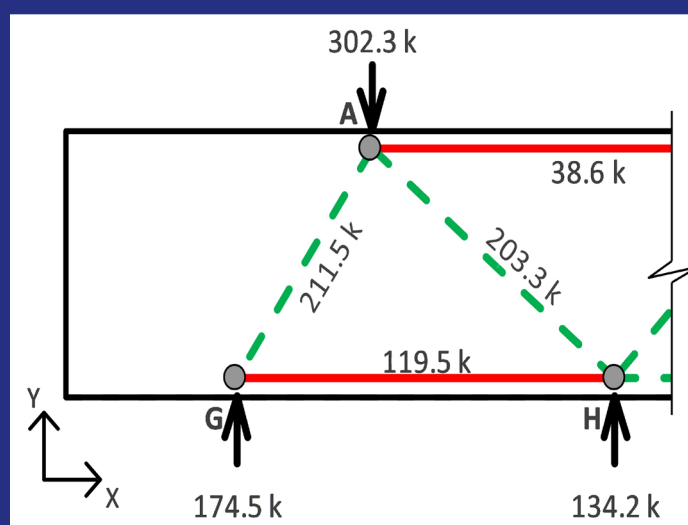


JOINT TRANSPORTATION RESEARCH PROGRAM

INDIANA DEPARTMENT OF TRANSPORTATION
AND PURDUE UNIVERSITY



Implementing the Strut-and-Tie Method for the Design of Bridge Components



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16. Abstract The strut-and-tie method (STM) is used for the design of D-regions (disturbed regions) of reinforced concrete structures. Many typical bridge substructure components consist of D-regions and warrant the use of the STM for design. Implementation of the STM is more complex than typical design methods, and engineers are often unfamiliar with the design process. As a result, designing using the STM is more time consuming than traditional design methods. The Indiana Department of Transportation (INDOT) identified a need for resources and tools that assist with the design of typical bridge substructure components using the STM. The primary tool created to fulfill this role is a computer program titled STEP (Strut-and-Tie Evaluation Program). To use the computer program, engineers input geometric conditions, material properties, reinforcement information, and loading conditions for a structural component. STEP uses this information to develop a strut-and-tie model and perform STM design procedures. A graphical representation of the model and a summary of the design results are provided as program outputs for the user. STEP, created using Excel VBA, is intended to aid in the design of multi-column bent caps, straddle bent caps, and integral and semi-integral end bent caps. To accompany the computer program, a guidebook was developed to provide basic STM guidance, describe the procedures that are executed by STEP, and present instructions for using the program for the design of bridge substructure components.			
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EXECUTIVE SUMMARY

Introduction

The strut-and-tie method (STM) is the appropriate procedure for the design of D-regions, or disturbed regions, in reinforced concrete structures. A majority of bridge substructure elements warrant the use of the STM. Despite the fact that STM design provisions were included in the *AASHTO LRFD Bridge Design Specifications* in 1994, the general need for guidance in the engineering community to appropriately implement the STM still exists. Furthermore, even for engineers familiar with the STM design process, implementation of the STM for the design of bridge components is time consuming. In response to these issues, a project was initiated to provide assistance with the implementation of the STM in Indiana. More specifically, the objectives of this project are to provide bridge engineers with STM guidance specific to Indiana needs and deliver resources and design tools to aid engineers with applying the STM to common bridge substructure components. The primary tool developed is a computer program titled STEP (Strut-and-Tie Evaluation Program). The program is capable of performing STM design procedures for multi-column bent caps, nonprestressed straddle bent caps, and integral and semi-integral end bent caps. Input from the project's Study Advisory Committee was instrumental in the development of a computer program applicable to the design of common substructure components. To use the program, a bridge designer first inputs information related to geometric conditions, material properties, reinforcement layout, and loading conditions for a given bridge component. Based on this information, the program outputs a strut-and-tie model and details of the corresponding design procedures and checks performed within STEP. Through the implementation of the computer program and the other deliverables of the project, engineers at the Indiana Department of Transportation (INDOT) are expected to have an increased familiarity with the STM and experience enhanced productivity in the design office.

Findings

Based on the project and the development of the deliverables, the following findings were identified:

- A set of rules based on the strut-and-tie method can be incorporated into a computer program to successfully develop viable strut-and-tie models for bridge substructure components with varying geometries and loading conditions. The resulting program is unique to any other known software available for design using the STM.
- Based on the strut-and-tie model developed by the program, performing design procedures, including strength checks,

can be successfully automated, and a summary of the procedures/checks performed by the program can be automatically generated within a few seconds.

