment task

### 4.8.2 TREND 2: INABILITY TO CONSTRUCT BENEFICIAL VISUAL REPRESENTATIONS OF WORD PROBLEMS

In order to assist the students to better understand the word-problems calculations under the Question 4 heading, I gave the advice: *In each problem, use a simple drawing of a fraction or ratio to show how you see the problem – how you solved the problem*. I wanted the students to use those sketches as scaffolds to construct viable solution strategies. Most of the students did not or could not fulfil this requirement.

While some initial resistance to drawing a mathematical model could be understood in light of the fact that it was a prior knowledge assessment task, contradicting this position, I also reminded myself that I had been working with visual models of ideas with the participants for the duration of the first quarter of 2016. In my classes, the students had seemed quite taken with the visual approaches and had begun to use such techniques themselves.

Most participants did not construct a single iconic representation or idea for any of the five sub-questions. The images presented in *Figure 4.48*, below, are indicative of the types of sketches that were drawn by those few students who did try to construct a visual scaffold.



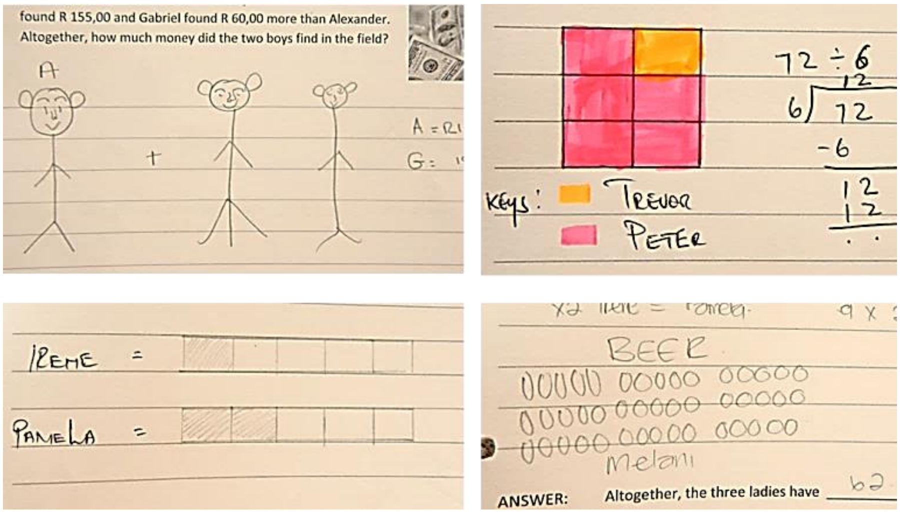


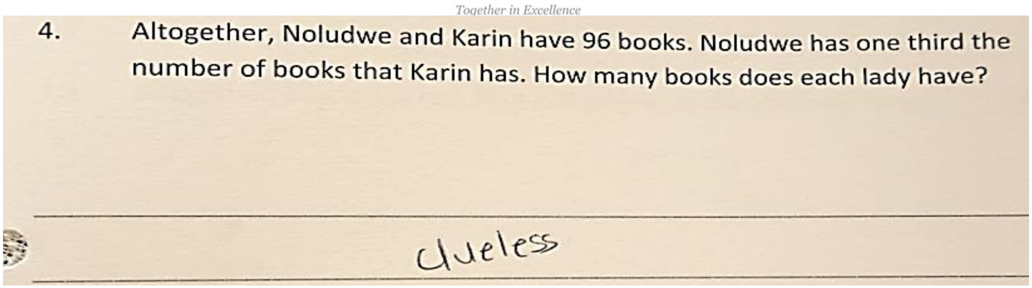

*Figure 4.48.* Examples of mathematical sketches as found in the Word Problem Calculations, Action Cycle 2, prior knowledge assessment task

The four photos in *Figure 4.48*, above, show that in their conception and construction of these drawings, some limited uptake - some transfer - of the visualising methodology that was used earlier in the IPS 413 E module had occurred. The top-left photograph taken from Q 4.1 was a simple pictorial interpretation of two children sharing money; however, by introducing a third person into the sketch, it was poorly conceptualised. The top-right photograph, as a visual scaffold for Q 4.2, reversed the fractional amounts accruing to Peter and Trevor. The bottom left and right photos demonstrated incorrect or incomplete applications of ribbon and array visualising techniques as used to identify the number of bottles of beer held by three persons. As was reported in the visual fractions and ratio concepts prior knowledge assessment tasks (4.7.11), a pattern was emerging which suggested that, albeit that the IPS – 413 E lessons contained visually strong activities, the uptake of the visual methodology was not bearing fruit.

## 4.8.3 TREND 3: FURTHER EVIDENCE OF LANGUAGE AS A BARRIER

Only two students achieved any marks for Q 4.4, and in each case each student was awarded 2 marks out of 4. So, Q 4.4 proved very problematic for the participants. *Figure 4.49*, below, succinctly summarizes the lot of the cohort.




4. Altogether, Noludwe and Karin have 96 books. Noludwe has one third the number of books that Karin has. How many books does each lady have?

clueless

*Figure 4.49.* A typical response for Q 4.4 as found in the Word Problem Calculations, Action Cycle 2, prior knowledge assessment task

Many students perceived Noludwe as having ⅓ of the total, that is 32 books, but in fact she has ¼ of the books, that is 24 books, and Karin has ¾ of the books, or 72 books. In *Figure 4.50*, below, it is useful to note that (1) the student reversed the order of box representations and the legend below it, and (2) did not perceive that the visual representation that had been generated suggested that Karin had half the number of books held by Noludwe.

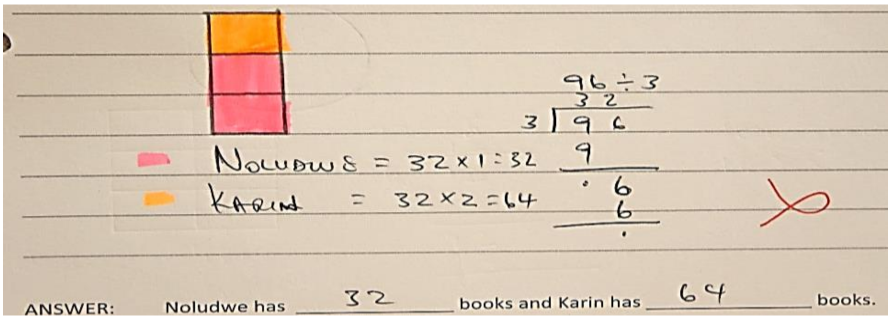



*Figure 4.50.* Example of misunderstanding in Q 4.4 in the Word Problem Calculations, Action Cycle 2, prior knowledge assessment task

Phrases such as, "Noludwe has one third of the number of books" unsettled many participants. Here, mathematics is not tested, but rather, one's proficiency in understanding the quirks found in the English language. A simpler account for the same question might read, "For every three books that Karin has, Noludwe has one."

## 4.8.4 TREND 4: FURTHER EVIDENCE OF DIFFICULTIES WITH RATIOS



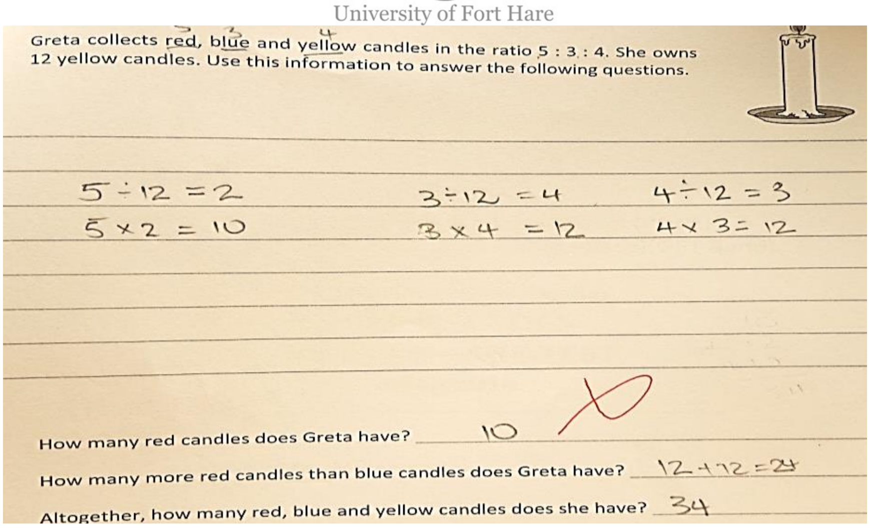



*Figure 4.51.* Example of a student's profound misunderstanding of mathematical concepts and routines as found in Q 4.5 in the Word Problem Calculations, Action Cycle 2, prior knowledge assessment task

In Q 4.5, 24/36 scored 0 for this question, 3/37 scored 1 mark and 9/37 scored 2 marks. Nobody scored either 3 or 4 marks. The class average was 15%. Only 5/36 students produced a visual scaffold for Q 4.5: of these, 3 got it wrong and 2 students achieved 2 marks out of 4. In *Figure 4.51,* above, for example, the exemplar shows that a student first divided 5 by 12 to achieve a quotient of 2, and then multiplied 5 by 2 to achieve 10. Thus, (1) the participant did not perceive that 5 ÷12 = $\frac{5}{12}$ , and (2) did not perceive that, should 5 ÷12 = 2 (it does not), then 2 x 5 = 12 (it is not).

### 4.8.5 TREND 5: FURTHER EVIDENCE OF DIFFICULTIES WITH FRACTIONS

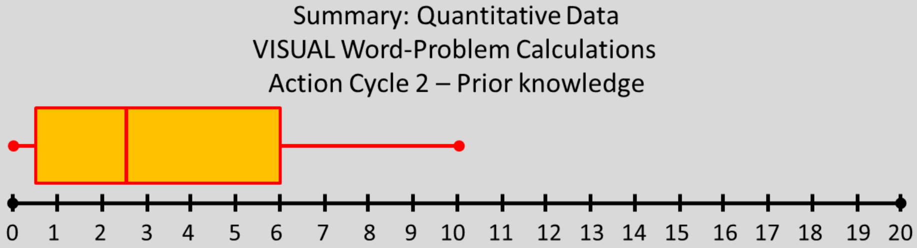
**Q 4.2** ***Altogether Trevor and Peter have 72 rabbits. Trevor owns ⅚ of the rabbits. How many rabbits does Trevor have? How many rabbits does Peter have?***

While the challenge of Q 4.2 did not seem particularly onerous, the participants fared badly with this word-problem and returned a 19% success rate. 26/36 scored 0; 6/36 got 2; 1/36 got 3 and 3/36 students achieved 4 marks for this task. Only 8 participants attempted to construct a visual scaffold. Of these participants, 3 scored 4 marks, that is, full marks, while 1 student obtained 3 marks and 4 students got it completely wrong.



### 4.8.6 SUMMARY OF QUANTITATIVE DATA FOUND IN WORD-PROBLEM CALCULATIONS, ACTION CYCLE 2 PRIOR KNOWLEDGE ASSESSMENT TASK

*Figure 4.52*, below, demonstrates that the participants scored poorly in this prior knowledge assessment task. 36 participants completed the 20 marks assignment, but no one scored more than 10 marks. On the low end, 9/36 participants scored 0; 2/36 got 1, and 7/36 students scored 2 marks each. On the high end, 1/36 got 7 marks; 3/36 participants got 8, and 1/36 achieved 10 marks. The bottom 25% of the students scored between 0 and 1 marks, 50% scored between 1 and 0,5 marks, and the top-performing 25% scored between 6 and 10 marks.  The class average for the word-problems part of the test was 3/20 or 15%. With 9 responses, the mode was 0. The median was 2,5; the range was 10, and the interquartile range was 5,5. The box and whisker plot visually demonstrates that the graph was strongly shifted to the left and that three-quarters of the students scored 5 marks or less for this 20 mark test.




| Quantitative Data | |
|---|---|
| Date of Test | 20/4/2016 |
| Number of Participants | 36 |
| Number who did not write | 2 |
| TOTAL | 38 |
| Mean | 3 |
| Mode (9 responses) | 0 |
| Median | 2,5 |
| Range (10 - 0) | 10 |
| Quartiles | 0,5; 2,5; 6 |
| Interquartile Range (6 – 0,5) | 5,5 |

*Figure 4.52.* Summary of quantitative data captured in the Visual Word Problem Calculations, Action Cycle 2, prior knowledge assessment tasks

Five major trends were revealed: the participants (1) struggled to solve highly-structured, Intermediate Phase word-problems that were set at a*pplying* and *analysing* on Bloom's Revised Taxonomy, (2) were unable to construct visual representations of word-problem scenarios, (3) idiosyncrasies in English language use created confusion, and the arithmetic skills and conceptions that are used in conjunction with (4) ratios and (5) fractions based word-problems were not in place.



Thus, as part answer to sub-research question 1: *What existing word-problem, problem-solving strategies do the students hold,* a meta-trend had begun to develop. Albeit that the questions were designed to review the problem-solving strategies held by the students *at the beginning* of the research, a continuing pattern of illogical solutions demonstrated that the participants had not developed a cogent tool-set of cognitive and practical skills to assist them to solve such problems.

## 4.9 VISUAL FRACTIONS AND RATIO CONCEPTS, ACTION CYCLE 2, PRIOR KNOWLEDGE ASSESSMENT

### 4.9.1 TREND 1: SMALL IMPROVEMENTS

Below, I have presented summaries of the Visual Fractions and Ratio Concepts, Action Cycle 2 prior knowledge assessment task, in Table 4.17, and re-evaluation, in Table 4.18. The full data-set for the re-evaluation s is appended (Appendix O)

Table 4.17

*Summary of data captured in the Visual Fractions and Ratio Concepts, Action Cycle 2 prior knowledge assessment task*

| ACTION CYCLE 2 - PRIOR KNOWLEDGE 36 PARTICIPANTS FRACTIONS AND RATIOS | REMEMBERING FRACTIONS AND RATIOS | | | | | | | | | | | REMEMBERING & UNDERSTANDING | | | | | | | | | | | UNDERSTANDING AND APPLYING | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| Correct responses per Sub Question | 36 | 29 | 26 | 18 | 18 | 24 | 14 | 11 | 12 | 1 | | 13 | 8 | 7 | 8 | 13 | 3 | 18 | 5 | 6 | 6 | | 25 | 2 | 0 | 12 | 22 | 13 | 2 | 8 | 8 | 15 |
| Sub-question Averages (%) | 100 | 81 | 72 | 50 | 50 | 67 | 39 | 31 | 33 | 3 | | 36 | 22 | 19 | 22 | 36 | 8 | 50 | 14 | 17 | 17 | | 69 | 6 | 0 | 33 | 61 | 36 | 6 | 22 | 22 | 42 |
| Correct responses per Main Question | 189 | | | | | | | | | | | 87 | | | | | | | | | | | 107 | | | | | | | | | |
| Main Question Averages (%) | 53 | | | | | | | | | | | 24 | | | | | | | | | | | 30 | | | | | | | | | |



Table 4.18

*Summary of data captured in the Visual Fractions and Ratio Concepts, Action Cycle 2, re-evaluation tasks*

| ACTION CYCLE 2 - RE-EVALUATION 38 PARTICIPANTS FRACTIONS AND RATIOS | REMEMBERING FRACTIONS AND RATIOS | | | | | | | | | | | REMEMBERING & UNDERSTANDING | | | | | | | | | | | UNDERSTANDING AND APPLYING | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| Correct responses per Sub Question | 38 | 8 | 15 | 15 | 28 | 36 | 36 | 21 | 12 | 18 | | 20 | 8 | 11 | 14 | 20 | 5 | 26 | 7 | 16 | 32 | | 36 | 24 | 3 | 16 | 29 | 23 | 22 | 34 | 33 | 30 |
| Sub-question Averages (%) | 100 | 21 | 39 | 39 | 74 | 95 | 95 | 55 | 32 | 47 | | 53 | 21 | 29 | 37 | 53 | 13 | 68 | 18 | 42 | 84 | | 95 | 63 | 8 | 42 | 76 | 61 | 58 | 89 | 87 | 79 |
| Correct responses per Main Question | 227 | | | | | | | | | | | 159 | | | | | | | | | | | 250 | | | | | | | | | |
| Main Question Averages (%) | 60 | | | | | | | | | | | 42 | | | | | | | | | | | 66 | | | | | | | | | |

A comparison between Table 4.17 and Table 4.18 demonstrates that in the re-evaluation task, the results improved, but only marginally so. In Main Question 1, Remembering Fractions and Ratios, the class average moved from 53% to 60%, a jump of 7% or a 13% improvement on the original result. Main Question 2, Remembering and Understanding, started at 24% and moved to 42%, a jump of 18%, or a 75% improvement on the prior knowledge. Main Question 3, Understanding and Applying, moved from 30% to 66%, generating a jump of 36% or an improvement of 220% on the prior knowledge assessment tasks result.

### 4.9.2 TREND 2: A SIGNIFICANT DECLINE

In Main Question 1, the result for Q 1.2 dropped from 29/36 (81%) to 8/38 (21%). While this reversal was initially startling, in truth, it occurred because I marked the question according to two different standards. In the prior knowledge task, I accepted 3/12 as a correct answer, but, by the time the class wrote the re-evaluation, I had worked through many ideas and examples of *equivalence of fractions* and the standardised requirement to always reduce fractions to their simplest form. Thus, in the re-evaluation, only $\frac{1}{4}$ as a final answer was marked correct.

### 4.9.3 TREND 3: MORE EVIDENCE OF LACK OF COMPRESSION OF FRACTIONS FACTS AND CONCEPTS

The prior knowledge assessment results for Q 1.10 were 1/36 (3%); the re-evaluation achieved 18/38 (47%). In Q 1.10, participants were required to establish how many times bigger 1⅔ was than ⅙. The answer was 12. A visual clue, in the form of two *boxes* was presented. Each of the boxes was sub-divided into thirds.

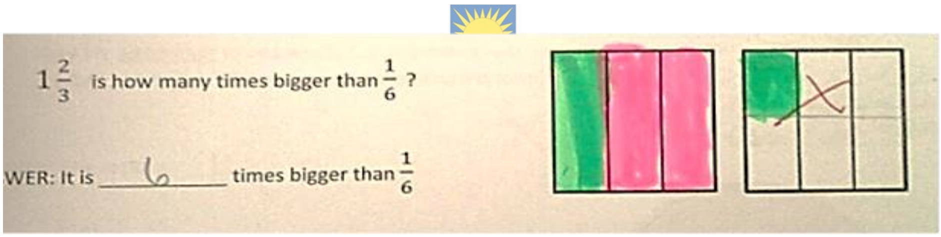



*Figure 4.53.* An exemplar which demonstrates an inability to use the supplied visual cues to solve Q 1.10 in the Visual Fractions and Ratio Concepts, Action Cycle 2, re-evaluation

I had hoped that the students would perceive the gestalt cues in the boxes and would use them as a visual-mental bridge to reconstruct the thirds as sixths. I anticipated that they would construct a robust visual representation of 1⅔ in ten of twelve equal parts. However, as seen in *Figure 4.53*, above, when the re-evaluation was written, many students had not assimilated, accommodated and compressed a fractions meta-concept. *This meant that after two action cycles, the students were unable to form either mental or physical images of routine arithmetic situations. Further, in the absence of the use of a visual solution, they could not construct an arithmetic algorithm nor carry out a simple mathematical calculation*.

### 4.9.4 TREND 4: ANOTHER, SOMEWHAT PUZZLING, DECLINE

The prior knowledge assessment result for Q 1.4 was 18/36 or 50%. In the re-evaluation, this score dropped to 15/38 or 39%. While I was not too sure why the return for Q 1.4 slumped at all, I believe that the newly acquired visualising techniques had not been properly assimilated, thus a cognitive dissonance – a disequilibrium – was in place.

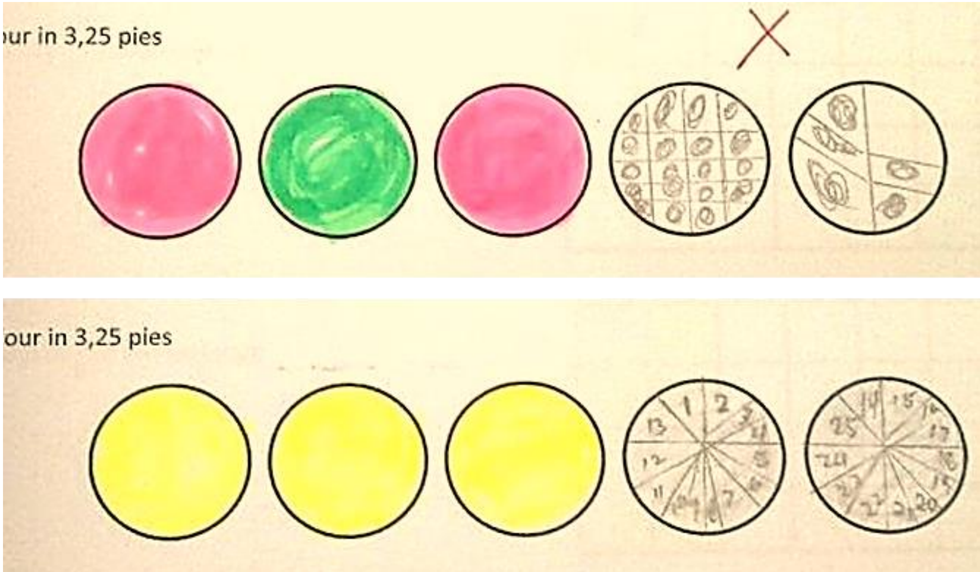





*Figure 4.54.* Two exemplars which demonstrate an inability to construct a visual presentation of a decimal fraction as required in Q 1.10 in the Visual Fractions and Ratio Concepts, Action Cycle 2 re-evaluation

The task required the students to colour in 3,25 circles. However, as seen in *Figure 4.54*, above, most students coped with colouring three wholes, but struggled with the display of decimal aspect of the construction.

Some students struggled with the concepts of fractions and ratios in general, thus, although it was disappointing to see a drop in the class mark, it restated a requirement to work continually at eliminating all foundational misunderstandings with fractions. In quiet reflection, I also conceded that it was possible that I had over-explained or over-simplified this work, thereby inadvertently obfuscating the very knowledge and skills bases needed to solve these simple questions. Nonetheless, the low return on investment proved a bitter pill to swallow.

### 4.9.5 TREND 5: MORE EVIDENCE OF INABILITY TO INTERPRET FIGURAL INFORMATION (IFI) IN A VISUAL REPRESENTATION

In Main Question 2, Q 2.6 attracted the lowest return. In the prior knowledge assessment, the return was 3/26 (8%) while the re-evaluation returned 5/38 (13%). Although there were many combinations of mistakes made in this sub-question, almost without fail the participants who got it wrong incorrectly selected the image of the square as representative of ⅖.

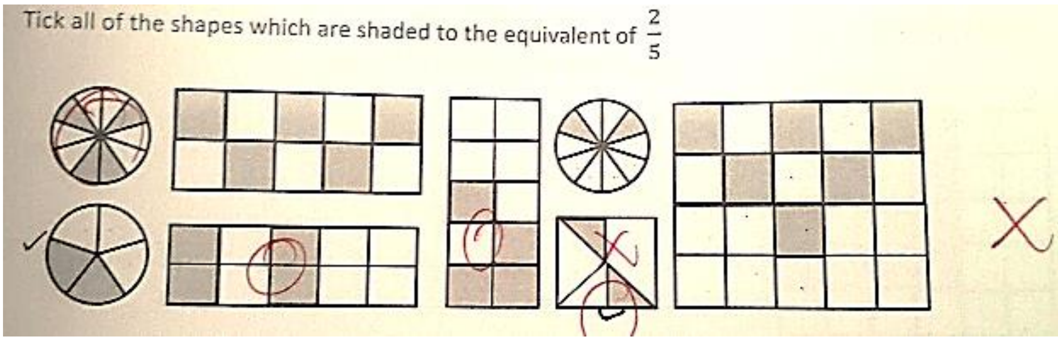



*Figure 4.55.* Misidentification of visual representations of $\frac{2}{5}$ in Q 2.6 in the Visual Fractions and Ratio Concepts, Action Cycle 2 re-evaluation



This particular question, its answer and many questions similar to it, was extensively covered in the IPS 413 E Mathematics module. However, *Figure 4.55,* above, showed that while the student had correctly identified one of the circles as representative of $\frac{2}{5}$, a second circle and two rectangular strips were ignored.

Sadly, the evidence captured from the responses to Q 2.6 suggested that many participants were not able to interpret the figural information that was presented to them in the sketches (IFI), that foundational low order thinking (LOT) was not applied, that simple remembering skills were not available to the students, that intuition and a combination of L-mode and R-mode thinking did not occur and that the compression of the concepts of *equal-parts* and *equivalence,* ($\frac{4}{10} = \frac{2}{5}$) in fractions had not been accommodated into a meta-concept of fractions. Further, it has to be remembered that this actual question was revised on a number of occasions, thus the poor return is difficult to understand.

### 4.9.6 TREND 6: MORE EVIDENCE OF AN INABILITY TO PLOT A NUMBER ON A NUMBER LINE

For Q 2.8, the prior knowledge assessment returned a score of 5/36 (14%); in the re-evaluation, it crept up slightly to 7/38 (18%).

Albeit that the class had been immersed in mathematical routines linked to operations with fractions throughout the semester, as seen in *Figure 4.56*, below, the calculation of a product of a fraction and whole number and the plotting of that product onto a number line continued to prove difficult for this cohort.

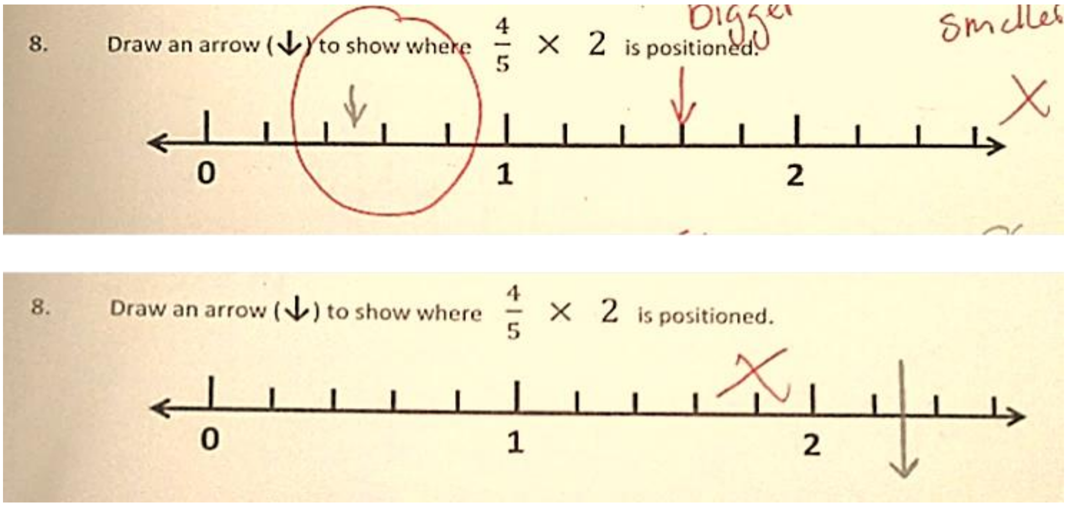



*Figure 4.56.* Incorrectly plotted representations of $\frac{4}{5} \times 2$ as found in Q 2.8 in the Visual Fractions and Ratio Concepts, Action Cycle 2 re-evaluation

### 4.9.7 TREND 7: MORE EVIDENCE OF POOR UNDERSTANDING OF RATIOS

The returns on Q 2.2, with 8/36 (22%) in the prior knowledge assessment and a result of 8/38 (21%) in the re-evaluation, gave rise to great concern. This poor uptake, as seen in *Figure 4.57*, below, was problematic for a number of reasons: (1) the actual asking requirement, cognitively speaking, was of a low order; (2) the *identical* question was offered in the prior knowledge assessment task and was subsequently carefully scaffolded in the classroom so as to give experiential and cognitive access to the problem; and (3), in class-time, using actual biscuits and other manipulatives and iconic stimuli, I spent a great deal of time exemplifying fractions and ratios in numerous, contextualised, sharing situations.

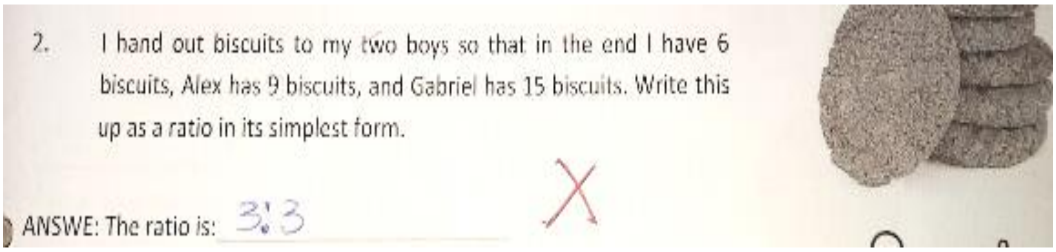



*Figure 4.57.* An incorrect interpretation of a ratio as found in Q 2.2 in the Visual Fractions and Ratio Concepts, Action Cycle 2 re-evaluation

Indeed, the participants used found materials to construct visual representations of fractions and ratios. I found that some participants did not simplify 6 : 9 : 15 into the ratio 2 : 3 : 5, and thereby did not obtain the mark. Many others, however, could not construct any form of cogent solution. Indeed, many students constructed two-part ratios and ignored the fact that the relationship was between three parties.

### 4.9.8 TREND 8: MORE EVIDENCE THAT STUDENTS CANNOT APPLY THE GRID METHOD FOR MULTIPLICATION

In the prior knowledge assessment, all of the students incorrectly answered Q 3.3; however, at that time the participants had not used the *Grid Method* as an application of the distributive law for multiplication. By the time the re-evaluation was applied, that situation had been corrected and yet, in the re-evaluation, achievement was only 3/38 (8%). Below, in *Figure 4.58*, I have included one exemplar of a correct set of possible solutions.



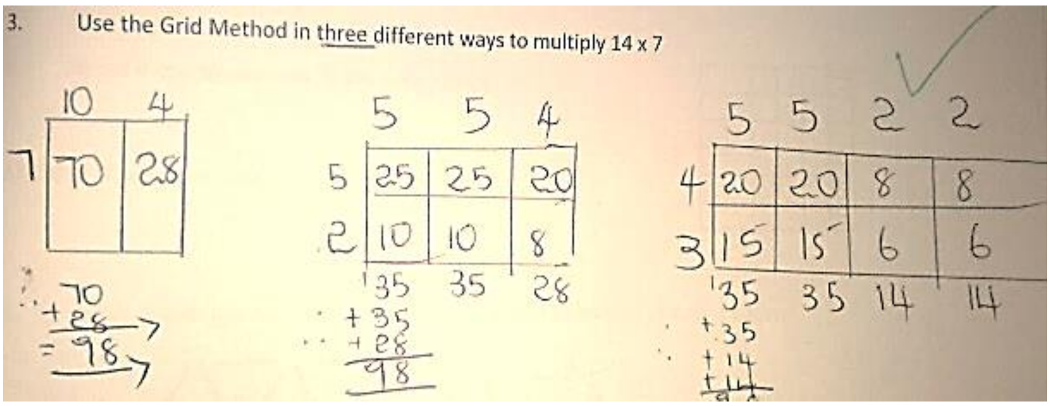



*Figure 4.58.* A set of correct responses to Q 3.3 in the Visual Fractions and Ratio Concepts, Action Cycle 2 re-evaluation

That so many students could not correctly complete this sub-question was problematic on at least two levels. First, the technique was presented to the students as a method for *teaching;* clearly, many would not be able to use this multiplication methodology in a classroom. Second, a further distress was that the participants continued to not be able to multiply accurately, nor could they judge the validity of their solutions.

This perplexing situation reinforced my view that the participants were doing little continuous self-study and instead might be cramming or learning by rote, and thereby overloading working memory.

### 4.9.9 TREND 9: MORE EVIDENCE OF LACK OF UNDERSTANDING OF POWERS AND BASES

Q 3.4 was presented as a number line onto which the numbers 1; 2; 4 and 8 had been encircled. Each of these numbers are consecutive powers: $2^0$; $2^1$; $2^2$ and $2^3$ respectively. I anticipated that the students would use the visual clues presented by the numbers on the line to establish 16, that is, $2^4$, as the next number to circle.

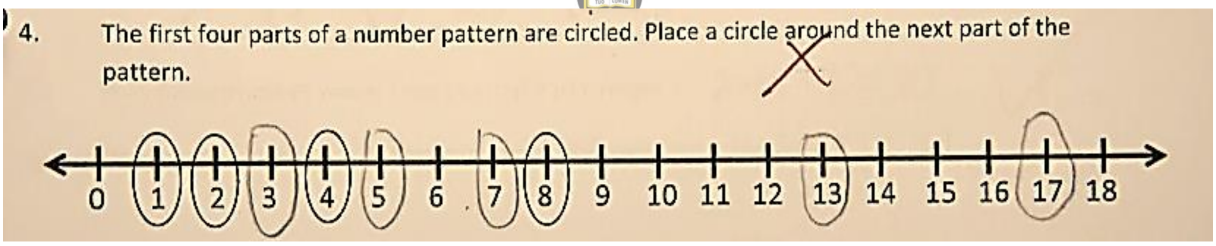



*Figure 4.59.* An incorrect response to Q3.4 in the Visual Fractions and Ratio Concepts, Action Cycle 2 re-evaluation

In *Figure 4.59*, above, the student appears to have no conception of number patterns and seemed to have arbitrarily inserted responses. Only 16/38 (42%) of the students managed to achieve the correct answer. While this was an improvement on the prior knowledge assessment, which came in at 12/36 (33%), the re-evaluation results suggested that the compression of important foundational arithmetic concepts was proving elusive.

