1965

The refinement of a test of quantitative judgment.

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THE REFINEMENT OF A TEST

OF

QUANTITATIVE JUDGMENT

by
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A problem presented in partial fulfillment
of the requirements for the
Master of Education Degree
School of Education
University of Massachusetts
1965

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INTRODUCTION

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INTRODUCTION

The scope and quantity (if not quality) of mathematics knowledge has been accelerated at an ever-increasing rate. The rapidity of this growth has affected the nation as a whole and the daily lives of its people, with far-reaching consequences. Among the latter has been the revision of mathematics curricula within educational institutions in both private and public schools. Outstanding among the changes which have occurred has been, according to Deans, the intensified emphasis upon the imaginative phases of modern mathematics, including topics which had formerly been excluded from the so-called traditional mathematics programs. These advancements for the mathematics student of today seem to be centered around an intensive study of the structure of number systems as a foundation for the revised mathematical learnings. The updating of mathematics curricula in response to the development of a concept of modern mathematics, is being evidenced on the elementary, secondary, and college levels.[1]

Accompanying the revisions of textbooks and programs

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[1]Edwina Deans, Elementary School Mathematics-New Directions (U.S. Dept. of Health, Education, and Welfare, Washington, D.C.: Government Printing Office, 1963), p.1.

of instruction, has been the upsurge in the testing movement; that is, no longer are the conventional standardized tests which measure, for a large part, the computational ability of students of mathematics, adequate to evaluate the objectives of modern mathematics. The revision of curricula has acted as a catalyst in the development of new phases in mathematics testing. It is precisely with the measurement aspect of modern mathematics, that this paper is concerned.

Historical Background.--Somewhere in the saga of mankind is a time unrecorded in the history of man--the evolution of a concept of number. Centuries after this, came the practical necessity of recording this concept of number; whether it be in connection with flocks of sheep, winter food storage, or the summation of some other objects in man's possession is not known. Grossnickle and Brueckner suggest that this early recording was carried out according to "handfuls, gourdfuls, ..., heaps, ..., pebbles, ..., fingers, or notches on sticks."[2] However, many, many centuries elapsed before the systematic answering of these problems of everyday living occurred.[3]

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[2]Foster E. Grossnickle and Leo J. Brueckner, Discovering Meanings in Arithmetic (Philadelphia: The John C. Winston Company, 1959), p. 13.

[3]Ibid.

> People are continually making references to how many, how far, how much, who was first, how long ago, how far into the future, etc. When a society becomes highly complex and industrialized, quantitative relationships become an indispensable part in the daily lives of everyone. The entire social, scientific, and technological milieu of the contemporary world is mathematical in character.[4]

The above paragraph summarizes the development of the specific aspect of mathematics with which this study is concerned--that of using "number in some way in dealing with the elements of a situation that lend themselves to mathematical analysis or description."[5] A simplified example of this type of understanding can be discerned when an adult or child attempts to measure an object or a situation; for when this happens, quantitative thinking has occurred.

Statement of the Problem.--This aspect of mathematical understanding or quantitative thinking lacks intensive investigation (as the research of this paper later reports). However, it is generally agreed, as stated by Martin, that children become more adept at handling concepts of quantity with increase in age.[6] Yet there has

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[4]John Jarolimek, "Teaching Quantitative Relationships in Social Studies," The Arithmetic Teacher, IV (March, 1957), pp. 70-74.

[5]Grossnickle and Brueckner, op. cit., p. 308.

[6]William E. Martin, "Quantitative Expression in Young Children," Genetic Psychology Monographs, XLIV (November, 1951), p. 214.

not been published to date any standardized test which purports to measure quantitative judgment.

However, there has been an unpublished investigation of this specific problem. In fact, a test instrument has been designed to measure aspects of quantitative judgment. This test--as a first of its kind--is still in a process of refinement. A preliminary pilot study followed by an administration of the test to over 700 students, and a statistical analysis upon the test scores and the test items has been carried out by Hall.[7] Nonetheless, this test of quantitative judgment is in its rudimentary stages, wherein many steps in refinement must occur preceding its readiness for factor analysis or any other advanced statistical examination.

The initial Test of Quantitative Judgment (Form H) is in a process of revision, expansion, and readministration to public school children. As a preliminary step to the development of a second Test of Quantitative Judgment (Form T), this writer chose to design a new series of test items. It was anticipated that items would be selected by means of item analysis from the Test of

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[7] Donald E. Hall, "The Ability of Intermediate Grade Children to Deal with Aspects of Quantitative Judgment" (unpublished Ed.D. dissertation, School of Education, Boston University, 1965), pp. 7-8.

Quantitative Judgment (Form H) and from the writer's test to form a second Test of Quantitative Judgment (Form T).

Purpose of the Study.--In accordance with the problem at hand--that of writing a new series of items which measure aspects of quantitative judgment--this study was an attempt to carry out the following ten main steps: (1) To discover the type of question which could be included in a test of quantitative judgment, (2) To write a series of test items which measure aspects of quantitative judgment, (3) To select from these test items those which appear to have the highest face validity, (4) To administer these select test items to pupils in the intermediate grades, (5) To determine statistical data from the test scores and to analyze the test items for item difficulty and item discrimination, (6) To compare the results of the writer's test of quantitative judgment with the first Test of Quantitative Judgment (Form H), (7) To select items from the initial Test of Quantitative Judgment (Form H) by means of item analysis, (8) To select items from the writer's test of quantitative judgment by means of item analysis, (9) To couple the selected test items from the first Test of Quantitative Judgment (Form H) with the selected test items of the writer's test of quantitative judgment, (10) To prepare the second Test of Quantitative Judgment (Form T) for administration to a new population for additional analysis and refinement.

<u>Assumptions and Limitations</u>.--It must be taken into consideration that this is a seldom-investigated area of mathematics. For this reason, there is a lack of literature and research which can provide technical knowledge to be applied to the basic problems of this study. For example, the construction of test items must be based upon trial and error, in many instances, rather than upon a prior awareness of the exact nature of the mathematical understanding measured by a test question.

Moreover, the writer intends to adhere to the definition of terms and construction of test items which formed the basis for the design of the first Test of Quantitative Judgment (From H). This will be a total attempt to build on to rather than away from the specific quantitative judgment test which has already been constructed. By working within the construction format of the initial Test of Quantitative Judgment (Form H), the items from this study will be analyzed according to the same procedures as the previous study and thus the established critera of test item selection for the Test of Quantitative Judgment (Form T) will be reasonably comparable.

Since this study consists of the preliminary steps in the construction of a new test, the actual test administration, for example, will encompass but a small sample of any population. Regardless of the limited scope of the population, however, the writer will follow the same procedure as would be encountered if this test were to be administered to a much larger population.

Term Defined.-- In this study the term *quantitative judgment* will be defined as it was in the initial Test of Quantitative Judgment (Form H). The explanations of this usage have been extracted as follows:

> By quantitative thinking, or quantitative judgment we refer to the individual's ability to apply number and mathematical concepts and processes to quantitative situations encountered socially both within and outside of the classroom environment. Quantitative judgment includes thinking about amounts, estimating or guessing intuitively relative to *how much*, *how many*, *how far* and/or *how large*....
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> For the purposes of this study, *quantitative judgment* is a restricted term. Children's ability to deal with quantitative judgment is here limited to an expression of sizes, weights, lengths, amounts, levels, distances, degrees or numbers as they appear in various social situations.[8]

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[8] *Ibid.*

CHAPTER II

REVIEW OF RELATED LITERATURE

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PREVIOUS RESEARCH

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PREVIOUS RESEARCH

Application of Principles of Mathematics to the Future.--One of the most important teachings in this century of rapid growth and discovery is, according to Evenson, the methods and approaches to the solving of problems faced by people in their daily lives.

However, it would be as impossible to prepare each person to deal effectively with every life problem, as it would be to predict the actual problems which would arise. Therefore, it is of utmost necessity that fundamental mathematical and problem-solving principles be taught in order that each person will be able to apply these basic concepts to his future life.[1] Moreover, Welmers maintains that since our civilization is scientifically advanced it becomes necessary for the student not only to comprehend the cultural forces which shape his daily life, but also to understand and gain an

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[1] A.B. Evenson, Modern Mathematics (Fair Lawn, N.J.: Scott, Foresman and Company, 1962), p. 7.

appreciation of the mathematics upon which society depends. This includes a knowledge of the relationship among quantities.[2]

Need for a Change in Curriculum.--The goals for teaching arithmetic in the elementary school have been revised in the past decade. Educational psychology and child development studies have provided insights to both administrators and teachers regarding both the methods of instruction in arithmetic and the materials for curriculum. Shane and McSwain emphasize the need to recognize that arithmetic is "a quantitative language and a science of numbers developed by man over the many centuries to express ideas of quantity, [and] to indicate the relationships existing among quantities...."[3] For this reason, one of the major objectives of the curriculum should be to have children "interpret arithmetic to be a language of quantitative thought and action."[4]

The Role of Quantitative Judgment.--Jarolimek

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[2]Everett T. Welmers, "Arithmetic in Today's Culture," Instruction in Arithmetic, Twenty-Fifth Yearbook of the National Council of Teachers of Mathematics (Washington, D.C.: 1960), p. 31.

[3]Harold G. Shane and E. T. McSwain, Evaluation and the Elementary Curriculum (New York: Henry Holt and Company, Inc., 1958), p. 229.

[4]Ibid.

points out that people are constantly using terms such as "how many, how far, how much, who was first, how long ago, how far into the future" in an attempt to express the social and the scientific aspects of their lives. In a society that is highly developed, quantitative relationships become an important part in the daily lives of the people.[5] Moreover, Reiss reports that in spite of the necessity for modern man to record his private financial status, to distinguish between more and less, and to recognize ratios among quantities, his natural capacity for these tasks is hardly greater than that of the animals. Although it is possible for man to deal with the concept of millions by the use of symbols, "he is not able to discriminate accurately between groups bigger than six, when counting is precluded."[6]

Anderson recognized this need when he experimented with methods of instructing children in arithmetic. He reports that children who are taught to understand meanings and relationships in the formation of generalizations will be better able to solve problems arising from

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[5] John Jarolimek, "Teaching Quantitative Relationships in Social Studies," The Arithmetic Teacher, IV (March, 1957), p. 71.

