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MAXIMIZING RETURNS FOR INVESTORS USING MODERN  
PORTFOLIO THEORY AND THE EFFICIENT FRONTIER

by

Charity Smith Parkinson

**Capstone submitted in partial fulfillment of  
the requirements for graduation with**

University Honors

**with a major in**

**Accounting  
in the School of Accountancy**

**Approved:**

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**Spring 2020**

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## **Abstract**

There exists an efficient frontier upon which there is an optimal point of allocation of an investor's assets among different types of investment vehicles. Identifying this point and allocating a portfolio accordingly allow an investor to capture the highest market return with the least amount of risk. This research study offers a model which can be used to find this optimal investment allocation and discusses the challenges and assumptions associated with using it. Using techniques discussed in Markowitz (1952), we obtain the optimal allocation of wealth for two portfolios of 13 and 12 assets, respectively. Such a model is not intended to portray the "perfect" portfolio allocation but provides context for decision making based upon the desire for high returns and investor's aversion to risk. This model allows for optimal allocation, both with and without constraints to short selling. The results from the models have important implications by providing investment advisors more sophistication when assigning allocation weights. Instead of assigning these weights arbitrarily, which is common in wealth advisory, our model provides direction for obtaining the weights corresponding to the efficient frontier.

## Acknowledgements

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## Final Written Product

Word Count: 4408

As investors approach financial planners or investment bankers for help managing their portfolios, there are many methods of risk measurement and portfolio allocation. Portfolio managers often have years of experience in the industry and thus often allocate the investments in their portfolio arbitrarily based on their past experience with, understanding of, and projections for the market.

The principles in the following literature on Modern Portfolio Theory will be applied during the creation of this model (hereafter referred to as “the Model”) and tested on a set of securities from an investment firm along the Wasatch Front (which shall remain unnamed due to privacy considerations and shall here in after be referred to as ‘The Firm’). The Firm has a portfolio of investments which include thirteen stock Exchange Traded Funds (Foucher & Gray, 2014). They also have a portfolio of investments which includes twelve bond ETFs and cash. They currently assign weights to each of the securities arbitrarily.

The Model was created with the intention of providing a means of portfolio allocation based on Markowitz’s (1952) and Sharpe’s (1964) theories for The Firm. It is intended for a user with a basic understanding of the ideas from the literature mentioned below, as well as a basic understanding of Microsoft (MS) Excel functions and the Solver data analysis tool “plug-in”.

## Literature Review

Markowitz (1952)

The foundations of Modern Portfolio Theory began with Henry Markowitz in 1952 with his discussion of Portfolio Selection. As published in the *Journal of Finance*, Markowitz explains his first two assumptions, that investors want to maximize discounted expected returns and that variance is undesirable. He points out that diversification is not implied within these assumptions (1952). It would make sense for an investor to invest in a security that will yield the highest return. Markowitz states a rule, “the investor does (or should) diversify his funds among all those securities which give maximum expected return” (79), but also explains that even diversification cannot eliminate all variance (1952).

Markowitz then explains that the number of securities within a portfolio and their covariance relationships have an impact on the total expected return and risk of the portfolio (Mangram, 2013). Covariance is the measure of the relation of two variables (or securities). This relation can be positive or negative. When two securities vary positively, their values both move in the same direction in response to market forces. When two securities vary negatively, their values move in inverse directions. Choosing securities that covary negatively with one another can reduce the amount of unsystematic risk (risk specific to the company or industry) in the portfolio (“Covariance”).

The foundation from Markowitz’s discussion on Portfolio Selection explains the basics behind the creation of this allocation model (1952). The mathematical formulas and ideas from Markowitz’s theory are shown below with their relation to the Model created.

Markowitz asks his readers to assume that  $Y$  is a random variable and that the probability that  $Y = y_1$  be  $p_1$  and the probability that  $Y = y_2$  be  $p_2$ . The expected value (or mean) of  $Y$  is (1952):

$$E(Y) = p_1y_1 + p_2y_2 + \dots + p_ny_n$$

As shown in the formula,  $Y$  could in fact be an infinite number of values. Calculating the expected value for  $Y$  would follow the same pattern as shown above for any number of possible  $Y$  values.

Next, Markowitz describes variance  $V$ , which is the average squared deviation of  $y_n$  from its expected value (1952). Standard deviation is another measure of dispersion and is the chosen measure of risk in the Model.

$$V = p_1 (y_1 - E(y_1))^2 + p_2 (y_2 - E(y_2))^2 + \dots + p_n (y_n - E(y_n))^2$$

$$\sigma = \sqrt{p_1 (y_1 - E(y_1))^2 + p_2 (y_2 - E(y_2))^2 + \dots + p_n (y_n - E(y_n))^2}$$

Supposing now that there are a number of random variables  $R_1, R_2$  etc., the weighted sum  $R$  would be (1952):

$$R = \alpha_1R_1 + \alpha_2R_2 + \dots + \alpha_nR_n$$

Therefore, if  $R$  is the weighted sum of random  $R$  variables, we can input  $R$  into the expected value equation resulting in:

$$E(R) = \alpha_1E(R_1) + \alpha_2E(R_2) + \dots + \alpha_nE(R_n)$$

where  $\alpha_n$  is the probability  $R_n$  occurs. Thus, the equation shows us how to calculate the expected return of a portfolio of  $R$  by summing the products of the  $E(R_1)$  and its probability  $\alpha_1$  up to the  $n$ th security. Markowitz explains, "The expected value of a weighted sum is the weighted sum of the expected values" (1952, 80). But, as he shows, the variance of  $R$ , the weighted sum, is not that simple. This is defined as the covariance and can be written as:

$$cov_{12} = E \{ [R_1 - E(R_1)] [R_2 - E(R_2)] \}$$

for the covariance of  $R_1$  and  $R_2$  (Markowitz, 1952).

