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Robust Design and Optimisation of a Radial Turbine Within a Supercritical CO_2 Solar Brayton Cycle

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1. Abstract
The generation of solar thermal power is dependent upon the amount of sunlight exposure, as influenced by the day-night cycle and seasonal variations. In this paper, robust optimisation is applied to the design of a power block and turbine, which is generating 30MWe from a concentrated solar resource of 560°C. The robust approach is important to attain a high average performance (minimum efficiency change) over the expected operating ranges of temperature, speed and mass flow. The final objective function combines the turbine performance and efficiency weighted by the off-design performance. The resulting robust optimisation methodology as presented in the paper gives further information that greatly aids in the design of non-classical power blocks through considering off-design conditions and resultant performance.
2. Keywords: Radial Turbine, Off-design, Supercritical CO2, Robust Optimisation.

3. Introduction
The power block is sensitive to operational parameters including inlet temperature, mass flow, which is reflected by the efficiency of the turbine [1]. Robust optimisation is an approach to solve optimisation problems under uncertainty in the knowledge of the underlying parameters. The application of this approach is necessary to determine the parametric factors for the optimisation of turbo machinery within a solar thermal power system. Optimisation of the mean performance is achieved by minimising its variation, through optimising the product and process design to make the performance minimally sensitive to the various causes of variation.
Literature covers many methods to perform robust optimisation and concentrates on the probability distribution near the mean values [2]. These parameters may include the variation of efficiency (η) in response to temperature, pressure, mass flow, rotational speed and turbine load. There are many approaches to consider off-design performance as demonstrated by Bonaiuti [3] who applied a multipoint approach applied to the inverse design method for the design of radial turbomachinery.
Studies of the off-design performance for turbomachinery has been done by authors at different levels from the analytical turbine model such as that by Fiaschi [4], to a full three-dimensional computational fluid dynamics analysis by Sauret [1]. These studies considered the variations of total-to-total pressure ratio (Π_{tt}), total temperature at inlet ($T_{T,in}$), rotational speed (ω) as well as some operational and design variables including blade-to-jet speed ratio. Many authors have applied full robust optimisation starting from a response surface approximation [5], the application of robust optimisation within evolutionary algorithms [6], to the extension to decision trade-offs [7].
This paper is part of the ongoing discussion on the gaps between the design of the thermodynamic cycle and the turbine design, addressing the robust optimisation approach useful to the analysis of both systems. The aim of this paper is to analyse the design of a radial turbine in a supercritical CO_2 cycle to determine which parameters the design is sensitive to when considering the variability of the solar-thermal resource.

4. System and Geometry Description
Radial turbomachinery are often used for low power systems, and are incredibly robust – there are many ongoing studies to determine their applicability to the current system and similar systems such as that by Sandia Lab [8]. Both supercritical CO_2 and radial turbomachinery have been identified based on their better overall plant economics due to the high power conversion efficiency given a moderate inlet temperature, compact size, and use of more common materials for construction [8]. The power block design for turbomachinery is a well-studied area, and the modelling using analytical heuristic models is well understood within literature [9].
The present work is considering the application of robust optimisation to a single turbine, fluid and working cycle configuration with operational parameters given in Table 1. Uncertainty is applied to the turbine speed, inlet temperature and mass flow to determine the response of the turbine similar to Sauret [1]. Each variable is studied at a range wide enough to capture the trend of the measured variables; the variation is indicated alongside the operating parameters (Table 1).

