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Fiscal Policy, the Sraffian Supermultiplier and Functional Finance

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Abstract: Sraffian supermultiplier models (SSM) try to identify autonomous components of demand. The most plausible candidate is government consumption. Descriptively, however, government consumption does not grow at a constant rate, and prescriptively there is no justification for keeping constant the growth rate of government consumption, irrespective of economic performance. An active fiscal policy guided by principles of functional finance can produce more powerful stabilization, avoid overheating and excessive utilization rates, and secure faster adjustments of the growth rate towards its target level.

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1. Introduction

The literature on 'Sraffian supermultipliers' (SSM) suggests that long-run growth is driven by autonomous, non-capacity generating demand. Several components of aggregate demand have been singled out as potentially autonomous, including capitalist consumption, residential investment, exports and government consumption. Descriptively it is questionable whether any of these components can be viewed as autonomous in the long run.[4] They are also -- with the exception of government consumption -- extremely volatile. Even if they were autonomous, it is therefore hard to see how these components could stabilize an economy that is subject to Harrodian instability.[5]

Government consumption could in principle be autonomous; in the absence of supply side constraints, policy makers could decide to raise government consumption at a fixed proportional rate every year. Mature economies may face labor constraints, but that is not the case for dual economies, and long-run capital constraints would be removed endogenously if the supermultiplier serves to stabilize the economy at a steady growth path with utilization at the normal (or desired) rate.[6] The SSM analysis might therefore seem to offer a promising approach to policy making: if an increase in the long-run growth rate is desired, it may be enough to raise the growth rate of government consumption to the new target rate. The Harrodian mechanism will ensure that accumulation adjusts, and a new long-run equilibrium will be established with utilization at the desired (normal) rate and a growth rate that is equal the growth rate of government consumption.

This SSM policy has the virtue of simplicity, but the policy also has important weaknesses. The Harrodian forces, first, must be very weak in order for the SSM policy to

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[4] There seems to be some ambiguity with respect to the meaning of the term autonomous. In the general acceptance of the term, autonomous means to be independent of the level and/or the rate of change of economic activity. The other meaning is the one given by Thirlwall (2002), for whom autonomous means to be exogenous to the economic system. Recent work on SSM such as Brochier and Macedo Silva (2019) makes autonomous consumption an endogenous variable dependent on rentier wealth. As argued by Oreiro, Silva and dos Santos (2020,p.527, n.18), however, the dynamics of rentier wealth is determined by rentiers' saving, which depends on the level of economic activity and rentiers' income. Thus, it is hard to see how spending that is determined by wealth can be autonomous in any meaningful sense, if the analysis is extended beyond the short run.

[5] The Sraffian supermultiplier was introduced by Serrano (1995a,b), Bortis (1997) and Dejuan (2005). Interest in the approach ballooned more recently following contributions by Freitas and Serrano (2015), Allain (2015) and Lavoie (2016). Skott (2017, 2019), Nikiforos (2018) and Oreiro, Silva and Santos (2020) are among the critics. A special issue of *Metroeconomica* (2019, vol 2) was devoted to the Sraffian supermultiplier and the role of autonomous demand.

[6] In economy is mature if the long-run rate of growth is constrained by the labor supply in efficiency units. Maturity does not imply 'full employment'; France, Japan or the US are mature in the sense that fast growth of aggregate demand at, say, 10 percent annually would lead to labor shortages within a few years. Dual economies – including almost all developing economies -- have large amounts of hidden unemployment, and labor constraints do not prevent Chinese-style growth rates.

stabilize the economy. Weak Harrodian forces, second, imply that accumulation rates adjust slowly to deviations of actual from desired utilization rates and that, consequently, the adjustment process towards the target rate of growth must be slow. The adjustment speed is needlessly retarded, however, by a policy that relies on the long-run effects of an increase in the growth rate of government consumption (the growth rate of autonomous consumption). The stimulus to accumulation only comes gradually as the utilization rate responds to the rise in the level of government consumption; an increase of two percentage points in the growth rate of government consumption is large from a long-run perspective, but the effects on the utilization rate are small in the short and medium run. For any significant increase in the targeted growth rate, third, the SSM policy generates a transition path with prolonged periods of very high utilization rates. Thus, the SSM policy may come up against binding capital constraints, and one would expect overheating and inflationary pressures (as well as balance of payments problems in open economies) long before the economy hits any such absolute capacity constraints. The SSM policy, finally, determines the long-run share of government consumption as a by-product of the growth process. It is not obvious why one would want to determine the share of resources going to health care, education and other public services in this way.

The weaknesses would not be important if there were no alternative policy options. But there are alternatives, and we can do better. If the aim is to raise capital accumulation, why not boost accumulation as quickly as possible, while taking into account capital constraints (constraints on the utilization rate) and the dangers of overheating?

Lerner's (1943) principle of functional finance is usually applied to mature economies with a well-defined notion of full-employment. In these economies, Lerner argued, fiscal and monetary policy should be set to achieve full employment and a target level of investment. In a growth context, these objectives translate into targets for the level and growth rate of output, and for the capital intensity of production, if there is a choice of technique (Ryoo and Skott 2013, Skott 2016).

In dual economies, the main supply side constraint comes from the capital stock rather than the supply of labor, and full employment (in the modern sector) is not a feasible short-run target. Policy makers have to define a growth target for the modern sector, weighing the benefits of fast accumulation against the cost of foregoing current consumption (Skott 2020). Once a target for the growth rate has been defined, aggregate demand policy is left to steer the economy to -- and then stabilize it at -- a growth path with accumulation at

the target rate and utilization rate at the desired rate. Functional finance mandates the continuous *adjustment of the policy instruments* to achieve these targets.

This article illustrates the differences between the perspectives of SSM and functional finance on fiscal policy. We consider two well-known benchmark models of autonomous demand, Allain (2015) and Serrano et al. (2019). Both models assume Harrodian instability, and both look to autonomous demand as the stabilizing force. The detailed specifications, however, are quite different. We simulate the effects of an increase in the growth rate of government consumption in each of the models. Using the same benchmark models, the SSM policies are juxtaposed against policies that follow principles of functional finance.

Section 2 outlines the two benchmark models. Section 3 describes our two policy regimes: an SSM regime with a constant growth rate of government consumption and a functional-finance regime with a state-dependent fiscal policy. The simulations are in section 4. Section 5 offers a few concluding comments.

2. Benchmark Models of Autonomous Demand

2.1.   Allain's Formulation

Allain (2015) focuses on government consumption as the autonomous component of demand and to avoid complications from public debt dynamics, he assumes a balanced budget. His investment function adds Harrodian dynamics to a simple Kaleckian short-run specification,

$$\frac{I}{K} = \gamma + \gamma_u(u - u_n) \tag{1}$$

$$\dot{\gamma} = \lambda(u - u_n) \tag{2}$$

where $u = Y/K$. Private saving (S) is taken to be proportional to after-tax profits,

$$\frac{S}{K} = (1 - \tau)s\pi u \tag{3}$$

where τ, s and π denote the tax rate, the saving rate out of aftertax profits and the profit share.