[6] Anita Reiss, "Numerical Quantification vs. Number Sense," Journal of Psychology, XV (January, 1943), pp. 99-108.

quantitative situations.[7] Buswell emphasizes the necessity for determining the relevance of quantitative reasoning in the elementary curriculum.[8]

Shane maintains that there is, without question, a need for quantitative analysis in the school curriculum. The growth of the nation from industrial to military areas has been dependent upon this quantitative understanding. Thus the elementary schools must combine meaning and skills to be applied to quantitative thinking.[9]

Dutton notes that not only is mathematics vital to a technological age, but also to the everyday living of people. He emphasizes that research findings have pointed to the necessity of differentiating between the mathematical and social phases of arithmetic and obtaining a balance between the two: "first, developing concepts and second, assisting pupils in understanding how

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[7]G. Lester Anderson, "Quantitative Thinking as Developed Under Connectionist and Field Theories of Learning," Learning Theory in School Situations, Studies in Education, No. 2 (The University of Minnesota Press, 1949), p. 69.

[8]Guy T. Buswell and Bert Y. Kersh, Patterns of Thinking in Solving Problems, XII, No. 2. (Los Angeles: University of California Press, 1956), p. 136.

[9]Shane, *op. cit.*, p. 254.

quantitative ideas are needed and used in school and society."[10] Hagaman appears to parallel this concept as he distinguishes between the <u>intrinsic</u> meanings which are found in quantitative relationships and are basic to mathematical understanding and <u>functional</u> meanings which evolve in the experiences of the children. This can also be spoken of as the abstract (intrinsic) as compared with the concrete (functional) in elementary arithmetic. Once again, Hagaman notes that these two functions or phases must be balanced.[11]

In the Twenty-Fourth Yearbook of the National Council of Teachers of Mathematics, Hildebrandt stresses the importance of developing the thinking power of the student. The field of mathematics can be utilized more effectively when the child develops the habits of exercising his mind at an early age and begins to apply the knowledges gained.[12] As a part of this development,

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[10] Wilbur H. Dutton and L.J. Adams, <u>Arithmetic for Teachers</u> (Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1961), pp. 1,3.

[11] A.P. Hagaman, "Word Problems in Elementary Mathematics," <u>The Arithmetic Teacher</u>, XI (January, 1964), pp. 10-11.

[12] E.H.C. Hildebrandt, "Mathematical Modes of Thought," <u>The Growth of Mathematical Ideas: K-12, Twenty-Fourth Yearbook of the National Council of Teachers</u> of Mathematics (Washington, D.C.: 1959), p. 403.

Collier states that educators of the present should accept as the theory of arithmetic teaching, the belief that experiences in quantitative thinking along with meanings and instruction should be provided within the elementary curriculum.[13]

Definition: Scope and Limitation of Quantitative Judgment.--Anderson defines quantitative thinking as "the ability to think mathematically in quantitative situations."[14]

Grossnickle in the Introduction to the Twenty-Fifth Yearbook of the National Council of Teachers of Mathematics points to the factors which comprise an effective elementary arithmetic program. Among these are (1) instruction in a manner that leads to a "unique, quantitative way of thinking," and (2) a means of acquiring skill in reading "quantitative statements."[15]

Brownell lists the scope of arithmetic in a program which provides for learnings in mathematical

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[13]Calhoun C. Collier, "The Development and Evaluation of a Non-Computational Mathematics Test for Grades Five and Six. Dissertation Abstracts, XVIII (1956), p. 1027.

[14]Anderson, op. cit., p. 40.

[15]Foster E. Grossnickle, "Introduction," Instruction in Arithmetic, Twenty-Fifth Yearbook of the National Council of Teachers of Mathematics (Washington, D.C.: 1960), p. 32.

understanding. He includes in this list (1) a vocabulary which enables the child to express himself in terms of quantitative ideas and relationships, (2) an ability to represent quantitative concepts in graphs and other forms of statistical presentation, (3) the facility to recognize, apply, and appreciate the role which quantity and quantitative reasoning have in social situations, and (4) the use of sound reasoning in concrete quantitative experiences.[16]

In addition, Douglass refers to the extension of the elementary curriculum at a rapid pace. He specifically notes that the "world is becoming steadily more quantitative, with new demands upon arithmetic and mathematics as a whole."[17]

Shane discusses the expectations of men in the latter half of the present century. He envisions the development of the technological fields ranging from sources of energy to means of communication. The results

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[16] William A. Brownell, "The Evaluation of Learning in Arithmetic," Arithmetic in General Education, Sixteenth Yearbook of the National Council of Teachers of Mathematics (New York: Bureau of Publications, Teachers College, Columbia University, 1941), pp. 231-232.

[17] Harl R. Douglass and Herbert F. Spitzer, "The Importance of Teaching for Understanding," Measurement of Understanding, Part I, Forty-Eighth Yearbook of the National Society for the Study of Education (Chicago: University of Chicago Press, 1946), p. 12.

of these scientific evolutions will require greater specialty in problem solving and quantitative observation. For this reason it becomes necessary for intensified research and investigation of the psychological and mathematical functions of the elementary school. The children so educated will initiate the needs and formulate the developments of the technological age upon which society is embarking.[18]

The Cambridge Report also places a role upon quantitative reasoning. It notes that students must live with problems which are far removed from the drill found in traditional texts. From a psychological standpoint it must be noted that "mathematics is something which people _do_; it is not something that they receive in a passive sense." Thus an emphasis is placed upon the active, intellectual stimulation which true mathematics incurs.[19] Weaver also is in accord with this view as he points to the necessity of recognizing that there are not only crucial issues in education in general, but also specific problems in the instruction of elementary

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[18] Shane, _op_. _cit_., p. 227

[19] The Report of the Cambridge Conference on School Mathematics, _Goals for School Mathematics_, Educational Services Inc. (Boston: Houghton Mifflin Co., 1963), pp. 27-29.

school arithmetic.[20]

Children and Quantitative Judgment.--Studies have revealed that the ability of children to deal with number concepts and relationships among quantities as measured by formal testing increases with age. Martin's research confirms that not only do children become more and more aware of quantitative relationships as they appear in environmental situations, but that they become more facile in acquiring a vocabulary which gives expression to these occurrences.[21]

Henderson maintains that conventional arithmetic programs stressing problem solving experiences were often inappropriate to the age and learning level of elementary school children. Moreover, much of the failure of children has been caused by the fact that few of the experiences have appeared realistic to children. However, there are several types of problems arising from the social and economic lives of the students which would require quantitative thinking if they were properly

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[20] J. Fred Weaver, "Six Years of Research on Arithmetic Instruction: 1951-1956," The Arithmetic Teacher, IV (April, 1957), pp. 39-90.

[21] William E. Martin, "Quantitative Expression in Young Children," Genetic Psychology Monographs, XLIIII (November, 1951), p. 214.

presented to the children.[22]

Hartung specifically denotes the types of questions which would fall into the above category. First, they would be indirect, rather than direct in form. Thus it would be necessary for the child to formulate the question in his or her own mind. Second, the data are not stated in precise terms. Third, the means of analyzing and interpreting the data are much more complex and require a higher level of reasoning than does computing an answer. Lastly, there may not necessarily be required a specific answer to each problem. The intent would be to involve the child in thinking mathematically, rather than in computing the answer.[23]

Need for Research.--Flournoy reports that children must increase their awareness of quantitative statements as they occur in reading, for inaccurate concepts are often a product of children's inability to carry on this type of reasoning. However, additional research is needed

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[22]Kenneth B. Henderson and Robert E. Pingry, "Problem-Solving in Mathematics," The Learning of Mathematics, Twenty-First Yearbook of the National Council of Teachers of Mathematics (Washington, D.C.: 1953), pp. 233-234.

[23]Maurice L. Hartung, "Advances in Teaching Problem-Solving," Arithmetic, Supplementary Educational Monographs, XLVIII, No. 66 (Chicago: University of Chicago Press, 1948), pp. 44-53.

to determine the means by which children develop greater skill in recognizing and analyzing quantitative statements as they appear in practical reading and listening experiences.[24]

Likewise, Dutton observes that little is known about the means by which children learn problem-solving techniques. This, too, is a vital area in mathematics which is frequently neglected when it comes to controlled research and investigation.[25]

Buswell reports that in past investigations attempts have been made to find specific means for teaching children to solve problems and think; however, relatively little success has resulted from this endeavor. Based upon that which is now known, it would be better to teach children to approach problems in "commonsense" ways, rather than by approved formulae.[26] The results of a study by Faulk concerning the techniques by which a child can learn to estimate answers are as follows:

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[24] Frances Flournoy, "Interpreting Definite Quantitative Statements Occurring in Reading Reference Materials," The Elementary School Journal, LVIII (January, 1958), p. 211.

[25] Dutton, op. cit., p. 182.

[26] Guy T. Buswell, "Solving Problems in Arithmetic," Education, LXXIX (January, 1959), p. 287.

> Teachers of the elementary grades can help children to keep abreast of current developments in mathematics by teaching them how to think quantitatively. Estimation of answers is one means toward that end....Through exercising mentally, rounding off numbers, searching for sensible answers, understanding processes in computation, clarifying estimates...the child becomes successful, and finds security. Consequently children become ready to face the challenge of the newer mathematics.[27]

Weaver in reporting on the Joint Commission on the Education of Teachers of Science and Mathematics indicates that research has been completed on number and quantity, grade placement of content, sequence of content, and methods of teaching arithmetic. Nevertheless, there is still a need for much more research in these areas.[28]

The Role of Evaluation in Arithmetic.--Dutton defines evaluation as "a process used in determining the amount and quality of pupil growth, development, and achievement based upon clearly defined purposes."[29] In The Cambridge Report it is remarked that with the increase in the intellectual difficulty of a material, there is the problem of valid testing of the mental process involved.

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[27] C.J. Faulk, "How Well Do Pupils Estimate Answers?" The Arithmetic Teacher, IX (December, 1962), p. 440.

[28] J. Fred Weaver, "Improving Elementary-School Mathematics Programs in American Schools," The Arithmetic Teacher, IX (January, 1962), p. 42.

[29] Dutton, op. cit., p. 354.

> Thus it is trivial to find out whether the student knows the date of the Pilgrims' landing on Plymouth Rock; but it is far more difficult to find out whether he understands what they did and why during the rest of the seventeenth century. Similarly, in mathematics, it is very easy to find out how fast and how accurately a student can multiply two three-digit numbers, but it is much harder to measure the extent and the depth of his grasp of mathematical ideas.[30]

Brueckner notes that in the last few years there have been devised tests to measure understandings of the number systems and skill in interpreting graphs. Yet Brueckner also points out that these are the measures which are most difficult to obtain; as, for example, "the ability to apply quantitative procedures and methods of thinking effectively in social situations."[31] The importance of this type of testing for mathematical growth is substantiated by Shane who reports that in order to develop ability in quantitative thinking there must be constant evaluation of arithmetical objectives and pupil progress.[32]

<u>Need for a Test of Quantitative Judgment</u>.--Among the limitations of standard achievement tests, Shane

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[30]The Report of the Cambridge Conference on School Mathematics, p. 29.