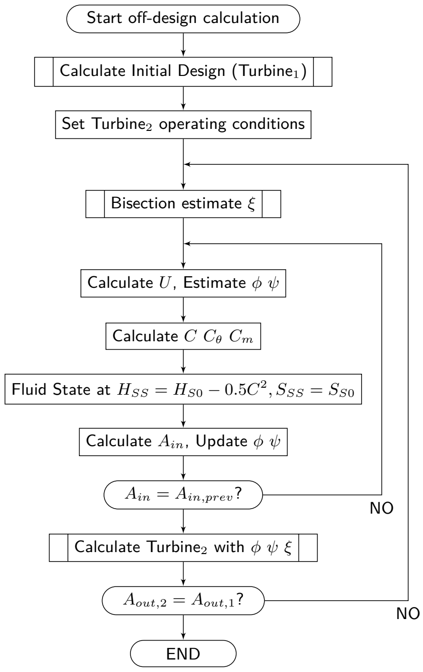

Figure 1: Off-design calculation process

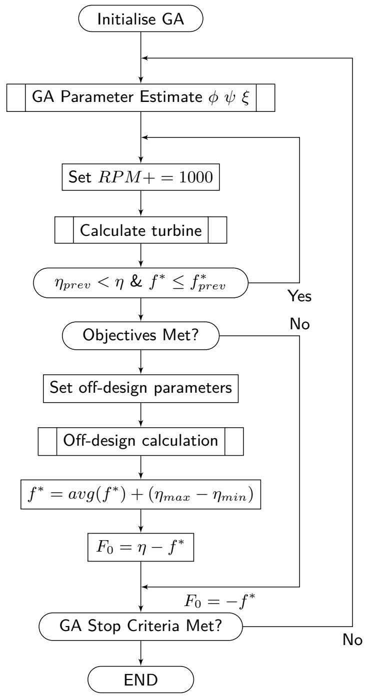

Figure 2: Robust optimisation process

The current power block design method was developed using the REFPROP library for thermodynamics [10] incorporating the real gas functions within the sizing and performance calculations [11]. The method primarily draws from that by Baines [9] with some modifications from Aungier [12] and Erbas [13] for the incidence and loss models respectively. There are numerous constraints on the design of radial inflow turbines by many authors as tabulated in Table 2 [9, 12, 13] in Section 6.1. The combined effect of the number of constraints is significant, and there is great difficulty when considering off-design performance being able to satisfy all of them especially the relative inflow angle ($\beta_{b,in}$). Constraint handling in the present work is implemented as an objective function with a penalty (f^*) value that is progressively reduced through the optimisation approach to zero (representing no deviation from expected value A_i to the present value F_i for objective i).

$$f^* = \sum_{i=1}^{n} f_i^*, \text{ where } f_i^* = e^{\left|\delta_{\text{objective,i}}\right|} \tag{1}$$

$$\delta_{\text{objective,i}} = \begin{cases} \dfrac{A_i - F_i}{A_i} & \text{for single bound} \\ \dfrac{A_i - F_i}{A_{max,i} - A_{min,i}} & \text{for range bound} \end{cases} \tag{2}$$

5. Problem Definition and Optimisation Approach

5.1. Single Objective Optimisation

The present work uses a combined single objective approach built from two separate approaches; firstly, a heuristic single objective optimisation algorithm PGAPACK [14] sets the non-dimensional geometric parameters (flow, stage and size coefficients). Secondly, a goal seeking method determines the ideal operation speed of the turbine that maximises efficiency without compromising the design as measured by a number of equality constraints (given in Table 1). The ideal operating speed is found through redesigning the turbine at different turbine speeds (RPM) within the optimisation loop, where the ideal RPM is determined as when:

- The efficiency is at a maximum value, and
- The penalty function does not deteriorate

The objective function is set based on the following conditions:

1. No deviation from expected parameter constraints and ranges given in Table 1, and

2. Minimum efficiency is met and does not exceed physical limits

If one of these should be violated the objective function is set to a negative value which is calculated as the exponential sum of constraint violations (see Section 4). Otherwise, the objective function is the turbine efficiency or a composition as described in Section 5.3.

Table 1: Parameter values and design range

Parameter	Nominal Value	Off-design range
Mass flow $\dot{m}$ [kg/s]	422.3	+- 100 kg/s
Temperature $T_{t,in}$ [C]	560	+- 10%
Pressure $P_{t,in}$ [MPa]	20	-
Turbine Speed [RPM]	-	+- 20 %

Initial optimisation was done with PGAPACK to determine the absolute best possible design given the inlet conditions and is presented in Table 3 (Section 6). A design that achieve approximately 95% total-to-static efficiency was developed which meets the design criteria at the nominal conditions. The high efficiency is to be expected from the heuristic approach given that there are a number of simplifications to a realistic design [12]. Similarly performing designs have been presented by many authors which then using computational fluid dynamics have an efficiency reduction such as that by Sauret [1].