## 4.9.10 TREND 10: MORE EVIDENCE OF ABSENT INTERPRETING FIGURAL INFORMATION CAPABILITY

In the prior knowledge assessment, the return for Q 3.6 was 13/36 (36%). In the re-evaluation, it was 23/38 (61%). Q 3.6, required participants to apply IFI skills to visualise powers of 4, namely $4^0$, $4^1$, $4^2$, $4^3$ to achieve the answer, that is, $4^4$ or 256.

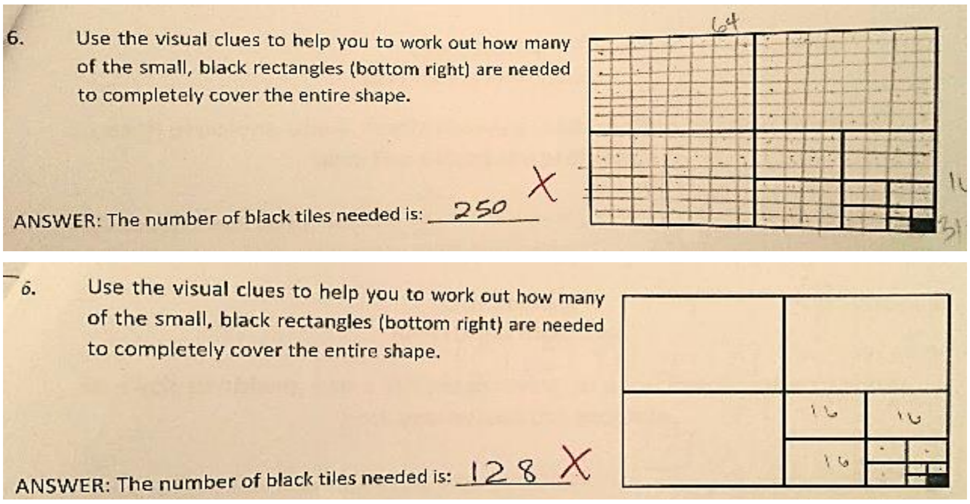



*Figure 4.60.* Two incorrect solutions to Q 3.6 in the Visual Fractions and Ratio Concepts, Action Cycle 2, re-evaluation tasks

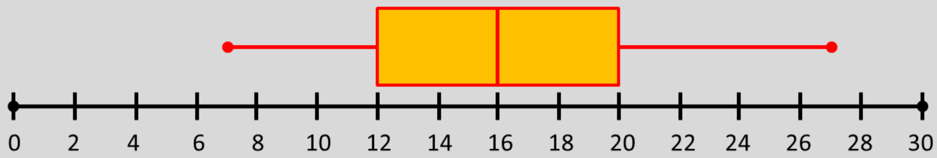


In the upper solution in *Figure 4.60*, above, a participant constructed an appropriate visual scaffold but incorrectly provided 250 as either the product of 16 x 64 or 64 x 4 or the sum of 64 + 64 + 64 +64. In the lower solution, the participant shows some sense of an IFI strategy, but failed to add or multiply correctly and achieved an incorrect answer of 128.

## 4.9.11 SUMMARY OF QUANTITATIVE DATA FOUND IN VISUAL FRACTIONS AND RATIO CONCEPTS, ACTION CYCLE 2, RE-EVALUATION

Overall, if not pleased, I was somewhat relieved with the summary of the data set for the Visual Fractions and Ratios – Action Cycle 2 – re-evaluation, as presented in *Figure 4.61,* below. When compared to the prior knowledge assessment tasks, the box and whisker plot for the re-evaluation shifted a long way to the right and that was good news. It suggested that the effects of the module and the visualising scaffolds that I had been using in the IPS 413 E Mathematics classes were beginning to bear fruit. Finally!

Summary: Quantitative Data
VISUAL FRACTIONS and RATIOS CONCEPTS
Action Cycle 2 – Re-evaluation




| Quantitative Data | |
|---|---|
| Date of Test | 16/5/2016 |
| Number of submitted Scripts | 38 |
| Number who did not write | 0 |
| TOTAL | 38 |
| Mean | 17 |
| Bi-Modal (4 occurrences each) | 12; 19 |
| Median | 16 |
| Range (27 - 7) | 20 |
| Quartiles | 12; 16; 20 |
| Interquartile Range (20 - 12) | 8 |

*Figure 4.61.* Summary of quantitative data captured in the Visual Fractions and Ratios Concepts, Action Cycle 2 re-evaluation



38 participants completed the 30 mark re-evaluation task. Low performing students, that is, the bottom 25% of the students, scored between 7 and 12 marks. 50% of the students obtained between 12 and 20 marks, and the top 25% scored between 20 and 27 marks out of a possible 30. The class average, the mean, was 17 marks. The data was bi-modal, 12 and 19 (4 occurrences each). The median was 16. The range was wide, coming in at 20 (27 – 7), and the interquartile range, at 8 (20 – 12) placed the middle 50% of the students within 8 marks of each other or a 38% spread.

However, with a median of 16 and a mean of 17, in a re-evaluation of mathematical content that was previously presented and which received considerable remedial attention, is problematic. Philosophically and ontologically, I had tried to create a nurturing and liberating teaching and learning environment. Pedagogically, methodologically and epistemologically, I had tried to create multi-pronged, activities-based, enactive and ironically rich learning experiences. The students had appeared to really appreciate these approaches, but the quantitative data above stubbornly portrays only limited improvements.

## 4.10 WORD-PROBLEM CALCULATIONS, ACTION CYCLE 2, RE-EVALUATION TASK

### 4.10.1 TREND 1: SOME BIG GAINS

For clarity and comparison, I have inserted a summary of the data-set for the prior knowledge assessment task, Table 4.19, and the re-evaluation, Table 4.20, of the word-problems calculations, below. The full data-set are appended (Appendix P).

Table 4.19

*Summary of data captured in the Word-Problem Calculations, Action Cycle 2, prior knowledge assessment*

| ACTION CYCLE 2 PRIOR KNOWLEDGE 36 PARTICIPANTS WORD PROBLEM CALCULATIONS | APPLYING AND ANALYSING | | | | |
|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 |
| Correct responses per Sub Question | 42 | 27 | 24 | 4 | 21 |
| Sub-question Averages (%) | 29 | 19 | 17 | 3 | 15 |
| Correct responses per Main Question | 118 | | | | |
| Main Question Averages (%) | 16 | | | | |



Table 4.20

*Summary of data captured in the Word Problem Calculations, Action Cycle 2, re-evaluation*

| ACTION CYCLE 2 - RE-EVALUATION 38 PARTICIPANTS WORD PROBLEMS | APPLYING AND ANALYSING | | | | |
|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 |
| Correct responses per Sub Question | 116 | 110 | 100 | 122 | 90 |
| Sub-question Averages (%) | 76 | 72 | 66 | 80 | 59 |
| Correct responses per Main Question | 538 | | | | |
| Main Question Averages (%) | 71 | | | | |

The prior knowledge assessment returned a success rate of 16% while the re-evaluation delivered 71%. *I am overjoyed by this result*. It constitutes a numerical jump of 55% or, compared with the prior knowledge assessment tasks, an overall improvement of 444%. In the prior knowledge assessment, the worst-answered question, Q 4.4, attracted a 3% success rate; when re-evaluated, this shot up to 80%. Similarly, in the prior knowledge assessment Q 4.5 returned 15% while, when re-evaluated, the success rate jumped to 59%.

### 4.10.2 TREND 2: LOW LEVELS OF CONSTRUCTION OF VISUAL SCAFFOLDS

The data-sets for Q 4.1 show that in the prior knowledge assessment, the participants achieved a 29% success rate. When re-evaluated, this figure rose to 76%. Below, Q 4.1 is transcribed and *Figure 4.62* presents an incorrect solution.

> ***Q 4.1: Alexander and Gabriel found money in the fields. Alexander found R 155,00 and Gabriel found R 60,00 more than Alexander. Altogether, how much money did the two boys find in the field?***

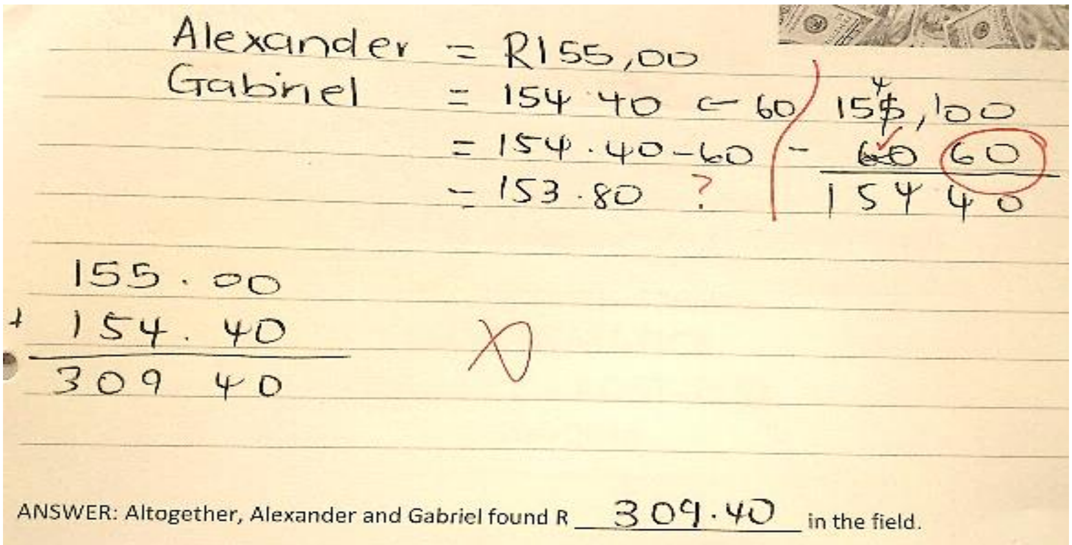



*Figure 4.62.* An incorrect response to Q 4.1 as captured in the Word-Problem Calculations, Action Cycle 2 re-evaluation

*Figure 4.62*, above, presents at least two causes for concern, those being that (1) although it was required, the participant, like others in the cohort, either did not or could not draw a visual representation of the word-problem, and (2) the computation suggests a misunderstanding of adding and subtracting with rands and cents.

Indeed, in one of a number of errors, as seen on the right side of Figure 4.62, it appears that the participant incorrectly subtracted R60,00 from R155,00 to achieve R154,40. Following this mistake, it can be seen that the participant then constructs a calculation in which 154,40 - 60 to–achieve 153,80.

### 4.10.3 TREND 3: SOME WELL CONSTRUCTED VISUAL REPRESENTATIONS

In the course of the semester, the participants encountered many visualising scaffolds – novel for many of them – so as to bring some *clarity and understanding* into their world-view of mathematics. An overview of their scripts suggested that this approach was gaining some success. In answering Q 4.4, most of the participant drew a visualising scaffold. Some students applied their visual scaffolds very fruitfully, while others were not as successful.

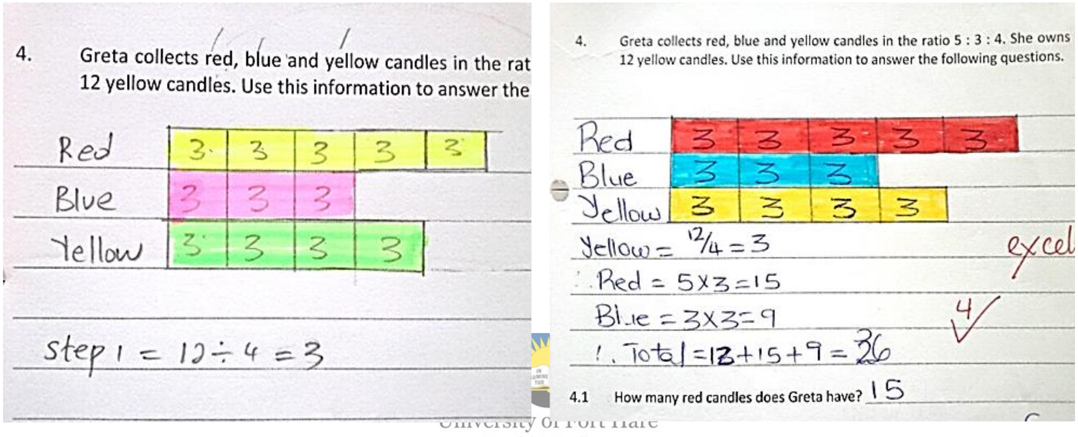





*Figure 4.63.* Two well-constructed responses to Q 4.4 as captured in the Word-Problem Calculations, Action Cycle 2 re-evaluation

In the prior knowledge assessment task, Q 4.4, achieved only a 3% success rate, but in the re-evaluation, the question achieved a much more respectable 80% return. I believe that the overall success can be attributed, in part, to the use of the visualising scaffolds that the students used in the classroom.

The correct solutions to Q 4.4, as seen in *Figure 4.63*, above, demonstrated the compression of ratios, fractions, multiplication and a capacity to represent a word-problem in a cogent, visual manner, such that the visual clues offered in the sketches were easily understood by self and others. By taking the time to quietly contemplate and then construct a visual scaffold, the students opened up cognitive space to facilitate cogent and logical thinking. For many, the approach paid off handsomely.

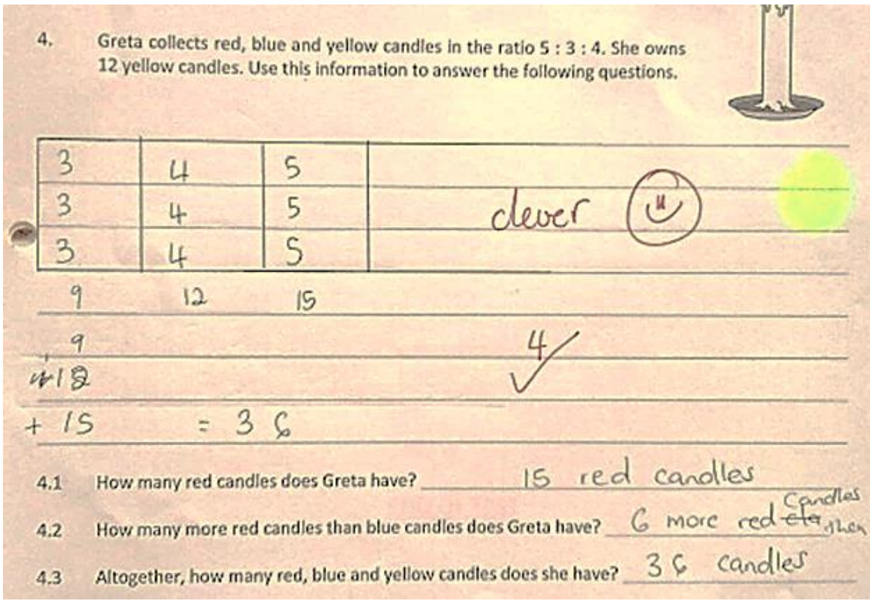


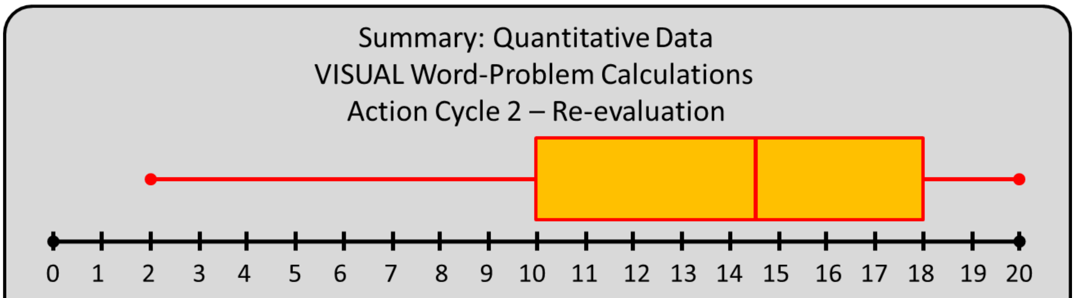

*Figure 4.64.* A novel response to Q 4.4 as captured in the Word-Problem Calculations, Action Cycle 2, re-evaluation

In *Figure 4.64*, above, a participant applied a different and novel visual solution strategy. And *that* was really exciting. This suggests that, in this student at least, the learning of fractions and ratios has been assimilated and accommodated, a robust meta-concept of fractions has been conceptualised, compression effected, and that the student was capable of evaluating and creating personalised visual solutions.



## 4.10.4 SUMMARY OF QUANTITATIVE DATA FOUND IN WORD-PROBLEMS CALCULATIONS, ACTION CYCLE 2, RE-EVALUATION TASK

Of all the box and whisker plots that I constructed for this action research, the graph below was the most satisfying. Although the bottom 25% of the students, that is, the lower whisker in *Figure 4.65,* below, continued to doggedly present low marks, many of the students were starting to show marked progress in the IPS 413 E Mathematics module and the graph was shifted markedly to the right. However, while the quantitative data presented a somewhat optimistic view of performance, I was also cognisant of the fact that the test was a repetition of the prior knowledge assessment tasks – any novelty in the questions had been eliminated by the action research – and the test questions were offered at an Intermediate Phase level.




| Quantitative Data | |
|---|---|
| Date of Test | 16/5/2016 |
| Number of Participants | 38 |
| Number who did not write | 0 |
| TOTAL | 38 |
| Mean | 14 |
| Bi-Modal (7 occurrences each) | 14; 18 |
| Median | 14,5 |
| Range (20-2) | 18 |
| Quartiles | 10; 14,5; 18 |
| Interquartile Range (18-10) | 8 |

*Figure 4.65.* Summary of quantitative data captured in the Word Problems-Calculations, Action Cycle 2, re-evaluation tasks

In total, 38 participants completed the 20 mark, re-evaluation task. Three-quarters of the participants scored at least half marks, and that was important because these marks contributed to the Duly Performed mark. The lowest 25% of the class obtained between 2 and 10 marks; 50% of the students achieved marks between 10 and 18, and the top-performing 25% of the students achieved 18 to 20 marks out of a possible 20 marks.



In terms of the extremes, on the low side, 1 student each scored 2/20 and 6/20 respectively, while 7 students scored 18/20, 1 got 19/20, and 5 students achieved 20/20 on the high side. The class mean was 14; a bi-modal effect was found at 14 and 18 (7 occurrences each), and the median was 14,5. The range was 18 (20 – 2), and the interquartile range was 8 (18 – 10). As part answers to sub-research question 1 (1.5), this data suggests that, through the auspices of the action research and the construction of underpinning arithmetic skills, relational understanding, compression of meta-concepts of fractions and ratios and visualisation, the students were able to offset Gooding's (2009) barriers and thereby enjoy a modicum of success in solving word-problems.

## 4.11 QUESTIONNAIRE 1

I planned to administer Questionnaire 1 in the first week of semester 1 in 2016. However, my ethical clearance certificate and number were not made available to me until 22 March 2016. After the Easter holidays and two weeks of practicums, the questionnaire was presented to participants on 22 April 2016.

The questionnaire contained two sections. Part A explored biographical and language information, and personal attitudes towards mathematics and English as the medium of instruction when learning mathematics. Part B answered to the 2nd sub-research question: *What barriers to solving word-problems do the students perceive that they hold?* Questionnaire 1 is appended (Appendix Q). The primary data is also appended (Appendix R). Albeit that there were 60 registered students in the IPS 413 E Mathematics class, at the time the questionnaire was administered only 33 students had become participants in the research, and so the data was confined to information gleaned from those participants.

### 4.11.1 QUESTIONNAIRE 1 – PART A



#### 4.11.1.1 PART A – A1 - BIOGRAPHICAL AND LANGUAGE INFORMATION

Part A – A1, contained four sub-strands, namely gender, home language, current age and highest pass in school in either mathematics or mathematical literacy. A summary of the data is presented in Table 4.21, below.

Table 4.21

*Summary of data captured for Part A – A1 of Questionnaire 1*

| PART A - A1 - BIOGRAPHICAL AND LANGUAGE INFORMATION | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 I am: | | 2 My home language is: | | | | 3 My current age is: | | | | | 4 My highest school pass in math or math literature is: | | | | |
| female | male | Afrikaans | English | isiXhosa | other | 20-25 | 26-30 | 31-35 | 36-40 | older | Grade 8 | Grade 9 | Grade 10 | Grade 11 | Grade 12 |
| 24 | 9 | 0 | 2 | 27 | 4 | 15 | 9 | 8 | 0 | 1 | 0 | 1 | 5 | 2 | 24 |

In Table 4.21, above, it can be seen that mostly the participants were female (24/33); were isiXhosa speaking (27/33), were between 20 and 25 years old (15/33), and had passed mathematics or mathematical literacy at a Grade 12 level (24/33).

### 4.11.1.2 PART A – A2 – PERSONAL ATTITUDE TOWARDS MATHEMATICS AND IPS 413 E - MATHEMATICS

Part A – A2 contained three Likert scale questions which were designed to provide some insights into the students' own perceptions of mathematics and of being in the IPS 413 E – Mathematics class. Table 4.22, below, presents a summary of that data.

Table 4.22

*Summary of data captured for Part A – A2 of Questionnaire 1*

| PART A - A2 - STUDENTS' PERCEPTIONS OF MATHEMATICS AND IPS 413 E | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 5 Generally, I enjoy doing mathematics | | | | | 6 Generally, I am good at doing mathematics | | | | | 7 I am excited to be doing IPS 413 E | | | | |
| Strongly disagree | Disagree | In the middle | Agree | Strongly agree | Strongly disagree | Disagree | In the middle | Agree | Strongly agree | Strongly disagree | Disagree | In the middle | Agree | Strongly agree |
| 1 | 1 | 10 | 15 | 6 | 1 | 6 | 18 | 6 | 2 | 0 | 1 | 5 | 13 | 14 |

It can be seen that, mainly, the participants were happy to be doing mathematics (15/33), were ambivalent about their own mathematical competence (18/33), and were very pleased to be in the IPS 413 E – Mathematics class (14/33).



Two distortions affected the data: (1) only 33 of the 60 strong cohort participated in the questionnaire – thus the attitude of the other students was unknown, and (2) instead of being issued in the first week of the semester, the questionnaire was administered in the second quarter of 2016. This meant that the contact sessions in the first quarter would have influenced the replies.

### 4.11.1.3 PART A – A3 – FLUENCY IN ENGLISH WHILE DOING MATHEMATICS

In 4.11.1.1, I have established that 27 of the participants and, in all likelihood, others who were not participating in the research, were isiXhosa speakers. Only 2 participants indicated English as a home language.

Part A – A3, asked three questions, each designed to elicit a sense of how well the participants might cope with instruction in English while studying mathematics. The primary data is summarised in Table 4.23, below.

Table 4.23

*Summary of data captured for Part A – A3 of Questionnaire 1*

| PART A - A3 - FLUENCY IN ENGLISH LANGUAGE WHEN DOING MATHEMATICS | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 8<br>I find it easy to listen to and understand English instructions about mathematics | | | | | 9<br>I find it easy to read to and understand English instructions about mathematics | | | | | 10<br>I find it easy to talk to and understand English instructions about mathematics | | | | |
| Strongly disagree | Disagree | In the middle | Agree | Strongly agree | Strongly disagree | Disagree | In the middle | Agree | Strongly agree | Strongly disagree | Disagree | In the middle | Agree | Strongly agree |
| 1 | 3 | 10 | 12 | 7 | 0 | 4 | 8 | 18 | 3 | 0 | 5 | 7 | 13 | 8 |

The data revealed that the students felt comfortable listening to and understood English instructions about mathematics (12/33); many indicated that they could read and understand English instructions about mathematics quite well (18/33), and many also suggested that they found it quite easy to talk about mathematical ideas (13/33).

## 4.11.2 QUESTIONNAIRE 1 – PART B

Part B contained five questions: each question contained two Likert scale questions and one specified-response component. The questions probed the barriers Gooding (2009, p. 5) suggested students might have when they solve word-problems.



### 4.11.2.1 PART B – B1 – READ AND UNDERSTAND WORD-PROBLEMS

Gooding (2009, p. 5) suggests that some students may not be able to read and understand word-problems. By default, this would preclude them from success. In Part B – B1, two questions asked the students to provide their own views of their reading and understanding competences. Table 4.24, below, provides a summary of the answers obtained from Likert scale questions 11 and 12.

Table 4.24

*Summary of data captured for Part B – B1 of Questionnaire 1*

| PART B - B1 - READ AND UNDERSTAND WORD PROBLEMS | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 11<br>I find it easy to read word problems | | | | | 12<br>I find it easy to understand what word problems require me to do | | | | |
| Strongly disagree | Disagree | In the middle | Agree | Strongly agree | Strongly disagree | Disagree | In the middle | Agree | Strongly agree |
| 0 | 3 | 19 | 10 | 1 | 2 | 5 | 16 | 9 | 1 |

In Q 11, it can be seen that, while a few students struggled (3/33) to read word problems, most students (19/33) were ambivalent regarding their understanding of what they read, while 10/33 suggested that were quite comfortable and one student (1/33) was strongly capable of reading word-problems. Similar results were obtained for Q 12. Three students' comments are transcribed below:

1. *Understanding word problems for me is sometimes confusing and I end up not getting what is required of me*
2. *I find it hard sometimes to understand the statement of the question especially when it has quarters and half quarters, 3 times of quarters*
3. *I find it easy to read but I lack understanding what is it that I have to do that is where I mess up and end up getting a zero*

4.11.2.2 PART B – B2 – FORM MENTAL IMAGES OF WORD PROBLEMS

Table 4.25, below, provides a summary of data captured for Part B – B2 in the questionnaire. The questions tried to establish the students' views of their ability to construct mental images, that is, to visualise mathematical word-problems.

Table 4.25

*Summary of data captured for Part B – B2 of Questionnaire 1*



| PART B - B2: FORM MENTAL IMAGES OF WORD PROBLEMS | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 13<br>I find it easy to construct a mental image of the requirements of word problems | | | | | 14<br>I find it easy to imagine a solution strategy for solving word problems | | | | |
| Strongly disagree | Disagree | In the middle | Agree | Strongly agree | Strongly disagree | Disagree | In the middle | Agree | Strongly agree |
| **2** | **8** | **12** | **10** | **1** | **2** | **11** | **13** | **6** | **1** |

In both questions, Q 13 and Q 14, the data consolidated about the *middle* option. The comments that the students wrote indicated that they found visualising scaffolds of mathematical scenarios and solution strategies confusing. Below, three comments present a range of views on their capacity to visualise word problems.

1. *I think if I can be able to construct mental images it can be easy to solve some problem*
2. *When it comes to word problems in mathematics, I just get confused and end up not knowing what to do.*
3. *I have no clue when it comes to construct a mental image; I am in the dark.*

#### 4.11.2.3 PART B – B3 – SUCCESSFULLY CONSTRUCT A NUMBER SENTENCE OR FORMULA TO START TO SOLVE WORD-PROBLEMS

Table 4.26, below, demonstrates that the students' perceptions of their ability to find starting points and to construct a number sentence or formula to solve word problems was somewhat cautious. Their caution was well-founded. In this chapter, (4.3; 4.4; 4.5 and 4.6) many trends in Action Cycle 1 attested to numerous underlying glitches that were connected with setting-out and an inadequate capacity to construct logical starting points for mathematical problems.

Table 4.26

*Summary of data captured for Part B – B3 of Questionnaire 1*

| PART B - B3: SUCCESSFULLY CONSTRUCT A NUMBER SENTENCE OR FORMULA TO START TO SOLVE WORD PROBLEMS | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 15<br>I find it easy to find a starting point to begin to solve work problems | | | | | 16<br>I find it easy to construct an appropriate number sentence or formula to solve problems | | | | |
| Strongly disagree | Disagree | In the middle | Agree | Strongly agree | Strongly disagree | Disagree | In the middle | Agree | Strongly agree |
| 1 | 10 | 16 | 5 | 1 | 2 | 8 | 18 | 4 | 1 |



Mostly, students suggested that they did not know where to begin to start to solve a problem, and many confessed that they were not able to construct algorithms and formulae. Three students' comments are presented below.

1. *I sometimes need an example that would unlock my thinking*
2. *No, I am struggling to begin in solving word problems*
3. *I am clueless, I am lost, and I am really in the dark*

#### 4.11.2.4 PART B – B4 – CAN DO THE ACTUAL CALCULATIONS NEEDED TO SOLVE WORD PROBLEMS

The data in Table 4.27, below, demonstrated that, generally, the students were undecided to slightly optimistic about their own capacity to work with numbers and mathematical operations and perform calculations. However, the data captured by the prior knowledge assessments and re-evaluation tasks in Action Cycle 1 (4.3; 4.4; 4.5 and 4.6) contradicted this position very strongly. That data demonstrated that most of the students were not able to do low-level, routines-based arithmetic.

Table 4.27

*Summary of data captured for Part B – B4 of Questionnaire 1*

| PART B - B4: CAN DO THE ACTUAL CALCULATIONS NEEDED TO SOLVE THE WORD PROBLEMS | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 17<br>I find it easy to work with numbers and operations (+ - x / √, etc.) | | | | | 18<br>I find it easy do to the calculations which are needed to solve word problems | | | | |
| Strongly disagree | Disagree | In the middle | Agree | Strongly agree | Strongly disagree | Disagree | In the middle | Agree | Strongly agree |
| 0 | 3 | 12 | 12 | 5 | 0 | 5 | 12 | 11 | 3 |

The students' comments, as seen below, were indicative of the true situation and buttressed the trends in the quantitative data that was captured in Action Cycle 1.

1. *When I am given a normal sum it is easier for me to understand and complete. And when doing the calculations for word problems I am never sure due to the fact that I am not sure if I am doing the correct thing or not.*
2. *With operations especially mixed I end up confused, I would do better if its one operation problem at a time*
3. *The operation of times I find it a bit workable however the square root or this √ it's a nightmare*



4.11.2.5 PART B – B5 – ABILITY TO JUDGE THE VALIDITY OF A CALCULATED ANSWER

As seen in Table 4.28, below, most of the participants (12/33) could not easily judge what the size and shape of the final answer of word-problems would look like. However, many (12/33) believed that once they achieved an answer, they were moderately secure that they would know if the answer was acceptable.

Table 4.28

*Summary of data captured for Part B – B5 of Questionnaire 1*

| PART B - B5: ABILITY TO JUDGE THE VALIDITY OF A CALCULATED ANSWER | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 19<br>I find it easy to judge approximately what my final answer will be | | | | | 20<br>I find it easy to look at my answers and judge if they look okay | | | | |
| Strongly disagree | Disagree | In the middle | Agree | Strongly agree | Strongly disagree | Disagree | In the middle | Agree | Strongly agree |
| 3 | 9 | 12 | 7 | 1 | 2 | 6 | 10 | 12 | 1 |

Many participants did not provide short-response answers for Q19 and Q20. Those that did, spoke to an inability to perceive the validity of a calculated answer. Three student responses are provided below.

1. *When it is a normal sum then yes but not with word problems as I am never sure if I am doing the correct thing.*
2. *When I am judging the validity of my it depends on how much I know the problem. When I am sure about it, I can reverse it and prove it. But if I am not sure, I just calculate and sometimes get it right. When a have no idea, I calculate and assume that it is correct.*
3. *I have to calculate before I get correct answer, no I cannot estimate.*

### 4.11.3 SUMMARY OF QUESTIONNAIRE 1

Part A of the questionnaire demonstrated that most of the participating students were female, isiXhosa speaking, 20 to 25 years old and had passed either Mathematics or Mathematics Literacy at a Grade 12 level. Many participants professed to enjoying and doing mathematics and were pleased to be in the IPS 413 E Mathematics module. Most participants indicated that they felt secure in listening, reading and talking about mathematics with understanding.



In Part B, Gooding's (2009, p. 5) five barriers were used to frame questions regarding the students' perceptions of their capability to attend to and solve word problems. The Likert scale tables suggested that the participants were ambivalent to moderately secure in their mathematical capabilities. The short specified responses provided a sober overview of the participants' insecurities, mixed feelings towards mathematics and reflections on their own mathematical capabilities. The textual responses triangulate closely with the quantitative data that was captured in the Action Cycle 1 prior knowledge and re-evaluation tasks.

## 4.12 QUESTIONNAIRE 2

Questionnaire 2 was designed to be administered towards the end of Action Cycle 2, that is, after the students had received instruction on visual scaffolds as a teaching and learning methodology. Questionnaire 2 was designed to address the 3rd sub-research question of the thesis, namely: *How effective do the students perceive the visualisation strategies to be?*

The participants received the questionnaire on 13 May 2016. It contained two main sections. Part 1 explored their views on the personal benefit that they perceived the visualising strategies they had learned about offered to them. Part 2 required the participants to consider whether, in the future, they might use visualisation as a methodology when dealing with word-problems in their own classes.

Part 1 contained 5 themes which addressed the five main concerns that students have when solving word-problems (Gooding, 2009, p. 5). Each theme contained two Likert scale questions and a specified response field. Part 2 contained five Likert scale questions and one specified response field. Questionnaire 2 is appended (Appendix S). The primary data is also appended (Appendix T).



There were 60 students in the IPS 413 E Mathematics class. By the time this questionnaire was written, 38 students had become participants in the research. However, because of ongoing absenteeism, only 34 received the questionnaire, and of these 34 students, 4 did not do Part 2 of the questionnaire.