- The computer program may not always develop the most efficient strut-and-tie model. Providing rules within the program that will consistently produce viable strut-and-tie models, while also considering the development of the most efficient model, presents challenges. However, if engineers identify changes to the model output by STEP that may result in added efficiency, they have the option to modify the model. The program will then perform design procedures based on the adjusted model.
- Creating STEP using Excel (2016) Visual Basic for Applications (VBA) allowed for a straightforward user interface. To use the program, engineers input required information directly into Excel and then click a button to run the program.
- Bridge engineers are eager to use STEP to aid in the design of bridge substructure components. After being exposed to the computer program during workshops, engineers provided positive feedback and expressed their anticipation for the release of the program for use in the design office.

Implementation

Implementation of STEP is recommended for the STM design of multi-column bent caps, nonprestressed straddle bent caps, and integral and semi-integral end bent caps. The project aligns with the Innovation and Technology strategic priority of the *Indiana Department of Transportation 2019 Strategic Plan*. The program is expected to result in significant time savings, potentially reducing the time to perform an STM design of a substructure component from hours to minutes. Reduced design time, in turn, is expected to result in cost savings. Furthermore, the program can help reduce potential errors when implementing the STM.

The project culminated in a series of workshops for bridge designers from INDOT and consulting companies in Indiana. Each workshop was approximately 3.5 hours in length and included an introduction to the strut-and-tie method, the presentation of an example demonstrating the use of the STM for the design of a multi-column bent cap, an overview of STEP, and a design example using STEP. As this design example was presented, the workshop participants were able to complete the example using STEP on their personal laptops. Based on the feedback received from the participants, widespread use of the program is expected due to the significant assistance the program can provide.

STEP will be posted on the INDOT website for download by practicing bridge engineers. A guidebook that explains how to use the program and that includes details of the procedures that are executed by the program will also be posted.

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1. INTRODUCTION

1.1 Background

The strut-and-tie method (STM) is a powerful procedure for the design of D-regions, or disturbed regions, of reinforced concrete structures. D-regions occur near applied concentrated loads/reactions or any geometric discontinuity (e.g., opening, corbel, dapped end, and frame corner) and are characterized by a nonlinear distribution of strains. The assumption within beam theory that plane sections remain plane is no longer valid (AASHTO LRFD, 2017; Birrcher et al., 2009; Williams et al., 2012). The STM allows the designer to visualize and model the distribution of stresses throughout the D-region by using axial force members, or truss members. Compression members are referred to as struts, and tension members are called ties. A strut-and-tie model is composed of struts and ties connected at nodes and represents the “flow” of forces from the applied loads to the supports. A strut-and-tie model for a beam is shown in Figure 1.1. Due to the close spacing of the loads and support reactions, the entire member is classified as a D-region. After developing a strut-and-tie model, a designer must ensure the structure has the required capacity to carry the tensile and compressive forces indicated by the model.

The strut-and-tie method was first introduced into design provisions in the United States in 1994 when it was included in the *AASHTO LRFD Bridge Design Specifications*. The STM later appeared as an appendix in ACI 318 in 2002 (ACI Committee 318, 2002). Despite its 25-year-long history in U.S. design provisions, engineers are often reluctant to apply the method in the design office. More recently, the STM was included in a chapter in the main body of ACI 318-14, and important updates to the provisions were incorporated into ACI 318-19 (ACI Committee 318, 2014, 2019). Furthermore, extensive changes to the STM

provisions were introduced in the 2016 Interim Revisions of the *AASHTO LRFD Bridge Design Specifications* (2014) to improve their accuracy and provide added simplicity (Birrcher et al., 2009; Williams et al., 2012). These updates are now included in the 8th edition of the AASHTO LRFD (2017) specifications. However, apprehension to apply the STM remains. The STM is less rigid and more complex than most traditional design procedures. Structural engineers are often unfamiliar with the STM design process, and questions frequently arise that are not clearly answered in available design resources. Moreover, designs using the STM often demand significantly more time than traditional methods, potentially taking hours, or even days, to design a single member. For this reason, performing detailed STM designs for every case for which it is required may not be possible.

Article 5.7.1.2 of AASHTO LRFD (2017) states that the shear and torsional design of D-regions should follow the strut-and-tie method or the “legacy methods” given in the specifications. However, the use of the legacy methods is limited (for example, refer to Article C5.8.4.1 of AASHTO LRFD, 2017). Furthermore, Article C5.5.1.2.3, in reference to the legacy methods, states, “Use of these methods is not preferred and is expected to decline with time.” The STM is the appropriate design method of many bridge components or regions of components.

