Detecting scalar fields with Extreme Mass Ratio Inspirals

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We study Extreme Mass Ratio Inspirals (EMRIs), during which a small body spirals into a supermassive black hole, in gravity theories with additional scalar fields. We first argue that no-hair theorems and the properties of known theories that manage to circumvent them introduce a drastic simplification to the problem: the effects of the scalar on supermassive black holes, if any, are mostly negligible for EMRIs in vast classes of theories. We then exploit this simplification to model the inspiral perturbatively and we demonstrate that the scalar charge of the small body leaves a significant imprint on gravitational wave emission. Although much higher precision is needed for waveform modelling, our results strongly suggest that this imprint is observable with LISA, rendering EMRIs promising probes of scalar fields.

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Introduction. The existence of additional gravitational wave (GW) polarizations with respect to general relativity (GR) is a generic feature of alternative theories of gravity. Direct observation of these extra polaritazions would be quite challenging, because they are expected to couple very weakly to detectors. Nonetheless, if they exist then they do affect the emission: any extra polarization is an additional channel for energy loss for a binary system. The latter generically loses energy at a different rate than in GR. This modifies the orbital dynamics and the GW frequency's evolution, leaving an imprint on standard polarizations.

For comparable mass binaries in the inspiral phase, the leading-order effect comes from dipolar emission [1–3]. The theory has an additional field and compact objects are "dressed" by it. One can think of them as carrying a "charge" – we use the term colloquially as we are not necessarily referring to gauge fields. Hence, to leading order each of the members of the binary acts like a monopole in the new field and the orbiting pair emits dipole radiation. The rate of emission, namely the energy loss, depends on how much "charge" the compact objects carry, and more specifically on the difference between charges (or sensitivities in post-Newtonian jargon [1]). Since the effects that are associated with the additional energy loss are cumulative, observing a long inspiral can lead to significant constraints on dipolar emission, assuming that the new field is massless or sufficiently light, so that the corresponding interaction has sufficiently long range.[1]

Dipole emission, or more precisely absence thereof, has yielded strong constraints on massless scalars using binary pulsar observations [7]. GW observations of binary neutrons star inspirals can significantly improve these constraints [8]. Moreover, they can probe the same effect at smaller separations and in principle detect or constrain more massive scalars. This is a major goal for ground-based detectors [9].

Can extreme mass ratio inspirals (EMRIs) onto supermassive black holes (BHs), which will be prime targets from LISA [10, 11], yield comparable constraints? No-hair theorems dictate that stationary BHs in most scalar-tensor theories will just be described by the Kerr metric [12–16]. Evading these theorems requires coupling the scalar to higher-order curvature invariants [17–20]. Indeed, the known BH solutions with scalar hair, *e.g.* [17–28], tend to have scalar "charges" that are not independent and are controlled by the mass of the BH. The more massive a BH is, the more weakly charged it is. This is because the "charge" is controlled by curvature and the curvature near the horizon tends to grow as the mass decreases. It is then tempting to conclude that systems that involve supermassive BHs will exhibit much smaller deviation from GR and hence will be less suitable for inspiral tests.

While this is true for comparable mass BH binaries (see *e.g.* Ref. [29]), it is incorrect in general. Consider an EMRI. So long as the companion carries a significant "charge", there should be emission in the corresponding polarisation. As we will see in more detail shortly, the fact that the supermassive BH in an EMRI carries no or very little charge is in fact a blessing in disguise from a technical perspective. Intuitively, the setup is not much different from an accelerated electric charge. One can think of the companion as a scalar monopole that is accelerated gravitationally by the supermassive BH and thus emits scalar (mostly dipolar) radiation. The main goal of this paper is to demonstrate that this emission has a very significant, cumulative effect during the long inspiral of an EMRI, which appears to be detectable by space interferometers as LISA (for similar computations for a specific class of scalar-tensor theories see [30, 31]). Moreover, we shall show that the additional energy loss

[1] A large mass would make the interaction short range, quenching the emission at large separations [4–6].

in an EMRI – and thus the dephasing of the gravitational waveform – mainly depends on the scalar charge of the object orbiting around the supermassive BH and has negligible dependence on other features of the underlying gravity theory. This makes EMRIs powerful tools for tests of gravity.