Calculating an expected value and variance can be done for returns, where  $R_i$  is the return on the  $i^{\text{th}}$  security.  $\mu_i$  will be the expected value of  $R_i$ .  $cov_{ij}$  is the covariance between  $R_i$  and  $R_j$  ( $cov_{ii}$  is the variance of  $R_i$ ). We use  $cov_{ij}$  rather than  $\sigma_{ij}$  (as Markowitz does) as a symbol for covariance because  $\sigma$  will be used later on as the symbol for standard deviation in the Model.  $X_i$  is the percentage of the investor's assets which are allocated to the  $i^{\text{th}}$  security and are not random but are chosen by the investor. The yield ( $R$ ) on the portfolio as a whole is:

$$R = \sum R_i X_i$$

which is a weighted sum of random variables (Markowitz, 1952). Thus, Markowitz (1952) then derives the expected return from the portfolio and the variance of the portfolio. He explains (1952), ‘The concepts “yield” and “risk” appear frequently in financial writings. Usually if the term “yield” were replaced by “expected yield” or “expected return” and “risk” by “variance of return,” little change of apparent meaning would result’ (89). For the Model, standard deviation is used, as it is the square root of the variance, and is thus also a measure of risk. The formula for standard deviation is also shown below (Bodie et al., 2019).

$$E(Y) = \sum_{i=1}^N X_i \mu_i$$

$$V = \sum_{i=1}^N \sum_{j=1}^N cov_{ij} X_i X_j$$

$$\sigma = \sqrt{\sum_{i=1}^N \sum_{j=1}^N cov_{ij} X_i X_j}$$

There are various combinations of  $E$  and  $V$  depending on the portfolio chosen ( $X_1, X_2, \dots, X_n$ ). All sets of obtainable  $E, V$  combinations can be plotted on a graph (Markowitz, 1952). The most efficient of these combinations are those which have the highest  $E$  for the lowest  $V$  or  $\sigma$  which will be shown in *Figure 1* in the following section.

The combination of  $E, V$  that is closest to Quadrant I (see Figure 1) and has the highest  $E$  for the lowest  $V$  (the most efficient  $E, V$  combination) is what we are solving for in the Model. It represents the portfolio allocation that The Firm could use to have the highest expected return with the least amount of risk.

Sharpe (1964)

Modern Portfolio Theory would not be complete without the work of William Sharpe (1964) on the Capital Asset Pricing Model. In this model, Sharpe assumes that an individual is only willing to act on the basis of two parameters when assessing investments, the expected value or return  $E(R)$  and the standard deviation  $\sigma$  which can be represented by a utility function (Sharpe, 1964):

$$U = g(E_R, \sigma_R)$$

Investors are assumed to prefer a higher expected return and lower standard deviation (as standard deviation is a measure of risk) (Sharpe, 1964). When these two variables are plotted on a graph, an individual investment can be shown as a point on the graph with a unique  $E_R$  and  $\sigma_R$ .

Sharpe then explains the Investment Opportunity Curve which is a curve on which all combinations of  $E_R$  and  $\sigma_R$  exist on a plane. Every investment plan available can be represented by a point on in the  $E_R, \sigma_R$  plane (Sharpe, 1964). But as he explains (1964), “A plan is said to be efficient if (and only if) there is no alternative with either (1) the same  $E_R$  and a lower  $\sigma_R$ , (2) the same  $\sigma_R$  and a higher  $E_R$  or (3) a higher  $E_R$  and a lower  $\sigma_R$ ” (Sharpe, 429).

Sharpe then shows how the expected return and standard deviation of a portfolio can be calculated:

$$E_{Rc} = \alpha E_{Ra} + (1 - \alpha) E_{Rb}$$

$$\sigma_{Rc} = \sqrt{\alpha^2 \sigma_{Ra}^2 + (1 - \alpha)^2 \sigma_{Rb}^2 + 2r_{ab}\alpha(1 - \alpha)\sigma_{Ra}\sigma_{Rb}}$$

where  $\sigma$  is the proportion of the individuals plan placed in plan A (Sharpe, 1964). The expected rate of return of the combination of two assets A and B in the portfolio should lie between the expected return of the two assets (Sharpe, 1964).

If the two assets are perfectly correlated,  $r_{ab}$  will reside on a straight line between the two points. If they are less than perfectly correlated, their standard deviation is also less than that of a perfect correlation, because  $r_{ab}$  would be lower (Sharpe, 1964). This is the situation we are hoping to create with our portfolio of assets. If the assets are negatively correlated, then by combining them together in a portfolio, an investor can lower the standard deviation or risk of the portfolio. We will show this as the efficient frontier in the figure below:

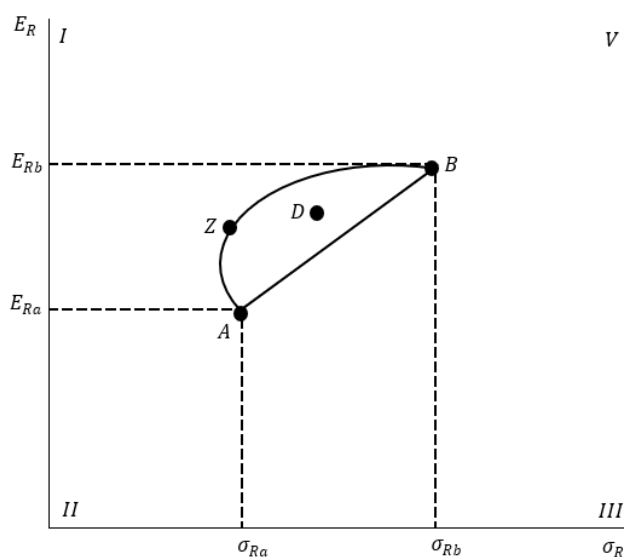


Figure 1: Efficient Frontier

When two securities are plotted on this graph, there exists a line between them depicting the efficient combinations of these securities (Line AZB). On the Line AZB there exists an infinite number of combinations of the securities with corresponding expected returns and standard deviations. This is called the *efficient frontier*. Point D is not on this line and is thus an inefficient way of allocating the portfolio between the two securities.

