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In the red shadow of the Earth

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Abstract

A technique is described for calculating the brightness of the atmosphere of the Earth that shines into the Earth's umbra during a total lunar eclipse making the Moon red. This 'Rim of Fire' is due to refracted un scattered light from all the sunrises and sunsets rimming the Earth. In this article, a photograph of the totally eclipsed Moon was compared with the Full Moon and the difference in brightness calculated taking into account the exposure time and ISO setting. The results show that the Full Moon is over 14 000 times brighter than the totally eclipsed Moon. The relative brightness of the eclipsed Moon can be used to estimate that the luminance of Rim of Fire is over 12 trillion watts. The experiment described in this paper would be suitable as a high school or university exercise.

Introduction

Total lunar eclipses are arguably the second most spectacular celestial phenomenon after total solar eclipses. Total lunar eclipses provide information about the Moon and the Earth. Images taken of the Moon during a total eclipse can be used to reveal information about the lunar atmosphere [1], the Earth's atmosphere [2], the lunar soil [3] and volcanic ash in the Earth's atmosphere [4,5].

Total lunar eclipses appear red and for this reasons are often called *blood moons*. An intriguing question is why the Moon has a red tint during a total eclipse and does not disappear in the shadow of the Earth. The reason is that when the Earth blocks the Sun, light is refracted by the atmosphere into the umbra (figure 1). This light is red since most of the blue light has been scattered, which is why the sky is blue.

The red shadow is always there but only becomes apparent during a total lunar eclipse. A technique for simulating the red sunset effect has been described in a previous publication [6]. Here, we describe a method for comparing the brightness of the eclipsed and Full Moons that can be used to calculated the brightness of the atmosphere that shines into the shadow of the Earth. Seen from within the shadow of the Earth the atmosphere would appear as a thin ring of orange-red light. In this paper, this ring of light has been called the 'Rim of Fire' to distinguish it from the 'Ring of Fire', which is the name given to the line of volcanos and fault lines around the edge of the Pacific.

The method would be suitable as a high school experiment and can be repeated every time a lunar eclipse is visible (or the images provided in supplementary materials can be used). At QUT, the students who witnessed the 15 April 2014 eclipse used the images to calculate the brightness of the Rim of Fire and submit a lab report. The 15 April and 8 Oct 2014 eclipse images are now used for an assignment that students do in their own time. Lunar and solar eclipses occur with about the same frequency, but lunar eclipses are observed much more often since they can be seen from the entire night side of the Earth at the same time (weather permitting of course). The dates for forthcoming lunar eclipses can be found from NASA eclipse web site: http://eclipse.gsfc.nasa.gov/eclipse.html .