General setup. To demonstrate this concretely, we start from the following action

$$S[\mathbf{g},\varphi,\Psi] = S_0[\mathbf{g},\varphi] + \alpha S_c[\mathbf{g},\varphi] + S_{\rm m}[\mathbf{g},\varphi,\Psi]\,, \quad (1)$$

where

$$S_0 = \int d^4x \frac{\sqrt{-g}}{16\pi}\left(R - \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi\right)\,, \quad (2)$$

R is the Ricci scalar, φ is a scalar field, and we use units in which $G = c = 1$; Greek indices run from 0 to 3, while Latin indices run from 1 to 3. αS_c describes nonminimal couplings between the metric tensor $\mathbf{g}$ and φ, and α is a coupling constant with dimensions $[\alpha] = (\text{mass})^{\rm n}$. $S_{\rm m}$ is the action of the matter fields Ψ.

We consider the inspiral of a body (the "particle") with mass $m_{\rm p}$ onto a BH of mass M. Since the inspiral is an EMRI, we assume that $m_{\rm p} \ll M$. We use the so-called "skeletonized approach" [1, 32–34], in which an extended body is treated as a point particle, replacing the matter action $S_{\rm m}$ with

$$S_{\rm p} = -\int m(\varphi)ds = -\int m(\varphi)\sqrt{g_{\mu\nu}\frac{dy^\mu_{\rm p}}{d\lambda}\frac{dy^\nu_{\rm p}}{d\lambda}}d\lambda\,. \quad (3)$$

Here $y^\mu_{\rm p}(\lambda)$ is the worldline of (the center of mass of) the particle in a given coordinate frame, and $m(\varphi)$ is a scalar function that depends on the value of the scalar field at the location of the particle.

In this approach it is assumed that the "skeletonized" body has a characteristic length-scale, l, which is much smaller that the length-scale L of the exterior spacetime, i.e. of the spacetime solution of the field equations in the absence of that body. The region of spacetime in which the gravitational field of the body is large is a world-tube with size $\sim l$, and can be treated as a worldline $y^\mu_{\rm p}(\lambda)$ in the exterior spacetime. The action $S_{\rm p}$ is obtained by integrating the matter action $S_{\rm m}$ over this world-tube. In the case of an EMRI, the skeletonized body and the exterior spacetime coincide with the "particle" of mass $m_{\rm p}$ and with the BH of mass M, respectively.

Let us consider the exterior spacetime. We assume that (perturbed) BHs in the theory under consideration are continuously connected to the corresponding GR solution as $\alpha \to 0$, and that $S_{\rm c}$ is analytic in φ. We identify two distinct cases in which one can describe an EMRI as the motion of a particle, described by the skeletonized action $S_{\rm p}$ given in Eq. (3), in the Kerr spacetime.

Case 1: The theory described by (1) satisfies a no-hair theorem [12–18] and, hence, stationary BHs are described by the Kerr metric.

Case 2: The theory evades no-hair theorems but the coupling constant α is dimensionful, with $n \geq 1$. Our assumption that the BH spacetime is continuously connected to the Kerr spacetime as $\alpha \to 0$ and the fact that the only dimensionful scale of the Kerr metric is its mass M, imply that any correction must depend on

$$\zeta \equiv \frac{\alpha}{M^n} = q^n \frac{\alpha}{m^n_{\rm p}}\,, \quad (4)$$

where $q = m_{\rm p}/M \ll 1$. Assuming that $\alpha/m^n_{\rm p} < 1$ (otherwise the modifications to GR would be too large to be consistent with current astrophysical observations [35]), it follows that $\zeta \ll 1$, being suppressed by the mass ratio, and thus the exterior spacetime can be approximated by the Kerr metric.

It should be stressed that Case 1 covers very wide classes of scalar-tensor theories. The theories that instead are known to evade no-hair theorems tend to belong to Case 2. A notable example is scalar Gauss-Bonnet (sGB) gravity [17–26], for which

$$S = S_0 + \frac{\alpha}{4}\int d^4x\frac{\sqrt{-g}}{16\pi}f(\varphi)\mathcal{G} + S_{\rm m}\,, \quad (5)$$

$n = 2$ and $f(\varphi)$ is a general function of the scalar field, specifying the coupling between the scalar field and the Gauss-Bonnet invariant $\mathcal{G} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}$. For massless scalars, which are expected to respect shift symmetry, this coupling is essential for evading no-hair theorems [17, 36]. However, action (1) is far more generic. For instance, it includes any theory in which a (pseudo)scalar couples to curvature invariants (e.g. generalized scalar-tensor theories or dynamical Chern-Simons gravity [37]). The analysis above can straightforwardly be extended to multiple coupling constants with different dimensions, and to theories with more than one scalar field [32, 38, 39]. The only crucial assumption is that of continuous connection to GR BHs as the new couplings tend to zero.

