Trajectory Optimization for the Virtual Telescope for X-Ray Observations

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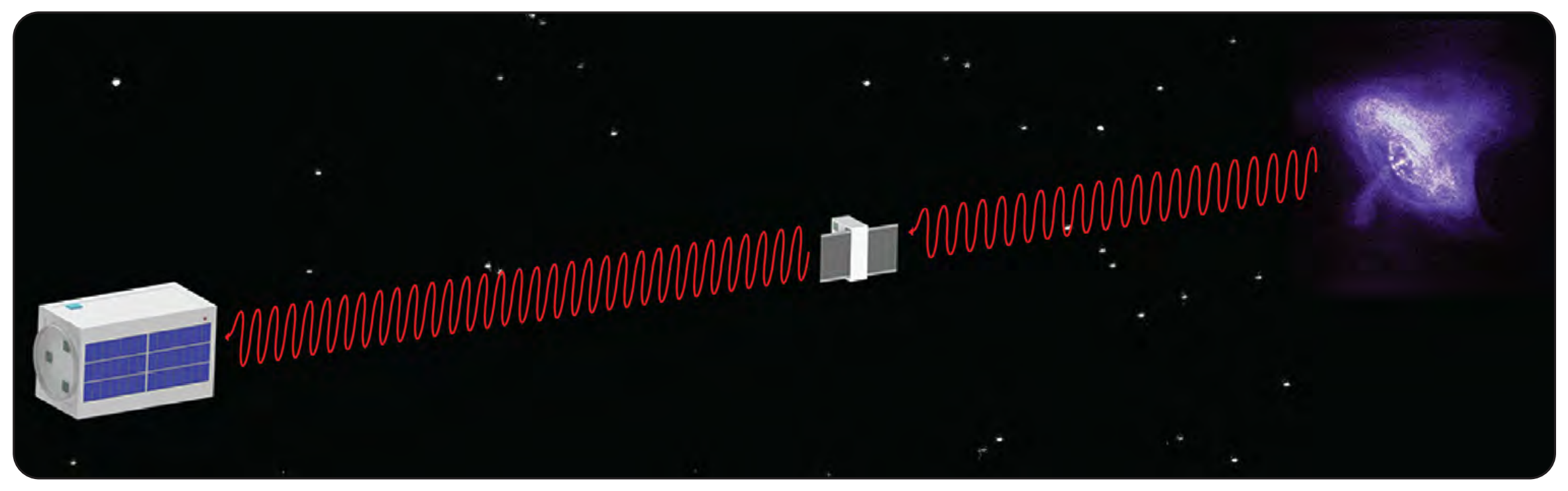
Introduction

The Virtual Telescope for X-Ray Observations (VTXO) is part of a new generation of distributed component, long focal length telescopes which promise to provide orders of magnitude improvement in angular resolution in the X-ray band over the current state of the art. VTXO will include Phased Fresnel Lenses (PFL), which provide nearly diffraction-limited imaging, with around a 1 km focal length carried by the Optics Spacecraft (OSC), which will fly in a precision formation with the Detector Spacecraft (DSC) approximating a rigid telescope body. In order to maintain the precise formation requirements, while pointing the telescope axis at the desired astronomical targets, one or both spacecraft will inherently be traveling on a non-natural orbit trajectory. These families of trajectories require one or both vehicles to maneuver regularly to maintain the desired path. If care is not taken in the trajectory design these paths can easily result in an unsustainably large propellent consumption.

Problem Statement

During astronomical observations VTXO's relative trajectories are strictly defined by the telescope focal length and pointing direction, as such there is little opportunity for optimization beyond observation scheduling, which is often driven by science requirements. However, there is a significant opportunity to optimize trajectories when re-arranging the formation to change pointing directions between different astronomical targets. This paper presents an optimization scheme for re-pointing the telescope, this scheme utilizes a non-traditional path-based cost function, along with a linearized relative dynamics model to solve for the propellent optimal trajectory for repositioning the spacecraft between different telescope pointing directions. These optimal trajectories are then tested in a well validated high-fidelity flight dynamics simulator to verify the propellent consumption relative to the linearized model.