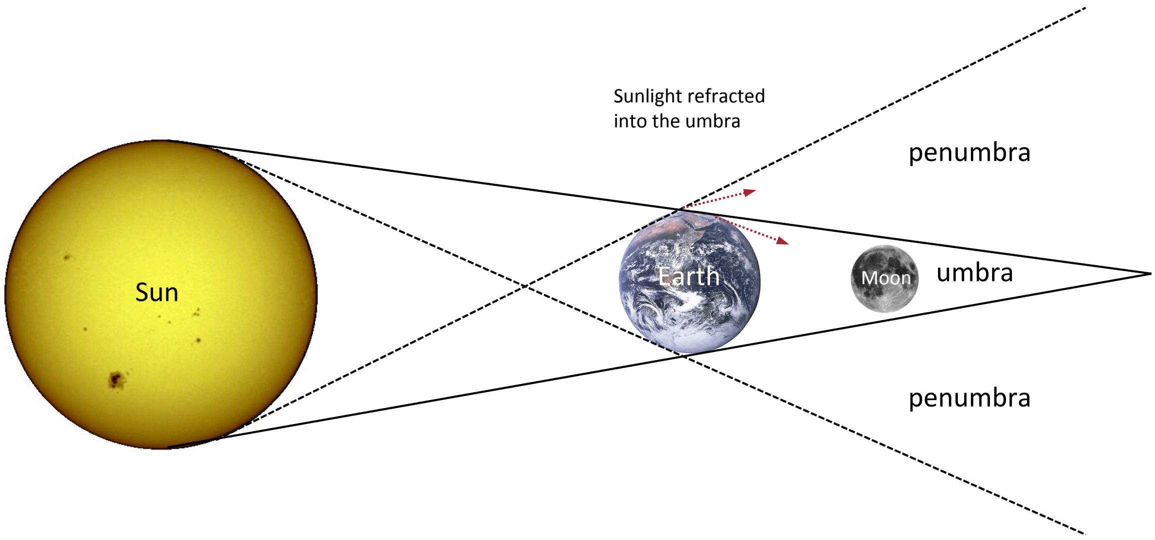

Figure 1. Schematic diagram of the shadow of the Earth. Not to scale. Unscattered light is refracted into the Earth's umbra. This scattering pattern is repeated around the circumference of the Earth. (Public domain images of the Sun, Earth and Moon were used to construct this diagram).

A reproduction of a painting by French astronomer and artist Lucien Rudaux (1874-1947) can be found here: http://en.wikipedia.org/wiki/Lucien_Rudaux#/media/File:Eclipse_from_moon.jpg

The image was painted either in the 1920's or 1930's, well before the age of space exploration. Rudaux's painting of the Moon has been compared favourably with Apollo images taken from the surface of the Moon (as seen in figure 6, for example) in that it depicts mountains and rocks with rounded surfaces from aeons of micrometeoroid weathering. Most artists' impressions of the lunar surface painted prior to the Moon landings depict sharp, angular features.

Photographing the Moon

Images of the Moon were obtained during the total lunar eclipses of 15 April and 8 October 2014 (images have been provided in the supplementary material). Images were obtained at the QUT observatory in the Brisbane CBD on 15 April using a Canon EOS 700D with a 500 mm telephoto lens at F6.3. The second set of images was photographed at the town of Seventeen Seventy about 400 km north of Brisbane on 8 October using a Sony SLT-A77V with a 135 mm telephoto lens at F5.6. These two eclipses were the first in a series of four spaced six months apart known as a 'tetrad' (15 April 2014, 8 October 2014, 4 April 2015 and 28 September 2015).

Calculating the brightness of the Rim of Fire

The images were loaded into a free image analysis program called ImageJ (http://imagej.nih.gov/ij/). Two circular regions of interest (ROI) were drawn – one fitted to the Moon and the other larger that included some sky (figure 2). (ROIs are provided in sthe upplementary material). The Measure option in ImageJ was used to obtain the integrated density and area for each ROI. The Moon pixels 'sit on' the sky pixels and therefore, for improved accuracy, this baseline should be subtracted. Proportionally, the sky pixels have a greater contribution in the eclipse images since the eclipsed Moon is much dimmer than the Full Moon. The eclipse images are also noisier owing to the higher ISO setting.

The average value of the sky pixels (*SP*) in each image was calculated by subtracting the Moon ROI integrated density (*MI*) from the sky ROI integrated density (*SI*) and then dividing by the area of the sky annulus, calculated by subtracting the area of Moon ROI (*SA*) from the sky ROI (*SA*).