Hence, for theories falling under Cases 1 and 2, one can describe an EMRI as the motion of a particle, described by the skeletonized action $S_{\rm p}$ given in Eq. (3), in the Kerr spacetime. This motion can be studied using spacetime perturbation theory, i.e. expanding the field equations in the mass ratio $q \ll 1$. Remarkably, the GR modifications affect the motion of the particle, but they *do not* affect the background spacetime. This results in a great simplification of the EMRI modelling, and, as we show below, it allows to make rather generic predictions of the corresponding phenomenology.

For the rest of this Letter we shall assume, for simplicity, that the BH with mass M is non-rotating, and thus described by the Schwarzschild metric. The case of a rotating BH will be studied in a forthcoming publication.

Field Equations. Varying the action with respect to the metric tensor we obtain the following modified Einstein equations:

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = T^{\rm scal}_{\mu\nu} + \alpha T^c_{\mu\nu} + T^{\rm p}_{\mu\nu}\,, \qquad (6)$$

where $T^{\rm scal}_{\mu\nu} = \frac{1}{2}\partial_\mu\varphi\partial_\nu\varphi - \frac{1}{4}g_{\mu\nu}(\partial\varphi)^2$ is the stress-energy tensor of the scalar field, and

$$T^c_{\mu\nu} = -\frac{16\pi}{\sqrt{-g}}\frac{\delta S_c}{\delta g^{\mu\nu}} \qquad (7)$$

is the stress-energy tensor associated to the coupling between gravity and the scalar field. Finally,

$$T^{{\rm p}\,\alpha\beta} = 8\pi\int m(\varphi)\frac{\delta^{(4)}(x-y_p(\lambda))}{\sqrt{-g}}\frac{dy_p^\alpha}{d\lambda}\frac{dy_p^\beta}{d\lambda}d\lambda \qquad (8)$$

is the particle's stress-energy tensor.

Variation with respect to the scalar field yields:

$$\Box\varphi + \frac{8\pi\alpha}{\sqrt{-g}}\frac{\delta S_c}{\delta\varphi} = 16\pi\int m'(\varphi)\frac{\delta^{(4)}(x-y_p(\lambda))}{\sqrt{-g}}d\lambda \quad (9)$$

and $m'(\varphi) = dm(\varphi)/d\varphi$.

In our units $[S_0] = ({\rm mass})^2$, $[S_c] = ({\rm mass})^{2-n}$. In an EMRI, S_c is evaluated on the background of the large, stationary BH, and since the only dimensionful scale in this background is the BH mass M, we expect that $S_c \sim M^{-n}S_0$. Therefore, $\alpha T^c_{\mu\nu} \sim \zeta G_{\mu\nu} \ll G_{\mu\nu}$ and $\alpha\frac{\delta S_c}{\delta\varphi} \sim \zeta\Box\varphi \ll \Box\varphi$. For an EMRI around a GR BH, the external scalar field has to be a constant, φ_0. Indeed, under our assumptions and without the contribution of the particle the field equations (6), (9) coincide to those of GR with a free scalar field, for which the no-hair theorem applies. $T^{\rm scal}_{\mu\nu}$ is quadratic in perturbations around $\varphi = \varphi_0$ and can also be neglected. Since S_c is analytical in φ, $\alpha T^c_{\mu\nu} \sim \zeta^2 G_{\mu\nu}$ and the corrections to the background metric due to the scalar field are of order $\sim\zeta^2$, i.e. they are suppressed at least by a factor q^{2n} with respect to the leading term. These terms can then be neglected with respect to the "particle" terms.