### 4.12.1 QUESTIONNAIRE 2 – PART 1 – PERSONAL BENEFIT – UNDERSTANDING AND SOLVING WORD-PROBLEMS

#### 4.12.1.1 PART 1 – 1 – READ AND UNDERSTAND WORD-PROBLEMS

In Table 4.29, below, is can be seen that the participants felt that the use of objects and images enabled them to read and understand word-problems. Indeed, most participants (18/34) strongly agreed that objects and images made it easier for them to *read* problems, while slightly less (17/34) agreed that using such scaffolds made it easier for them to *understand* what word-problems required of them.

Table 4.29

*Summary of quantitative data captured for Part 1 – 1 of Questionnaire 2*

| PART 1 - 1: READ AND UNDERSTAND WORD PROBLEMS | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 I have found that the use of objects and images makes it easy for me to read word-problems | | | | | 2 I have found that the use of objects and images makes it easy to understand what word-problems require me to do | | | | |
| Strongly disagree | Disagree | In the middle | Agree | Strongly agree | Strongly disagree | Disagree | In the middle | Agree | Strongly agree |
| 0 | 0 | 1 | 15 | 18 | 0 | 0 | 3 | 17 | 14 |

Approximately half (18/34) of the students supplied a specified response. Three positive responses are provided below.

1. *We did the first test arithmetic well it was or has been more than ten years of not doing maths and it was a bit difficult to grasp the concepts, however this images and objects has brought understanding and I wish I was taught maths this way from my elementary school.*
2. *If the use of objects and images in mathematics widely continues as a full time teaching in South Africa every learner will highly pass maths without any doubts*
3. *The visualisation method is just going to work for everyone, it is perfect especially for the young children, it will be a good foundation to their maths knowledge*



4.12.1.2 PART 1 – 2 – FORM MENTAL IMAGES OF WORD-PROBLEMS

The storyline in a word-problem can, for the prepared mind, offer opportunities to imagine scenarios and thereby provide a context for problem-solving.

The data in Table 4.30, below, suggested that, generally, most of the cohort (19/34) agreed that objects and images assisted the participants to *form* a mental image of word-problems, and most students (22/34) agreed that they could use said images to assist them to *imagine* a problem-solving strategy for word-problems.

Table 4.30

*Summary of quantitative data captured for Part 1 – 2 of Questionnaire 2*

| PART 1 - 2: FORM MENTAL IMAGES OF WORD PROBLEMS | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 3 I have found that the use of objects and images assists me to form a mental image of the requirements of word problems | | | | | 4 I have found that the use of objects and images helps me to imagine a solution strategy for solving word problems | | | | |
| Strongly disagree | Disagree | In the middle | Agree | Strongly agree | Strongly disagree | Disagree | In the middle | Agree | Strongly agree |
| **1** | **0** | **4** | **19** | **10** | **1** | **1** | **0** | **22** | **10** |

Very few participants (13/34) constructed a written comment for Part 1 – 2. Mostly, those comments suggested that the participants used visualising techniques to conceptualise potential solution strategies for solving word-problems. Three such comments are presented below.

1. *The ribbon method and the pie-chart drawing is great to help imagine a solution.*
2. *Yes, I form mental image and try to draw them and try to find the answer. Images are a perfect guide to what is required.*
3. *By just having the picture on your mind, you do see which one will be bigger even though you don't know the actual outcome.*



However, as indicated below, some of the participants did not view the use of visualising techniques favourably.

1. *Sometimes I would know what the question ask for but it is difficult to construct the image using the given question.*
2. *I imagine wrong solutions.*
3. *Unfortunately I don't have the same sentiment. To me it's all about the numbers given nothing more.*

### 4.12.1.3 PART 1 – 3 – SUCCESSFULLY CONSTRUCT A NUMBER SENTENCE TO START TO SOLVE THE PROBLEM

Gooding (2009, p. 5) suggests that many students struggle with word-problems because they are unable to construct a number sentence to solve the problem. Below, in Table 4.31, the data suggests that most participants (18/34) agreed that visual scaffolds enable them to find *starting points*; and similarly (16/34) students agreed that visualising techniques enabled them to *construct* number sentences.

Table 4.31

*Summary of quantitative data captured for Part 1 – 3 of Questionnaire 2*

| PART 1 - 3: SUCCESSFULLY CONSTRUCT A NUMBER SENTENCE TO START TO SOLVE THE PROBLEM | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 5 I have found that the use of objects and images makes it easy for me to find a starting point to begin to solve word problems | | | | | 6 I have found that the use of objects and images makes it easy for me to construct an appropriate number sentence or formula to solve word problems | | | | |
| Strongly disagree | Disagree | In the middle | Agree | Strongly agree | Strongly disagree | Disagree | In the middle | Agree | Strongly agree |
| 1 | 0 | 6 | 18 | 9 | 1 | 1 | 5 | 16 | 11 |



The students delivered 14/34 written replies to this part of the questionnaire. The three comments below are indicative of the perspective that most students held.

1. *How to find a starting point to begin to solve word problems. Now images makes it easy for me to construct an appropriate number sentence or formula to solve word problems.*
2. *Use of objects and images gives one a direction of how to solve a problem.*
3. *The images are very helpful because you get to see what is required of you, and also it gives a direction of where to start.*

However, the optimistic views listed above were not universally held by all of the participants. Two contradictory viewpoints are presented below.

1. *They just complicate everything and sometime I feel they are a waste of time.*
2. *It is difficult for me to find the starting point, although I have an idea/solution in mind. It is challenging using objects/images.*

#### 4.12.1.4 PART 1 – 4 – CAN DO THE ACTUAL CALCULATIONS NEEDED TO SOLVE THE WORD-PROBLEM

Albeit that Table 4.32, below suggested that the participants' perceptions of their own ability to perform arithmetic computations were quite buoyant, the quantitative evidence captured in two action cycles suggested that, mostly, deep understanding, relational understanding, HOT skills, and compression had not occurred.

That notwithstanding, most of the participants (18/34) agreed that objects and images had made it *easier to work* with numbers, and 22/34 of the participants agreed that visual scaffolds assisted them in *doing the calculations* for solving he word problems.

Table 4.32

*Summary of quantitative data captured for Part 1 – 4 of Questionnaire 2*

| PART 1 - 4: CAN DO THE ACTUAL CALCULATIONS NEEDED TO SOLVE THE WORD PROBLEM | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 7 I have found that the use of objects and images makes it easy for me to work with numbers | | | | | 8 I have found that the use of objects and images makes it easy for me to do the calculations which are needed to solve word problems | | | | |
| Strongly disagree | Disagree | In the middle | Agree | Strongly agree | Strongly disagree | Disagree | In the middle | Agree | Strongly agree |
| 0 | 1 | 2 | 18 | 13 | 1 | 0 | 3 | 22 | 8 |



Roughly one third of the participants (12/34) provided a written comment. All of those comments were positive.

1. *Able to do the maths problem better when I work with images and objects.*
2. *I have established a solid link between numbers and objects and this makes me feel comfortable in my understanding.*
3. *With maths being my weakest subject since high school I have come to find working with numbers a lot easier because of images and objects used.*
4. *They help me solve problems. They make fractions look easier than just using numbers and symbols.*
5. *Do the maths problem better when I work with images and objects.*

4.12.1.5 PART 1 – 5 – ABILITY TO JUDGE THE VALIDITY OF A CALCULATED ANSWER

In Table 4.33, below, it can be seen that almost half the students (15/34) agreed that visualisation assisted them to *anticipate* the size and shape of final answers for word problems, and 19/34 agreed that visual scaffolds assisted them to *review* whether or not their final answers looked to be correct.

Table 4.33

*Summary of quantitative data captured for Part 1 – 5 of Questionnaire 2*

| PART 1 - 5: ABILITY TO JUDGE THE VALIDITY OF A CALCULATED ANSWER | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 9 I have found that the use of objects and images makes it easy for me to approximately judge what my final answer will be | | | | | 10 I have found that the use of objects and images makes it easy for me to look at my final answers and judge if they look okay | | | | |
| Strongly disagree | Disagree | In the middle | Agree | Strongly agree | Strongly disagree | Disagree | In the middle | Agree | Strongly agree |
| 1 | 0 | 7 | 15 | 11 | 1 | 0 | 3 | 19 | 11 |

Only 12/34 students provided a textual reply. Some positive comments are presented below.



1. *Even though I might not know the final answer, but it is just easy to judge.*
2. *LOL I used to just write just for the sake of writing and finishing but now I look back and see what I might have done wrong, I can easily note my mistakes and correct them, unlike when I used to just write numbers which I did not even understand myself.*
3. *I know if I practice more I will do best, I know I am one of the people who are very careless and lazy but this technique has become more easier and it make counting faster, e.g. The grid method, in it you come up with a solution more faster you don't have to memorise everything. Thank you Mr Shaw.*
4. *As a learner you are able to know when your answer is not correct even before the teacher tells you.*

One student, however, rallied against visualisation.

1. *They don't assist me because I have my simple ways of validating my answer.*

### 4.12.2 QUESTIONNAIRE 2 – PART 2 – METHODOLOGICAL BENEFIT – USING VISUALISATION WHEN TEACHING CHILDREN IN YOUR CLASSES TO SOLVE WORD-PROBLEMS

Part 2 of the questionnaire contained five Likert scale questions. The questions were similar to the questions posed in Part 1 and addressed the five hurdles for solving word-problems as identified by Gooding (2005). However, in Part 2, the focus of the answers shifted from the participants' use of visualising scaffolds – as research participants – to their use as a methodological teaching strategy – as teachers. A single field for a specified written response was included.

#### 4.12.2.1 PERCEIVED METHODOLOGICAL BENEFIT OF USING VISUALISATION AS A TEACHING AND LEARNING STRATEGY

Only 30 of the 34 participants who received the questionnaire completed Part 2. As seen in Table 3.34, below, across the questions, the consensus was that the use of objects and images, as a methodological strategy for developing understanding and doing word-problems, was perceived to be of great importance.



Table 4.34

*Summary of quantitative data captured for Part 2 of Questionnaire 2*

| PART 2 - USING VISUALISATION WHEN TEACHING CHILDREN IN YOUR CLASSES TO SOLVE WORD PROBLEMS | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 11. I believe that the use of objects and images (visualization) will help my pupils to read and understand word problems | | | | | 12. I believe that the use of objects and images (visualization) will help my pupils to form mental images of word problems | | | | | 13. I believe that the use of objects and images (visualization) will help my pupils to successfully construct a number sentence or formula to start to solve word problems | | | | | 14. I believe that the use of objects and images (visualization) will help my pupils to do the actual calculations needed to solv word problems | | | | | 15. I believe that the use of objects and images (visualization) will help my pupils to judge the validity of their calculated answers | | | | |
| Strongly disagree | Disagree | In the middle | Agree | Strongly agree | Strongly disagree | Disagree | In the middle | Agree | Strongly agree | Strongly disagree | Disagree | In the middle | Agree | Strongly agree | Strongly disagree | Disagree | In the middle | Agree | Strongly agree | Strongly disagree | Disagree | In the middle | Agree | Strongly agree |
| 0 | 0 | 0 | 8 | 22 | 0 | 0 | 0 | 10 | 20 | 0 | 1 | 0 | 14 | 15 | 0 | 0 | 1 | 11 | 18 | 0 | 0 | 3 | 8 | 19 |

Just over half of the participants (17/30) constructed a short written statement. To a person, each of the comments carried very positive overtones. Some of the participants' views are listed below.

1. *It has become easy for me to read and understand word-problems because I first visualised it and seeing it made it easier for me to solve a problem after*

*struggling for a long time. If it worked for me surely it will work for my pupil: things are much clear now; if maths was taught in images we would have perhaps continued with maths to matric level. I dropped math because it was a nightmare. My belief is that if mathematics is taught this way the love of numbers will be revived.*

2. *I tutor Grade 4 and 5 pupils and have tried a few of these methodologies and they seem to enjoy visualisation more and understand concepts more (faster) with the use of colours. So I would like you to know that visualisation has the potential to work in schools. With this from a young age, they are able to form mental images and predict answers. Compared to our generation who weren't taught these techniques from a young age.*
3. *I strongly agree with all of the above sentences because I do not see any easier way that my pupils would understand (in their age) without the use of images and objects. Thank you Mr Shaw, your lessons, all of them, showed that you are really passionate not just about maths but also making sure that the rest of the class understands. :)*
4. *I the prospective teacher will make sure I use visualisation when teaching my pupils so that they can read and understand word-problems. And this will also help my pupils to do the calculations needed to solve word-problems. Therefore at the end they can judge the correctness of their calculated answers.*

### 4.12.3 SUMMARY OF QUESTIONNAIRE 2

Offered at the end of Action Cycle 2, Questionnaire 2 was positioned to provide an answer to the third, sub-research question. In Part 1, Gooding's (2009, p. 5) barriers were used as pivots in five pairs of Likert scale questions and five specified responses. The participants were requested to share how effectively they had perceived the visualisation strategies that they were taught had been in assisting them to solve word-problems. Overwhelmingly, their replies were positive. Part 2 was similar to Part 1, but in this case, the participants were asked if, as teachers, they might find any use for visualisation in their own mathematics classes. As in Part 1, again, the responses were very positive.

## 4.13 FOCUS GROUP INTERVIEW

On Thursday 5 May 2016, at about 12h45, seven participants presented themselves at our faculty boardroom; they had been invited to take part in a focus group interview. I had arrived at my final selection of participants by drilling through my research data sets in the following way:

1. I identified all the students who had agreed to participate in the research;
2. I identified participants who had completed all the research tasks;
3. I interrogated the data from the first action cycle Foundational Arithmetic Concepts prior knowledge assessment tasks. In that test, the class average was 7 out of 20.
4. I used items 1, 2 and 3, above, as foci to construct a homogenous group of 12 potential student-participants, clustered around that average of 7.

In the end, as seen in Table 4.35, below, 12 participants were considered. Of these candidates, 7 agreed to take part in the exercise.



Table 4.35

*Potential participants for Focus Group Interview*

| Score (out of 20) | Potential Participants |
|---|---|
| 3 | 1 |
| 4 | 3 |
| 5 | 1 |
| 6 | 1 |
| 7 | 3 |
| 8 | 1 |
| 9 | 1 |
| 10 | 1 |

On the 5th May, I had a discussion with the interviewer; she was an academic, a mathematical expert and was fluent in isiXhosa and English. We discussed the six leading questions in the interview questions and tried to consider potential issues, conflicts and solutions for matters that might manifest in the meeting. At midday, I set up the meeting venue and welcomed the 7 participants. The interviewees were invited to enjoy a light luncheon, and were then each given a copy of the interview guide, attached (Appendix U). I introduced the participants to the interviewer, switched on the audio recorder and exited the meeting. I scheduled 30 to 40 minutes for the interview, but, in fact, it was almost one hour long. At its conclusion, the interviewer presented me with the audio recording, which I transcribed (Appendix V). The formal introduction for the interview took about three and a half minutes. After that, the focus group questions were presented to the group. Below, I recount some of the important viewpoints which were raised in the meeting.

### 4.13.1 ENGAGEMENT QUESTION: 1. SO, HOW IS IT GOING IN YOUR IPS 413 E- MATHEMATICS MODULE?

The engagement question was an icebreaker. Responses suggested the participants came from a wide range of homesteads: Amalinda and Quigney in East London, Cape Town and Lady Frere were mentioned. Three of the participants suggested that they liked reading to stay informed; two suggested they liked being with and helping others, two liked attending church, and three enjoyed cooking and baking. Three participants spoke of physical activity, going to a gym, walks, and swimming.



### 4.13.2 EXPLORATION QUESTION: 2. IN WHAT WAYS HAS THE REVISION OF BASIC ARITHMETIC SKILLS AND CONCEPTS HELPED OR NOT HELPED YOU TO COPE WITH THE IPS 413 E – MATHEMATICS MODULE?

The participants, rather than answering the exploration question directly, took their lead from the response offered by Number 5 who was the first student to respond. Indeed, throughout the interview, Number 5 was often the first voice.

*Number 5.* *Um, from the first instance, when I was working for Mr Shaw's class, um, the way, or the manner in which he gives things is very different. Um, it was new, it was vibrant, and it was to, my knowledge, it made me think as well, like, this is a creative man, um, using creative ways in order to attain what he wants to attain, and that is for us to know and love mathematics, um, as far as like, the class, um, how it's going right* *now, for me, personally, I, at first, I, I'm not a person who real likes mathematics, necessarily likes mathematics.*

The response above served as a rallying point for subsequent replies with other students suggesting a historic fear of mathematics, of finding the introductory lessons to this module interesting and completely different to their school-based methodologies and experiences, and of finding the lecturer (me) for the IPS 413 E – Mathematics module easily approachable. Some comments are presented below.

*Number 5* *…I have never experienced anything like that before in my life…*

*Number 6* *…and he used different teaching strategies to, like, accommodate the whole class… so at first it was fearful and scary, but now it's… we're getting there…*

*Number 2* *When, when you are in Mr Shaw's class it not like, you don't feel dumb because you don't understand something because you know he is going to explain it further and he, he takes his time to explain so that we understand properly… he doesn't make you feel scared of anything of the subject, not him, you actually enjoy going to his class.*

*Number 3* *…because Peter tries to accommodate everyone by using resources like bottle tops, biscuits to explain something, and at first I normally, don't understand what he is saying but by the time he is using resources… Yes, yes, I understand by the use of resources.*

Albeit that the interviewer restated the question, the students did not offer much to support a direct answer to the question other than to express interest in the use of tactiles and other resources as a novel, interesting and accessible teaching and learning methodology. An exemplar discussion is provided below.

*Number 4.* *Ja! I can say that it has helped me a lot that, that, ah, revision of the mathematics because I also had a problem; I didn't like maths, so by him explaining and using some examples that were practical sometimes I did, I end up, um, being more interested in mathematics.*

*Interviewer* *So in a way you are saying those practical examples can help somebody to understand mathematics?*

*Number 4* *Yes*

*Interviewer* *Do you think that by using practicality when we teach mathematics it can help in all topics of mathematics?*

*Number 4* *Yes, I think so. [inaudible] because maths is is is is kind of difficult*

Further, a theme of being enabled to understand their mathematics in the IPS 413 E class came through; an exemplar of this often stated view is presented below.

*Number 2* *And when he explains it, he makes you feel like, I do know this, it's not like I don't know what I'm doing, I've just forgotten what it is that I have to do. And the way that he does this is he uses that, I don't know what you call it, that camera that he connects to his laptop and then [inaudible] you can see everything that he's doing, so he's busy writing here and is explaining, and you can see what he's actually explaining. It's not like he's just explaining and you can't see what he's doing, and that also made it a lot easier for all of us to understand the work.*

The focus group strongly endorsed the visualising techniques that were used to liberate memories of arithmetic skills and processes. This contradicted my classroom based experience with the cohort in which I had constantly implored students to actively participate in constructing their own understanding. Many students had resisted using the opportunities made available to them to use tactiles and visual aids and had preferred to be lectured to, that is, to be recipients of Freire's (2007) transmission-mode, banking method of education.



### 4.13.3 EXPLORATION QUESTION: 3. HOW DO YOU FEEL ABOUT THE NEW TECHNIQUES AND SUPPORT THAT WERE GIVEN TO YOU IN THE IPS 413 E – MATHEMATICS MODULE?

Initially, the responses to this question referred to negative, past, school-based experiences of mathematics with some small, but positive, reference to the visualising methodology used in the IPS 413 E – Mathematics classroom.

*Number 6* *…when we get to high school, we say, "Oh my God, I hate Maths," and you have that bad attitude… this new technique, is going to change the, the understanding of mathematics…*

One of the participants held a view that served as a counterpoint to the general view that the support and visualising techniques assisted understanding.

*Number 2.* *I, I agree with the fact that there's a new approach that we're being taught and stuff, but I'm not a big fan of it. I don't know if it's because I don't understand it quite yet, or I don't grasp it, grasp it yet, or whatever it is, but I still prefer the older techniques. Maybe it's because of what I know and what I understand… not that the change is bad, but for me personally, I'm not a big fan of it.*

The interviewer, noting the continuing commentary on past experiences, made the following adjustment:

*Interviewer:* *You are all talking about your experiences with high school; why don't we look at these techniques and see if we will be able to apply them into the primary school?*



The first response to this altered perspective was that a student tried to explain the rationale for the continuing references to past experiences. An excerpt of that response is transcribed below.

*Number 5* *I believe the reason my responses have been about high school have been because we have received a lot of those bad experiences in high school… I would go to use the word dictatorship kind of attitude, ummm, because, because what I found in high school is that, it was, it was for the teachers it was my way or the highway.*

*Interviewer* *I wonder why.*

*Number 5* *I wonder why. And and and, but ahhh [Laughter]*

*Number 6* *Number 6*

*Number 5* *I see. [laughter] ahh, Number 6 made a very important point in the fact that um if, if, if, if, we, the people who are in the IPS 413 class get to* *absorb, um, understand what Mr Shaw, Mr Shaw is trying to do, we will become better teachers and a result of that, mam, is, is there will be better mathematics students in primary school, in high school, where ever you want to go. It is with the, the attitude, it is with the approach, umm, that has that has failed us, um, and predominantly in our higher levels of education.*

Participant Number 5 continued to speak for some time on this matter, so clearly, for this participant, many negative past experiences of mathematics continued to haunt his perception of the subject. Participant Number 7, however, did respond directly to the exploratory question.

*Number 7* *…I'm very, I'm really looking forward to the strategies that he is going to use for this term with us because now that I know that he is someone who is creative he doesn't leave anyone without understanding the content. He will even try different strategies, for example he introduced us, he introduced the topic of fractions, ne, he gave us the examples with bottle tops and then he, he feeled that no, we are not we are not understanding this thing, he changed and gave out the different example, and then he introduced another thing like, he introduced the coins and then, even from there, he changed from that one…*



This response, above, the only direct response to the question, spoke to the many concrete and iconic visual scaffolds that had been a part of the teaching and learning process in the IPS 413 E – Mathematics module.

### 4.13.4 EXPLORATION QUESTION: 4. HOW DO YOU FEEL ABOUT THE VIEW THAT TEACHING AND LEARNING TO SOLVE WORD-PROBLEMS IS IN PART DEPENDENT UPON THE APPLICATION OF BASIC ARITHMETIC KNOWLEDGE?

Participant Number 1 enunciated a response that I commonly hear to the question.

*Number 1* *Yoh, I struggle with this one. I, I don't know the problem is there. Now which, I don't know where to start when I am faced with word-problems. I see them, but I struggle to figure out where do I start. When Mr Shaw is explaining I understand, oh it's very easy to understand it when he is explaining, but when I'm, he given the question, I don't know where to start with it.*

In the short, in the light-hearted discussion which followed this statement, it was agreed that more practice would be of benefit to the students. Participants Number 5 and Number 1 spoke to the complexity of understanding the nuances found in the structure of English sentences. Number 5 suggested that English posed no barrier; however, reading for context and imagining the story, he suggested, would make it easier to negotiate the arithmetic requirements of word-problems. When prompted for a response by the interviewer, Number 1, as seen below, suggested that her inability to understand the English sentences acted as a barrier to her success.

*Number 1* *…I would say it's a English in in the way the question, a story is structured to understand what is said in the story. I would say it is English, yes.*



Participant Number 5 endorsed Number 1.

*Number 5* *… I believe that English and Mathematics find a meeting place in word-problems. They find a very distinct meeting place and if one is lacking, then the application of mathematics will not suffice. If there is, I don't know, maybe, lack in English, then the mathematical application will suffice. It, it does not really mean that one does not know mathematics or one does not know English, but the [inaudible] can throw people off sometimes. But if one has better understanding of English and the English language, I think they are able to deal with it from my experiences and word-problems better.*

This, too, was supported by comments from other students.

*Number 2* *Okay. Number 2. I don't think it's a problem with the language, but it's a problem with the wording... so, word-problems is difficult, I never liked word-problems but it's the way that they structure the sentence, regardless of the language that makes it a problem because that is also when you don't know where to start, you don't know do you start in the middle, or do you start at the end or start at the beginning.*

At this point, the words *mathematics* and *language* were interrogated, with various positions being advanced.

*Number 5* *…I met another gentleman who said to me, a very old gentleman, I forgot his name who said to me that mathematics is the language of God, that's how he, that how God speaks, that's how God understands things…*

Consequently, Number 2 clarified her position and Number 7 offered an opinion:



*Number 2* *Number 2, sorry, um, I don't think my, my point was made quite clear because, its, I didn't mean language as in the language that we speak, it could be English, it could be Afrikaans, it could be Xhosa whatever, it doesn't matter what that specific language, what the sentence is written out in…*

*Interviewer* *Okay*

*Number 2* *…it's the way that the words are put in different languages that would still confuse a person about the word sum.*

*Number 7* *[beeping from reversing truck in the background]. Number 7 Yes [pause] ahhh [pause] I understand that mathematics is a language on its own, ne, because we find that and that's where our problem lies, in word, in word-problems, ne, because we are given that text and then you have to read it and you find there are, there are two thirds, at first I didn't even know what is a third, [laughter] I have think first, what is a third and here it is said that there are two thirds, so I think the problem lies, inyani, in mathe…, in language yaba-mathematics…*

The various excerpts presented above demonstrated that, in this focus group at least, most of the students perceived language, and particularly the idiosyncrasies of the English and mathematical forms of it, to be a barrier to their success.

### 4.13.5 EXPLORATION QUESTION: 5. HOW DO YOU FEEL ABOUT THE USE OF VISUALISATION PRACTICES AS A METHODOLOGY USED TO ASSIST STUDENTS TO UNDERSTAND WORD-PROBLEMS?

The initial responses to this question were light-hearted and somewhat ambivalent. Participant Number 6, for example, suggested that, while she found the visualisation approach exciting, there might be some resistance to using visualisation as a teaching and learning methodology.

*Number 6* *We not flexible, not all of us are keen to change, or not all of us are want to change…*



Answering the same question somewhat obliquely, Number 5 referred to an IPS 413 E – Mathematics lesson in which we had discussed the construction of memory, of how we learn and recall that which is learned. He referred to the school-based practice of not being allowed to use hands as tactile scaffolds and contrasted this with his IPS 413 E experiences.

*Number 5* *…like we were told, I am sure Mr Shaw mentioned this as well, don't count with your hands, you are geared to be like a child, you know, don't count with your hands, you are geared as if someone who does not know how to count, don't use your hands when you count, hide those hands away. Um, I think that culture has, has done us a world of wrong because that is, that is the culture that I also absorbed as well, that, you know what, I should not count with my hands, I'm doing Grade 6 now, we are not in Grade 2 anymore, you know. Whereas Mr Shaw revealed to us that the way in which our minds work, they use our hands, our hands are very important when we count, um, because there, there are hands that* *are mapped in our heads. My hand is a part of my body so my brain understands my hand better than anything else [laughter]*

While this explanation was not a perfect paraphrase of our lesson, what was pleasing was that he had formed links between the external and physiological world and the inner psychological and conceptual world. Participant Number 7 stated that:

*Number 7* *…when you see something, ne, it's easy for you to remember it, when, even though you are no longer seeing it, so I think this approach is going to help us a lot and I am really hoping that Mr Shaw, ne, is going to bring it like that, even in the exams.*

And Number 2 suggested that she had witnessed an ex-PGCE student successfully using visualisation methodologies in a local primary school.

*Number 2* *…because it was a very young teacher as well that also graduated from doing or did a PGCE last year, she finished last year, so I don't know if, but, if the approach seems to be working with those children also and for us as …*

As a final comment on the question, Number 6 had the following to say:

*Number 6* *… so now when you do, when like with yesterday when we did maybe the grid method or we had to, we had to section out a quarter, a quarter, a quarter and things like that, you will be able to see what you are doing and you'll also be able to estimate but something doesn't look right here or something, somethings not, doesn't seem okay and you will be able to check what you can do…*

These comments suggest that, after some initial misgivings, this group had strongly embrace visualisation as a scaffold for solving mathematical word-problems.

### 4.13.6 EXIT QUESTION: 6. IS THERE ANYTHING ELSE THAT YOU WOULD LIKE TO SAY THAT MIGHT ADD TO OUR UNDERSTANDING OF YOUR EXPERIENCES OF IPS 413 E MATHEMATICS?

A first answer to this question was offered by Number 5 who suggested that he would like to have full-class attendance in the IPS 413 E lessons. No doubt, as discussed in 4.1.1, this was a veiled reference to the swathes of students who often missed classes. He also suggested that he would have appreciated more contact time for the IPS 413 E class and concluded:

*Number 5* *Ah, I hope Mr Shaw, I hope, um, I hope whatever data he is collecting through this work, I, what I have experienced, you know, what I, what I feel, and what I've seen worked for me, you know, that, you know when I see it, I become better at it, you know when you see it, I, it becomes very, very applicable, I apply all that I know when I see things better so I would like Mr Shaw to continue, um, with, this method of teaching.*



Below, I have transcribed the last few interactions in the focus group interview.

*Interviewer* *Okay. [pause] Any other end? [long pause] Anybody wants to add, say anything?*

*Number 4* *Number 4. Well, um, I can say that I loved being in this class of IPS and I would love to teach like Mr Shaw when I go out to school, he's very good.*

*Interviewer* *Okay. Any other persons?*

*Number 6* *Number 6. I am just going to say I hope this new technique works. [laughter] I am just praying for that because it's, it's going to make things simpler for some of us…*

*Interviewer* *Okay [mumble]*

*Number 6* *Maybe, it's going to create [inaudible] for us [unknown - let's hope] let's hope [laughter] that's all.*

*Interviewer* *Okay*

*Number 6* *Not forgetting though to accommodate all of us [laughter]*

*Interviewer* *To accommodate everybody, ne, in the class, okay. Okay guys, thank you very much, Thank you.*

*Number 5* *Are we allowed to take the eats?*

*Interviewer* *Yes*

*END*

### 4.13.7 SUMMARY OF FOCUS GROUP INTERVIEW

The narrative between the interviewer and the focus group participants was conducted in a cordial and professional manner. The students seemed to be at ease and seemed to answer the six questions freely and honestly. My interpretation of the qualitative data suggests that the students enjoyed coming to my classes and felt that their lessons in the IPS 413 E module had proved beneficial.

The focus group commented positively on the novelty and accessibility of the visualisation strategies, suggesting that the approach caused them to think about their working and thereby improved their understanding in solving word-problems

It is important, however, to recall that the use of visual scaffolds did not meet with universal approval and it is also important to note that the use of English, used to teach and learn mathematics, and used to compose word-problems, was repeatedly flagged by the students as a significant barrier to their success in this subject.



## 4.14 QUESTIONNAIRE 3

In term four of 2016, I asked the participants to complete one last questionnaire. Although I was no longer teaching them, I felt that the questionnaire might offer closure for the research. The questionnaire, appended (Appendix W), contained five questions. Questions 1 through 4 contained selected responses. Question 5, contained four cross-referenced sub-questions.

13 of the participants completed the questionnaire. Their responses are appended (Appendix X). Below, I recount some of the reflective, qualitative data gleaned from their responses.

### 4.14.1 QUESTION 1 - HAS THE IPS 413 E MATHEMATICS MODULE IN ANY WAY CHANGED YOUR ATTITUDE TOWARDS MATHEMATICS, AND IF SO, HOW?

The question garnered 13 responses, all endorsing the IPS 413 E – Mathematics module as an effective and liberating experience. Some comments are listed below:

1. *It has greatly, because I "hated" Mathematics because of a bad experience during my school years but now I got the chance to enjoy maths as a subject and be able to understand it.*
2. *Yes, I have seen that mathematics can be fun if you take time to understand it and work on it with other people. It made me realise that it can help in critical thinking and problem solving.*
3. *Mathematics has been a nightmare and scared to do it, however after the ten years gap without doing mathematics from the poor background of the subject, in IPS 413 E with Mr Peter Shaw it was like doing Maths for the first time the wall between me and maths was removed and my hatred and attitude has change for the better towards maths.*



### 4.14.2 QUESTION 2 - DID YOU FIND THE KNOWLEDGE, ACTIVITIES AND METHODS USED IN THE IPS 413 E MATHEMATICS MODULE USEFUL TO YOU AND, IF SO, HOW?

This question also received 13 responses. The written replies endorsed the visual approaches used in the module. Three students' personal views are presented:

1. *Yes, it helped me a lot, especially the visualisation method it made it easier to understand what was required of me.*
2. *Yes, very. They were simpler making it easier to comprehend and I could use my hands to count something my past school teachers hated and did not allow.*
3. *Yes, I love it. It made it easier for me to be able to explain to learners in school experience.*

### 4.14.3 QUESTION 3 - DID YOUR EXPERIENCES IN IPS 413 E MATHEMATICS HELP YOU TO TEACH MATHEMATICAL IDEAS TO CHILDREN DURING SCHOOL EXPERIENCE AND, IF SO, HOW?

13 responses were received: 1 was negative, 2 cautious and 10 suggested that the experiences in the IPS 413 E module were beneficial. The question was aimed at the broad experience of the IPS 413 E module; however, the responses mainly spoke to the visualisation methodology. In the case of the negative response, the response indicates that a practicum host teacher followed a prescriptive programme:

> *Not really, because my host teacher was not welcoming to anything that was different from CAPS and did not allow me to let the learner experience another way of doing maths and I only did limited lessons.*

While I cannot be certain why the host teacher would hold such a position, in 1.2.1.10 and in 2.3.1, this thesis has demonstrated that the CAPS curriculum (Department of Basic Education, 2011) embraces visualising methodologies. Intuitively, I imagine that the host teacher might have been worried that, in applying a novel strategy for at most four or five weeks and then leaving, the student might disrupt the processes of teaching and learning in the host teacher's class.



