

Power Spectral Density Computation and Dominant Frequencies Identification from the Vibration Sensor Output under Random Vibration Environment

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ABSTRACT

The objective of the modal and spectral analysis is to determine the vibration characteristics of structures such as natural frequencies, dominant frequencies and mode shapes. Such modal and spectral analyses have major relevance to the study of the dynamic properties of the structures undergoing dynamic vibration. Methods for the estimation of the power spectral density and identification of the dominant frequencies from the sensor responses under random vibrating environment are presented in this paper. Periodogram using FFT, Welch Method and MUSIC algorithm are used to analyse the known frequency sinusoids with additive white noise and output of the vibration sensor mounted on the test object. The resultant spectra obtained using the methods and their corresponding errors with the reference spectrum are analysed. The Welch method is further studied with three different windows, namely, Hann, Hamming and Blackman-Harris and with three different overlapping criteria viz. 0%, 25% and 50%. The same algorithm and methodology were adopted and compared in two different platforms: Mathematical Model Simulation and Hardware-In-Loop-Simulation. It is observed from the results that Welch Method with 25% overlap used in combination either with Hann or Blackman-Harris window yields more accurate results, compared to other combinations. Also, 25% overlap provides better execution time trade-off compared to 50% overlap.

Keywords: Random signal; Welch method; Windowing; Power spectral density; Modal analysis; Spectral frequency; Hardware-in-loop simulation

1. INTRODUCTION

Every physical entity has mass and elasticity, causing the body to vibrate. The vibration characteristics exhibit oscillatory behaviour. The vibrations of these objects are generally classified as linear or non-linear. Besides, the vibration phenomenon can be classified further into free vibration and forced vibration. When the object vibrates in the absence of any external forces, but under the action of the inherent forces in the system, it is said to be free vibration. One or more frequencies of oscillation under free vibration are known as natural frequency of the object. On the other hand, when external forces are applied to the system for excitation, it is labeled forced vibration. If the frequency of the external excitation coincides with the natural frequency of the object, then the condition is termed resonance. Resonance in turn may result in damage to the structure because of the dangerously large oscillations. The structure's vibration characteristics i.e. the natural frequency and the mode shapes can be determined by modal analysis. The dynamic properties of the structures are analysed using modal analysis. The dominant frequencies and the associated energy content along with the overall RMS value can be analysed using spectral analysis of vibration signals. Most of the physical signals vary with time and are random

in nature. Being random, these signals are non-periodic and non-deterministic. Hence, these signals cannot be treated in the same way as deterministic signals and analysed by assuming a periodicity of infinite time, because these signals do not obey the Dirichlet condition¹. This frequency variation with time cannot be obtained using Fourier transform as this transform simply expands a signal as a linear combination of single frequencies that exist over all time². For a single component, time variant signal,

$$s(t) = a(t)e^{j2\pi c\phi(t)}$$

the instantaneous frequency, i.e. frequency at a particular time can be described

$$\zeta S(t) = c \frac{d\phi(t)}{dt}$$

where $a(t)$ is the signal's amplitude, $\phi(t)$ is its phase, and c is its rate of frequency change with respect to time. Information about the propagation in an unknown environment may be derived from the instantaneous frequency estimation of the signal. The instantaneous frequency estimation is highly useful in many applications in underwater acoustics, radar, active vibration control.

There are three main parameter estimation problems which involve frequency estimation³.

- Single tone frequency estimation: the signal is a single, constant-frequency sinusoid. This is the simplest

frequency estimation problem.

- Multi-harmonic frequency estimation: the signal is composed of the sum of harmonically related sinusoids. This case is important because rotational or periodic phenomena rarely generate sinusoidal waveforms.
- Multi-tone frequency estimation: there are several tones of unrelated frequencies present.

In statistical signal processing and time series analysis, the primary focus is on estimating the sinusoidal signal frequency when it is coupled with environmental noises⁴. For a random process of length n and time series of $\{x_1, \dots, x_n\}$,

$$x_t = \sum_{k=1}^{\infty} (\beta \cos(2\pi f_k t + \phi_k)) + \epsilon_t$$

accurate frequency estimates can be obtained by maximising the Periodogram,

$$I_n(f) = n^{-1} \left| \sum x_t e^{-2\pi f t} \right|^2$$

as a continuous function of f . The asymptotic standard error of this estimator can be expressed as $O(n^{-3/2})$.