Let us now consider Eq. (9) in a "buffer" region close enough to the body to be inside the world-tube, but faraway enough to have a metric which can be written as a perturbation of flat spacetime. In this region, since the coupling term is negligible the scalar field equation takes the form $\Box\varphi = 0$. Hence, in a reference frame $\{\tilde{x}^\mu\}$ centered on the body, its solution has the simple form

$$\varphi = \varphi_0 + \frac{m_{\rm p}\,d}{\tilde{r}} + O\left(\frac{m_{\rm p}^2}{\tilde{r}^2}\right) \qquad (10)$$

where d is the *dimensionless scalar charge* of the body with mass $m_{\rm p}$. At $\tilde{r} \gg m_{\rm p}$ the solution (10) tends to the asymptotic value φ_0, which is also the value of the external scalar field near the location of the worldtube; thus, in the particle action (3) (and in the source terms) the scalar function $m(\varphi)$ and its derivative should be evaluated at $\varphi = \varphi_0$.

The value of φ_0 is determined by asymptotics, so in a realistic scenario it is fixed by cosmological considerations and it will be theory dependent. However, it turns out to be irrelevant for our analysis. For convenience we set $\varphi_0 = 0$, which amounts to the redefinition $\varphi \to \varphi - \varphi_0$ in equations (6), (9).

Replacing the expression in Eq. (10) into Eq. (9) after our approximations yields the relation $m'(0)/m_{\rm p} = -d/4$. Finally, in the weak-field limit the (tt)-component of the particle's stress-energy tensor reduces to the matter density of the particle $\rho = m_{\rm p}\delta^{(3)}\left(x^i - y^i_{\rm p}(\lambda)\right)$, and since (see Eq. (8))

$$T^{{\rm p}\,tt} = 8\pi m(0)\delta^{(3)}\left(x^i - y^i_{\rm p}(\lambda)\right) + O\left(\frac{m_{\rm p}}{\tilde{r}}\right)\,, \qquad (11)$$

we also have $m(0) = m_{\rm p}$. We can conclude that in the class of theories considered in this paper, the perturbed Einstein's equations and scalar field equations for EMRIs have the form

$$G_{\mu\nu} = T^{\rm p}_{\mu\nu} = 8\pi m_{\rm p}\int\frac{\delta^{(4)}(x-y_p(\lambda))}{\sqrt{-g}}\frac{dy_p^\alpha}{d\lambda}\frac{dy_p^\beta}{d\lambda}d\lambda \quad (12)$$

$$\Box\varphi = -4\pi d\, m_{\rm p}\int\frac{\delta^{(4)}(x-y_p(\lambda))}{\sqrt{-g}}d\lambda\,. \qquad (13)$$

While the perturbed Einstein's equations (12) coincide with the corresponding equations in GR, the perturbed scalar field equations (13) have a source term which is proportional to the scalar charge d. All information about the underlying gravity theory is encoded in the scalar charge d, which thus universally captures the changes in the EMRI dynamics.

Perturbations. To study EMRI's evolution in theories specified by Eq. (1) and belonging to the Cases 1 and 2 discussed above, we compute the perturbations around a Schwarzschild BH induced by a particle with mass $m_{\rm p}$ which takes into account beyond-GR corrections in the source term. We consider linear order perturbations to the gravitational and the scalar sector, i.e. we expand both the metric tensor and φ around a background, $g_{\alpha\beta} = g^0_{\alpha\beta} + h_{\alpha\beta}$ and $\varphi = \varphi_0 + \varphi_1$, where $g^0_{\alpha\beta}$ describes the Schwarzschild spacetime (in Schwarzschild coordinates (t, r, θ, ϕ)), and – as discussed above – $\varphi_0 = 0$. We decompose $h_{\mu\nu}$ and φ_1 in tensor and scalar spherical harmonics, respectively. The metric perturbations decouple in two classes, known as *polar* and *axial* perturbations, $h_{\alpha\beta} = h^{\rm pol}_{\alpha\beta} + h^{\rm ax}_{\alpha\beta}$, according to their properties under parity transformation [40–42]. For binaries in circular orbits both sectors are excited. In the Schwarzschild background, the metric and the scalar field perturbations are decoupled. In this set-up, and working within the so-called Regge-Wheeler gauge, the components of the metric perturbations $h^{\rm pol}_{\mu\nu}$ ($h^{\rm ax}_{\mu\nu}$) reduce to a single function $Z_{\ell m}$ ($R_{\ell m}$). In the frequency domain tensor and scalar perturbations satisfy the wave equation

