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Increasing Senior High School Students' Mathematical Problem-Solving Skills through Improving Their Spatial Visualization Skills by Learning and Practicing Three-Dimensional Dynamic Geometry with Cabri 3D

A Dissertation Presented

By

NURUL YAQIN

Submitted to the Graduate School of the
University of Massachusetts Amherst in partial fulfillment
of the requirements for the degree of

DOCTOR OF EDUCATION

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College of Education

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NURUL YAQIN

Approved as to style and content by:

Joseph Berger, Chair

Claire Hamilton, Member

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DEDICATION

For my family, my father Nadlir Bambang Muslichin, my mother Fauziah Nurbaiti, my wife Hj. Siti Munawaroh, and my son H. Muhammad Nur Reza A'Masyi

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ABSTRACT

INCREASING SENIOR HIGH SCHOOL STUDENTS' MATHEMATICAL PROBLEM-SOLVING SKILLS THROUGH IMPROVING THEIR SPATIAL VISUALIZATION SKILLS BY LEARNING AND PRACTICING THREE-DIMENSIONAL DYNAMIC GEOMETRY WITH CABRI 3D

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Problem-solving is one of the standards of mathematics processes developed by the National Council of Teachers of Mathematics. Many research studies indicated that there was a relationship between mathematics achievement and spatial visualization skills. But the research results still didn't indicate whether or not there was a causal mediation relationship between the achievement of learning-and-practicing three-dimensional geometry and mathematical problem-solving skills mediated by spatial visualization skills. Therefore, it's still necessary for us to specifically investigate whether or not spatial visualization skills can be a medial factor between the increase of mathematical problem-solving skills and the effects of learning and practicing three-dimensional geometry.

For effectively increasing mathematical problem-solving skills and enhancing spatial visualization skills through learning and practicing three-dimensional geometry, we

need an effective tool. And for investigating which tool was effective for learning and practicing three-dimensional geometry, experimental research was conducted at one of the prestigious public senior high schools in Indonesia. This experimentation applied Cabri 3D (dynamic geometry software) for the treatment group and traditional tools (non-dynamic geometry software) for the control group. The results of this experimental research indicated that both kinds of tools were significantly able to develop and enhance students' spatial visualization skills, but with different effect sizes. The effect size in the treatment group was bigger than the effect size in the control group. The students in the treatment group outperformed the students in the control group in spatial visualization skills and mathematical problem-solving skills.

Further investigation was to investigate the causal mediation relationship between achievement in learning-and-practicing three-dimensional geometry with Cabri 3D and mathematical problem-solving skills mediated by spatial visualization skills. Mediation analysis in this research indicated that there was a significant indirect effect between achievement in learning-and-practicing three-dimensional geometry with Cabri 3D and the increase of mathematical problem-solving skills mediated by the enhanced spatial visualization skills. This experimental research was then completed with questionnaires (students' feedback).

The conclusions of this experimental research are Cabri 3D (dynamic geometry software) is an effective tool for learning and practicing three-dimensional geometry, developing and enhancing spatial visualization skills, and indirectly increasing mathematical problem-solving skills.

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CHAPTER 1

CONTEXT AND RATIONALE

Context

There is great variation in the demonstrated ability of some students in senior high schools to excel in mathematics while others struggle to do so. Such a great variation can be the result of varying interests in math as a subject "… there is a significant difference in interest in mathematics and academic achievement of high school level students … the students should train and exposed to various problem-solving skills as a supportive technique to reinforce the learning of the subject mathematics." (Illiyas & Charles, 2017, p. 264)

For those who struggle with math achievement, a variety of approaches have been investigated regarding how best to help them succeed. To better understand how these students can be assisted with improvement in their mathematical skills, we need to examine cognitive factors that significantly influence students' mathematical problem-solving skills. This approach is essential given that mathematical problem-solving activities are some of the most important processes in mathematics education, as mentioned in Gresham's study "… using problem-solving situations … [is] very important for teaching and motivating students to learn mathematics." (Gresham, 2018, p. 97). Furthermore, the NCTM (2014a) encourages teaching mathematical understanding and reasoning, the making of meaningful connections, and the understanding of concepts and procedures through problem-solving. Some studies have shown that spatial visualization skills have positive relationships with problem-solving skills in geometry and mathematics (Fennema & Sherman, 1977; Tartre, 1990; Tartre & Fennema, 1995).