5.2. Off-Design method

The design of radial turbomachinery is based on the fluid change in velocity (Euler turbine equations), which are expected to change when operating at different conditions (e.g different speed). The method presented below follows the design point calculation method from Baines [9] which is then rearranged to determine the three non-dimensional design parameters flow coefficient (ϕ), loading coefficient (ψ) and sizing coefficient (ϵ). Additional parameters in the design include the turbine power, speed ratio (ξ), mass flow ($\dot{m}$), pitch speed (U), area (A) and the derived parameters are absolute flow vector ($\vec{C} = [C_m, C_\theta]$).

$$\Delta H_0 = \tfrac{\text{power}}{\dot{m}}, \quad \psi = \Delta H_0 / U_{in}^2, \quad \phi = \tfrac{\dot{m}}{A\rho\xi U_{in}} \tag{3}$$

$$\begin{array}{c} U_{in} = r \times \omega \\ C_\theta = U_{in}\psi, \quad C_m = \xi\psi U_{in}, \quad C = \sqrt{C_\theta^2 + C_m^2} \\ A = \dot{m}/\rho C_m \end{array} \tag{4}$$

The method is effective at matching exiting turbine parameters, however the value of ξ in Equation (4) is unable to be derived through a simple numerical approach and is calculated through the bisection method iterating the full turbine calculation the outlet sizing is matched. The resulting off-design process is shown in Figure 1.

5.3. Robust Optimisation

Design optimisation is a decision-making problem and involves the selection of the optimum design based on a certain criteria with a progressively refined design evolving from information about the turbine fitness and performance. The primary goal of the turbine optimisation is to determine not only the best design, but also the design that is most unaffected by variations in how the turbine is used. Variations are often minor as the turbines and the use of robust optimisation as described by [15] is formulated based on:
1. The data are uncertain / inexact,
2. The optimal solution, even if computed very accurately, may be difficult to implement accurately,
3. The constraints must remain feasible for all meaningful realisations of the data,
4. ‘Bad’ optimal solutions which are very sensitive to operating conditions are not uncommon

The present work adopts the same approach as [16], and integrates it into the genetic algorithm as described earlier. Here $x \in \mathbb{R}^n$ is a vector of design variables, f_0, f_i are constraint values, $u_i \in \mathbb{R}^k$ are disturbance vectors or parameter uncertainties. $U_i \subseteq \mathbb{R}^k$ is the uncertainty sets, will be a vector of known quantities with respect to the nominal case (See Table 1).

$$\begin{array}{ll} \text{maximise} & f_0(x) \\ \text{subject to} & f_i(x, u_i) <= 0, \forall u_i \in U_i, i = 1 \ldots m. \end{array} \tag{5}$$

Robust optimisation as implemented in the present code considers the optimality to be a function of the overall change (difference between minimum and maximum) of the decision variable over the range of parameter values tested. The present method is similar to Deb [17] (although only applied to a single objective) and considers the off-design efficiency response (η^*) to varied parameters:

$$\begin{array}{ll} \text{minimise} & \eta^*_{max} - \eta^*_{max}(x) \\ \text{subject to} & F_0^*(x, u_i) > 0, \forall u_i \in U_i, i = 1 \dots m. \end{array} \tag{6}$$

The method is subject to a single objective function (F_0) which captures the response of η and f^*. In the present work we are only concerned when the efficiency is lower than the current efficiency – it is observed from Figure 3 and Figure 4 that the turbine is sensitive to turbine speed and mass flow. The final objective therefore includes a composition between the design point efficiency, the off-design efficiency change (η^*). Finally, the constraint violation for the off-design method was included into the F_0 reduction where the average value of the cost function (Section 4) across the variation was subtracted from the objective function.

$$F_0 = \eta_{ts} - (\eta^*_{max} - \eta_{min}) - f^*_{avg} \tag{7}$$

The method for robust optimisation is captured in Figure 2 showing the inclusion of the robust objective F_0 into the initial optimisation routine.