$$\frac{d^2\psi_{\ell m}}{dr_\star^2} + \left(\omega^2 - e^{-\lambda}V\right)\psi_{\ell m} = J\,, \qquad (14)$$

where $e^{-\lambda} = 1 - \frac{2M}{r}$, $\psi_{\ell m} = (Z_{\ell m}, R_{\ell m}, \delta\varphi_{\ell m})$, $r_\star = r + 2M\log(r/2M - 1)$ is the tortoise coordinate, V is a 3×3 diagonal matrix with

$$V_{11} = 2\frac{9M^3 + 9M^2 r\Lambda + 3Mr^2\Lambda^2 + r^3\Lambda^2(1+\Lambda)}{r^3(3M + r\Lambda)}\,, \quad (15)$$

$V_{22} = \frac{\ell(\ell+1)}{r^2} - \frac{6M}{r^3}$, $V_{33} = \frac{\ell(\ell+1)}{r^2} + \frac{2M}{r^3}$ and $\Lambda = \ell(\ell+1)/2 - 1$. The source's terms J_Z, J_R are explicitly given in [43], while the scalar field component reads:

$$J_\varphi = -d\, m_{\rm p}\frac{4\pi P_{\ell m}(\frac{\pi}{2})}{r^{3/2}e^\lambda}\sqrt{r - 3M}\,\delta(r - r_{\rm p})\delta(\omega - m\omega_{\rm p})\,. \quad (16)$$

Here $r_{\rm p}$ is the particle's coordinate radius, $\omega_{\rm p} = (M/r_p^3)^{1/2}$ and $P_{\ell m}(\theta)$ the Legendre polynomials.

We numerically integrate the wave equations (14) by first finding the homogeneous solutions at the horizon $\psi^{(-)}_{\ell m}$ and at infinity $\psi^{(+)}_{\ell m}$, which satisfy the boundary condition of purely ingoing and outgoing waves, respectively, i.e. $\psi^{(\pm)}_{\ell m} \sim e^{\mp i\omega r_\star}$. The non-homogeneous solution $\psi_{\ell m}(r_\star)$ is obtained by integrating the homogenous part over the source terms. Evaluating the solution at the horizon and at infinity we get

$$\psi^\pm_{\ell m} \equiv \lim_{r_\star\to\pm\infty}\psi_{\ell m}(r_*) = e^{\pm i\omega r_\star}\int_{2M}^\infty \frac{\psi^{(\mp)}_{\ell m}J}{W}dr_\star\,, \quad (17)$$

where $W = \psi'^{(+)}_{\ell m}\psi^{(-)}_{\ell m} - \psi'^{(-)}_{\ell m}\psi^{(+)}_{\ell m}$ is the Wronskian and the prime denotes derivative with respect to $r_\star$. From Eqns. (17) we can compute the gravitational and scalar energy flux at the horizon and at infinity [44, 45]:

$$\dot{E}^\pm_{\rm grav} = \frac{1}{64\pi}\sum_{\ell=2}^\infty\sum_{m=-\ell}^{\ell}\frac{(\ell+2)!}{(\ell-2)!}(\omega^2|Z^\pm_{\ell m}|^2 + 4|R^\pm_{\ell m}|^2)\,,$$
$$\dot{E}^\pm_{\rm scal} = \frac{1}{32\pi}\sum_{\ell=1}^\infty\sum_{m=-\ell}^{\ell}\omega^2|\delta\varphi^\pm_{\ell m}|^2\,. \quad (18)$$

Results. We compute the the total energy flux

$$\dot{E} = \dot{E}^+_{\rm grav} + \dot{E}^-_{\rm grav} + \dot{E}^+_{\rm scal} + \dot{E}^-_{\rm scal} = \dot{E}_{\rm GR} + \delta\dot{E}_d \quad (19)$$