One of the two hesitant responses is transcribed below. It does not demonstrate antipathy towards the IPS 413 E module but, rather, lack of self confidence:

> *Yes, but I was not confident enough on myself to teach although I did teach a few.*

The other replies all suggested that the application of skills and ideas that they have learned in the IPS 414 E module proved beneficial in their practicums.

1. *Yes especially the ratio and fractions which learners had problem differentiating.*
2. *Yes, definitely. I bought sweets, biscuits, bottle-tops and stones for topics such as fractions and ratios. The learners participated because of tangible resources.*

3. *With the method of visualisation I first used the Mr Peter Shaw biscuits methods and it worked as the learners did understand. They learn more with what they see rather than me talking to them.*

### 4.14.4 QUESTION 4 - DID YOU FIND THE USE OF APPARATUS (E.G. BISCUITS) AND IMAGES (E.G. DRAWINGS OF MATHEMATICAL IDEAS) USEFUL AND, IF SO, HOW?

All 13 responses to this question were supportive of and endorsed the use of apparatus. Variously, different responses spoke to the PGCEs' own improvement in understanding, better understanding through visualisation for their pupils, and the capacity this technique has to offset language barriers.

1. *Yes, they were VERY useful because they made me understand the lesson content better. They helped me more on ratios, with the use of them I am happy to say that I now have an understanding of ratios.*
2. *Yes, this made it easy for learners to understand the work because they could visualise it and also the examples. This made activities to be fun for learners and it did not feel like they were doing mathematics.*
3. *Yes, mathematical language can be confusing. Using apparatus especially at different colours helps in solving the problem step-by-step.*



### 4.14.5 QUESTION 5

Question 5 contained multiple parts – these are discussed below.

#### 4.14.5.1 QUESTION 5.A - DURING SCHOOL EXPERIENCE, DID YOU TEACH CHILDREN TO SOLVE MATHEMATICAL WORD-PROBLEMS? (YES/NO)

Eleven students suggested that they did teach children to solve word-problems during their school experience practicums. Two others replied that they did not.

### 4.14.5.2 QUESTION 5.B - IF YOUR ANSWER TO 5.A. IS YES, DID YOU USE VISUALISING TECHNIQUES (E.G. BISCUITS, DRAWINGS, ETC.)? (YES/NO)

Eleven responses were received; all of then suggested that they had used visualising techniques to assist pupils to solve word-problems.

### 4.14.5.3 QUESTION 5.C - IF YOUR ANSWER TO 5.B. IS YES, PLEASE EXPLAIN THE VISUAL METHODS THAT YOU USED.

Unfortunately, the eleven responses were typically very short, each containing a few words, rather than any deep explanation. However, the use of apparatus and iconic stimuli are repeatedly presented in the responses.

1. *I used bottle tops and drawings to calculate, visual pictures; visual strips*
2. *I used bottle tops to teach ratios and fractions which they were struggling with when it comes to solving word problems.*
3. *I used charts, sweets, bottle tops, biscuits and match sticks*



### 4.14.5.4 QUESTION 5.D - IF YOUR ANSWER TO 5.B. IS YES, DID THE CHILDREN IN YOUR CLASS LIKE THE APPROACH?

Possibly, this last question, squeezed in as it is at the end of the last data-collection instrument, is the most important question in the entire data collecting exercise of this thesis. If the ultimate recipients of this visualising methodology reject it, all of the work, no matter its good intentions, comes to nought. However, as seen in the exemplars below, the responses to this question suggest that the pupils who were exposed to visualisation scaffolds were enthusiastic about the methodology.

1. *They loved the approach and understood the content.*
2. *They enjoyed it a lot and wanted more mathematical word problems.*
3. *Yes, because children, I think, learn best from seeing and visual methods work well for them.*
4. *Yes, they liked the approach because they were active during the lesson and able to answer questions based on visual that I brought in class.*

### 4.14.6 SUMMARY OF QUESTIONNAIRE 3

Only 13 participants responded to the request to complete this questionnaire and of these students 2 did not answer all of the questions. The responses generally supported the epistemological and methodological approaches that were used within the IPS 413 E Mathematics module. Some students suggested that the techniques that had been used in our classroom had been influential in breaking down mental barriers that they had carried of mathematics. The participants strongly endorsed the use of visualising scaffolds, suggesting that such scaffolds made their school experience practicum work enjoyable and made their explanations to pupils effective. In this regard, they spoke to the use of concrete apparatus – enactive representation – rather than the construction of enabling sketches – iconic representation. The participants suggested that in their practicum classes pupils embraced the visual and liberating approaches that they used in the mathematics periods.

## 4.15 SUMMARY OF CHAPTER 4

The field notes were presented so as to give some sense of highs and low, frustrations and breakthroughs, which occurred within the 2016, IPS 413 E Mathematics module. The Thinkboard activity provided a wealth of qualitative data which demonstrated that members of the PGCE cohort, while often fearful of mathematics, also attached great importance to the subject and were keen to become *great* mathematics teachers.



Analysis of the prior knowledge and re-evaluation tasks for both action cycles showed that, for the duration of the semester, many low order facts, routines and concepts remained uncompressed into schema of meta-concepts of fractions and ratios and thus precluded the construction of relational knowledge. Some uptake of the use of enactive and iconic visual scaffolds did liberate understanding, but this was a hard-won prize. Participants struggled to apply cogent visual scaffolds to word-problems; in such situations, illogically constructed visualisations impeded understanding. For some, the English language and phrasing in the word-problems constituted a barrier.

Questionnaires 1 and 2 produced rich biographical information and provided insights regarding the students' perceptions of mathematics. The focus group interview provided thick, personal accounts of their opinions of the IPS 413 E module, of the methodologies used and of the use of visualising scaffolds in arithmetic work and word-problems. These accounts endorsed the content, methodology and pedagogy used in the module and suggested that the programme had liberated many suppressed skills and had improved the students' dispositions towards mathematics.

Upon their return from their second semester practicums, participants were invited to complete a final questionnaire. The qualitative data showed that the students continued to endorse the methodological and pedagogical approaches that had been used in the IPS 413 E module. The students reported that they had found the methodologies and liberating pedagogy that was infused into the module, highly beneficial in their own teaching and suggested that children in their practicums embraced these novel and liberating epistemological approaches.

Next, in Chapter 5, I will consolidate all of the trends that were discussed in Chapter 4 and I will raise implications that these findings have revealed to me. I will discuss problems which occurred in the action research and indicate recommendations for further research. I will also indicate the contributions made by this research and I will present a short autobiographical account of my experience of the thesis. I will close the chapter with a conclusion to the work.

# CHAPTER 5 – REVIEW, IMPLICATIONS AND CONCLUSION

## 5.0 SUMMARY OF THESIS

Chapter 1 opened with a prologue to the thesis. It described my first memory of purposefully using visualisation as a learning scaffold, in the 1970s, in an English language classroom. In that class, colourful pencil icons were used to assist children to assimilate and accommodate parts of speech. The prologue is important because it provides a context for my life-long passion for making the obscure, tangible; the invisible, visible. Visualisation has influenced all aspects of my teaching and learning career and its application in solving mathematical word-problems has become a central focus of this thesis. Indeed, the purpose of the thesis was to evaluate how the use of visualisation strategies, applied in a university classroom setting, would assist PGCE students to better understand and solve mathematical word-problems.

I demonstrated that, in principle, South Africa, like many countries, has adopted an understanding-based approach for teaching and learning mathematics. However, many PGCE students, that is, students who have matriculated through South Africa's education system and are university graduates in different fields, are often perplexed by the low order arithmetic procedural requirements and linguistic complexities that are characteristics of Intermediate Phase (Grade 4 – 6) word-problems.



Word-problems petition the use of many interwoven skills and understandings. However, where deeply conceptualised understanding is not available, cognitive overload can thwart success. As a remedial measure, I suggested that the purposeful use of visualisation methodologies can assist students to solve word-problems. In this thesis, my contention has been that a strongly visual approach – one which is rich in the use of concrete and iconic stimuli – can offset some of the barriers which preclude students from success and can facilitate better understanding of word problems.

Because action research requires one to become contemplative, reflective and reflexive, in the first section of Chapter 2 I have articulated my deeply held view that the purpose of a good education must be to liberate human potentials. I referred to Plato and Freire to construct an argument that suggests that, where human potentials are inhibited, psychological damage is done to people and societies.

In Plato's allegory of the cave, I found a philosophical foothold for my work: the allegory is used to outline a dual-level state of reality. Plato posits that humans often limit their view of the world to day-to-day experiences, sensory stimulation and naive belief systems that are found in the lower-level realm of sensations. But, he suggests, true emancipation only occurs on a higher plain, in the realm of forms, when meta-cognitive thinking is brought to bear on our conceptions of the world. Plato suggests that although our progression towards deep understanding and emancipation is largely driven by personal intrinsic motivations, teachers can act as conduits to facilitate the liberalization of latent potentials in their students. Teachers do this by engaging with students and encouraging them to think deeply, to be morally upstanding, to take calculated risks and to strive to conceptualise relationships and big, overarching ideas.

Freire suggests that because the ontological disposition of humans is to become more human, any oppressive practices which thwart that desire will result in conflict. Further, in any society in which an elite few subjugate others, the oppressor's violence will always result in some form of reciprocation. Oppressors will work to maintain the *status quo* but as oppressed people become aware of their own dehumanisation, they will agitate for their liberation. Oppressed people cannot initiate violence; they can only react to it. Freire suggests that although teachers are often unaware of their *benevolent oppression*, educators who use transmission mode teaching practices disenfranchise students, turning them into receivers of educational titbits, facts and routines. Freire rejects transmission mode, *tabula rasa* teaching practices – he calls this a banking model of education – in favour of a liberating model for education which in turn opens up possibilities for action, problem-solving and critical thinking.



Building on the above, I have demonstrated that South Africa's school-going pupils, measured against international and regional mathematical benchmarks, fare badly. In our country, poorly qualified teachers continue to use out-dated transmission-mode teaching practices. These practices have done a lot of damage to otherwise competent people. Because our teachers continue to use banking models of education, we have not delivered on the liberating promise that is contained within the definition of mathematics as espoused within the CAPS curriculum for mathematics.

In Chapter 2, aspects of mathematical problem-solving and word-problems, attributes of good versus poor problem-solvers, problem-solving techniques, hierarchies of problems, foundational and relational knowledge and compression were discussed. Foundation mathematical knowledge, on its own, limits students to working at a rote and instrumental level in mathematics. For more advanced work, relational understanding enables students to cogently apply a variety of mathematical abilities to multi-step, non-routine mathematical problems. Compression indicates automated mastery of interrelated mathematical skills and knowledge, of knowing not just a fact or skill, but knowing its relationship and applicability to other aspects of mathematics.

The literature review also investigated salient features of mathematics teacher education and looked at how we construct our conceptions of mathematics, a requirement for specialised teaching knowledge for mathematics and principles which inform effective mathematics teaching practice. As exemplars of PCK applied to mathematics teaching, multiplicative and fractional reasoning were explored.

Visualisation is a strategic teaching and learning tool in mathematics. Visualisation *mediates*: it acts as an intermediary scaffold between phenomena and symbolic representations of the world. Because it plays a pivotal role in my thesis, I unpacked many features of visualisation so as to make a sober case that demonstrates that pedagogically, while it may not prove to be a best fit for all students, visualising scaffolds do offer many significant benefits for many students.



All of the above brought me to the theoretical framework for this thesis which draws broadly on Jerome Bruner's opinions about the processes of education and specifically on his three-phase theory of deep learning. In unpacking my understanding of Bruner's work, I have tried to demonstrate its potential for practical application in classrooms. Bruner believed that knowledge construction requires active participation by the learner. He suggested that deep learning occurs when we link new ideas and information to our existing knowledge structures. He supported socially-mediated, non-routine, problem-solving based and intrinsically-motivated discovery learning. He proposed that teachers should apply a spiral approach in their educational programmes and suggested that structured, sequenced and embodied iterations of mathematical skills and ideas would facilitate deep understanding. To achieve the goal of deep understanding, Bruner suggested that all learning should begin in the world of enactive representations, followed by visual or iconic modes of representation, and finally, subsuming the others, by symbolic notations of our understanding. He perceived that a strength of his three-phase learning theory was that students would loop-back through these different forms of representations so as to pick a most suitable scaffold to make sense of a particular problem. Indeed, all of the teaching and learning in the research was deeply informed by Bruner's views.

I began Chapter 3 with an explanation about the strengths and limitations of action research. I unpacked the epistemological qualities that are found in action research and introduced my research approach, mixed methods. I explained that I wanted to combine different data sources to construct a rich tapestry of the benefits and limitations, highlights and low-points which might occur during the actual action research exercise. Following this, I explained the action research paradigm as being interpretive and I suggested that my research adopted a naturalistic perspective and was driven by a relativist ontology.



In Chapter 4, even after multiple edits and refinement of the primary data, the consolidated summaries of data remained a large and somewhat unwieldy challenge to put down onto paper. Because it covered so much ground, the captured data was discussed in a number of sequenced revelations of qualitative and quantitative data-sets. Qualitative data was quantitized; similarly, quantitative data was qualitized. From my fieldwork notes, I constructed a narrative which included features of important day-to-day events which occurred during the action research. I presented three problematic issues which conspired against progress in the research and, as exemplars of praxis, I constructed five explanations of educational actions that were taken within the interventions. This section was followed by an in-depth discussion of the qualitative findings from the Thinkboard activity.

In Action Cycle 1, quantitative data was captured via two prior knowledge tasks and two re-evaluations. That data was grouped together and interrogated. Trends which manifested were accounted for and often iconic exemplars of problems and solutions were inserted into the text so as to construct a strong evidence trail. The data in Action Cycle 2 was attended to in the same way as Action Cycle 1. Qualitative data was captured from two questionnaires. That data was interrogated and consolidated and the most important features were presented as trends. I then moved on to a discussion of the findings that were obtained from the focus group interview. I included quotations of the opinions and ideas of the participants' answers to a number of semi-structured questions that were posed to them by an interviewer.

In the fourth quarter of 2016, I asked the participants to give a retrospective account of their experiences of the IPS 413 E module: I did this by inviting the participants to complete one last questionnaire. The data was consolidated to establish important trends, and the trends were introduced into the chapter.

## 5.1 SUMMARY OF RESEARCH DATA

Below, the research data is consolidated in two ways. First, in 5.1.1, I have presented a series of box and whisker plots, which act as a summary of the quantitative data that was captured in the two action cycles. I have also constructed a statistical analysis of the quantitative data, and in 5.1.2 I have constructed summaries of qualitative data that was captured from the three questionnaires that the research participants returned to me. Second, in 5.1.3 – 5.1.6, I have linked the data findings to the three sub-research questions and to the main research question.

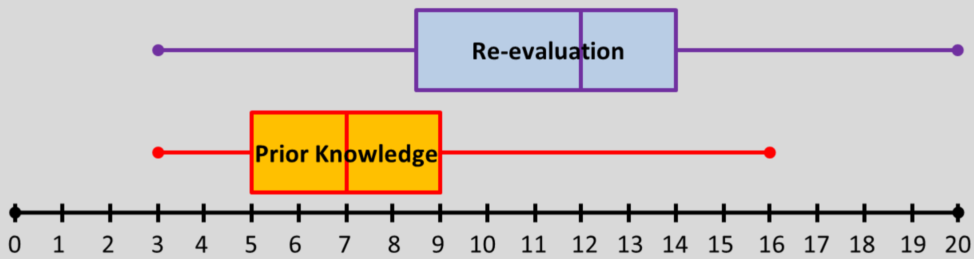


### 5.1.1 SUMMARY OF QUANTITATIVE RESULTS

The graphs below already exist in Chapter 4 (4.3.9; 4.4.5; 4.5.10; 4.6.5; 4.7.11; 4.8.6; 4.9.11; 4.10.4; 4.14.6); however, I believe that when they are read in conjunction with each other, the visual displays offered by the various box and whisker plots provide a succinct and powerful summary of the slow but steady progress that was made by the participants who competed their studies in IPS 313 E – Mathematics, in June 2016. In *Figures 5.1,* and *5.2* below, I have combined the data for the prior-knowledge and re-evaluation tasks for each action cycle into a series of four graphs.

## Action Cycle 1

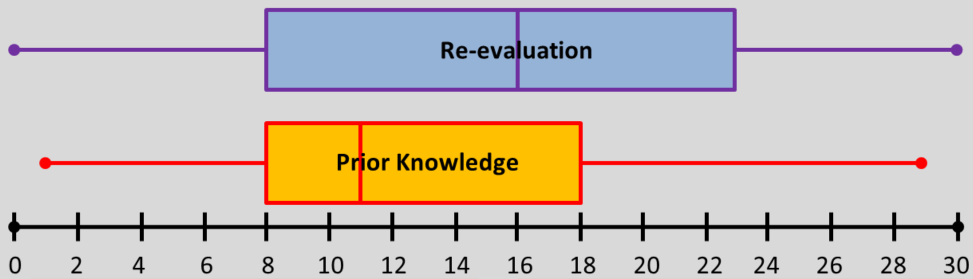
Summary: Quantitative Data
FOUNDATIONAL ARITHMETIC CONCEPTS
Action Cycle 1




| Quantitative Data | |
|---|---|
| Date of Test | 10/2/2016 |
| Number of submitted Scripts | 34 |
| Number who did not write | 4 |
| TOTAL | 38 |
| Mean | 7 |
| Mode (6 occurrences) | 4 |
| Median | 7 |
| Range (16-3) | 13 |
| Quartiles | 5; 7; 9 |
| Interquartile Range (9-5) | 4 |

| Quantitative Data | |
|---|---|
| Date of Test | 16/3/2016 |
| Number of submitted Scripts | 38 |
| Number who did not write | 0 |
| TOTAL | 38 |
| Mean | 12 |
| Mode (5 occurrences) | 14 |
| Median | 12 |
| Range (20-3) | 17 |
| Quartiles | 8,5; 12; 14 |
| Interquartile Range (14-8,5) | 5,5 |

Summary: Quantitative Data
WORD-PROBLEM CALCULATIONS
Action Cycle 1



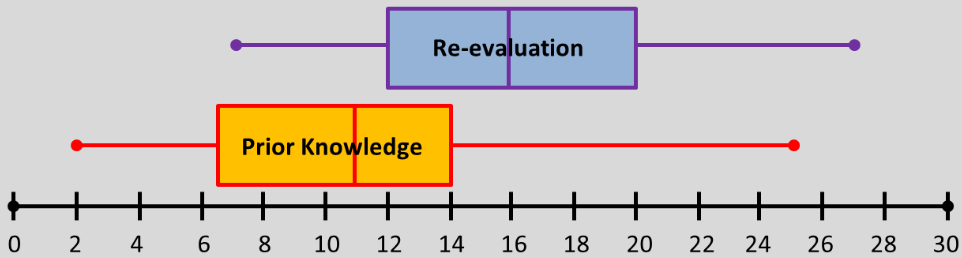

| Quantitative Data | |
|---|---|
| Date of Test | 22/2/2016 |
| Number of submitted Scripts | 37 |
| Number who did not write | 1 |
| TOTAL | 38 |
| Mean | 12 |
| Tri-Modal (3 occurrences each) | 8; 11; 19 |
| Median | 11 |
| Range (29-1) | 28 |
| Quartiles | 8; 11; 18 |
| Interquartile Range (18-8) | 10 |

| Quantitative Data | |
|---|---|
| Date of Test | 16/3/2016 |
| Number of submitted Scripts | 37 |
| Number who did not write | 1 |
| TOTAL | 38 |
| Mean | 15 |
| Bi-Modal (4 occurrences each) | 11; 25 |
| Median | 16 |
| Range (30-0) | 30 |
| Quartiles | 8; 16; 23 |
| Interquartile Range (23-8) | 15 |

*Figure 5.1.* Summary of quantitative data captured in Action Cycle 1

## Action Cycle 2

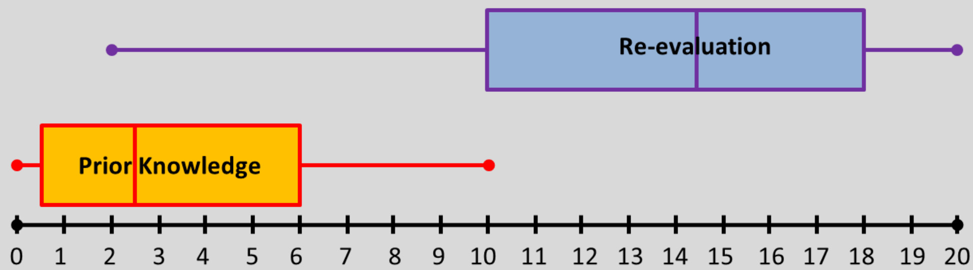
Summary: Quantitative Data
VISUAL FRACTIONS AND RATIOS CONCEPTS
Action Cycle 2




| Quantitative Data | |
|---|---|
| Date of Test | 20/4/2016 |
| Number of submitted Scripts | 36 |
| Number who did not write | 2 |
| TOTAL | 38 |
| Mean | 11 |
| Bi-Modal (5 occurrences each) | 6; 9 |
| Median | 10 |
| Range (25-2) | 23 |
| Quartiles | 6,5; 11; 14 |
| Interquartile Range (14-6,5) | 7,5 |

| Quantitative Data | |
|---|---|
| Date of Test | 16/5/2016 |
| Number of submitted Scripts | 38 |
| Number who did not write | 0 |
| TOTAL | 38 |
| Mean | 17 |
| Bi-Modal (4 occurrences each) | 12; 19 |
| Median | 16 |
| Range (27 - 7) | 20 |
| Quartiles | 12; 16; 20 |
| Interquartile Range (20 - 12) | 8 |

Summary: Quantitative Data
VISUAL WORD-PROBLEM CALCULATIONS
Action Cycle 2




| Quantitative Data | |
|---|---|
| Date of Test | 20/4/2016 |
| Number of Participants | 36 |
| Number who did not write | 2 |
| TOTAL | 38 |
| Mean | 3 |
| Mode (9 responses) | 0 |
| Median | 2,5 |
| Range (10 - 0) | 10 |
| Quartiles | 0,5; 2,5; 6 |
| Interquartile Range (6 – 0,5) | 5,5 |

| Quantitative Data | |
|---|---|
| Date of Test | 16/5/2016 |
| Number of Participants | 38 |
| Number who did not write | 0 |
| TOTAL | 38 |
| Mean | 14 |
| Bi-Modal (7 occurrences each) | 14; 18 |
| Median | 14,5 |
| Range (20-2) | 18 |
| Quartiles | 10; 14,5; 18 |
| Interquartile Range (18-10) | 8 |

*Figure 5.2.* Summary of quantitative data captured in Action Cycle 2

Four, inferential *t-Test: Paired Two Sample of Means* analyses were used to compare the four prior knowledge tasks with their corresponding re-evaluation tasks.

When one conducts t-Tests, a sample size of 10 is regarded as being the minimum, thus, with data sets of 34 to 37 natural pairings and df's of 33 to 36, the data used in the t-Tests would be considered as stable representations. In the analyses, in cases where a participant did not complete one of the two assessments, the data for that participant was excluded from the analysis. MS Excel© was employed to compute the statistical analyses.

The t-Tests were driven by two competing hypotheses, those being a null hypothesis which assumed that the mean differences between the pairings was zero, and an alternate hypothesis, which assumed that the mean differences between the pairings was not zero. In this research, a two-tailed test was used, thus the two competing hypotheses can be defined as:

$H_o$: $\mu_o = 0$ that is, there will be no change between the means in the data sets

$H_a$: $\mu_o \neq 0$ that is, there will be a change between the means in the data sets.



While a simple comparison of the means of two sets of data, for example 78% and 83% might show that 83% is indeed larger (better) than 78%, the t-Test: Paired Two Sample of Means eliminates the skewing effects that factors such as outliers might effect on the findings.

Although MS Excel© generates a comprehensive set of results for the t-Test: Paired Two Sample of Means, for purposes of analysis:

1. the alpha level (0,05) is compared with the P(T,=t) two-tail value. If the P(T,=t) two-tail value is less than alpha (0,05) the null hypothesis can be rejected.
2. Further, if the t Stat-value is larger than the t Critical two-tail value, the null hypothesis can be rejected.
3. The outcomes of both conditions must correlate in order to reject the null hypothesis.

Table 5.1

*Summary of statistical analysis of Action Cycle 1 – Foundational Arithmetic Concepts*

| t-Test: Paired Two Sample for Means | | | | | |
|---|---|---|---|---|---|
| ACTION CYCLE 1 - FOUNDATIONAL ARITHMETIC CONCEPTS | | | | | |
| | *RE-EVAL* | *PRIOR* | | | |
| Mean | 12,265 | 7,471 | | Mean Difference | 4,794 |
| Variance | 14,928 | 10,499 | | Standard Deviation of difference | 2,837 |
| Observations | 34,000 | 34,000 | | Standard Error of difference | 0,487 |
| Pearson Correlation | 0,694 | | | T alpha half 95% Confidence Interval | 2,035 |
| Hypothesized Mean Difference | 0,000 | | | Lower Confidence Level | 3,804 |
| df | 33,000 | | | Upper Confidence Level | 5,784 |
| t Stat | 9,854 | | | α | 0,05 |
| P(T<=t) one-tail | 0,000 | | | | |
| t Critical one-tail | 1,692 | | | | |
| P(T<=t) two-tail | 0,000 | | | $H_o$: $\mu_o = 0$ | |
| t Critical two-tail | 2,035 | | | $H_a$: $\mu_o \neq 0$ | |

In Table 5.1, above, the P(T,=t) two-tail value (0,000) is less than alpha (0,05). Also, the t-Stat value (9,854) is larger than the t Critical two-tail value (2,035). Therefore, the null hypothesis is rejected.

Table 5.2

*Summary of statistical analysis of Action Cycle 1 – Word Problems*



| t-Test: Paired Two Sample for Means | | | | | |
|---|---|---|---|---|---|
| ACTION CYCLE 1 - WORD PROBLEMS | | | | | |
| | *RE-EVAL* | *PRIOR* | | | |
| Mean | 15,556 | 13,056 | | Mean Difference | 2,500 |
| Variance | 75,625 | 50,454 | | Standard Deviation of difference | 5,107 |
| Observations | 36,000 | 36,000 | | Standard Error of difference | 0,851 |
| Pearson Correlation | 0,809 | | | T alpha half 95% Confidence Interval | 2,030 |
| Hypothesized Mean Difference | 0,000 | | | Lower Confidence Level | 0,772 |
| df | 35,000 | | | Upper Confidence Level | 4,228 |
| t Stat | 2,937 | | | α | 0,05 |
| P(T<=t) one-tail | 0,003 | | | | |
| t Critical one-tail | 1,690 | | | | |
| P(T<=t) two-tail | 0,006 | | | $H_o$: $\mu_o = 0$ | |
| t Critical two-tail | 2,030 | | | $H_a$: $\mu_o \neq 0$ | |

In Table 5.2, above, the P(T,=t) two-tail value (0,006) is less than alpha (0,05). Also, the t-Stat value (2,937) is larger than the t Critical two-tail value (2,030). Therefore, the null hypothesis is rejected.

Table 5.3

*Summary of statistical analysis of Action Cycle 2 – Visual Fractions and Ratios*

| t-Test: Paired Two Sample for Means | | | | | |
|---|---|---|---|---|---|
| ACTION CYCLE 2 - VISUAL FRACTIONS & RATIOS | | | | | |
| | *PRIOR* | *RE-EVAL* | | | |
| Mean | 17,111 | 10,639 | | Mean Difference | 6,132 |
| Variance | 29,930 | 28,409 | | Standard Deviation of difference | 3,363 |
| Observations | 36,000 | 36,000 | | Standard Error of difference | 0,560 |
| Pearson Correlation | 0,834 | | | T alpha half 95% Confidence Interval | 2,030 |
| Hypothesized Mean Difference | 0,000 | | | Lower Confidence Level | 4,994 |
| df | 35,000 | | | Upper Confidence Level | 7,269 |
| t Stat | 12,478 | | | α | 0,05 |
| P(T<=t) one-tail | 0,000 | | | | |
| t Critical one-tail | 1,690 | | | | |
| P(T<=t) two-tail | 0,000 | | | $H_o: \mu_o = 0$ | |
| t Critical two-tail | 2,030 | | | $H_a: \mu_o \neq 0$ | |

In Table 5.3, above, the P(T,=t) two-tail value (0,000) is less than alpha (0,05). Also, the t-Stat value (-12,478) is larger than the t Critical two-tail value (2,030). Therefore, the null hypothesis is rejected.

Table 5.4

*Summary of statistical analysis of Action Cycle 2 – Word Problems*



| t-Test: Paired Two Sample for Means | | | | | |
|---|---|---|---|---|---|
| ACTION CYCLE 2 - WORD PROBLEMS | | | | | |
| | *RE-EVAL* | *PRIOR* | | | |
| Mean | 14,486 | 3,189 | | Mean Difference | 11,000 |
| Variance | 21,146 | 8,324 | | Standard Deviation of difference | 4,748 |
| Observations | 37,000 | 37,000 | | Standard Error of difference | 0,781 |
| Pearson Correlation | 0,368 | | | T alpha half 95% Confidence Interval | 2,028 |
| Hypothesized Mean Difference | 0,000 | | | Lower Confidence Level | 9,417 |
| df | 36,000 | | | Upper Confidence Level | 12,583 |
| t Stat | 15,477 | | | α | 0,05 |
| P(T<=t) one-tail | 0,000 | | | | |
| t Critical one-tail | 1,688 | | | | |
| P(T<=t) two-tail | 0,000 | | | $H_o: \mu_o = 0$ | |
| t Critical two-tail | 2,028 | | | $H_a: \mu_o \neq 0$ | |

In Table 5.4, above, the P(T,=t) two-tail value (0,000) is less than alpha (0,05). Also, the t-Stat value (15,477) is larger than the t Critical two-tail value (2,028). Therefore, the null hypothesis is rejected.

It can therefore be inferred that a statistically significant improvement occurred in each pair of prior knowledge and re-evaluation assessments.

### 5.1.2 SUMMARY OF QUALITATIVE RESULTS

A Thinkboard and three questionnaires provided useful insights into the views that were held by the participants. While Questionnaire 1 provided a mix of quantitative and qualitative data, in the main, the Thinkboard and the questionnaires were designed to extract qualitative data.

That data has been discussed extensively in Chapter 4 and is now summarised in a series of tables (Table 5.5 - Table 5.10), below.