Optimization

SYSTEM DYNAMICS [4]

Where $\vec{r}$ is the vector from the Optics Sat to the Detector Sat, and $\vec{R}_D$ $\vec{R}_O$ are the vectors from the Earth to the Detector Sat, and Optics Sat Respectively.

$$\vec{r} = \vec{R}_D - \vec{R}_O \tag{1}$$

$$\ddot{\vec{R}}_D = -\frac{\mu}{\left\|\vec{R}_D\right\|^3}\vec{R}_D + \vec{a}_T \tag{2}$$

$$\ddot{\vec{R}}_O = -\frac{\mu}{\left\|\vec{R}_O\right\|^3}\vec{R}_O \tag{3}$$

$$\left\|\vec{R}_O\right\| \gg \|\vec{r}\| \tag{4}$$

$$\ddot{\vec{r}} = [\Gamma_{GG}]\,\vec{r} \tag{5}$$

$$[\Gamma_{GG}] = -\frac{\mu}{\left\|\vec{R}_O\right\|^3}\left([I] - 3\left[\hat{R}_O\right]\left[\hat{R}_O\right]^T\right) \tag{6}$$

Equation 5 shows the gravitational acceleration in the relitive frame. $[\Gamma_{GG}]$ is given by Equation 6.

ΔV ESTIMATION

The acceleration function for a trajectory can be described as the sum of the acceleration due to gravity $\ddot{\vec{r}}_g$, acceleration due to the thruster $\vec{a}_T$, and acceleration due to disturbance forces $\ddot{\vec{r}}_d$ as show in Equation 7.

$$\ddot{\vec{r}} = \ddot{\vec{r}}_g + \vec{a}_T + \ddot{\vec{r}}_d \tag{7}$$

ΔV can then be found by integrating over $\|\vec{a}_T\|$.

$$\Delta V = \int_{t_0}^{t} \left\|(\ddot{\vec{r}}(t) - [\Gamma_{GG}]\,\vec{r}(t) - \ddot{\vec{r}}_d(t)\right\| dt \tag{8}$$