Mathematical problem solving is one of the standards of five mathematics processes developed by the National Council of Teachers of Mathematics, e.g. problem-solving, reasoning and proof, connections, communications, and representations NCTM (2000). Among the skills that are found to be related to mathematical learning and achievement are spatial skills (Tartre, 1990). Some research studies such as in Bishop's (1980) spatial visualization training indicate that there is a relationship between mathematics achievements and spatial visualization skills. Spatial visualization skills can be developed through instructions and practices (Alam, 2009). Again Allam (2009) notes that "spatial visualization skills in young adults can be enhanced through instruction and practice." (p.1). In other words, spatial visualization skills can be developed and enhanced by learning and practicing. Bishop (1980) developed his spatial visualization training with three-dimensional geometry. So, according to Bishop's study, high school students' spatial visualization skills might be developed and enhanced through learning geometry, especially through learning and practicing three-dimensional geometry.

The mathematical problem-solving process necessitates reasoning and sense-making because reasoning and sense-making are the foundation of problem-solving skills (National Council of Teachers of Mathematics, 2010). "In the most general term, reasoning can be thought of as the process of drawing conclusions on the basis of evidence or stated assumptions" (National Council of Teachers of Mathematics, 2010, p. 4), whereas "sense-making [be thought of] as developing [an] understanding of a situation, contexts, or concepts by connecting it with existing knowledge" (National Council of Teachers of Mathematics, 2010, p. 4). As identified in the book *Focus in High School Mathematics: Reasoning and Sense-Making in Geometry* (National Council of Teachers of Mathematics, 2010, p. 10) the key elements of reasoning and sense-making with geometry can be described as follows:

- Conjecturing about geometric objects: reasoning inductively and deductively by using a variety of representations.

- Construction and evaluation of geometric arguments: developing arguments (informal) to justify a conjecture.

- Multiple geometric approaches: analyzing a situation by using transformational and synthetic approaches.

In practice, reasoning and sense-making are intertwined across the continuum from informal observations to formal deduction as shown in Figure 1.1. It identifies sense-making with the informal end of the continuum and reasoning (especially doing proofs) with a more formal end (National Council of Teachers of Mathematics, 2010). Therefore, we can understand that mathematical reasoning and sense-making are both important outcomes of mathematics instruction as well as important means by which students understand mathematics (National Council of Teachers of Mathematics, 2010).

Figure 1.1 The Intertwining of Reasoning and Sense-Making

Source: from NCTM (2010)

The type of this dissertation research is experimental research. In this research, the treatment group used Cabri 3D (DGS) and the control group used traditional tools (non-DGS) for learning and practicing three-dimensional geometry. DGS is a dynamic geometry software; and traditional tools are non-DGS such as paper-pencil, rulers, protractors, compass, set squares, and dividers. In a teaching-and-learning environment, DGS is the teaching-and-learning tool in a dynamic geometry environment (DGE), and traditional tools are the teaching-and-learning tools in a paper-pencil environment (PPE).

Further investigation in this experimental research was to investigate whether or not there was a direct or indirect causal relationship between the achievement of learning-and-practicing three-dimensional geometry with Cabri 3D and mathematical problem-solving skills mediated by spatial visualization skills.

Learning and practicing three-dimensional geometry with Cabri 3D in the DGS-based learning program could be very important associated with effectively increasing students' problem-solving skills. "The effective role of the DGS-based lessons on students' learning experiences is assumed to be associated with the aspects of motivation, … , conceptual understanding, and problem-solving strategies" (Denbel, 2015, p. 24).

However, many teachers and students in senior high schools are reluctant to teach and learn three-dimensional geometry due to the limitations of traditional tools' features and properties. Traditionally, students in schools use papers, pencils, rulers, compasses, set squares, dividers, and protractors for learning and practicing three-dimensional geometry. The use of these traditional tools is time-consuming, and also with these tools students and teachers cannot freely manipulate and construct various three-dimensional objects.