[31]L.J. Brueckner, "Evaluation in Arithmetic," <u>Education,</u> LXXIX (January, 1959), p. 292.

[32]Shane, <u>op</u>. <u>cit</u>., p. 227.

lists the inadequate evaluation of quantitative thinking.[33] Collier states that in spite of attempts to measure the understandings and reasoning possessed by children, most elementary schools "measure little more than speed and accuracy in computation."[34] Koenker notes that the greater depth of understanding necessary to carry out a process, the more important it becomes to test for meaning. Nevertheless, there have been few endeavors to evaluate the meaningful methods of teaching mathematics.[35]

Brueckner reports that the "higher the grade level, the greater is the range in pupil performance and mastery."[36] Chase adds that if it is assumed that there are developmental levels or stages in the acquisition of a concept, then a test should be possible which will note the growth of this concept.[37] These investigations

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[33] Ibid., p. 247.

[34] Collier, op. cit., p. 1027.

[35] R.H. Koenker, "Measuring the Meanings of Arithmetic," The Arithmetic Teacher, VII (February, 1960), pp. 93, 95.

[36] Brueckner, op. cit., p. 292.

[37] C.I. Chase, "Application of Levels of Concept Formation to Measurement of Vocabulary," Journal of Educational Research, LV (October, 1961), p. 77.

appear to have implications to the measurement of quantitative judgment.

Wrightstone describes a developmental program in concepts as one which ranges from "vague, approximate, and descriptive levels--for example, concepts of far, near, and many to the more specific and exact concepts of 23 feet, 19½ pounds, or two 2's are 4."[38] In spite of the fact that many more tests than ever before are attempting to measure these, Rappaport writes that the "lack of a good test has already had its effect upon teachers and administrators who are questioning the value of a meaningful arithmetic program because they are unable to measure its results."[39]

Factors Needed to Answer a Test of Quantitative Judgment.--Young maintains that whether or not a child is able to read and understand a page in mathematics is dependent upon the child's previous experience with the ideas that are present within that writing. Thus, for example, if a child can make up a problem which exemplifies the concept being taught the teacher has one means

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[38] J. Wayne Wrightstone, "Constructing Tests of Mathematical Concepts for Young Children," The Arithmetic Teacher, III, No. 3 (April, 1956), p. 81.

[39] D. Rappaport, "Tests for Meanings in Arithmetic," The Arithmetic Teacher, VI (April, 1959), p. 140.

of measuring his understanding.[40] According to Anderson, the "tests of quantitative thinking place a premium, not upon ability to recall learning directly, but upon ability to adapt learning to new situations." Moreover, these tests demonstrate that the method by which the child is taught influences his ability to transfer his learnings.[41]

Test Construction.--Epstein and Myers cite the criteria which a mathematics test must fulfill and the details of its construction. First, such a test whether of a diagnostic nature or for an appraisal of growth must have a specific purpose. Thus one of the primary steps in test construction is to determine the content, difficulty and the like of the test in relationship to its purpose. Secondly, the questions must be written, reviewed, and pretested. Finally, the formal test can be established and checked once more.

Each question which is written for the test should measure a concept which cannot be evaluated in an easier way. The answer to the items when multiple-choice should contain, besides the correct response, possible inaccurate

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[40]William E. Young, "Teaching Quantitative Language," Education Digest, XXII (January, 1957), pp. 48-49.

[41]G.L. Anderson, "Quantitative Thinking," Research in the Three R's, ed, C.W. Hunnicutt and W.J. Iverson (New York: Harper, 1958), p. 363.

replies and misunderstandings or misconceptions, rather than errors in arithmetic, tricks, or tedious computations.

One of the most vital parts of test construction is the reviewing of the questions and the test. It is a frequent technique to use comparable groups of pupils and analyze each question for index of difficulty and index of discrimination on a pretest. The index of difficulty is based on the percent who responded correctly to an item. The index of discrimination indicates how well the item differentiates between the pupils who obtain a high score on the test compared with those who do not. To determine this, a statistic is derived based upon the number of high and low students who choose each answer choice on a question. The above data is conveniently analyzed by use of the IBM operations.[42]

In compiling the final test it is not desirable to have a high mean index of discrimination as this would indicate a test which is homogeneous in its content and which is measuring a limited number of skills. On the other hand, it must not be too low because of the complexity of interpretation. Nevertheless, it is at times necessary to use low indices of discrimination on tests in their developmental stages which are appraising very

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[42] M. Epstein and S. Myers, "How a Mathematics Test is Born," The Mathematics Teacher, LI (April, 1958) pp. 299-301.

"worthwhile" concepts. When these indices of difficulty and discrimination are controlled "it is possible to achieve desired reliabilities within particular score regions or to produce different score distributions for given groups."[43]

If the test is carefully constructed it will be possible to obtain validity which designates that the test is measuring what it is intending to measure. Likewise, reliability can be determined by the degree to which the test is consistent; that is, students can be expected to achieve the same relative scores if they were to repeat a parallel form of the test. Reliability is a prerequisite to validity, but the converse is not true.[44]

Thiele indicates that a problem may occur when the student attempts to symbolize quantitative relationships which are in a language other than the one used in daily conversation and/or reading. In this instance, there exists a problem in translation. For this reason it becomes necessary for the teacher to judge the actual depth of understanding which the pupil seems to possess, for he will not be speaking in the language

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[43] Ibid., p. 301.

[44] Ibid.

of the textbook.[45] Eads maintains that it is a prerequisite to appraise the students facility to comprehend the significance of or relationships among mathematical principles. At the same time it is also valuable to obtain a measure of the pupil's ability to place an application of these principles upon new situations.[46]

Wrightstone comments that it is not necessary to use a guessing factor in the determination of test scores in instances when each pupil answers each question, for it can be demonstrated that the pupils will fall into the same relative positions with or without the use of guessing factors.

In addition, Wrightstone points to the effectiveness of the multiple-choice item which is

> "relatively free from 'absolutes' in that the 'best' statement of several that are given is to be selected as the 'correct' answer. The 'correct' answer, therefore, is relative to several other given statements rather than to all possible 'not given' statements, as in true-false questions."[47]

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[45] C.L. Thiele, "Arithmetic in the Middle Grades," The Teaching of Arithmetic, Fiftieth Yearbook of the National Society for the Study of Education, Part II (Chicago: University of Chicago Press, 1951), p. 79.

[46] L.K. Eads, "Evaluation of Learning in Arithmetic," National Association of Secondary School Principals Bulletin, XLIII (May, 1959), p. 130.

[47] J. Wayne Wrightstone et al., Evaluation in Modern Education, (New York: American Book Company, 1956), p. 85.

Multiple-choice questions call for the recognition of the answer; yet they reduce the opportunity for guessing as compared to the true-false item. In addition, the scoring for these is also objective.

Moreover, Wrightstone outlines the form of a multiple-choice test item. The question should contain the main concept with the distractors (or choices of answers) of approximately equal length and in a concise form which is grammatically accordant. The foils (or distractors) should be realistic; otherwise, they will merely attract the poorer student and will not provide any value to measurement. In addition, the distractors must be plausible and such that they represent the type of faulty reasoning in which the child might actually engage.[48]

Research-Relationship of Quantitative Judgment to Other Factors.--Treacy did extensive research on the relationship between reading skills and the ability to solve problems in arithmetic. This was an attempt to answer two specific questions: (1) Does the reading level of the child assist or hinder his arithmetical problem-solving experiences? (2) Are there definite reading skills which are singularly related to problem-

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[48] Ibid., pp. 81, 85-87.

solving ability?

Treacy maintains that problem solving is life itself and has a common core of attributes which are similar regardless of the area of learning in which it persists. Thus, by definition, problem solving in arithmetic, for example, can be transferred as a positive or a negative factor to the solution of life perplexities. For this reason, Treacy emphasizes the necessity of teaching with the intent of encouraging and providing for successful problem solving experiences, instead of permitting failures to occur because of pitfalls in areas such as reading. Among arithmetical factors which are pertinent to the successful solution of problems such as level of general intelligence and skill in computation, Treacy includes the use of "quantitative terms."[49]

The results of Treacy's study indicate that good achievers in problem solving are significantly superior to poor achievers in reading skills and mental ability. Likewise, good achievers obtain scores which are significantly higher at the one percent level in quantitative relationships.[50]

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[49] John P. Treacy, "The Relationship of Reading Skills to the Ability to Solve Arithmetic Problems," Journal of Educational Research, XXXVIII (September 1944-May 1945), pp. 86-88.

[50] *Ibid.*, p. 92

Muscio reports that there is a significant difference between boys and girls in quantitative understanding as measured by his study. The boys show a definite superiority which is not attributable to either general intelligence nor computational skill.

He does find, however, that there is a correlation between achievement on measures of quantitative understanding and achievement in arithmetic computation, arithmetic reasoning, and mathematical vocabulary. It is indicated that this result can be accounted for by the overlap of skills involved in quantitative understanding.

Although there is no close relationship between achievement on measures of quantitative understanding and specific reading skills, there is a correlation between quantitative understanding and general reading achievement.

There is also a positive relationship between quantitative understanding and intellectual capacity. This is about the same for general intellectual ability as it is for specific mental factors. Mental language factors are more directly associated with quantitative understanding then nonlanguage factors. It seems evident that certain verbal skills should be recommended for inclusion in an elementary arithmetic program. The importance of this language preparation is a necessary consideration in the planning of appropriate learning experiences for children.

The child's attitude toward arithmetic is not an adequate predictor of his ability in quantitative understanding. The content of an elementary program as recommended by Muscio must be founded upon its significance to the learner, rather than to its general appeal.

It is also substantiated by this study that high achievers are younger in chronological age than low achievers. This finding has been generally reported in regard to total school achievement. It might be logically explained by the assumption that the poorer student could be a repeater, although this would obviously not account for every case.

There are many instances of variability in this study in spite of the correlations which are evident. For this reason, it is concluded on the basis of individual achievement records, that there is a lack of any "general arithmetic ability." This is manifested when children are grouped according to any one factor such as quantitative understanding, and an attempt is made to isolate general and/or specific common patterns. Similar results are found for extremes as well as individual cases.[51]

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[51] R.D. Muscio, "Factors Related to Quantitative Understanding in the Sixth Grade," The Arithmetic Teacher, IX (May, 1962), pp. 261-262.

Wozencraft in the area of sex differences finds somewhat different results than does Muscio. She notes that sex variations are in favor of the girls when the total group is considered. There are no statistically significant differences when the children are grouped according to high or low intelligence quotients. Specifically, there is similarity in ability between bright boys and bright girls as well as slow boys and slow girls.