Unlike Allain we include depreciation; I and S are gross investment and gross saving, and the growth rate of the capital stock is given by $\hat{K} = I/K - \delta$. Two reasons motivate this slight modification of the model. The proportional saving rate, first, seems more plausible as

a description of the relation between gross income and gross saving, rather than between net income and net saving. Using gross variables, second, empirical calibration yields a higher saving rate which favors the model: it becomes possible to allow a higher value of γ_u without jeopardizing short-run stability, and an increase in the value of γ_u enhances the stabilizing effect of autonomous demand.

Government consumption is predetermined in the short run but grows at a constant rate,

$$\hat{G} = \alpha \tag{4}$$

The ratio of government consumption to capital $(z = G/K)$ therefore follows a differential equation,

$$\hat{z} = \hat{G} - \hat{K} = \alpha - g \tag{5}$$

where $g = I/K - \delta$ is the net accumulation rate. All incomes are taxed at the same rate, and the balanced budget assumption implies that

$$G = \tau Y = T \tag{6}$$

Thus, the tax rate satisfies the condition

$$\tau = z/u \tag{7}$$

where $z = G/K$ is the ratio of government consumption to capital (the ratio of autonomous demand to capital).

Short-run equilibrium requires that $(S + T - G)/K = I/K$, and using equations (1)-(7) we have:

$$(1 - \tau)s\pi u = s\pi u - s\pi z = \gamma + \gamma_u(u - u_n) \tag{8}$$

Hence,

$$
\begin{aligned}
u &= \frac{\gamma - \gamma_u u_n + s\pi z}{s\pi - \gamma_u} \\
&= \frac{1}{s\pi - \gamma_u}\gamma + \frac{s\pi}{s\pi - \gamma_u}z - \frac{\gamma_u u_n}{s\pi - \gamma_u}
\end{aligned} \tag{9}
$$

$$
\begin{aligned}
g &= \frac{I}{K} - \delta = \gamma + \gamma_u\left(\frac{\gamma - \gamma_u u_n + s\pi z}{s.\pi - \gamma_u} - u_n\right) - \delta \\
&= \frac{s\pi}{s\pi - \gamma_u}\gamma + \frac{s\pi.\gamma_u}{s\pi - \gamma_u}z - \frac{s\pi.\gamma_u u_n}{s\pi - \gamma_u} - \delta
\end{aligned} \tag{10}
$$

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The dynamics of the economy can now be described by a 2D system of differential equations. Substituting (9)-(10) into equations (2) and (4), we have:

$$\dot{\gamma} = \lambda(u - u_n) = \lambda\left(\frac{\gamma - \gamma_u u_n + s\pi z}{s\pi - \gamma_u} - u_n\right) \tag{11}$$

$$\dot{z} = z(\alpha - g) = z\left[\alpha - \frac{s\pi}{s\pi - \gamma_u}\gamma - \frac{s\pi.\gamma_u}{s\pi - \gamma_u}z + \frac{s\pi.\gamma_u.u_n}{s\pi - \gamma_u} + \delta\right] \tag{12}$$

Equations (10)-(11) always have a stationary solution (a steady growth path) with $z = 0$ and $\gamma = s\pi u_n$. This stationary solution describes the standard Harrodian warranted growth path in an economy without autonomous demand. The more interesting case arises when the system allows for a second solution with $z > 0$; this happens if $s\pi u_n > \alpha + \delta$.

Assuming that the existence condition is satisfied, the second stationary solution is given by $(\gamma^*, z^*) = \left(\alpha + \delta, u_n - \frac{\alpha + \delta}{s\pi}\right)$. The local stability of this solution is determined by the Jacobian matrix evaluated at the stationary state. We have:

$$J(\gamma, z) = \begin{bmatrix} \frac{\lambda}{s\pi - \gamma_u} & \frac{\lambda s\pi}{s\pi - \gamma_u} \\ -z^*\frac{s\pi}{s\pi - \gamma_u} & -z^*\frac{s\pi\gamma_u}{s\pi - \gamma_u} \end{bmatrix} \tag{13}$$

The determinant is unambiguously positive,

$$\det J = z^*\frac{\lambda s\pi}{(s\pi - \gamma_u)} > 0 \tag{14}$$

Thus, local stability of the steady growth path is ensured if the trace of Jacobian is negative; formally, if

$$tr\, J = \frac{\lambda - s\pi\gamma_u z^*}{s\pi - \gamma_u} < 0 \tag{15}$$

By assumption, the short-run equilibrium is stable $(s\pi > \gamma_u)$. The stability condition (15) therefore imposes an upper limit on the Harrodian adjustment parameter,

$$\lambda < s\pi\gamma_u z^* \tag{16}$$

For comparison with the Serrano-Freitas version, it is useful to rewrite the Harrodian equation (2). The investment equation (1) implies that

$$(g + \delta) - \gamma = \gamma_u(u - u_n) \tag{17}$$

Combining (2) and (17), we have

$$\dot{\gamma} = \frac{\lambda}{\gamma_u}(g + \delta - \gamma) \tag{18}$$

Thus, if $\beta = \lambda/\gamma_u$ denotes the sensitivity of the change in γ to deviations of the accumulation rate from its steady growth value, the limit on the stability condition in equation (16) can expressed as

$$\beta < s\pi z^* \tag{19}$$

2.2. A Serrano-Freitas version

Serrano et al. (2019) (SFB) assume that saving and investment are given by the following equations:

$$S = sY - Z \tag{20}$$

$$I = hY \tag{21}$$

where Z is autonomous demand.[7] As in the Allain example, let government consumption be the autonomous component $(Z = G)$ and assume that the government budget is balanced,

$$T = G \tag{22}$$

The equilibrium condition for the goods markets can be written

$$S + T - G = I \tag{23}$$

or simply, using equation (22),

$$S = I \tag{23}$$

Assuming a constant saving rate $\tilde{s}$ out of disposable income, we have

$$S = \tilde{s}(Y - T) \tag{24}$$

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[7] Curiously, in the SFB specification of the investment function there is no predetermined investment, even in the short run. Investment adjusts instantaneously to any short-run increase in the level of output. This determination of investment by the value of current output implies that non-capacity generating demand, by construction, becomes the only predetermined variable. Moreover, it denies any influence of uncertainty and animal spirits on investment spending In fact, Sraffian or Neo-Ricardian Keynesians do not seem to attach much importance to uncertainty, expectations and animal spirits for economic analysis. In the words of Eatwell and Milgate : "If, on the other hand, an attempt is made to locate uncertainty and expectations within the class of the persistent, systematic forces characterizing the workings of the economy, then the analysis becomes bereft of any definite result—the behavior of the economy being as arbitrary as the hypothesis made about the formation of expectations" (2011, p. 301)

As in our version of the Allain model, I and S denote gross investment and gross saving.

Combining equations (21)-(24), the level of output in short-run equilibrium is given by

$$Y = \frac{\tilde{s}}{\tilde{s}-h}G \qquad (25)$$

The Keynesian stability condition is satisfied, and the equilibrium solution is positive if $\tilde{s} > h$. If this condition fails to be met, the model becomes economically meaningless.

Unlike most of the literature on autonomous demand, Serrano et al. (2019) cast their model in discrete time. To facilitate comparison with other models, we recast it in continuous time. This change from discrete to continuous time relaxes the stability condition slightly because instability through overshooting cannot occur in continuous time. The basic properties of the model are unchanged, however.