6. Results

Preliminary variation study was performed to determine which parameters had the greatest effect on turbine performance. It is seen in Figure 3 that the turbine speed (RPM) has the greatest and most predictable effect on the performance. Proceeding the preliminary study, a parametric analysis was performed that explored how efficiency changed with RPM over the maximum range of flow and stage loading coefficients as illustrated in Figure 4. The parametric study shows regions where the designs are available and regions where those designs have high sensitivity of efficiency with respect to RPM ($\partial^2\eta/\partial RPM^2$). Figure 4 highlights regions of low sensitivity in yellow and higher sensitivities are deeper colours. The off-design method as detailed in Section 5.2 was then integrated into the optimisation routine to find a design with the least sensitivity to changes in RPM.

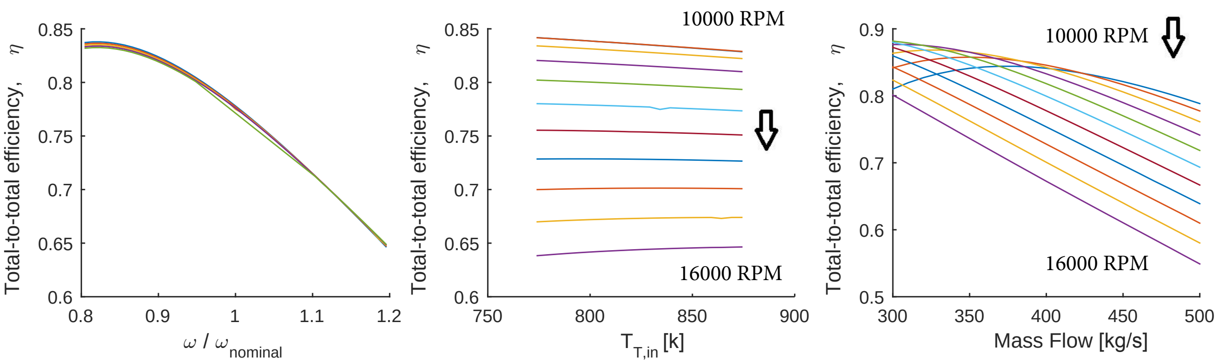

Figure 3: Results from initial off-design study

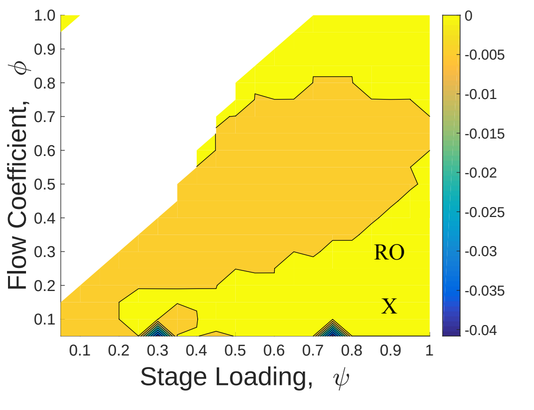

Figure 4: $\partial^2\eta/\partial RPM^2$ at design conditions

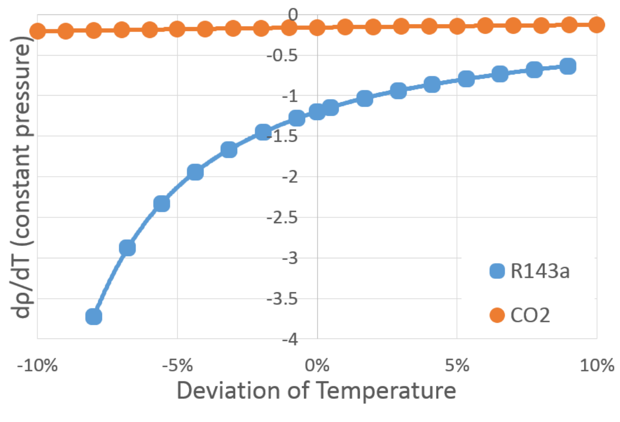

Figure 5: $d\rho/dT$ for CO_2 and R143a [10]

Table 2: Design constraints for nominal case

Design Variables	Minimum	Maximum
Velocity Constraints		
C_{in} [Mach]	-	0.9
C_{out} [Mach]	-	0.9
W_{in} [Mach]	-	0.9
Angle Constraints		
β_{in} [deg]	-44	-18
α_{in} [deg]	30	80
α_{exit} [deg]	-30	30
Size Constraints		
$r_{in}/r_{out,tip}$ [-]	1.2	-
$r_{out,hub}/r_{out,tip}$ [-]	0.4	-
$r_{in,hub}$ [mm]	10.1	-
b_{in} [mm]	5	50
Z_r (even) [-]	14	20
Non-Dimensional Constraints		
ϕ [-]	0.1	0.4
ψ [-]	-	1.2
ξ [-]	0.75	-
ϵ [-]	-	1.0