$$SP = \frac{SI - MI}{SA - MA} \tag{1}$$

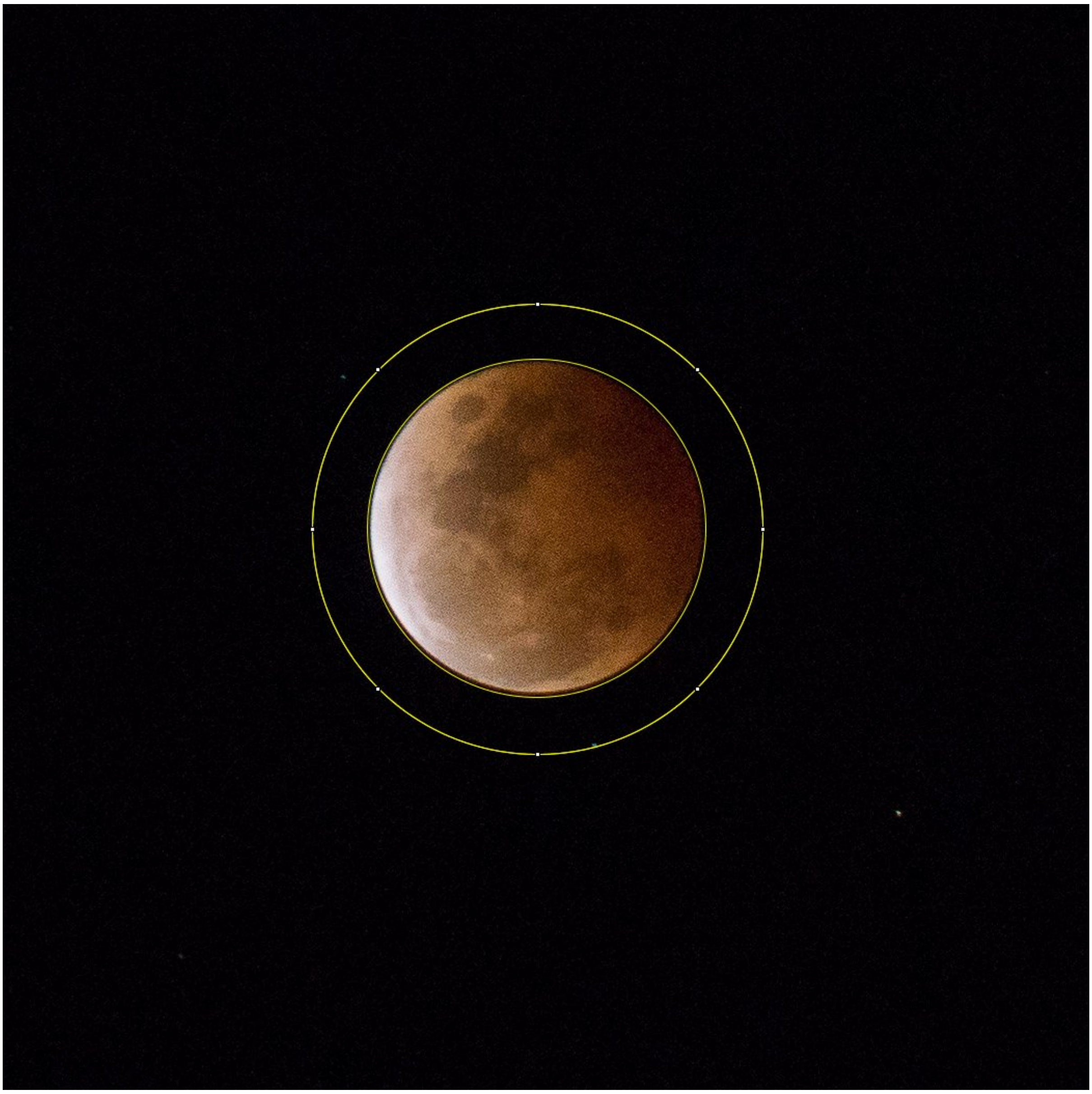

Figure 2. Eclipsed Moon on 8 October with inner and outer ROI.

The exposures need to be placed on an equal footing to allow quantitative comparison. This can be effected either by correcting the integrated density of the eclipsed Moon to the value it would have if the Full Moon exposure were applied, or *vice versa*. So, for example, the exposure for the Full Moon on 15 April 2014 was 1/331 of a second compared to1/10 s for the eclipsed Moon.

This means that if the photo of the Full Moon was taken with the longer exposure the image would be 33.1 times brighter. The ISO setting was 100 for the Full Moon and 6 400 for the eclipsed Moon. Therefore, if a photo of the Full Moon were taken with the same ISO as the eclipsed Moon it would be 64 times brighter. We need to multiply the Full Moon brightness by a brightness conversion factor (BCF) of 33.1 × 64 = 2 118. Ideally the exposure of the Moon should be such that none of the pixels reach the maximum value. The BCF can be calculated as follows:

$$BCF = \frac{T_R}{T_F} \times \frac{ISO_R}{ISO_F}$$

(2)

Where T_R is the exposure time for the red Moon, T_F the exposure time for the Full Moon, ISO_R the ISO setting for the red Moon exposure and ISO_R the ISO setting for the Full Moon exposure.