2. VIBRATION IN LAUNCH VEHICLES

The flight vehicles used for placing the payload onto the targeted trajectory encounter propulsion forces, aerodynamic forces, acoustic and shock loads during the launch of the spacecraft. These forces strongly interact with the flight structures and cause mechanical vibrations throughout the length of the launch vehicle and also at the interface with the payload. When the steady state accelerations are subtracted, these mechanical vibrations are generally categorised as⁴.

- | | |
|----------------------------------|--------------|
| (i) Sinusoidal vibrations | 5-100 Hz |
| (ii) Random vibrations | 20 - 2000 Hz |
| (iii) Shock loads, accelerations | 100 - 5000Hz |

The shock loads have very short time periods in comparison to the time cycles associated with the lowest and lower natural frequencies of the launch vehicle and are generally repressed by the shock response spectrum. Acoustic excitation of the spacecraft causes mechanical vibrations of a random nature on top of random mechanical vibrations. Many of the failures of satellites in the initial days are due to vibro-acoustics during the vehicle launch phase⁵. The calculation of dynamics response in the spacecraft, the primary structure, the secondary structures, the instruments, and the equipment etc. is called dynamic analysis. MIL-STD-810G emphasizes the tailoring of flight components' design, test limits of the conditions and test methods. The Environmental Engineering Guidelines, the laboratory test methods and the Climatic region guidance are envisaged in MIL-STD-810G. For the missile and launch vehicles, the applicable test method IV (in MIL-STD-810G) defines the details of the spectrum. Generally, flattened spectrum is used for the vibration analysis of air-borne systems, with provision for fine-tuning the same using shaped spectrum based on flight requirements and flight data. Besides the dynamic responses, the modal properties of the complete launch vehicle and subsystems, e.g. natural frequencies, mode shapes, effective masses are also of great interest when discussing vibration in launch vehicles.

Launch vehicles are high mass, high speed vehicles

which are subjected to difficult and extreme atmospheric conditions during the flight. Since the operating conditions of the spacecraft/launch vehicles vary, the disturbances due to vibrations are considered random. These random vibrations are generated due to propulsion load, aerodynamic load, acoustic load, control forces, wind gusts, etc. Most of the launch vehicles are highly flexible and modal frequencies are quite low while the spectral frequencies are completely random in nature. Among the modal characteristics of the dynamic system, the structural frequency of the vehicle is generally contributed by aerodynamic load and propulsion load added by structural loads viz. geometry, joint rotation constant and material properties. This structural frequency and the energy contained at those frequencies affect the stability of the flight vehicle and performance of the avionics packages. Hence, the prediction of the mode shape and spectral frequencies during the design stage using simulation techniques is very important in arriving at the optimum configuration of the launch vehicle.

3. METHODS OF ANALYSIS

The methods by which the modal frequencies can be identified are broadly classified as :

- Frequency domain or time domain
- Bayesian or Non-Bayesian

The known theoretical properties such as correlation function or spectral density of the measured vibrations are used with statistical estimators in Non-Bayesian methods. When the number of modal frequencies in the vibration data is large, the time domain approach provides the best results. However, the limitation is that these estimates are within the range of analysis. This does not take into account the residual effects of those frequencies lying outside the range. On the other hand, the frequency domain approaches provide the best results when the frequency range of interest is limited and the number of modes is relatively small. But Frequency resolution, Leakage and High modal densities are the disadvantages of the Frequency domain analysis. These problems, however, can be addressed by increasing the order of the model.

The requirement of on-board analysis of the modal and spectral vibration signals is gaining importance with high optimization of flight vehicles and the need for better and on-board updating of the control coefficients. Hence, providing a solution to analyse the vibration signals on-board, generating power spectral density and identifying the dominant frequencies within the time bounds of the data updates in flight vehicles are of paramount importance for better control and stability. Acree⁶, *et al.* analysed the flight flutter data using Fourier transformation and computing the spectral data using Chirp-Z transformation. The method was tested for the initial test flight structure of XV-15 tilt rotor wing at baseline speed.

To extract the instantaneous undamped frequency and the real non-linear elastic force characteristics, Feldman⁷ had proposed the Hilbert Transform analysis in time-domain. For Hilbert Transformation method, the vibration signal should be a mono-component signal from a S-DoF system or from a Multi-DoF system after special decomposition. Freevib method was used for non-linear frequency response function

and the decomposition/scaling technique was used for force characteristic interpretation, which involves intense numerical calculations that may not be suitable for real-time on-board applications.

