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Stability analysis of three exoplanet systems

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ABSTRACT

The orbital solutions of published multiplanet systems are not necessarily dynamically stable on time-scales comparable to the lifetime of the system as a whole. For this reason, dynamical tests of the architectures of proposed exoplanetary systems are a critical tool to probe the stability and feasibility of the candidate planetary systems, with the potential to point the way towards refined orbital parameters of those planets. Such studies can even help in the identification of additional companions in such systems. Here, we examine the dynamical stability of three planetary systems, orbiting HD 67087, HD 110014, and HD 133131A. We use the published radial velocity measurements of the target stars to determine the best-fitting orbital solutions for these planetary systems using the SYSTEMIC console. We then employ the *N*-body integrator MERCURY to test the stability of a range of orbital solutions lying within 3σ of the nominal best fit for a duration of 100 Myr. From the results of the *N*-body integrations, we infer the best-fitting orbital parameters using the Bayesian package ASTROEMPEROR. We find that both HD 110014 and HD 133131A have long-term stable architectures that lie within the 1σ uncertainties of the nominal best fit to their previously determined orbital solutions. However, the HD 67087 system exhibits a strong tendency towards instability on short time-scales. We compare these results to the predictions made from consideration of the angular momentum deficit criterion, and find that its predictions are consistent with our findings.

Key words: planets and satellites: dynamical evolution and stability – stars: individual: (HD 67087) – stars: individual: (HD 110014) – stars: individual: (HD 133131A) – stars: planetary systems.

1 INTRODUCTION

Exoplanets – planets orbiting stars other than the Sun – are most often identified through indirect means. We observe a star with periodic behaviour that would otherwise be unexpected and conclude that the best explanation is the presence of one (or more) planet(s). We direct the interested reader to Perryman (2018) for a summary of the various exoplanet detection techniques. By piecing together the observations of the unexpected behaviour, it is possible to constrain, to some degree, the orbit and physical nature of the planets in question. Such inference is, however, not perfect – particularly when the planets in question have been detected through observations of the ‘wobble’ of their host star, as is the case for planets found using the radial velocity technique (e.g. Mayor & Queloz 1995; Butler & Marcy 1996; Bonfils et al. 2013; Wittenmyer et al. 2014b; Christiansen et al. 2017), or candidate planets claimed on the basis of binary star eclipse timing variability (e.g. Lee et al. 2009; Beuermann et al. 2010; Potter et al. 2011; Qian et al. 2011).

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The accurate determination of the (minimum) masses and orbits of newly discovered exoplanets provides the key data by which we can understand the variety of outcomes of the planet formation process. As such, it behooves us to ensure that exoplanet catalogues contain information that is as accurate and realistic as possible. Such accurate solutions do not just enable us to properly ascertain the distribution of planets at the current epoch – they also provide an important window into the history of the planetary systems we discover (e.g. Ford 2014; Pu & Wu 2015; Fulton et al. 2017; Wu et al. 2019), and allow us to predict and plan follow-up observations through population synthesis models (e.g. Hasegawa & Pudritz 2013; Mordasini 2018; Dulz et al. 2020). For example, the migration and mutual gravitational interaction of planets have been identified as being of critical importance to both the observed architectures and predicted long-term stability of the menagerie of known multiplanet systems, heretofore identified through radial velocity and transit surveys (e.g. Pierens & Nelson 2008; Wittenmyer, Horner & Tinney 2012c; Mills et al. 2016; Gillon et al. 2017; Hamers et al. 2017; Mustill, Davies & Johansen 2017; Childs et al. 2019).

However, the accuracy of orbital parameters of the planetary companions presented in discovery works is frequently limited by the time period covered by the observations that led to the discovery, which are often enough to claim detection and little more (Wittenmyer et al. 2016b, 2020). Long-term follow-up of known planet host systems is therefore desirable to refine the orbital parameters for known companions, to infer the presence of additional companions at lower masses and/or larger semimajor axes (e.g. Becker & Adams 2017; Denham et al. 2019; Horner et al. 2019; Kane et al. 2019; Rickman et al. 2019; Wittenmyer et al. 2019), and to disentangle the complex signals produced by planets on resonant (e.g. Anglada-Escudé, López-Morales & Chambers 2010; Wittenmyer et al. 2012c, 2014a, 2016a) or eccentric orbits (e.g. Wittenmyer et al. 2013, 2017c). Equally, due to the relative paucity of data on which planet discoveries are often based, it is possible for those initial solutions to change markedly as more data are acquired. The ultimate extension of this is that, on occasion, the process by which planetary solutions are fit to observational data can yield false solutions – essentially finding local minima in the phase space of all possible orbital solutions that represent a good theoretical fit to the data while being unphysical. It is therefore important to check the dynamical feasibility of multiplanet solutions that appear to present a good fit to observational data – particularly in those cases where such solutions invoke planets on orbits that offer the potential for close encounters between the candidate planets.