Cabri 3D is a learning tool in DGE that has made learning and practicing three-dimensional geometry easier and less time-consuming than with traditional tools. In his article, Denbel (2015) also said that "Cabri offers more opportunities to construct and justify geometrical concepts than the pencil and paper settings." (Denbel, 2015, p. 26), and it can help students learn, animate, and construct three-dimensional objects that cannot be easily done by traditional tools. "This process is thus facilitated by the dynamic features of the geometry software." (Denbel, 2015, p. 24)

In the context of developing and enhancing spatial visualization skills and increasing mathematical problem-solving skills, the learning tools must be able to help students learn and practice easily in at least three basic problems, namely developments, rotations, and views. Furthermore, the learning tools have to be able to support students in doing explorations, making conjectures, and verifications. Hence, Cabri 3D can be an important learning tool for developing and enhancing students' spatial skills and increasing their mathematical problem-solving skills as well.

"The general objectives of geometry education can be summarized as: [The] students should use geometry within the process of problem-solving, understanding and explaining the physical world around them ….” (Guven & Kosa, 2008, p. 100). It has been clear that the physical world around us cannot only be explained with two-dimensional geometry but also with three-dimensional geometry. Because of that, “NCTM (2000) recommends that geometry instruction should include the study of three-dimensional geometry and provide students opportunities to use spatial abilities to solve problems.” (Guven & Kosa, 2008, p. 100)

Spatial ability consists of spatial visualization skills, such as changing, rotating, bending, and reversing an object presented for stimulation in the mind (McGee, 1979). According to Yurt and Tunkler (2016) “There is no consensus in the relevant literature about the components of spatial ability.” (p. 966). Spatial visualization seems to tap the ability of mental integration, which is done while consolidating several orthographic views into a single image of perspective. Given the obvious connection between spatial thinking and transformational geometry, which is the base of engineering drawing, one might hypothesize that work with the latter would improve skills in the former (Clements, Michael, & Battista, 1992). Spatial visualization skills are important not only for mathematicians but also for other professionals such as engineers and architects. Spatial visualization skills are needed for adequately modeling and representing three-dimensional objects.

Generally, we know that some students can easily reach high achievement in mathematics while others cannot. In response to this situation, a few questions have emerged such as: how to increase students’ achievement in mathematics and mathematical problem-solving skills (MPS); whether or not there is a causal mediation relationship between achievement in learning and practicing three-dimensional geometry with Cabri 3D (C3D) and mathematical problem-solving skills (MPS) mediated by spatial visualization skills (SVS); and whether or not learning and practicing three-dimensional geometry with Cabri 3D can effectively develop and enhance spatial visualization skills.

In general, spatial skills are very important for understanding and representing mathematical concepts, and spatial skills can be considered to be the mental skills concerned with understanding, manipulating, reorganizing, or interpreting mathematical relationships visually (Tartre, 1990).

Motivating Factors

The ability to create and project mental representations is often referred to as spatial visualization skills. We need to conduct a research study for investigating spatial visualization skills that can be developed and enhanced through learning and practicing three-dimensional geometry with Cabri 3D. Spatial visualization skills are very important skills for learning geometry, particularly in geometrical problem-solving. Furthermore, we think that there could be a causal mediation relationship between mathematical problem-solving skills (MPS) and the achievement of learning and practicing three-dimensional geometry with Cabri 3D (C3D) mediated by spatial visualization skills (SVS).

Previous research studies only indicated there was a relationship between mathematics achievement and spatial visualization skills (SVS), and SVS could be enhanced by instruction and practice. The research still didn't indicate about causal mediation relationships among the three factors, namely: learning and practicing three-dimensional geometry with Cabri 3D (C3D) for developing spatial visualization skills

(SVS), in turn, increasing mathematical problem-solving skills (MPS). Figure 3.4 shows the causal mediation relationship between achievement in learning and practicing three-dimensional geometry with Cabri 3D (C3D) and mathematical problem-solving skills (MPS) mediated by spatial visualization skills (SVS).

Most mathematics teachers find that mathematical problem-solving is a big problem that their students face when learning mathematics, "that is, students without conceptual understanding may get better at performing procedures based on rote memorization in solving routine problems. However, when facing non-routine problems involving strategic skills, such students become at a loss to develop a new appropriate technique in solving them. Hence, a conceptual understanding was [needed] … for developing problem-solving strategies …." (Denbel, 2015, p. 28). Therefore, the enhancement of students' mathematical problem-solving skills ought to be the center of mathematics education in senior high schools. This experimental research also found that senior high school students' mathematical problem-solving skills significantly improved through systematic training in spatial visualization skills.