Tadeusz⁸, *et al.* proposed a method based on the linearisation of the relation between ARMA model parameter variance and the standard deviation of the modal parameters. The time domain method was formulated for linear systems with time-varying parameters. A Morlet type wavelet was used in the proposed algorithm. Confidence intervals for all parameters were relatively small. Ruzzene⁹, *et al.* used the wavelet transform of the system's free response for the estimation of the modal parameters. Trapero¹⁰, *et al.* had suggested the use of pure sinusoidal model in combination with the algebraic derivative method for parameter identification. This algebraic method is also capable of estimating more than two frequencies but its real-time implementation for actual signals could be computationally difficult. This method is generally suited for condition monitoring of mechanical moving structures. León¹¹, *et al.* proposed the use of complex continuous wavelet transformation for the fundamental frequency estimation. The method was applied for the base frequency estimation of audio signals in single channel. Maximum likelihood estimator for real valued signals was proposed by Nielsen¹², *et al.* as a viable alternative to the autocorrelation-based methods for audio signal processing. Different methods had already been studied and subsequent recommendations for use in different applications have been published in scholarly journals^{13,14}. Many of these methods are generally applicable for off-line analysis of the vibration signals.

Hence, the objective of on-board identification of the dominant frequency of flight vehicle vibration signal in frequency domain was simulated with mathematical models

using three methods :

- (i) Fast Fourier Transformation
- (ii) Welch Method of Periodogram
- (iii) Correlogram using Multiple Signal Classification (MUSIC) algorithm.

In the periodogram, simple FFT calculations are carried out, while the method suggested by Peter D Welch, is an average of periodograms across time with rectangular window and formed from non-overlapping successive blocks of data. The MUSIC algorithm is a super-resolution approach. Initially, known sinusoids of frequencies 5 Hz, 50 Hz, 100 Hz, 500 Hz, 1000Hz, 1500 Hz and 2000Hz with amplitude of 1 V are generated. The summated signal is then added with random Gaussian white noise taken as the reference input for simulation. The peaking frequencies are identified for the known input by three methods and the corresponding error with the reference signal frequencies is analysed for correctness. The same known sinusoidal inputs were tested in hardware-in-loop-simulation (HILS) setup using data acquisition platform (NI's LabVIEW). The data flow diagram¹⁵ for HILS is as shown in Fig. 1 and the corresponding observed frequencies are as shown in Table 1.

It may be noted that the amplitude of the signal is selected as 1 volt in the simulation setup. This value is not varied while carrying out simulations as the work involves determining the frequencies.

It is observed from the above simulations that the Welch method provides better approximation to the expected frequency peaks than the other two methods. The processing time for the MUSIC algorithm is more and this method is used for time-frequency analysis of the signals. MUSIC algorithm involves the computational intensive matrix decomposition operations and is not suitable for real-time applications. As the periodogram doesn't involve windowing and averaging