Although bridge engineers routinely face designs that warrant the use of the STM, the general need for guidance in the engineering community to appropriately implement the method still exists. For the Indiana Department of Transportation (INDOT), a primary need is design aids that provide significant time savings for the design of bridge substructure members using the STM. The tool that would provide the most benefit is a computer program that performs STM procedures based on basic user inputs related to the component being designed. This report describes a project that was

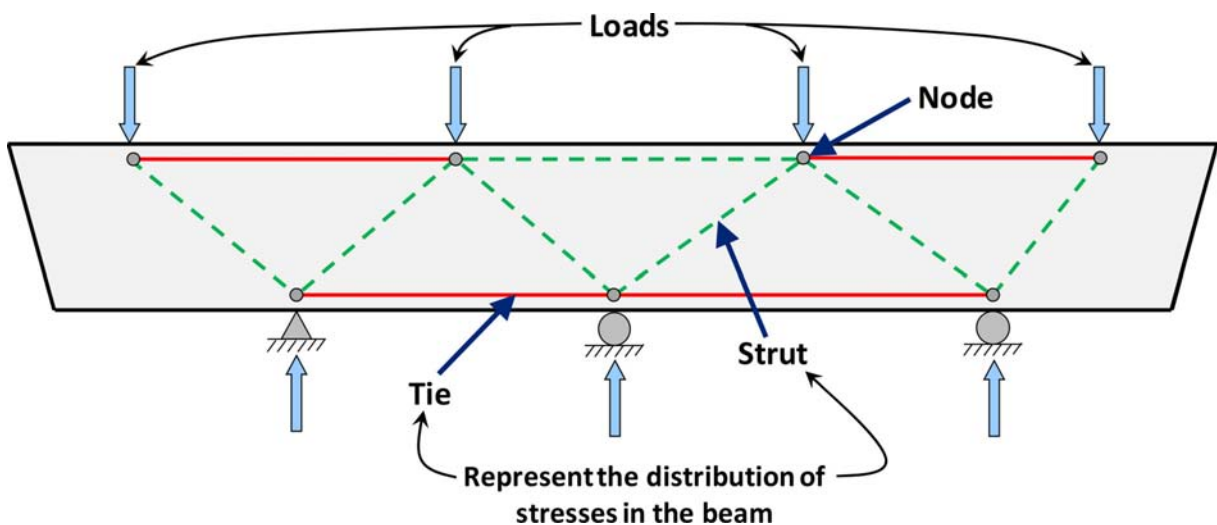


Figure 1.1 Strut-and-tie model for a beam (adapted from Williams et al., 2012).

developed to address the STM design needs in Indiana, including the creation of a computer program that helps to fulfill these needs.

1.2 Objective, Scope, and Deliverables

The objectives of this project are to provide bridge engineers with STM guidance specific to Indiana needs and deliver resources and design tools to aid engineers with applying the STM to common bridge substructure components. Four deliverables resulted from the project.

1. A computer program capable of performing STM design procedures for common bridge substructure components dominated by D-regions. The program is titled STEP (Strut-and-Tie Evaluation Program).
2. An STM guidebook tailored specifically toward Indiana needs and focused on the design of typical bridge components. In addition to basic STM design guidance, the guidebook focuses on the implementation of STEP. The guidebook is provided in Appendix A.
3. STM design workshops and accompanying presentation slides that introduce bridge engineers in Indiana to the STM and provide training for the use of STEP.
4. Poster presentations at Purdue Road School, the JTRP Poster Session in Indianapolis, and a JTRP Executive Committee meeting to promote awareness of the design resources produced by the project within the bridge design community and gather feedback.

The computer program and guidebook focus on three common bridge substructure components: multi-column bent caps, straddle bent caps, and integral and semi-integral end bents. Because these components are generally dominated by D-regions, the STM is typically the appropriate method for the design of these elements.

Successful implementation of the guidance and resources that were developed as a result of the project will provide bridge engineers with the means to routinely perform STM designs that satisfy code specifications. It is expected that engineers will be able to design using the STM in a fraction of the time than would be required prior to receiving the project deliverables, potentially reducing the time to perform an STM design of a typical bridge substructure component from hours to minutes.