The SFB specification in discrete time implies the following Harrodian equation for the share of investment in output (using their equations $(B2)$-$(B3)$):

$$h_t - h_{t-1} = \beta[v(y_{t-1} + \delta) - h_{t-1}] \qquad (26)$$

where $y_t = (Y_t - Y_{t-1})/Y_t$ is the growth rate of output. The parameter v denotes the capital output ratio at normal utilization. The parameter β represents the speed of adjustment of the expected growth rate of output towards actual the growth rate; expected growth influences investment, and this adjustment captures the Harrodian dynamics in the model[8].

From the equilibrium condition (25) it follows that (their equation B6).

$$y_t = \alpha_t + \frac{h_t - h_{t-1}}{\tilde{s}-h}(1 + \alpha_t) \qquad (27)$$

where $\alpha_t = (G_t - G_{t-1})/G_{t-1}$ is the growth rate of government consumption. Subtracting y_{t-1} from both sides, this equation can be rewritten

$$y_t - y_{t-1} = \alpha_t + \frac{h_t - h_{t-1}}{\tilde{s}-h_{t-1}-(h_t-h_{t-1})}(1 + \alpha_t) - y_{t-1} \qquad (28)$$

The continuous-time version of equations (26) and (28) are[9]

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[8] A similar interpretation of the Harrodian dynamics has been suggested by Lavoie (1995). But unlike SFB and seemingly motivated by a desire to simplify the analysis, Lavoie uses the accumulation rate $\widehat{K}$ as an approximation for $\widehat{Y}$.

[9] The interaction term $(h_t - h_{t-1}).(G_t - G_{t-1})$ vanishes in the continuous-time version.

$$\dot{h} = \beta[v(y + \delta) - h] \tag{29}$$

$$\dot{y} = \alpha + \frac{\dot{h}}{\tilde{s} - h - \dot{h}} - y = \alpha + \frac{\beta[v(y+\delta)-h]}{\tilde{s}-h-\beta[v(y+\delta)-h]} - y \tag{30}$$

This two-dimensional dynamic system has no economically meaningful stationary solutions if $\tilde{s} < v(\alpha + \delta) = h^*$.

Focusing on the meaningful case with $\tilde{s} > v(\alpha + \delta)$, equations (29)-(30) have a unique stationary solution with $y = \alpha$ and $h = v(\alpha + \delta)$. Evaluated at the stationary point, the Jacobian matrix is given by

$$J(h, y) = \begin{bmatrix} -\beta & v\beta \\ -\frac{\beta}{\tilde{s}-v(\alpha+\delta)} & \frac{v\beta}{\tilde{s}-v(\alpha+\delta)} - 1 \end{bmatrix} \tag{31}$$

The determinant is unambiguously positive $(\det J = \beta)$ and the stationary state will be locally stable if the trace is negative. As in the Allain model, the instability condition can be expressed as an upper limit on the adjustment speed; β must satisfy the condition[10]

$$\beta < \frac{\tilde{s} - v(\alpha + \delta)}{v - [\tilde{s} - v(\alpha + \delta)]} = \frac{\tilde{s}(G/Y)^*}{v - \tilde{s}(G/Y)^*} \tag{29}$$

The SFB and Allain specifications are more closely related than it might seem. Using the notation in the Allain model, the output capital ratio is u and its value at normal utilization is u_n; thus, $vu_n = 1$. Thus, setting $h = \gamma_u$ and $\gamma = \gamma_u u_n$, the SFB specification of investment can be written as

$$\frac{I}{K} = \frac{h}{v} = \gamma_u u = \gamma + \gamma_u(u - u_n) \tag{30}$$

The SFB dynamics in equation (29) can also be rewritten in a similar form to the Allain dynamics in equation (19):

$$\dot{\gamma} = u_n \dot{\gamma}_u = u_n \beta[v(y + \delta) - h] = \beta[(y + \delta) - u_n h] \tag{31}$$

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[10] We have $\tilde{s}(G/Y) = \tilde{s} - v(\alpha + \delta)$ in steady growth. This result follows from
$$\tilde{s}(Y - T) + T = I + G$$
$$T = G$$
$$I = (\alpha + \delta)K$$
$$\frac{Y}{K} = \frac{1}{v} = u_n$$
Hence,
$$\tilde{s}(1 - G/Y) = v(\alpha + \delta)$$
or
$$\tilde{s}G/Y = \tilde{s} - v(\alpha + \delta)$$

Equation (31) can be compared with Allain's specification. Setting $\lambda = \beta u_n$, the Allain specification relates the change in γ to the difference between actual accumulation and the steady growth rate. SFB assume that both γ and γ_u change and relate the change to the difference between output growth and the accumulation rate that would have been obtained with the given investment output ratio if utilization had been at the normal rate.

Given these similarities, the correspondence between the stability conditions in (19) and (29) may not be surprising. The saving rate out of disposable income is $s\pi$ in Allain and $\tilde{s}$ in SFB, and the ratio $z^* = \left(\frac{G}{K}\right)^* = \left(\frac{G}{Y}\right)^* u_n$ in the Allain notation is equal to $\left(\frac{G}{Y}\right)^* \frac{1}{v}$ in SFB. Thus, the local stability conditions are identical, except for the appearance of the (very small) term $\tilde{s}\left(\frac{G}{Y}\right)^*$ in the denominator of the expression in (29).

3. Two Policy Regimes

Our SSM regime is straightforward: the growth rate of government consumption (α) is set equal to the target rate of growth in both the Allain and SFB models. Once α has been chosen, no further adjustments are made on the spending side; following Allain, tax rates are adjusted to maintain a balanced budget.

The functional finance approach advocates an active fiscal policy, rather than a passive, Friedmanite rule that keeps the growth rate of government consumption constant. The fiscal parameters are adjusted continuously in response to changes in the state of the economy. We assume a balanced budget at all times but unlike in the SSM approach, the growth rate of government consumption is not kept constant. There is no reason why fiscal policies based on functional finance should maintain a balanced budget; it may be desirable to run deficits in some periods but surpluses if conditions change. The balanced-budget assumption facilitates comparison with the SSM scenarios, however.

Suppose that the economy is initially following a steady growth path with output and government consumption growing at the rate α_0 and utilization at the desired rate. Policy makers now want to raise the growth rate to α_1. Suppose, moreover, that they wish to implement this increase as fast as possible, but that utilization rates above $\bar{u}$ (where $\bar{u} > u_n$) will lead to overheating and bottlenecks with adverse effects on inflation (and, in an open economy, on the current account). Given these targets and constraints, functional finance

prescribes an expansionary fiscal policy that raises actual utilization rates to the upper limit $\bar{u}$ as quickly as possible. Once $u = \bar{u}$, any further increase must be avoided, and fiscal adjustments now aim to keep utilization at the safe rate $\bar{u}$ until the accumulation rate has increased sufficiently, at which point utilization rates can be brought back down to the desired rate u_n.

The implementation of this general principle is slightly different in the two benchmark models. In the Allain model output is a jump variable, the Harrodian dynamics determine the change in the investment parameter γ as a function of the utilization rate, and the short-run solution for $u = Y/K$ is given by

$$u = \frac{\gamma - \gamma_u u_n + s\pi z}{s\pi - \gamma_u} \tag{32}$$

Changes in the level of G affect the ratio z of government consumption to capital and thereby also the utilization rate u. Implementing the expansion of the modern sector as fast as possible translates into an instantaneous jump in z to get $u = \bar{u}$.