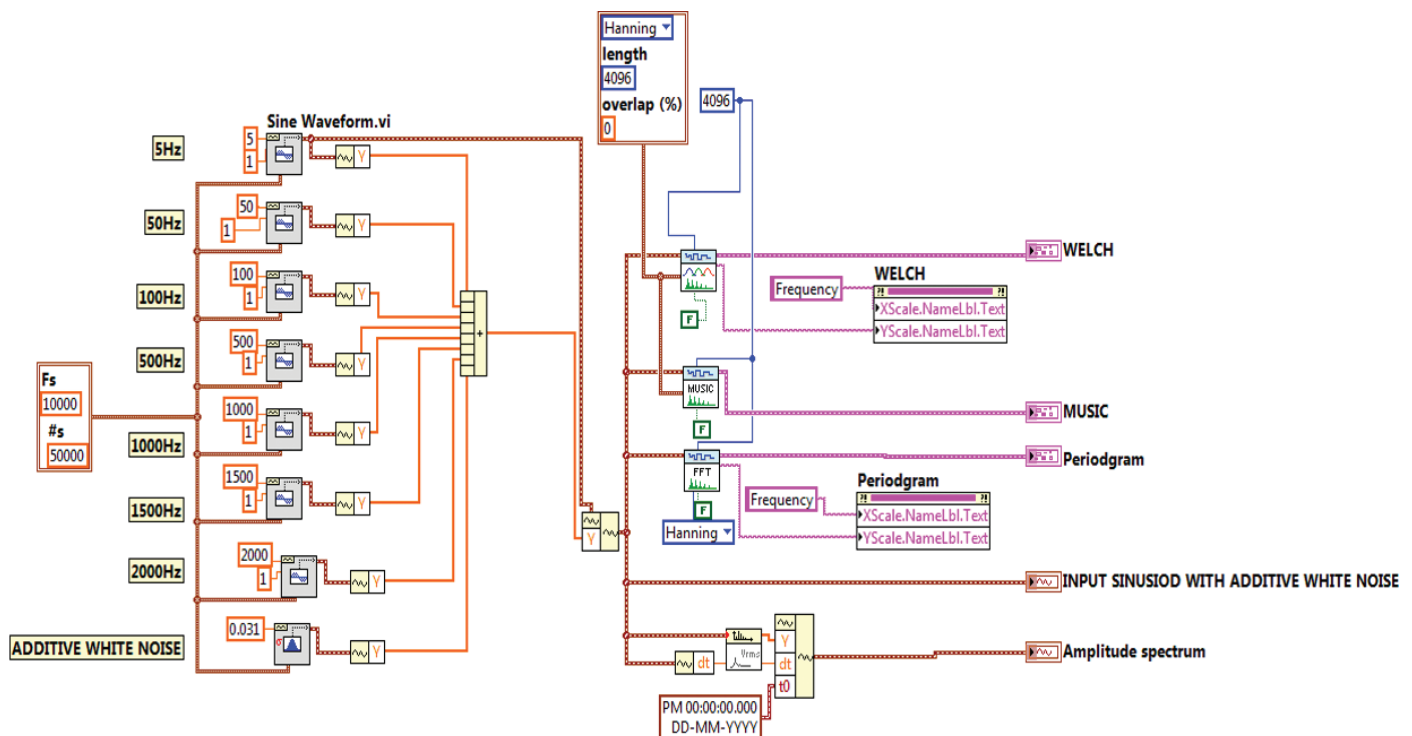


Figure 1. Data flow diagram for hardware-in-loop-simulation.

Table 1. Comparative frequency estimates – HILS (All units are in Hz)

Input Spectrum	5	50	100	500	1000	1500	2000
Fourier Transformation	4.871	48.5	100.176	500.488	1000.98	1501.49	1999.53
Error (Input - Fourier Transform)	0.129	1.5	-0.176	-0.488	-0.98	-1.49	0.47
Welch Method	5.06	49.627	100.092	499.767	1000.93	1499.07	1999.67
Error (Input - Welch)	-0.06	0.373	-0.092	0.233	-0.93	0.93	0.33
MUSIC Algorithm	4.913	48.837	100.093	500.051	1000	1499.07	1999.67
Error (Input - MUSIC)	0.087	1.163	-0.093	-0.051	0	0.93	0.33

method, the error in the estimation is more. And in HILS platform, the statistical data are compared and thus it was decided to proceed with the Welch method for frequency domain analysis of random vibration sensor signal to calculate the PSD and identify the peaking frequencies.

4. RANDOM VIBRATION TEST SETUP

The test section is subjected to random vibration for a level of 0.02 g²/Hz i.e. 5.35 g_{rms} for a duration of 200 s in the longitudinal direction as per the input vibration signal profile. The random vibration test setup is as shown in Fig. 2(a) and the input spectrum for random vibration is shown in Fig. 2(b) as per standard guidelines in MIL-STD-810G.

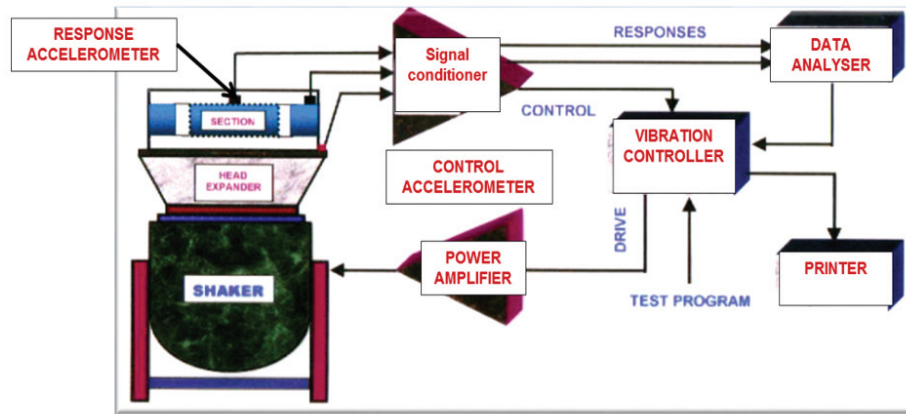
ICP Accelerometer 353B14 with the following specifications is used to measure the vibration response from the test article.

