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Digital Simulations for Grade 7 to 10 Mathematics

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This article describes a Department of Science and Technology – Philippine Council for Industry, Energy and Emerging Technology (DOST-PCIEERD) project aimed to facilitate the implementation of the mathematical objectives raised by the Department of Education's (DepEd) K to 12 program in the Philippines through the use of innovative digital technologies. In particular, a selection of application software ("apps") were created for Grade 7 to 10 mathematics that covered topics indicated in the five strands outlined in the K to 12 program – namely (1) number, (2) geometry, (3) measurement, (4) patterns and algebra, and (5) statistics and probability. The design of the apps was informed by an amalgamated framework of the Cognitive Theory of Multimedia Learning (Mayer 2005) and Mathematical Theories of Representation (Goldin 1998). The design was informed by how students learn and how students learn mathematics. The project also aimed to design manipulable software that allows learners to construct and grapple with their mental representations of mathematical concepts. This paper describes a selection of the apps designed by the project and how their features were informed by the theoretical framework. It also presents results from pilot studies that demonstrate the apps' potential to increase performance, facilitate conceptual development, and increase learners' engagement.

Keywords: application, K to 12 program, mathematical software, mobile technology, technology in mathematics education

INTRODUCTION

Technology has become an important – if not a necessary – part of the mathematics classroom not only because it enhances learning, but also because the ability to use technological tools is a necessary skill in today's world. The Philippine K to 12 mathematics curriculum framework recognizes that "the use of appropriate tools is needed in teaching mathematics. These include manipulative objects, measuring devices, calculators, computers, smartphones and tablet PCs (personal computers), and the internet" (DepEd 2012). Various computing and graphing devices make lessons more interesting and make exploration and mathematical discovery possible (NCTM 2000). However, the extent and the way technology is used in the teaching of mathematics depend not only on the availability of resources (software, technological tools, infrastructure (Internet), and computer laboratories) but also on the teacher's ability and disposition to use these devices. The primary objective of this project, which was funded by DOST-PCIEERD for the period 2015–2018, was to augment available technological tools for Grade 7 to 10

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mathematics by creating interactive apps. Despite a large number of existing apps in the market, only a very small portion provides opportunities for the user to manipulate objects, form conjectures, and refine ideas (Goodwin and Highfield 2013). Instead, most apps are instructive, founded on "drill-and-practice" pedagogic design.

This paper discusses the design and development process of the apps created under the research project. The scientific framework guiding this process is also presented. The designed apps include manipulable components and visual feedback to guide students towards mathematical learning. The potential effect of adopting these technologies on students' learning outcomes is also discussed. It is hoped that this work will be helpful to researchers who want to develop their own apps using the scientific framework highlighted in this paper.

The apps have the features of mobile technology (MT) and are easily accessible via Android devices, but it is also possible to use these on computers and laptops using a free Android emulator such as Genymotion (https://www.genymotion.com/). To enhance teachers' competency, the software is supported by a set of teaching guides. These teaching guides are linked to the learning competencies prescribed by the current DepEd mathematics curriculum (DepEd 2012). For greater accessibility, the apps will only require an internet connection during installation but not during use. The mobile apps and teaching guides are freely available for download from the MathPlus Resources website (https://mathplusresources.wordpress.com/). A total of 14 Android apps and 41 *GeoGebra* applets were developed under the project.

The developed resources and the project website were launched in a teacher training workshop-seminar attended by Grade 7 to 10 teachers from schools in Metro Manila. Throughout the app development phase, the mobile apps were pilot-tested with public and private school students to gather input for refining the apps. Two mobile apps, namely *ProveIt* and *AlgeOps*, were tested in more depth to assess their impact on students' understanding of specific mathematical concepts. The results of the teachers' evaluation of mobile apps and the testing of *ProveIt* and *AlgeOps* are discussed later in this paper.

Review of Related Literature

Research on the use of technology in the classroom has provided evidence that it is a tool for capacity building of students across levels and disciplines, including mathematics. It improves educational quality by affording more opportunities to develop higher-order thinking skills (Fu 2013). This is further enhanced by the development of dynamic geometry and computer algebra systems. Various studies have explored how to enhance students' proving skills in a range of areas such as geometry, abstract algebra, and calculus using dynamic geometry (de las Peñas and Bautista 2008, 2011). Further, these dynamic tools have also been used to facilitate the understanding of fundamental ideas of calculus (Verzosa *et al.* 2014), to introduce algebra concepts such as the concept of variable and symbolic representations between variables (Mackrell 2012), to facilitate visualization of mathematical concepts (Taka *et al.* 2010), and to develop students creativity (Shelomovskiy and Nosulya 2012).

In recent years we have seen the rise in the use of MT (*e.g.*, smartphones, tablets) in mathematical learning. Accompanying the rising popularity of MT is the growth in the number of mobile apps in mathematics (Larkin 2015) and studies have been carried out to measure their impact on mathematics education (Calder *et al.* 2018, Fabian *et al.* 2018, Larkin and Calder 2016). MT also has the potential to transform the learning experience and enhance mathematics learning opportunities. It is portable, easy to boot up, and can easily fit in one's hand. Mobile apps are more easily accessible and popular among students. Students may thus tend to carry and use them constantly (Norris *et al.* 2011). Among the potential benefits of mobile learning are encouraging learning anytime, anywhere; enabling a personalized learning experience (Shuler 2009); and inducing a positive change in students' attitudes and engagement (Fabian *et al.* 2018). In Latin America, MT (termed pocket school) is used as a sustainable literacy option for underserved indigenous children (Kima *et al.* 2008). MT's ease of operation, allied with the students' interaction being focused primarily on touch and sight, can make their use intuitive for learners (Calder and Campbell 2016). Research studies have also highlighted the effectiveness of MT in other aspects of mathematics learning [*e.g.*, those in Calder *et al.* (2018)].

In the Philippines, the K to 12 Curriculum of the DepEd recognizes that calculators, computers, smartphones, tablets, and PCs can be appropriate tools in teaching mathematics. These can help learners demonstrate understanding and appreciation of key concepts and principles of mathematics as applied in problem-solving, communicating, reasoning, and making connections, representations, and decisions in real life (DepEd 2012).

In the past year, different strategies have been suggested to maximize the use of existing technology in Philippine schools (Bautista and de las Peñas 2006). School administrators have invested in affordable graphics calculator kits for use of faculty and students. Free software packages, such as Wingeom and GRAPES, have provided opportunities for experimentation and discovery with topics in geometry, analytic geometry, trigonometry, and calculus (de las Peñas and Bautista 2008). *GeoGebra* – free dynamic software that has capabilities for geometry, algebra, and statistics – has been a popular choice for

Philippine schools (de las Peñas and Bautista 2011). In recent years, MT has emerged as a tool in studying and teaching mathematics. In 2012, the Science Education Institute and Advanced Science Technology Institute under the DOST launched the project entitled "Technology Package for Student Learning Empowerment: PC Tablet Math Lesson Courseware Development," which involved the design of interactive learning materials in Grade 1 Mathematics that run on tablet PCs (Lee-Chua 2012). To date, there is a courseware involving particular math topics in Grades 1 to 8 that run on Android devices (DOST Courseware may be accessed at www.dost.sei.gov.ph). Studies have also been conducted on the use of MT in high school mathematics (*e.g.*, Etcuban and Pantinople 2018).