summing all the multipole contributions up to $\ell = 5$, where $\dot{E}_{\rm GR}$ is the energy flux emitted in GR by a binary system with the same masses $m_{\rm p}$, M. Since, as discussed above, the perturbed Einstein's equations coincide with the corresponding equations in GR, the correction to the energy flux is only due to the scalar field emission at infinity and at the horizon, $\delta\dot{E}_d = \dot{E}^+_{\rm scal} + \dot{E}^-_{\rm scal}$. Figure 1 shows the relative correction $\delta\dot{E}_d/\dot{E}_{\rm GR}$ as a function of the orbital velocity $v = (M\omega_{\rm p})^{1/3}$, while the inset provides the value of $\delta\dot{E}_d$. Note that $\delta\dot{E}_d$ formally enters at the same order in the mass ratio q as the GR contribution: for a given orbital configuration the normalized flux $q^2\delta\dot{E}_d$ only depends on dimensionless scalar charge d. The scalar flux increases as the binary inspirals towards the ISCO at $r = 6M$, accelerating the coalescence due to the extra leakage of energy. The ratio $\delta\dot{E}_d/\dot{E}_{\rm GR}$ decreases for smaller orbital separations, since the gravitational term $\dot{E}^\pm_{\rm grav}$ grows faster than the scalar field contribution at large frequencies. The relative difference between the total flux in GR and in the modified gravity theory can be $\sim 1\%$ close to the plunge.

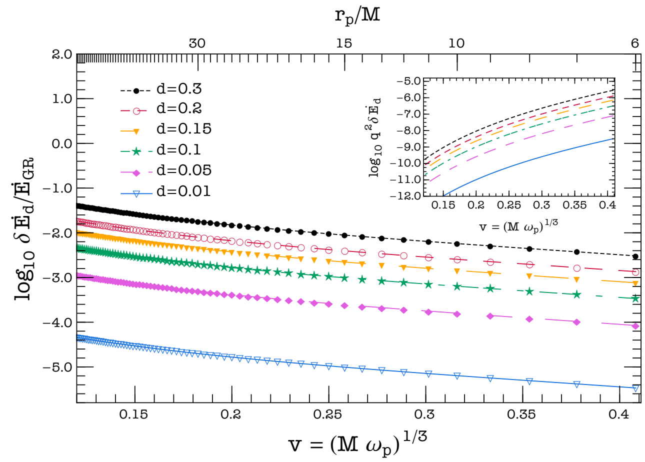

FIG. 1. Relative difference between the GW flux in modified gravity and in GR as a function of the orbital velocity $v = (M\omega_p)^{1/3}$ (or radius r_p/M), and of the scalar charge d. The inset shows the values of $q^2\delta\dot{E}_d$.

Having computed the emitted energy flux, we can determine the EMRI's adiabatic evolution, i.e. the GW phase ϕ as a function of the frequency $f = \omega_p/\pi$:

$$\frac{d\phi}{df} = \frac{f}{\dot{f}}\,,\quad \dot{f} = \frac{3}{2}\frac{f}{r_p}\frac{dr}{dE_{orb}}\bigg|_{r_p}\dot{E}\,, \quad (20)$$

with E_{orb} particle's orbital energy. The total phase can be written as $\phi(f) = \phi_{\rm GR}(f) + \delta\phi_d(f)$ where both the GR and the scalar field contribution are of the order $\mathcal{O}(1/q)$. The correction $q\delta\phi_d$ is indeed universal, and depends only on the normalized charge d.

To quantify the impact of GR modifications on possible GW detections by future interferometers like LISA we compute the number of cycles accumulated before the merger [2]:

$$\mathcal{N} = \int_{f_{min}}^{f_{\max}}\frac{f}{\dot{f}}df\,. \quad (21)$$

We choose $f_{max} = (6^{3/2}\pi M)^{-1}$ and $f_{min} = \max[f_{\rm T}, 10^{-4}]$, where $f_{\rm T}$ is the GW frequency 4 years before the ISCO [31], which represents the typical observing time of LISA [10]. Figure 2 shows $\Delta\mathcal{N} = \mathcal{N}_{\rm GR} - \mathcal{N}_d$ for some prototype systems with $\mu = 10M_\odot$. The difference is always positive, since the scalar field emission increases the energy loss by the binary. $\Delta\mathcal{N}$ decreases monotonically as the mass of the central object grows, and it is strongly dependent on the scalar charge. We find that for $d \gtrsim 0.01$ the dephasing can be larger than 1 radiant (the standard conservative value for a detectable dephasing) for $M \sim 4 \cdot 10^6 M_\odot$. For lighter BHs with $M \sim 10^5 M_\odot$ and large d, $\Delta\mathcal{N}$ is significantly higher and can be as large as $\sim 10^3$ radians.