- Make : PCB Piezotronics, NY
- Sensitivity : 5 mV/g
- Measurement Range : +/- 1000g peak
- Frequency Range : 1-10000 Hz
- Resonant Frequency : > 70 kHz

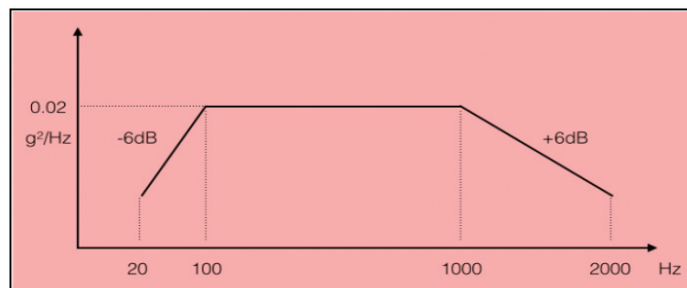
- Broadband Resolution : 0.01 g_{rms}
- Nonlinearity : 1 %
- Spectral Noise : 1Hz - 6400 μg/√Hz,
10 Hz - 1400 μg/√Hz
100 Hz - 360 μg/√Hz
1 KHz - 128 μg/√Hz

Sensing Element : Quartz with Shear Sensing geometry
The reference controller with data analyser (DEWE-4010) has a throughput of 45 Mb/s and maximum sampling frequency of 10 KHz. The data analyser uses 1024 point analysis with exponential type averaging of 64 length. The window and the overlap are set in AUTO as a standard practice. The number of points and averaging length are so decided that the output is of 6-sigma accuracy. The data analyser results are considered as reference input for all structural designs. The raw signal data from one vibration sensor digitised in the controller is taken as the reference input for processing through different algorithms. The processed PSD output from the data analyser is used as the benchmark for comparison among the outputs of the algorithms used.

Of the multiple sensors mounted on the test object, one



(a)



(b)

Figure 2. (a) Test set-up, (b) Input spectrum.

sensor is selected in such a way that the peaking frequencies are distributed across the range of interest, as shown in Fig. 3. From the PSD computation of the sensor response, it is observed that there are two dominant frequencies where the structure is getting excited, these being at $f_1 = 293$ Hz with an amplitude of $A_1 = 0.8882$ g^2/Hz and $f_2 = 839.8$ Hz, $A_2 = 0.7978$ g^2/Hz . The RMS value of the response is at $9.883g_{rms}$ indicating an overall amplification factor of 1.85 induced due to mechanical interface and structural integrity of the test article.

5. RESULTS AND ANALYSIS

Based on the previous simulation results of the known sinusoidal signals, application requirements, necessity and adequacy of the methodology, Welch method with 4096 point analysis is chosen for further study and simulation. Three windowing methods viz. Hann, Hamm and Blackman-Harris are applied with 0%, 25% and 50% overlap over the sensor response data in frequency domain analysis. The study and simulations are carried out both in Mathematical Model and Hardware- in-Loop-Simulation setup. The raw data of the sensor response signal from the output of the ADC of the Data Analyser is retrieved and used as input array for processing selected algorithms with different windowing techniques and data point overlapping. The discrete data are first simulated in the mathematical model of Welch algorithm to verify the performance and matching of the PSD profile with normalised amplitudes. Confirming the response profile in all the three methods of windowing in Welch method, the same simulation algorithm is used in HILS platform. The overall sensor output spectrum of the PSD plot is similar in all three windows and three overlapping methods and comparable with the reference controller PSD plots. However, on close examination in the

frequency range of interest, variations are observed in the peaking values.

Firstly, the Welch method with Hann Window and three overlaps of 0%, 25% and 50% are processed in HILS setup. The peaking frequency zones at low frequency and high frequency ranges are zoomed out and shown in Fig. 4 and Fig. 5. It is observed that the overall spectrum and RMS value remains the same in all the three overlapping procedures. But the number of dominant frequency peaks are varying with difference overlaps. As the energy is distributed at nearby frequencies, the amplitudes are marginally reduced. The number of peaks observed in both the frequency range of interest, their values in Hz and the average frequency are as shown in Table 2.

Similarly, the sensor output signal is processed through the Hamming Window with 0%, 25% and 50% overlapping. The identified frequencies are tabulated in Table 3.

Thirdly, to probe further, Blackman-Harris window is configured with three overlaps of 0%, 25% and 50%. The sensor response is processed with the configured windows and overlaps. Table 4 summarises the frequency peaks identified with Blackman-Harris window.

The frequency peaks identified which are closer to the reference values estimated through the DAQ controller are highlighted in their respective tables.