The basic results are shown in tables 1 and 2. Table 1 shows the exposure time and ISO setting for each photo and the area and integrated density for the sky and Moon ROIs. Table 2 shows the average sky pixel value for each image calculated using equation (1). The average sky pixel value was multiplied by the area of each Moon ROI and the result subtracted from the Moon integrated density to provide sky corrected integrated densities. The sky corrected integrated density of each full moon was divided by the sky corrected integrated density of the corresponding red Moon (*F/R*). This ratio was then multiplied by the brightness correction factor (BCF) for each eclipse to provide a dimming factor (DF) for each eclipse.

$$DF = (F/R) \times BCF$$

(3)

Table 1. Basic values associated with the eclipses. R = red, F = Full, *SI* = sky ROI integrated density, *SA* = area of sky ROI, *MI* = Moon ROI integrated density, *MA* = area of Moon ROI.

Date	**Moon**	**Exposure (s)**	**ISO**	**Sky ROI ID (SI)**	**Moon ROI ID (MI)**	**Sky ROI area (SA)**	**Moon ROI area (MA)**
15 Apr	R	1/331	100	81057181	56241141	1535008	885788
“	F	1/10	6400	108640517	108214806	”	“
8 Oct	R	1	100	7013019	6741502	147948	83492
“	F	1/800	800	14546651	14440140	“	“

Table 2. Intermediate values obtained in calculating the brightness of the Rim of Fire. The last column shows the dimming factor.

Date	**Moon**	**Average Sky Pixel value**	**Sky corrected ID**	**Sky corrected F/R**	**BCF**	**DF (Sky corrected F/R × BCF)**
15 Apr	R	0.65	22382433	4.81	2118	10185
“	F	38.2	107633971			
8 Oct	R	1.62	6389797	2.24	6400	14325
“	F	4.21	14302172			

On 8 October the eclipsed Moon was 14 325 times dimmer than the Full Moon. If the Sun were dimmed by this factor, the Moon would appear about as bright as it does during a total eclipse, ignoring the colour difference. The reduced solar output can then be used to calculate the flux of light falling on the Moon. The DF for 15 April is less than for 8 October since the Moon was still within the penumbra when the Full Moon photograph was taken.

The eclipsed Moon is red throughout totality indicating that the light from the Rim of Fire is distributed across the umbra. The eclipsed Moon is not uniformly lit, but the integrated density can be taken as a fair sample of the umbra. The area of the umbra (A_u) multiplied by the dimmed solar flux at the position of Moon during the eclipse will give the brightness of the Rim of Fire (b_r):

$$b_r = \frac{b_S \times A_u}{DF}$$

(4)

The last step is to calculate the area of the umbra (A_u). During a total lunar eclipse the centres of Sun, Earth and Moon are in line and are said to be in syzygy.

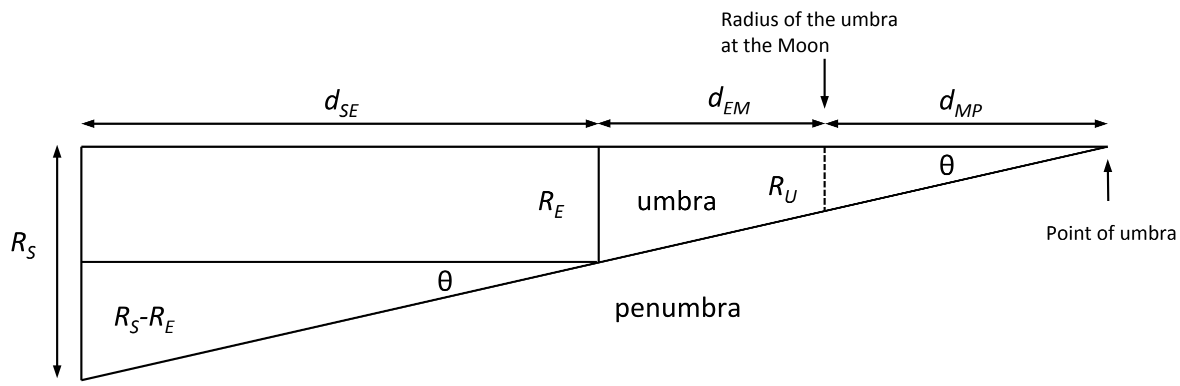

Figure 3. Schematic diagram of the geometry required to calculated the area of the umbra at the position of the Moon.