Following this logic, we have in the past tested the stability of multiplanet systems in a variety of environments, including around main-sequence (Marshall, Horner & Carter 2010; Wittenmyer et al. 2012b, 2015, 2017a), evolved (Wittenmyer et al. 2017b,c; Marshall et al. 2019), and post-main-sequence stars (Horner et al. 2011, 2012, 2013; Wittenmyer et al. 2012a; Mustill et al. 2013). In some cases, our results confirmed that the proposed systems were dynamically feasible as presented in the discovery work, while in others our analysis demonstrated that alternative explanations must be sought for the observed behaviour of the claimed 'planet host' star (e.g. Horner et al. 2011, 2013). To ensure that our own work remains robust, we have incorporated such analysis as a standard part of our own exoplanet discovery papers. We test all published multiplanet solutions for dynamical stability before placing too great a confidence in a particular outcome. As an extension to this approach, we presented a revised Bayesian method to the previously adopted frequentist stability analysis in Marshall et al. (2019), and demonstrated the consistency between these approaches.

Rather than using direct dynamical simulations, the stability of a planetary system can also be inferred from a criterion derived from the planetary masses, semimajor axes, and conservation of the angular momentum deficit (AMD; Laskar 2000; Laskar & Petit 2017). AMD can be interpreted as measuring the degree of excitation of planetary orbits, with less excited orbits implying greater stability. The definition of AMD stability has been revised to account for the effect of mean motion resonances and close encounters on orbital stability (Petit, Laskar & Boué 2017, 2018). Of the systems examined in this work, HD 110014 has been identified as being weakly stable, while HD 67087 and HD 133131A are both considered unstable according to AMD (see figs 6 and 7 of Laskar & Petit 2017). In our previous dynamical studies, we find good agreement between the stability inferred from AMD and our dynamical simulations with 13 systems in common between them, of which 9 were classified unstable and 4 stable, 1 marginally so (Horner et al. 2012; Robertson et al. 2012; Wittenmyer et al. 2012a,b,c, 2014a,c, 2015, 2016a; Endl et al. 2016). In this paper, we examine the dynamical stability of the three multiplanet systems, HD 67087, HD 110014, and HD 133131A, as a critical examination of their stability and a further test of the reliability of AMD for the identification of instability in exoplanet systems.

Table 1. Table of references for the radial velocities data used in this work.

Target	References
HD 67087	Harakawa et al. (2015)
HD 110114	de Medeiros et al. (2009)
HD 133131A	Teske et al. (2016)

HD 67087, observed as a part of the Japanese Okayama Planet Search programme (Sato et al. 2005), was discovered to host a pair of exoplanets by Harakawa et al. (2015). The candidate planets are super-Jupiters, with $m \sin i$ of $3.1M_{\rm Jup}$ and $4.9M_{\rm Jup}$, respectively. They move on orbits with (a, e) of (1.08, 0.17) and (3.86, 0.76), respectively, which would place the outer planet among the most eccentric Jovian planets identified thus far. The authors noted that the orbit and mass of the outer planet are poorly constrained.

HD 110014 was found to host a planet by de Medeiros et al. (2009); the second companion was identified through re-analysis of archival spectra taken by the FEROS instrument (Kaufer & Pasquini 1998) looking to derive an updated orbit for planet b (Soto, Jenkins & Jones 2015). The two candidate planets have super-Jupiter masses, and Soto et al. (2015) cautioned that the proposed second planet was worryingly close in period to the typical rotation period of K giant stars. However, their analysis of the stellar photometry was inconclusive in identifying its activity as the root cause for the secondary signal.

HD 133131A's planetary companions were reported in Teske et al. (2016), based on precise radial velocities primarily from the Magellan Planet Finder Spectrograph (Crane, Shectman & Butler 2006; Crane et al. 2008, 2010). Their data supported the presence of two planets, where the outer planet is poorly constrained due to its long period. Teske et al. (2016) ran a single dynamical stability simulation on the adopted solution and found it to remain stable for the full 10^5 yr duration. The authors presented both a low- and high-eccentricity solution, reasoning that in a formal sense the two solutions were essentially indistinguishable. They favoured the low-eccentricity model ($e_2 = 0.2$) for dynamical stability reasons. There is precedent in the literature for this choice, since it does happen that the formal best fit can be dynamically unfeasible while a slightly worse fit pushes the system into a region of stability (e.g. Mustill et al. 2013; Trifonov et al. 2014; Wittenmyer et al. 2017a).

The remainder of the paper is laid out as follows. We present a brief summary of the radial velocity observations and other data (e.g. stellar parameters) used for our re-analysis in Section 2 along with an explanation of our modelling approach. The results of the reanalyses for each target are shown in Section 3. A brief discussion of our findings in comparison to previous work on these systems is presented in Section 4. Finally, we present our conclusions in Section 5.

2 OBSERVATIONS AND METHODS

2.1 Radial velocity data

We compiled radial velocity values from the literature for the three systems examined in this work; the origins of these data are summarized in Table 1.

Table 2. Planetary orbital parameters based on SYSTEMIC fits to radial velocity data. Semimajor axes were calculated using measured orbital periods and stellar masses taken from the NASA Exoplanet Archive.