Scientific Basis and Framework

The creation of the apps in this study was informed by amalgamating the Cognitive Theory of Multimedia Learning (Mayer 2005) with Mathematical Theories of Representation (Goldin 1998).

Mayer's (2005) Cognitive Theory of Multimedia Learning hinges on theoretical and empirical evidence that learning is maximized when the instructional medium is aligned with how the human mind works. It further argues that the instructional medium is always a reflection of the designer's understanding of how the human mind processes information. As an example in mathematics, computer games that mostly consist of drill and practice sessions reflect the designer's view of the learner from a behavioral perspective. That is, learning is often equated to observable responses to certain tasks or stimuli, with little attention to how a learner's internal representations or mental models facilitate or stifle problem-solving success.

Based on the Cognitive Theory of Multimedia Learning, instructional learning outcomes were designed, informed by how students learn, and – in particular – how students learn mathematics. Mathematical ideas are essentially communicated through representations, which are inventions and reinventions that are indicative of the cultural, semiotic, and technological advancement of a given society (Bu *et al.* 2011). Goldin (1998) proposed a unified psychological model of mathematical learning and problem solving based on a theory of representations. He states, "the overarching goal should be to foster in students the *construction of powerful, internal systems of representation*" (p. 159; italics in original). Also, according to Bu *et al.* (2011), "all mathematical ideas are dynamic in the mind of a mathematically competent person" (p. 23). In this sense, technologies may promote mathematical learning if it provides learners the opportunity to construct and manipulate representations in a way that supports interactions between the learner and the representations themselves.

Despite a large number of commercial or freely available educational software, very few of these allow learners to construct and grapple with their mental representations of mathematical concepts (Goodwin and Highfield 2013). To address this gap, we designed a manipulable software where students can construct representations, test conjectures, reason hypothetically, visualize, analyze, interpret, and discover. A second important feature is the provision of visual feedback. The provision of immediate formative feedback is integrated with a wide range of technology-based tools and game-based apps and is consistent with experiential learning theory (Kiili 2005). Instantaneous feedback allows software or app to act as a tutor (Johnston-Wilder and Pimm 2005), help students modify their conceptions (Avraamidou *et al.* 2015), and increase their motivation (Gros 2015). The feedback should not just inform students that they are correct or not. Instead, the feedback includes animations that demonstrate why a particular response is correct or incorrect.

METHODOLOGY AND DESIGN OF THE APPLICATION SOFTWARE

The team involved in this project consisted of mathematicians and mathematics educators from the Department of Mathematics, Ateneo de Manila University. The process of app development involved the steps given below. This can be compared to existing app development frameworks in the literature, such as Pugh's design process model (Wong *et al.* 2012) and the NCA Model of Design and Development (Nisiyatussani *et al.* 2018). Two types of apps were designed to enhance the teaching and learning of mathematics in Grade 7 to 10 – software apps for Android devices and applets created using the free dynamic geometry software *GeoGebra* (https://www.geogebra.org/).

1. ***Data collection and review of related literature.*** The first few months of the project included a thorough review and survey of related literature on currently available software and new technologies and various methods on the use of technology in teaching mathematics. The DepEd Grade 7 to 10 curriculum in mathematics and prescribed competencies were reviewed and studied. The team also studied the National Achievement Test results that were obtained upon request from the DepEd's National Education Testing and Research Center.
2. ***Identification of topics.*** With the materials gathered in the previous step, the team members identified the grade levels and topics in the different mathematics strands that could be taught and studied with interactive software, which had a huge potential for supporting student learning. Topics that are linked to the least learned competencies (provided by DepEd) were taken into consideration. The apps covered

topics indicated in the five strands of mathematics outlined in the K to 12 Program – namely (1) number, (2) geometry (3) measurement, (4) patterns and algebra, and (5) statistics and probability.

3. ***Conceptualization of design.*** Team members conceptualize the design on a topic or activity that will be implemented on the app. An example is shown in Figure 1 on the design of Triangle Congruence on app *ProveIt.* The teaching guide is also written at this stage.

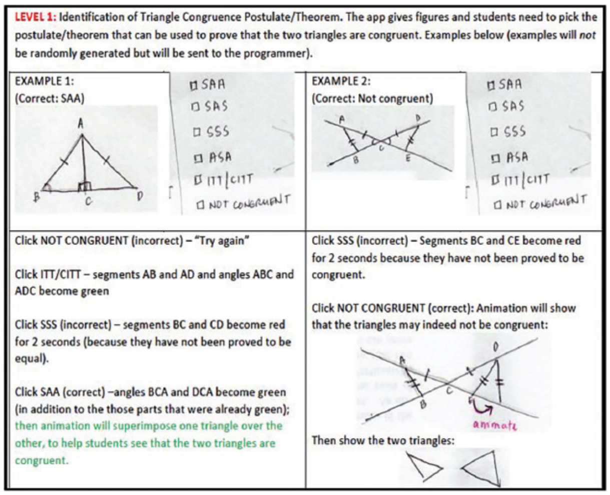

Figure 1. Conceptual design of Triangle Congruence in *ProveIt*.

4. ***Detail of the design and prototyping.*** Another team member writes the idea in the format of a computer program that will be sent to the programmer. This describes the flow of the app. See Figure 2 on suggested commands for the Multiply feature of *AlgeTiles.* A prototype will then be shown by the programmer and, after suggestions and discussions with the team, the app will be created.

a. App generates question
   i. If 1-variable, App generates (ax+b)(cx+d) as follows
      1. generate a and c from the set {-3,-1,0,1,3}
      2. generate b from the set
         a. {-9 to 9-3a} if a >=0
         b. {-9-3a to 9} if a < 0
      3. generate d from the set
         a. {-9 to 9-3c} if c >=0
         b. {-9-3c to 9} if c < 0
   ii. If 2-variables, App generates (ax+by+e)(cx+dy+f) where
      1. generate a and c from the set {-2,-1,0,1,2}
      2. generate b from the set
         a. {-2 to 2-a} if a >=0
         b. {-2-2a to 2} if a < 0
      3. generate d from the set
         a. {-2 to 2-c} if c >=0
         b. {-2-2c to 2} if c < 0
      4. generate e from the set
         a. {-8 to 8-3a-3b} if a >=0 and b >=0
         b. {-8-3b to 8-3a} if a >=0 and b < 0
         c. {-8-3a to 8-3b} if a < 0 and b >= 0
         d. {-8-3a-3b to 8} if a < 0 and b < 0

Figure 2. Suggested flow for the Multiply feature of *AlgeTiles*.