Sharpe's (1964) studies explain that if these securities have less than a positive covariance, their combination will result in a standard deviation less than that obtained from a perfect correlation. Thus, the resulting portfolio standard deviation and expected return must lie above the line AB. An investor is able to reduce the risk of an entire portfolio by combining two securities with negative covariance, without reducing the expected return.

$$E_R / \sigma_R$$

We will refer to this ratio as the Sharpe Ratio, which is the ratio our model is maximizing with respect to specific constraints.

## Other Literature

*Ian Foucher and Kyle Gray (2014)*

The Firm has an investment strategy for many of its clients based solely in Exchange Traded Funds. ETFs are popular among investors because they value the benefits the products provide, including low expense ratios and liquidity. Most ETFs are considered a passive investment because they are trying to replicate their benchmark (Foucher & Gray, 2014). While not all ETFs are replicating a well-known index, they do often track a group of diversified investments in an effort to replicate them on a smaller, more liquid scale. This allows investors to diversify their portfolio, without having to purchase one of each stock in the index, much like a mutual fund.

*Myles E. Mangram (2013)*

There are many theoretical limitations of Modern Portfolio Theory that could influence the results of this model. Mangram (2013) gives an extensive, but not exhaustive list. These limitations include:

- Investor 'Irrationality'- not all investors act rationally (minimizing risk and maximizing returns)
- Higher Risk = Higher Return – It's assumed that investors only accept higher amounts of risk if they are compensated by higher returns
- Perfect Information – In most cases, there exists information asymmetry where one party has superior information over another
- Unlimited Access to Capital – This is not the case, as investors and firms have credit limits

- Efficient Markets – Modern Portfolio Theory is based on the assumption that all markets are perfectly efficient, which is not always the case
- No Taxes or Transaction Costs – There is no consideration for taxes or transaction costs
- Investment Independence – This assumption is that the performance of one security has no effect upon the performance of other securities within the portfolio (Mangram, 2013).

*Robert Battalio, Hamid Mehran, and Paul Schultz (2012)*

Short selling is the practice of borrowing shares and then selling them with the intention of purchasing them later at a lower price (Bodie et al., 2019). There are many risks involved in short selling, the most evident being that after you sell the shares, you may not be able to purchase them back at a lower price and may end up spending more for the shares than you sold them for. Some market observers are also concerned that short selling may drive stock prices to artificially low levels (Battalio et al., 2012). While there are risks to doing so, short selling is a strategy that can be used to create a diversified and high returning portfolio and will thus be considered in our analysis. Making unconstrained variables non-negative will yield different results than allowing for short selling.

## Data

For the purpose of building this model using Modern Portfolio Theory, the thirteen stock ETFs and twelve bond ETFs used in The Firm's investment strategy were included in two separate models. Cash was excluded as it refers to cash held in a liquid account with earnings that are immaterial to the model. The model was tested using weekly return historical data for two years for each of these securities. The following explanation of the model's creation and usage will reference only the Stock ETF's data for simplicity. The same process was applied to create the Bond Model using the same assumptions. The Solver tool in excel was used to find the optimal allocation of the investments in the portfolio that are consistent with Markowitz's (1952) theory.

*Table 1: Stock and Bond ETFs*

<b>Stock ETFs</b>	<b>Bond ETFs</b>
SPDR S&P 500 ETF Trust (SPY)	Vanguard Short-Term Bond Index Fund ETF Shares (BSV)
Vanguard S&P 500 ETF (VOO)	Vanguard Short-Term Corporate Bond Index Fund ETF Shares (VCSH)
ALPS Sector Dividend Dogs ETF (SDOG)	iShares Core U.S. Aggregate Bond ETF (AGG)
Vanguard Mid-Cap Index Fund ETF Shares (VO)	Vanguard Intermediate-Term Bond Index Fund ETF Shares (BIV)
Schwab U.S. Mid-Cap ETF (SCHM)	Vanguard Total International Bond Index Fund ETF Shares (BNDX)
Vanguard Small-Cap Index Fund ETF Shares (VB)	SPDR Bloomberg Barclays International Corporate Bond ETF (IBND)
Schwab U.S. Small-Cap ETF (SCHA)	Schwab U.S. TIPS ETF (SCHP)
iShares Edge MSCI Min Vol EAFE ETF (EFAV)	iShares TIPS Bond ETF (TIP)

Xtrackers MSCI EAFE Hedged Equity ETF (DBEF)	SPDR Bloomberg Barclays High Yield Bond ETF (JNK)
iShares Edge MSCI Min Vol Emerging Markets ETF (EEMV)	iShares iBoxx \$ High Yield Corporate Bond ETF (HYG)
Vanguard FTSE Emerging Markets Index Fund ETF Shares (VWO)	Invesco Emerging Markets Sovereign Debt ETF (PCY)
Vanguard Global ex-U.S. Real Estate Index Fund ETF Shares (VNQI)	iShares J.P. Morgan USD Emerging Markets Bond ETF (EMB)
Vanguard Real Estate Index Fund ETF Shares (VNQ)	

The Data used as inputs for this model was collected from <https://finance.yahoo.com>. This data was chosen because of its accessibility for consumers. It is assumed that if a company were to use such a model, they would need a regular source from which they extract returns data. Because of this assumption, it was essential that the model would be able to calculate the ideal portfolio weights for any set of returns (weekly, monthly, yearly, etc.) and for any time frame. To make the model functional, however, we have limited the number of individual returns that can be input into the model to 4,995 per security.

After choosing a set of returns and a time frame (weekly returns taken over two years – for this example), the data were downloaded into individual excel files for each security.