6. OBSERVATIONS

With reference to the identified peaks at 293 Hz and 839.8 Hz, it is observed from Tables 3 - 5 that the 25% overlapping for windowing provides a closer estimation in two cases viz. Hann Window and Blackman-Harris Window and in case of Hamming window, the values are varying widely. Also, Hann Window and Blackman-Harris Window techniques are more

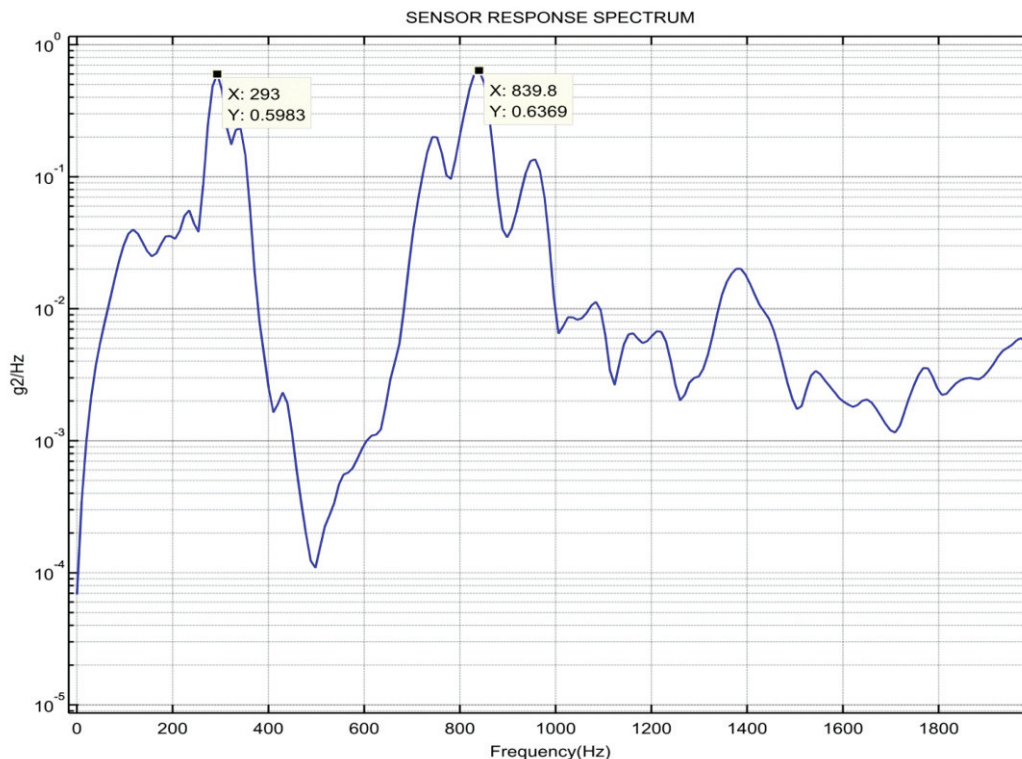


Figure 3. Sensor response spectrum.