5. ***App delivery.*** The app is delivered by the programmer and presented to the author. The app undergoes revisions after comments from the author and/another team member. The teaching guide is rewritten in response to the full capabilities and limitations of the apps.
6. ***Field test.*** The apps were field-tested on teachers and students from a private and a public high school before finalization. We tested the apps on groups of students from various grade levels – both those who have experience using tablets in their lessons and those who were new to the use of technology in the classroom. The app developers and the team conceptualized the questions to be asked during the field-testing for the teachers and students to ensure that substantial comments can be gathered on the following aspects: (i) functionality and routines present in the app, (ii) ease of use of the apps, (iii) aesthetic look and layout, (iv) retention of user's interest, and (v) ability of the app to address the required learning competency. Questions ranged from the interface and responsiveness of the app to touch, ease of use, clarity of instructions, design, display and colors, and experience when using the app.
7. ***Consolidation of field test results.*** The comments from the field test and other team members are assessed by author and the programmer and incorporated into the app. Though the *GeoGebra* apps were not field-tested, comments from team member/s on *GeoGebra* apps were noted and will be taken into consideration during the refinement process.
8. ***Refinement and finalization.*** The apps undergo further refinement and are finalized. The teaching guides are also finalized. The app and teaching guide are uploaded into a website. Teacher training is conducted on the use of apps and teaching guides.

To ensure that the app design is guided by the indicated scientific framework, team members discussed developments in research on mathematical learning. The discussions included broad theoretical perspectives so that the entrenched behaviorist position, which values observable procedural skill and deemphasizes the development of a learner's internal representations, does not dominate the app design. These discussions were crucial because apps reflect the designer's view of mathematical learning (Mayer 2005). Early on, the team members had to understand that providing learners an opportunity to construct representational systems is a key goal of mathematics education (Goldin 1998). In connection, it was decided that each app needed to provide (1) manipulable representations and (2) visual feedback, as shown in Table 1. These features can facilitate exploration, visualization, and justification. In Table 1, we also describe the novelty of each of the app in accordance with these features.

Table 1. List of mobile apps with descriptions of the manipulable representation and visual feedback.

Subject	App	Competencies[a]	Manipulable representation	Visual feedback	Novelty
Algebra	*AlgeTiles*	Factors completely general trinomials Uses models and algebraic methods to find the: (a) product of two binomials; (b) product of the sum and difference of two terms; (c) square of a binomial; (d) cube of a binomial; and (e) product of a binomial and a trinomial.	Students can use virtual algebra tiles to help them in multiplying (Figure 3a) and factoring (Figure 3b) polynomials. The app simulates the filling in of quadrilaterals using these tiles. Students must fill in the quadrilaterals using the correct counts and sizes of tiles	Students use a button (chk) to check if they have done each phase of the solution correctly; at the end, the student inputs an answer and they can verify if it is correct using the chk button	Distinguishing features include a grid as scaffolding for beginning students, a robust handling of negative numbers, and instant feedback
	LinearX	Finds the solution of linear equations of inequalities in one variable	Students use the buttons so that all boxes (representing the variable) are on one side of the equation. When all boxes are on one side, students figure out how many balls are in each box (assuming each box contains the same number of balls) and enter their result on the number line provided (Figure 3c)	When students enter an incorrect answer, the app shows that the boxes would not contain the same number of balls, as initially required	The app allows students to work on diagrams that represent linear equations. Both correct and incorrect answers are simulated by the app so that students see for themselves why their answer is correct or incorrect; for example, suppose there are 3 unknowns (represented by boxes) and 18 objects that have to be divided equally among the 3 boxes: if the student enters the incorrect answer 5, the app simulates placing 5 balls in each box, and reveals that there are 3 balls that are yet to be placed inside the box
	DrawLine	Graphs a linear equation given (a) any two points; (b) the x- and y-intercepts; and (c) the slope and a point on the line Finds the equation of a line given (a) two points; (b) the slope and a point; and(c) the slope and its intercepts	Students can construct their own lines while choosing parameters such as two points, a slope and a y-intercept, or a point and a slope (Figure 3d)	The students formulate the equation of a line and feedback is given if this is correct or not	The app uses separately the three forms of the equation of a line (two-point, slope-intercept, and point-slope) and allows the student to input one representation given another; feedback is provided as well
Geometry	*Angle Xplore*	Derive relationships of adjacent angles using measurements and inductive reasoning Derive relationships of complementary and supplementary angles using measurements and inductive reasoning	Students are required not just to identify complementary or supplementary angles, but it requires them to construct their own, given a set of angles (Figure 4a)	When students choose two such angles, an animation shows whether the two angles can be used to form a 90- or 180-degree angle; when two angles are neither supplementary nor complementary, this is also displayed in the app.	The app allows the students to construct their own representations of complementary and supplementary angles so that they can deduce the definition and properties of complementary and supplementary angles
	ProveIt Triangle Congruence (Level 2)[b]	Proves two triangles are congruent	Students are shown two triangles, which they must show are congruent. The app provides students two enabling prompts: (1) the triangle congruence theorem/postulate that must be used, and (2) the theorems that are needed so that the appropriate triangle congruence theorem/postulate may be applied (Figure 4b)	If students identify the correct property/ theorem and the correct associated triangle parts, feedback is provided.; if incorrect parts are selected, the app notifies the student and gives a hint about the correct parts	The app provides an interactive visual representation of the triangle congruence statement to be proved; using the mobile device's touch screen, the user can select or highlight parts of the figure when answering the proving exercise; the app also provides immediate feedback so that in the case of an incorrect answer the student can amend his/her answer (Verzosa *et al.* 2019)
	GeoGebra Applet: Two-column proof: prove that the given quadrilateral is a parallelogram	Determines the conditions that guarantee a quadrilateral is a parallelogram	Students are asked to prove the theorem "if a quadrilateral is a parallelogram, then its opposite sides are congruent" using the *GeoGebra* app; to navigate the proof, the student clicks the right arrow button. A line of the proof appears and the student fills in the missing details in the proof (Figure 4c)	By ticking a checkbox beside a step in the proof, the students see a visualization or animation of that step	There is no known app on two-column proofs where visual feedback is given for each line of the proof
	GeoGebra Applet: Intersecting Chords	Proves theorems on intersecting chords of a circle	Students can change the diagrams by dragging the points of the configuration (Figure 4d)	When the students drag a point of the configuration, the lengths of the segments change together with the product of the lengths; moreover, the highlighted triangles help the students prove the relationship between the lengths (Figure 4d)	The applet features scaffolding measures that help the student understand and prove the theorems

Table 1 continuation . . .