The members of the Study Advisory Committee (SAC) for the project were instrumental to achieving the stated objectives. The SAC provided feedback throughout the development of the computer program, ensuring that it would be applicable to common bridge substructure components encountered in the design office. Invaluable assistance from the SAC also ensured that the STM design workshops were a success. The following sections include an overview of the STM computer program and details of the implementation of the project. The guidebook is then provided as an appendix.

2. STRUT-AND-TIE EVALUATION PROGRAM (STEP) OVERVIEW

Despite the complexity of the STM procedure, a set of rules were able to be defined that can be used within

a computer program to develop viable strut-and-tie models for members of varying geometries and loading conditions. The computer program can then use information from the strut-and-tie models to perform design procedures, including strength checks.

As previously stated, the computer program, STEP, was created with feedback from the Study Advisory Committee. Additionally, during its development, a request for input was sent to bridge engineering consulting firms in Indiana. The responses provided suggestions for desired capabilities of the program. The comments were considered to ensure that the program would be valuable in typical design scenarios while also being user-friendly.

STEP was created using Excel (2016) Visual Basic for Applications (VBA). Excel VBA was selected as the programming language to ensure that the program is user-friendly and easy to navigate. Engineers are generally familiar with Excel and are not required to learn a completely new program to use the STM software. VBA is the coding language within Microsoft products that allows users to automate tasks. Creating the program in Excel allowed for graphical outputs of the strut-and-tie model as well as tables that display the results of the design procedures that were performed.

When using STEP, designers input information directly into an Excel spreadsheet based on the structural component being designed. The users then click a “Run Program” button for the program to perform the calculations. The program then creates a strut-and-tie model and performs STM design procedures for the structural component based on the user inputs. The program does not perform every step of the structural design process, such as sizing the member, determining the number and sizes of longitudinal reinforcing bars, and deciding the most appropriate size of stirrups and the distribution of stirrup legs through the member width. Because many variables impact these aspects of design, they are left to the discretion of the engineer. Therefore, the designer using the program is responsible for developing parameters for the structural component, such as geometric conditions, material properties, and reinforcement information. The designer must also determine the critical load cases. The program will use these parameters to develop a strut-and-tie model and perform design procedures according to Article 5.8.2 of AASHTO LRFD (2017). The program will inform the user whether the design input into STEP satisfies the design checks performed by the program or if the design does not meet STM code requirements.

The computer program consists of the three primary components presented in Figure 2.1. The program initially reads inputs provided by the designer. This information is then used to perform design procedures according to the STM. After the program is run, the strut-and-tie model is output along with a summary of the design procedures that were performed. An example of a strut-and-tie model developed by the program for a multi-column bent cap is presented in Figure 2.2. It is

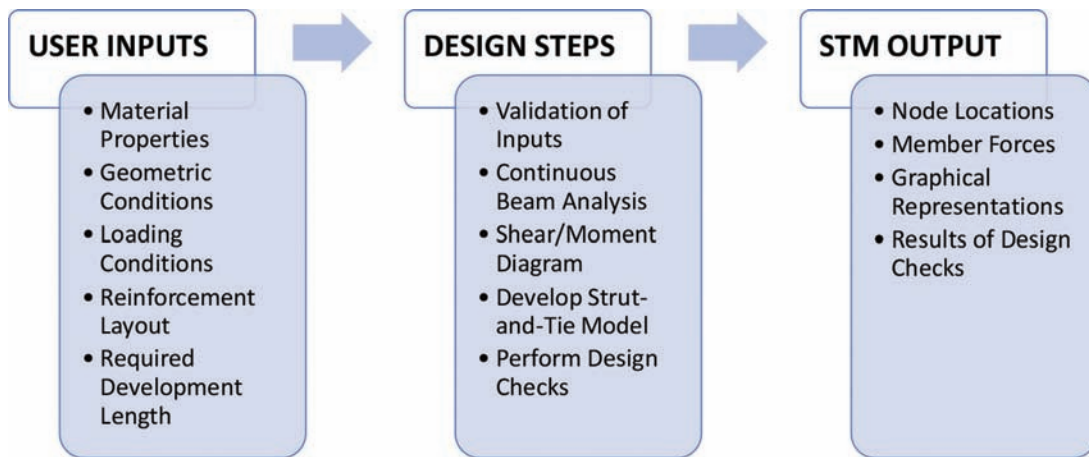


Figure 2.1 Computer program procedure overview.