> The difference between the mean scores of the bright boys and of the slow boys is over one and one-half years in the third grade in arithmetic reasoning, and over two and one-half years in the sixth grade. For the girls, the difference between the high and low levels is about 1.1 years in the third grade and 2.7 years in the sixth grade for the trait of Arithmetic Reasoning. Differences for Arithmetic Computation and Arithmetic Average between high and low levels for both sexes in the sixth grade are equally significant.[52]

Welker made a study of the interrelationships between the ability to do arithmetic verbal problems, the ability to determine procedures for these problems, and the ability to carry through the computation. He states that the ability to solve verbal numerical problems is greater than the ability to solve non-numerical problems which consist of nearly identical problem-solving conditions. In turn, the ability to carry out the calculations for these problems when accepted procedures are indicated

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[52]Marian Wozencraft, "Are Boys Better Than Girls in Arithmetic?" The Arithmetic Teacher, X (December, 1963), pp. 489-490.

is greater than the ability to solve the numerical problems. There is a higher correlation between success in solving numerical problems and success in solving non-numerical problems than there is between numerical problems and computation ability.

Welker also reports that the computation ability that is taught in a traditional mathematics program in the elementary school does not provide transfer of training to problem solving.

The factor of academic aptitude is positively related to non-numerical problem solving, numerical problem solving, and computation. As academic aptitude increases, the scores which likewise increase the most are in the area of numerical problems.[53]

Collier conducted an investigation in which he constructed a non-computational mathematics test for the fifth and sixth grades as an evaluation of arithmetical understanding and reasoning ability. This test consisted of four-option-multiple-choice type items.

From this Collier concludes that the difference between arithmetical understanding and reasoning ability of boys and girls in the fifth and sixth grades is not

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[53] Latney C. Welker, Jr., "A Study of Interrelationships in Arithmetical Problem Solving," Dissertation Abstracts, XXIII (April, 1963), pp. 3750-3751.

significant. However, children at both these levels do evidence lack of strength in making judgments regarding quantitative data.

It is also reported that there exists a specific relationship between arithmetical understandings as measured by this instrument and both mental and reading abilities.

There is a statistically significant difference between the fifth and sixth grade scores in this study. In addition, those pupils from a higher socio-economic environment score higher on arithmetical understanding and reasoning ability when compared with those pupils from a lower socio-economic background.[54]

Butler's study of the relationships between children's understanding of computational skills and their ability to solve problems reveals that correlations between computing and understanding are related to problem-solving, but not intelligence.

Contributing Factor	% Contribution
computing ability	39.69
understanding	33.14
intelligence	00.70
all others	26.47

For this reason, Butler highly recommends the promotion of understanding with its strong influence upon verbal

[54]Collier, op. cit., p. 1027.

problem-solving in the elementary schools.[55]

Corle's study of the quantitative values of fifth and sixth grade pupils provides evidence that the estimation of quantities is developmental; that is, sixth graders are superior to fifth graders in this ability. In addition, this study demonstrates that skill in estimation can be improved within the school. It is also reported that boys are better at estimating quantities than are girls. It is suggested that boys have had more experience with measurement and measurement devices.[56]

It often appears that there are conflicting views from research concerning the relationship between various forms of problem-solving ability and reading ability. For example, Fay finds that there is no statistically significant difference between superior and inferior readers in arithmetic when chronological and mental ages are held constant.[57] Corle reports that lack of an

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[55]Charles C. Butler, "A Study of the Relation Between Children's Understanding of Computational Skills and Their Ability to Solve Verbal Problems in Arithmetic," Dissertation Abstracts, XVI (1956), p. 2400.

[56]Clyde G. Corle, "A Study of the Quantitative Values of Fifth and Sixth Grade Pupils," The Arithmetic Teacher, VII (November, 1960), pp. 333-340.

[57]Leo C. Fay, "The Relationship between Specific Reading Skills and Selected Areas of Sixth Grade Achievement," Journal of Educational Research, XLIII (March, 1950), p. 544.

adequate vocabulary and comprehension of it can attribute to failure in problem solving.[58] Spache states that some reading skills have a relationship to achievement in problem-solving.[59] Balow concludes that general reading ability does have an effect on the ability to solve problems. "The findings of this study may differ from previous studies because the total range of reading ability was used rather than two groups defined as good and poor readers. Also, the effect of intelligence was controlled."[60] Streby maintains that a different type of reading is required in the solution of problems in arithmetic as compared to other subject-areas.[61] The conclusion drawn by Russell in his review of several studies ties together these contrasting reports.

> It seems safe to conclude, then, that problem-solving ability is not related to <u>any</u> specific reading skills. Rather, it is closely related to certain reading and thinking abilities such as general and mathematical

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[58] C.G. Corle, "Thought Processes in Grade Six Problems, <u>The Arithmetic Teacher</u>, V (October, 1958), p. 202.

[59] George Spache, "A Test of Abilities in Arithmetic Reasoning," <u>The Elementary School Journal</u>, XLVII (March, 1947), p. 445.

[60] Irving H. Balow, "Reading and Computation Ability as Determinants of Problem Solving," <u>The Arithmetic Teacher</u>, II (January, 1964), p. 22.

[61] G.W. Streby, "Reading in Mathematics," <u>The Arithmetic Teacher</u>, IV (March, 1957), pp. 80-81.

> vocabularies, ability to grasp quantitative relationships, and ability in drawing inferences and otherwise integrating scattered ideas. Arithmetic is not related to reading to predict and to nonspecialized ability in using reference materials. Furthermore its relationship to general level of reading comprehension and to ability to read for details seems contradictory in different studies. In general, reading abilities closely related to quantitative thinking are the ones that count. The specific abilities needed appear more clearly in the analysis of problem solving....[62]

Vander Linde reports that students who have been exposed to direct study of quantitative vocabulary achieve higher on tests of problem-solving ability than do pupils who have not received this teaching.[63]

Piaget and Concept Formation.--Elkind and Coxford both confirm Piaget's findings in the area of concept formation. "Piaget assumes that success in comparing quantity earmarks developmental changes in both the form and the content of children's quantitative thinking."[64]

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[62]David H. Russell, "Arithmetic Power Through Reading," Instruction in Arithmetic, Twenty-Fifth Yearbook of the National Council of Teachers of Mathematics (Washington, 1960), p. 214.

[63]Louis F. Vander Linde, "An Experimental Study of the Effect of the Direct Study of Quantitative Vocabulary on Arithmetic Problem Solving Ability of Fifth Grade Pupils," Dissertation Abstracts, XXIII (April, 1963), p. 3701.

[64]D. Elkind, "The Development of Quantitative Thinking; a Systematic Replication of Piaget's Studies," Journal of Genetic Psychology, XCVIII (1961), pp. 44-45 and Arthur F. Coxford, Jr., "Piaget: Number and Measurement," The Arithmetic Teacher, X (November, 1963), pp. 419-20.

The first stage in the recognition of quantity by a child consists of a generalized impression (<u>global</u> conception). In the second stage children become more exacting in their breakdown of quantity. This is referred to by Piaget as the phase of <u>intuitive</u> conception. The final stage is marked by the child envisioning the concept as a logical whole and is termed <u>abstract</u> conception.[65]

[65] Elkind, <u>op</u>. <u>cit</u>., p. 45.

CHAPTER III

PROCEDURE

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PROCEDURE

The first step to be taken consisted of (1) becoming generally familiar with the scope of quantitative judgment, and (2) acquiring a specific understanding of quantitative judgment as defined in the construction of the Test of Quantitative Judgment (Form H). The test items to be developed for this pilot study were modeled after the established test both in form and content.

From the first Test of Quantitative Judgment (Form H), thirty test items were selected on the basis of data which had been previously compiled by Hall using Chung Teh-Fan's Item-Analysis Table.[1] These test items were not administered in this study, but were set aside to constitute one-half the second Test of Quantitative Judgment (Form T). It was the explicit purpose of this study to couple the test items devised by Hall with questions designed by the writer and subjected to analysis for indices of item difficulty and item discrimination.

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[1] Donald E. Hall, "The Ability of Intermediate Grade Children to Deal with Aspects of Quantitative Judgment" (unpublished Ed.D. dissertation, School of Education, Boston University, 1965), pp. 63-64.

Over one hundred items were initially constructed by the writer and exposed to extensive review and rewriting. Each question was multiple-choice with four alternative answers. As an idea for a test item evolved, the tentative question was written on a 3x5 card; these could then be shuffled by topics, position of correct answers, and feasibility as actual test items.

The questions were then scrutinized for face validity, as defined by Helmstadter. Face validity as a form of content validity is a step in the initial writing of a test item. It is concerned with determining whether or not the test appears to measure the desired objectives. Although this beginning evaluation is subjective, it does provide a means of selecting and discarding original items.[2]

In this manner, sixty items which constituted the writer's test were developed. Thirty were originally constructed by the writer and were selected on the basis of content validity from the one hundred crude test items previously described. The other thirty items were miscellaneous in source; some were revised from notes on questions not used in the first Test of Quantitative

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[2] G.C. Helmstadter, Principles of Psychological Measurement (New York: Appleton-Century-Crofts, 1964), p. 89.

Judgment (Form H), some came from suggestions of questions written by other students, and six were rewritten from the first Test of Quantitative Judgment (Form H). In this latter group, which consisted of items 1, 18, 19, 22, 27 and 28 from the first test (Form H), the roots and stems of the test items were reversed.

An attempt was made to control variables which might distort test score results. For example, the positions of the correct alternatives were distributed as follows.

TABLE 1

DISTRIBUTION OF TEST ANSWER POSITIONS

Position of Answer (1)	Q.J. (Form H) (2)	Writer's Items (3)	Miscellaneous (Defined above) (4)	Totals (5)
A	7	7	8	22
B	8	8	7	23
C	7	8	7	22
D	8	7	8	23
Totals	30	30	30	90

This insured that the positions of the correct alternatives would not influence the selection of the answers. Likewise, the order of occurrence of A's, B's, C's, and D's as correct choices was not in any discernible pattern which could affect test score results.

At the same time the topics or subjects covered by each test item were classified and the following categories obtained.

TABLE 2

SUBJECT CATEGORIES OF TEST ITEMS

Subjects	Q.J. (Form H)	Writer's Items	Miscellaneous (Defined above)	Total
Volume	8	4	10	22
Distance	5	3	4	12
Length or Width	2	4	5	11
Weight	4	4	3	11
Height	2	4	1	7
Area	1	3	3	7
Time	4		2	6
Money	3	2		5
Thickness	2	1		3
Radius		2		2
Circumference		1	1	2
Pythagorean Theorem		1		1
Temperature			1	1
Total				90

The assignment of test items to a subject area was subjective. Although an endeavor was made by the above listing to provide an estimate of the range of types of measures included, there was no attempt to balance the number of questions found in each subject area, nor to assign more importance to one topic over the others.