Geometry	*Geomatch*	Illustrates triangles-angles and sides; illustrates and applies the properties of the different kinds of quadrilaterals	Students can select from two modes (Triangles or Quadrilaterals) and from three speed modes (Slow, Normal, Fast); triangles or quadrilaterals start to appear on the screen one by one; students need to select the triangle or quadrilateral that satisfies the description, which appears at the lower part of the screen (Figure 4e)	Note that the student has three lives to play; an incorrect selection results in a decrease in the number of lives. The game ends when the screen is filled with 16 shapes	The learning app emphasizes the definitions and characteristics of the various types of triangles and quadrilaterals as well as related theorems involving angles, sides, and diagonals
	GeoGebra Applet: Angles of Elevation and Depression	Describes and illustrates angles of elevation and angles of depression	This app contains three examples, which have accompanying drawings; the student can use built-in functions in *GeoGebra* to draw the line of sight of an observer and measure angles of elevation or depression (Figure 4f)	Upon the use of appropriate functions, the line of sight and angle measures are displayed on the screen	In this *GeoGebra* App, the student is tasked to construct the angle of elevation or depression instead of just seeing these labeled in textbook diagrams; this app is believed to be the first app to focus specifically on angles of elevation and depression
Number	*AlgeOps*	Performs fundamental operations on integers Adds and subtracts polynomials	Students can add or remove red/green boxes and balloons until they obtain the appropriate configuration to solve a given problem (Figure 5a)	After the app animates the fading of positive-negative pairs, the student inputs an answer to the given problem; the app verifies if the answer is correct or not	While the app's main focus is polynomial addition and subtraction, the app can also facilitate students' understanding of integer addition and subtraction via the neutralization model and the number line model (Verzosa *et al.* 2018) The app also provides immediate visual and auditory feedback and applies animations to emphasize the neutralization aspect of polynomial or integer addition and subtraction
	Catch the Carrot	Arranges rational numbers on a number line	Students can freely adjust the position of the carrot basket along the number line (Figure 5b)	The student's answer is verified by the animation of a falling carrot; the answer is correct if the carrot falls into the basket positioned by the student	The app is similar to some existing software that requires students to locate certain numbers on the number line; the novelty of this app is in its availability on Android and mobile devices
	GeoGebra Applet: Absolute Value as Distance	Represents the absolute value of a number on a number line as the distance of a number from 0	Students can drag two numbers a and b along the real number line (Figure 5c)	As the numbers a and b are dragged, the app displays the value of $\|a - b\|$ and how it is computed	This app limits the representation of the points to the one-dimensional number line, unlike most other apps that show distances on a 2-dimensional plane; the aim is to draw attention to the concept of linear distance and its relationship to the absolute value
Statistics and Probability	*Statistics*	Organizes data in a frequency distribution table Uses appropriate graphs to represent organized data: pie chart, bar graph, line graph, histogram, and ogive Calculates the measures of central tendency of ungrouped and grouped data; calculates the measures of variability of grouped and ungrouped data Uses appropriate statistical measures in analyzing and interpreting statistical data Draws conclusions from graphic and tabular data and measures of central tendency and variability	Students can adjust the number of classes to be used in the frequency distribution tables and histograms. In the built-in scenarios, the students may also partition the data into subclasses; importing of custom data is also supported (Figure 6a)	Depending on the student's input, the app displays a histogram, frequency distribution table, and measures of central tendency and variability.	The app features two realistic scenarios that help students learn how to do statistics but also where it can be used The app has a very simple interface, requiring only a few inputs from the user, that allows even first-time users to make the most out of the app
	Counting	Counts the number of occurrences of an outcome in an experiment: (a) table; (b) tree diagram; (c) systematic listing; and (d) fundamental counting principle Finds the probability of a simple event; solves problems involving probabilities of simple events	Students can choose which of the ice cream flavors, cones, and toppings are available (Figure 6b)	The app displays a tree diagram showing all possible combinations of the available ice cream flavors, cones, and toppings	This app is the possibly the first that aims to help students discover on their own the Fundamental Principle of Counting; it features a realistic setting (an ice cream shop) and immerses the students in a scenario (choosing which ice cream flavors, cones, and toppings are available)

[a]Based on the DepEd mathematics curriculum for Grade 7 to 10 (DepEd 2012)
[b]The app has three other modes: Triangle Congruence (Level 1), Triangle Similarity, and Quadrilaterals.

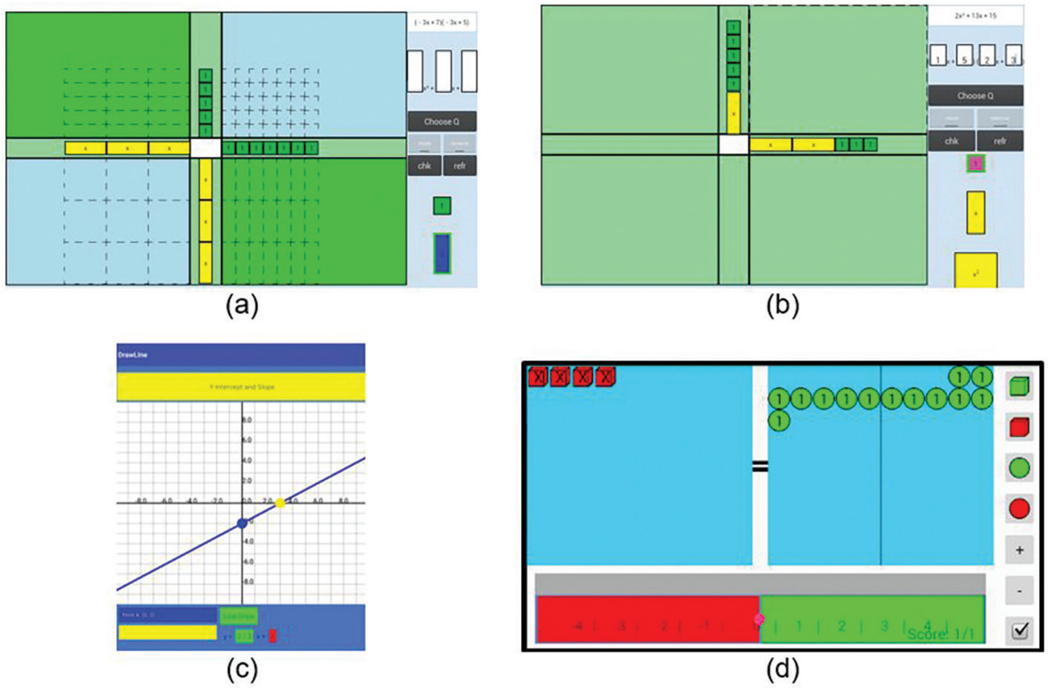

Figure 3. (a) AlgeTiles: Guide gridlines to facilitate the multiplication of two binomials. (b) AlgeTiles: Final phase of a factoring exercise where the algetiles corresponding to the factors of a trinomial are shown. (c) DrawLine: A line is graphed using input y-intercept and slope. (d) LinearX: A linear equation is visualized using four boxes (representing 4x) shown to be equal to thirteen circles.