OPTIMIZATION

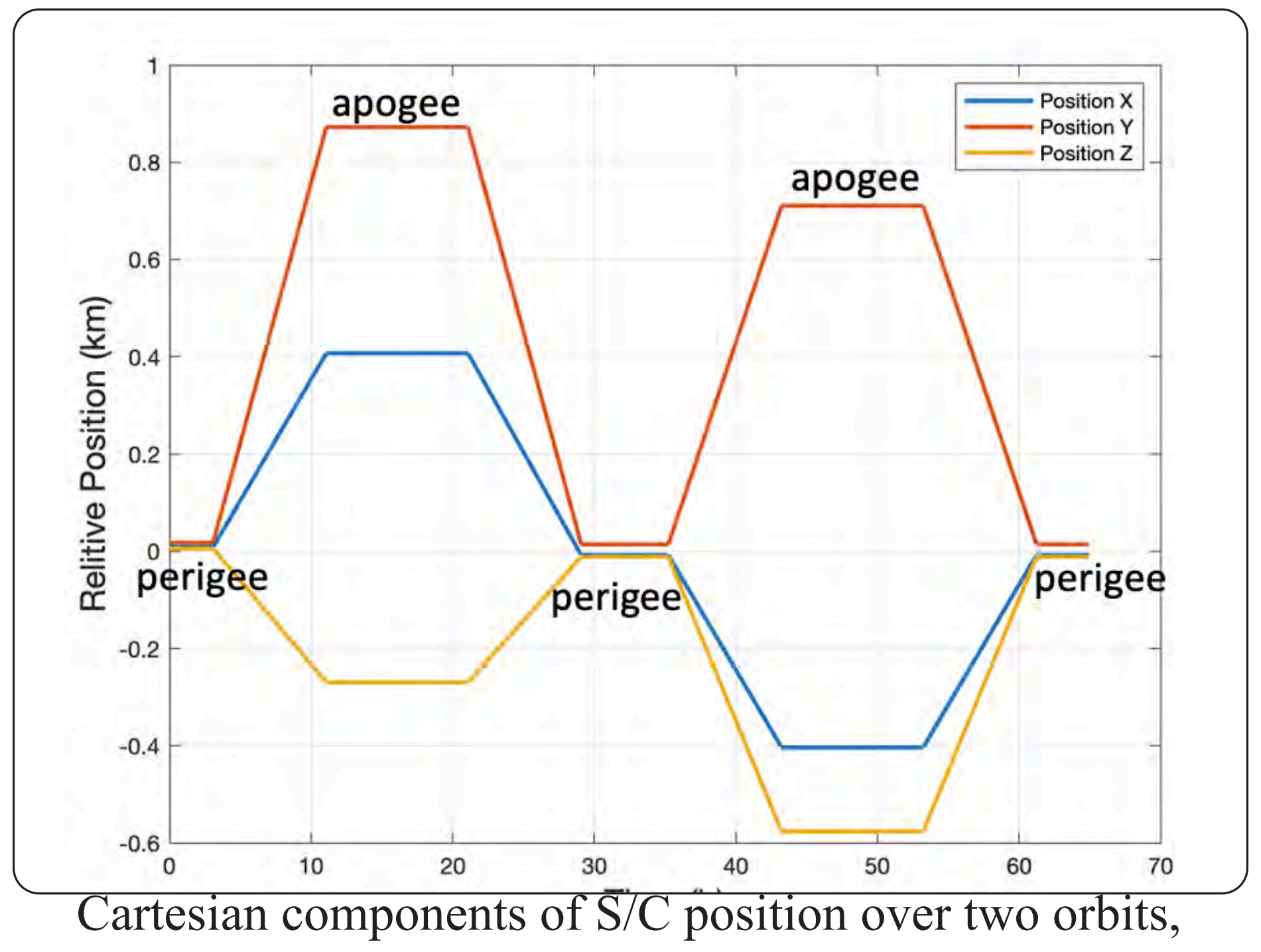
The propellant optimal trajectory can then be found by minimizing Equation 8 subject to the following constraints. Which ensure the solution can be flow utilizing a real propulsion system, and prevents the spacecraft from colliding.