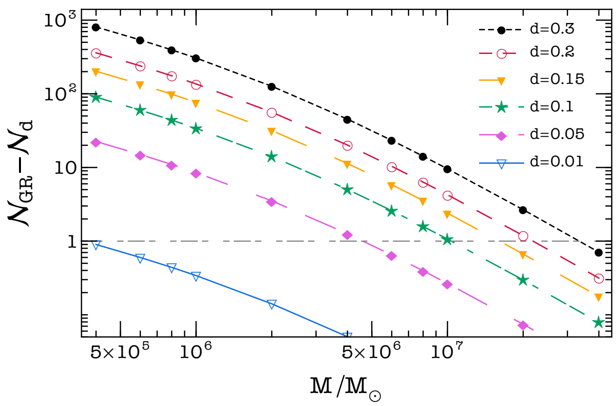

FIG. 2. Difference in the number of GW cycles accumulated by EMRIs on circular orbits with $\mu = 10M_\odot$, $M \in [5 \times 10^5, 10^8]M_\odot$, and different values of the dimensionless scalar charge d. All binaries are observed four years before merger.

In principle, a potential degeneracy between the scalar charge and the black hole masses may affect a detector's ability to distinguish a non-GR signal from a GR signal with different parameters. Even if such degeneracy plagues single observables (*e.g.* number of cycles), one expects it to be lifted by more detailed waveform analysis. Preliminary results in this direction confirm this expectation and a systematic analysis will be presented elsewhere [46].

A case study: sGB gravity. Figures 1, 2 show that EMRIs are a probe of the particle's scalar charge. Typically, this quantity is related to the fundamental coupling constant α of the modified gravity theory.

Let us consider, for instance, the case of sGB gravity with $f'(0) \neq 0$ (i.e., excluding theories which allow for BH scalarization [18, 19]). If the body is a BH, its dimensionless scalar charge is proportional to the dimensionless coupling constant of the theory $\beta \equiv q^{-2}\zeta = \alpha/m_{\rm p}^2$ [23, 47, 48]. The explicit form of $d(\beta)$ has been derived in [48]. Taking into account the different normalization conventions, one finds that, for instance, $d = 2\beta + \frac{73}{30}\beta^2 + \frac{15577}{2520}\beta^3 + O(\beta^4)$ for Einstein-dilaton Gauss-Bonnet gravity [23, 47] ($f(\varphi) = e^\varphi$), while $d = 2\beta + \frac{73}{60}\beta^3$ for shift-symmetric sGB gravity [17, 26] ($f(\varphi) = \varphi$).

Conclusions. EMRIs are golden binaries to test fundamental physics in the strong-gravity regime [8, 49–53]. In gravity theories with additional scalar fields, the enhanced energy emission during the inspiral leads to a cumulative dephasing of the gravitational waveform. We showed that for theories satisfying no-hair theorems or having dimensionful coupling constants, the central BH of an EMRI can be taken to be the Kerr metric, and the modification of the waveform only depends on the scalar charge of the inspiraling body. Using this significant simplification, we demonstrated that the corresponding dephasing should be detectable by LISA. Interestingly, the vast majority of known theories with additional scalar field satisfy our assumptions [9, 54]. Our results imply that EMRIs can be excellent systems for probing the existence of scalar fields and constrain fundamental physics.

For any given modified gravity theory, a bound (or a measurement) of the scalar charge obtained from the detection of an EMRI waveform can be translated in a bound (or a measurement) of the fundamental coupling constant of the theory. Forecasts on scalar charge constraints for a given detector can act as a theory-independent assessment of its potential to test GR.

The approach and key simplifications we have introduced here are a critical first step towards a consistent description of EMRIs beyond GR. Our formalism can be straightforwardly extended to rotating BHs, as well as to generic (*e.g.* eccentric) orbits, and to theories with further polarizations. A more challenging problem is to study the effect of extra degrees of freedom on self-force corrections. The latter are essential for accurate EMRI modelling [55, 56], and are currently under intense studies aimed to provide second-order corrections to the binary dynamical evolution [57]. The separation of scales discussed in this paper is expected to greatly reduce the complexity of self-force description beyond GR.

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