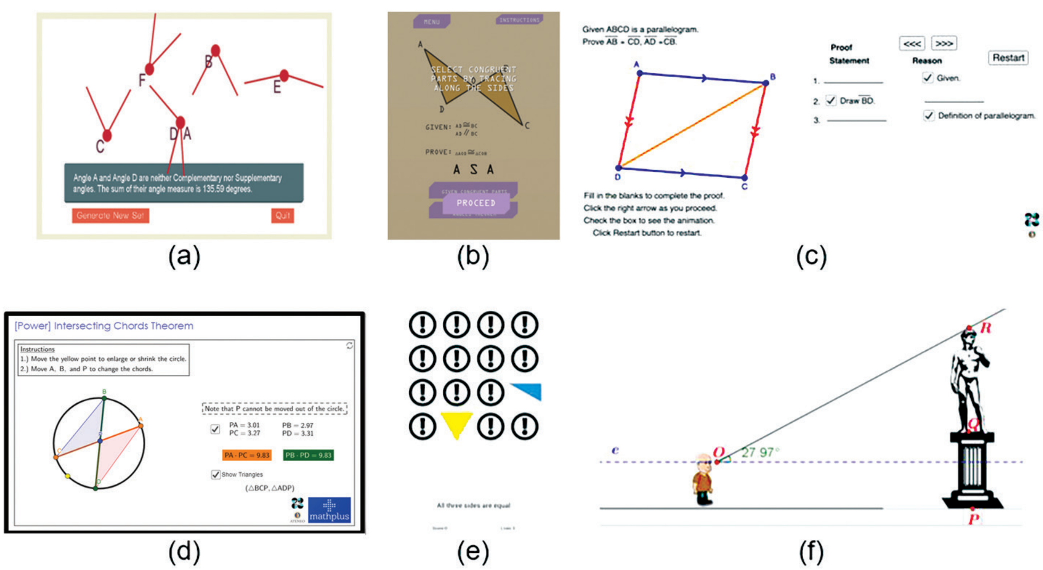

Figure 4. (a) Angle Xplore: A pair of adjacent angles, neither complementary nor supplementary, has been formed by the user by dragging some of the given angles. (b) ProveIt: A prompt showing an instruction to facilitate the solution of the given proving problem. (c) A *GeoGebra* app facilitating a two-column proof of a theorem involving a parallelogram. (d) A *GeoGebra* app illustrating the Intersecting Chords Theorem. (e) Geomatch: As shapes are revealed on the screen, the user has to tap one that matches the description given at the bottom. (f) A *GeoGebra* app where the observer's line of sight is drawn and the angle of elevation to the top of a statue is measured.

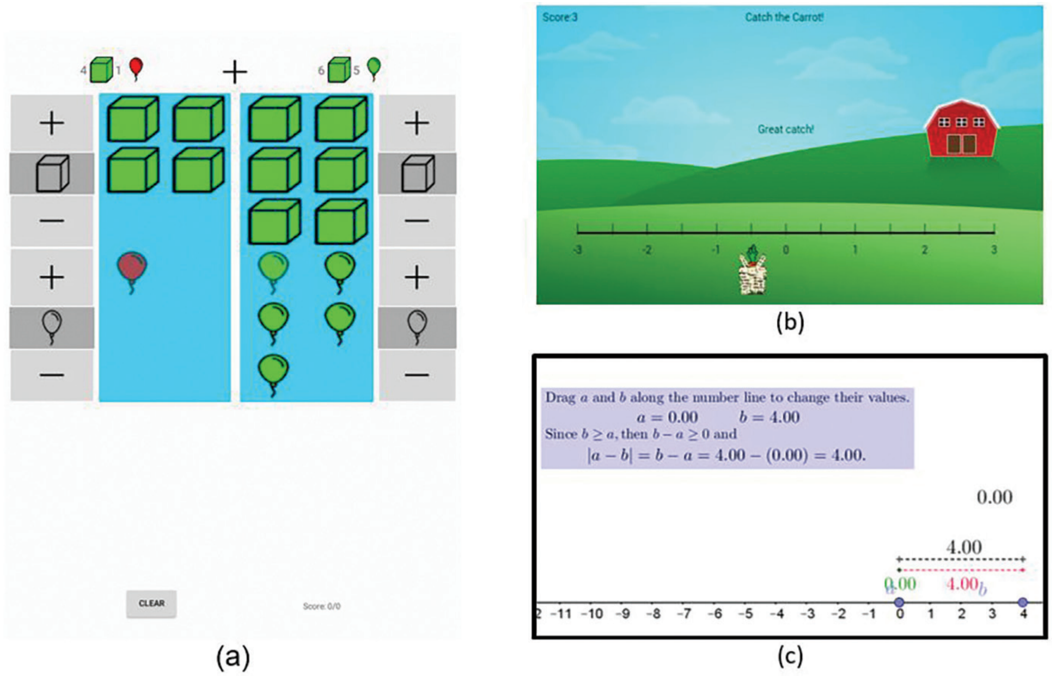

Figure 5. (a) AlgeOps: Addition of two expressions is visualized using boxes and balloons; a pair of balloons (red and green) is shown to fade to signify that they cancel each other. (b) CatchTheCarrot: The basket was correctly positioned at –1/2 so the falling carrot was successfully caught. (c) A *GeoGebra* app for exploring the idea that absolute value can be understood as distance.

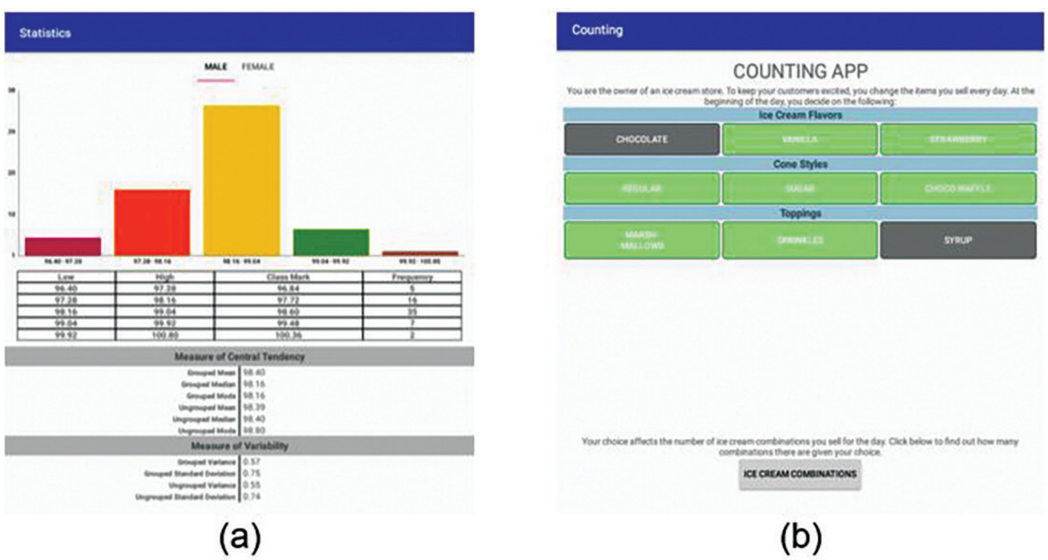

Figure 6. (a) Statistics: After inputting the desired number of classes, the app displays a histogram, a frequency distribution table, and measures of central tendency and variation. (b) Counting: The user makes a selection of which ice cream ingredients are currently available in an ice cream store.