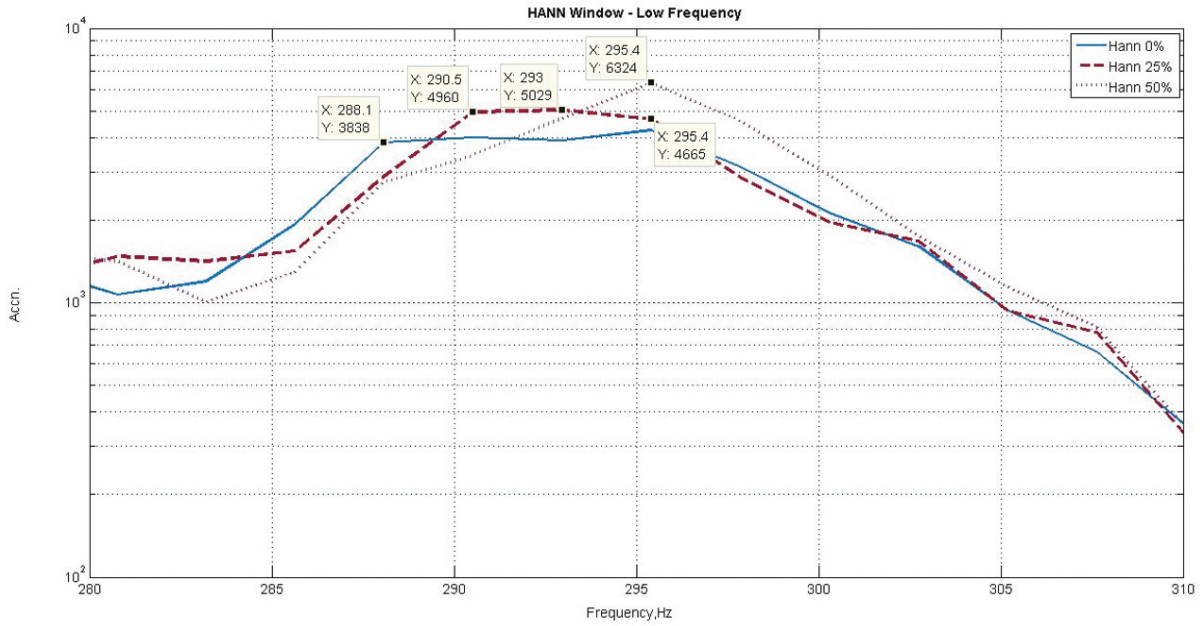


Figure 4. Hann window zoomed out plot – low frequency range.

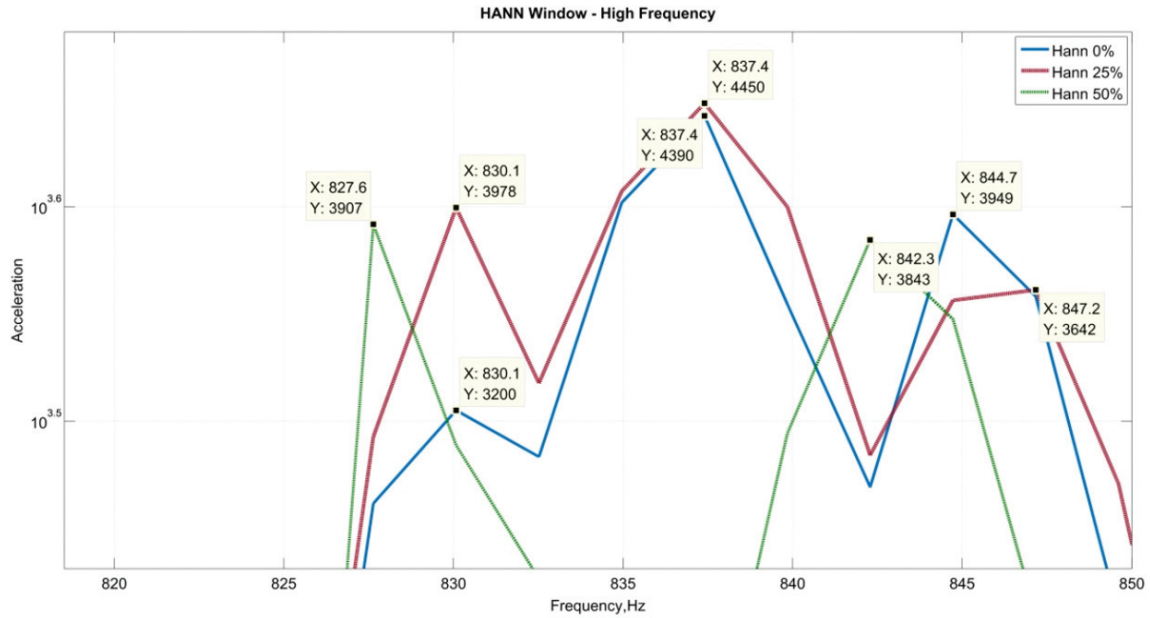


Figure 5. Hann window zoomed out plot – high frequency range.

Table 2. Hann window – identified frequencies

Overlap	Lower frequency			higher Frequency		
	No. of peaks	Frequency (Hz)	Average frequency (Hz)	No. of peaks	Frequency (Hz)	Average frequency (Hz)
0%	2	288.1	291.75	3	830.1	837.4
		295.4			844.7	
		290.5			830.1	
25%	2	295.4	292.95	3	837.4	839.85
		295.4			847.2	
50%	1	295.4	295.4	2	827.6	834.95
		295.4			842.3	

Table 3. Hamming window – identified frequencies

Overlap	Lower frequency			Higher frequency		
	No. of peaks	Frequency (Hz)	Average frequency (Hz)	No. of peaks	Frequency (Hz)	Average frequency (Hz)
0%	1	295.4	295.4	3	827.6	838.2
		842.3			844.7	
		830.1				
25%	3	288.1	291.33	3	837.4	837.4
		290.5			844.7	
		295.4			830.1	
		290.5				
50%	2	295.4	292.95	3	835.0	836.6
		844.7				
		844.7				