Setting $u = \bar{u}$ and solving for z, we get

$$z = \frac{G}{K} = \bar{u} - \frac{\gamma + \gamma_u \cdot (\bar{u} - u_n)}{s\pi} \tag{33}$$

The utilization rate now exceeds normal utilization; the value of γ will start increasing, and policy makers reduce z gradually as γ increases in order to keep actual utilization at the upper bound $(u = \bar{u})$. When γ has reached the target value for the gross accumulation rate $(\gamma = \alpha + \delta)$, the expansionary policy is abandoned. The government spending ratio z is adjusted to the level that is consistent with $u = u_n$ and the target growth rate, that is,

$$z = z^* = u_n - \frac{\alpha + \delta}{s\pi} \tag{34}$$

The length of the adjustment period can be found analytically in this model: the dynamic equation for γ implies that the transition from α_0 to α_1 will take $(\alpha_1 - \alpha_0)/[\lambda(\bar{u} - u_n)]$ periods.

The SFB model treats output as a state variable; it cannot jump and the growth rate of output determines the change in the investment share. Adapting the functional finance approach to this setting, we assume that the general objective is unchanged: to raise the long-run growth rate as quickly as possible without violating the constraint on utilization $(u \leq \bar{u})$. In the SFB model the policy solution takes the form of four phases.

During the first phase the growth rate of output is set 'as high as possible' ($y = M$ for some large M) until u reaches $\bar{u}$. Government consumption and output do not jump, as in the Allain model. Instead, high growth rates of output during the initial phase are achieved by setting a high growth rate of government consumption (cf. equation (30)). This is the SFB version of raising u to $\bar{u}$ instantaneously in the Allain version. In phase two, fiscal policy is adjusted to keep $u = \bar{u}$; that is, the growth rate of output is now set equal to the net accumulation rate, $y = \widehat{K} = h\bar{u} - \delta$. The investment ratio h increases gradually during this phase which comes to an end when h reaches a threshold value $\bar{h}$. During phase three output is reduced as 'fast as possible' ($y = -M$) until $u = u_n$. The threshold value $\bar{h}$ is calibrated to ensure that at the moment when u returns to u_n we also have $h = v(g^T + \delta)$. If the maximum growth rate M goes to infinity, the threshold value of h is given by $\bar{h} = v(g^T + \delta) + v\beta \ln(\bar{u}/u_n)$; this expression serves as a good approximation for large finite values of M. Phase four now starts: the target has been achieved, and the task of fiscal policy will be to stabilize the economy at $u = u_n$.

4. Simulations

All our simulations consider an initial steady-growth path which is disturbed by a permanent shock to the growth rate of government consumption. The simulations use the Runge-Kutta method for numerical integration of ODEs. [11]

4.1. Allain-SSM

Allain's model is simulated using the parameters in table 1. The parameter α -- growth rate of public spending -- is raised from 0.03 to 0.05. Given the values of the other parameters, the condition for local stability of the new steady growth path requires that $\lambda < 0.005$.

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[11] We also analyzed the paths using Euler's method for numerical integration. No relevant differences appeared among the simulated models.

Table 1 – Parameters for Allain-SSM simulation

Parameter	Value	Parameter	Value
α_0	0.03	λ	0.0025, 0.02
α_1	0.05	π	0.5
δ	0.07	s	0.6
γ_u	0.166	u_n	0.5

Figures 1a-1d: Allain-SSM trajectories for $\lambda = 0.0025$
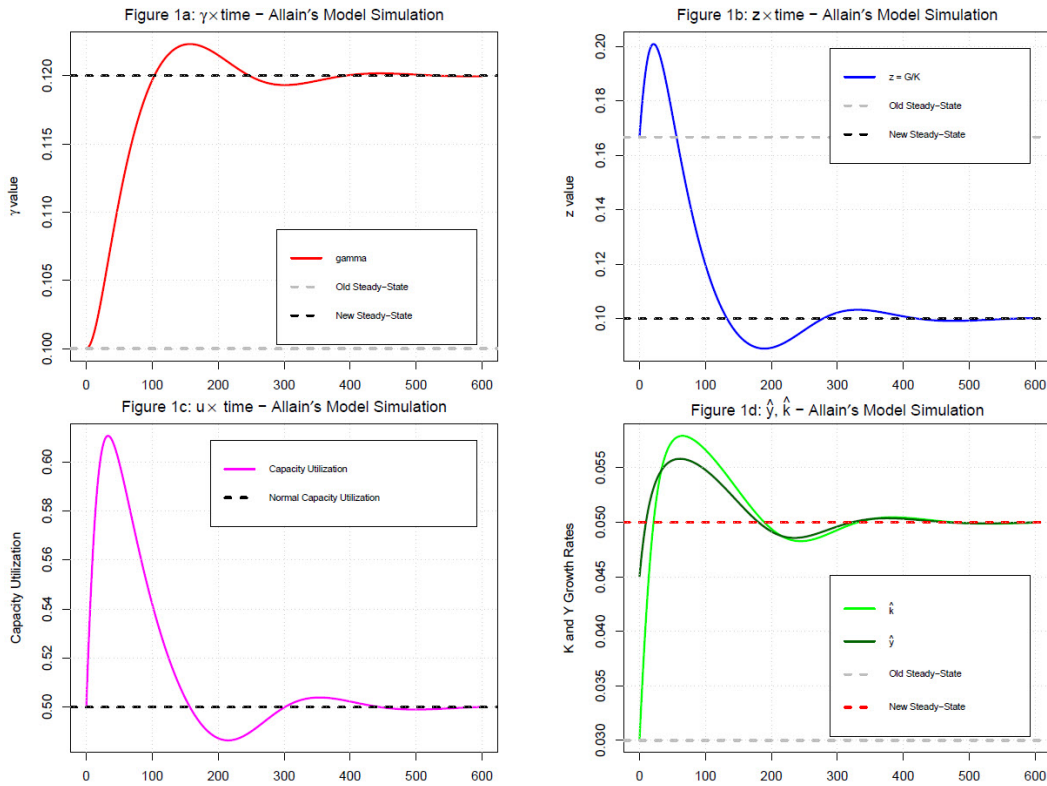
Figures 1a-1d depict a stable case with $\lambda = 0.0025$; figures 2a-2d illustrate the instability that follows from raising the adjustment speed. The extremely slow rate of convergence is explained by the low value of the adjustment parameter. If the adjustment parameter is interpreted as reflecting adjustments in expected growth, this low value ($\lambda = 0.0025$) implies that the half-life of deviations of expected growth from a constant steady-

growth value is about 46 years, that is, if accumulation rates were to increase permanently from 3 percent to 5 percent, it would take 46 years before expected growth rates have adjusted from 3 to 4 percent.[12] Slow convergence is coupled with a prolonged period – more than 70 years – in which utilization rates exceed the normal rate by more than ten percent.

The high adjustment speed in figures 2a-2d $(\lambda = 0.02)$ gives a half-life of about 6 years which still would seem on the low side for the adjustment of expectations. Since the adjustment speed now exceeds the critical value, the steady growth path becomes unstable.