Many of the apps developed focused on providing students a means of visualizing abstract mathematical concepts and operations while also giving them opportunities to explore these on their own. By using these apps, students can deepen their understanding and come up with observations, extensions, or generalizations on their own instead of simply accepting lectured materials. Examples of such apps are *LinearX, AlgeTiles*, *Counting*, *Statistics*, and a *GeoGebra* applet for visualizing the idea of absolute value as distance.

DrawLine and *AngleXplore* are visualization and exploration apps that make use of dynamic geometry, which has been a natural platform for students to test conjectures as it allows them to manipulate representations, observe the consequences, and formulate conjectures. Some *GeoGebra* applets that make use of dynamic geometry were also developed. For instance, one *GeoGebra* app focuses on the Intersecting Chords Theorem while another focuses on Angles of Elevation and Depression.

Apps, such as *CatchTheCarrot*, *GeoMatch*, and *AlgeOps,* not only afford visualization of the concepts but also focus on reviewing and practicing students. To encourage students to use the app even outside classes, these apps were made with a game environment.

Proving in geometry has also been one of the key points focused on the development of the apps since proving is one of the most difficult topics encountered by students. Very few students can be said to have mastered proof writing, even of proofs involving elementary geometric concepts (Senk 1985). In disadvantaged areas in the Philippines, students enter secondary school without having mastered elementary mathematics concepts (Verzosa *et al.* 2017) and, consequently, proofs end up being skipped altogether in favor of easier topics. The app *ProveIt* as well as several *GeoGebra* apps were designed to assist students in understanding and developing proofs for theorems in geometry.

Two of the apps, namely *ProveIt* and *AlgeOps*, were selected for testing for their potential impact on students' understanding of certain topics in geometry (in the case of *ProveIt*) and algebra (in the case of *AlgeOps*). In algebra, we focused on the topic addition and subtraction of integers, since mastery of integer operations forms one of the basic skills in secondary education. In the Philippines, it is quite common for students to reach their last year of high school without knowing how to perform operations on integers (Verzosa *et al.* 2017). *AlgeOps* is a visualization and exploration app that aims to anchor students' understanding of addition and subtraction of integers and polynomials. In particular, the app emphasizes the idea that positive and negative expressions cancel each other as shown by the fading animation of pairs of red and green boxes or balloons. Among the topics in geometry, we focused on proving since it is one of the topics most students have difficulty with. *ProveIt* is an app that provides users a visual representation and interactive learning tool to facilitate the construction of proofs involving congruent triangles, similar triangles, and quadrilaterals. One of the app's most important features is that it allows the user to select parts of the geometric figures on the VMR itself using the mobile device's touch-screen interface. As opposed to usual textbook exercises on these topics, *ProveIt* provides immediate feedback, allowing the user to quickly amend any incorrect responses provided or to understand why the provided response is correct. The results of testing these two apps will be discussed in the next section.

To accompany the mobile apps and *GeoGebra* applets, teaching guides were also developed to optimize the use and benefits of the software and to suggest ways to incorporate these technologies into the lesson. The teaching guides are expected to help teachers design lessons on the mathematical strands (*i.e.*, number sense, algebra, geometry, and probability and statistics) and to promote critical thinking and problem-solving in the classroom. The teaching guides also provide guide questions as prescribed by the topic addressed. In addition to teaching guides accompanying the apps, teaching guides on geometric constructions using a compass and straightedge was also developed. A total of 101 teaching guides were written on selected topics in Grade 7 to 10 mathematics. Note that each mobile app or *GeoGebra* applet may cater to different grade levels and/or topics and may address various learning competencies accompanied by several teaching guides. For example, *ProveIt* may be used in conjunction with lessons on triangle congruence (a topic in Grade 8) and conditions for parallelograms (a topic in Grade 9).

EVALUATION AND DISCUSSION

The apps were presented to 116 teachers from Metro Manila during a teacher training workshop-seminar organized by the researchers. The seminar aimed to disseminate information about the apps and their corresponding teaching guides. The call for participants for this workshop was disseminated through the DepEd local Schools Division Superintendent of cities in Metro Manila and through the research team members' personal network of high school teachers. At the end of the seminar, the teachers were asked to complete a survey to determine their perspective on the apps' usefulness and relevance to the curriculum.

To complement survey results on the apps' potential for increasing engagement and motivation as seen from the teachers' perspective, the research team also performed pilot studies on two of its apps, *ProveIt* and *AlgeOps*, to gain evidence on increased mathematical performance and conceptual development. The team chose to conduct a pilot test for *ProveIt* as we were interested to see the extent to which the app's interactive and visual capabilities can help develop students' competency in writing two-column proofs in contrast to usual textbook-based exercises. The team also launched a pilot study for *AlgeOps* to see how two models for integer operations, namely the neutralization model and the number line model, when implemented via a mobile app can facilitate students' conceptual development in integer addition and subtraction. The two pilot studies and their corresponding research designs are described below, but more details can be found elsewhere (Verzosa *et al.* 2018, 2019).

Teacher's Evaluation of the Apps

The first part of the survey required the teachers to rate their degree of agreement to a given statement (see Table 2 for a list of these statements) on a scale from 1 to 5 (where 5 indicates strong agreement and 1 indicates strong disagreement). The second part asked the teachers the following open-ended questions:

1. Describe your overall experience when using mobile apps.
2. How do you think the apps can be improved?
3. What are the advantages and disadvantages of the apps compared to similar tasks in textbooks and workbooks?

In developing the survey questions for the teacher training-workshop, the research team was primarily interested in the teachers' reception of the mobile apps developed during the project, especially in terms of usefulness in the classroom. Furthermore, their input could be used to further refine the apps, among other uses. As such, the feedback forms distributed during the event asked the teachers for their feedback, comments, and suggestions on the mobile apps only.

Table 2 provides summary statistics on the teachers' responses to the first part of the survey. Representative responses to the second part of the survey are also cited below to supplement the results in Table 2.