	HD 67087 b	HD 67087 c	HD 110014 b	HD 110014 c	HD 133131A b	HD 133131A c
Amplitude (m s^{-1})	74.0 ± 3.0	54.0 ± 4.0	36.949 ± 0.750	5.956 ± 1.617	135.315 ± 3.640	64.660 ± 3.966
Period (d)	352.3 ± 1.7	2380^{+167}_{-141}	877.5 ± 5.2	130.125 ± 0.096	648. ± 3	5342^{+7783}_{-2009}
Mean anomaly (deg)	35^{+20}_{-16}	94^{+28}_{-35}	155 ± 4	231 ± 3	265 ± 11	188^{+104}_{-123}
Longitude (deg)	281^{+18}_{-15}	256 (fixed)	41 ± 3	302 ± 4	$16.6^{+4.7}_{-4.5}$	110^{+25}_{-43}
Eccentricity	$0.18^{+0.07}_{-0.06}$	0.51 (fixed)	0.259 ± 0.017	0.410 ± 0.022	0.340 ± 0.032	$0.63^{+0.25}_{-0.20}$
$M \sin i$ ($M_{\rm Jup}$)	$3.10^{+0.15}_{-0.14}$	$3.73^{+0.47}_{-0.45}$	10.61 ± 0.25	3.228 ± 0.098	1.418 ± 0.036	$0.52^{+0.45}_{-0.17}$
Semimajor axis (au)	1.08	3.87	2.32	0.65	1.44	5.88

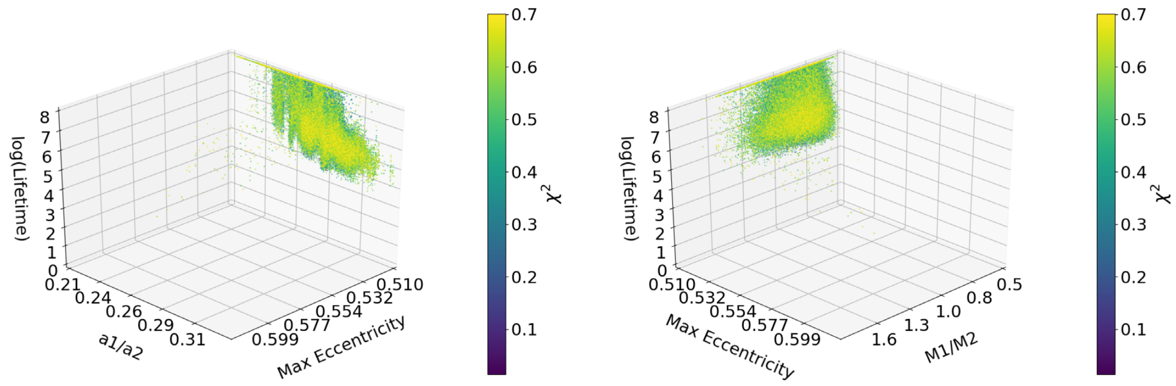

Figure 1. Visualization of the dynamical stability of the HD 67087 planetary system. On the left we show the log(lifetime) as a function of the largest initial eccentricity fit to HD 67087 b and HD 67087 c, and the ratio of their orbital semimajor axes, while on the right we show the log(lifetime) as a function of the largest initial eccentricity fit and the mass ratio between HD 67087 b and HD 67087 c. The colour bar shows the goodness of fit (χ^2) of each solution tested. We find no stable solutions that last the full 100 Myr duration of the dynamical simulations close to the nominal best-fitting orbital solution for the planets, with the only stable solutions lying at the extreme edges of the parameter space towards low eccentricities, large separations, and low mass ratios.

2.2 Modelling

To test the dynamical stability of these proposed planetary systems, we follow the updated dynamical methodology outlined in our previous work (Wittenmyer et al. 2017a; Marshall et al. 2019).

In brief, we perform a fit to the published velocity data using the SYSTEMIC console (Meschiari et al. 2009), the results of which are presented in Table 2. We then use the Markov Chain Monte Carlo (MCMC) tool within SYSTEMIC to explore the parameter space about the best fit. The MCMC chain runs for 10^7 steps, discarding the first 10 000, and we then draw the trial solutions for our dynamical stability simulations from these posteriors. Using these data, we populate three 'annuli' in χ^2 space corresponding to the ranges $0-1\sigma$, $1-2\sigma$, and $2-3\sigma$ from the best fit. Each annulus contains 42 025 unique realizations drawn from the MCMC chain. The innermost annulus was drawn from the lowest 68.3 per cent of all χ^2 values, the middle annulus contained the next best 27.2 per cent of values, and the outer annulus contained the worst 4.5 per cent of solutions (i.e. those falling $2-3\sigma$ away from the best fit). The result is a set of 'clones' that fall within 3σ of the best-fitting solution, thus representing a reasonable region of parameter space within which we explore the dynamical stability of the proposed planetary system, using the constraints afforded by the existing observational data.

We then proceed to perform lengthy dynamical simulations of each of the 126 075 solutions generated by this method. We used the hybrid integrator within the *N*-body dynamics package MERCURY (Chambers 1999) to integrate the solutions forward in time for a period of 100 Myr. The simulations are brought to a premature end if either of the planets being simulated is ejected from the system, is flung in to the central star, or if the two planets collide with one another. When such events occur, the time at which the collision or ejection occurred is recorded, giving us the lifetime for that particular run. As such, our suite of simulations yields 126 075 tests of the candidate planetary system, allowing us to study how its stability varies as a function of the particular details of the solution chosen to explain the observational data.