The sixty questions were then arranged indeterminately as to occurrence of position of answers and topics; this also was a subjective process. Both the format of the test and the style of the answer sheets were based upon the established form of the first Test of Quantitative Judgment (Form H). (See Appendix.)

The pilot test was then administered to six classes of pupils in grades 4, 5 and 6. The following table shows the distribution of the population by grade and sex.

TABLE 3

DISTRIBUTION OF POPULATION BY GRADE AND SEX

Grade	Boys	Girls	Total
4	32	18	50
5	24	23	47
6	30	24	54
Total	86	65	151

CHAPTER IV

ANALYSIS OF DATA

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ANALYSIS OF DATA

Test Scores.--The tests for the 151 pupils in this study were hand-scored. The raw score for each test was the number of correct items per test. The distribution of the raw test scores by grade and sex is illustrated in Table 4.

TABLE 4

DISTRIBUTION OF RAW TEST SCORES BY GRADE AND SEX

Grade 4*

																				xh					
											x		x	x	x					xh					
						ol		xl	xl		x		x	x	x					xh	xh				
					xl	ol		xl	xl		x	o	o	x	x			x	xh	xh	xh			xh	
xl			xl		xl	ol	ol	ol	xl		x	o	o	o	o	o	x	o	oh	oh	xh	oh		oh	oh
...15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40...

Grade 5*

														x											
														x		x	xh								
														x		xh	xh								
														x	x	xh	xh								
							xl		xl		o			o	x	o	xh								
					xl		ol	xl	o		o		x	o	o	o	oh								
ol				xl	ol	xl	ol	ol	ol	xl	o	x	x	o	o	oh	oh	oh	oh	oh					xh
...19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44...

Grade 6*

```
                        x
                        x  x  x  x
         xl             o  x  x  x        x  xh
      xl xl       xl xl o  x  x  x  x     xh xh
   xl xl ol       ol o  o  o  o  o  oh x  xh xh                   xh
   ol ol ol    xl ol o  o  o  o  o  oh oh oh oh xh       xh       oh
...25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46          ...
```

* The raw test scores of boys are designated by x.
The raw test scores of girls are designated by o.
A raw test score in the top 27% is designated by h.
A raw test score in the bottom 27% is designated by l.

<u>Means and Standard Deviations</u>.--From the preceding distributions the mean scores were computed by grade and sex.

TABLE 5

MEAN TEST SCORES BY GRADE AND SEX

Grade	Boys	Girls	Total
4	28.7	29.2	28.8
5	32.1	31.6	31.8
6	34.1	33.0	33.6

To determine whether or not the mean test scores were significant by grade and sex, tests of significance were computed. These are reported in Table 6 and Table 7.

TABLE 6

TESTS OF SIGNIFICANCE BY GRADE

Grade (1)	4-5 (2)	5-6 (3)	4-6 (4)
Z TEST	2.7**	2.0*	4.4**

**Statistically significant at the .01 confidence level.
*Statistically significant at the .05 confidence level.

TABLE 7

TESTS OF SIGNIFICANCE BY SEX

Grade (1)	4 (2)	5 (3)	6 (4)
Boys-Girls (t)	.102	.060	.153

The mean test score of grade 4 when compared to the mean test score of grade 5 and the mean test score of grade 4 when compared to the mean test score of grade 6 were statistically significant at the 1 per cent confidence level. The mean test score of grade 4 when compared to the mean test score of grade 5 was statistically significant at the 5 per cent confidence level. These findings are consistent with Hall's study. The data shows a difficulty increase by grade.

There was no statistically significant difference between sexes in any one grade. Hall's study reported this same result.[1]

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[1]Donald E. Hall, "The Ability of Intermediate Grade Children to Deal with Aspects of Quantitative Judgment" (unpublished Ed.D. dissertation, School of Education, Boston University, 1965), p. 85.

Standard deviations were also computed by grade and sex.

TABLE 8

STANDARD DEVIATIONS BY GRADE AND SEX

Grade (1)	Boys (2)	Girls (3)	Total (4)
4	6.0	6.2	6.0
5	4.8	5.0	4.8
6	5.1	4.6	4.9

[2]

[2]Statistical computations in this chapter are based on procedures found in John E. Fruend, Modern Elementary Statistics, (Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1960).

Item Analysis.--Item analysis was carried out by use of Chung Teh Fan's Item Analysis Table. By this method, each test item is evaluated for item difficulty and item discrimination. These two statistics are determined by a comparison of the proportion of successes of the top 27 per cent high scorers with the bottom 27 per cent low scorers. For purposes of this study, the total population was broken down by grade and sex for separate item analysis of each test item.

In Fan's Table, the *p* represents the proportion of successes or correct answer choices. The *r* is the discrimination index which is "the correlation between the criterion score, which forms the basis for the selection of high and low 27-per-cent groups, and the continuous score assumed to underlie responses to the items."[3] The statistic delta (Δ) is expressed by the following:

$$\Delta = 13 + 4x$$

In this formula x is considered positive when p is less than .50 and negative when p is greater than .50. The delta increases in direct proportion to item difficulty.[4]

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[3]Chung-Teh Fan, Item-Analysis Table (Princeton: Educational Testing Service, 1952), p. 3.

[4]*Ibid.*, pp. 3-4.

The proportion of successes by grade and sex were determined as follows. Cards (3x5) were set up according to the form below for each item.

Item #		A	B	C	D
4 Upper 27%	B G				
Lower 27%	B G				
5 Upper 27%	B G				
Lower 27%	B G				
6 Upper 27%	B G				
Lower 27%	B G				

where 4, 5, 6 indicate the grade level
where B represents boys
where G represents girls

In each square the number of pupils selecting each answer on the item was recorded. From this data, the proportion of successes and failures could be determined. Then by turning to the Fan Table and entering the high-low statistics, a p, r, and Δ, was recorded for each item by grade and sex. Table 9 reports these.

TABLE 9*

ITEM DISCRIMINATION AND ITEM DIFFICULTY BY

GRADE AND SEX

Grade 4 - Boys

Item	p	r	Δ	Item	p	r	Δ
1.	.61	.13	11.9	31.	.38	.36x	14.2
2.	.87	.63	8.5	32.	.44	.24	13.6
3.	.73	-.13	10.6	33.	.74	.41	10.4
4.	.38	.12	14.2	34.	.56	.24	12.2
5.	.26	.41	15.6	35.	.50	.34x	13.0
6.	.73	.13	10.6	36.	.44	.36x	13.6
7.	.56	.24	12.4	37.	.56	.46	12.4
8.	.32	.51	14.9	38.	.50	.12	13.0
9.	.56	.24	12.4	39.	.62	.36x	11.8
10.	.67	.25	11.2	40.	.38	.36x	14.2
11.	.44	.00	13.6	41.	.74	.79	10.5
12.	.73	.13	10.6	42.	.79	.31x	9.8
13.	.89	.00	8.1	43.	.43	.67	13.7
14.	.26	.79	15.5	44.	.78	.75	9.9
15.	.56	.24	12.4	45.	.74	.79	10.5
16.	.38	-.12	14.2	46.	.91	.55	7.6
17.	.27	.13	15.4	47.	.26	.41	15.6
18.	.27	-.13	15.4	48.	.21	.31x	16.2
19.	.16	.18	16.9	49.	.50	.12	13.0
20.	.09	.55	18.4**	50.	.67	.25	11.2
21.	.78	.75	9.9	51.	.63	.59	11.7
22.	.50	.12	13.0	52.	.37	.59	14.3
23.	.21	-.31	16.2	53.	.83	.69	9.2
24.	.22	.00	16.1	54.	.50	.55	13.0
25.	.37	.59	14.3	55.	.27	.13	15.4
26.	.68	.51	11.1	56.	.38	.36	14.2
27.	.16	.18	16.9	57.	.56	.24	12.4
28.***				58.	.16	.18	16.9
29.	.33	.25	14.8	59.	.21	.31x	16.2
30.	.33	.00	14.8	60.	.56	.24	12.4

Grade 4 - Girls

Item	p	r	△	Item	p	r	△
1.	.61	.42	11.9	31.	.33	.84	14.8
2.	.61	.42	11.9	32.	.60	.00	12.0
3.	.85	.67	8.9	33.	.77	.76	10.1
4.	.20	.00	16.4	34.	.33	.84	14.8
5.	.67	.84	11.2	35.	.50	.59	13.0
6.	.61	.42	11.9	36.	.30	.24	15.1
7.	.61	.42	11.9	37.	.40	.00	14.0
8.	.30	.24	15.1	38.	.30	.24	15.1
9.	.33	.84	14.8	39.	.85	.67	8.9
10.	.50	.20	13.0	40.	.61	.42	11.9
11.	.60	.00	12.0	41.	.85	.67	8.9
12.	.77	.76	10.1	42.	.70	.24	10.9
13.	.61	.42	11.9	43.***			
14.	.50	.59	13.0	44.	.50	.20	13.0
15.	.92	.53	7.4	45.	.92	-.53	7.4
16.	.20	.00	16.4	46.	.92	-.53	7.4
17.	.20	.00	16.4	47.	.40	.00	14.0
18.	.30	-.24	15.1	48.	.15	.67	17.1
19.	.08	.53	18.6**	49.	.20	.00	16.4
20.***				50.	.50	.20	13.0
21.	.92	.53	7.4	51.	.61	.42	11.9
22.	.61	.42	11.9	52.	.20	.00	16.4
23.	.08	.53	18.6**	53.	.67	.84	11.2
24.	.61	.42	11.9	54.	.60	.00	12.0
25.	.50	.93	13.0	55.	.70	.24	10.9
26.	.92	.53	7.4	56.	.61	.42	11.9
27.	.08	.53	18.6**	57.	.15	.67	17.1
28.	.30	-.24	15.1	58.	.33	.84	14.8
29.	.50	.59	13.0	59.	.20	.00	16.4
30.	.50	-.20	13.0	60.	.40	.00	14.0