The motivating factors in this study are:

- Development and enhancement of spatial visualization skills will help students increase their achievement in learning mathematics.
- Enhancement of spatial visualization skills will affect the results of mathematics processes – problem-solving, reasoning and proof, connections, communications, and representations.

- Enhancement of spatial visualization skills will increase ability and skills in the understanding of mathematics and science.

Furthermore, high spatial visualization skills and mathematics achievement will facilitate students to learn other related knowledge like physics, chemistry, engineering, and architecture.

Learning Theory

This section will present the main learning theories, namely: Constructivism learning theory, Brain-based learning theory, and Multiple-intelligences theory. And these learning theories will be further described in the next chapter.

Constructivism Learning Theory

Constructivism is a philosophy of learning founded on the premise that people learn best through experiences. That is, people create their meaning through experience. "The idea of using constructivism theory is not a new theory in educational teaching and learning in Indonesia." (Suhendi & Purwarno, 2018, p. 87). Indonesia is a country where this experimental research conducted.

Dewey's theory states that constructivism can build personal and social knowledge. The essence of the constructivism theory is in the meaning of a learning process. "In the perspective of constructivism theory, students are motivated and directed to learn the main idea through discovery learning." (Suhendi & Purwarno, 2018, p. 89)

Constructivist thinking is rooted in several aspects of Piaget and Vygotsky's cognitive theories. "From Piaget, we get active learning, create schemes, assimilate, and accommodate all forms of science, etc. From Vygotsky, we get social constructivism, group work, internships, and so on. Constructivism embraces a 'top-down' rather than a ' bottom-up' instructional methodology …." (Aljohani, 2017, p. 98). The top-down instructional methodology means that the teachers give their students the main problem, and the students learn the main problem and then derive the detail. "The main principles in constructivism are: Knowledge is Constructed; the Learner is an Active Creator." (Aljohani, 2017, p. 98). In recent decades, the constructivism learning theory has become more popular in the school system.

Brain-Based Learning Theory

The brain is a primary organ that human has. A part of its functions is to control human activities like teaching and learning processes. Therefore, for that necessity, brain-based learning theory is developed. Although this learning theory is relatively new, this learning theory has helped design a learning program in the classroom, for example, learning with calming music to decrease stress or create comfortable reading and study areas. "Brain-based learning refers to teaching methods, lesson designs, and school programs that are based on the latest scientific research about how the brain learns, including such factors as cognitive development—how students learn differently as they age, grow, and mature socially, emotionally, and cognitively" (Glossary of Education Reform, 2013, para. 1)

> Brain-based learning is motivated by the general belief that learning can be accelerated and improved if educators base how and what they teach on the science of learning, rather than on past educational practices, established conventions, or assumptions about the learning process. For example, it was commonly believed that intelligence is a fixed characteristic that remains largely unchanged throughout a person’s life. However, recent discoveries in cognitive science have revealed that the human brain physically changes when it learns and that after practicing certain skills it becomes increasingly easier to continue learning and improving those skills. This finding—that learning effectively improves brain functioning, resiliency, and working intelligence—has potentially far-reaching implications for how schools can design their academic programs and how teachers could structure educational experiences in the classroom. (Glossary of Education Reform, 2013, para. 2)

As mentioned above, the human's brain physically changes because of the learning process and practice with specific skills and knowledge. Such learning and practicing will effectively improve brain functioning like problem-solving and spatial reasoning. Furthermore, learning and practicing three-dimensional geometry with Cabri 3D might enhance spatial visualization skills and increase mathematical problem-solving skills.

Multiple-Intelligences Theory

The theory of multiple-intelligences was introduced in 1983 by psychologist Howard Gardner in his book “Frames of Mind.” This theory suggests that traditional psychometric views of intelligence are too limited. The essence of the Multiple-intelligences theory is that each person has nine types of intelligence, and the core of intelligence is the ability to learn and solve problems.