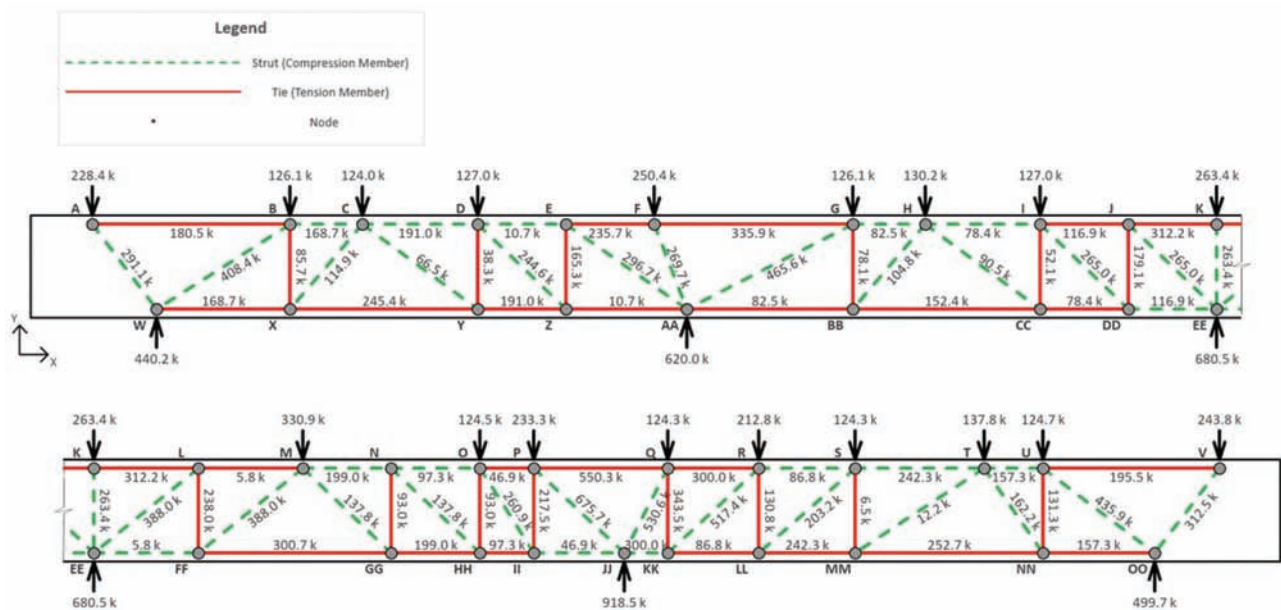


Figure 2.2 Example strut-and-tie model developed by STEP.

the responsibility of the designer to ensure the accuracy of the results and interpret whether the design is structurally sound. The computer program only performs design procedures required by the STM provisions in Article 5.8.2 of AASHTO LRFD (2017). The designer may need to perform additional design checks not specified in the STM provisions.

Due to the difference in procedures for pier caps and end bent caps, two different Excel (2016) workbooks have been created for the computer program. The first workbook, titled *Pier Cap STM Design*, is to be used for the design of multi-column bent caps and straddle bent caps. The second workbook, titled *End Bent Cap STM Design*, is to be used for the design of integral and semi-integral end bent caps. Both programs are limited to the design of nonprestressed members.

3. IMPLEMENTATION

In the following sections, the contributions of the project toward fulfilling the priorities of the INDOT Strategic Plan are described. Then, details of each deliverable resulting from the project are provided.

3.1 INDOT Strategic Plan

The implementation of this project aligns with the Innovation and Technology strategic priority of the *Indiana Department of Transportation 2019 Strategic Plan*. This project, funded through JTRP, resulted in tools that will provide significant time savings for bridge designers. Most importantly, the project led to the development of an innovative computer program

that will increase efficiency in the design office. For each substructure component designed using the computer program, hours of design time are expected to be saved compared to an STM design completed without the aid of the program. The computer program along with the other deliverables will result in bridge engineers gaining an improved understanding of the STM. This improved understanding coupled with the implementation of the computer program is expected to reduce the possibility for design errors, resulting in safe, durable bridge substructure elements. There is also an expected cost savings due to the reduced design time for consultants and in-house designers.

3.2 STEP

STEP will be posted on the INDOT website for public download. Both Excel (2016) workbooks (*Pier Cap STM Design* and *End Bent Cap STM Design*) will be available. Bridge engineers will be able to download the workbooks from the website and use STEP for the design of bridge substructure elements. The VBA code (i.e., macros) within the workbooks will be password protected. INDOT's Management Information Systems will be responsible for managing accessibility to the program and posting the most up-to-date version of STEP.