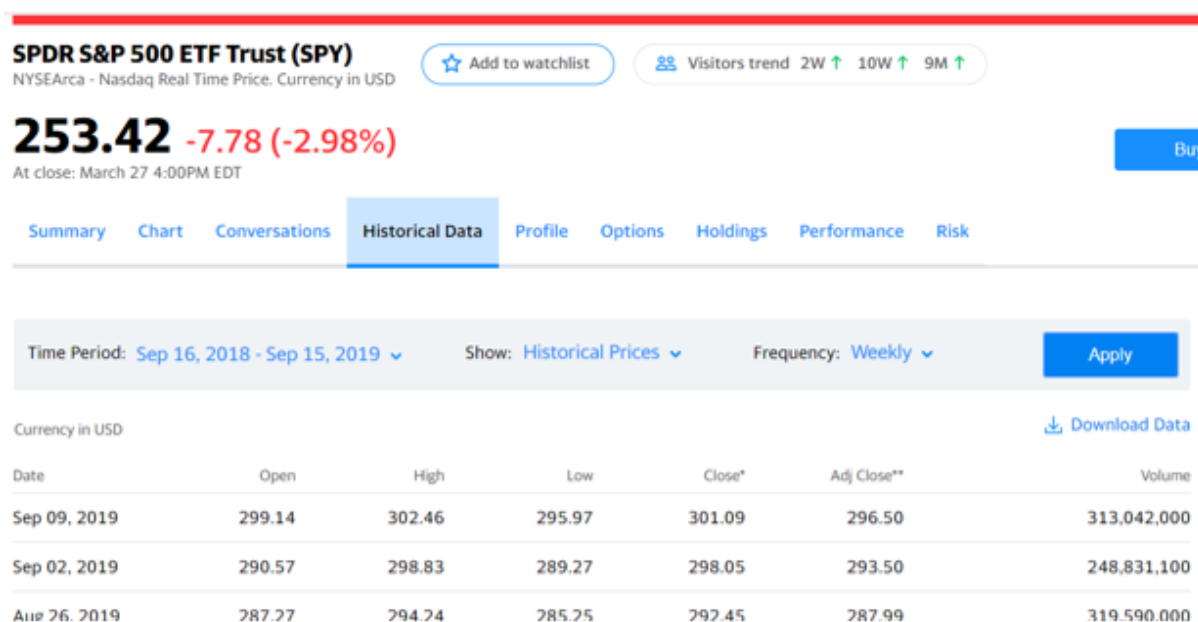


Figure 2: Yahoofinance.com Data

Additional calculation was needed at this point to determine the individual returns. The reported prices for each security are shown in the excel file represented below:

	A	B	C	D	E	F	G
1	Date	Open	High	Low	Close	Adj Close	Volume
2	9/17/2018	289.58	293.94	289.55	291.99	286.3266	3.17E+08
3	9/24/2018	291.34	292.24	289.41	290.72	286.3718	3.07E+08
4	10/1/2018	292.11	293.21	286.22	287.82	283.5152	3.92E+08
5	10/8/2018	287.05	288.86	270.36	275.95	271.8228	8.35E+08
6	10/15/2018	275.55	281.15	274.3	276.25	272.1183	6.06E+08
7	10/22/2018	277	277.36	262.29	265.33	261.3615	7.46E+08
8	10/29/2018	268.8	275.23	259.85	271.89	267.8235	6.68E+08
9	11/5/2018	272.44	281.22	271.35	277.76	273.6057	3.93E+08
10	11/12/2018	277.19	277.46	267.01	273.73	269.6359	5.85E+08
11	11/19/2018	273.05	273.38	263.07	263.25	259.3127	3.57E+08

Figure 3: Raw Data

Returns are then calculated using the adjusted close for each reported week. Thus, the return for the first weeklong period,  $R_{p1}$ , would be calculated using the following formula:

$$R_{p1} = \frac{Adj. Close_2 - Adj. Close_1}{Adj. Close_1}$$

The formula is then copied to all the weekly prices for the two-year period shown in column H of Figure 3:

	A	B	C	D	E	F	G	H
1	Date	Open	High	Low	Close	Adj Close	Volume	Returns
2	9/17/2018	289.58	293.94	289.55	291.99	286.3266	3.17E+08	
3	9/24/2018	291.34	292.24	289.41	290.72	286.3718	3.07E+08	0.000158
4	10/1/2018	292.11	293.21	286.22	287.82	283.5152	3.92E+08	-0.00998
5	10/8/2018	287.05	288.86	270.36	275.95	271.8228	8.35E+08	-0.04124
6	10/15/2018	275.55	281.15	274.3	276.25	272.1183	6.06E+08	0.001087
7	10/22/2018	277	277.36	262.29	265.33	261.3615	7.46E+08	-0.03953
8	10/29/2018	268.8	275.23	259.85	271.89	267.8235	6.68E+08	0.024724
9	11/5/2018	272.44	281.22	271.35	277.76	273.6057	3.93E+08	0.02159
10	11/12/2018	277.19	277.46	267.01	273.73	269.6359	5.85E+08	-0.01451
11	11/19/2018	273.05	273.38	263.07	263.25	259.3127	3.57E+08	-0.03829
12	11/26/2018	265.78	276.28	265.34	275.65	271.5272	4.64E+08	0.047104
13	12/3/2018	280.28	280.4	262.44	263.57	259.6279	6.46E+08	-0.04382
14	12/10/2018	263.37	269	258.62	260.47	256.5742	5.85E+08	-0.01176
15	12/17/2018	259.4	260.65	239.98	240.7	237.0999	1.02E+09	-0.0759
16	12/24/2018	239.04	251.4	233.76	247.75	245.4696	7.05E+08	0.0353
17	12/31/2018	249.56	253.11	243.67	252.39	250.0669	5.58E+08	0.018729

Figure 4: Raw Data with Calculated Return

## Methods

These returns are then input into the model on the Stock ETF Returns sheet in their corresponding columns:



Stock ETF Return Input Data													
Instructions: Insert in each column the historical returns for each ticker symbol. The model can hold 4995 values for returns. Returns can be in any time increment (daily, monthly, annually, etc.)													
	SPY	VOO	SDOG	VO	SCHM	VB	SCHA	EFAV	DBEF	EEMV	VWO	VNQI	VNQ
1	-0.02007987	-0.02070405	-0.0364	0.04062	0.00818	0.01474	0.01018	-0.004983	0.01148	0.00443	-0.0222	-0.0121	0.01127
2	0.01434166	0.01519549	0.04967	-0.019	-0.0083	-0.02	-0.0153	0.032023	0.00768	0.05457	0.076	0.04213	-0.0528
3	0.01285617	0.01250663	-0.0073	-0.004	0.02016	0.02016	0.02194	-0.004118	0.01921	-0.0347	-0.0355	-0.0148	-0.003
4	-0.02505466	-0.02403247	-0.0426	-0.0113	-0.0202	-0.0082	-0.0048	-0.035883	-0.0699	-0.0398	-0.0345	-0.0397	-0.0562
5	0.02756349	0.02654116	0.00229	0.00593	0.01185	-0.0029	-0.0034	0.047176	0.06197	-0.0281	-0.0539	0.00041	0.0684
6	-0.05387173	-0.06136316	-0.034	-0.0459	-0.0526	-0.0583	-0.0651	-0.043607	-0.079	-0.0757	-0.0986	-0.0744	-0.0396
7	-0.03780879	-0.02981585	-0.0381	-0.0695	-0.0444	-0.0536	-0.0489	-0.032413	-0.044	-0.0287	-0.042	-0.0006	-0.0043
8	0.09573643	0.09028237	0.09233	0.09055	0.0638	0.06594	0.06283	0.054977	0.06902	0.05058	0.06735	0.05576	0.06816
9	-0.0009576	0.00425052	-0.0109	0.00299	0.00776	0.01797	0.01989	-0.00942	0.01686	-0.039	-0.023	-0.0233	-0.0063
10	-0.02309622	-0.0232011	-0.0196	-0.0314	-0.0361	-0.0473	-0.0505	-0.005061	-0.042	-0.0326	-0.0391	-0.0229	0.00428
11	-0.04413154	-0.04344453	-0.0337	-0.0714	-0.0657	-0.0702	-0.0759	-0.01083	-0.0321	-0.0213	-0.0529	-0.0327	-0.0209
12	-0.000826	-0.00208171	0.05551	0.0137	0.011	0.00772	0.00647	-0.005354	-0.0514	0.00064	-0.0032	0.00945	-0.0036
13	0.06178962	0.06302851	0.08319	0.0777	0.07723	0.08159	0.079	0.051932	0.03275	0.09228	0.12528	0.08937	0.09243
14	0.00911928	0.00888621	0.02286	0.00759	0.01942	0.02054	0.02382	0.01174	0.00431	0.00175	0.01185	0.02245	-0.0126
15	0.0170116	0.01754552	0.00887	0.01838	0.02356	0.01886	0.01923	-0.003124	0.02807	-0.0251	-0.0324	-0.017	0.02249

Figure 5: Stock ETF Return Input Data Sheet

After all returns are loaded into the model, the Stock ETF Analysis tab is then used to run the model. The mean, standard deviation, and variance of each investment are automatically calculated using excel formulas. The mean or average was calculated using the =AVERAGE function. This function divides the summation of all selected returns by the number of returns. The standard deviation was calculated using the =STDEV.S function. This formula assumes the standard deviation is for a sample, rather than the entire population. The variance was calculated using the =VAR.S function. This formula assumes the variance is for a sample, rather than for the entire population. The Sharpe Ratio for each individual security was calculated by dividing the mean of the security by its standard deviation (calculated using the descriptions above). These are shown in Table 1S for each security and in *Figure 2* below:

Column1	SPY	VOO	SDOG	VO	SCHM	VB	SCHA	EFAV	DBEF	EEMV	VWO	VNQI	VNQ
Average (mean)	0.0095749	0.0096403	0.007087	0.007761	0.007845	0.007568	0.0070506	0.004809	0.005418	0.002493	0.004303	0.004388	0.006028
Standard Deviation (std)	0.0353027	0.0350148	0.04073	0.039012	0.039411	0.042851	0.0448377	0.027665	0.036279	0.035679	0.046227	0.038635	0.038812
Variance (var)	0.0012463	0.001226	0.001659	0.001522	0.001553	0.001836	0.0020104	0.000765	0.001316	0.001273	0.002137	0.001493	0.001506
Sharpe Ratio	0.2712231	0.2753196	0.173991	0.198941	0.199061	0.176622	0.1572483	0.173822	0.149351	0.069881	0.09309	0.113569	0.155324

Figure 6: Table 1S- Stock ETF Analysis

Because calculating the expected return, variance, and standard deviation of a portfolio of securities requires numerous individual calculations and could become difficult if done by hand, we will use matrix multiplication in excel to do so.

A Variance-Covariance Matrix was created using the returns for each security. The matrix shows each security's variance with each of the other securities, including itself. When a security varies with itself, this is the variance. It was calculated by using the formula:

$$=VAR.S(\text{Security's Returns from the Stock ETF Returns Sheet})$$

The variances are highlighted in Table 2S in a medium grey along a diagonal. The covariances of each of the securities with all others were also calculated. This was done using the formula:

$$=COVARIANCE.S(\text{SPY Returns from the Stock ETF Returns Sheet}, \text{VOO Returns from the Stock ETF Returns Sheet})$$

These calculations were completed for all securities and is shown in Table 2S as light grey values. The values on either side of the medium grey variances are mirror images of each other:

**Table 2S**

Ticker	SPY	VOO	SDOG	VO	SCHM	VB	SCHA	EFAV	DBEF	EEMV	VWO	VNQI	VNQ
SPY	0.0012	0.0012	0.0013	0.0013	0.0013	0.0014	0.0014	0.0007	0.0010	0.0008	0.0011	0.0009	0.0008
VOO	0.0012	0.0012	0.0013	0.0013	0.0013	0.0013	0.0014	0.0007	0.0010	0.0008	0.0011	0.0009	0.0008
SDOG	0.0013	0.0013	0.0017	0.0014	0.0014	0.0015	0.0015	0.0007	0.0010	0.0009	0.0013	0.0010	0.0008
VO	0.0013	0.0013	0.0014	0.0015	0.0015	0.0016	0.0016	0.0007	0.0011	0.0009	0.0012	0.0010	0.0009
SCHM	0.0013	0.0013	0.0014	0.0015	0.0016	0.0017	0.0017	0.0007	0.0011	0.0008	0.0011	0.0009	0.0009
VB	0.0014	0.0013	0.0015	0.0016	0.0017	0.0018	0.0019	0.0006	0.0011	0.0008	0.0011	0.0009	0.0010
SCHA	0.0014	0.0014	0.0015	0.0016	0.0017	0.0019	0.0020	0.0006	0.0011	0.0008	0.0011	0.0009	0.0010
EFAV	0.0007	0.0007	0.0007	0.0007	0.0007	0.0006	0.0006	0.0008	0.0007	0.0008	0.0009	0.0009	0.0006
DBEF	0.0010	0.0010	0.0010	0.0011	0.0011	0.0011	0.0011	0.0007	0.0013	0.0007	0.0009	0.0008	0.0007
EEMV	0.0008	0.0008	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0013	0.0015	0.0012	0.0006
VWO	0.0011	0.0011	0.0013	0.0012	0.0011	0.0011	0.0011	0.0009	0.0009	0.0015	0.0021	0.0015	0.0007
VNQI	0.0009	0.0009	0.0010	0.0010	0.0009	0.0009	0.0009	0.0009	0.0008	0.0012	0.0015	0.0015	0.0009
VNQ	0.0008	0.0008	0.0008	0.0009	0.0009	0.0010	0.0010	0.0006	0.0007	0.0006	0.0007	0.0009	0.0015

Figure 7: Table 2S- Stock ETF Analysis

Table 3S shows each security with a corresponding weight and return. The returns are average returns that were calculated in Table 1S. The weights will be calculated using the Excel Solver data analysis tool and should sum to 1 in Total weights. This means that the weight of each security in the portfolio adds up to 100% of the portfolio as illustrated in Table 3S:

**Table 3S**

Ticker	Weight	Return
SPY	-7.90	0.96%
VOO	12.06	0.96%
SDOG	-0.82	0.71%
VO	-0.14	0.78%
SCHM	-0.72	0.78%
VB	5.60	0.76%
SCHA	-5.53	0.71%
EFAV	0.60	0.48%
DBEF	-1.07	0.54%
EEMV	-0.60	0.25%
VWO	-0.30	0.43%
VNQI	-0.10	0.44%
VNQ	-0.09	0.60%
<b>TOTAL WEIGHTS</b>	<b>1</b>	

Figure 8: Table 3S- Stock ETF Analysis

Table 4S includes estimated return of the portfolio  $E(R_p)$ , standard deviation of the portfolio, and the Sharpe ratio of the portfolio. The  $E(R_p)$  is calculated using matrix multiplication. The formula used is:

$$=MMULT(TRANSPOSE(Weight Vector), Return Vector)$$

The  $\sigma_p$  is also calculated using matrix multiplication. The formula used is:

$$=\text{SQRT}(\text{MMULT}(\text{MMULT}(\text{TRANSPOSE}(\text{Weight Vector}), \text{Variance-Covariance Matrix}), \text{Weight Vector}))$$

The Sharpe Ratio $_p$  is calculated by dividing the  $E(R_p)$  by  $\sigma_p$ .

<b>Estimated Return</b>	E[R <sub>p</sub> ]	0.024854834
<b>Standard Deviation</b>	σ(p)	0.052237167
<b>Sharpe Ratio</b>	Sharpe (p)	0.47580746

Figure 9: Table 4S- Stock ETF Analysis

The cells included in the vectors for matrix multiplication are referenced on the Stock ETF Analysis Sheet below Table 2S (as shown in *Figure 10* below):

Cell References		
Description	Cells	Table
Aggregations (mean, standard deviation, variance, and Sharpe ratio) of each security based on input data	A13:N17	Table 1S
Variance-Covariance Matrix With Headers	A19: N32	Table 2S
Variance-Covariance Matrix	B20:N32	Table 2S
Weight and Return Vectors	P13:R27	Table 3S
Weight Vector	Q14:Q26	Table 3S
Return Vector	R14:R26	Table 3S
Estimated return, standard deviation, and Sharpe Ratio of the Portfolio	P29: R31	Table 4S

Figure 10: Cell References

The Model also allows for constraints to short selling. The implications of short selling will be explained later. The results will be calculated both with and without constraints to short selling.

To use the model, instructions are listed on the Stock ETF Analysis Sheet:

Stock ETF Analysis and Portfolio
<p>Instructions: Do not change any data on this tab. Only use the Solver tool. Complete the following steps.</p> <ol style="list-style-type: none"> <li>1. Ensure that the weights in the weight vector = 1. It does not matter the allocation of the weights as they will be changing.</li> <li>2. Click on the Solver data analysis tool on the Data ribbon</li> <li>3. Set Objective: Sharpe Ratio (p)</li> <li>4. To: Max</li> <li>5. By Changing Variable Cells: Weight Vector (see description below)</li> <li>6. Subject to the Constraint: Set constraint so that the total weight must = 1</li> <li>7. Make Unconstrained Variables Non-Negative: To allow for short selling, leave this box unchecked. To disallow short selling, check this box</li> <li>8. Click Solve: Solver may take a minute to compute, choose "Keep Solver Solution" to see the solved solution in the excel sheet</li> </ol>

Figure 11: Instructions

A user will click on the Solver data analysis tool on the Data ribbon of Microsoft Excel. The inputs to the solver are as follows:

Solver Parameters

Set Objective:

To:  Max  Min  Value Of:

By Changing Variable Cells:

Subject to the Constraints:

Make Unconstrained Variables Non-Negative

Select a Solving Method:

Solving Method

Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.