Figures 2a-2d: Allain-SSM trajectories for $\lambda = 0.02$.
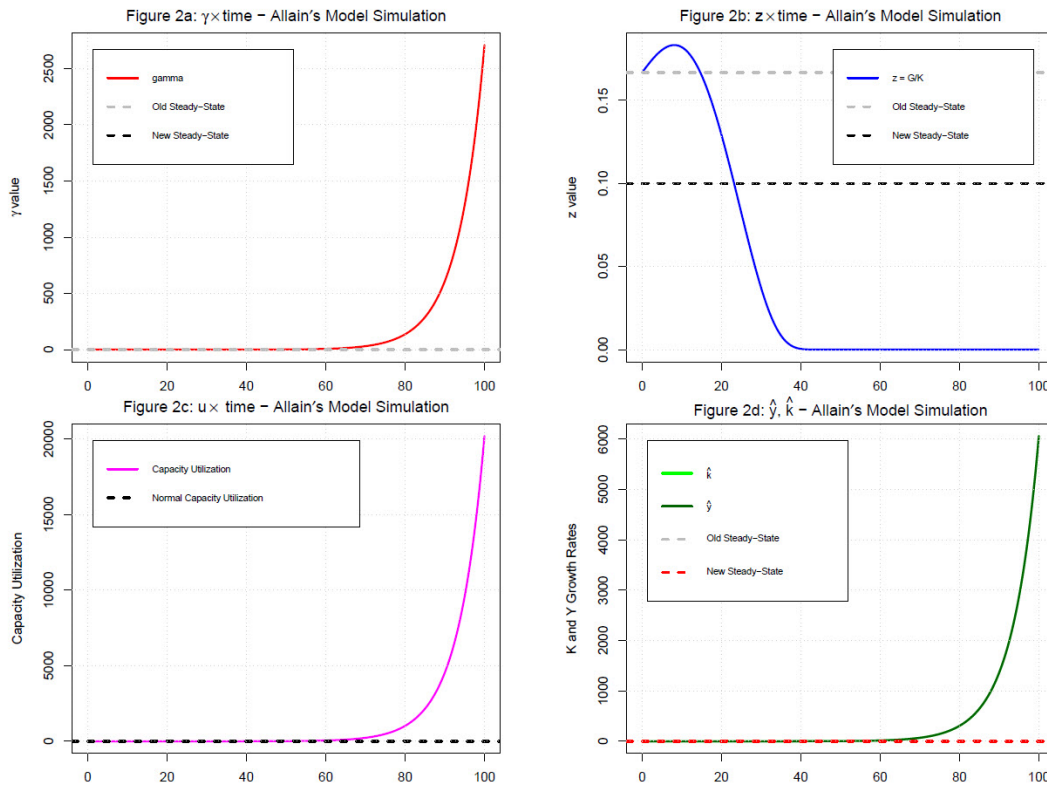
4.2.   Allain-FF

The simulations in figures 3a-3d for the functional finance version of the Allain model use the same parameters as in Allain-SSM simulation in Figures 1a-1d. The convergence is still slow, but the profile has changed: the utilization rate jumps immediately to the upper limit of the safe range and stays at this upper limit during the transition process. The accumulation rate therefore increases more quickly than in the Allain-SSM specification during the early stage of the transition. The relative speeds are reversed during the later stages as the utilization rates increases above the safe rate in the SSM specification. The SSM trajectory overshoots the new steady growth path, and the times to full convergence have the same order of magnitude in the two cases.

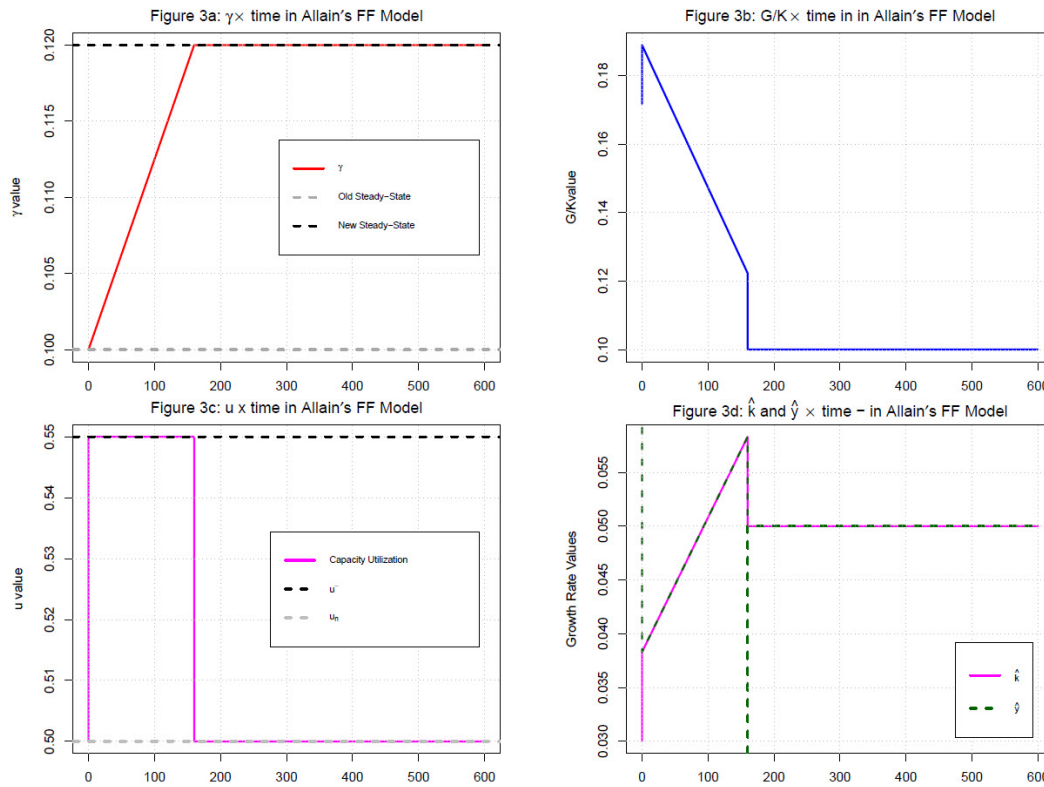Figures 3a-3d: Allain-FF trajectories for $\lambda = 0.0025$

Figure 4a-4d corresponds to figure 2a-2d for the Allain SSM model. In the FF version we get stability, and full convergence to the new steady growth path happens in 20 years. Stability conditions do not limit the permissible adjustment speeds; the time to full convergence would be reduced to 5 years if the speed were increased to $\lambda = 0.08$ which corresponds to a half-life of expectation adjustment of about 1.5 years.