We determine the best-fitting parameters and uncertainties for each system using the code Exoplanet Mcmc Parallel tEmpering Radial velOcity fitteR[1] (ASTROEMPEROR), which uses thermodynamic methods combined with MCMC. Our approach has previously been established and described in Marshall et al. (2019) and Wittenmyer et al. (2019). We summarize the input values and constraints used in the fitting presented in this work for the sake of reproducibility. Given that our goal was to test the feasibility of the exoplanetary systems as presented in the literature, we restricted ASTROEMPEROR to consider zero, one, or two planetary signals in the radial velocity data; dynamical configurations with additional planetary companions in orbits that could mimic a single planetary companion, e.g. two resonant planets looking like a single eccentric planet (for a total of three planetary companions), were not

[1] https://github.com/ReddTea/astroEMPEROR

considered in this analysis. The planetary fitting parameters were the orbital period (P), line-of-sight mass ($M \sin i$), orbital eccentricity (e), longitude of periastron (ω), and mean anomaly (M). We also include an additional jitter term when fitting the data. We initialized the locations of the walkers in the MCMC fitting at their best-fitting values from the SYSTEMIC console fit, plus a small random scatter. The priors on each parameter were flat and unbounded, i.e. with uniform probability between $\pm\infty$, except for the orbital eccentricities that had folded Gaussian priors, and the jitter term, which was a Jeffries function (but still unbound between $\pm\infty$). The parameter space was surveyed by 150 walkers at 5 temperatures over 15 000 steps, with the first 5000 steps being discarded as the burn-in phase.

3 RESULTS

3.1 HD 67087

The HD 67087 system is catastrophically unstable, as illustrated by the results of our stability analysis in Fig. 1. In this plot, it is clear that the most stable solutions cluster towards the largest ratios of semimajor axes and the smallest eccentricities. Even in this limit, the longest lived solutions that plausibly represent the observations are still only stable for 10^6 yr, out of a total integration time of 10^8 yr. This leads us to the interpretation that the HD 67087 system, as inferred from the available radial velocity data, is dynamically infeasible. Given this high degree of instability, we do not attempt to determine a global best-fitting solution for the system parameters.

3.2 HD 110014

The HD 110014 system is found to be dynamically stable, with a broad swathe of parameter space centred on the nominal solution producing system architectures that last for the full 10^8 yr of our dynamical integrations. We show the results of the stability analysis, sampling the 3σ parameter space around the nominal orbital solution determined from the radial velocities in Fig. 2. The results of the Bayesian analysis, showing what we infer to be the global best-fitting parameters for the system, are presented in Fig. 3.

3.3 HD 133131A

The HD 133131A system shows a very complex parameter space in the stability plots. As one would expect, the stability of the system generally increases towards lower orbital eccentricities and lower mass ratios between the two planetary components. The overall stability appears to be insensitive to the ratio of the semimajor axes for the planets, with long-lived solutions possible across the full range of values probed for this parameter. Interestingly, we demonstrate that stable architectures for the planetary system exist in both the high and low orbital eccentricity scenarios for the system. We show the results of the stability analysis, sampling the 3σ parameter space around the nominal orbital solution determined from the radial velocities, in Fig. 4. The results of the Bayesian analysis, showing what we infer to be the global best-fitting parameters for the system, are presented in Fig. 5. Further observations to refine the planet properties of this system will be required to definitively characterize its dynamical stability.

4 DISCUSSION

The results of our dynamical modelling for the three systems considered in this work, HD 67087, HD 110014, and HD 133131A, show three distinctly different outcomes. For the first system tested, HD 67087, we find no orbital solutions that exhibit long-term dynamical stability. As a result, we are forced to conclude that, if the planets proposed to orbit that star are real, they must move on orbits significantly different from those proposed in the discovery work and sampled in our simulations. It seems likely that new radial velocity observations of HD 67087, extending the temporal baseline over which the star has been observed, will yield fresh insights to the system – either significantly constraining and altering the proposed orbit for the outermost planet or even revealing that the eccentric solution is in fact the result of multiple unresolved planets at large orbital radii. Such an outcome is far from unusual – and, indeed, it is often the case that, with more data, a single eccentric planet seen in RV data is resolved to actually be two planets moving on near-circular orbits (e.g. Wittenmyer et al. 2013, 2019). For now, however, we can do no more than to call the existence of HD 67087 c into question, pending the acquisition of such additional data.

In contrast to the instability of HD 67087, our simulations of the HD 110014 system reveal that the best-fitting solution for that two-planet system lies in a broad region of strong dynamical stability. In this case, our simulations simply reveal that the system, as proposed in the discovery work, is dynamically feasible – and in a sense, the simulations add little beyond that.

The case of HD 133131A is somewhat more interesting. Here, our simulations reveal that solutions that fit the observational data can exhibit both strong dynamical stability and extreme instability (with dynamical lifetimes of just a few years). Both the high- and low-eccentricity solutions considered in Teske et al. (2016) can produce scenarios that are stable for the full 100 Myr of our simulations. In both the high- and low-eccentricity cases, the stable solutions cluster around the least eccentric available scenarios. The more widely separated the two planets, the more eccentric their orbits can be before instability occurs – a natural result of the stability being driven by the minimum separation between the planets, rather than their orbital semimajor axes. The more widely the semimajor axes of the orbits are spaced, the more eccentric they must be to bring the planets into close proximity. These results show once again the benefits inherent to such dynamical analysis – reminding us how studying the dynamical evolution of a given system can help to provide stronger constraints on the orbits of the planets contained therein than is possible by studying the observational data on their own.