Buttons: Add, Change, Delete, Reset All, Load/Save, Options, Help, Solve, Close

Figure 12: Solver Parameters

**Set Objective:** Sharpe Ratio<sub>p</sub> in Table 4S

**To:** Max

**By Changing Variable Cells:** Weight Vector in Table 3S (does not include the total weight)

**Subject to the Constraint:** Total Weight in Table 3S must equal 1

**Make Unconstrained Variables Non-Negative:** Check this box to disallow short selling

**Solve:** Solver will compute the optimal weights of the portfolio to maximize the Sharpe ( $p$ ) ratio (highest return with the lowest risk)

**Keep Solver Solution:** Replaces weights currently on the excel sheet with the optimal weights calculated using the solver

## Results

After using the Solver data analysis tool, the model calculates the optimal weights of the portfolio that maximize the Sharpe Ratio<sub>p</sub>. If you select Keep Solver Solution, Excel will replace the current values in the weight vector of Table 3S and the values for  $E(R_p)$ ,  $\sigma_p$ , and Sharpe Ratio<sub>p</sub> in Table 4S. This calculation will generate the highest return with the least amount of risk, based off the historical returns used to create the model by maximizing the Sharpe Ratio<sub>p</sub>.

Results will vary based on the time frame and frequency of returns used and constraints for short selling.

The following figures show the results of the model run on weekly returns from 9/17/2018 to 09/16/2019. *Figure 13* shows the results with constraints to short selling. *Figure 14* shows the results without constraints to short selling.

<b>Table 3S</b>		
Ticker	Weight	Return
SPY	0.00	0.96%
VOO	1.00	0.96%
SDOG	0.00	0.71%
VO	0.00	0.78%
SCHM	0.00	0.78%
VB	0.00	0.76%
SCHA	0.00	0.71%
EFAV	0.00	0.48%
DBEF	0.00	0.54%
EEMV	0.00	0.25%
VWO	0.00	0.43%
VNQI	0.00	0.44%
VNQ	0.00	0.60%
<b>TOTAL WEIGHTS</b>	<b>1</b>	
<b>Table 4S</b>		
Estimated Return	$E[R_p]$	0.009640264
Standard Deviation	$\sigma(p)$	0.035014817
Sharpe Ratio	Sharpe (p)	0.275319567

*Figure 13: Results with Constraints to Short Selling*

<b>Table 3S</b>		
Ticker	Weight	Return
SPY	-7.90	0.96%
VOO	12.06	0.96%
SDOG	-0.82	0.71%
VO	-0.14	0.78%
SCHM	-0.72	0.78%
VB	5.60	0.76%
SCHA	-5.53	0.71%
EFAV	0.60	0.48%
DBEF	-1.07	0.54%
EEMV	-0.60	0.25%
VWO	-0.30	0.43%
VNQI	-0.10	0.44%
VNQ	-0.09	0.60%
<b>TOTAL WEIGHTS</b>	<b>1</b>	

<b>Table 4S</b>		
Estimated Return	E[R <sub>p</sub> ]	0.024854702
Standard Deviation	σ(p)	0.052236889
Sharpe Ratio	Sharpe (p)	0.47580746

Figure 14: Results with No Constraints to Short Selling

As illustrated in the figures 13 and 14 above, the results of the model vary drastically for the two years of weekly returns for these ETFs when calculated with and without short selling capabilities. It was expected that the model without short selling would recommend a portfolio of at least two securities. However, this was not the case. The model instead recommended that the investor place the entire portfolio into VOO. This is likely because it has a high Sharpe ratio and combining it with any other security does not increase this ratio.

A theory as to why this is the case has to do with the nature of the securities themselves. The model is designed to help an investor diversify a portfolio of securities.

“An ETF is an investment fund that is traded on a stock exchange. Its popularity is largely attributable to the benefits it provides to investors: the liquidity, ease of trade, and lower cost associated with an exchange traded product, but with the diversification of a mutual fund. The structure of ETFs also shares certain characteristics with mutual funds; for example, the returns of both these investments are based on the performance of an underlying basket of securities, less a management fee” (Foucher & Gray, 2014).

ETFs are the investment vehicle of choice for the Firm because of their attractive benefits of liquidity, low cost, and ease of trade. ETFs are a great choice for many investors that wish to diversify their investments. This diversification may be one of the underlying issues related to the unexpected results of the model. Because the ETFs are already diversified, further diversification through the model is ineffective, as shown by *Figure 13*.

*Figure 14* shows the results of the model with no constraints to short selling. This result shows much more variety in the weights of the securities. It is consistent with the results from *Figure 13* because it suggests that a large amount of the portfolio be placed in VOO. In essence, the model suggests short selling many of the other securities to purchase VOO.

Short selling may also have implications for the effectiveness of this model. In *Figure 14*, we see that allowing for short selling in the model results in suggested selling of many other assets and purchasing VOO. If it were the case that this strategy was effective, we would see a large market surge to short sell other securities and a large demand for purchasing VOO. This demand would change prices, and thus returns of these securities, creating an inefficient allocation.

Another concern with short selling securities is the idea of transaction costs and taxes. Mangram (2013) explains that the lack of consideration of transaction costs and taxes is a limitation of Modern Portfolio Theory. If taxes and transaction costs were considered in the model allocation that allows for short selling, there would be a large number of transaction costs that would effectively decrease the overall return of the portfolio.

### **Limitations**

This model, while capable of handling any number of returns from 1 to 4,995, it is not capable of adding an infinite number of returns. This decision was made to limit the file size, but to still retain functionality. It is expected that for the data to be useful, a company would likely aggregate their returns by weeks, months, or years, rather than days, or seconds. It was also assumed that most often companies make investment decisions based on more recent years, rather than all years of historical returns. Thus, within these assumptions, the model was created to accommodate a likely amount of returns.

In addition to the limitation, results shown in this model are not for the  $n$ -security case; instead they are presented for the thirteen and twelve security cases for simplicity, understanding, and relevance to The Firm. Their current portfolio consists of thirteen Stock ETFs and twelve Bond ETFs. While currently functional, the model will only be of use when thirteen and twelve securities, respectively, are placed in each portfolio. This is a large limitation to the model when considering it for public use because an investor may choose a portfolio with fewer than twelve securities. While it is not impossible to increase or decrease the scale of this model (as was done when expanded from twelve to thirteen securities), it is a manual process that is done by hand. In the future, if the model were adapted for more widespread use, it would need to be created in such a manner that any number of securities could be accommodated.