Figures 4a-4d: Allain-FF trajectories for $\lambda = 0.02$
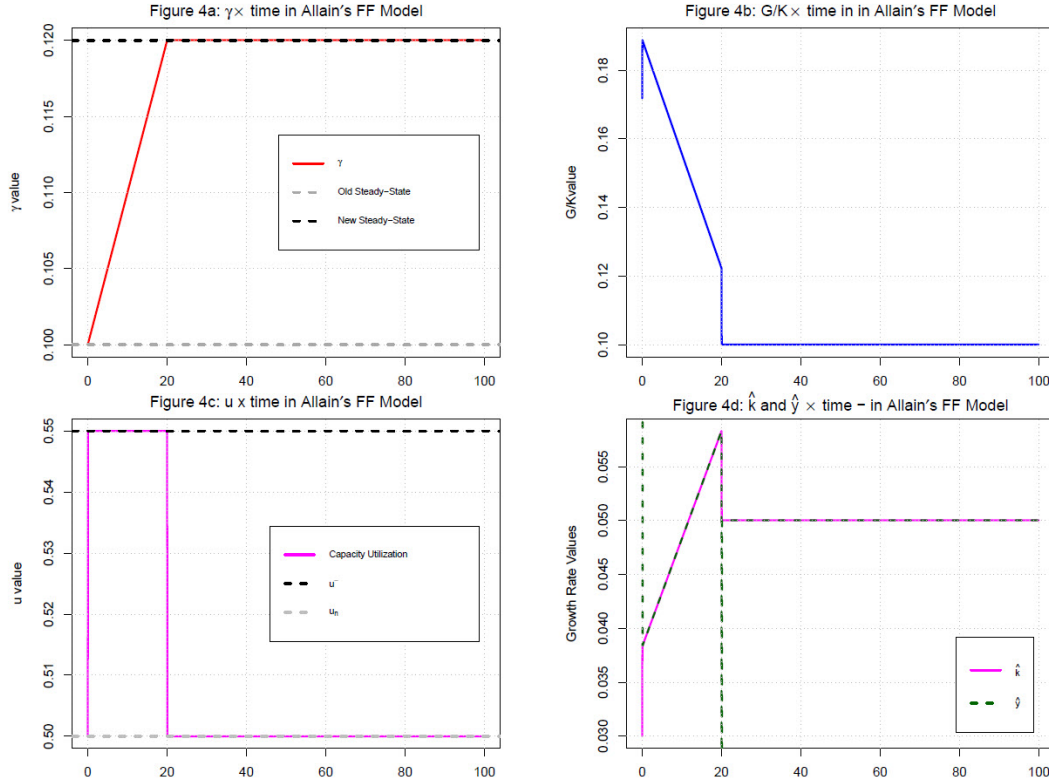
4.3. SFB-SSM

As in the Allain calibration, we assume a capital output ratio of 2 (at the normal utilization rate) and a saving rate of 0.3 out of disposable income. The full set of parameter values are listed in Table 2. Using (29), the local stability of the new steady growth path requires that $\beta < 0.0309$.

Table 2 – Parameters for SFB-SSM Simulation

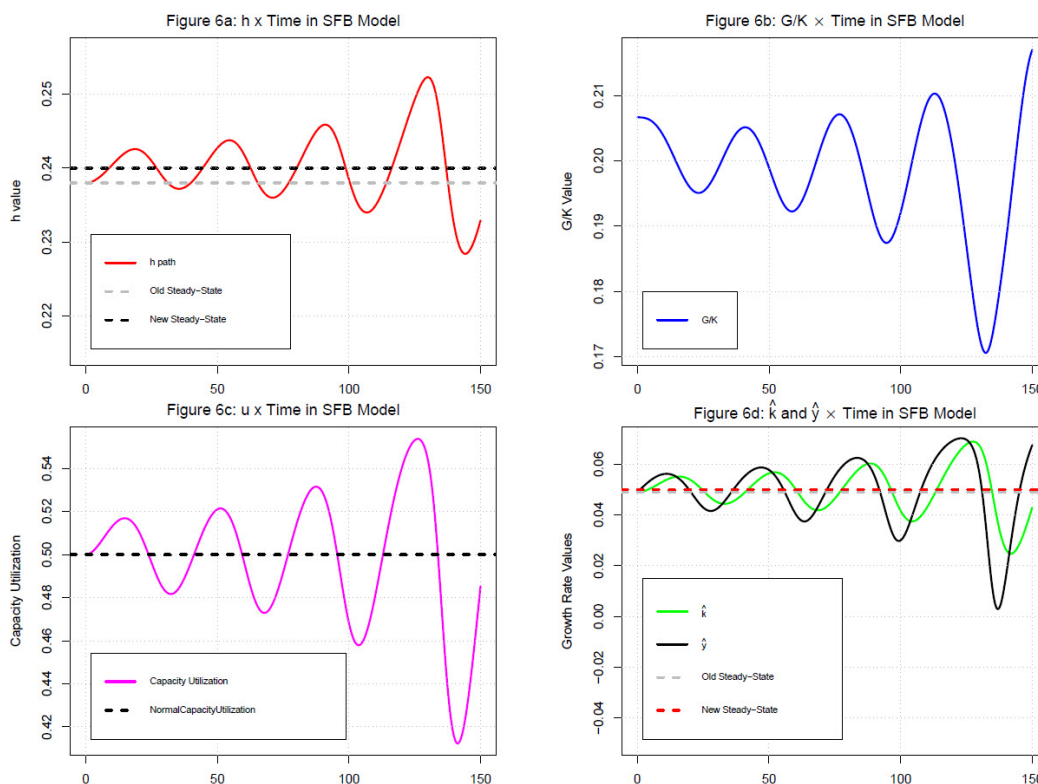| Parameter	Value	Parameter	Value
α_0	0.03, 0.049	δ	0.07
α_1	0.05	$\tilde{s}$	0.3
β	0.015, 0.0315	$v = 1/u_n$	2.0

The low adjustment speed for h in figures 5a-5d implies convergence but – as in figures 1a-1d – the convergence is slow, and utilization rates remain more than ten percent above the normal rate for more than 50 years.

Figures 5a-5d: SFB-SSM trajectories for $\beta = 0.015$
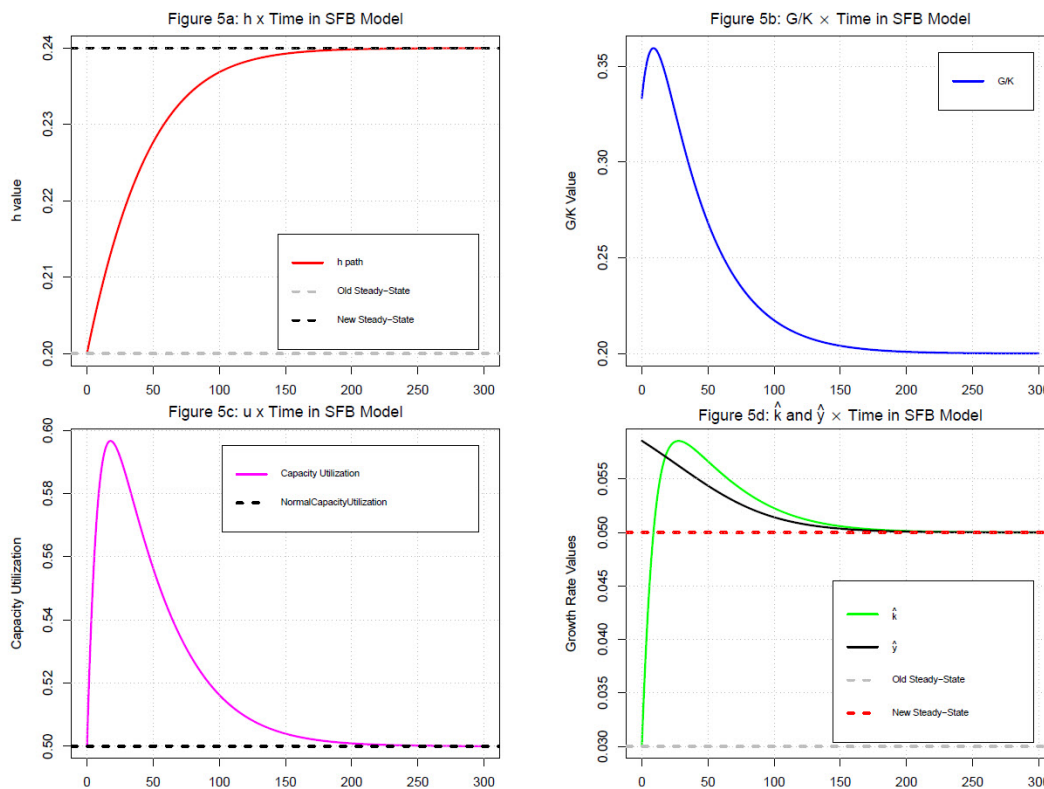
Especially when combined with large shocks, high values of the adjustment speed β lead to violent instability, and – given the discrete approximations used in the simulations – it is difficult to get the algorithms to produce sensible results. Figures 6a-6d depicts a marginally unstable case with $\beta = 0.0315$ and an initial steady growth path with $\alpha = 0.049$, just marginally below the new target.