$$\|\vec{r}\| > MinimumSeparationDistance \tag{9}$$

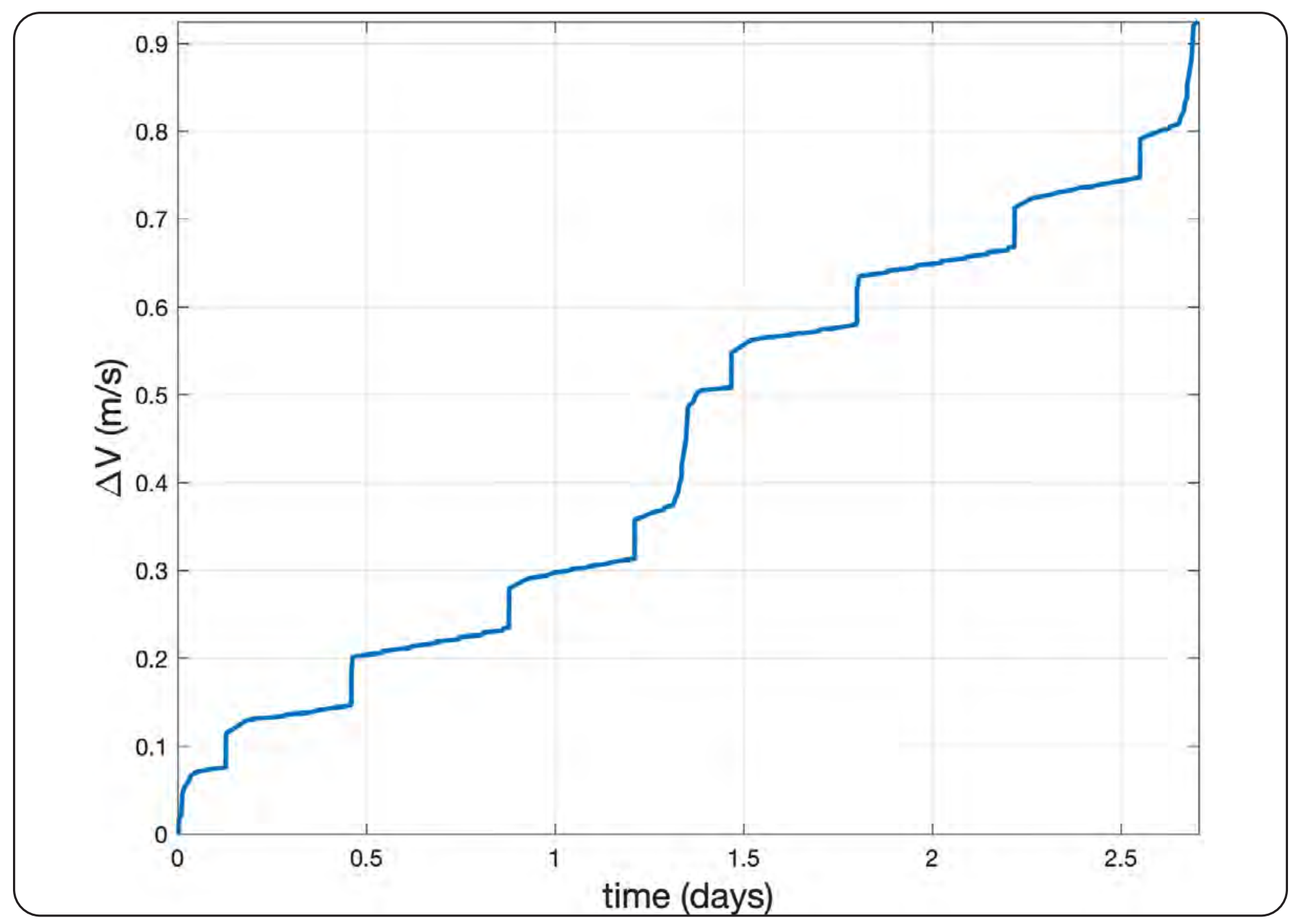
$$\|\vec{a}_T\| < \frac{MaxThrust}{mass} \tag{10}$$

Baseline

- * Baseline DeltaV ~360 mm/s per orbit.
- * Maintains 1km observation formation at apogee.
- * Moves to 20m propellent optimal trajectory at perigee.
- * Flies straight line constant velocity trajectory between apogee, and perigee formations.