Grade 4 - Total

Item	p	r	Δ	Item	p	r	Δ
1.	.61	.22	11.9	31.	.37	.52	14.3
2.	.77	.48	10.0	32.	.50	.14	13.0
3.	.75	.10	10.3	33.	.74	.53	10.4
4.	.32	.08	14.8	34.	.50	.42	13.0
5.	.37	.52	14.3	35.	.50	.42	13.0
6.	.68	.08	11.2	36.	.38	.38x	14.2
7.	.57	.29	12.3	37.	.50	.28	13.0
8.	.31	.24	15.0	38.	.43	.15	13.7
9.	.50	.42	13.0	39.	.70	.41	11.0
10.	.61	.22	11.9	40.	.46	.35x	13.4
11.	.50	.00	13.0	41.	.78	.75	9.9
12.	.72	.35x	10.6	42.	.76	.29	10.2
13.	.79	.21	9.8	43.	.26	.53	15.6
14.	.36	.67	14.5	44.	.66	.47	11.4
15.	.68	.25	11.1	45.	.77	.48	10.0
16.	.32	-.08	14.8	46.	.89	.19	8.1
17.	.25	.10	15.7	47.	.32	.25	14.9
18.	.28	-.18	15.3	48.	.20	.42	16.4
19.	.13	.27	17.4	49.	.39	.07	14.1
20.	.06	.46	19.2**	50.	.61	.22	11.9
21.	.83	.69	9.1	51.	.63	.52	11.7
22.	.54	.21	12.6	52.	.30	.41	15.0
23.	.17	-.11	16.8	53.	.78	.75	9.9
24.	.36	.15	14.5	54.	.54	.35x	12.6
25.	.40	.72	14.0	55.	.43	.15	13.7
26.	.77	.48	10.0	56.	.46	-.07	13.4
27.	.13	.27	17.4	57.	.43	.29	13.7
28.	.10	-.16	18.1**	58.	.23	.48	16.0
29.	.38	.38x	14.2	59.	.21	.21	16.2
30.	.39	-.07	14.1	60.	.50	.14	13.0

Grade 5 - Boys

Item	p	r	△	Item	p	r	△
1.	.50	-.42	13.0	31.	.28	.35x	15.4
2.	.86	.00	8.7	32.	.79	.21	9.8
3.	.59	.58	12.1	33.	.50	.70	13.0
4.	.28	.81	15.3	34.	.66	.47	11.4
5.	.57	.00	12.3	35.	.66	.47	11.4
6.	.57	.29	12.3	36.	.66	.47	11.4
7.	.72	.35x	10.6	37.	.57	-.29	12.3
8.	.57	.00	12.3	38.	.43	.00	13.7
9.	.17	.69	16.9	39.	.57	.00	12.3
10.	.50	.42	13.0	40.	.36	.15	14.5
11.	.59	.58	12.1	41.	.89	.61	8.2
12.	.72	-.35	10.6	42.****			
13.	.94	.46	6.8	43.	.50	-.14	13.0
14.	.72	-.35	10.6	44.	.50	.42	13.0
15.	.94	.46	6.8	45.	.86	.00	8.7
16.	.36	.15	14.5	46.****			
17.	.29	.00	15.2	47.	.36	.15	14.5
18.***				48.	.28	.35x	15.4
19.	.41	.58	13.9	49.	.21	.21	16.2
20.***				50.	.66	.47	11.4
21.	.64	.15	11.5	51.	.64	.15	11.5
22.	.59	.58	12.1	52.	.36	.15	14.5
23.	.11	.61	17.8	53.	.94	.46	6.8
24.	.29	.00	15.2	54.	.34	.47	14.6
25.	.57	-.29	12.3	55.	.89	.61	8.2
26.	.83	-.69	9.1	56.	.36	.15	14.5
27.	.29	.00	15.2	57.	.57	.00	12.3
28.	.28	.35x	15.4	58.	.06	-.46	19.2**
29.	.79	.21	9.8	59.	.28	.35x	15.4
30.	.28	.35x	15.4	60.	.43	.29	13.7

Grade 5 - Girls

Item	p	r	△	Item	p	r	△
1.	.59	-.51	12.1	31.	.33	.37x	14.8
2.	.93	.50	7.1	32.	.67	-.37	11.2
3.	.41	.51	13.9	33.	.67	.37x	11.2
4.	.33	.00	14.8	34.	.41	.18	13.9
5.	.17	.00	16.8	35.	.66	.85	11.4
6.	.66	.85	11.4	36.	.41	.18	13.9
7.	.50	.34x	13.0	37.	.41	-.18	13.9
8.	.59	.51	12.1	38.	.41	-.18	13.9
9.	.41	.18	13.9	39.	.50	.34x	13.0
10.	.74	.79	10.5	40.	.59	.18	12.1
11.	.50	.00	13.0	41.	.93	-.50	7.1
12.	.50	.00	13.0	42.	.87	.63	8.5
13.	.81	.72	9.5	43.	.50	.34x	13.0
14.	.67	.00	11.2	44.	.93	.50	7.1
15.****				45.****			
16.	.50	.34x	13.0	46.****			
17.	.13	.63	17.5	47.	.33	.37x	14.8
18.	.13	-.63	17.5	48.	.13	.63	17.5
19.	.41	-.18	13.9	49.	.50	.34x	13.0
20.	.13	.63	17.5	50.	.59	.18	12.1
21.	.75	.21	10.3	51.	.67	.37x	11.2
22.	.33	.00	14.8	52.***			
23.	.07	.50	18.9**	53.****			
24.	.34	.85	14.6	54.	.50	.34x	13.0
25.	.59	.51	12.1	55.	.59	.51	12.1
26.****				56.	.50	.65	13.0
27.	.33	.37x	14.8	57.	.41	.18	13.9
28.	.07	.50	18.9**	58.	.07	.50	18.9**
29.	.50	.00	13.0	59.	.50	.00	13.0
30.	.26	-.79	15.5	60.	.50	.65	13.0

Grade 5 - Total

Item	p	r	△	Item	p	r	△
1.	.54	-.46	12.6	31.	.30	.36x	15.2
2.	.89	.15	8.2	32.	.73	-.10	10.5
3.	.50	.54	13.0	33.	.59	.55	12.1
4.	.33	.43	14.7	34.	.54	.32x	12.6
5.	.38	.00	14.2	35.	.64	.64	11.5
6.	.59	.55	12.1	36.	.54	.32x	12.6
7.	.59	.26	12.1	37.	.50	-.24	13.0
8.	.58	.24	12.2	38.	.42	-.08	13.8
9.	.30	.36x	15.2	39.	.54	.16	12.6
10.	.59	.55	12.1	40.	.46	.16	13.4
11.	.49	.23	13.1	41.	.89	.15	8.2
12.	.62	-.16	11.8	42.	.94	.47	6.9
13.	.88	.62	8.3	43.	.50	.08	13.0
14.	.62	.33x	11.8	44.	.70	.36x	10.8
15.	.95	.26	6.3	45.	.92	.00	7.4
16.	.50	.08	13.0	46.****			
17.	.23	.22	16.0	47.	.34	.25	14.6
18.	.06	-.47	19.1**	48.	.21	.42	16.2
19.	.42	.24	13.8	49.	.34	.25	14.6
20.	.06	.47	19.1	50.	.62	.33x	11.8
21.	.70	.18	10.9	51.	.66	.25	11.4
22.	.46	.32x	13.4	52.	.19	.12	16.5
23.	.09	.56	18.3**	53.****			
24.	.33	.43	14.7	54.	.42	.40	13.8
25.	.58	-.08	12.2	55.	.72	.54	10.7
26.	.91	-.56	7.7	56.	.42	.40	13.8
27.	.30	.18	15.1	57.	.50	.08	13.0
28.	.18	.35x	16.6	58.	.06	-.47	19.1**
29.	.66	.08	11.4	59.	.38	.16	14.2
30.	.27	-.10	15.5	60.	.46	.46	13.4

Grade 6 - Boys

Item	p	r	△	Item	p	r	△
1.	.75	.00	10.3	31.	.49	.73	13.1
2.	.94	.45	6.7	32.	.70	-.43	10.9
3.	.85	.66	8.8	33.	.50	.50	13.0
4.	.37	.27	14.3	34.	.56	.39x	12.4
5.	.37	.00	14.3	35.	.58	.62	12.2
6.	.56	.39x	12.4	36.	.56	-.12	12.4
7.	.75	.00	10.3	37.	.35	.54	14.5
8.	.63	.27	11.7	38.	.43	.13	13.7
9.	.37	.27	14.3	39.	.62	.00	11.8
10.	.85	.66	8.8	40.	.49	.25	13.1
11.	.50	.50	13.0	41.	.70	.82	10.9
12.	.81	.72	9.5	42.	.75	.00	10.3
13.	.85	.66	8.8	43.	.63	-.27	11.7
14.	.75	.32x	10.3	44.	.58	.62	12.2
15.	.94	.45	6.7	45.	.81	.18	9.4
16.	.56	-.12	12.4	46.	.90	.57	7.9
17.	.25	.00	15.7	47.	.37	.00	14.3
18.	.25	.00	15.7	48.	.33	.37x	14.8
19.	.31	.14	15.0	49.	.37	.27	14.3
20.	.43	.38x	13.7	50.	.85	.66	8.8
21.	.81	.18	9.4	51.	.75	.32x	10.3
22.	.81	.72	9.5	52.	.56	-.12	12.4
23.	.18	.20	16.6	53.	.85	.66	8.8
24.	.29	.44	15.2	54.	.56	-.12	12.4
25.	.31	.14	15.0	55.	.64	.53	11.6
26.	.94	.45	6.7	56.	.43	.38x	13.7
27.	.50	.50	13.0	57.	.49	.25	13.1
28.	.31	-.14	15.0	58.	.25	.00	15.7
29.	.75	.32x	10.3	59.	.31	-.14	15.0
30.	.56	-.12	12.4	60.	.64	.53	11.6

Grade 6 - Girls

Item	p	r	Δ	Item	p	r	Δ
1.	.93	.50	7.1	31.	.50	.65	13.0
2.****				32.	.59	.51	12.1
3.	.59	.51	12.1	33.	.81	.72	9.5
4.	.75	.21	10.3	34.	.67	.37x	11.2
5.	.25	-.21	15.7	35.	.67	.00	11.2
6.	.59	.51	12.1	36.	.50	.65	13.0
7.	.67	.37x	11.2	37.	.41	.18	13.9
8.	.59	.18	12.1	38.	.59	.18	12.1
9.	.50	-.34	13.0	39.	.67	-.37	11.2
10.	.83	.00	9.2	40.	.25	-.21	15.7
11.	.41	.51	13.9	41.	.81	.72	9.5
12.	.83	.00	9.2	42.	.87	.63	8.5
13.	.81	.72	9.5	43.	.50	.00	13.0
14.	.50	.65	13.0	44.	.93	.50	7.1
15.	.87	.63	8.5	45.	.93	-.50	7.1
16.	.13	.63	17.5	46.****			
17.	.50	-.34	13.0	47.	.33	.37x	14.8
18.	.25	.21	15.7	48.	.07	.50	18.9**
19.	.25	-.21	15.7	49.	.50	.34x	13.0
20.	.07	.50	18.9**	50.	.67	.37x	11.2
21.	.75	.21	10.3	51.	.59	.18	2.1
22.	.33	.37x	14.8	52.	.41	.18	13.9
23.	.33	.00	14.8	53.****			
24.	.87	-.63	8.5	54.	.50	.65	13.0
25.	.59	-.51	12.1	55.	.50	.65	13.0
26.	.93	.50	7.1	56.	.25	-.21	15.7
27.	.13	.63	17.5	57.	.67	.00	11.2
28.	.17	.00	16.8	58.	.07	.50	18.9**
29.	.50	-.34	13.0	59.	.25	.21	15.7
30.	.33	.00	14.8	60.	.50	.93	13.0