Figures 6a-6d: SFB-SSM trajectories for $\beta = 0.0315$

Figure 6a: h x Time in SFB Model

Figure 6b: G/K × Time in SFB Model

Figure 6c: u x Time in SFB Model

Figure 6d: $\hat{k}$ and $\hat{y}$ × Time in SFB Model

4.4. SFB-FF

The simulations of the functional-finance version of the SFB model are in figures 7a-7d and 8a-8d. Figures 7a-7d use the same parameters as in figure 5a-5d. The small adjustment parameter makes for slow convergence and, as in figures 3a-3d, utilization rates are at the upper limit during most of the transition process.

The adjustment parameter in figures 8a-8d is significantly higher than in figures 6a-6d, and the initial growth rate is 3 percent, as in all other simulations except for figures 6a-

6d where is was set at 4.9 percent. Full convergence is achieved in 8 years, again with utilization rates at the upper limit during (most of) the transition process.
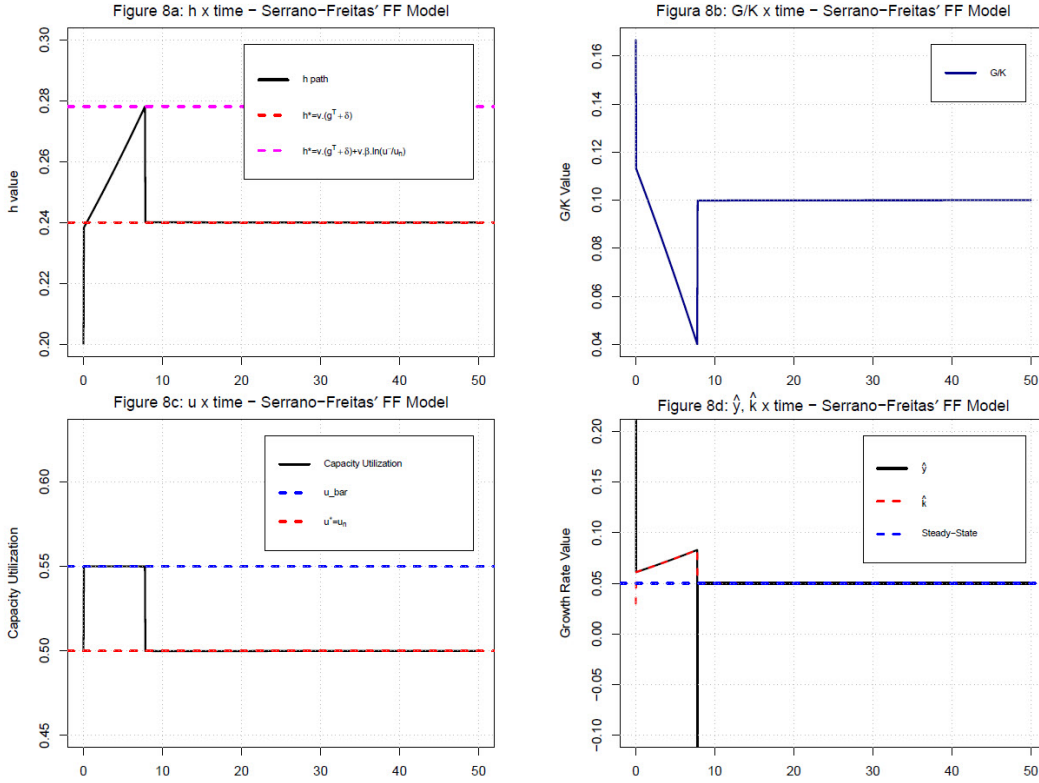
Table 3 – Parameters for SFB-FF simulation

Parameter	Value	Parameter	Value
M	2.0	δ	0.07
β	0.015, 0.2	α_0	0.03
$\bar{u}$	0.55	α_1	0.05
$1/v = u_n$	0.50	$\tilde{s}$	0.3

Figures 7a-7d: SFB-FF trajectories for $\beta = 0.0015$
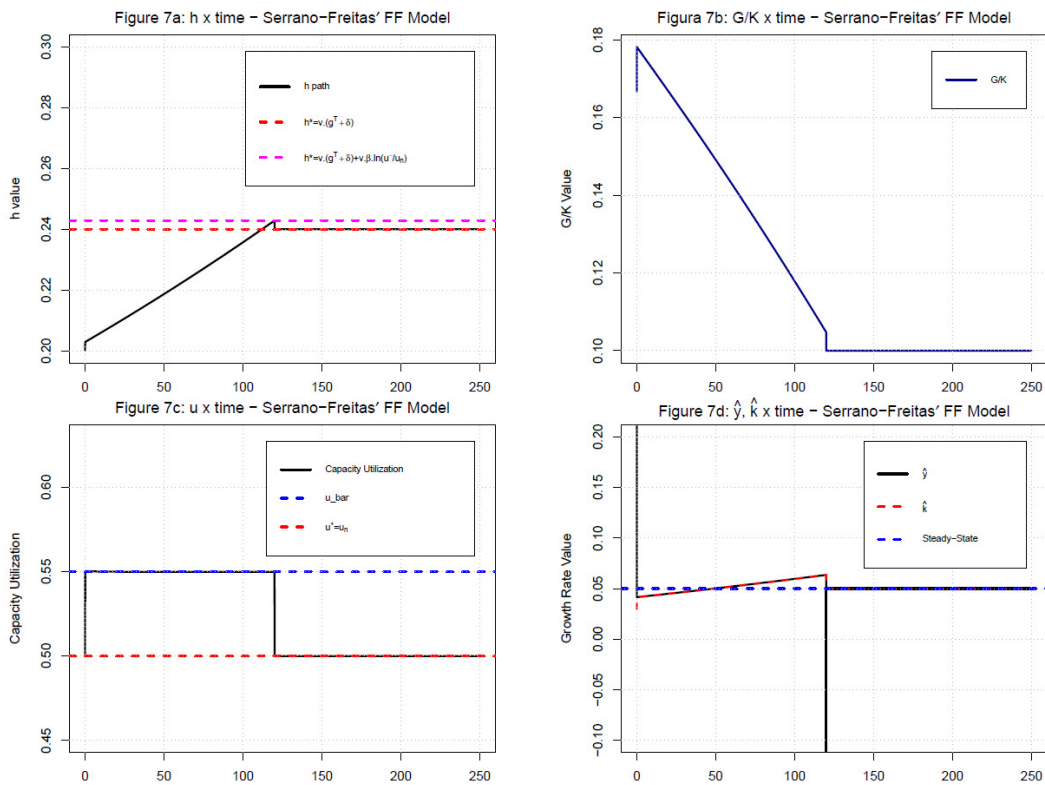
Figures 8a-8d: SFB-FF trajectories for $\beta = 0.2$