### **Conclusion and Further Study**

The results from the model has important implications by providing investment advisors more sophistication when assigning allocation weights. Instead of assigning these weights arbitrarily, which is common in wealth advisory, our model provides direction for obtaining the weights corresponding to the efficient frontier. This can provide direction in wealth advisory as well as personal portfolio allocation. This is not to say that the model is the perfect and most efficient method of allocation if the assumptions of the model become invalid.

As discovered in the Results section, the model does not provide useful allocations when used with the thirteen and twelve respective ETFs. This can be attributed to the use of ETFs as securities in the portfolio. If different individual securities were used, the model would likely show results more as expected with weights assigned to each security to maximize the Sharpe Ratio. This is an area of further exploration that has presented itself during this research. Testing this model on differing types of securities, such as individuals' stocks, mutual funds, hedge funds, etc. would provide a deeper look into the effect the type of security has on the output of the model.



**Reflective Writing**

Word Count:1012

When considering an experience for my capstone project that would be the culmination of my undergraduate education, my mind immediately looked toward investing. I have had interest in investing from a very young age as my father showed me his retirement plan and helped me set up an Individual Retirement Account (IRA). As an Accounting major, I have gained some exposure to basic investing principles and have enjoyed applying them in my life. During my education, my mind was drawn specifically toward the combination of accounting and finance. To assist me in my future goals and career aspirations, I determined that pursuing a capstone project that was related to finance would be ideal. It would allow me the opportunity to see how the principles I had learned in the study of accounting could be useful in application toward a project in finance.

In the future, I hope to assist others in financial matters. This could be in the form of education (professorship or community education, wealth management, or financial or estate planning). This capstone allowed me to experience the creation of a model using financial principles that could be applied to financial planning or wealth management specifically. Understanding the assumptions used in the model and its limitations has given me practice and preparation for using similar financial skills in my future career.

My mentor, Benjamin Blau, has been very helpful in the motivation for this capstone work. It was in his undergraduate finance course that I was first introduced to the principles used in this model. That course has sparked my interest ever since and working with Professor Blau has given me the opportunity to understand Modern Portfolio Theory and its application at a deeper level. I remember being fascinated at the possibility of reducing risk and increasing return by combining assets into a portfolio. I had always been familiar with the suggestion to “diversify

your portfolio” but learning about Modern Portfolio Theory has allowed me to better understand how this advice can be applied. I also appreciate the advice of my Departmental Honors Advisor, Dr. James Cannon. He has been helpful in assisting me in the actual model creation. I consulted him when I became stressed and nervous about creating a model that could actually be useful and applicable to an individual or company, rather than a model that just “worked”. He helped me understand several options for portions of my model that would make it both effective and efficient. I appreciate his knowledge and expertise in this area.

Accounting itself is not a research-intensive major but relies on research behind a lot of the principles and theories used. This capstone has given me an opportunity to read the research behind the principles of Modern Portfolio Theory and to use it in application. I have learned how to bring together research and model creation into one project, which was a great experience for me.

This capstone also required that I think critically about how investments are recorded for businesses. I have learned these types of accounting principles in my courses but have never considered how a model such as the one I created could change the type or method of accounting used by a company that adopts it. This has also sparked my curiosity to further research how these types of investments are recorded in a company and the reporting requirements for models such as the one I had created. I wonder if all the securities are required to be reported separately, or if all of a company’s investments can be pooled. I look forward to the possibility of learning more about such reporting as I pursue my graduate degree in accounting.

I have broadened my experience across disciplines. This capstone has introduced me to more economics and finance concepts than I would normally be exposed to in the course of my major courses. I have enjoyed this experience as the principles that I learned in basic economics,

finance, and accounting are all inter-related. If I had additional time left in my undergraduate education, I would consider adding minors in finance and economics because of the valuable principles I have learned.

Using financial information from a real firm along the Wasatch Front in my capstone project allowed me to see what benefit my knowledge could have on a real business. It wasn't theoretical for me anymore, it became real. If my model works correctly, it would be something that could be beneficial for a business. That was a very motivating factor for me. Understanding that the portfolio I was using for my model creation was a set of securities used to invest for other individuals was thrilling. It made me wonder how I could improve the investments they were already making. Basing my model off a real investment strategy was one of my favorite parts of my Capstone Creation.

The process of completing this capstone project has been both difficult and very rewarding. There were challenges during the creation of the model that were unexpected. As discussed earlier, one specific challenge was making my model useful and effective, but still applicable to consumers or businesses. I had to find a way to make the model adjustable for a larger number of returns, rather than a set number. This was more easily done than I expected, but I had to consult with Professor Blau on the best way to do so. There was also additional learning required to create the model and to make it efficient, especially in Excel, which Dr. Cannon was very kind to assist me with. I learned a lot from his suggestions, and I hope to continue learning more from him in the future.

I have enjoyed the opportunity to complete this Capstone project through Utah State's Honors Program. I appreciate all the support and help I have received from fellow students, professors, mentors, and Honors Staff. The Honors community is full of incredibly brilliant

individuals and I am thankful to have had the opportunity to discuss my Capstone with some of them and to have received feedback.

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### Professional Author Biography



Charity Parkinson is a Senior in the Accounting program at Utah State University with a minor in Family Finance. Maintaining a 4.0 GPA throughout her Undergraduate career, she has also been involved outside of her courses. Awarded Undergraduate Teaching Fellow of the Year of the Huntsman School of Business during 2019-2020 school year, she assists students in their Intermediate Accounting Course.

She is currently a member of the President's Cabinet where she assists the student body president, Sami Ahmed, in fulfilling his initiatives and planning student events. She was a Business Ambassador for the Huntsman School and enjoys spending her time helping other students as a Student Mentor for the FJ Management Center for Student Success. She is also a member of the Huntsman Scholar Program and a mentor for students within that program. With a fascination for markets and investing, she plans to complete an internship with JP Morgan Private Bank and then continue her education by pursuing her MAcc at Utah State University.