A comparison of our results to the analysis of the AMD stability criterion presented in Laskar & Petit (2017) shows agreement between the two different techniques for the dynamical stability of the three systems. While HD 67087 and HD 110014 are, respectively, very clear-cut cases of an unstable and a stable system, HD 133131A exhibits a more complex behaviour. HD 133131A may be dynamically stable, but the inferred lifetime for the planetary system as proposed is sensitive to the chosen initial conditions; this system therefore represents an edge case of stability where limitations of available data and the respective analyses provide no clear answer to the veracity of the previously inferred planetary system.

Combining these new results with our previous dynamical analyses, as summarized in the introduction, we may consider that the AMD criterion is a reliable estimator of stability for planetary

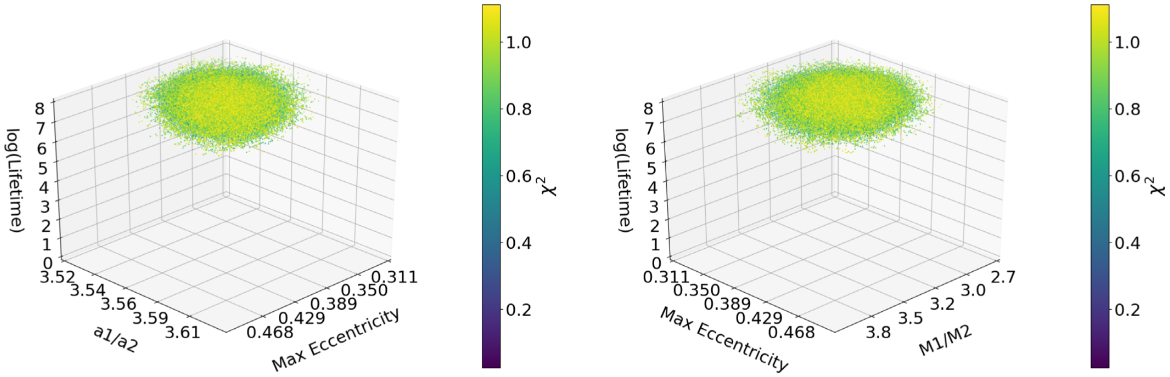

Figure 2. Visualization of the dynamical stability of the HD 110014 planetary system. On the left we show the log (lifetime) as a function of the largest initial eccentricity fit to HD 110014 b and HD 110014 c and the ratio of their orbital semimajor axes, while on the right we show the log (lifetime) as a function of the largest initial eccentricity fit and the mass ratio between HD 110014 b and HD 110014 c. The colour bar shows the goodness of fit (χ^2) of each solution tested. We find stable solutions that last the full 100 Myr duration of the dynamical simulations close to the nominal best-fitting orbital solution for the planets.