In figure 3, R_S is the solar radius, R_E the Earth's radius, R_U the radius of the umbra at the position of the Moon, d_{SE} the Earth-Sun distance, d_{EM} the Earth-Moon distance and d_{UP} the distance between the Moon and point of the umbra, and d_{MP} the distance between the Moon and the point of the umbra. The radius of the umbra at the position of the Moon can be calculated in three steps.

$$\theta = tan^{-1}\left(\frac{R_S - R_E}{d_{SE}}\right)$$

(5)

$$d_{MP} = \frac{R_E}{\tan\theta} - d_{EM}$$

(6)

(R_E/θ is also the total length of the umbra)

$$R_U = \tan\theta \, d_{MP}$$

(7)

Using standard values for the required quantities ($R_s = 6.958 \times 10^8$ m, $R_E = 6.371 \times 10^6$ m, $d_{SE} = 1.496 \times 10^{11}$ m, $d_{EM} = 3.844 \times 10^8$ m) gives an umbra length of 1.38 million km and an umbra radius at the position of the Moon of 4.6×10^6 m. So, the umbra is 2.65 times the diameter of the Moon, and 7.0 times the area of the Moon. The area of the umbra is 1.33×10^{14} m^2. The brightness of Sun shining on the Full Moon (b_s) can be calculated from:

$$b_s = \frac{L}{4\pi R^2}$$

(8)

where L is the solar luminance (3.846×10^{26} W), and R the distance between the Sun and Moon, which can be calculated by adding the Earth-Sun and Earth-Moon distance at the time of the eclipse.

A planetarium program such as Stellarium (www.stellarium.org) can be used to find the Earth-Moon and Earth-Sun distance during an eclipse. On 15 April 2014 the Earth–Moon distance was 383 900 km and Earth-Sun distance 150 078 000 km making a total of 1.505×10^{11} km. On 8 October 2014 the Earth-Moon distance was 361 450 km and Earth–Sun distance 149 078 475 km making a total of 1.495×10^{11} km. In equation (8) these values give a $b_s = 1.363$ kW m^{-2} at the Moon at the time of the eclipse. Using the value for A_U in equation (4) gives

$$b_r = \frac{b_S \times A_u}{DF} = \frac{1.36\times10^3\times1.3\times10^{14}}{14325} = 1.23\times10^{13}\ \text{W}$$

The brightness of the Rim of Fire is about 12.3 trillion watts.

Discussion

It is interesting to compare this value with the brightest light in the world, which apparently is the Luxor Sky Beam in Las Vegas, comprising 39 xenon lamps each with a power of 7 kW, making a total power of 273 kW. Assuming that these lamps are 100% efficient, which of course they are not, the Rim of Fire is equivalent to at least 46 million Sky Beams.

Another question of interest is how large the Rim of Fire would look in the lunar sky during a lunar eclipse. Figure 4 shows a composite photo of the Earth placed next to the rising Full Moon. The Earth has been scaled to show how large it would look on the surface of the Moon. Another way of imagining how big the Earth would look from the Moon is to note that the Moon would just fit into Australia, since the diameter of the Moon is 3 475 km and the Great Circle distance between the most westerly and easterly points in Australia (Steep Point, WA, and Cape Byron, NSW, respectively) is 3 994 km.

Figure 4. Composite image of the scaled Earth against the Full Moon. The colour of the Moon has been adjusted to give an idea how earthrise would look if there were a second Earth at the same position as the Moon. This image gives an idea of the size of the Rim of Fire seen from the surface of the Moon. (The base image was taken by author SWH and the image of the Earth was taken from Apollo 17 and is courtesy of NASA).