Table 2 shows that teachers strongly agreed with most statements presented in the evaluation survey. On one hand, teachers highly rated (with a fair consensus) the mobile apps' interactivity and the ease of use. One teacher highlighted the increasing relevance of mobile technologies in teaching schoolchildren who are nowadays increasingly tech-savvy, saying that "the learners will really participate in class if these apps will be presented because the learners these days can easily adapt to technology." Another respondent echoed this sentiment, saying that "[the apps] are very timely for us teachers. They can enhance our class discussion and are very exploratory and interactive." A similar comment was made by another respondent who said: "These mobile apps are very interactive and are appropriate to our millennial students." Other respondents also noted that the use of mobile apps provides a novel and interesting way of approaching classroom discussions and helps increase student motivation in studying for lessons. One respondent compared the use of mobile apps to the use of textbooks, saying "using a textbook is very traditional; millennial students want an interactive way of learning."

On the other hand, the teachers gave a relatively low rating for the apps' accommodation of students of all ability levels and its adaptability to classroom discussions. One teacher said that the apps are "not for advanced students – they would find it boring." Another teacher mentioned that the apps may also be somewhat confusing for beginner students. Several respondents also highlighted the issue of accessibility of these mobile technologies especially for public school students, citing that not all schools are Wi-Fi-ready and that not all students own a mobile device on which the apps can be used. However, other teachers said that if their students did have access to a mobile device, "students can practice on their own (at home or anywhere) … therefore increasing the students' retention of the lesson."

Table 2 Summary of teachers' degree of agreement to given statements on the apps' applicability to classroom use (n = 116, 1 = strongly disagree, 5 = strongly agree).

Statement	Mean	Median	Mode	SD
The mobile apps are interactive.	4.61	5.00	5.00	0.52
The mobile apps are easy to learn and to use.	4.57	5.00	5.00	0.53
I feel students will be able to practice mathematical skills through the apps.	4.53	5.00	5.00	0.66
I feel students will be able to explore mathematical concepts through the apps.	4.52	5.00	5.00	0.61
The apps can cater to beginning, average, and advanced students.	4.47	5.00	5.00	0.60
The mobile apps can be adapted to my classroom discussions.	4.32	4.00	4.00	0.71

Several respondents also highlighted the visualizations the mobile apps offer and how these aid in increasing students' understanding of mathematical concepts discussed in class. One cited this aspect as an advantage of using mobile apps in classroom discussion, saying that "[the apps] are effective and interactive because of the illustrations and graphics." Another teacher said that "the understanding of the topic will be more concrete since it utilizes manipulatives." The visual aspect is also useful in helping the students explore the topic, with one teacher saying "students can visualize the given activities or exercises, it is good to use in exploratory activities."

Overall, the teachers gave a positive evaluation of the mobile apps, especially in relation to increasing students' interest in the topic and to helping teachers prepare teaching aids more efficiently for their classroom discussions. There was also a strong agreement among the respondents that these mobile technologies are helpful and timely with regards to elevating high school mathematics teaching in an increasingly technology-dependent generation of students.

Increased Performance in Writing Two-Column Proofs

In the study by Verzosa *et al.* (2019), the researchers investigated the use of the *ProveIt* app in relation to students' competency in writing two-column proofs. Non-Android technologies for facilitating two-column proofs require the user to make conjectures based on figures and then enter statements to their proof. By contrast, *ProveIt* is Android-based and allows the user to trace over parts of the figure. In the Triangle Congruence mode of *ProveIt*, a user determines – through logic – a pair of congruent parts in the figure and traces over this pair. Visual feedback is provided through color flashes. The pair of parts becomes green for correct identification, and red if otherwise. The goal is to identify enough pairs of congruent parts so that the displayed triangle congruence postulate (*e.g.*, Side-Angle-Side) is satisfied.

To a sample of 12 private school students with various degrees of mathematical ability (*i.e.*, low-achievers, high-achievers, and average performers, as assessed by their respective teachers), the researchers introduced the *ProveIt* app as an intervention to help students construct two-column proofs for statements on triangle congruence. A pre-test consisting of three proving tasks on triangle congruence was first administered. After the pre-test, the students were given 30 minutes to use the *ProveIt* app, with the time split into periods of guided and unguided use of the app. A post-test – consisting of three proving tasks which were different from, but of the same difficulty level as, the pre-test – was then given to the students after the 30-minute period. The students' responses in the pre- and post-tests were marked using a rubric patterned after Senk (1985).

Using a Friedman two-way analysis of variance, the researchers found a significant improvement in the students' post-test scores when compared to their pre-test scores (p-value $= 0.043$). Further, the improvement tended to be more pronounced among students who already had a preliminary grasp of writing two-column proofs, although this must be investigated further. This preliminary study showed that the *ProveIt* app, even after only a short session on the use of the app, has the potential to improve students' ability to construct two-column proofs.

In a focus group discussion that was conducted after the post-test, the students lauded the interactive and visual elements of the app. Several students commented that using the app was markedly different from the usual textbook experience in that ProveIt requires user inputs via a touch screen. Specifically, students said that the act of tracing line segments and angles to identify them was a novel experience. The students also appreciated the immediate feedback given by the app when they entered their answers in the app and the ease with which the app was used.

Concept Development

A second study (Verzosa *et al.* 2018) presented a quantitative and qualitative analysis of how the *AlgeOps* app facilitated students' understanding of integer addition and subtraction. Earlier technologies involving the addition and subtraction of integers highlight the idea of canceling integers of opposite sign (neutralization model) or the idea of motion on a number line (number line model). The *AlgeOps* app is an integration of these two models. While it was designed to represent addition and subtraction of polynomials, it may also be used for teaching integer addition and subtraction. In the initial screen in the Add mode of *AlgeOps,* a set of red and green objects is presented as a target, with a blank panel below it. The user must use the plus and minus buttons so that the same number of red and green objects is shown on the panel. When the user successfully constructs the target number of objects on the panel, a number line with a highlighted point is shown. The user must move the point to the correct number representing the set of objects on the panel. As the user moves the point along the number line, visual feedback is shown through objects that appear over each number in the number line (see Figure 7).