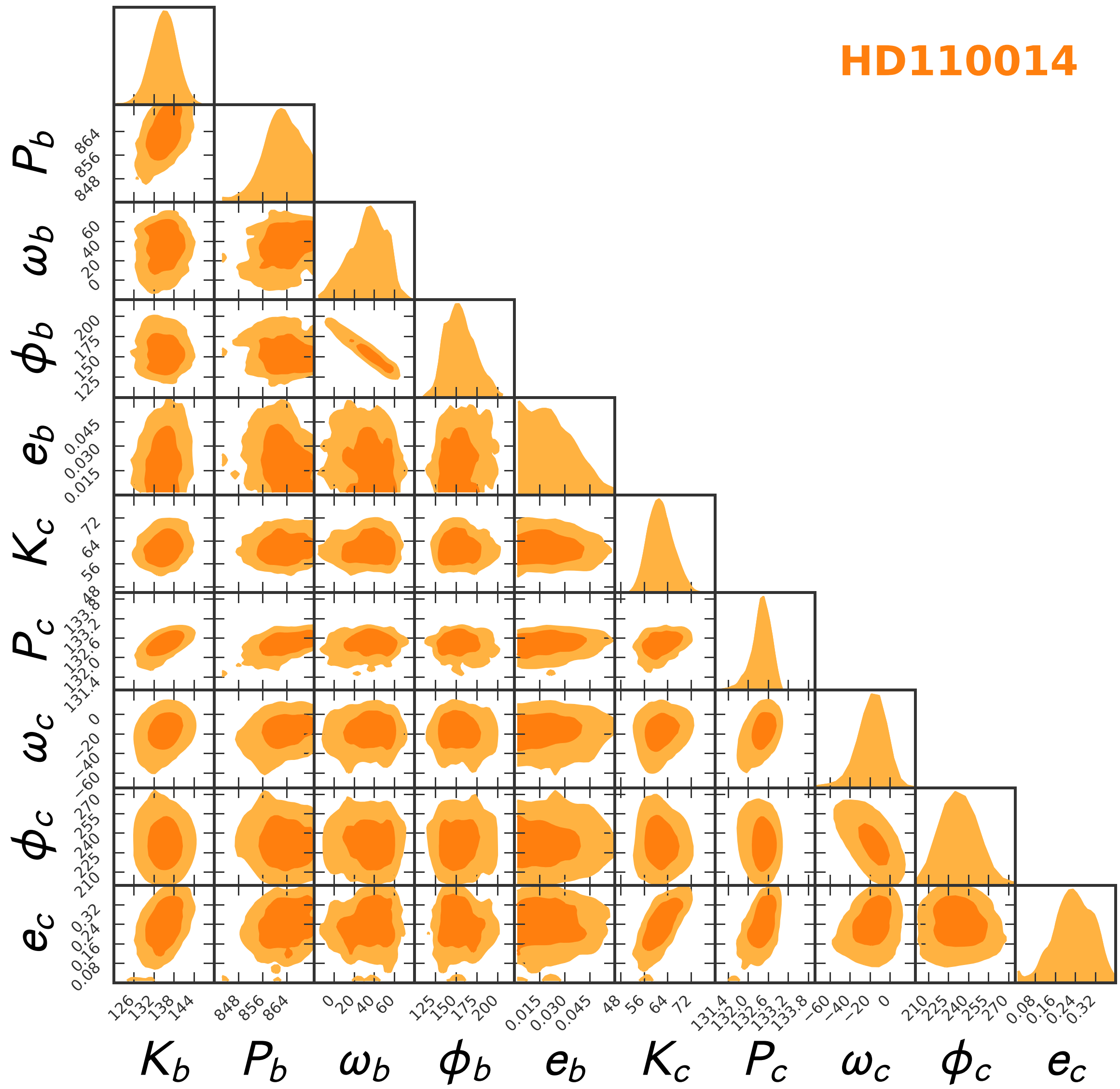

Figure 3. Bayesian posterior distributions of HD 110014 b's and HD 110014 c's orbital parameters derived from ASTROEMPEROR. From left to right (top to bottom), the parameters are K_b, P_b, ω_b, ϕ_b, e_b, K_c, P_c, ω_c, ϕ_c, and e_c. Credible intervals are denoted by the solid contours with increments of 1σ.

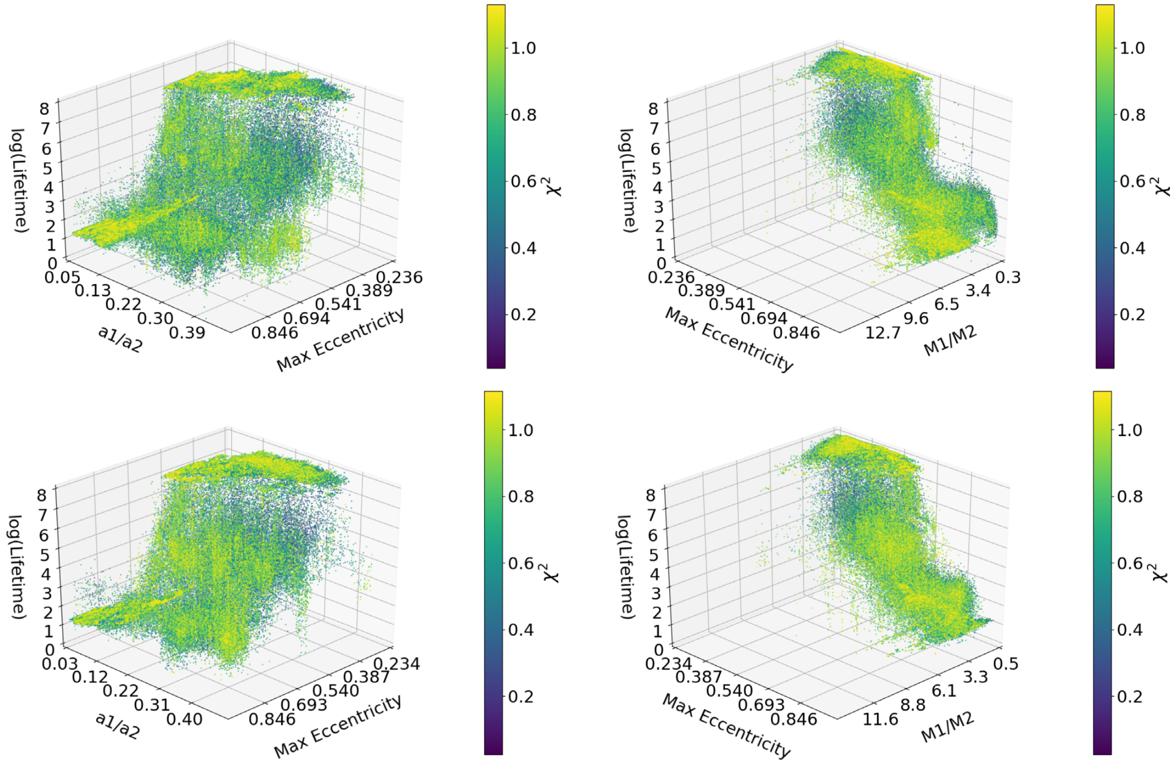

Figure 4. Plots of the dynamical stability of the HD 133131A planetary system for both the high-eccentricity (top) and low-eccentricity (bottom) orbital solutions. On the left we show the log(lifetime) as a function of the largest initial eccentricity fit to HD 133131A b and HD 133131A c, and the ratio of their orbital semimajor axes, while on the right we show the log(lifetime) as a function of the largest initial eccentricity fit and the mass ratio between HD 133131A b and HD 133131A c. The colour bar shows the goodness of fit (χ^2) of each solution tested. The stability revealed by our dynamical simulations is complex, with regions of both extreme stability (log(lifetime) $\sim$ 100 Myr) and instability (log(lifetime) $\sim$ 100 yr) lying within the 3σ reach of the nominal best-fitting orbital parameters.