An attempt was made to recreate Lucien Rudaux's paining using 'natural ingredients'. The colour of the Rim of Fire was estimated using an image of a sunset taken from the International Space Station (ISS) shown in figure 5.

Figure 5. Photograph taken of a sunset from the ISS. Image courtesy of NASA Johnson Space Center: Earth Science and Remote Sensing Unit, NASA-Johnson Space Center. "The Gateway to Astronaut Photography of Earth."
http://eol.jsc.nasa.gov/scripts/sseop/photo.pl?mission=ISS001&roll=421&frame=24. The orange band is the troposphere and the upper blue band the stratosphere.

The official a start of space is the Karman line, 100 km above the surface of the Earth. Spacecraft re-entering the atmosphere begin to feel the effects of the atmosphere at an altitude of 120 km. Therefore we will assume that the band of colour between the ground and space in figure 6 is 100 km thick. This is 1.6% of the Earth's radius. A circle was constructed with an orange line equal to the average colour of the troposphere and a blue line equal to the average colour of the stratosphere in figure 5. The width of the line was approximately equal to 1.6% of the radius of the circle. This is shown in figure 6.

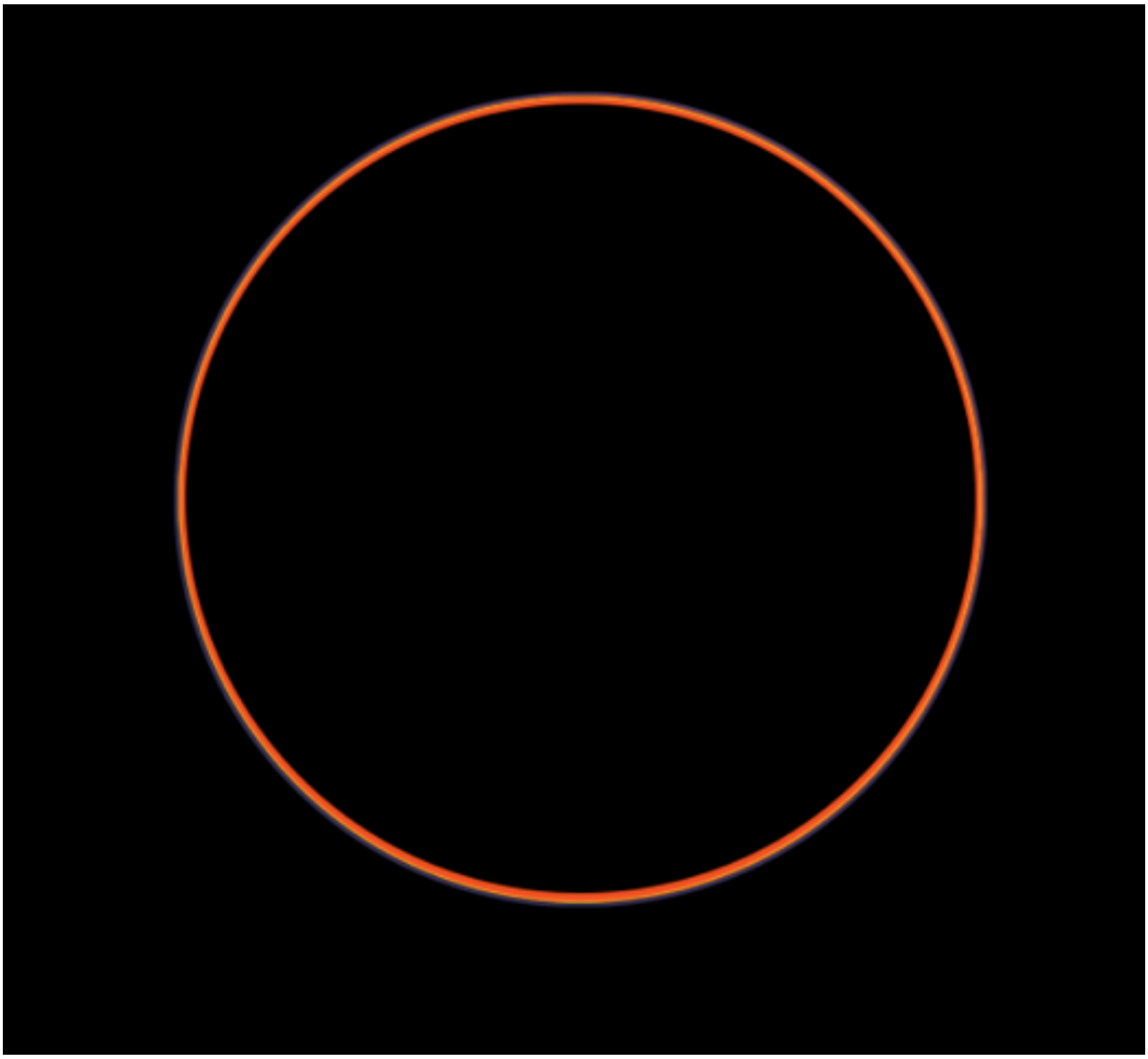

Figure 6. Simulation of the Rim of Fire using the image of the sunset image taken from the ISS shown in figure 5. The thickness of the ring is assumed to be 100 km.