Table 3: Initial and robust optimisation results

Rotor Variables	Initial optimisation	Robust optimisation
Design Variables		
ϕ [-]	0.1765	0.2957
ψ [-]	0.9385	0.7146
ξ [-]	1	1
ϵ [-]	0.32847	0.3246
Ω [RPM]	13000	20000
Turbine Information		
r_4 [mm]	248.62	185.199
b_4 [mm]	47.771	32.183
r_6^h [mm]	81.667	60.118
C_4 [m/s]	338.47	299.995
Z_r	18	14
Π_{ts} [-]	2.109	2.204
η_{tt} [%]	95.75	90.89
η_{ts} [%]	94.25	86.10
$f*$ Information	(-10%, +10%)	(-10%, +10%)
β_{in} [deg]	66.09, -59.16	19.53, -60.81
α_{in}[deg]	82.03, -	-, -
C_{in} [Mach]	0.932, -	-, -

6.1. Preliminary variation study

The preliminary variation study considered the off-design ranges and parameters based on work by Sauret [1] to determine which variables most affect performance. It was seen in Figure 1 that the off-design performance follows a similar trend for turbine speed; however the variation of the operational temperature does not affect the performance in contrast to the study by Sauret [1] which showed significant variation on this metric. A plot of density vs temperature is given in Figure 5 over the same temperature deviation (range) CO_2 behaves much closer to an ideal gas compared to R143a, and the change in density with respect to pressure is minimal. The plot suggests that the behaviour of CO_2 is much simpler compared to R143a explaining the reduced sensitivity of the present turbine to temperature variations.

6.2. Robust optimisation

Robust optimisation was then performed observing the change of efficiency over the range of expected rotational speeds. It was found that the efficiency would change by approximately 10% from the nominal case to the extreme cases. However, the constraints were typically difficult to meet fully and dictated the form of the objective function given in Section 5.3 with the final robust design generating 90% total-to-total and 86% total-to-static efficiency as given in Table 3.

6.3. Comparison Robust to Initial Optimisation

There is minimal differences between the robust optimisation and initial single objective optimisation given the minimal variance of CO_2 in the current operation range with geometric design shown in Table 3. However, when including the robust optimisation the efficiency (total-to-static) does reduce as the flow coefficient raised to ensure minimal deviation from constraints. The robust optimisation was unable to find a turbine that was able to completely satisfy all constraints at +-10% variation of turbine speed, Table 3 shows the constraints that were not satisfied at these turbine speeds. The robust optimisation case had a deviation from expected for the relative inflow angle (β_{in}) of 37^o (at 16000 RPM) and 16^o (at 24000 RPM); this clearly indicates that the new turbine performs poorly at the extreme ends of the off-design range. In comparison, the robust turbine is significantly better than the original design which had deviations from the expected inlet Mach number (C_{in}), inlet relative flow angle and inlet absolute flow angle (α_{in}).

The location of the robust design (RO) is marked alongside the location of the initial design (X) on Figure 4 where the flow coefficient is higher for the robust design. It is interesting to note that the robust design is closer to a region where there is significant sensitivity of the design conditions on the efficiency at different turbine speeds on design conditions.

7. Conclusions

A robust optimisation method has been developed including an off-design performance to the initial optimisation of radial-inflow turbines and successfully applied. It was found that for the present design using supercritical CO2 (a near ideal gas) as a working fluid, the off-design performance varies little with wide variations in operating parameters. The method used is a fast, heuristic approach for turbomachinery design and has great potential in the initial design of turbomachinery when considering the computational cost of higher order methods such as coupled computational fluid dynamics and inverse design. Future work building from this will include the analysis of working fluids such as R143a as presented by Sauret [1] which are useful in quite a number novel designs. Furthermore the analysis will be extended to consider Axial turbomachinery and components of the cycle in order to develop a model that is able to help deliver the best Levelised Cost of Electricity.

8. Acknowledgements

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