Table 5.5

*Summary of Thinkboard data*

| | THINKBOARD ACTIVITY - 10 FEBRUARY 2016 | |
|---|---|---|
| | **Field 1: My View of Mathematics** | |
| | Number of Participants | 33 |
| | Number of Responses | 97/99 |
| | Return Rate | 98% |
| | **Major Trends** | **Number of submissions** |
| 1 | Practice makes perfect | 15 |
| 2 | Maths is integrated into daily life | 17 |
| 3 | Maths is perceived as just calculations | 11 |
| 4 | Maths promotes brain activity | 12 |
| 5 | It is interesting but challenging | 13 |
| 6 | A tricky, complicated and confusing subject | 9 |
| 7 | Maths prepares one for success in life | 8 |
| 8 | Keep away from this subject | 7 |
| | **Field 2: My Biggest Hopes for Mathematics** | |
| | Number of Participants | 33 |
| | Number of Responses | 97/99 |
| | Return Rate | 98% |
| | **Major Trends** | **Number of submissions** |
| 1 | A desire to be a great mathematics teacher | 23 |
| 2 | A need to understand mathematics | 27 |
| 3 | Committed to overcoming a fear of maths | 22 |
| 4 | I just want to pass | 19 |
| | **Field 3: My Biggest Fears for Mathematics** | |
| 1 | Number of Participants | 33 |
| 2 | Number of Responses | 93/99 |
| 3 | Return Rate | 94% |
| | **Major Trends** | **Number of submissions** |
| 1 | A deep-seated fear of calculations | 28 |
| 2 | A fear of not teaching mathematics well | 23 |
| 3 | Memories of past failures in mathematics | 15 |
| 4 | A fear of failing the IPS 413 E module | 11 |
| 5 | A fear of giving up on the subject | 9 |
| | **Field 4: A Drawing of how I Feel at this Moment** | |
| 1 | Number of Participants | 33 |
| 2 | Number of Responses | 33/33 |
| 3 | Return Rate | 100% |
| | **Major Trends** | **Number of submissions** |
| 1 | So many faces | 33 |
| 2 | Happy/ambivalent/anxious/shocked/ serene/ crying/angry | 20/5/3/3/1/1 |
| 3 | Simplistic representations | mostly |

Table 5.6

*Summary of Questionnaire 1 – Part A – A1*

| | | |
|---|---|---|
| **QUESTIONNAIRE 1 - 22 APRIL 2016** | | |
| **Part A - A1 - Biographical and Language Information** | | |
| | Number of Participants | 33 |
| **1** | **I am:** | |
| | Female | 24 |
| | Male | 9 |
| **2** | **My Home Language is:** | |
| | Afrikaans | 0 |
| | English | 2 |
| | isiXhosa | 27 |
| | Other | 4 |
| **3** | **My Current Age is:** | |
| | 20 – 25 | 15 |
| | 26 – 30 | 9 |
| | 31 – 35 | 8 |
| | 36 – 40 | 0 |
| | Older | 1 |
| **4** | **My highest school pass in math or math literature is:** | |
| | Grade 8 | 0 |
| | Grade 9 | 1 |
| | Grade 10 | 5 |
| | Grade 11 | 2 |
| | Grade 12 | 24 |

Table 5.7

*Summary of Questionnaire 1 – Part A – A2 and Part B – B1-5*

| | | | | | | |
|---|---|---|---|---|---|---|
| **QUESTIONNAIRE 1 - 22 APRIL 2016** | | | | | | |
| **Part A - A2 - Students' perceptions of mathematics and IPS 413 E** | | | | | | |
| | | **Strongly disagree** | **Disagree** | **In the middle** | **Agree** | **Strongly agree** |
| **5** | Generally, I enjoy doing mathematics | 1 | 1 | 10 | 15 | 6 |
| **6** | Generally, I am good at doing mathematics | 1 | 6 | 18 | 6 | 2 |
| **7** | I am excited to be doing IPS 413 E | 0 | 1 | 5 | 13 | 14 |
| | | | | | | |
| **Part A - A3 - Fluency in English Language when doing mathematics** | | | | | | |
| **8** | I find it easy to listen to and understand English instructions about mathematics | 1 | 3 | 10 | 12 | 7 |
| **9** | I find it easy to read to and understand English instructions about mathematics | 0 | 4 | 8 | 18 | 3 |
| **10** | I find it easy to talk to and understand English instructions about mathematics | 0 | 5 | 7 | 13 | 8 |
| **Part B - B1 - Read and Understand Word Problems** | | | | | | |
| **11** | I find it easy to read word-problems | 0 | 3 | 19 | 10 | 1 |
| **12** | I find it easy to understand what word-problems require me to do | 2 | 5 | 16 | 9 | 1 |
| **Part B - B2 - Form Mental Images of Word Problems** | | | | | | |
| **13** | I find it easy to construct a mental image of the requirements of word-problems | 2 | 8 | 12 | 10 | 1 |
| **14** | I find it easy to imagine a solution strategy for solving word-problems | 2 | 11 | 13 | 6 | 1 |
| **Part B - B3 - Construct a Number Sentence or Formula to start to solve a Word Problem** | | | | | | |
| **15** | I find it easy to find a starting point to begin to solve word-problems | 1 | 10 | 16 | 5 | 1 |
| **16** | I find it easy to construct an appropriate number sentence or formula to solve word-problems | 2 | 8 | 18 | 4 | 1 |
| **Part B - B4 - Can do the Actual Calculations needed to solve the Word Problems** | | | | | | |
| **17** | I find it easy to work with numbers and operations ( + - x / √) (1 response missing) | 0 | 3 | 12 | 12 | 5 |
| **18** | I find it easy to do calculations which are used to solve word-problems (2 responses missing) | 0 | 5 | 12 | 11 | 3 |
| **Part B - B5 - Ability to Judge the Validity of a Calculated Answer** | | | | | | |
| **19** | I find it easy to judge approximately what my final answer will be | 3 | 9 | 12 | 7 | 1 |
| **20** | I find it easy to look at my answers and judge if they look okay | 2 | 6 | 10 | 12 | 1 |

Table 5.8

*Summary of Questionnaire 2 – Part 1*

| | | | | | | |
|---|---|---|---|---|---|---|
| **QUESTIONNAIRE 2 - 13 MAY 2016** | | | | | | |
| | Number of participants | | | | | 34 |
| **Part 1 - Personal Benefit - Understanding and Solving Word Problems** | | | | | | |
| **Part 1 - 1 - Read and understand word problems** | | | | | | |
| | | **Strongly disagree** | **Disagree** | **In the middle** | **Agree** | **Strongly agree** |
| **1** | I have found the use of objects & images makes it easy for me to read word-problems | 0 | 0 | 1 | 15 | 18 |
| **2** | I have found that the use of objects and images makes it easy to understand what word-problems require me to do | 0 | 0 | 3 | 17 | 14 |
| **Part 1 - 2 - Form mental images of word problems** | | | | | | |
| **3** | I have found that the use of objects and images assists me to form a mental image of the requirements of word-problems | 1 | 0 | 4 | 19 | 10 |
| **4** | I have found that the use of objects and images helps me to imagine a solution strategy for solving word-problems | 1 | 1 | 0 | 22 | 10 |
| **Part 1 - 3 - Successfully construct a number sentence to start to solve the problem** | | | | | | |
| **5** | I have found that the use of objects and images makes it easy for me to find a starting point to begin to solve word-problems | 1 | 0 | 6 | 18 | 9 |
| **6** | I have found that the use of objects and images makes it easy for me to construct an appropriate number sentence or formula to solve word-problems | 1 | 1 | 5 | 16 | 11 |
| **Part 1 - 4 - Can do the actual calculations needed to solve the word problem** | | | | | | |
| **7** | I have found the use of objects & images makes it easy for me to work with numbers | 0 | 1 | 2 | 18 | 13 |
| **8** | have found the use of objects & images makes it easy for me to do the calculations which are needed to solve word-problems | 1 | 0 | 3 | 22 | 8 |
| **Part 1 - 5 - Ability to judge the viability of a calculated answer** | | | | | | |
| **9** | I have found the use of objects & images makes it easy for me to approximately judge what my final answer will be | 1 | 0 | 7 | 15 | 11 |
| **10** | I have found that the use of objects and images makes it easy for me to look at my final answers and judge if they look okay | 1 | 0 | 3 | 19 | 11 |

Table 5.9

*Summary of Questionnaire 2 – Part 2*

| QUESTIONNAIRE 2 - 13 MAY 2016 | | | | | | |
|---|---|---|---|---|---|---|
| | Number of participants (4 did not complete this part of the questionnaire) | | | | | 30/34 |
| **Part 2 - Methodological Benefit - Using Visualisation when Teaching Children to Solve Word Problems** | | | | | | |
| | | **Strongly disagree** | **Disagree** | **In the middle** | **Agree** | **Strongly agree** |
| **11** | I believe that the use of objects and images (visualization) will help my pupils to read and understand word-problems | 0 | 0 | 0 | 8 | 22 |
| **12** | I believe that the use of objects and images (visualization) will help my pupils to form mental images of word-problems | 0 | 0 | 0 | 10 | 20 |
| **13** | I believe that the use of objects and images (visualization) will help my pupils to successfully construct a number sentence or formula to start to solve word-problems | 0 | 1 | 0 | 14 | 15 |
| **14** | I believe that the use of objects and images (visualization) will help my pupils to do the actual calculations needed to solve word-problems | 0 | 0 | 1 | 11 | 18 |
| **15** | I believe that the use of objects and images (visualization) will help my pupils to judge the validity of their calculated answers | 0 | 0 | 3 | 8 | 19 |

Table 5.10

*Summary of Questionnaire 3*

| | | | | |
|---|---|---|---|---|
| **QUESTIONNAIRE 3 - 14 SEPTEMBER 2016** | | | | |
| | Number of participants | | | 13 |
| **Completed upon return from schools-based practicums** | | | | |
| | | **Negative Response** | **Neutral Response** | **Positive Response** |
| **1** | Has the IPS 413 E Mathematics module (the first semester module) in any way changes your attitude towards mathematics and, if so, how? | 0 | 0 | 13 |
| **2** | Did you find the knowledge, activities and methods used in the IPS 413 E Mathematics module useful to you and, if so, how? | 0 | 0 | 13 |
| **3** | Did your experiences in IPS 413 E Mathematics help you to teach mathematical ideas to children during School Experience and, if so, how? | 1 | 2 | 10 |
| **4** | Did you find the use of apparatus (e.g. biscuits, buttons, bottle-tops) and images (e.g. drawings of mathematical ideas) useful and, if so, how? | 0 | 0 | 13 |
| | | | | |
| | | | No | Yes |
| **5a** | During school experience, did you teach children to solve mathematical problems? | | 2 | 11 |
| **5b** | If your answer to 5.a. is yes, did you use visualising techniques (e.g. biscuits, bottle-tops, drawings, etc.)? (2 non-responses) | | 0 | 11 |
| **5c** | If your answer to 5.b is yes, please explain the visual methods that you used. (2 non-responses) | bottle tops, drawings, charts, sweets, biscuits match sticks, visual pictures, visual strips, stones | | |
| **5d** | If your answer to 5.b is yes, did the children in your class like the approach? (2 non-responses) | | 0 | 11 |

## 5.2 SYNTHESIS OF FINDINGS

This action research was employed to answer one pivotal research question and three supplementary, or sub-research questions. In order to fully explain the richness of the findings of the data that was captured by the summary of data in 5.1.1 and 5.1.2, I will first reflect upon data which addresses the sub-questions and after that, I will use those findings to build up a case to answer the main research question.

### 5.2.1 SUB 1: WHAT EXISTING WORD-PROBLEM, PROBLEM-SOLVING STRATEGIES DO THE STUDENTS HOLD?

The Word-Problem Calculations, Action Cycle 1, prior knowledge assessment task was specifically designed to uncover the *existing* problem-solving techniques that were employed by the participants. The assessment task was administered to the students before they participated in any activities in the IPS 413 E – Mathematics module. Further, *preceding* that word problems assessment task, the participants completed a Foundational Arithmetic Concepts assessment task.



The data captured from the arithmetic assessment task provided evidence which suggested that many participants were unable to construct procedurally logical solutions to low order, routines-based, arithmetic operations work. Strong, recurring features of arithmetic ineptitude were found and included an inability to neatly set out a cogent solution to arithmetic problems (see 4.3.1); low success rates in solving single operations, Intermediate Phase level arithmetic tasks (4.3.2); inconsistency in applying the BODMAS mnemonic to multi-operations calculations (4.3.4); an inability to identify patterns of perfect squares (4.3.5); low understanding of fractions (4.3.6 and 4.3.7); and an inability to work with ratios (4.3.8). Collectively, these types of difficulties were consistent with at least three of the five problem-solving strategies that Gooding (2009, p. 5) has identified, in that, (1) the data demonstrated that the participants were unable to successfully construct number sentences or algorithms, (2) they could not carry out mathematical calculations and, (3) the participants demonstrated an inability to judge the validity of the answers that they had calculated. The absence of multiplicative and fractional thinking (2.5.3 and 2.5.4) precluded cogent instrumental understanding and compression of mathematical meta-ideas.

When I turned my attention to the word-problems assessment task, I found that many of the error trends in that task were consistent with the findings in the arithmetic task. Many solutions were messy, disorganised and procedurally incorrect (4.4.1); the participants had struggled enormously with fractions, often demonstrating absolutely no understanding of the roles of numerators and denominators in fractions and/or their relationship to the larger task at hand (4.4.3); very few students constructed any form of visual scaffold (4.4.3.3); and many solutions in the test papers demonstrated that idiosyncrasies in the English language had quite easily become a barrier to success (4.4.4). The data revealed that, in addition to the three types of difficulties that were revealed by the arithmetic task, in the word-problems assessment task the participants were (4) unable to read and understand word-problems nor (5) could they judge the validity of their answers.

Thus, a concise answer to sub-research question 1 is that at the commencement of the action research, within the cohort, there was generally very little evidence of any consistent application of cogent or strategic problem-solving technique. Instead, the quantitative evidence captured from the two prior knowledge assessment tasks suggested that most participants seemed to have little or no access to basic arithmetic foundational knowledge and skills and showed that the routine arithmetic procedures that are used in highly structured and moderately structured problems (Zanele, 2015), as discussed in (2.4.4), were largely unavailable to the participants.



### 5.2.2 SUB 2: WHAT BARRIERS TO SOLVING WORD-PROBLEMS DO THE STUDENTS PERCEIVE THAT THEY HAVE?

Questionnaire 1 (4.11) was purposely constructed to identify the participants' perceptions of the barriers that they perceived might hinder their success with mathematical word-problems.

Data captured from Part A of Questionnaire 1 (see 4.11.1, Table 5.6 and Table 5.7), suggested that generally the participants were female isiXhosa speakers, who were moderately pleased to be involved in mathematics and moderately comfortable with the use of English as the medium of instruction when doing mathematics.

In Part B of Questionnaire 1 (4.11.2 and Table 5.7), I used Gooding's (2009, p. 5) classification of five types of difficulties that students encounter when they solve word-problems to formulate my questions. Part B contained five pairs of Likert scale questions. Each pair of questions contained below them a short specified response field. The data revealed that the majority of the participants were ambivalent (in the middle) to moderately confident that they could read and understand word-problems and form mental images of word-problems. When asked to reflect upon their ability to construct starting points to solve word-problems, the majority of participants were in the middle or felt moderately incapable of creating starting-points to solve problems. However, when responding to questions regarding their abilities to actually do calculations, that is, the procedural and computational work, the majority of participants perceived themselves as in the middle to moderately capable of doing this work. With regard to their ability to judge the validity of their calculated answers, the participants were mainly ambivalent; a small number of participants were moderately happy that they could judge the validity of their answers while a slightly larger number of respondents were moderately unable to judge the validity of their answers.



Overall, the participants' perceptions of their problem-solving abilities were neutral to optimistic. However, when their perceptions were contrasted with their performance in the arithmetic and word-problems prior knowledge and re-evaluation assessment tasks in Action Cycle 1, (4.3; 4.4; 4.5; 4.6), that data painted a more gloomy picture. In those tasks, the participants enjoyed only a modicum of success when attending to *remembering* questions: such questions occupy the lowest level of Bloom's Revised Taxonomy (Krathwohl & Anderson, 2010, Maphalala, 2016, p. 94 - 97). Indeed, the success rates quickly diminished as layers of *understanding, applying* and *analysing* from the taxonomy were introduced to the questions.

Thus, the participants' perceptions of their abilities and the quantitative data occupied two different planes: the participants perceived that they were fairly capable of solving word-problems while the quantitative data from the four assessment tasks contradicted their views.

### 5.2.3 SUB 3: HOW EFFECTIVE DO THE STUDENTS PERCEIVE THE VISUALISATION STRATEGIES TO BE?

Towards the end of Action Cycle 2, the participants were asked to complete Questionnaire 2, (4.12). That questionnaire was used to ascertain their perceptions of the efficacy of the visualisation strategies that they had encountered during their studies in the IPS 413 E – Mathematics module. Like Questionnaire 1, it employed questions that were based on Gooding's (2009, p. 5) barriers that can impede success in solving word-problems.

In Part 1 of the questionnaire (4.12.1 and Table 5.8), Gooding's (2009, p. 5) questions were posed in a manner which required to the participants to reflect on their own sense of the efficacy of the visualisation strategies. Analysis of the data showed that, with only a handful of dissenting opinions, the replies found in the Likert scales and the short specified responses were moderately in agreement to strongly in agreement; that is, the participants endorsed the use of visualising methodologies as techniques suitable for alleviating Gooding's (2009) barriers. This was a very pleasing outcome.

In Part 2 (4.12.2), looking to the future, and again using Gooding's (2009) classification of problems, the participants were encouraged to imagine the potential efficacy that visualisation methodologies might derive to pupils as a teaching and learning methodology. In congruence with Part 1, the Likert scale answers were strongly supportive of visualising methodologies that the participants had learned to use and their short, textual replies were very complimentary of this type of work.

Also, towards the end of the second action cycle, a group of participants was invited to a focus group interview (4.13). In the interview, exploration question 5 (4.13.5), in particular, was constructed to ascertain the participants' views of any benefit that they felt that might have accrued to them by using the visualising methodologies in the IPS 413 E module. While some of the students indicated that initially they were apprehensive and doubtful of its role, in the main, the students strongly endorsed the visual approach.

Further, those participants who completed the post-action research questionnaire (4.14), were almost unanimous in their view that, during their school-based practicums, their own use of visualising scaffolds, as a teaching methodology, had enabled them to teach mathematics in a liberating way to their pupils. Both they and their pupils perceived the construction of visual scaffolds as being user-friendly and educationally beneficial. This was a very pleasing outcome.

Thus, all in all, the PGCE students perceived the use of visualisation strategies as being of high benefit.

### 5.2.4 MAIN: HOW WILL VISUALISATION STRATEGIES ASSIST STUDENT-TEACHERS TO BETTER UNDERSTAND AND SOLVE MATHEMATICAL WORD-PROBLEMS?

Beginning with my interrogation of the qualitative data in the Thinkboard activity, (4.2), and throughout the course of the research period, it become clear to me although the participants were generally keen to do and understand mathematics, many were fearful that they would not be up to the task. This deep desire within the participants to do better, to do *good*, produced an emancipatory reaction in me, one which oriented me towards the release of their untapped human potentials. I already held a practical interest in the thesis; I knew that I wanted to understand, interpret and thereby generate knowledge that would inform and guide the teaching and learning in our lessons. But, layered on top of that, I wanted to emancipate the participants from the rules and mindlessly learned practices, habits and self-deception (Carr & Kemmis, 1986) that was infused into their writing.



Largely, this fear was realised in the Action Cycle 1 arithmetic (4.3), and word-problems (4.4), prior knowledge assessment tasks which revealed that high numbers of the participants suffered high levels of mathematical ineptitude. The re-evaluation tasks (4.5 and 4.6) contained the same questions as the prior knowledge tasks. Although the questions were revisited and attention was given to constructing and reconstructing many arithmetic skills and concepts, vis-à-vis, for example, multiplicative and fractional thinking, the results in the re-evaluation tasks were only marginally better than in the prior knowledge tasks.

The consolidation of the quantitative data from the first action cycle suggested to me that, possibly, the confluence of a novel methodology – visualisation – superimposed upon an existing, *deeply flawed mathematical framework*, might rend the research asunder. Essentially, it appeared as if the limited number of visualisation strategies that were introduced in Action Cycle 1 had offered no assistance to the participants and, in fact, may have obfuscated their understanding. Maybe, conceptually, it was too large a step for the students to take. However, I can also aware that the first quarter of the 2016 academic year has been assaulted by numerous distractions (4.1.1; 4.1.2 and 4.1.3), and thereby we had lost a great deal of teaching and learning time. Often times, because of the mischievous nature of the sporadic interruptions, the few bigger mathematical ideas that I was trying to help the participants to understand, had to be continuously revisited. Every time a kernel of understanding was being sown, whether it be multiplicative facts or reasoning, or fractional representations and thinking, etcetera, a disruptive distraction thwarted its development, and thereby conspired against the compression of those big mathematical ideas.

With the problematic features of Action Cycle 1 uppermost in mind, I felt that it was important that I redouble my efforts to assist the participants to move away from the flawed instrumental conceptions of mathematics which had been revealed to be their *modus operandi.* I realised that I would only have a handful of lessons available to me for teaching and learning in the 2nd quarter of 2016, thus I concentrated my efforts on the operations, fractions and ratios. I also set about constructing courseware that would link enactive, iconic and symbolic representations of these mathematical ideas together (Bruner, 1966), (see 2.7.3; 2.7.4 and 2.7.5).



In Action Cycle 2, in both the arithmetic (4.7) and the word-problems components (4.8), the prior knowledge assessment tasks delivered very disappointing sets of results. However, the results from the re-evaluation tasks showed good improvement for the arithmetic task (4.9), and great improvement for the word-problems task (4.10). Thus, clearly, Action Cycle 2 benefited from the introspection of Action Cycle 1 and the fit-for-purpose courseware and visualising methodology that were incorporated into the second-quarter teaching and learning contact sessions.

Thus, in answering *how* the participants benefited, from an analysis of the quantitative data, it can be seen that the sustained use of visualising scaffolds, employed as a teaching and learning methodology with a cohort of IPS 413 E participants who had earlier demonstrated high levels of mathematical ineptitude, had proved to be academically beneficial to the cohort. Improvements were noted in areas of the operations, of which multiplicative reasoning is but one aspect, in fractional thinking and the compression of foundational arithmetic concepts, and the understanding of pertinent mathematical vocabulary and phrases. Further, the participants became quite adept at using visual scaffolds such as arrays, fractional circles and ribbons, and bar models as representations of their proportional mathematical thinking. Enactive and iconic representations were linked to symbolic mathematical notations and computations. All of these LOT and HOT skills contributed to big improvements in the participants' ability to work through word problems.

Further, in answering the sub-research questions (5.1.1, 5.1.2, and 5.1.3) above, I found that quantitative analysis of the assessment data shows a strong correlation with the quantitative data that was obtained from the Thinkboard, questionnaires and focus group interviews. This correlation was substantiated further by the questionnaire (4.15) that was offered to participants in the second semester of 2016, that is, *after* they had completed their second-semester school experience practicums. By that time, they had had opportunities to apply the visualising methodologies in their own teaching practice. Overwhelmingly, in their comments in the questionnaire, the participants presented views that suggested that they, as teachers, had felt that (1) the exemplification of problems through the use of tactile and iconic devices had improved the quality of their teaching practice and (2) the pupils in their classes had enjoyed and benefitted by using visual scaffolds to solve word-problems.



From a qualitative perspective, if one follows the qualitative trends that started with the Thinkboard activity through to the quantitative data trends that are presented in the post action research questionnaire, it can be seen that, in their conception of mathematics, the students moved away from a banking and rote-learning conception of mathematics, towards a more liberal, self-actualising understanding of the subject.

I think that it is fair to suggest that when contrasting the starting points of the action research with the end point, the quantitative data demonstrates sustained improvement in arithmetic and word-problem competences and the qualitative data demonstrates a healthy, positive and somewhat ebullient psychological outlook towards teaching and learning mathematics (4.14). In other words, the use of visual scaffolds has liberated the students academically and emotionally. Of these two benefits, possibly the emancipating benefit is of higher importance: indeed, because the students now look upon mathematics in a more favourable light, they might be predisposed to labour longer at mathematical tasks even when they are confronted with difficult or tricky non-routine word-problems.

Directly, the research data has shown that the visualising strategies that were used in the research have assisted the student-teachers to understand and solve mathematical word problems in the following ways:

1. Enactive apparatus and iconic representations of word problems were used to negotiate a better understanding of the mathematical vocabulary and turns of phrase which had misdirected their interpretation of the word problems. In the focus group interview this novel approach, the participants suggest, (4.13.2), made access to the meaning that was implied in the sentence structures of the word problems easier to navigate. However, it has to be acknowledged that throughout the research period, on many occasions, language barriers manifested as a continuing impediment to success (4.13.4).
2. With the benefit of a better understanding of the asking requirements implied by the vocabulary and turns of phrase, the visual scaffolds also made it easier for the participants to form mental images of the contexts within which their word problems were operating. As seen, for example in the focus group interview, (4.13.2), the participants used visualising scaffolds as practical exemplifications of their mental conceptions which in turn helped them to better understand and conceptualise their word problems. The participants mention the use of bottle tops, biscuits and my document camera. These devices and other stimuli were used to construct arrays and area models, to draw fractional ribbons and number lines and so forth. These visual representations facilitated multiplicative and fractional reasoning and made it easier for the participants to form mental images of the sharing situations found in word problems which might necessitate the use of fractions and ratio calculations, etcetera.

3. The participants perceived that it was important that they had to construct and compress many foundational arithmetic concepts and have easy access to them. The acquisition of such a toolbox of foundational knowledge and skills, in turn, made it easier for the participants to enjoy the full use of the cognitive benefits that the visual scaffolds provided and thereby enabled them to construct suitable algorithms and carry out the computational aspects of the word problems. The visual models also allowed the participants to make fair judgement calls – estimations – of the size and shape of the answers they were seeking (4.13.5). However, it is acknowledged that some participants did not perceive any benefit in using the visual scaffolds (4.13.3), preferring rather to rely on purely symbolic representations of mathematical ideas.

The research data also revealed that the participants perceived the visualizing methodology which was used in the module as user-friendly, accessible, connectivist and capable of freeing-up potentials (4.13.5). They were able to contrast past experiences of routines, rules and rote inspired learning – banking models – with the more activities-based, socially-mediated lessons – liberating model – in the IPS 413 E module, and were highly complementary of their university-based lessons (4.13.5). They particularly enjoyed the pollinating connections model that was used – words linked to pictures linked to practice linked to symbols used in our lessons (Haylock & Manning, 2014). Comments such as*, I apply all that I know when I see things better so I would like Mr Shaw to continue, um, with, this method of teaching* and *I can say that I loved being in this class of IPS and I would love to teach like Mr Shaw when I go out to school,* attest to strong support for the continued use of visualising methodologies in assisting learners to make sense of big mathematical ideas.

Further, while the participants did not use terms such as banking model, instrumental and rote learning as contrasted with Plato’s realms of sensation and forms, relational learning and compression, it was found that conceptions of these ideas were appropriated by the participants who mainly, now endorse the more ontologically liberating model that was used in the IPS 413 E classroom. Further, in our lessons, the participants benefited from a research experience which occurred in a natural setting – our classroom – and was modelled on Bruner’s theory of teaching and learning and, in particular, which was driven by Bruner’s conception of three phases of learning, these, of course, tied to this recursive enactive, iconic and symbolic forms of representation. This strategy enabled the participants to use words, gestures, artefacts, pictures, symbols, language and life experiences to express and demonstrate mathematical ideas.

However, I also acknowledge that, the generous support of the participants' comments aside (4.13), this model was only partially successful in that all too often, I reverted to a teacher-tell methodology. This directed or explicit instruction (Killen, 2015), is recognised as having many educational benefits, but often, in quiet reflection, I also realised that various pressures – slow uptake of mathematical ideas, missed lessons, imminent exams, etcetera – seemed to force my hand, thus, in truth, on occasion I reverted to unvarnished transmission mode practices.

## 5.3 RELATIONSHIP WITH PREVIOUS RESEARCH

Data that was captured from the Action Cycle 1, Foundational Arithmetic Concepts, prior knowledge assessment task, demonstrated high levels of ineptitude. Indeed, for the duration of the entire semester-long module, the participants typically had great difficulty in assimilating and accommodating low order, mathematical ideas and skills. The IPS 413 E participants were all products of a South African education who mostly, could not apply routines, rote-learned, and instrumental mathematical skills to numerous foundational arithmetic computations. This finding was broadly in line with ideas found in the literature review. Such incompetence is consistent with a banking model of education (Freire, 1997), and reinforces Block's (2009) view that, in part, because our teachers hold low pedagogical knowledge, South African education is in crisis. Similarly, the data confirms international data-sets – TIMSS (Mullis et al., 2016), regional data-sets – SACMEQ (Spaull, 2016), local data sets – NSC (Centre for Development and Enterprise, 2010); matriculation pass rates (Department of Education, 2013); and ANA test scores (Department of Basic Education, 2014) which all consistently reveal our struggle to teach and learn mathematics.



Similarly, data captured from the Action Cycle 1, Word-problems, prior knowledge assessment tasks demonstrated that the participants could not solve well-defined (Galotti, 2014) to moderately structured (Zanele, 2015) Intermediate Phase word-problems. In part, *and students attested to this*, many were fearful of such problems (Murray, 2012, p. 55). They suggested that when confronted with word-problems, they set up mental blocks (Belbase, 2013), and thereby indirectly ran the risk of disempowering themselves.

Further, as was already discovered in the foundational arithmetic concepts prior knowledge assessment task, the participants did not have access to a toolbox of practiced instrumental mathematical skills and concepts (Skemp, 2006), and thus were automatically precluded from perceiving the word-problems from a heighted relational-understanding level (Siemon et al., 2012; Reason, 2003). This meant that they could not draw support from any mathematical meta-concepts (Plotz, Froneman & Nieuwoudt, 2012). They did not have any schemata in place (Biehler & Snowman, 1997), nor did they have access to compressed mathematical ideas (Chin, 2013).

Because these features were missing from their conception of mathematics, when they were confronted with the word-problems, their underlying fears of mathematics, coupled with the flood of incoming sensory data found in the questions, may have caused students to lose focused-attention and may have overwhelmed their working memory. Lacking as they were in even foundational procedural knowledge (Mayer, 1997; Gray & Tall, 2007), it seems likely that flawed LOT skills and an absence of HOT skills (Bezuidenhout & Alt, 2011), was visited upon their attempts to solve the word-problems. Certainly, all five of Gooding's (2009, p. 5) barriers to success in word-problem, problem-solving manifested in the data. The participants struggled to conceptualise and compute solutions to word problems for the duration of the semester and only began to show marked score-card improvements in the re-evaluation of the second action cycle.

The negative impact of teaching and learning the IPS 413 E module in English – which was *not* the mother-tongue for most of the participants – was repeatedly found to be a barrier that obstructed their understanding of word-problems. This finding was consistent with previous research. Other researchers have discussed the hurdles that confront learners when they attend to word-problems which are written in languages other than their mother-tongue (Kasule & Mapolelo, 2016), and have indicated how language-use is perceived and applied differently in different communities (Lerman, 1996). As revealed often in this research, (4.4.4; 4.6.3 and 4.8.3), any misunderstanding of a single word or phrase can alter the entire meaning of a word-problem (Sherman & Gabriel, 2016) and ambiguous nuances in text structure can restrict non-mother-tongue students from access to viable solutions (Mahofa & Adendorff, 2014). Barbu and Beal (2010) and Murray (2012) found that the linguistic complexity of word-problems directly impacts non-mother-tongue students' perceptions of these problems and thereby may inhibit them and negatively impact upon their problem-solving capability. Indeed, Tobias (2006) questions whether it is even acceptable to pose mathematical word-problems, written in English, to students for whom, English is not their mother-tongue.

Albeit that initially, some students in the IPS 413 E cohort were hesitant to *get involved* with visualisation (4.14.3), this research showed that visual scaffolds provided epistemological benefits for many students in the cohort (4.14.2). This finding is congruent with those of other researchers. Visual scaffolds – bottle-tops, pebbles, diagrams, etcetera (Naidoo, 2012) – presented as enactive and iconic representations of mathematical ideas (Bruner, 1966) act as building blocks (Novak & Tassell, 2017) which provide a methodological technique that facilitates cognition and effective problem-solving (Siemon et al., 2012; Boaler, Chen, Williams & Cordero, 2016; Mnguni, 2014; Fong & Lee, 2004). While it is accepted that visualisation is not a best-fit for all students (Jordaan & Jordaan, 2013; Barmby et al., 2013), many researchers have suggested that visualisation supports sense-making (Rivera, Steinbring, & Aravi, 2014) and provides students with opportunities to deeply understand mathematical ideas and make meaningful – relational – connections (Mhlolo et al., 2012). Such opportunities to integrate enactive, iconic and symbolic representations has a multiplier-effect on cognition (Siemens, 2006; Bruner, 1980).

Further, data captured from *particularly* Questionnaires 2 and 3 and the focus group interview provide evidence that the participants derived great psychological benefit from the visualisation-based methodologies that were applied in the 2016, IPS 413 E Mathematics module. Their ebullient dispositions can be explained in terms of philosophical positions espoused by Plato and Freire and constructivist stand-points held by Bruner. Although the participants were sometimes/often hesitant to accept the offer to become self-actualising (Maslow, 1943), nonetheless, our classroom atmosphere was characterised by values of truth, beauty, social justice, independence, co-construction and humour.

I also acknowledge that we, researcher and participants alike, may *not* have fully realised Plato's promise of a good education (Plato, as cited in Curren, 2007), *nor* may we have fully ontologically freed ourselves from our positions of oppressor and oppressed. I think that such turning around, such redirection, requires much more sustained effort than can be effected within one university semester-long module. But, I also believe the participants made progress towards rejecting a banking model (Freire, 1997) of mathematical facts and routines in favour of becoming active, inquiry-based, discovery-oriented and intrinsically motivated problem solvers (Bruner, 1960). In adopting these transformational positions for themselves, the participants have joined me on a journey to fully liberate our human potential (Curren, 2007).

## 5.4 IMPLICATIONS OF THE FINDINGS OF THIS RESEARCH

In this thesis I have demonstrated that, across the world, modern approaches to teaching and learning mathematics are moving towards understanding-based conceptions of mathematics (1.1.1): evidence of this is presented in nine exemplars (1.2.1), and I have suggested that the characteristics of word-problems (1.2.2) make them appropriate mechanisms for constructing understanding-based teaching and learning opportunities. However, notwithstanding their educational benefits, word-problems present at least five types of difficulties to learners which act as barriers to success (1.2.2.3). I have suggested that the incorporation of visualising scaffolds into the methodological practices of teaching and learning (1.2.2.4) might offer a beneficial therapy, that is, a way to offset the effects of the barriers.



I have also suggested that in the preparation of new teachers, of whom graduated PGCEs are a prime example, universities are urged to apply considerable attention to the development of subject content knowledge and *innovative and novel* pedagogy and methodology practices (1.8 and 1.9). This is encouraged so as to offset the existing South African malaise in mathematical competence as found in both pupils and their teachers (2.2.1), and to provide support for the 21st century approach to mathematics that is espoused in the national curriculum (2.3). Through the operationalisation of this action research project, I have tried to apply my understanding of problem-solving in mathematics (2.4), teaching for understanding (2.5), and visualisation of mathematical ideas (2.6). Largely, all of my research really seems to condense down to actualising Bruner's (1915 – 2016) theories on good teaching and learning habits (2.7).

With the above in mind, the data that was captured early in my study offered compelling evidence of the participants' inability to work successfully through many low level routines and operations based arithmetic computations. At the commencement of the study, impediments that were experienced by large numbers of the participants included an inability to set out a solution, to recall the products of the times-tables, to work with single operations and common fractions, decimal fractions and percentages, identify factors, place mixed numbers onto a number line, exercise multiplicative and fractional thinking, construct ratios, to write large numbers, identify perfect squares and square roots, and so forth (4.3). That data was congruent with the literature (2.2; 5.3) which consistently exposes South Africa's school-going pupils as having very poor arithmetic skills.