systems. There are 13 systems (out of 131 considered in that work) from Laskar & Petit (2017) that have had dynamical modelling of their stability. In Laskar & Petit (2017), a planetary system is considered strongly stable if all planet pairs have β values less than 1, such that collisions are impossible while weakly stable planetary systems are those in which the innermost planet might collide with the star without disrupting the remainder of the planetary system. In five systems, both the AMD criterion and dynamical modelling agree on their dynamical stability (HD 142, HD 159868, NN Ser (AB), GJ 832, and HD 110014); the planets in each of these systems are dynamically well separated and therefore not strongly interacting (Horner et al. 2011; Wittenmyer et al. 2012b, 2014c; this work). Six systems are unstable according to the AMD criterion with values of β in the range 1–5 for the planet pair (HD 155358, 24 Sex, HD 200964, HD 73526, HD 33844, and HD 47366), but all are in mean motion resonances and have been demonstrated to be dynamically stable through *N*-body simulations (Robertson et al. 2012; Wittenmyer et al. 2012c, 2014a, 2016a; Marshall et al. 2019). The remaining two systems (HD 67087 and HD 133131A) are dynamically unstable in both the AMD and dynamical analysis (this work). However, dynamical analysis of the HD 133131A system reveals regions of dynamical stability consistent with the observed radial velocities, prompting the need for further investigation of this system and its architecture. Neither of these two unstable planetary systems have β values radically different from those of the planetary systems in resonance, or each other, such that determining their stability can only be carried out using dynamical simulations. The existence of such systems in the known planet population as demonstrated in our analysis therefore showcases the necessity of performing long duration dynamical analyses of proposed planetary system architectures to reveal the complex dynamical interplay between high-mass planets, the evolution of their orbital elements, and determine what constraints this places on the available parameter space for the endurance of the proposed planetary system over its lifetime.

5 CONCLUSIONS

We re-analysed the dynamical stability of the exoplanet systems around HD 67087, HD 110014, and HD 133131A, using available radial velocity data. These three planetary systems have poorly constrained orbital parameters and had previously been identified as being potentially unstable. We combine a determination of the best-fitting orbital parameters from least-squares fitting to the data with *N*-body simulations to determine the global best-fitting solution for the planetary system architectures, and thereafter determine the probability distribution of the orbital solutions through Bayesian inference.

Our dynamical analysis confirms that the published planetary system parameters for HD 67087 b and HD 67087 c are dynamically unstable on very short time-scales, and we must conclude that the system, as published, is dynamically unfeasible. As more data are collected for the HD 67087 system, it seems likely that the true nature of the candidate planets therein will be revealed, and that fu-