3.3 Guidebook

The STM guidebook provided in Appendix A accompanies the computer program and provides detailed guidance and STM design examples. The guidebook will be posted online to be used alongside STEP. The document is both a general design aid for implementing the STM and a manual for using STEP. More specifically, the guidebook includes the following components:

1. A detailed introduction to the strut-and-tie method, including background information on the theory of the method and an explanation for each step of the design process.
2. An overview of STEP, including the layout and formatting of the program. Required user inputs are described along with explanations of the program outputs.
3. Explanations of the procedures executed by STEP in order to perform STM design processes for each of the three types of bridge substructure components for which the program was designed (multi-column bent caps, straddle bent caps, and integral and semi-integral end bent caps). Descriptions of the VBA code are included.
4. Design examples that demonstrate the use of STEP to perform STM design procedures for each of the three types of substructure components.

Prior to using STEP, bridge designers should be familiar with the information outlined in the guidebook. The guidebook should then be used as a reference document while operating the computer program.

3.4 Presentations

Poster presentations were given at various events while STEP was being developed to disseminate information about the program and gather feedback from bridge engineers. The presentations were given at Purdue Road School, the JTRP Poster Session in Indianapolis, and a JTRP Executive Committee meeting at Bowen Laboratory at Purdue University. A podium presentation that introduced STEP was also given at Purdue Road School.

3.5 Workshops

The project culminated in three workshops for bridge engineers in Indiana. Two of the workshops were held at Beam, Longest and Neff (BLN) in Indianapolis on July 18, 2019. A total of 53 participants, a majority of whom were design consultants, attended the workshops held at BLN. Figure 3.1 is a photograph from one of these workshops. Due to the popularity of the workshops and the limited space, all INDOT engineers wishing to attend could be accommodated. A separate workshop was therefore held at the INDOT Central Office on July 24, 2019. This workshop consisted of 11 participants.

The workshops began with an overview of the STM, including relevant theoretical concepts, the proper development of strut-and-tie models, and design procedures that should be followed for a given strut-and-tie model. A design example for a five-column bent cap was then presented. The remainder of the workshop focused on STEP. An overview of the computer program was provided, and important limitations and assumptions associated with the program were highlighted. The procedures that are executed by the computer program were presented next. During the last part of the workshop, the participants downloaded a trial version of STEP onto their personal laptops. Using the program, the participants worked through a design example for the same five-column bent cap that was previously presented. With guidance, they entered all required user inputs into STEP. After running the program, the resulting program outputs were reviewed. At the end of the workshop, the participants were asked to provide feedback on the computer program. Many suggestions provided by the engineers were incorporated into STEP.



Figure 3.1 Workshop held at BLN.

4. SUMMARY

Because of the geometries and loading conditions of bridge substructure components, the members are often dominated by D-regions and the use of the strut-and-tie method is therefore necessary for their proper design. The assumptions within traditional sectional design methods are not compatible with the distribution of stresses within components dominated by D-regions, making the methods invalid. Bridge designers, however, are often unfamiliar with the implementation of the STM. Furthermore, the use of the STM is generally time consuming. Through the project described in this report, design resources, including a computer program titled STEP, were created to aid in the STM design of bridge substructure components. When implemented, the computer program is expected to result in significant time savings and accurate designs. The guidebook that accompanies STEP is provided in Appendix A. Designers should be familiar with the information presented in the guidebook prior to using STEP.

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APPENDICES

The appendices are available for download at <https://doi.org/10.5703/1288284317112>

About the Joint Transportation Research Program (JTRP)

On March 11, 1937, the Indiana Legislature passed an act which authorized the Indiana State Highway Commission to cooperate with and assist Purdue University in developing the best methods of improving and maintaining the highways of the state and the respective counties thereof. That collaborative effort was called the Joint Highway Research Project (JHRP). In 1997 the collaborative venture was renamed as the Joint Transportation Research Program (JTRP) to reflect the state and national efforts to integrate the management and operation of various transportation modes.

The first studies of JHRP were concerned with Test Road No. 1 — evaluation of the weathering characteristics of stabilized materials. After World War II, the JHRP program grew substantially and was regularly producing technical reports. Over 1,600 technical reports are now available, published as part of the JHRP and subsequently JTRP collaborative venture between Purdue University and what is now the Indiana Department of Transportation.

Free online access to all reports is provided through a unique collaboration between JTRP and Purdue Libraries. These are available at <http://docs.lib.purdue.edu/jtrp>.

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