It follows that in cases where participants operate off a flawed understanding of routine arithmetical procedures, their work with word-problems would be compromised. And, indeed, early in the research study this view was confirmed by the data that was captured in the Action Cycle 1, Word Problems prior knowledge assessment task (4.4). Further, the participants' solutions to the word-problems produced an abundance of evidence of all five of Gooding's (2009, p. 5) hurdles. In particular, the data demonstrated that the subtleties of the English language that were used in the word-problems confused many participants. And, as it turned out, for the duration of the research study, the English language often played a role in precluding many participants from gaining access to understanding the asking requirements of these questions (14.11; 14.12 and 14.13).

However, with the benefit of the passage of time, the participants progressed through the two action cycles and over time, the sustained application of the visualising methodology began to bear fruit. The research data began to produce evidence of gradual improvements in arithmetic and word-problem competences. Indeed, multiple data sets (4.5 through 4.10) provide strong and compelling evidence which supports my view that the deliberate application of a visualising methodology, applied iteratively and cogently to a limited body of mathematical competences, has the potential to ameliorate the effect of Gooding's (2009, p. 5) mathematical hurdles (4.9 through 4.15).

Thus, the implication drawn from the above suggests that, in educational settings where the ontological fulfilment of teachers and students is facilitated within a liberating teaching and learning environment, where socially-mediated and intuitive reasoning are welcomed and a spiral curriculum applied to the course of study, many mathematical potentials can be liberated. Indeed, this study has demonstrated that in situations where participants are provided with opportunities to use enactive and iconic representations of ideas, those opportunities facilitate the construction of robust and connected understanding that then facilitates the application of the abstraction – the symbolic construction – of mathematical solutions (2.7.4).

This study has shown that visual scaffolds assist students to link the physical world to the psychological world and elevate knowledge uptake from the application of routines, LOT (3.6.1.1), and rote learning to understanding-based, HOT, problem-solving. And the fact that participants *like* this philosophical, methodological and epistemological approach is an additional windfall.



## 5.5 LIMITATIONS OF THIS RESEARCH

I believe that notwithstanding the successful outcome that the students enjoyed in their learning, these data must be treated with some circumspection. While the triangulated data-sets demonstrate evidence of microgenesis (3.2.4), the successes must be tempered against at least two truths, namely that:

1. the mathematics content in the IPS 413 E module comprised only a few, important, cherry-picked aspects of mathematics which were drawn from the *Numbers, Operations and Relationships*, and *Patterns, Functions and Algebra* Content Areas of the CAPS, Intermediate Phase Mathematics curriculum (Department of Education, 2012, p. 5): there simply was not enough time in the university semester to cover more content, (1.9), and,
2. albeit that the application of visualising methodologies was novel for most of the cohort and ultimately proved to be beneficial to many of them, some participants struggled with the arithmetic/visual requirements imposed upon them by the

module. And this, in spite of the fact that the content was limited to mostly low level, routines, procedures, operations and simple arithmetic computations and well-defined (Galotti, 2014) or highly-structured (Zanele, 2015) word-problems (2.4.4).

In Chapter 4 of this thesis, I have produced data that shows that the computational and conceptual uptake of mathematical ideas and conventions proved somewhat elusive for many of the participants in the IPS 413 E class. The slow, incremental development of deep understanding of this work, as demonstrated throughout the two action cycles, suggests to me that many participants may still not hold compressed conceptions (Gray & Tall, 2007; Chin, 2013; Tall, 2014), of the big mathematical ideas which were used to solve the word problems.

In writing the above, I do not aim to pour cold water on my participants' efforts – indeed, wearing my teacher's cap, I am rather proud of them – rather, my concern is based on my own intuitive and experiential knowledge, obtained as it is, through many years of working as a teacher.



Mayer (1997, p. 387 – 398) differentiates between expert and novice problem-solvers, suggesting that while experts may not have greater cognitive skills than novices, experts in a particular field hold more specific domain knowledge of the field than novices, and can easily chunk together related aspects of their field into useful configurations. But largely, for the duration of the module, the participants did not easily find links between the various, *linked* mathematical ideas that were covered by the IPS 413 E module.

Further, Mayer (1997) also suggests that experts typically hold at least 50 000 chunks of domain-specific knowledge and he suggests that, as a minimum, such knowledge is rarely acquired in less than about ten years of intensive study. Thus, because they have accumulated large stores of factual knowledge, semantic knowledge, schematic knowledge, strategic knowledge and syntactic knowledge, experts excel where their domain-specific knowledge is required.

Thus, operating as they were off the low arithmetic and word-problem, problem-solving bases that were uncovered by the Action Cycle 1 prior knowledge assessment tasks

(4.3 and 4.4), I hesitate to believe that the students can have turned around those deficits in the course of one, semester-long mathematics module. I think that *they think that they know* their work, but in fact, the fabric of their understanding remains thin. However, I also anticipate that time and the accretion of learning experiences will gradually propel the students towards mastery of their subject.

Therefore, in short, a severe limitation of this research is an acknowledgement that it is not possible to turn around in one semester, manifold misunderstandings of mathematical ideas and word-problems. Indeed, given the problematic nature of particularly the first quarter of the 2016 academic year, and thereby the limited time that was made available for teaching and learning mathematics, I believe that a third action cycle, should that have been possible, would have proved very beneficial.

This view finds agreement with Herr and Anderson (2005, p. xvi) who write, "In fact, it is often difficult to think of action research as a linear product with a finite ending, as successful projects can spiral on for years." In effect, a more substantial and sustained investigation into the potential applicability for supporting and enhancing understanding of word problems would produce a more trustworthy outcome.



Another limitation must be that the research was undertaken by one researcher with only one cohort of participants which thus brings the question of generalisability into focus. Further, the IPS 413 E – Mathematics cohort comprised 60 persons of whom only 38 became participants and of those participants, on occasion, only 13 students responded to the data collection instruments.

So, even within the participants, it is not possible to categorically make any form of broad inference. Thus, clearly, the work is not generalizable. However, as discussed earlier (1.10 and 3.12) albeit that this research offers an account of only one action research project, the real strength of this type of work lies in the construction of many similar research undertakings, conducted in different settings and undertaken by different researchers, such that the whole is greater than the sum of its parts.

## 5.6 PROBLEMS ARISING DURING THE RESEARCH

Three, somewhat calamitous problems arose during the research, and all of them manifested at the point of delivery, within and throughout the two action cycles.

A first problem, one which came into sharp focus during the administration of the action cycles, was the hugely disruptive nature of interruptions that were visited upon our classes (4.1.1). Each interruption affected *all* university classes and thereby fragmented the entire teaching and learning programme: where the cause of the disruption was a boycott, such action brought with it various forms of intimidation, damage to property, and on occasion, police action. The first semester of 2016 also enjoyed many public holidays. And, after the Easter holidays, the students were unexpectedly placed into local schools for a two-week practicum experience.

To place the above into a context, phase studies modules such as IPS 413 E typically enjoy a maximum of about 30 contact sessions per semester. In 2016, after discounting the Easter holidays, the semester 1 module might have had 29 lessons. However, the IPS 413 E module lost four lessons to class boycotts and a further five lessons enjoyed very low and very late attendance figures. The coincidence of public holidays cost the module an additional three lessons, and the school practicums accounted for a further four lessons away from IPS 413 E classes. In the end, only 13 full and unimpeded lessons were retained; these were supplemented by three extra lessons that most students did attend.



In 4.1.2, I have explained that a second big problem lay in the difficulty that I had in obtaining an ethical clearance certificate for the thesis. Institutional mistakes prevented it being released and so, other than recording day-to-day experiences with the IPS 413 E cohort for the duration of Action Cycle 1, no formal qualitative research could be undertaken. The ethical clearance certificate was eventually presented to me on 23 March 2016, but I was only able to finally get consent forms signed off by willing participants on 18 April 2016.

A third problem, one which confronted the IPS 413 E cohort directly (4.1.3), was a three-way standoff between the Department of Basic Education – Eastern Cape, previously graduated PGCE students and some, included my own, regional universities. While officials from the department took a position that many of the

PGCEs' qualifications were invalid, the universities held a different position. The previously qualified PGCE students were threatened with expulsion from their teaching posts while, in our classroom, the 2016, IPS 413 E participants looked on in dismay. This situation remained unresolved for the duration of the semester and thus weighed heavily on the research participants.

## 5.7 RECOMMENDATIONS

This thesis has demonstrated that many of the PGCE participants who registered for the 2016, IPS 413 E Mathematics module entered the programme holding low arithmetic and word-problem, problem-solving skills sets. These participants' knowledge of mathematics were steeped in mathematical traditions which promoted transmission mode teaching, rote learning and the use of only symbolic representations of mathematical ideas.

The thesis has subsequently demonstrated that, although initially met with some scepticism and misunderstanding (Barmby et al., 2013; Poch, van Garderen, & Scheuermann, 2015), when the participants became fully engaged in a mathematics module which used many manipulatives and iconic stimuli to explain mathematical ideas, most participants found the visual methodology refreshing, educationally beneficial, and accessible.



Indeed, while it is tragic that something as simple as a packet of biscuits shared in a ratio of, for example, 2 : 5, or a sketch of a bisected circle, as a visual representation of 1/2, should be perceived as novel and deeply enriching by the PGCE participants, nonetheless, such simple apparatus have liberated arithmetic and word-problem mathematical potentials (Presmeg, 2009; Duval, 2013; Clements, 2014; Wilkie & Clarke, 2014; Mhlolo et al., 2012).

I think it is fair to suggest that the participants who were registered for the 2016, 413 E Mathematics module were typical of PGCE students found in other universities in South Africa. That being true, I think it is fair to suggest that students who register for PGCE programmes in the foreseeable future will present levels of mathematical distress and ineptitude which are similar to those in this research group: largely, they will be bound together by similar histories and experiences of mathematics.

If this is true, then those students might also respond well to the use of visual tools (Naidoo, 2012), (2.6.3.3) which foster embodied cognition (Boaler et al., 2016), (2.6.3.4), and can mediate for the complexity of mathematical explanations (Mudaly, 2010), (2.6.3.5). I think that visualisation-oriented research projects which foster, for example, a deep relational understanding of multiple applications for number-lines or fractions and ratios, etcetera, may uncover liberating, novel and innovative learning opportunities for other PGCE students.

## 5.8 CONTRIBUTIONS MADE BY THIS RESEARCH

Authors such as Adler et al. (2009, p. 29 - 36) and Parker (2009, p. 19) encourage research into primary mathematics teacher education in South Africa and hold views that PGCE students may hold a key to repairing the poor state of mathematics teaching in our country (1.9). Such is the breadth and depth of the crisis in teaching and learning mathematics in South Africa that Bloch (2009, p. 58) referred to it as a "national disaster."

One small contribution made by this thesis is that it has demonstrated that, albeit that the PGCE participants may have many other skills-sets, in this cohort, and most-likely in others, their understanding of low-level, routines-based arithmetic and their word-problem mathematical capacities were muddled. In fact, because of the prevalence of such misunderstandings, often the pivotal research task – visualisation applied to word-problems – had to give way to first reconstructing numerous unpinning arithmetic skills and conceptions. Thus, it is important that university academics do not underestimate the magnitude of the mathematics deficit held by PGCE students and take steps to ameliorate mathematical myths and misunderstanding.



A second small contribution made by this thesis is that the research data has demonstrated that the application of visualising scaffolds as a teaching and learning methodology, can enhance PGCE students' understanding and improve success rates in solving Intermediate Phase word-problems. The research has been shown to ameliorate Gooding's (2009) difficulties with word-problems and has confirmed the positive views regarding the benefits of visualization as held by Presmeg (2009); Duval (2013); Clements (2014); and Wilkie and Clarke (2014), among others. It has also shown congruence with modern approaches to teaching mathematics (Mhlolo, Venkat and Schäfer, 2012), and has used easily sourced visual tools, (Naidoo, 2012; du Toit and Kruger, 1993; and Boaler et al., 2016). Over time, many participants began to embrace the visual methodology (4.15). Thus, I think that it is fair to suggest that this research introduced the PGCE participants to a productive methodology and liberating pedagogy (Adler et al., 2009).

## 5.9 AUTOBIOGRAPHICAL REFLECTION

The thesis has enjoyed the complementary and intertwined aspirations of emancipatory and practical goals, one infused into the other. The emancipatory goal sought to liberate new mathematical potentials for the participants while the practical goal sought to improve my own practice. My conception of these goals is of their being in harmony, one balancing with and fortifying the other; any improvement in one, beneficially effecting the other.

The first section in Chapter 2 (2.1) was created as a consequence of my reflections about the Thinkboard activity which I administered in the very first lesson of the 2016 academic year. The thinkboards presented copious evidence of much hurt in previous mathematics experiences. I felt that this hurt needed a comprehensive philosophical response. Thus, I tried to construct a robust explanatory framework of my own philosophy of a *good* education. Such a philosophy, I believe, must influence one's epistemology and ontology and would also guide pedagogical and methodological practice and habits.



This writing (and thinking) spoke to the emancipatory goal of my research and could be fruitfully located within the significance of the study (1.8), in which the research was conceived to find "productive ways for teachers [vis-à-vis, student teachers] to learn and/or re-learn subject content to prepare them for teaching" and would try to find, "ways of offering and inducting teachers [student-teachers] into new experiences of teaching to broaden their pedagogical imagination" (Adler et al., 2009, p. 29). Such *productive ways* and *new experiences*, I believed, would create liberating educational experiences for the participants. An additional motivation is found in the rationale of this thesis (1.9) which broadly, speaks to a desire to find ways to improve mathematics understanding in South Africa.

These considerations informed the writing in the literature review which came under the heading, *Two Affective Influences on my Personal Perspective (2.1).*

In (2.1.1), I reflected upon Plato's *Allegory of the Cave* which was used as a metaphor to explain, on one hand, humankind's often distorted perception and understanding of their world and their pervasive sense of helplessness and, on the other hand, the risks and ultimate joys and fulfilment which are encountered when one seeks out knowledge, liberty and self-actualization. I used the allegory to suggest that transmission mode teaching methods shackle students' learning opportunities while liberal teaching methods encourage self-sustained effort and foster deep understanding.

Further, from Freire, (2.1.2), I indicated that we are all ontologically driven to become more human. As a result of this ontological disposition, in any situation where some are oppressed by others, revolt by the oppressed over their oppressors will occur. Transmission mode teaching practices are oppressive in nature – Freire calls these *banking mode practices* – and as such they disenfranchise students: he calls for their overthrow in favour of a liberating model of education in which cognition supplants rote learning and understanding replaces the habit of low-order memorization. Thus, is can be seen that philosophically, Plato and Freire hold similar ambitions for the outcomes of the educational process.



With the above in mind and now, with the benefit of having completed the face-to-face parts of the action research, I have come to realise that the philosophical intentions – the emancipatory intentions - have remained somewhat unrealised. I have had to acknowledge that often I/we, teacher and participants alike, stymied our own progression towards liberation. In our journey we fell into the traps of our past habits: all too easily and all too often, the participants turned to me to seek advice on the next step, the next calculation, and all too effortlessly, I obliged.

Notwithstanding the fact that every PGCE participant entered the programme as a university graduate, their prior exposure to transmission-mode teaching and banking-mode learning approaches made it difficult for them to move forward independently. Typically, most participants could not or would not seek support from their peers nor a text-book nor an online reference. Instead, they turned too easily to me. And, all too easily, I readily provided support, thereby contributing to their dependency. Thus, unhappily, I have realised that often, far from stretching and liberating potentials, I have acted as a *benevolent* oppressor of deep learning. And this, a complete contradiction of my philosophical aspiration.

In Chapter 2 (2.1.1), I wrote that Plato suggests that the, "eye that cannot be turned around from darkness to light *without turning the whole body* [my emphasis]" (Plato as cited in Curren, 2007, p. 22). However, when I reflect on the overall circumstance of the action cycles, I find that this full turning of the whole body has not occurred. I think that three impediments diluted that ambition.

1. I think that the pressures brought to bear on the programme by the boycotts, partial and late attendance in class, the coincidence of public-holidays falling on teaching days and the unanticipated School Experience practicums put enormous pressure on us all to, at the very least, *get through it*.
2. Further, my own ineptitude may have conspired against the liberalisation of my students' potentials. I have often stated to colleagues and students that I remain an *emerging* social constructivist who, too easily, falls back on transmission mode teaching practices: I think, I reverted to this fall-back position too easily in the 2016, IPS 413 E Mathematics module.
3. I think that some progression *towards* liberalisation was effected and is evident in the discussions contained within the focus group interview (4.13), and the post research questionnaire (4.15). However, my own ineptitude aside, I found it difficult to overcome the previously inculcated dependency habits that the participants had acquired and brought into our classroom.



Continuing with item 3, above, and not to play around with the semantics of phrases, I realised that it would be inaccurate to suggest that I had partly liberated the participants from the yoke of banking model practices. Plato wrote of *turning the whole body* and Freire of our *ontological disposition to become more human by liberating ourselves from oppression.* One cannot be a little bit pregnant, a little bit dead, a little bit liberated. And so this is why I chose the words in (3) above, that we had made *some progression towards liberalisation.*

Albeit that I pay a great deal of attention to my teaching practice, this research opportunity has highlighted problematic issues which have now become foregrounded in my consciousness. This heightened awareness has enabled me to grow as a person and to thereby improve some aspects of my practice, thus enabling me to fulfil the practical goal of my thesis. Below, I discuss two of these problematic issues, and stratagems that were used to work through the problems, and after that, I also discuss my sense of the high and low points in attending to linking visual scaffolds, understanding and word-problems together.

**1**

I have already written about the fact that the first semester of 2016 was disrupted by stay-aways, intimidation, political rallies, hot and humid working environments, water shortages, questions raised about the status of the PGCE qualification and future employability, fees, queues and administration issues (4.1.1 – 4.1.3). All of these upsetting features talk directly to a pervasive oppression of the ontological well-being of the participants (Freire, 1997). However, if I set aside issues of philosophy, pedagogy, methodology and epistemology for a moment, common sense suggests that these constant threats to the participants' well-being (Maslow, 1943), might possibly have constantly been uppermost in their minds. And, it is quite conceivable that for the greater part of the semester the participants might have been preoccupied with such manner of things thereby precluding focused attention on their mathematics. Below, I refer to issues pertaining to the development of additive reasoning in our classroom. I use this text to offer an example of the absentmindedness which plagued our lessons. By extrapolation, this exemplar can be expanded to include many other mathematical facts, operations, processes and ideas that proved difficult to assimilate.



In our classroom meetings, I was struck by the fact that I had to *repeatedly* visit many foundational additive ideas, even to the point of demonstrating to the participants, as an example, how it is that we might add *big* numbers. The point I am trying to raise here is not *de facto* that I had to revise columns-based addition, rather, it was the fact that this revision extended even to the point of *repeatedly* negotiating the alignment of the digits - units in the units column, tens in the tens column, etcetera. *Figure 4.13*, in (4.3.1) provides a good example of this columns misalignment.

In the classroom, and from the data in the assessment tasks, it became clear that the poor setting out habits that the participants used, in part demonstrated a haphazard approach to the exposition of their thinking and also, in part, demonstrated that the participants did not understand why and what they were doing. Maintaining the example of adding large numbers for the moment, I realised that the difficulties that the participants had with addition in columns and of not attaching meaning to the positions of the digits in the columns was confirmed by their inability to rewrite a textual number in digits. An example is provided in *Figure 4.25* in (4.5.6).

In the course of the first quarter, I realised that firstly, a meta-concept of addition (and of the other operations) was not in place, and secondly, that the limited support that I had offered to brush aside cobwebs had not been effective. As the magnitude of this misunderstanding of addition, and many other intersecting misunderstandings of LOT foundational arithmetic concepts dawned on me, I had to pause to consider a route to follow to improve the participants' additive understanding.

At the back of my mind, I was constantly mulling over the fact that any investigation into word problems, whether by novel or traditional means, required first, a good conception of number sense and arithmetic competence. I also knew that albeit that informal and intuitive ways of knowing and solving mathematical word problems could take the participants a long way, in the end, these techniques must be absorbed into more formal mathematical conceptions and computations and that these computations were underpinned by many foundational arithmetic competences.



Also, because of the shrinking number of lessons that were available to me in the first quarter, I had to acknowledge that most of the support I would offer would occur in the second quarter of 2016. And, that itself, the second quarter, was a *short* quarter.

In the second quarter, more often that I would have wanted to, or care to admit to, I often adopted a direct instruction teaching strategy. I had not set out to do this – in my preparations for the thesis, I had envisaged a great deal of interaction and self-directed study – indeed, I had created many resources to support this liberating objective, but in class, it just did not happen.

Direct instruction, also called explicit instruction, offered me many benefits. These benefits include enabling me to (1) provide a broad overview of new areas of study; (2) to provide engaging and accessible demonstrations of ideas such that the participants would become curious to know more about a topic; (3) to teach explicit facts and skills to struggling participants; (4) to create a non-threatening teaching and learning environment in the classroom; and (5) to assist participants to learn more deeply from *guided learning* rather than self-discovery (Killen, 2015).

Directed instruction is also useful in situations where teacher demonstrations can offset the absence of sufficient materials for the learners to use, and, directed instruction facilitates teacher reflection (Killen, 2015). In my own classes with the participants, a manifestation of this absence of sufficient materials was occasioned by the fact that the participants were encouraged to bring bottle tops to class (of which there are an abundance) but steadfastly seemed unwilling to do so (4.1.6). Thus, my own use of bottle tops, as a directed instruction demonstration served a purposeful role.
However, these good points about directed learning aside, I have also indicated that occasionally, to move things forward, the teaching and learning reverted to a transmission-mode, banking model (5.2.4), which has become a bitter pill to swallow.



The activities that were occasioned by the participants' inability to add large numbers were handled in a manner congruent with my exposition on multiplicative reasoning (4.1.6) and fractional reasoning (4.1.7). For addition, (1) we explored aggregation – demonstrated and visualised in our class as groups of objects which are brought together and linked with stem questions such as, *altogether, how many*; (2) addition as augmentation, presented on structured and unstructured number lines and linked visually to doubling and halving and directed numbers; and (3) the commutative law.

The big takeaway here is not the mathematical approach or the activities and exercises, but rather, the stubborn persistence of low uptake of mathematical ideas. I have stated before that the participants were a lovely group of seemingly committed students. However, their inattention to addition and the continuing issues that they had with addition, became a red flag in my consciousness. The participants were constantly in need of support – of the same issues – such that even though certain additive points might be considered as LOT skills or facts, each time they encountered that particular arithmetic problem, it was as if it was first time that they encountered that problem.

In class, the short explanations about ideas that had been unpacked in previous lessons were typically met with silence or obscure questions. In our classes, time seemed to *drag,* even to the point where it might take minutes for some participants to put pen to paper: nor was this delay because deep contemplative thinking preceded writing or even opening a book. These factors alerted me to the fact that very few of the participants were engaging in any form of self-study. And, pursuing this train of thought, I picked up a sense that the participants were distracted, which indeed, they attested to. Participants complained that the various strike actions, etcetera (4.1.1) were *wasting a great deal of time*; others, speaking about the problems with the Department of Basic Education, (4.1.3), questioned the viability of the PGCE programme. Thus, it is fair to suggest that at least in part, this distraction was an answer to the oppressive, ongoing problems which occurred during the action research period in semester 1 of 2016.



Of deep concern is the fact that as I write this, in June 2018, the upheavals which were experienced in semester 1 of 2016, continue unabated. In my own institution, *for example,* semester 1 of 2018 began with a three-week boycott of classes by the students; today, 14 June 2018, many university staff have entered into their third day of strike action: they have closed down the university and put a halt to the mid-year exam processes. One can imagine the negative ramifications which extend outwards from this action… Further, this type of turmoil is being experienced by higher learning institutions across South Africa. It is conceivable that these distractions will have a long-term negative impact on the quality of teaching and learning.

**2**

While I have always been obliquely aware of the difficulties that English Second Language (ESL), (Nel & Nel, 2016) learners might have when they try to negotiate and attach meaning to English words and phrases in mathematical word problems, this thesis has provided me with an opportunity to closely scrutinise this obstacle.

Due diligence was applied to the construction of the numerous text documents that the participants received in the course. These documents included assessment tasks, questionnaires, support texts, etcetera. However, the written replies in the assessment tasks, (4.8.3), demonstrated to me that seeming innocent turns of phrase such as *one third the number of, double, twice as many,* etcetera, are actually heavily laden with meaning-filled nuances of the English language that were not easily available to participants who, as a matter of course, did not think and speak in English.

In as much as *udibaniso, uthabatho, uphinda-phindo,* and *ulwahlula-hlulu* are meaningless arrangements of letters to the uninformed mind, but to isiXhosa speakers imply addition, subtraction, multiplication and division respectively, so too, I realised that unintentionally, terms such as *rounding off* and *carrying over*, could create cognitive havoc in ESL learners.

Thus, I was faced with a conundrum: words such as *doubling* and *halving*, *estimating* and *compensating* are operationalised terms in the CAPS documents (Department of Basic Education, 2011). Further, in South Africa, we have adopted a learning strategy called *additive bilingualism* which suggests that while children's home languages are cherished, in our schools, an additional language is introduced into their learning experiences (Nel & Nel, 2016). An expression of this additive bilingualism is that in the Intermediate Phase, teachers and learners in South African schools are required to conduct mathematics lessons in either English or Afrikaans. In other words, such words are part of the learned mathematics curriculum.

In the participants assessment tasks (4.4.4), in the written replies in the questionnaires (4.11.2.1), in their discussion in the focus group interview (4.13.4), and in their chats with me in class, it became very clear that in word problems, a single word or misinterpreted term could throw participants off balance. Indeed, as has been mentioned before, the phrase *has half the number of* had proved to be a sticking point. I knew that there were numerous other ways to describe the same situation – *Thabo has twice the number of…; Thabo has double…; Thabo has two times as much,* and so forth. And I knew that different turns of phrase might offer some respite. So, indeed, these alternatives were used in my classroom. But I also knew that there was no getting away from the sticky words and phrases: thus, those terms had to be scaffolded and worked into the participants' vocabulary and conceptual framework of mathematical understanding. To do any less, would be to short-change the participants of valuable mathematical knowledge.

Rickard (2013, p. 3) suggests that because mathematical communication and reasoning is underpinned by very a specific use of vocabulary, “learning mathematics is somewhat akin to learning a foreign language.” Such vocabulary has to become a part of a *metalanguage* of mathematics (Killen, 2015), and opportunities have to be built into lessons to specifically address the mathematically-contextualised meanings of problematic vocabulary. Haylock (2002), suggests many student-teachers struggle with the technical jargon in mathematics. Killen, (2015) explains how terms such as *integration* and *equality*, used in mathematics and social sciences classes, in different contexts come to mean different things. Because the vocabulary that is used in mathematics is tightly linked to mathematical concepts, symbols, diagrams and processes, each term must be deeply understood in terms of its mathematical application and context (Van De Walle, Karp & Bay-Williams, 2010; Siemon, et al., 2013).



Tipps, Johnson and Kennedy, (2011), suggest that when we are teaching LEP students – that is, students with limited English proficiency – it is important to avoid using slang and colloquialisms. Further, when we discuss mathematical ideas, teachers should be even-paced, enunciate clearly, use gestures, facial expressions and visual scaffolds. They should speak and write in short sentences and offer many summaries. In such classrooms, of which there are an abundance in South Africa, our learners should be encouraged to use code-switching. Code-switching encourages ESL to seamlessly revert from the language of learning and teaching (LoLT) to their mother-tongue and back. The view we hold is that by discussing mathematical ideas in both languages, ESL learners can make deep connections with vocabulary and concepts (Nel & Nel, 2016) and thereby can come to “know how, and also know why” (Ma, 2010, p. 108).

With the above in mind and coupled with the other considerations and actions that were discussed in (4.1.6) multiplicative reasoning, and (4.1.7), fractional reasoning, I realised with fresh eyes that action had to be taken to assist the participants to make sense of problematic words and phrases. In this regard, the visualisation strategies which were conceived to be worked into assisting the participants to solve word problems, were roped in to unpack difficult words and phrases.

A handful of bottle-tops, pulled out of a pocket and dumped onto a display table, enabled me to explain, for example, *estimation* and *rounding off*. Similarly, a sharing of biscuits, in different ratios, with different numbers of participants, allowed us to explore vocabulary and notions of *common denominators, numerators, ratios* and so forth, and also enabled me to enjoin these ideas with phrases such as *double, twice, one third the amount as,* etcetera. Thus, with a portable chalkboard at my side, and many, numerous found resources at my disposal, it became quite easy to explain and integrate words and ideas in enactive, iconic and symbolic ways.

The vocabulary and concepts linked to *squares a*nd *square roots* serves as example of how guided learning and sustained attention to problematic mathematical ideas can eventually bear fruit. In the first action cycle, as discussed in (4.5.7), a LOT question (Q4.3) on squares and square roots returned scores of 3% and 32% in the prior knowledge and re-evaluation tasks, respectively. In the second action cycle, a similar question (Q3.8) returned 22% and 89%. In each action cycle, in the periods between the prior knowledge and re-evaluation assessments, I discussed these problematic questions with the participants and demonstrated how it was that squares and square roots are incorporated into our mathematical thinking. I found that the words themselves were as problematic as the concepts they represent (4.11.2.4). Indeed, my directed teaching revealed two big unanticipated misconceptions that informed many participants misunderstanding of squares.

The first misconception that was addressed was the mistaken view that some participants had that any rectangular quadrilateral was in fact a square. Thus, the defining properties of squares has to be re-established. A second misconception, indeed an IFI based conception, enjoined participants to perceive the unit squares contained within a larger square of, for example 9 units of area, as being of a congruent size. In order to facilitate that uptake, I literally used small, square wall tiles, to tile squares of 1 unit of area, 4, 9 and 16 units of area. This visual methodology enabled us to clarify the vocabulary that is linked to squares and square roots, and as an extension, enabled the participants to conceptualise and estimate the square roots of for example, 7 or 15.

Thus the big takeaway here was that the misunderstanding of many pieces of mathematical vocabulary was quite easily renegotiated for understanding through the application of a visually-rich directed teaching and learning mechanism. And of course, with this renewed understanding of pivotal mathematical vocabulary, the participants were empowered to concentrate more attentively on the asking requirements of the word problems that were presented to them.

I that I believe that, as a result of reflexive practice, I managed to create a course which blended contact lessons and learning experiences with outside-of-class tasks; I also designed into the programme multiple opportunities for the participants to engage with the materials, discuss ideas and construct their own knowledge and all of this done reflexively, as practical manifestations of my attempts to improve my own practice, and hence, provide improved support to the participants.

I also believe that in as much as many opportunities for personal growth were presented to us, for different reasons we all missed some of those opportunities, and thereby, must acknowledge that our transit must continue. I believe that, had there been more time to co-operate with each other – a third action cycle perhaps – we might have begun to make real inroads into developing a comprehensive, visualising strategy for solving word problems, but as it stands, that work is unfinished.



**3**

I first started to purposefully look at visualising word problems in Intermediate Phase mathematics about ten years ago. At that time in South Africa, a circumstance of new versions of mathematics curricula called for constant reinvention and thereby offered us opportunities for exploration. Further, data from international, regional and local assessments demonstrated the parlous state of mathematics teaching and learning in South Africa. And, in my *geometry* classes, consecutive cohorts of undergrad students were displaying less and less competence in perceiving three dimensional objects, drawn in two dimensions, in either a textbook or on a chalkboard. This visualisation problem, in turn, drew me to Bishop's (1983), conception of IFI – interpreting figural information and VP – Visual Processing.

As such confluences of intersecting issues can do, an unintended consequence of the above revealed to me that countries which had successfully modernised their mathematics curricula had placed emphases on relational understanding and contextualised learning. I think, in principle, we too recognized the importance of such aspirations, but we were, and continue to be, less successful. Also, in those countries, in primary school education, enactive and iconic modes of representation were used to great effect to liberate learners' mathematical understanding. Again, to be fair, I think that most South African mathematics teachers have always made use of some concrete apparatus, some posters, some chalkboard sketches.

I also found that in other countries, teachers were taking the lead offered by the Singapore Mathematics method and applying it, in different ways, into their own work (Cheong, 2002; Ng & Lee, 2005; Looi & Lim, 2009; Naroth & Luneta, 2015; Dennis, Knight & Jerman, 2016). I found that visualisation was beginning to play an increasingly important part in developing mathematical reasoning and understanding. And indeed, I found that visual models were being used to solve arithmetic word problems. Often presented in the form of rectangular bars, learners constructed and used these visual scaffolds to solve mathematical word problems. To be fair, in South Africa, many teachers picked up on these novel ideas. Indeed, at one stage, my own work used a number of open-source Mathematics Learning and Teaching Initiative (MALATI) teaching ideas that were developed in South Africa. That resource too, was heavily oriented towards relational understanding and incorporated many visual scaffolds into its teaching and learning programmes.



At about that time, ten years ago, the demographic in my mathematics classes started to change. From classes of maybe twenty to thirty undergrad students, and occasionally sixty, we began to move towards classes of one hundred-and-fifty to two hundred and often, over three hundred students. Similarly, the mathematics profile of the students began to change from a position where most students had had a satisfactory to excellent school experience of mathematics to one where many students now enter our classrooms holding very low, instrumentally learned, conceptions of mathematics, and from students who mostly came from English speaking homes to now, mostly isiXhosa.