The simulated Rim of Fire shown in figure 6 was inserted into the lunar sky above a high-resolution photograph of the lunar surface, taken Eugene Cernan commander of the Apollo 17 crew (figure 7). Under artistic license, the Milky Way star field was added as a backdrop. The lunar landscape and astronaut were re coloured with the coppery-red colour of the Moon during a total eclipse. The result is shown in figure 8.

Figure 7. Original photograph taken by Eugene Cernan, commander of Apollo 17, of geologist Harrison Schmidt with his rock scoop. The photo has been abstracted from a panoramic image: http://www.hq.nasa.gov/alsj/a17/a17.1464906_gpan.jpg. Image courtesy of NASA.

An interesting question is whether the thickness of the Rim of Fire would be resolvable with the naked eye from the Moon. The angular resolution (θ) of an optical device with a circular aperture is given by the expression

$$\theta = 1.22\frac{\lambda}{D} \tag{9}$$

where λ is the wavelength of the radiation being imaged and D is the diameter of the optical device. The angle in radians subtended by an object of size s at a distance r is s/r. Therefore, the maximum size of an object that can be resolved on the Moon from the Earth, or *vice versa*, is given by the expression

$$s = 1.22\frac{\lambda r}{D} \tag{10}$$

If the eye is our optical instrument, and we assume a pupil diameter of 7 mm, a wavelength of 550 nm and an Earth-Moon distance of 384 000 km the maximum resolvable size is 36 km.

This is less than the 100 km thickness of the atmosphere and therefore the Rim of Fire would be perceived to have a thickness.