Table 4. Blackman-Harris window – identified frequencies

Overlap	Lower frequency			Higher frequency		
	No. of peaks	Frequency (Hz)	Average frequency (Hz)	No. of peaks	Frequency (Hz)	Average frequency (Hz)
0%	1	295.4	295.4	2	827.6	834.95
		842.3			830.1	
		290.5				
25%	3	293	292.96	3	837.4	838.23
		295.4			847.2	
		288.1			830.1	
		290.5				
50%	3	290.5	291.33	3	837.4	837.4
		844.7				
		295.4				

precise than the Hamming Window in the test setup. As is well known, the more the number of points, the smaller ΔF , resulting in more frequency points for analysis. In the analysis with the sampling frequency of 10000 samples/s in data analyser and 4096 points analysis, the frequency distribution is closer. It is also observed that with increasing overlapping, the number of peaks increased in the frequency range of interest, but the peak amplitudes are lower than the reference amplitude. The reason for the same can be attributed to the distribution of energy over nearby frequencies. But, it is seen that the overall PSD spectrum of the sensor output remained the same. Table 5 summarizes the execution time for each of the methods studied. The execution time tabulated is only for the execution of the algorithms used for study and analysis. This does not include the time required for A-to-D conversion and other pre-processing/post-processing requirements for specific application. The preprocessing was carried out in the external electronics and charge amplifiers. As the processing circuit design is common for all the methods and the time taken is the same irrespective of the algorithm used, the hardware loop-time can be considered constant in all routines.

Filters are not considered necessary as all the sensor responses are compensated signals. During the controller setting and sensor calibration, offset is defined and limited to omit the noise component introduced by the cables and environment.

Table 5. Execution time for windowing and overlapping

Overlap	Hann window	Hamming window	Blackman-Harris Window
0%	1047-1052 μ s	1047-1051 μ s	1049-1053 μ s
25%	1050-1059 μ s	1051-1055 μ s	1049-1077 μ s
50%	1090-1096 μ s	1092-1107 μ s	1089-1096 μ s

Statistical bounds are limited to the closeness to the peaking frequency points and the corresponding amplitudes with the reference PSD plot.

7. CONCLUSION

From the analysis of the multiple sinusoids with additive noise in the HILS setup, it is observed that the Welch method provides much closer and/or accurate frequency values compared to the other two methods viz. Periodogram and Music. Also, the execution time and the resource requirement for the MUSIC algorithm is much higher compared to the Welch method, as MUSIC is generally employed for Time-Frequency analysis of the signals and is well documented. Windowing technique is used as a standard practice to avoid rounding off of the signal and the associated side-lobes. Subsequently, from the analysis of the random vibration sensor response

signal in hardware environment with different windowing and overlapping techniques, it is observed that the algorithm for Welch method with Hann and Blackman-Harris window provided much better frequency identification. Also, there is not much difference between the execution time taken for 0% and 25% overlapping. But, the execution time difference is far more when compared between 25% and 50%. With 50% overlap, the peak frequency identified is as accurate as 25% when compared with reference estimates. Exact Accuracy, Trade-off requirements and compromise if any, can be clearly identified based on the application, boundary conditions of the requirements and the dedicated architecture.

The above study and analysis are to be further extended to design architecture for on-board analysis of vibration signals and PSD computation. The execution time of the process is a key factor in finalizing the architecture for on-board applications as it involves pre- and post- processing of the sensor response signal and also in fine tuning the control coefficients of the autopilot algorithm. The knowledge of the structural mode shape frequency is critical in designing the filter for the autopilot control loop. It is important that the mode shape frequency does not enter the control loop. The vehicle becomes unstable and non-controllable in case structure-control interaction occurs. The bandwidth of the controllers and the sensors are selected such a way that the interaction is minimum, and the structural noises are filtered out from the auto-pilot control loop. The major points to consider and finalise are the error in the identified frequency to increase confidence level, the update rate suitable for hardware implementation for on-board application, total loop time in arriving at the auto-pilot coefficients and the fault-tolerant modes.

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Dr (Mrs) Lakshmi Bopanna, received her MTech (Electronics and Instrumentation Engineering) from the National Institute of Technology-Warangal, and PhD from the Indian Institute of Technology-Kharagpur. She is currently working as an Associate Professor at the National Institute of Technology-Warangal, India. Her areas of interests include digital system design, Microprocessor systems, FPGA Design, VLSI Architecture and IoT.

In the current study, she has extended supervision to the main author for the analysis, outline of the research work, analysis of the results, literature survey and final scrutiny of the paper.