Grade 6 - Total

Item	p	r	△	Item	p	r	△
1.	.83	.11	9.2	31.	.50	.70	13.0
2.****				32.	.64	.00	11.6
3.	.74	.53	10.4	33.	.63	.52	11.7
4.	.54	.21	12.6	34.	.62	.38x	11.8
5.	.32	-.08	14.8	35.	.62	.38x	11.8
6.	.58	.44	12.2	36.	.54	.21	12.6
7.	.72	.18	10.7	37.	.38	.38x	14.2
8.	.61	.22	11.9	38.	.57	.29	12.3
9.	.43	.00	13.7	39.	.64	-.15	11.5
10.	.83	.36x	9.2	40.	.39	.07	14.1
11.	.38	.38x	14.2	41.	.75	.78	10.3
12.	.80	.42	9.6	42.	.79	.21	9.8
13.	.83	.69	9.1	43.	.57	-.15	12.3
14.	.66	.47	11.4	44.	.74	.53	10.4
15.	.91	.54	7.5	45.	.86	.00	8.7
16.	.39	.07	14.1	46.	.94	.46	6.8
17.	.36	-.15	14.5	47.	.36	.15	14.5
18.	.25	.10	15.7	48.	.20	.42	16.4
19.	.29	.00	15.2	49.	.43	.29	13.7
20.	.28	.35x	15.4	50.	.77	.48	10.0
21.	.79	.21	9.8	51.	.68	.25	11.1
22.	.58	.44	12.2	52.	.50	.00	13.0
23.	.25	.10	15.7	53.	.91	.54	7.5
24.	.54	.07	12.6	54.	.54	.21	12.6
25.	.43	-.15	13.7	55.	.59	.58	12.1
26.	.94	.46	6.8	56.	.36	.15	14.5
27.	.34	.47	14.6	57.	.57	.15	12.3
28.	.25	-.10	15.7	58.	.17	.11	16.8
29.	.68	.08	11.2	59.	.29	.00	15.2
30.	.46	-.07	13.4	60.	.60	.72	12.0

* The values of r and △ are not listed when p is greater than .95 or less than .05.

** The item is difficult.

*** The item is very difficult and has a low discrimination index.

**** The item is very easy and has a low discrimination index.

____ The item has an excellent discrimination index between high and low scorers.

x The item has a good discrimination index between higg and low scorers.

The criteria for indices are based on the Hall Study.

Table 9 shows that 58 out of 60 items (or 96 2/3 percent) are either good or excellent high-low discrimination indices at one or more grade levels for either one or both sexes. (Criteria for these were based on the Hall Study.) There were nine *r*'s for each item since the analysis was by grade and sex. The percentage of items in which there ranged from one to nine good or excellent discriminant *r*'s are summarized in Table 10 which follows.

TABLE 10

NUMBER AND PERCENTAGE OF ITEMS WITH 1 to 9 STRONG DISCRIMINANTS

Number of Strong Discriminants	0 (1)	1 (2)	2 (3)	3 (4)	4 (5)	5 (6)	6 (7)	7 (8)	8 (9)	9 (10)
Number of Items	2	3	10	10	9	9	6	5	3	3
Percentage of Items	3 1/3	5	16 2/3	16 2/3	15	15	10	8 1/3	5	5

Each item was examined for its discrimination power by grade and sex and its relative difficulty level. Following this process, thirty items were selected to be included in the second Test of Quantitative Judgment (Form T).

Preparation of the New Test.--The writer had two sets of thirty test items. These when coupled together would constitute the second Test of Quantitative Judgment (Form T).

The thirty items designed by Hall were in rank order from easiest to most difficult. This arrangement of items was based on data compiled in Hall's study.

Each of the thirty items from the writer's test were then analyzed for the total population to determine the percentage of successes on each item. The thirty test items were then ranked from easiest to most difficult. For example, of the thirty test questions, the item with the highest percentage of successes was considered the easiest and the item with the lowest percentage of successes was considered the most difficult.

There were at this point two lists of thirty test questions each in rank order from easiest to most difficult and numbered from 1 to 30, respectively.

Then these two parts were dove-tailed according to the following procedure.

Item #1. The first ranked item from Form H.
Item #2. The sixteenth ranked item from the writer's test.
Item #3. The first ranked item from the writer's test.
Item #4. The sixteenth ranked item from Form H.

.
.
.

Item #57. The fifteenth ranked item from Form H.
Item #58. The thirtieth ranked item from the writer's test.
Item #59. The fifteenth ranked item from the writer's test.
Item #60. The thirtieth ranked item from Form H.

A final check was made to insure randomness in position of answers. The test was then ready for administration to a new population.

CHAPTER V

SUMMARY AND CONCLUSIONS

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SUMMARY AND CONCLUSIONS

Summary.--This study was designed as a follow-up to Hall's investigation of a Test of Quantitative Judgment (Form H) for intermediate grade children. The scope of the writer's work encompassed the initial steps in the construction of a new Test of Quantitative Judgement (Form T).

From the Hall test, thirty items were selected by analysis and were set aside to constitute one-half of the new test (Form T). The writer then endeavored to prepare a comparable set of thirty items which would become the second half of the new test (Form T).

Initially, the writer developed over one hundred items which were written and rewritten. From these, sixty items which appeared strong at face validity were administered to 151 boys and girls in grades 4, 5, and 6.

The analysis of data then made it possible to select thirty items for the new test (Form T). Indices of item difficulty and item discrimination were obtained for each item by grade and sex.

The procedure followed in both the design of questions and analysis of data closely paralleled the methodology of Hall. For this reason, the two parts of the new test were considered relatively comparable.

Conclusions.--The study carried on by this writer did enable the preparation of a new Test of Quantitative Judgment (Form T). The data obtained from the Test of Quantitative Judgment (Form H) and the writer's test made possible the selection of items which measured aspects of quantitative judgment. Hall demonstrated further, that these types of items are measuring something other than general intelligence or mathematical ability.[1]

The analysis of data in this pilot study supported the findings of Hall. It was once again demonstrated that the ability to make quantitative judgments as measured by the devised test increases by grade. There were statistically significant mean differences in test scores between grade 4 and grade 5, grade 4 and grade 6, grade 5 and grade 6. However, there was no statistically significant difference by sex within any one grade. It would then seem apparent that the aspect of quantitative judgment measured by this testing device is not peculiar to

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[1]Donald E. Hall, "The Ability of Intermediate Grade Children to Deal with Aspects of Quantitative Judgment," (unpublished Ed.D. dissertation, School of Education, Boston University, 1965), p. 88.

either sex, but is a type of reasoning perceived by both sexes. There is implied, moreover, an experiential factor which must be a part of the background of the pupil for success in quantitative judgment. Since the mean test scores increased by grade, this experiential factor appears, in part, to be learned.

Implications for Further Research.--It is clearly evident that only the preliminary steps have been taken in the design of a final test instrument. The initial Test of Quantitative Judgment (Form H) was in a very developmental stage. This study is a part of an attempt at refinement in the preparation of the Test of Quantitative Judgment (Form T). This second test should evidence a higher reliability and validity.

Through this process of refinement, factor analysis will eventually be applicable. Before this analysis is possible, however, the test must be equated with other criteria such as general intelligence, mathematical ability, computational skill, and reading ability.

Once a clearer picture of quantitative judgment has been obtained, it should then become feasible to use tests of this nature as predictors of success in certain mathematical areas. This would lead to the design of quantitative judgment tests for the primary grades, as well as for the secondary and college levels.

If it is determined that quantitative judgment is a learned phenomenon, then investigation into the experiential background which must be a part of educational programs, can be made. The measuring of this ability, should reveal what type of a thinking process can be developed through learning programs which will lead toward, if not transfer to, quantitative judgment. However, before the above can occur, there needs to be extensive analysis and investigation of this aspect of quantitative judgment with larger and varied populations.

APPENDIX

QUANTITATIVE EXERCISE

DIRECTIONS:

Read the question and the answers that are below it. Choose the answer you think is correct and place its letter in the space on the answer sheet. Look at the sample questions and see how they are done.

SAMPLE QUESTIONS

1. Twenty-five cents is about enough to buy:
   A. a fur coat
   B. a quart of milk
   C. a new silk dress
   D. two pounds of steak

2. To hard-cook an egg, boil it for:
   A. 10-12 minutes
   B. 3 minutes
   C. 2-3 minutes
   D. 1 minute

SAMPLE ANSWER SHEET

1. B
2. A

1. With one dollar and some change you could buy: 1.
   A. 3 gallons of milk C. 4 pounds of coffee
   B. 5 pounds of turkey D. 2 dozen eggs

2. Mr. Jones found the bunk-bed in his cabin too small for him so he had to sleep on the floor. The length of the bed was probably: 2.
   A. 5 feet C. 1 foot
   B. 10 feet D. 7 feet

3. Which of the following would take up the most room on a shelf? 3.
   A. a box of dried prunes C. a pound of mercury
   B. a tube of toothpaste D. a gallon of milk

4. Mary ran down the walk in 12 steps and along the sidewalk to the mailbox in 9 steps. Then she ran from the mailbox right across the grass to her front door in about: 4.
   A. 5 steps C. 15 steps
   B. 10 steps D. 25 steps

5. A boy in the sixth grade read a 55-page book. It probably took him: 5.
   A. 30 minutes C. 3 days
   B. 2 hours D. 55 seconds

6. Fran filled one quart jar with grapes and the other with apples. He had: 6.
   A. more grapes than apples C. the same amount of each
   B. more apples than grapes D. none of these

7. Jim built a house which measured 6 by 4 by 4 feet in length, width, and height for his pet: 7.
   A. bird C. horse
   B. cat D. dog

8. Jack's piggy bank holds about 75 pennies. It is probably the size of a: 8.
   A. golf ball C. baseball
   B. football D. basketball

9. Jack had his own set of measures: a box of sugar for 1 pound; a large bag of flour for 10 pounds. He used a brick for: 9.
   A. 2 pounds C. 15 pounds
   B. 5 pounds D. 20 pounds