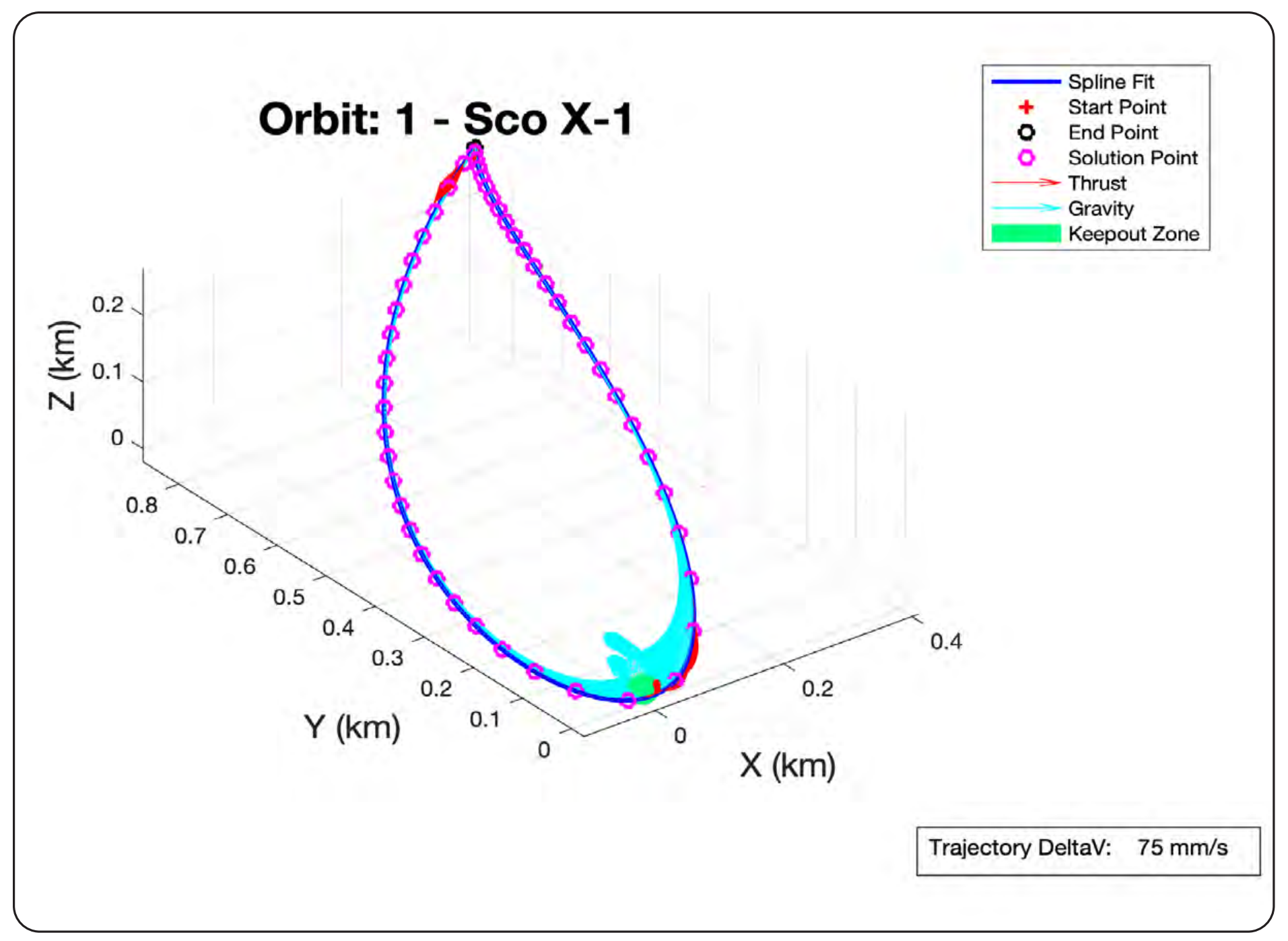
Cartesian components of S/C position over two orbits, showing the trajectory scheme.