The study was conducted on a sample of 26 students (Grades 5 to 7, aged 11 to 13 years old) consisting of public and private school students. The pre- and post-tests consisted of eight questions on integer addition and eight

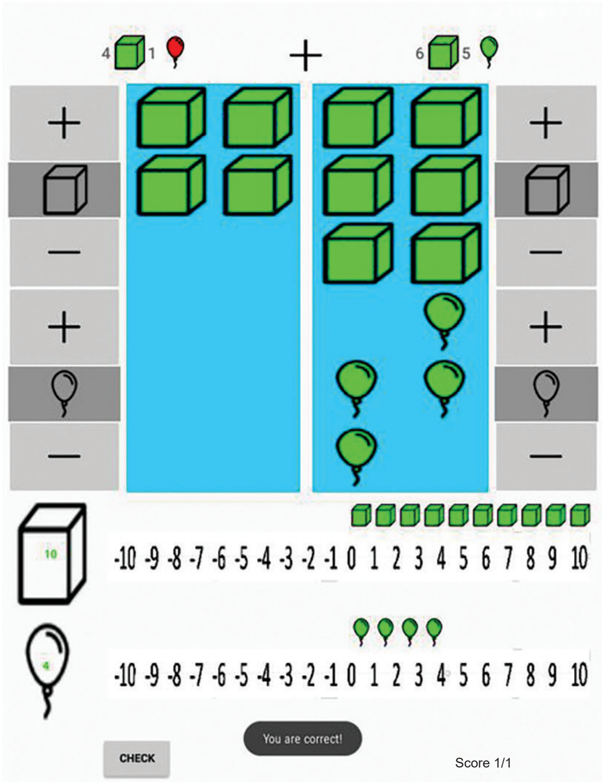

Figure 7. Confirming if the answer is correct in *AlgeOps* "Add" Level 2. The answer entered by the user is visualized using boxes and balloons that appear over the number lines.

questions on integer subtraction. Although different items were used in the post-test, the two tests were of equal difficulty. After the pre-test, students were asked to use and explore *AlgeOps* for 15 minutes, and the post-test was then administered.

The researchers found that post-test scores were generally higher than pre-test scores after a brief guided session on the use of *AlgeOps*. A significant improvement (p-value $= 0.009$) was observed among the public school students in integer addition, whereas a significant improvement (p-value $= 0.045$) was found for integer subtraction among the private school students in the sample. This shows that *AlgeOps* also has the potential to be an effective tool in developing students' competency in integer addition and subtraction.

The quantitative results were complemented with a qualitative analysis to provide an in-depth analysis of the causal processes (Maxwell 2004) behind how the app contributed to conceptual development. A framework informed by instrumental genesis (Guin and Trouche 1999) and a dialogic relationship between voices (Kazak *et al.* 2015) was used to analyze how two students developed their conception integers and integer operations as they used the app.

The two students were asked to use the plus and minus buttons on the app to construct the required number of objects shown on the screen (for example, three red boxes and five green boxes, representing negative and positive integers, respectively). They had no difficulties adding green objects – they simply needed to click the plus sign. However, they had difficulties adding red objects to the panel. When they clicked on the plus sign, only green objects appeared, so they undid their actions by clicking the minus sign. At this point, they still could not conceptualize quantities less than zero, so they stopped clicking the minus sign when there were zero objects on the screen. One student wondered, "How do you change the color?" They clicked other parts of the screen, but still had not succeeded in adding a red object to the panel. Instrumentalization was still incomplete – the users still had not learned the relevant commands in the app. When there were zero objects, one student clicked on various parts of the screen and ended up clicking the minus button and a red box appeared. That child said, "That's it!" and laughs. However, because of the accidental nature of this action, the students still struggled with the next problem. In a succeeding problem, they realized that red objects appeared when they clicked on a minus sign when there were zero objects on the panel. At the moment of realization, they both shouted, "Whoa!" signifying that at this instant, the app became an instrument that was integrated into their mental schemes. This "Whoa!" moment signaled a shift in perspective as the app became a voice that altered their primitive notions of numbers. It was a major turning point – the students took several minutes to answer the first two problems, but their newly learned concept allowed them to complete all subsequent problems more quickly.

CONCLUSION AND FUTURE DIRECTION

Under this project, we designed apps for various learning competencies in Grade 7 to 10 mathematics, guided by the framework of multiple representations and multimedia learning. An investigation of the effectiveness of these mobile apps, such as *ProveIt* and *AlgeOps*, shows the potential of these technologies to improve student learning outcomes in mathematics. The resources we developed have also garnered a positive reception from teachers who stated that these mobile technologies provide a more interactive means of instruction (as opposed to traditional paper-based media) and show potential to help increase students' motivation and retention of lessons. We invite the readers to visit the project website (https://mathplusresources.wordpress.com/) as future updates and refinements to the developed resources will be made available here.

A further investigation may be considered toward the development of digital learning resources on mathematics topics for both primary and secondary levels. Given the feedback gathered from teachers and students during the field-testing and teacher training-workshop, further refinements are being considered for the mobile apps for Grade 7 to 10. In addition, since this project focused on Grades 7 to 10, future work may be considered for Grades 1 to 6 mathematics. This will address the learning of number sense, which is the foundation for higher mathematical learning. The focus on primary mathematics is additionally critical because Filipino children rely on superficially learned strategies from an early age (Verzosa 2015).

The thrust towards the development of learning resources for statistics is timely, as the new K to 12 curriculum specifies a very rigorous statistical curriculum, with many topics not included in the previous basic education curriculum. A framework on mathematical modeling may be considered as the basis of the development of mobile apps on statistics. Specifically, these apps may be designed to take advantage of real-life data to impart a better understanding of statistical concepts and their applications. To this end, developing a database for statistical learning (*e.g.*, Census-at-School, which is implemented in Australia) may also be a worthwhile pursuit in connection with the heavier emphasis on statistics in the new K to 12 curriculum and with the need to build stronger statistical competencies among students.

In addition to developing digital resources for mathematics learning and statistical thinking, future work may also be done in the development of database infrastructure for assessment and statistical reasoning. With a database of student assessment, educators may then measure students' progress across a larger scale and over time.

The creation of resources for Grades 1 to 6, the refinement of the apps for Grades 7 to 10, the development of the database and teaching guides for statistical learning, and the development of an assessment database are the major components of an ongoing project entitled "Technology Innovations for Mathematical Reasoning, Statistical Thinking, and Assessment" being conducted by the researchers. It is suggested that alongside the task of refining the existing apps for Grades 7 to 10, additional studies on the potential impact of these apps for student learning will be carried out. It is further proposed that other apps be developed using the scientific frameworks and methodologies presented in this study. It is also recommended that feedback from teachers regarding the teaching guides accompanying the existing apps be collected in future studies.

There is still a lot of work and questions to be addressed in the study of the use of MT in learning mathematics. While the results of our initial studies yield positive results, further research is needed to help identify which situations and learning environments are most suitable for mobile apps to be most beneficial. There are also other issues, such as the following, that need to be addressed. Should schools in the Philippines allow the use of technology in the study of mathematics in the classrooms in secondary mathematics? Should students bring their mobile phones and other devices? There are also technical considerations. Are schools equipped with wireless internet and mobile devices for teachers to use? Can teachers cope with the technical aspects and difficulties of varying devices? How are teachers being prepared in schools when the field of learning with MT is new, but the technology that supports it is changing very fast?

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STATEMENT ON CONFLICT OF INTEREST

No potential conflict of interest was reported by the authors.

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