10. The elevator held fifteen people. The floor measured about: 10.
    A. 1 foot by 3 feet C. 5 feet by 8 feet
    B. 2 feet by 4 feet D. 16 feet by 20 feet

11. Stan decided to sell eggs for 70 cents a pound instead of 70 cents a dozen. He probably made: 11.
A. the same amount of money C. more money
B. less money D. none of the above

12. A half pint of milk would just about fill: 12.
A. a gasoline tank C. a ketchup bottle
B. an empty toothpaste tube D. a glass

13. Jim's father was stopped for speeding on the turnpike. How many miles per hour was his car probably going? 13.
A. 25 C. 75
B. 50 D. 125

14. An apple sliced in halves measured 3 inches on one of the flat surfaces. To hold the apple together again you would need one piece of tape just over: 14.
A. 3 inches C. 16 inches
B. 1 inch D. 9 inches

15. A person with normal eyesight usually holds a book about how far away from him while reading? 15.
A. 12-14 inches C. 1-2 inches
B. 36-38 inches D. 24-26 inches

16. Tommy found that 12 issues of the Reader's Digest could be stacked up on the shelf. The shelf must have been at least as high as: 16.
A. a milk bottle C. a 6-inch ruler
B. a yard stick D. a bumble bee

17. To wrap a birthday gift 7 inches long, 3 inches wide, and 2 inches tall, you would need a piece of ribbon as long as: 17.
A. a tape measure C. an elastic band
B. a rubber ruler D. a shoe lace

18. George measured around the face of the clock and found that it was 12 inches. The hand of the clock was a little less than: 18.
A. $\frac{1}{2}$ inch C. 4 inches
B. 2 inches D. 6 inches

19. Tim cut a hole in his old basketball and filled it full of water. He found that it held about: 19.
A. 2 quarts C. 10 quarts
B. 7 quarts D. 13 quarts

20. Tom can mow his own lawn in 20 minutes. His grandmother's lawn is twice as wide and twice as long. He can probably mow her lawn in: 20.
A. 20 minutes C. 60 minutes
B. 40 minutes D. 80 minutes

21. John can walk to Junior High School in 10 minutes. John walks a distance of: 21.
A. 3 miles
B. 10 miles
C. 1 mile
D. ½ mile

22. The cookie dish was 3 ½ feet around the outside. It was about how far across? 22.
A. 1 foot
B. 2 feet
C. 3 feet
D. 4 feet

23. Fred - a fifth grader - works after school lifting heavy boxes onto a truck. They probably weigh: 23.
A. 5 pounds
B. 25 pounds
C. 50 pounds
D. 100 pounds

24. A boy traveled from Iowa to Boston - a 1300-mile trip - in three days. He probably went by: 24.
A. car
B. airplane
C. train
D. foot

25. Lou punched holes close to one another around the outside edge of a post card. She found that she could punch about: 25.
A. 1-5 holes
B. 15 holes
C. 35 holes
D. 100 holes

26. A quart jar filled with which of the following would be heaviest to carry? 26.
A. leaves
B. sand
C. water
D. milk

27. There are three trees. If the third tree is twice as high as the second, and the second tree is twice as high as the first, the third tree is ________________ as high as the first. 27.
A. three times
B. four times
C. six times
D. eight times

28. John used to row across the pond in one hour. Half of the water was let out; it now takes him: 28.
A. 45 minutes
B. 30 minutes
C. 15 minutes
D. 2 hours

29. A milk bottle is shorter than: 29.
A. a teaspoon
B. a jar of mustard
C. a ruler
D. a coke bottle

30. Harold has two square boxes. One holds one quart of sand, the other holds four quarts of sand. How long is the side of the larger in comparison to the smaller? 30.
A. the same
B. twice as long
C. three times as long
D. four times as long

31. Tom had a map of hidden treasures on which 6 inches equaled one mile. He drew another map twice as large. Then 6 inches equaled:' 31.
A. 4 miles C. $\frac{1}{2}$ mile
B. 2 miles D. $\frac{1}{4}$ mile

32. Bill used 2 quarts of water to: 32.
A. fill his goldfish bowl C. water the garden
B. wash the dishes D. clean the car

33. John was sick from eating too much ice cream. He probably ate: 33.
A. 2 cones C. 3 dishfuls
B. 6 spoonfuls D. 7 quarts

34. Robert rode his bicycle to his friend's house a mile away. It probably took him: 34.
A. 3 minutes C. 30 minutes
B. 6 minutes D. 60 minutes

35. The desk was 3 feet long. Its width was about: 35.
A. 1 foot C. 3 feet
B. 2 feet D. 4 feet

36. The back seat of a car holds 3 people. Its width is probably: 36.
A. 3 feet C. 5 feet
B. 9 feet D. 7 feet

37. John put four blackboard erasers on one side of the science scales and found that these were balanced by which weight? 37.
A. $\frac{1}{2}$ pound C. $1\frac{1}{2}$ pound
B. 1 pound D. 2 pound

38. Phil built a castle by laying his mother's shoe boxes on top of one another. The castle was 3 boxes high and measured about: 38.
A. 1 foot C. 1 yard
B. 6 inches D. 4 feet

39. Ric stared out his hotel window and saw the cars which looked like toys. He was staying on the: 39.
A. second floor C. eighth floor
B. fourth floor D. sixteenth floor

40. John measured around the telephone pole with his mother's tape measure and found that it was: 40
A. 1 foot C. 3 feet
B. 2 feet D. 4 feet

41. Cindy pushed the window open a crack. To her surprise a guest came in. It was probably: 41.
A. an elephant
B. a sparrow
C. an airplane
D. a bee

42. A pound of tomatoes might fit into the same sized bag as: 42.
A. a pound of nails
B. a pound of apples
C. a pound of feathers
D. all of the above

43. Frank decided to spend his $5.00 in pennies for Christmas shopping. He probably carried his money in: 43.
A. an egg box
B. a wallet
C. a gallon pail
D. a milk carton

44. Sally - a fifth grader - made a jump rope. For this she needed a piece of rope about how long? 44.
A. 1 mile
B. 2 yards
C. 3 feet
D. 12 inches

45. Ralph climbed a tree one hundred feet high. He could se: 45.
A. a baseball on the ground
B. a key in the front door
C. an ant
D. a nickel on the lawn

46. The front of the automobile was smashed. The car had probably hit: 46.
A. a wastebasket
B. a haystack
C. an airplane
D. a telephone pole

47. A birthday candle takes about how many minutes to burn down: 47.
A. 5 minutes
B. 10 minutes
C. 15 minutes:
D. 20 minutes

48. Sam read a 300-page book. It was about as thick as: 48.
A. a potato chip
B. a pancake
C. a brick
D. a cheese sandwich

49. Mr. Brown painted a <u>FOR SALE</u> sign for his house on a board 2 feet long and 1 foot wide. The letters were how tall? 49.
A. 9-10 inches
B. 1-2 inches
C. 15-16 inches
D. 4-5 inches

50. David looked out the window and saw an airplane flying about a mile away. It probably looked like: 50.
A. an elephant with wings
B. a fly
C. a balloon
D. a bird

51. Nancy started cracking eggs into a measuring cup. She found that it was full after she had cracked about: 51.
A. 1 egg C. 4 eggs
B. 2 eggs D. 8 eggs

52. Jon tried to kick his basketball into the doghouse but the basketball stuck in the door. How wide was the door? 52.
A. 1 foot C. 3 feet
B. $\frac{1}{2}$ foot D. 2 feet

53. Jill's mother told her to wear her winter coat because the thermometer read: 53.
A. 100 degrees C. 50 degrees
B. 70 degrees D. 20 degrees

54. A tailor made a suit for a man. About how much wool did he use? 54.
A. 1 yard C. 6 yards
B. 3 yards D. 12 yards

55. Don and his father drove directly from Boston to Florida in: 55.
A. two minutes C. two days
B. two hours D. two weeks

56. Tim saved his popsicle sticks in a pile on the window sill until they measured an inch high. He then had about: 56.
A. 8 sticks C. 32 sticks
B. 16 sticks D. 64 sticks

57. John counted his marbles and found that he had a total of 160. These just about filled: 57.
A. a glass C. a pail
B. a match box D. a milk bottle

58. Jim wanted an 8 by 10 foot rug for the clubhouse floor. His mother had some old pieces of carpet which were 4 by 5 each. He used how many of these? 58.
A. 2 pieces C. 6 pieces
B. 4 pieces D. 8 pieces

59. A dollar bill is about as long as: 59.
A. a 6-inch ruler C. an elephant's nose
B. a new pencil D. a postcard

60. Box A is 2 feet high, 2 feet wide, and 6 feet long. Box B is 6 feet high, 2 feet wide, and 2 feet long. If Box A is filled with sand and then poured into Box B, the latter is: 60.
A. exactly full C. half full
B. overflowing D. two-thirds full

QUANTITATIVE EXERCISE
GRADES 4,5, and 6
(Circle your grade.)

NAME__________________________BOY____GIRL____DATE____________

SCHOOL__________________________CITY__________________________

AGE____________BIRTHDATE________________TEACHER_______________
YEAR MONTH DAY

THIS IS TO THE STUDENT,PLEASE READ IT:

This is a test that can show how well you make quantitative judgment about ordinary things that are all around you everywhere.

Each question has four answers after it. Think what the question means and use your best judgment in choosing the answer. If you cannot think of the answer to a question, go on to the rest and come back to the ones that give you trouble when you have finished all the others. If you do not know the answer leave the space blank; but if you think you know, then answer what you think it should be. Work as fast as you can but be careful when placing your answers. Do not guess, but read carefully and think.

THE BACK OF THIS PAGE IS YOUR ANSWER SHEET, PLACE ALL ANSWERS ON IT. DO NOT MARK THE TEST BOOKLET. Turn this sheet and place it under the edge of the test booklet. Slide the sheet up or down so that the numbers line up evenly.

QUANTITATIVE EXERCISE

Page 1	Page 2	Page 3	Page 4	Page 5	Page 6
1.____	11.____	21.____	31.____	41.____	51.____
2.____	12.____	22.____	32.____	42.____	52.____
3.____	13.____	23.____	33.____	43.____	53.____
4.____	14.____	24.____	34.____	44.____	54.____
5.____	15.____	25.____	35.____	45.____	55.____
6.____	16.____	26.____	36.____	46.____	56.____
7.____	17.____	27.____	37.____	47.____	57.____
8.____	18.____	28.____	38.____	48.____	58.____
9.____	19.____	29.____	39.____	49.____	59.____
10.____	20.____	30.____	40.____	50.____	60.____

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Date 6-10-65