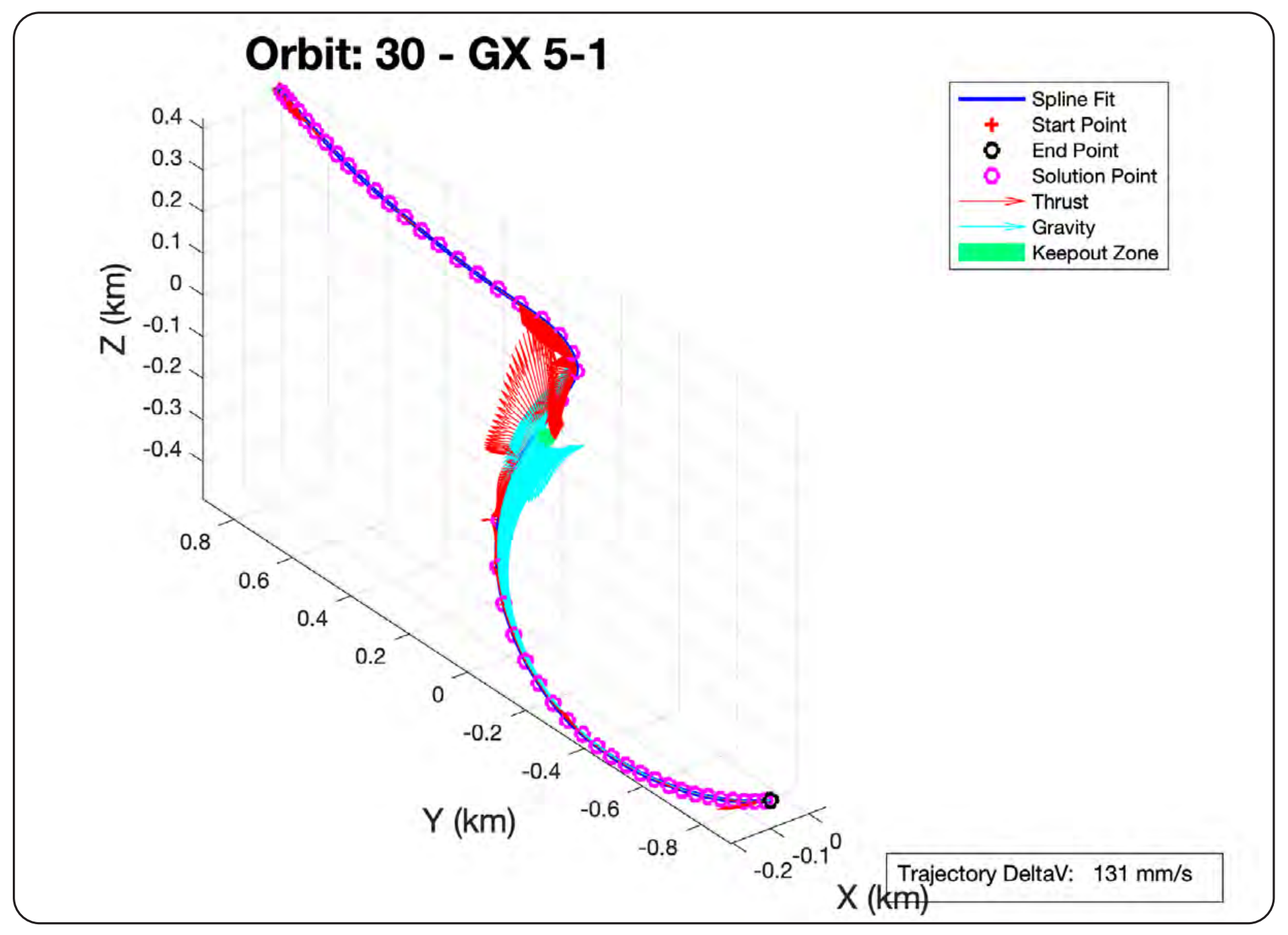
Cumulative DeltaV over the same two orbits as shown above.