With all of the above in mind, I was drawn to the potentially liberating promise that was offered by the visual approaches that were being used in other countries. And so I began to saturate myself in readings which reported on applications of these approaches and then began to create my own courseware for delivery in my own classrooms. My first big foray into the deliberate use of visual scaffolds to solve word-problems occurred in 2011/12 when I was a member of a university/NGO collaboration with a mathematical group of professional teachers and academics at AIMSSEC. In this programme, I introduced the visual teaching and learning strategy to carefully selected, small groups (±25 per class) of highly talented teachers in a four-year long, mathematics outreach programme. The programme offered these teachers an additional professional qualification – the Advanced Certificate in Education (Mathematics)(Course Code: 50042) and carried an honour-based stipulation that the participants shared – we used the term, *cascade* – their newfound knowledge with other teachers in their communities.



The work was very successful but I learned that some those participants perceived the visual approach and the vocabulary that I used as a layer on top of our CAPS informed studies in mathematics, so much so that it was perceived by some as – *we are doing CAPS and Singapore Mathematics.* However, (and I picked up this train of thought in other, university-based classes), that was not the intention, thus I backed off a bit. I had introduced my explanations about Singapore Mathematics and the use of terms such as Part-Whole, Comparison, Change and Remainder Concepts, etcetera, into our lessons to contextualise our work, but for some students, the strategy was perceived to usurp CAPS. But that was not the intention *so I backed off*.

However, I was very taken with the way that the visual models could be tied into fraction walls and fraction ribbons – in fact, in many foundational applications of the model method that is all that the bars really are. Further, in the Intermediate Phase, arithmetic word problems typically invoke the use of the operations, common and decimal fractions, percentages and ratios.  I knew that these types of computations lent themselves to visual representations and my experiential knowledge showed me that students in different classes had found visual conceptions of fractions and the operations novel and invigorating. So, I remained confident that visual stimuli, in the forms of found resources and suitable diagrams – enactive and iconic representations – could assist students to solve word problems.

So, largely, I backed off *making a thing* of Singapore Mathematics, the model method and the attendant turns of phrase. But, with renewed understanding, I began to invest a lot of energy into reconstructing my own coursework and introduced activities which contained layers of found resources, arrays, number lines, area models, fraction wall and ribbons and so forth into my conversations with my students. And this thesis, and its aspirations, are a product of that work. What I have tried to do is find out what works, is problematic, and so forth so that the participants and researcher/teacher alike, benefit from the this new knowledge.

At the very beginning of Action Cycle 1, the evidence that was presented to me in the thinkboards and prior knowledge assessment tasks suggested that many previously learned but misremembered foundational facts and processes would need to be repaired. I was aware that the successful solution of the arithmetic word problems rested on the foundational skills and knowledge. Thus, it suited my purposes to invest time and energy in assisting the research participants to deeply understand the operations, common fractions, ratios and so forth. And that work, in turn, enabled me to link many enactive and iconic representations of mathematical ideas and processes to the participants' flawed symbolic understanding.



Because the problems that were encountered in this work have been discussed at many points in the thesis, I will not discuss them at length here. Rather, it is important to comment on the slow, but certain, uptake of the visual scaffolds by the participants. Their understanding of fraction walls, fraction ribbons, equivalence and the like took a long time to form, much longer than I had anticipated. However, as their embryonic understanding produced more sustainable schema, and as their instrumental learning gave way to a more relational understanding of their mathematics, so too, their ability to construct beneficial visual scaffolds moved from mimicry to the conception and construction of thought-filled representations. Of course, I am very pleased that this happened, and indeed, the data demonstrates these improvements nicely, but I am always cognisant of the low level at which we work working and the limited number of mathematical ideas that were applied to our studies.

In the early stages of this research, when I was assisting the participants to make sense of and draw visual representations – for example, as an array, area model or fraction strip – I was insistent that they work neatly and drew the representations in fair proportion. In adopting this requirement, I was informed by (1) my previous geometry work with many other students who had been found to not be able to perceive, for example, a pyramid within a wire-frame isometric drawing, and (2) by my realisation that many participants held incorrect conceptions of fractional equivalence, that is, that there are eight eights in a whole and each eighth is the sa*me size* as the others. Indeed, Cheong (2002, p. 62), suggests that learners need to understand that "a mathematical model – an 'accurate diagram' – provides a powerful visual aid," so bar models should realistically represent ideas in sensible proportions. Petit, Laird and Marsden (2010), found that drawing inaccuracies are often due to misunderstanding of wholes and fractional reasoning. Like Siemon, et al. (2013), I wanted the structures of the visual scaffolds to reveal meaning-filled relationships and patterns. I wanted to bed these ideas down and I felt that neat representations would facilitate cognitive uptake. I also believed that these IFI skills would foster VP reasoning that would be useful later, in the word problems calculations.

When I introduced the participants to this work, typically I used models and directed teaching methods to unpack the requisite skills and ideas. In the initial stages, the participants used apparatus such as small boxes of Smarties©, grid paper, pencils, highlighters and rulers to make sense of area and bar models. For example, in one early activity, using direct correspondence, the participants constructed bar models which contained the same number of cells as there were in the contents of their Smarties© boxes. These cells were shaded in, to accurately represent the colourful contents of the Smarties© box. Extensions on this work included drawing sketches of, for example, adding the yellow and the red Smarties© together, or subtracting the green ones from the whole box, or comparing the number of blue with the number of orange Smarties©, etcetera. Because the sort-size was small and the context – Smarties© – well understood, this early work was quickly and easily linked to common fractions, ratios and so forth.

Building on the above, when it became obvious that the direct correspondence of items and cells within bars would be ineffective with larger numbers, proportional reasoning enabled the participants to assign congruent groups to representative cells. Thereafter, as the meaning attached to the bars became better understood, and the prescriptive neatness and proportional accuracy of the bars found in our earlier work retreated, the cells within the bar came to represent many larger and/or smaller amounts. All of this work was analogous with model methods that are used in other countries (Ng & Lee, 2005; Naroth & Luneta, 2015; Dennis, Knight & Jerman, 2016).

However, even in situations where, for example, I presented the participants with grid paper and boxes of Smarties©, very often, I found it difficult to get all of the participants to fully engage with their activities. Nor could I push this too hard. From an ethical perspective, I did not want to be perceived as being exploitative, of using the participants for my own means (Robson, 2002), but as their teacher, I wanted to encourage the participants to engage with the activities. Thus, I did not go beyond a quiet nudge here, a gentle suggestion there, to suggest more involvement. But often, reticent participants did not sustain their efforts. And so this presented me with conundrum that I never did completely unravel. Although the foot dragging to class, rates of absenteeism, and some of the body-language and gestures that the participants used in class, indicated some (internal) resistance, they were always polite and cooperative – it was not in the nature of this group of students to be otherwise – but, for some, the suggestion that they immerse themselves in their mathematical activities, was possibly just a step too far for them to take.



Q4.2, taken from the word problems calculations assessment task in the second action cycle can serve as an example to show how visual scaffolds can assist learners to solve word problems: **Altogether Trevor and Peter have 72 rabbits. Trevor owns $\frac{5}{6}$ of the rabbits. How many rabbits does Trevor have? How many rabbits does Peter have?** It was accompanied by a photograph of rabbits.

In the prior knowledge assessment it achieved an 18% success rate; upon re-evaluation this improved to 72%. This question, and others like it, was plagued by language and arithmetic issues, discussed earlier in the thesis: these problems included making sense of the word altogether and linking the word to a concept of whole; of understanding the common fraction and proportional reasoning, and of computing verifiable answers. However, those problems notwithstanding, the question itself lent itself to many visual interpretations.

Visually, as a drawn representation of the word problem Q4.2 lends itself to being constructed as a fraction circle. It would contain six congruent sectors; five of these sectors might be shaded. The circle would represent the whole – 72 – while each sector would represent a equivalent proportion – $\frac{1}{6}$ – of the whole. By deduction, the observer then perceives that $72 \div 6 = 12$, and that releases the answers, one 12 to Peter and five 12's to Trevor, etcetera.
Instead of using a fraction circle, a sixth's fraction strip might be constructed and the word problem could be solved in the same fashion. Alternatively, a comparison – a ratio – might be conceived. If this is the case, the participants had been taught to draw two bars, one above the other, and in fair proportion, such that in this example, one bar would contain five congruent sub-sections, and the other bar, one cell of congruent size to the sub-sections in the first bar. The ratio that this visual representation presents, reveal 6 parts in the ratio, which encourages division of 72 by 6, and so forth.



The big takeaway for the participants was that as these visualisations began to make sense and were understood, their compression and relational understanding could be transferred into similar word problems. The numbers and contexts might change, and the computations might be different, but the meta-idea would remain intact. Further, it then became possible for the participants to integrate enactive and iconic conceptions with into symbolic ways of knowing mathematics. Initially, the visual scaffolds were used to prop up symbolic understanding, and later, to consolidate it.

Time constrains within the research period precluded properly establishing the successful uptake of the ideas, however, from the data in the questionnaire that the participants completed after their school-based practicums, I got the sense that the participants had found the use of the visual representations useful for themselves and their learners. But, the sustainability of this novel approach has not been established.

## 5.10 CONCLUSION

In the prologue in this thesis, I acknowledged my own positive inclination favouring the use of visualisation as a powerful exemplification tool. Further, in teaching and learning mathematical ideas in general and understanding and solving mathematical word-problems in particular, I hold an opinion that visualisation is an especially useful expository ally to both teacher and student. Indeed, in mathematics, I believe that properly conceptualised visual representations foster deep, relational understanding and can offset barriers which might otherwise preclude students from success. Such scaffolds might be constructed by using objects, by drawing representative sketches or by conjecturing mental images in the mind. However, I have not set out to prove the efficacy of visual scaffolds, that benefit has long been established. Rather, I wanted to (1) understand how PGCE participants perceived this, *for them*, novel approach, to perhaps liberate untapped potentials, and (2) through reflexive action, I wanted to improve my own understanding.

From an emancipatory and liberating perspective, the data has revealed that the participants found the mathematics classes helpful and learning oriented; they enjoyed the supportive roles offered by the enactive and iconic representations of mathematical ideas and the links that these scaffolds made to symbolic representation; they enjoyed the activities-based approaches, explanations and discussions that were the stuff of our lessons, and for some participants, a negative predisposition towards mathematics was changed to a more optimistic view.



Similarly, the quantitative data has shown that, while it was slow, progress was made in assimilating and accommodating many foundational arithmetic skills and processes. The participants made inroads into constructing relational understanding of these LOT skills and were able to apply this knowledge to solve word problems.

My own work had been improved in many subtle ways. Teaching is a lot more difficult than lecturing! Because I was trying to *teach* the participants how to make sense of foundational arithmetic concepts, use visual scaffolds and so forth, often I had to set-aside my quiet thoughts that – *it's obvious* – so as to try to properly understand *their* problems. In effect, their problems become my/our problems.

In my activities and in my written correspondence with the participants, I set the language and vocabulary levels at the upper end of our Intermediate Phase and lower end of our Senior Phase. Somewhat nonchalantly, I believed that this covered the bases. However, through the auspices of the thesis, I have learned that I made a mistake – I found that seemingly *safe* English words and phrases and mathematical jargon, are in fact saturated in meanings which are not easily understood by participants who do not use English as a home language. Because of this revelation, I have developed a deep respect for the participants who, daily and without complaint, negotiated this mountain. And, I have relooked my own use of the problematic words and phrases, not so much to avoid their use, but so as to make sure that they are flagged in my consciousness and unpacked in my classrooms.

I am pleased that the participants made sense of our discussions on multiplicative and fraction reasoning and could link them to different visual scaffolds that were introduced to the lessons. By the end of the semester, were able to use them to good effect. But the journey to that end point revealed many conceptual errors which have required me to rethink what it is that one can take for granted in my classes.



The facts that initially, the participants could not multiply three digit numbers nor add mixed numbers together, etcetera, required a shifting of position within me. These foundational problems caused me to more precisely demonstrate/construct-meaning with the participants. In many of my courses, I rely mainly on slideshows and a document camera but do not use a chalkboard at all. Slideshows and the document camera were used for exposition in this research, but in our lessons the chalkboard became a constant source of support, a place where we unpacked and discussed ideas. The chalkboard offered me a focus and rallying point which is not available to me when I use the other expository tools, and thus suggested to me that I should, in future classes, make use of its immediacy more often.

Through the academic lens of the Literature Review (Chapter 2) which informed this thesis, I have demonstrated that there is now much support for methodologies which cast aside *tabula rasa* teaching and learning methods in favour of techniques which liberate meaningful mathematical understanding. Visualisation facilitates such liberation: visualisation assists students to construct deep, connected understandings of mathematics. Indeed, within this thesis, evidence of the intellectual beneficiation offered by visualising is evidenced in modern-day international and regional mathematics curricula and the South African CAPS curriculum, in academic research conducted by others, in understanding how we come to know mathematics, and in a theory of teaching and learning espoused by Bruner.

The qualitative and quantitative research data, as presented in the Data Collection and Analysis chapter (Chapter 4), has revealed that there was some initial hesitation, even reluctance, and ineptitude in using visualisation scaffolds by the research participants at the beginning of the action research. However, the participants developed an emancipating understanding of the methodology and were able to cogently demonstrate this understanding in re-evaluation assessment situations. And, *of their own accord*, the participants introduced visualisation scaffolds as a teaching and learning methodology into their practicum school experience praxis. In turn, they reported that their pupils warmly embraced the visualisation methodology.

This thesis does not make a claim that visualisation, by itself, is a silver bullet, a panacea for all of the ills which may befall student-teachers' attempts to understand and teach mathematical word-problems. Nor do I believe that one six-month long intervention can offset years of accumulated myths and misunderstandings of mathematical practices and conceptions. However, in broad agreement with the literature which informed the research and was actualised using Bruner's (1980) theory of learning (2.7.4), this research has demonstrated that visualisation offers a liberating starting point, and in the context of the participants' previous experiences of mathematics, it offers a novel teaching and learning methodology. Crucially, visualisation engages both low order thinking (LOT) and high order thinking (HOT), (Bezuidenhout & Alt, 2011; Mohammadi et al., 2015); it facilitates instrumental and relational thinking; and it promotes deep understanding and compression.



For the duration of the action research, the assimilation and accommodation of visually mediated representations of mathematical ideas did not come easily to many of the participants in the research. However, with practice and sustained effort, initial, floundering attempts and, by implication, misunderstanding, incrementally gave way as deeper and more cogent conceptions of arithmetic ideas such as multiplication, fractions and ratios were formed. Further, the confluence of the quantitative and qualitative data converge – triangulate – positively, thus it is fair to suggest that within the observed period of research, microgenesis occurred. In the end, the main research question is answered: *Visualisation assisted the student-teachers to better understand and solve mathematical word problems* by improving their mental disposition to mathematics, by applying liberating conceptions of teaching and learning mathematics, by introducing novel, visual ways of knowing mathematics, by improving foundational arithmetic skills, by assisting participants to construct relational meta-concepts – compression – of big mathematical ideas.

# REFERENCES

Abma, T., & Schwandt, T. A. (2011). The purpose, practice, and politics of sponsored evaluations. In B, Somekh & C, Lewin (Eds.), *Theory and methods in social research* (2nd ed.). (pp. 102-109). London: Sage.

Ackermann, E. K. (2004). Construction Knowledge and transforming the world. In M. Tokoro & L. Steels. (Eds.), *A learning zone of one's own: Sharing representations and flow in collaborative learning environments* (pp. 15-37). Amsterdam: IOS Press.

Adjiage, R., & Pluvinage, F. (2007). An Experiment in teaching ratio and proportion. *Educational Studies in Mathematics, 65* (2), 149-175.

Adler, J., Pournara, D., Taylor, D., Thorne, B., & Moletsane, G. (2009**).** Mathematics and science teacher education in South Africa: A review of research, policy and practice in times of change. *African Journal of Research in Mathematics, Science and Technology Education.* Special Issue 2009, 28–46.

Adler, J. (2017). Mathematics in mathematics education*. South African Journal of Science*, 113 (3/4). Art. #a0201, 3 pages. Retrieved from http://dx.doi.org/10.17159/sajs.2017/a0201

Akinsola, M. K. (2008). Relationship of some psychological variables in predicting problem solving ability of in-service mathematics teachers. *The Montana Mathematics Enthusiast, 5* (1), 79-100.

Alibali, M. W., & Nathan, M. J. (2011). Embodiment in mathematics teaching and learning: *Evidence form learners' and teachers' gestures. Journal of the Learning Sciences, 21* (2), 247 – 286).

Altrichter, H., Posch, P., & Somekh, B. (1993). *Teachers investigate their work. An introduction to the methods of action research.* London: Routeledge.

Anderson, J. A. (1996). Experiments to investigate instrumental and relational understanding among undergraduates. *International Journal of Mathematical Education in Science and Technology, 27* (6), 813-819.

Anghileri, J. (2007). *Developing number sense – progression in the middle years*. London: Continuum.

Antonini, S., Premeg, N. C., Mariotti, M. A., & Zaslavsky, O. (2011). On examples in mathematical thinking and learning. *ZDM Mathematics Education, 43,* 191-194.

Asli Özgün-Koca, S., & Edwards, T. G. (2010). Drawing a regression line between spaghetti & basketball. *Mathematics Teaching in the Middle School, 16* (1), 50-55.

Australia Curriculum (2015). Mathematics. Retrieved from http://www.australiancurriculum.edu.au/mathematics/curriculum/f-10?layout=1

Ausubel, D. P. (1968). Educational psychology: A cognitive view. New York: Holt, Rinehart and Winston.

Badger, M. S., Sanguin, C. J., Hawkes, T. O., Burn, R. P., Mason, J., & Pope, S. (2012). *Teaching problem-solving in undergraduate mathematics.* Creative Commons Attribution No Derivatives (CC BY-ND).

Ball, D. L., Hill, H. C., & Bass, H. (2005).Knowing mathematics for teaching: Who knows mathematics well enough to teach third grade, and how can we decide? *American Educator*, Fall (2005), 14-46.

Ball, D. L. (2009). *Out of the blue – episode 102 – Deborah Ball* [Video file]. Retrieved from https://www.youtube.com/watch?v=PYgg5WA6ORE

Ball, D. L. (2015). *Mathematical knowledge for teaching - Dr. Deborah Loewenberg Ball* [Video file]. Retrieved from https://www.youtube.com/watch?v=leasL_kk8XM

Bantwini, B., & Arowolo, O. (2015). Presentation of qualitative research findings. In C. Okeke & M. van Wyk (Eds.), *Educational research: An African approach* (pp. 496-512). Cape Town: Oxford University Press.

Barbu, O. C., & Beal, C. R. (2010). Effects of linguistic complexity and math difficulty on word problem solving by English learners. *International Journal of Education, 2* (2), E6.

Barmby, P., Bolden, D., Raine, S., & Thompson, L. (2013). Developing the use of visual representations in the primary classroom. Nuffield Foundation. Retrieved from http://www.nuffieldfoundation.org/sites/default/files/files/Developing%20the%20use%20of%20visual%20representations%20in%20the%20primary%20classroom%2013Jun13.pdf



Belbase, S. (2013). Images, anxieties, and attitudes toward mathematics. *International Journal of Education in Mathematics, Science and Technology, 1* (4), 230-237.

Bennie, K. (2005). The MATH taxonomy as a tool for analysing course material in mathematics: A study of its usefulness and its potential as a tool for curriculum development. *African Journal of Research in SMT Education, 9* (2), 81-95.

Beswick, K. (2005). The beliefs/practice connection in broadly defined contexts. *Mathematics Education Research Journal*, 17 (2), 39-68.

Berry, R. Q., Bol, L., & McKinney, S. E. (2009). Addressing the principles for school mathematics: A case study of elementary teachers' pedagogy and practices in an urban high-poverty school. *International Electronic Journal of Mathematics Education, 4* (1), 1-22.

Bezuidenhout, M. J., & Alt, H. (2011). ‘Assessment drives learning’: Do assessments promote high-level cognitive processing? *South African Journal of Higher Education, 25* (6), 1062-1076.

Bishop, A. (1983). Space and geometry. In R. Lesh & M. Landau (Eds.), Acquisition of mathematics concepts and processes (pp. 175-203). New York: Academic Press.

Birjandi, P., & Ebadi, S. (2012), Microgenesis in dynamic assessment of L2 learners’ socio-cognitive development via web 2.0. *Procedia – Social and Behavioral Sciences, 32*, 34-39.

Blaikie, N. (2010). *Designing social research* (2nd ed.). Cambridge: Polity Press.

Bloch, G. (2009). *The toxic mix: What’s wrong with South Africa’s schools and how to fix it*. Cape Town: Tafelberg.



Bloom, B. S., Max, D. E., Furst, E. J., Hill, W. H., & Krathwohl, D. R. (1956). *Taxonomy of educational objectives, handbook 1: The cognitive domain.* New York: David McKay.

Boaler, J. Chen, L., Williams, C., & Cordero, M. (2016). Seeing as understanding: The importance of visual mathematics for our brain and learning. *Journal of Applied and Computational Mathematics, 5* (5), 1-6.

Bodner, G. M. (1986). Constructivism: A theory of knowledge. *Journal of Chemical Education, 63* (10), 873–878.

Bodrova, A. & Leong, D. J. (1996). *Tools of the mind – the Vygotskian approach to early childhood education.* Columbus: Merrill.

Booth, S., Osman. R. & Venkat, H. (2016). The experience of research in pre-service teacher education: A phenomenographic study of students' experienced meaning of research in educational and professional contexts. In R. Osman & H. Venkat (Eds.), *Research-led teacher education*, (pp. 69-87). Cape Town: Pearson.

Bradbury, H. (2015.). Introduction: How to situate and define action research. In H. Bradbury (Ed.), *The SAGE Handbook of Action Research* (3rd ed.). (pp. 1-9). London: Sage.

Breen, C. (2001). Coping with fear of mathematics in a group of preservice primary school teachers. *Pythagoras, 54*, 42-50.

Brown, C. M. & Murphy, T. J. (2000). Research in undergraduate mathematics education. *Reference Services Review, 28* (1), 65-81.

Brown, T., & Heggs, D. (2011). From hermeneutics to post-structuralism to psychoanalysis. In B, Somekh & C, Lewin (Eds.), *Theory and methods in social research* (2nd ed.). (pp. 295-303). London: Sage.

Bruner, J. S. (1960). *The process of education*. Cambridge: Harvard University Press.

Bruner, J. S. (1966). *Towards a theory of Instruction.* Cambridge: Harvard University Press.

Bruner, J. S. (1980). *Beyond the information given*. London: George Allen & Unwin.

Bruner, J. S. (1980). The act of discovery. In J. M. Anglin (Ed.), *Beyond information given: studies in the psychology of knowing* (pp. 401-412). London: George Allen & Unwin.

Bruner, J. S. (1980). The course of cognitive growth. In J. M. Anglin (Ed.), *Beyond information given: studies in the psychology of knowing* (pp. 325-351). London: George Allen & Unwin.

Bruner, J. S. (1986). *Actual minds, possible worlds.* Cambridge: Harvard University Press.

Bruner, J. S. (2007). On learning mathematics. *The Mathematics Teacher*, 100, 48-55.

Bryman, A. (2012). *Social Research Methods.* (4th ed.). Oxford: Oxford University Press.

Bukatko, D. & Daehler, M. W. (1995). *Child development: A thematic approach* (2nd ed.). Boston: Houghton Mifflin.

Bush, H. M., Daddysman, J., & Charnigo, R. (2014). Improving outcomes with Bloom's taxonomy: from statistics to research partnerships. *J Biomet Biostat*, *5* (4), 1-3.

Calas, M. & Smircich, L. (1999). Past postmodernism? Reflections and tentative directions. *Academy of Management Review*, *24* (4), 649–671.

Carr, W. & Kemmis, S. (1986). *Becoming critical: Education, knowledge and action research.* London: Falmer Press.

Carroll, S. (2017). The epistemological gap (between teaching and learning). *The National Teaching and Learning Forum, 26* (2), 1-4.

Carson, J. (2007). A problem with problem solving: Teaching thinking without teaching knowledge. *The Mathematics Educator, 17* (2), 7-14.

Chamberlin, M., & Powers, R. (2010). The promise of differentiated instruction for enhancing the mathematical understandings of college students. *Teaching Mathematics and its Applications, 29,* 113-139.

Chauraya, M., & Mhlolo, M. K. (2008). An analysis of in-service mathematics teachers' conceptions of problem solving in the subject at Midlands State University in Zimbabwe. *African Journal of Research in SMT Education, 12* (1), 75 – 86.

Cheong, Y. K. (2002). The model method in Singapore. *The Mathematics Educator, 6* (2), 47-64.

Chin, K. E. (2013). *Making sense of mathematics: supportive and problematic conceptions with special reference to trigonometry.* (Unpublished doctoral dissertation). University of Warwick, England.

Christ, T. W. (2010). Teaching mix methods and action research: Pedagogical, practical and evaluative considerations. In A. Tashakkori & C. Teddie (Eds.), *SAGE handbook of mixed methods in social and behavioural research* (2nd ed.), (pp. 643-676). Thousand Oaks: Sage.

Christie, P. (1988). *The right to learn; the struggle for education in South Africa*. Goodwood: Raven Press.

Clements, M. A. (2014). Fifty years of thinking about visualization and visualizing in mathematics education: A historical overview. In M. N. Fried & T. Dreyfus (Eds.), *Mathematics & mathematics education: Searching for common ground* (pp. 177-192). Dordrecht: Springer.

Coetzee, S., le Roux, C., & Mohangi, K. (2016). Legal and ethical considerations for barrier-free education. E. Landsberg, D. Kruger & E Swart (Eds.), *Addressing barriers to learning: A South African perspective* (3rd ed.). (pp. 29 – 47). Pretoria: Van Schaik.

Cohen, L., Manion, L., & Morrison, K. (2001). *Research methods in education* (5th ed.). London: RouteledgeFalmer.

Collins, R. (2014). Skills for the 21st Century: teaching higher-order thinking. *Curriculum & Leadership Journal*, 12 (14). Retrieved from http://www.curriculum.edu.au/leader/teaching_higher_order_thinking,37431.html?issueID=12910

Cooke, B., & Cox, J. W. (2005). Introduction. In B. Cooke & L. W. Cox (Eds.), *Fundamentals of action research: Volume 1*. (pp. xix-xliii). London: Sage.

Cornbleth, C. (1987). The persistence of myth in teacher education and teaching. In T. S. Popkewitz (Ed.), *Critical studies in teacher education: Its folklore, theory and practice*, pp 186-210.

Courtney-Clarke, M., & Wessels, H. (2014). Number sense of final year pre-service primary school teachers. *Pythagoras*, 35 (1). Retrieved from http://pythagoras.org.za/index.php/pythagoras/article/view/244



Craig, T. S. (2016). The role of expository writing in mathematical problem solving. *African Journal in Mathematics, Science and Technology Education, 20* (1), 57-66.

Creswell, J. W. (2014). *Research design: Qualitative, quantitative and mixed methods approaches* (4th ed.). Los Angeles: SAGE.

Dalgarno, N., & Colgan, L. (2007). Supporting novice elementary mathematics teachers' induction in professional communities and providing innovative forms of pedagogical content knowledge development through information and communication technology. *Teaching and Teacher Education, 23,* 1051-1065.

Davenport, P. (1999). Conceptual gain and successful problem-solving in primary school mathematics. *Educational Studies, 25* (1), 55-78.

Dennis, M. S., Knight, J., & Jerman, O. (2016). Teaching high school students with learning difficulties to use model drawing strategy to solve fraction and percentage word problems. *Preventing School Failure: Alternative Education for Children and Youth, 60* (1), 10-21.

Department of Basic Education. (2011). *Curriculum and Assessment Policy Statement, (CAPS), Intermediate Phase, Grades 4 – 6, Mathematics.* Pretoria: Department of Basic Education.

Department of Basic Education. (2013). *Education statistics in South Africa 2011*. Retrieved from http://www.education.gov.za/Portals/0/Documents/Reports/2011%20Education%20Statistics%20WEB.pdf?ver=2015-01-29-161525-400

Department of Education. (2002). *Education statistics in South Africa at a glance in 2000.* Retrieved from https://www.dhet.gov.za/DHET%20Statistics%20Publication/DoE%20Stats%20at%20a%20Glance%202000.pdf



Department of Education. (2006). *The National Policy Framework for Teacher Education and Development in South Africa: More Teachers; Better Teachers.* Retrieved from http://www.kzneducation.gov.za/LinkClick.aspx?fileticket=8ksfbr8jU_g%3D&tabid=106&mid=391

Department of Education. (2013). *National curriculum in England: Mathematics programmes of study.* Retrieved from https://www.gov.uk/government/publications/national-curriculum-in-england-mathematics-programmes-of-study/national-curriculum-in-england-mathematics-programmes-of-study

De Vos, A. S., Strydom, H., Fouché, C. B., & Delport, C. S. L. (2011). *Research at grass roots: For the social sciences and human services professions*. (4th ed.). Pretoria: Van Schaik.

Doyle, K. M. (2006). *Mathematical modelling through top-level structure.* (Unpublished masters dissertation). Queensland University of Technology, Brisbane.

Drake, M. (2015). Objectivity, values and subjectivity: Issues of values and ethics in the research process. In C. Okeke & M. van Wyk (Eds.), *Educational research: An African approach* (pp. 96-111). Cape Town: Oxford University Press.

du Toit, S. J., & Kruger, N. (1993). *The child: an educational perspective*. Durban: Butterworths.

Duval, R. (2014). Commentary: Linking epistemology and semio-cognitive modelling in visualisation. *ZDM Mathematics Education, 46* (1), 159-170.

Eastaway, R., & Askew, M. (2010). *Maths for mums and dads.* London: Square Peg.

Edwards, L. D. (2009). Gestures and conceptual integration in mathematical talk. *Educational Studies in Mathematics, 70* (2), 127 – 141.



Elgin, C. Z. (2007). Education and the Advancement of Understanding. In R. Curren (Ed.), *Philosophy of Education, An Anthology* (pp.417 – 422). Oxford: Blackwell.

Ellis, J. H. M., & Kiely, J. A. (2005). Action inquiry strategies: Taking stock and moving forward. In B. Cooke & J. W. Cox (Eds.), *Fundamentals of Action Research, Volume IV* (pp. 85-100). London: Sage.

English, L. D., Lesh., & Fennewald, T. (2008). *Future directions and perspectives for problem solving research and curriculum development*. In: 11th International Congress on Mathematical Education, 6-13, July 2008, Monterrey, Mexico. (Unpublished).

Etchberger, M. L., & Shaw, K. L. (1992). Teacher change as a progression of transitional images: a chronology of a developing constructivist teacher. *School Science and Mathematics, 92* (8), 411–417.

Ethics. (1999). In *Collins Concise Dictionary* ($4^{th}$ ed.). (p. 488). Glasgow: HarperCollins.

Evans, B. R., Ardito, G., & Kim, S. (2017). Editor's perspective article: Mathematics problem solving, literacy, and ELL for alternative certification teachers. *Journal of the National Association for Alternative Certification, 12* (1), 29-33.

Feza, N. (2015). Qualitative data analysis. In C. Okeke & M. van Wyk (Eds.), *Educational research: An African approach* (pp. 458-476). Cape Town: Oxford University Press.

Fong, N. S., & L, K. (2004). Word problems – Can the model method help?  In A Rogerson (Ed.). *The Mathematics Education into the $21^{st}$ Century Project: Proceedings of the International Conference: The future of Mathematics Education* (pp. 104 – 108). Poland.

Fosnot. C. T. (1989). *Enquiring teachers, enquiring learners: A constructivist approach for teaching*. New York: Teachers College Press.



Fraenkel, J. R., & Wallen, N. E. (2009). *How to design and evaluate research in education* ($7^{th}$ ed.). New York: McGraw-Hill Higher Education.

Freire, P. (1997). *Pedagogy of the oppressed.  New Revised $20^{th}$ Anniversary Edition.* New York: Continuum.

Friedrich, A., Jonkmann, K., Nagengast, B., Schmitz, B,. & Trautwein, U. (2013). Teachers' and students' perceptions of self-regulated learning and math competence: Differentiation and agreement. *Learning and individual differences*, 27 (2013), 26-34.

Gardner, H. (1993). *Frames of mind: The theories of multiple intelligences* ($2^{nd}$ ed.). London: Fontana Press.

Gardner, H. (1997). Extraordinary minds: Portraits of 4 exceptional individuals and an examination of our own extraordinariness. New York: Basic Books.

Galotti, K. M. (2014). *Cognitive psychology in and out of the laboratory* (5th ed.). Los Angeles: Sage.

Geellan, D. (2003). *Weaving Narrative Nets to Capture Classrooms: Multimethod Qualitative Approaches for Educational Research.* Dordrecht: Kluwer Academic Publishers.

Gierdien, F. (2012). Pre-service teachers' views about their mathematics teacher education modules. *Pythagoras*, 33 (1). Retrieved from http://www.pythagoras.org.za/index.php/pythagoras/article/view/134/182

Gillham, B. (2000a). *Case Study Research Methods*. London: Continuum.

Gillham, B. (2000b). *Developing a questionnaire*. London: Continuum.



Goldschmidt, G. (2003). The Backtalk of Self-Generated Sketches. *Massachusetts Institute of Technology, 19* (1), 72-88.

Gonnerman, C., O'Rourke, M., Crowley, S. J., & Hall, T. E. (2015). Discovering philosophical assumptions the guide action research: The reflexive toolbox approach. In H. Bradbury (Ed.), The SAGE Handbook of Action Research (3rd ed.). (pp. 373-380). London: Sage.