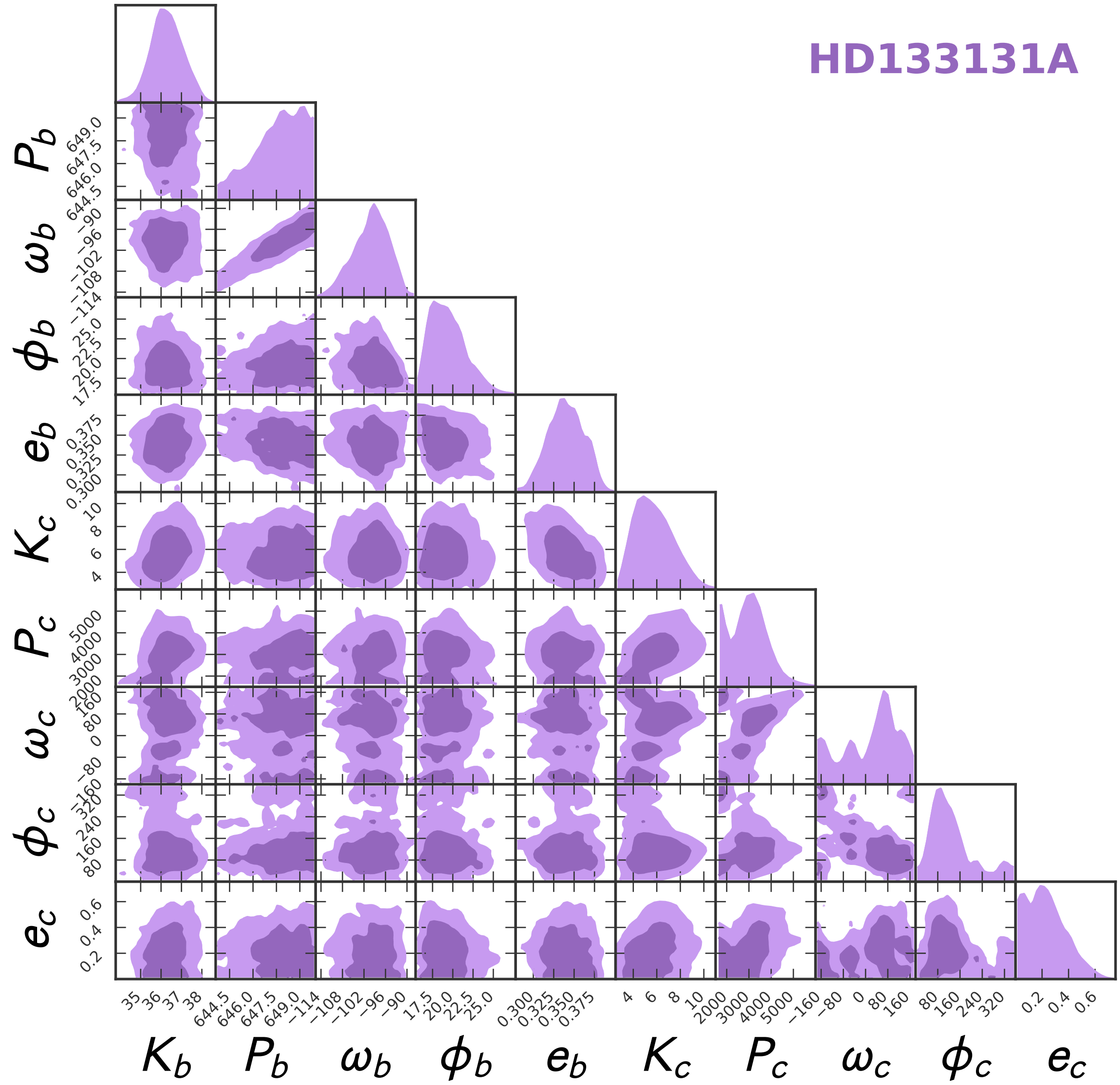

Figure 5. Bayesian posterior distributions of HD 133131A b's and HD 133131A c's orbital parameters derived from ASTROEMPEROR. From left to right (top to bottom), the parameters are K_{b}, P_{b}, ω_{b}, ϕ_{b}, e_{b}, K_{c}, P_{c}, ω_{c}, ϕ_{c}, and e_{c}. Credible intervals are denoted by the solid contours with increments of 1σ.

Table 3. Results from ASTROEMPEROR exploration of parameter space around SYSTEMIC nominal best-fitting values for planetary companions to HD 133131A and HD 110014.

	HD 133131A b	HD 133131A c	HD 110014 b	HD 110014 c
Amplitude (m s^{-1})	36.949 ± 0.750	5.956 ± 1.617	135.315 ± 3.640	64.660 ± 3.966
Period (d)	647.816 ± 1.575	3205.648 ± 948.063	865.206 ± 6.170	132.431 ± 0.279
Phase (deg)	261.620 ± 4.850	31.734 ± 98.433	43.753 ± 72.179	341.373 ± 64.346
Longitude (deg)	18.550 ± 2.165	113.777 ± 81.302	146.633 ± 17.229	236.903 ± 16.452
Eccentricity	0.341 ± 0.021	0.263 ± 0.145	0.011 ± 0.015	0.294 ± 0.076
$M \sin i$ (M_{Jup})	1.428 ± 0.099	0.388 ± 0.124	10.622 ± 0.757	2.581 ± 0.247
Semimajor axis (au)	1.435 ± 0.046	4.153 ± 0.800	2.350 ± 0.075	0.668 ± 0.023
Jitter (m s^{-1})	3.557 ± 1.254	0.466 ± 0.419	6.060 ± 1.856	13.350 ± 1.492
Offset (m s^{-1})	-9.333 ± 4.787	12.321 ± 7.543	52.575 ± 4.737	72.198 ± 4.541
MA coefficient	0.714 ± 0.531	0.466 ± 0.419	0.697 ± 0.214	13.350 ± 1.492
MA time-scale (d)	4.158 ± 2.815	12.321 ± 7.543	9.793 ± 2.488	72.198 ± 4.541
Acceleration (m s^{-1} yr^{-1})	−1.435	–	−21.620	–

ture planetary solutions for that system will veer towards dynamical stability as the planetary orbits become better constrained.

In the case of HD 110014 b and HD 110014 c we demonstrate that the system parameters can be dynamically stable for the full duration of our 100 Myr integrations. The third system, HD 133131A, exhibits much more complex behaviour, with HD 133131A b and HD 133131A c being strongly unstable over much of the parameter space exhibited in this work, including the region encompassing the nominal best fit to the orbital parameters. In agreement with previous analysis of this system, we strongly disfavour a high-eccentricity orbital solution for planet c. Additional observations of this system will be required to more precisely determine the planetary properties for HD 133131A b and HD 133131A c and thereby categorically rule on the plausibility of the proposed planetary system.

These results demonstrate the complementarity of various techniques to deduce the stability of planetary systems, with good agreement between the results of our various works and those of the AMD approach. We highlight the appropriateness of dynamical simulations for determining the long-term stability of planetary systems in the presence of strongly interacting planets, which although costly in a computing sense capture the full essence of planetary interaction in such systems that is not possible with other techniques. We finally assert that the orbital parameters for these three systems that have been determined in this work (as summarized in Table 3) should be the accepted values adopted by exoplanet archives or elsewhere. This work is thus one additional thread in the tapestry of cross-checking of published results through various means that ensures the reliability of archival information on planetary properties and the architectures of planetary systems that are essential to inform models of the formation and evolution of the exoplanet population (e.g. Childs et al. 2019; Denham et al. 2019; He, Ford & Ragozzine 2020; Volk & Malhotra 2020).

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