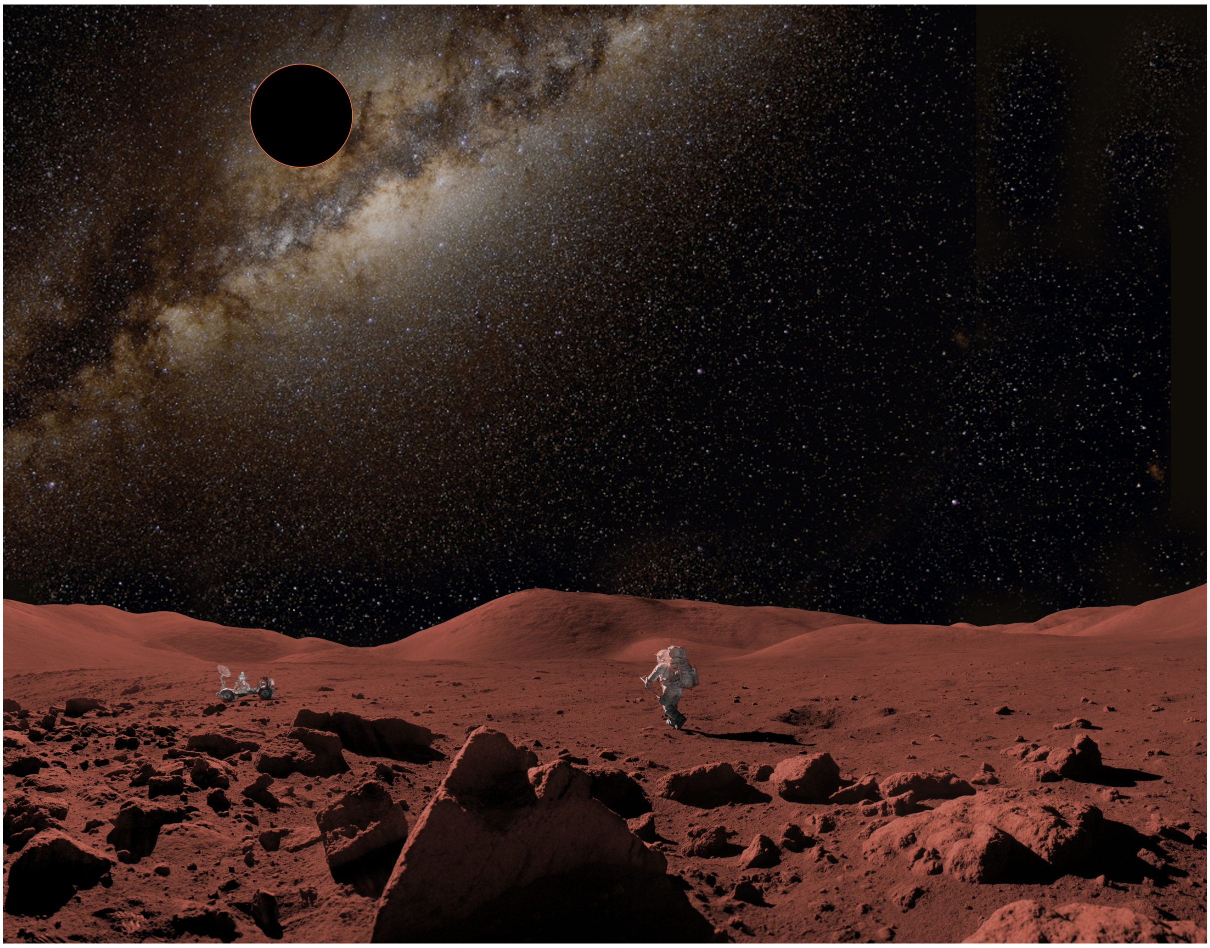

Figure 8. Composite photo giving an impression of the Rim of Fire seen from the surface of the Moon during a total lunar eclipse. The colour of the Moon has been adjusted to be a similar colour to the eclipsed Moon. (Image assembled by author DS)

An interesting class exercise would be to use a length of orange electroluminescent (EL) wire to simulate the Rim of Fire. If the EL wire is 2 mm in diameter, and formed into a circle with a radius of 12.5 cm, the wire will be in the same proportion as the Rim of Fire. When viewed from 7.5 m the simulated Rim of Fire will appear as it would on the Moon.

Figure 9 shows a photo of an EL wire Rim of Fire suspended from a hanging pot plant outside the kitchen of author S.H. A length of fencing wire was formed into a circle 25 cm in diameter to serve as scaffold for the EL wire, which was attached using Scotch tape. This could be placed on the wall of a darkened classroom to give students some idea of what it would be like to look at the Earth during a total lunar eclipse on the surface of the Moon. The EL wire and inverter were obtained from partyroo, in Sydney, Australia (www.partyroo.com) and cost about $10 in total (with an extra $10 for postage).

The exercise in this paper would be suitable as a high school or university project and the level of complexity could be adjusted to suit the academic level of the students. The exercise could be used to compliment lessons on eclipses and phases of the Moon, which are concepts often difficult to grasp [7,8].

Figure 9. A simulated Rim of Fire made from a 1 m electroluminescent (EL) wire 2 mm in thickness formed into a circle 25 cm in diameter. The inverter, and part of the EL wire are hidden amongst the leaves of the plant in the hanging basket.

Acknowledgements

David Sawell generated figures five and eight. Kazuyuki Hosokawa and Joshua Carroll acquired photos of the 15 April eclipse and Colin Wilson photo of the 8 October eclipse.

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