Optimized Results

- * Optimized DeltaV averages ~91 mm/s per orbit



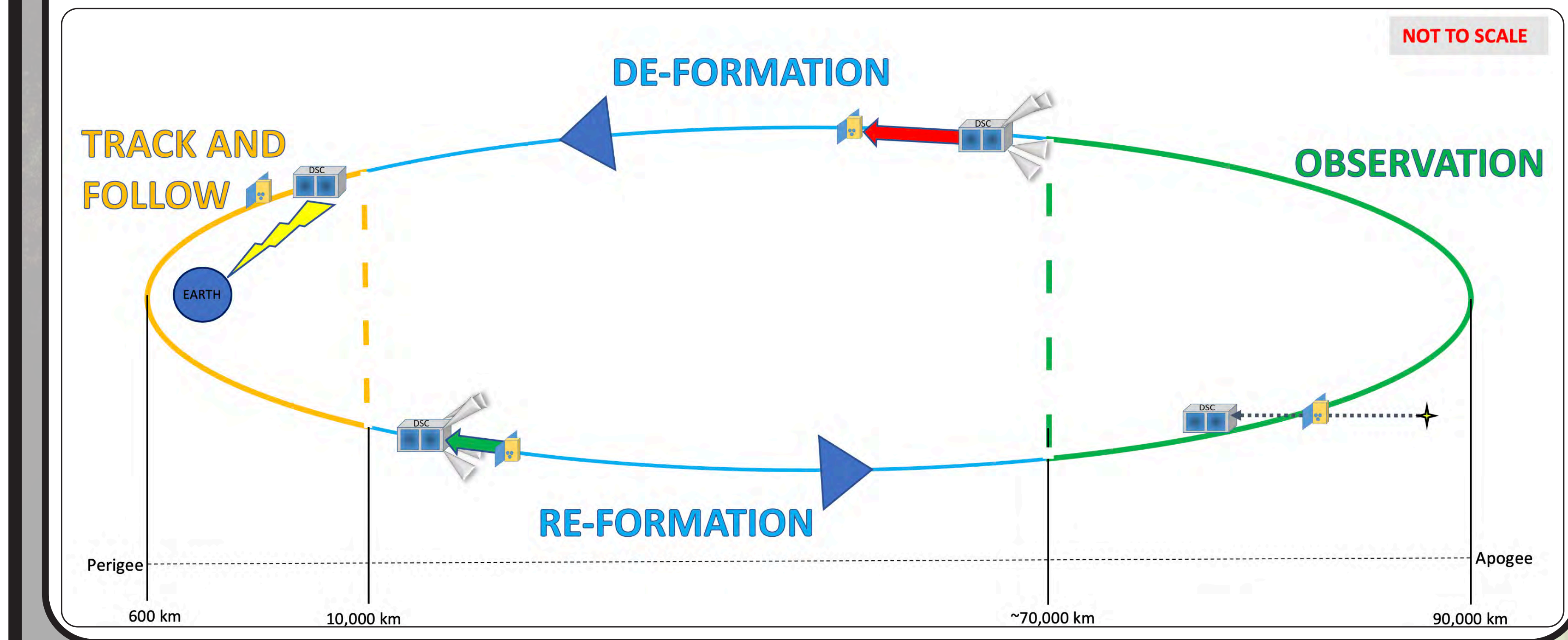
Shows propellent optimal trajectory for moving VTXO between the first two observation formations, both targeting Sco X-1. Total DeltaV for this sequence is 75 mm/s.



Shows propellent optimal trajectory moving from VTXO's 30th observation formation targeting GX 5-1 to the 31st observation formation targeting the Crab Pulsar. Total DeltaV for this sequence is 131 mm/s.

ConOps

- * Mission performs observations near apogee where gravity gradient is minimized.
- * Observations last about 10h.
- * Observation formation is broken at end of observation window.
- * Propellent optimal trajectory is followed until through perigee until beginning of next observation period.



Conclusions

- * Average Delta V of ~90 mm/s ~ factor of 4 improvement over baseline.
- * Need to add a realistic thruster model
- * Consider alternate optimization algorithms
- * Need to refine constraints on optimization
- * Potential applications to other trajectories with fixed start and end points

Citations

[1] K. Rankin, S. Stochaj, N. Shah, J. Krizmanic and A. Naseri, "VTXO - VIRTUAL TELESCOPE FOR X-RAY OBSERVATIONS," in 9th International Workshop on Satellite Constellations and Formation Flying, Boulder Colorado, 2017.

[2] H. Schaub and L. J. Junkins, Analytical Mechanics of Space Systems, Reston VA.: AIAA Education Series, 2014.

[3] H. D. Curtis, Orbital Mechanics for Engineering Students, Elsevier, 2014.

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[5] K. Rankin, S. Stochaj, J. Krizmanic, N. Shah, and A. Naseri, "SSC19-WK VII-09 Virtual Telescope for X-Ray Observations Conference on Small Satellites," in Small Satellite Conference, Logan UT, 2019.(Logan UT), (Logan UT), 2019.and A. Naseri, \SSC19-WK VII-09 Virtual Telescope for X-Ray Observations Conference on Small Satellites," in Small Satellite Conference, (Logan UT), 2019.