Gonzales, P., Calsyn, C., Leslie Jocelyn, L., Mak, K., Kastberg, D., Arafeh, S., Williams, T., & Tsen, W. (2000). *Pursuing Excellence: Comparisons of International Eighth-Grade Mathematics and Science Achievement from a U.S. Perspective, 1995 and 1999. Initial Findings from the Third International Mathematics and Science Study–Repeat.* Retrieved from http://nces.ed.gov/pubs2001/2001028.pdf

Gooding, S. (2009). Children's difficulties with mathematical word problems. In M. Joubert (Ed.), *Proceedings of the British Society for Research into Learning Mathematics, 29* (3), (pp. 31 – 36). Retrieved from http://www.bsrlm.org.uk/publications/proceedings-of-day-conference/ip29-3/

Grandy, R. E. (2007). Constructivisms and objectivity: Disentangling metaphysics from pedagogy. In R. Curren (Ed.), *Philosophy of Education* (pp. 410-417). Oxford: Blackwell.

Graven, M. (2015). Going back in order to go forward – the recovery of mathematical foundations for intermediate phase improvement. *Journal of Educational Studies, Special Issue 1, 15* (2), 1-15.

Gray, E., & Tall, D. (2007). Abstraction as a natural process of mental compression. *Mathematics Education Research Journal, 19* (2), 23–40.

Greene, J. C., Kreider, H., & Mayer, E. (2011). Combining qualitative and quantitative methods in social inquiry. In B. Somekh & C. Lewin (Eds.), *Theory and methods in social research* (2nd ed.). (pp. 259-266). London: Sage.

Greenwood, D. J. (2005). Action Research: Unfulfilled promises and unmet challenges. In B. Cooke & J. W. Cox (Eds.), *Fundamentals of action research, Volume IV* (pp. 173-195). London: Sage.

Hartanto, A., Yang, H., & Yang, S. (2018). Bilingualism positively predicts mathematical competence: Evidence from two large-scale studies. *Learning and individual differences*, 61 (2018), 216-227.

Haylock, D. (2002). Mathematics explained for primary teachers (2nd ed.). Thousand Oaks: Sage.

Haylock, D & Manning, R. (2014). *Mathematics explained for primary teachers* (5th ed.). London: Sage.

Henderson, S. (2012). Why the journey to mathematical excellence may be so long in Scotland's primary schools. *Scottish Educational Review*, 44 (1), 46-56.

Hammersley, M. (2012). *Methodological paradigms in educational research*. British Educational Research Association on-line resource. Retrieved from https://martynhammersley.files.wordpress.com/2013/11/methodological-paradigms-4.pdf

Heikkinen, H. L. T., Huttunen, R., & Syrjälä, L. (2005). On the Problem of Quality in Narratives of Action Research. *Proceedings from The European Research Association Annual Conference,* University College, Dublin, Ireland. Retrieved from http://www.eera-ecer.de/ecer-programmes/conference/16/contribution/28188/

Henning, J. E., Stone, J. M., & Kelly, J. L. (2009). *Using action research to improve instruction: An interactive guide for teachers.* New York: Routledge.

Hennink, M., Hutter, I., & Bailey, A. (2011). *Qualitative Research Methods*. London: Sage.



Herr, K., & Anderson, G. L. (2005). *The Action Research Dissertation: A Guide for Students and Faculty.* Thousand Oaks: Sage.

Hodgen, J., Marks, R., & Pepper, D. (2013). *Towards universal participation in post-16 mathematics: Lessons from high-performing countries*. Retrieved from www.nuffieldfoundation.org

Hofstee, E. (2006). *Constructing a good dissertation: A practical guide to finishing a Master's, MBA or PhD on schedule*. Sandton: EPE

Holmes, A. F. (1987). *The idea of a Christian college* (Revised Edition). Grand Rapids: Eerdmans.

Holtman, l., Julie, C., Mbekwa, M., Mtetwa D., & Ngcobo, M. (2011). A comparison of preferences for real-life situations that could be used in school mathematics in three SADC countries. *Southern African Review of Education*, 17, pp. 120 – 137.

Howie, S. J. (n.d.). *Third International Mathematics and Science Study-Repeat (TIMSS-R): What has changed in South African pupils' performance in mathematics between 1995-1998?* Retrieved from http://www.amesa.org.za/TIMSSR.htm

Hughes, J. (1990). *The philosophy of social research* (2nd ed.). Harlow: Longman.

Hungi, N., Makuwa, D., Ross, K., Saito, M., Dolata, S., van Cappella, F., Paviot, L., & Vellien. (2010). *SACMEC III - Project Results: Pupil Achievement Levels in Reading and Mathematics.* Southern and Eastern Africa Consortium for Monitoring Educational Quality. Retrieved from http://www.sacmeq.org/sites/default/files/sacmeq/reports/sacmeq-iii/working-documents/wd01_sacmeq_iii_results_pupil_achievement.pdf



Hyde, A. A., & Bizar, M. (1989). *Thinking in Context: Teaching cognitive processes across the elementary school curriculum.* White Plains: Longman.

Issitt, J. (2007). Evidence and metacognition in the new regime of truth: Figures of the Autonomous learner on the walls of Plato's cave. *Journal of Philosophy of Education*, *41* (3), 381 – 393.

Johnson, A. P. (2014). *Educational Psychology: Theories of Learning and Human Development.* National Science Press. Retrieved from www.nsspress.com

Jones, M. G., Minogue, J., Tretter, T. R., Negishi, A., & Taylor, R. (2006). Haptic augmentation of science instruction: Does touch matter? In G. J. Kelley & R. E. Mayer (Eds). *Science Education*, *90* (1). 111-123.

Jordaan, C., & Jordaan, D. (2013). The case for formal visual literacy teaching in higher education. *South African Journal of Higher Education*, *27* (1). 76 – 92.

Kadunz, G., & Sträßer, R. (2004). Image – metaphor – diagram: Visualisation in learning mathematics. *Proceedings of the 27th Conference of the International Group for the Psychology of Mathematics Education, 4*, 241-248.

Kasule, D., & Mapolela, D. (2013). Prospective teachers' perspectives on the use of English in the solving and teaching of mathematics word problems: A brief cross-country survey. *African Journal of Research in Mathematics, Science and Technology Education*, 17 (3), 265-274. Retrieved from http://dx.doi.org/10.1080/10288457.2013.848539

Kemmis, S., & Wilkinson, M. (1998). Participatory action research and the study of practice. In B. Atweb, B. Kemmis & P. Weeks (Eds.), *Action Research in Practice – Partnerships for social justice in education,* (pp. 21-36). London: Routledge.



Killen, R. (2015). *Teaching strategies for quality teaching and learning*, (2nd ed.). CapeTown: Juta.

Kohan, W. O. (2013). Plato and Socrates: From an educator of childhood to a childlike educator. *Studies in Philosophy & Education, 32* (3). 313 – 325.

Koshy, V. (2010). *Action research for improving educational practice.* London: SAGE.

Krathwohl, D. R., & Anderson, L. W. (2010). Melin C. Wittrock and the revision of Bloom's taxonomy. *Educational Psychologist, 45* (1), 64-65.

Kribs-Zaleta, C. (2008). Oranges, posters, ribbons, and lemonade: Concrete computational strategies for dividing fractions. *Mathematics Teaching in the Middle School, 13* (8), 453-457.

Krulik, S., & Rudnick, J. A. (1980). *Problem Solving: A handbook for teachers* (2nd ed.). Boston: Pearson Education.

Kumar, R. (2014). *Research methodology: A step-by-step guide for beginners* (4th ed.). London: Sage.

Leitch, R. & Day, C. (2000). Action research and reflective practice: Towards a holistic view. *Educational Action Research,* 8 (1), 179-193).

Lerman, S. (1996). Intersubjectivity in mathematics learning: A challenge to the radical constructivist paradigm? *Journal for Research in Mathematics Education, 27* (2), 133-150.

Leung, F. K. S., Graf, K. D., & Lopez-Real, F. J. (2006). *Mathematics education in different cultural traditions: a comparative study of East Asia and the West.* New York: Springer.

Looi, C. K., & Lim, K. S. (2009). From bar diagrams to letter-symbolic algebra: A technology-enabled bridging. *Journal of Computer Assisted Learning, 25*, 358 – 374.

Losin, P. (1996). Education and Plato's parable of the cave. *Journal of Education. 178* (3). 49 – 65.

Ludhra, G. (2008). The complexities of creativity within Initial Teacher Education. *Perspectives in Education, 26* (2), 53-65.

Ma, L. (2010). *Knowing and teaching elementary mathematics: Teaching understanding of fundamental mathematics in China and the United States.* New York: Routledge.

Macnab, J. S., Phillips, L. M., & Norris, S. P. (2012). Visualizations and visualization in mathematics education. In S. P. Norris (Ed.), *Reading for Evidence and Interpreting Visualizations in Mathematics and Science Education,* (pp. 103-122). Rotterdam: Sense.

Mahofa, E., & Adendorff, S. A. (2014). Code switching in the learning of mathematics word problems in Grade 10. *Journal for Educational Studies, 13* (2), 84 –112.

Makina, A., & Wessels, D. (2009). The role of visualisation in data handling in Grade 9 within a problem-centred context. *Pythagoras*, 69, 56 – 68. Retrieved from http://www.pythagoras.org.za/index.php/pythagoras/article/viewFile/46/36

Makuwa, D. K. (2010). UNESCO International Institute for Educational Planning. (2010). iiep News Letter. XXVII (3), September – December 2010, 1–5. Retrieved from http://www.iiep.unesco.org/sites/default/files/nl_2010-3_en.pdf



Malan, S. B., Ndlovu, M., & Engelbrecht, (2014). Introducing problem-based learning (PBL) into a foundation programme to develop self-directed learning skills. *South African Journal of Education, 34* (1). Art. # 457, 16 pages. Retrieved from https://www.ajol.info/index.php/saje/article/view/106630/96541

Malley, L., Neidorf, T., Arora, A., & Kroeger, T. (2016). In *TIMSS 2015 encyclopaedia: United States.* Retrieved from http://timssandpirls.bc.edu/timss2015/encyclopedia/countries/united-states/

Maphalala, M. (2016). Bloom's taxonomy and implications for teacher preparation. In C. Okeke, J. Abongdia, E. O. Adu, M. van Wyk & C. Wolhuter (Eds.), *Learn to Teach: A handbook for Teaching Practice.* Cape Town: Oxford.

Maslow, A. H. (1943). A theory of human motivation. *Psychological Review, 50* (4), 370 – 396.

Masole, T. M., Gabalebatse, M., Guga, T., & Pharithi, M. (2016). In *TIMSS 2015 encyclopaedia: Botswana*. Retrieved from http://timssandpirls.bc.edu/timss2015/encyclopedia/countries/botswana/

Mayer, R. E. (1997). *Thinking, problem solving, cognition* (2nd ed.). New York: W. H. Freeman and Company.

McAteer, M. (2013). *Action research in education*. Los Angeles: Sage.

McNiff, J., & Whitehead, J. (2006). *Action research for teachers: A practical guide.* London: David Fulton.

Mertens, D. M. (2005). *Research and evaluation in education and psychology: Integrating diversity with quantitative, qualitative, and mixed methods (*2ns ed.) Thousand Oaks: Sage.

Mertens, D. M., Bledsoe, K. L., Sullivan, M., & Wilson, A. (2010). Utilization of mixed methods for transformative purposes. In A. Tashakkori & C. Teddie (Eds.), *SAGE handbook of mixed methods in social and behavioural research* (2nd ed.), (pp. 193-214). Thousand Oaks: Sage.

Mertler, C. A. (2012). *Action Research – Improving Schools and Empowering Educators.* Thousand Oaks: Sage.

Mhlolo, M. K., Venkat, H., & Schäfer, M. (2012). The nature and quality of the mathematical connections teachers make. Pythagoras, 33 (1), Art. #22, 9 pages. Retrieved from http://pythagoras.org.za/index.php/pythagoras/article/view/22

Ministry of Education & Skills Development, (2002*). Lower primary school syllabus: Standard one to four*. Gaborone: Government Printer.

Ministry of Education, Namibia. (2010). *Mathematics upper primary phase syllabus, grades 5 -7. National Institute for Educational Development (NIED).* Okahandja: NIED.

Ministry of Education, Science, Vocational Training and Early Education, (2013). *Mathematics Syllabus, Grades 1 – 7.* Lusaka: Curriculum Development Centre.

Ministry of Education, Singapore, (2012). *Primary mathematics teaching and learning syllabus.* Retrieved from http://mgs.moe.edu.sg/qql/slot/u500/Primary/MG%20Curriculum/Academic/Mathematics/maths-primary-2013.pdf

Ministry of Education, Singapore, (2012). *Mathematics syllabus, primary one to four.* Retrieved from https://www.moe.gov.sg/docs/default-source/document/education/syllabuses/sciences/files/mathematics-syllabus-(primary-1-to-4).pdf



Ministry of Education, Sport and Culture, (2006). *Primary school mathematics syllabus: Grades 6-7.* Harare: Curriculum Development Unit.

Mnguni, L. E. (2014). The theoretical cognitive process of visualization for science education. *SpringerPlus,* 3 (184).

Mohammadi, E., Kiany, G. R., Samar, R. G., & Akbari, R. (2015). Appraising pre-service EFL teachers' assessment in language testing course using revised Bloom's taxonomy. *International Journal of Applied Linguistics and English Literature, 4* (4), 8-20.

Montague, M. (2012). Math problem solving for middle school students with disabilities. *Research report of the Access Centre: Improving outcomes for All Students K-8*. Retrieved from www.k8accesscenter.org

Moreno, R., & Valdez, A. (2005). Cognitive load and learning effects of having students organise pictures and words in multimedia environments: The role of student interactivity and feedback. *Educational Technology Research and Development*, *53* (3). 35-45.

Moschkovich, J. (2013). Principles and guidelines for equitable mathematics teaching practices and materials for English language learners. *Journal of Urban Mathematics Education, 6* (1), 45-57.

Mouton, J. (2003). *How to succeed in our master's and doctoral studies. A South African guide and resource book.* Pretoria: van Schaik.

Moyo, N., Modiba, M., & Simwa, K. (2015). Critical research: Understanding material constraints and engaging in transformative action research. In C. Okeke & M. van Wyk (Eds.), *Educational research: An African approach* (pp. 57-76). Cape Town: Oxford University Press.

Mudulay, V. (2010). Thinking with diagrams whilst writing with words. *Pythagoras*, *71*, 65-75.



Muir, T. (2008). Principles of practice and teacher actions: Influences on effective teaching of numeracy. *Mathematics Education Research Journal*. 20 (3), 78-101.

Mullis, I. V. S., Martin, M. O., Gonzalez, E. J., & Chrostowski, S. J. (2004). *TIMSS 2003 International Mathematics Report.* Retrieved from http://timss.bc.edu/PDF/t03_download/T03INTLMATRPT.pdf

Mullis, I. V. S., Martin, M. O., Foy, P., & Aroro, A. (2012). *TIMSS 2011 International Results in Mathematics.* Retrieved from http://timssandpirls.bc.edu/timss2011/downloads/T11_IR_Mathematics_FullBook.pdf

Mullis, I. V. S., Martin, M. O., Foy, P., & Hooper, M. (2016). *TIMSS 2015 International Results in Mathematics.* Retrieved from http://timssandpirls.bc.edu/timss2015/international-results/

Mukwambo, M., Ngcoza, K., & Chikunda, C. (2015). How researchers define and measure variables. In C. Okeke & M. van Wyk (Eds.), *Educational research: An African approach* (pp. 186-206). Cape Town: Oxford University Press.

Munez, D., Orrantia, J., & Rosales, J. (2013). The effect of external representations on compare word problems: Supporting mental model construction. *The Journal of Experimental Education*, *81* (3). 337 – 355.

Murray, H. (2012). Problems with Word Problems in Mathematics. *Learning and Teaching Mathematics, 13*, 55-58. Retrieved from http://www.amesa.org.za/Hanlie.pdf

Mwakapenda, W. (2004). Understanding student understanding in mathematics *Pythagoras*, 60, 28-35.



Naidoo, J. (2012). Teacher reflection: the use of visual tools in mathematics classrooms. *Pythagoras, 33* (1), Art. #54, 9 pages. Retrieved from http://pythagoras.org.za/index.php/pythagoras/article/viewFile/54/177

Nardi, E. (2014). Reflections on visualization in mathematics and in mathematics education. In M. N. Fried & T. Dreyfus (Eds.), *Mathematics & Mathematics Education: Searching for Common Ground,* (pp. 193-222). Dordrecht: Springer.

Naroth, C., & Luneta, K. (2015). Implementing the Singapore Mathematics Curriculum in South Africa: Experiences of Foundation Phase Teachers. *African Journal of Research in Mathematics, Science and Technology Education, 19* (3), 267-277.

National Governors Association Center for Best Practices, Council of Chief State School Officers. (2017a). Common core state standards initiative, Grade 1, Introduction. Retrieved from http://www.corestandards.org/Math/Content/1/introduction/

National Governors Association Center for Best Practices, Council of Chief State School Officers. (2017b). Common core state standards initiative, High schools, Functions, Introduction.  Retrieved from http://www.corestandards.org/Math/Content/HSF/introduction/

Nel, M. & Nel, N. (2016). Second-language difficulties in a South African context. In E. Landsberg, D. Krüger & E. Swart. *Addressing barriers to learning: A South African perspective*, (3rd ed.). (pp. 195-213). Pretoria: Van Schaik.

Ndlovu, M. (2014). The effectiveness of a teacher professional learning programme: The perceptions and performance of mathematics teachers. *Pythagoras, 35* (2). Art # 237, 10 pages. Retrieved from http://dx.doi.org/10.4102/pythagoras.v35i2.237

Nelsen, L. L. (2014). Out of Plato's cave: The role of mathematics in the Christian liberal arts curriculum. *Christian Higher Education*, *13* (2), 101 – 117.

News24 archives. (2010). Matric results approved by Umalusi. Retrieved from http://www.news24.com/SouthAfrica/News/Matric-results-approved-by-Umalusi-20101230

Newton Suter, W. (2012). *Introduction to educational research – A critical thinking approach* (2nd ed.). Thousand Oaks: Sage.

Ng, S. F. & Lee, K. (2005). How primary five pupils use the model method to solve word problems. *The Mathematics Educator, 9* (1), 60 – 83.

Noffke, S., & Somekh, B. (2011). Action Research. In B, Somekh & C, Lewin (Eds.), *Theory and methods in social research* (2nd ed.). (pp. 94-101). London: Sage.

Novak, E., & Tassell, J. L. (2017). Studying preservice teacher math anxiety and mathematics performance in geometry, word, and not-word problem solving. *Learning and Individual Differences, 54,* 20 – 29.

Ojose, B. (2008). Applying Piaget's theory of cognitive development to mathematics instruction. *The Mathematics Educator, 18* (1), 26-30.

Özgün-Koca, S. A., & Edwards, T. G. (2010). Drawing a regression line between spaghetti & basketball. *Mathematics Teaching in the Middle School, 16* (1), 50-55.

Parker, D. (2009). Access to [pre-service] teacher education.. Umalusi/CEPD Seminar *Access to Pre-service Education* - 3 September 2009, 10-23. Retrieved from http://www.cepd.org.za/files/pictures/Final%20Report-Sept%202009%20CEPD%20Umalusi%20seminar%20-%20Access%20to%20preservice%20teacher%20education.pdf

Pesek, D. D., & Kirshner, D. (2000). Interference of instrumental instruction in subsequent relational learning. *Journal for Research in Mathematics Education, 31* (5), 524 – 540.

Petit, M. M., Laird, R. E., & Marsden, E. L. (2010). *A focus on fractions: Bringing research to the classroom.* New York: Routledge.

Piaget, J. (1986). *The psychology of intelligence*. London: Routledge & Kegan Paul.

Piaget, J. (2000). Piaget's theory. In K. Lee (Ed.), *Childhood cognitive development: The essential readings*. Malden: Blackwall. pp. 33-47.

Plotz, M., Froneman, S., & Hieuwoudt, H. (2012). A model for the development and transformation of teachers' mathematical content knowledge. *African Journal of Research in MST Education, 16* (1), 69 – 81.

Poch, A. L., van Garderen, D., & Scheuermann, A. M. (2015). Students' understanding of diagrams for solving word problems: A framework for assessing diagram proficiency. *Teaching Exceptional Children, 47* (3), 153-162. Retrieved from http://journals.sagepub.com/doi/pdf/10.1177/0040059914558947

Presno, C. (1997). Bruner's three forms of representation revisited: Action, pictures and words for effective computer instruction. *Journal of Instructional Psychology, 24* (2), 112-118.



Presmeg, N. C. (1986). Visualisation and mathematical giftedness. *Educational Studies in Mathematics, 17* (3), 297-311.

Presmeg, N. C. (2014). Mathematics education research embracing arts and sciences. In M. N. Fried & T. Dreyfus (Eds.), *Mathematics & Mathematics Education: Searching for Common Ground,* (pp. 361-375). Dordrecht: Springer.

Reason, M. (2003). Relational, Instrumental and Creative Understanding. *Mathematics Teaching*, 184, 5-7.

Reason, P., & Bradbury, H. (2008). Introduction. In P. Reason & H. Bradbury (Eds.), *The SAGE Handbook of Action Research – Participative Inquiry and Practice* (pp. 1-13). London: Sage.

Reddy, V. (2012). *SA and Honduras at bottom of TIMSS rankings – HSRC*. Retrieved from http://www.politicsweb.co.za/politicsweb/view/politicsweb/en/page71654?oid=346605&sn=Detail&pid=71616

Reddy, V., Arends, F., Juan., & Prinsloo, P. (2015). In *TIMSS 2015 encyclopaedia: South Africa*. Retrieved from http://timssandpirls.bc.edu/timss2015/encyclopedia/countries/southafrica/

Reddy, V., Kivilu, K., Cosser, M., & Frempong, G. (2011*). Comments on the 2010 matric results: From the research programme on education and skills development, Human Sciences Research Council (HSRC)*. Retrieved from http://www.hsrc.ac.za/Media_Release-410.phtml

Rellensmann, J., Schukajlow, S., & Leopold, C. (2017). Making a drawing. Effects of strategic knowledge, drawing accuracy, and type of drawing on students' mathematical modelling performance. *Education Studies in Mathematics*, 95 (2017), 53-78.

Rembe, S., Shumba, J., Maphosa, C., & Musesengwe, E. (2016). Change theory – the teacher as reflective and change agent. In C. Okeke, J. Abongdia, E. Olusola Adu, M. van Wyk & C. Wolhuter (Eds.), *Learn to Teach – a Handbook for Teaching Practice,* (pp. 32-48). Cape Town: Oxford University Press.

Richardson, M., & Isaacs, T. (2016). In *TIMSS 2015 encyclopaedia: England,* TIMSS 2015. Retrieved from http://timssandpirls.bc.edu/timss2015/encyclopedia/countries/england/

Rickard, C. (2013). *Essential primary mathematics*. Maidenhead: Open University Press.

Risma, D. A., Putri, R., I., I., & Hartono. (2013). On developing students' spatial visualisation ability. *International Education Studies*, *6* (9), 1-12.

Rivera, F. D., Steinbring, H., & and Arcavi, A. (2014). Visualization as an epistemological learning tool: an introduction. *ZDM – The International Journal on Mathematics Education*, *46* (1), 1-2. Retrieved from https://doi.org/10.1007/s11858-013-0552-0

Robinson, V. M. J. (2005). Current controversies in action research. In B. Cooke & J. W. Cox (Eds.), *Fundamentals of action research, Volume IV* (pp. 59-84). London: Sage.

Robson, C. (2002). *Real world research: A resource for social scientists and practitioner-researchers,* (2nd ed.). Malden: Blackwell.

Rockswold, G. K. (2014). College algebra with modelling and visualization (5th ed.). London: Pearson.

Rosken, B., & Rolka, K. (2006). A picture is worth a 1000 words – the role of visualization in mathematical learning. In J. Novotná, H. Moraová, M. Krátká & N. Stehlíková (Eds.), *Proceedings 30th Conference of the International Group for the Psychology of Mathematics Education, Vol. 4* (pp. 457-464). Prague: PME. Retrieved from http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.518.7984&rep=rep1&type=pdf

Roth, G., & Bradbury, H. (2013). Learning history: An action research practice in support of actionable learning. In P. Reason & H. Bradbury (Eds.), *The SAGE Handbook of Action Research – Participative Inquiry and Practice* (pp. 1-13). London: Sage.

Sadoski, M. (2005). A dual coding view of vocabulary learning. *Reading & Writing Quarterly, 21* (3). 221-238.

Sajadi, M., Amiripour, P., & Rostamy-Malkhalalifeh, M. (2013). The examining mathematical word problems solving ability under efficient representation aspect. *Mathematics Education Trends and Research,* 2013. 1–11. Retrieved from http://www.ispacs.com/journals/metr/2013/metr-00007/article.pdf

Sandelowski, M. (2000). Focus on research methods: Combining qualitative and quantitative sampling, data collection and analysis techniques in mixed-method studies. *Research in Nursing & Health, 23*. 246–255.

Sanford, N. (2005). Whatever happened to action research? In B. Cooke & J. W. Cox (Eds.), *Fundamentals of action research, Volume IV* (pp. 3-23). London: Sage.

Schäfer, J. (2010). *An investigation of how visual arts can be used to teach mathematical concepts of space and shape in grade R.* (Unpublished masters dissertation). Rhodes University, South Africa.

Schunk, D. (1996). *Learning theories* (2nd ed.). Columbus: Prentice Hall.



Scott Baumann, A., Bloomfield, A., & Roughton, L. (1997). *Becoming a secondary school teacher.* London: Hodder & Stoughton.

Sepeng, P. (2015). Strategies used by Grade 6 learners when solving mathematical story problems. *Journal of Educational Studies, 14* (1), 101 – 124.

Sepeng, P., & Webb, P. (2012). Exploring mathematical discussion in word problem-solving. *Pythagoras, 33* (1), Art.#60, 8 pages. Retrieved from http://dx.doi.org/10.4102/pythagoras.v33i1.60

Shenton, A. K. (2004). Strategies for ensuring trustworthiness in qualitative research projects. *Education for Information, 22*, 63–75. Retrieved from https://pdfs.semanticscholar.org/452e/3393e3ecc34f913e8c49d8faf19b9f89b75d.pdf

Sherman, K., & Gabriel, R. (2016). Math word problems: Reading math situations from the start. *The Reading Teacher*, *70* (4), 473-477.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15* (2), 4-14.

Schulman, L. S. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review*, 57 (1), 1-22.

Siemens, G. (2006). Knowing Knowledge. Retrieved from http://www.elearnspace.org/KnowingKnowledge_LowRes.pdf

Siemon, D., Adendorff, S., Austin, P., Fransman, J., Hobden, S., Kaino, L., … Warren, E. (2013). *Teaching mathematics: Foundation and intermediate phase.* Cape Town: Oxford University Press.

Siemon, D., Beswick, K., Brady, K., Clark, J., Faragher, R., & Warren, E. (2012). *Teaching mathematics: foundations to middle years.* South Melbourne: Oxford University Press.

Siew Yin, H. (2010). *Seeing the value of visualization*. Retrieved from http://singteach.nie.edu.sg/issue22-mathed/

Silverman, D. (2002). *Interpreting qualitative data - methods for analysing talk, text and interaction* (2nd ed.). London: Sage.

Skemp, R. R. (1976). Relational understanding and instrumental understanding. *Mathematics Teaching*, *77,* 20–26.

Skemp, R. R. (2006). Relational understanding and instrumental understanding. *Mathematics Teaching in the Middle School,* 12 (2), 88 – 95.

Smith, M. K. (2002). Jerome S. Bruner and the process of education. *The encyclopaedia of informal education*. Retrieved from http://infed.org/mobi/jerome-bruner-and-the-process-of-education/

Sotuku, N., Duku, S. (2015). Ethics in human science research. In C. Okeke & M. van Wyk (Eds.), *Educational research: An African approach* (pp. 112-130). Cape Town: Oxford University Press.

Southwood, S., & Spanneberg, R. (2000). *Rethinking the teaching and learning of mathematics.* Pretoria: Via Afrika.

Sowder, J. T., Philipp, R. A., Armstrong, B. E., & Schappelle, B. P. (1998). *Middle-grade teachers' mathematical knowledge and its relationship to instruction: A research monograph.* Albany: State University of New York.

Spaull, N. (2016, September 14). Serious technical concerns about SACMEQ IV results presented to parliament. [Web log comment]. Retrieved from https://nicspaull.com/2016/09/14/serious-technical-concerns-about-sacmeq-iv-results-presented-to-parliament/

Splitter, L. J. (1991). Critical thinking: What, why, when and how. *Journal of Educational Philosophy and Theory, 23* (1), 89-108.

Stringer, E. T. (2014). *Action Research*, (4th ed.). Thousand Oaks: Sage.

Taber, S. B. (2009). Mathematics teaching in the middle school. *National Council of Teachers of Mathematics, 15* (3), 148-155.

Takaya, K. (2008). Jerome Bruner's theory of education: From early Bruner to later Bruner. *Interchange*, 39 (1), 1-19. Retrieved from http://ocw.metu.edu.tr/pluginfile.php/8931/mod_resource/content/1/7su.pdf

Tall, D. (2014). Making sense of mathematical reasoning and proof. In M. N. Fried & T. Drefus (Eds.), *Mathematics & Mathematics Education: Searching for Common Ground,* (pp. 223-235). Dordrecht: Springer.

Teddlie, C., & Tashakkori, A. (2009). *Foundations of mixed methods research: Integrating quantitative and qualitative approaches in the social and behavioural sciences.* Thousand Oaks: Sage.

Teddlie, C., & Tashakkori, A. (2010). Overview of contemporary issues in mixed methods research. In A. Tashakkori & C. Teddie (Eds.), *SAGE handbook of mixed methods in social and behavioural research* (2nd ed.), (pp. 1-41). Thousand Oaks: Sage.

The Centre for Development and Enterprise. (2010). *The mathematics and science performance of South Africa's public schools: Some lessons from the past*. Johannesburg: Centre for Development and Enterprise.



Thompson, T. (2008). Mathematics teachers' interpretation of higher-order thinking in Bloom's taxonomy. *International Electronic Journal of Mathematics Education, 3* (2), 96-109. Retrieved from file:///C:/Users/pshaw/Downloads/IEJME_314_article_5757c1ac8e4f4%20(1).pdf

Tipps, S., Johnson, A., & Kennedy, L. M. (2011). *Guiding children's learning of mathematics: International edition* (12th ed.). United States of America: Wadsworth

Tobias, B. (2006). Mathematical word problems: Understanding how secondary students position themselves. *African Journal of Research in SMT Education*, *10* (2), 1-14.

van der Berg, S., & Spaull, N. (2011). SA education: The poorest choice*. The Mail and Guardian online.* Retrieved from http://mg.co.za/article/2011-04-08-sa-education-the-poorest-choice/

van der Sandt, S. (2007). Research framework on mathematics teacher behaviour: Koehler and Grouws' framework revisited. *Eurasia Journal of Mathematics, Science and Technology Education, 3* (4), 343-350.

Van de Walle, J. A., Karp, K. S., & Bay-Williams, J. M. (2010). Elementary & middle school mathematics: Teaching developmentally (7th ed.). Boston: Pearson Education.

Venkat, H., Mathews, C., & du Plessi, J. (2012). Researching primary mathematics teacher content knowledge in classroom practice: Understandings for teacher education. In R. Osman & H. VenKay (Eds.), *Research-led teacher education*. Cape Town: Pearson.

von Glasersfeld, E. (1996). Learning and adaptation in the theory of constructivism. In L. Smith (Ed.), *Critical readings on Piaget*. New York: Routledge, pp. 20-27.

Wadsworth, B. J. (1989). *Piaget's theory of cognitive and affective development* (4th ed.). White Plains: Longman.

Watt, D. (2007). On Becoming a Qualitative Researcher: The Value of Reflexivity. *The Qualitative Report, 12* (1), 82–101.

Wernert, N., & Thomson, S. (2016). In *TIMSS 2015 encyclopaedia: Australia. TIMSS 2015.* Retrieved from http://timssandpirls.bc.edu/timss2015/encyclopedia/countries/australia/references/

Wilkie, K. J., & Clarke, D. M. (2014). Developing student's functional thinking in algebra through different visualisations of a growing pattern's structure. *Mathematics Education Research Group of Australasia* (pp. 637-644). Australia: Springer.

Wilson, D. (2012). Training the mind's eye: "Brain Movies" support comprehension and recall. *The Reading Teacher*, *66* (3). 189-194.

Witzel, B., Smith, S. W., & Brownell, M. T. (2001). How can I help students with learning disabilities in algebra? *Intervention in School and Clinic, 37* (2). 101 – 104.

Wood, D. (1998). *How children think and learn.* London: Blackwell.

Woodwell, D. (2014). *Research foundations: How do we know what we know?* Thousand Oaks: Sage.

Woolfolk, A. (2010). *Educational psychology, Global edition* (11th ed). London: Pearson International Edition.

Worthy. (1999). In *Collins Concise Dictionary* (4th ed.). (p. 1720). Glasgow: HarperCollins.

Zanele, S. (2015). *Factors affecting the teaching of Grade 10 and 12 mathematical problem-solving: towards needs-driven context-sensitive continuous professional teacher development model.* (Unpublished doctoral dissertation). University of Fort Hare, South Africa.