5. Conclusion

The post-Keynesian literature increasingly recognizes the potential significance of Harrodian instability. The SSM approach has addressed the instability issue by emphasizing the stabilizing forces of components of demand that do not create capacity and are 'autonomous', that is, independent of past, current and expected future movements in output.

This approach has been attractive to many researchers because allegedly it preserves a 'Keynesian position' in which investment is independent of saving and long-run growth is driven by autonomous demand (e.g. Garegnani 1992, pp.47-48, Serrano and Freitas 2015,

p.17). This argument is peculiar. Why would one insist that changes in the rate of saving have no effects on current or future investment? Saving rates influence aggregate demand and the utilization rate, and firms react to changes in capacity utilization. There is nothing 'un-Keynesian' about feedback effects from saving behavior to investment. In fact, in the SSM models autonomous demand allows utilization to converge to the desired rate precisely because the trajectory of autonomous demand generates changes in the average saving rate; these changes endogenize Harrod's warranted growth and influence investment, thereby reversing the cumulative process by which actual growth rate diverges from the warranted growth rate (Oreiro, Silva and Santos 2020; Skott 2019b).

Clearly, some demand components are autonomous in the short run. This, indeed, is a standard element of all short-run Keynesian models. Fractions of both consumption and investment can plausibly be seen as predetermined (exogenously given) in the short run, and we routinely analyze the short-run effects of shifts in consumer confidence or animal spirits. It is also true that some components of demand can be semi-autonomous in the medium run. Relaxations of credit constraints, for instance, can generate asset bubbles that feed on themselves and influence aggregate demand (Oreiro 2005). But asset bubbles do not continue forever, and feedback effects from output are important, both when it comes to sustain the bubbles and for an understanding of why the bubbles burst at some point. These autonomous short-run components and semiautonomous medium-run processes are irrelevant for the analysis of the local stability properties of the steady growth path in the SSM models.

Government consumption could play the role of autonomous demand in the long run. It is autonomous in the sense that it need not be closely tied to movements in output, and it could be set to grow at an exogenous rate. A sensible policy, however, will adjust fiscal (and monetary) policy so as to achieve the policy makers' targets. In a mature economy, as Lerner (1943) argued, economic growth with full employment would seem an obvious and fairly uncontroversial target. To be sure, 'full employment' may not be precisely defined, even in a mature economy, and in dual economies there is no similar, obvious target for the growth rate. But the principle of functional finance still applies: if a growth target has been decided upon, we can analyze the implications for fiscal and monetary policy. Policy makers still face a double challenge: to adjust the warranted growth rate to the target rate and prevent divergence from the warranted growth path. To best meet this challenge, they must respond to movements in output and, more generally, to economic performance.

The simulations in this paper illustrate the difference between the two approaches to economic policy. The rate of convergence is extremely slow in all scenarios, when the Harrodian adjustment parameter is kept within the range that will ensure stability in the SSM cases. The range is very narrow, however, and the implied half-life of adjustments in expectations is unreasonable large. Plausible adjustment speeds generate fast convergence in the FF cases, but divergence in the SSM cases.

Even in the stable cases, the SSM simulations produce prolonged periods with very high utilization rates. Empirically, utilization rates exhibit large cyclical fluctuations, but it would be unprecedented to have utilization rates stay 10-20 percent above normal for a period of more than 50 years; yet this is what happens in the SSM simulations of a two percentage point increase in growth rate of autonomous demand. The FF simulations capped the utilization rate at 10 percent above normal; if 20 percent is safe – in the sense that it does not lead to bottlenecks and inflation -- the cap could be raised, and the convergence would be faster.

SSM proponents could object that the cards have been stacked against the SSM policy: the simulations of functional finance presume an unrealistic ability of policy makers to control and fine tune the economy. This is a fair point. But it does not affect the main argument: we may not be able to fine tune the economy in the precise way suggested by the simulations, but surely we can do much better than keep $\hat{G}$ constant. If an economy is in deep recession, then presumably we would all recommend aggressive stimulus, rather than balanced budgets and the continuation of the previous trend in government spending.

The emphasis in SSM models on the stabilizing influence of autonomous demand suggests a simple rule for fiscal policy: set the growth rate of G and rely on the long-run convergence of accumulation and economic growth to this growth rate. This is the policy rule that we have simulated and that we criticize in this note. But proponents of SSM have not, to our knowledge, been explicitly advocating this rule, and maybe we have misinterpreted their position. We would be delighted if in fact they recommend a much more active policy. But if active policy – perhaps along the lines of functional finance -- is what they recommend, there would seem to be neither a need for autonomous demand to stabilize the economy nor any good candidates for the role of long-run autonomous demand. Instead

of a vain search for any such candidates, we can focus on discussing how policy should be designed to meet the challenges we face.[13]

The simple models and simulations in this paper have many limitations. We have completely eschewed open economy complications, except for a brief reference to overheating and balance of payments problems. We have also said nothing about monetary policy or industrial policy, and we have restricted the types of fiscal policy under consideration. Sensible fiscal policies may require a non-balanced budget, and we excluded this possibility by assumption. The exclusion forced us to treat government consumption and the share of government consumption in total income as mere policy instruments to control the accumulation rate. The government consumption ratio G/Y should not, however, be treated as an accommodating variable whose value has no independent interest aside from its effects on aggregate demand.

Fast growth may necessitate a squeeze on consumption (this happens in the SSM models through the decline in the share of autonomous demand). But it does not have to be government consumption that is squeezed; in fact, large parts of government spending may be essential for long-term development. Decisions must be made about how many teachers and roads are needed; taxes can then be used to adjust aggregate demand (Skott 2020). The restricted policy space and the balanced-budget assumption are harmless for the purposes of this paper. But policy discussions need to consider a much richer menu, once we abandon fiscal rules of constant growth rates for government consumption.

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[13] Some recent contributions point in this direction. Fazzari et al. (2020, p.602), for instance, suggest that "[w]hen private autonomous demand falters public demand can help avoid stagnation", and Allain (2020) link movements in what he still labels 'autonomous demand' to changes in the unemployment rate. Franke (2018) and Ryoo and Skott (2019) examine stabilization policy in Harrodian models.

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