The nine types of intelligence according to Gardner in his Multiple-intelligences theory, namely: visual-spatial intelligence, linguistic-verbal intelligence, logical-mathematical intelligence, bodily-kinesthetic intelligence, musical intelligence, interpersonal intelligence, intrapersonal intelligence, naturalistic intelligence, and existentialist intelligence. Naturalistic intelligence is the most recent intelligence addition to Gardner's theory and met with more resistance than his original seven-intelligences.

When we hear the word intelligence, the concept of IQ testing may immediately come to mind. Intelligence is our intellectual potential, something we are born with, something that can be measured, and a capacity that is difficult to change. In recent years, however, other views of intelligence have emerged. One such concept is the theory of multiple intelligences that was proposed by Howard Gardner.

To capture the full range of abilities and talents that people possess, Gardner theorizes that people do not have just an intellectual capacity, but have many kinds of intelligence, including musical, interpersonal, spatial-visual, and linguistic intelligence. While a person might be particularly strong in a specific area like musical intelligence, the person might be also strong in verbal, musical, and naturalistic intelligence.

Gardner's theory has come under criticism from both psychologists and educators. These critics argue that Gardner's definition of intelligence is too broad and that his nine different-intelligences only represent talents, personality traits, and abilities. Cerruti (2013) said, "I criticize MI [Multiple Intelligences] for doing little to clarify for teachers a core misunderstanding, specifically that MI [Multiple Intelligences] was only an anatomical map of the mind but not a functional theory that detailed how the mind … processes information.” (p. 1)

Cerruti’s core intention is plain, namely to advance the utility of Multiple-intelligences theory to both teachers and researchers by building a functional Multiple-intelligences theory. He has argued that as the field of educational neuroscience grows, Multiple-intelligences theory may be a particularly useful foundation upon which to build a proper scientific theory of neurocognitive learning processes. However, “the theory of Multiple-intelligences enjoys considerable popularity with educators. Many teachers utilize multiple intelligences in their teaching philosophies and work to integrate Gardner’s theory into the classroom.” (Cherry, 2019, para. 5)

Although there are nine Gardner’s different-intelligences, only two Gardner’s different-intelligences correlate to this study, namely: Logical-Mathematical Intelligence and Visual-Spatial Intelligence.

- Logical-Mathematical Intelligence:

  The strength of this intelligence is on analyzing problems and mathematical operations. People who are strong in logical-mathematical intelligence are good at reasoning, recognizing patterns, and logically analyzing problems. These individuals tend to think conceptually about numbers, relationships, and patterns (Singh et al., 2017). Thinking logically and reasoning deductively are most often associated with scientific and mathematical thinking or problem-solving skills.

- Visual-Spatial Intelligence:

> The strength of this intelligence is on visual and spatial judgments. People who are strong in visual-spatial intelligence are good at visualizing things. These individuals are often good with directions as well as maps, charts, videos, and pictures (Levine, Ratliff, Huttenlocher, & Cannon, 2012). And people who are strong in this intelligence also enjoy drawing three-dimensional objects with dynamic geometry software and recognizing patterns easily. Hence, visual-spatial intelligence has a close relationship with spatial reasoning in geometry and spatial visualization skills.

The description of the three learning theories above indicates that spatial visualization skills and problem-solving skills can develop and enhance through learning, practicing, and connecting brain capacity to life experience. Discovery learning and explorations in the field of three-dimensional dynamic geometry might develop spatial visualization skills and increasing mathematical problem-solving skills. These learning theories, including Piaget's theory of spatial development and Van Hiele Level, will be reviewed in Chapter 2.

Problem Statement

Problem-solving is one of the mathematics process standards. Mathematical problem-solving skills are primary skills for solving problems in mathematics and real-world situation. These skills relate to spatial visualization skills. Some research mentioned that there was a relationship between mathematics achievement and spatial visualization skills, and spatial visualization skills could be improved by practice. The problem is how to increase mathematical problem-solving skills effectively in the context of developing and enhancing spatial visualization skills. Spatial visualization skills are the cognitive abilities that can be developed and enhanced by learning and practicing three-dimensional geometry with traditional tools and dynamic geometry software. By experimentation, we investigate which tool is more effective for developing and enhancing spatial visualization skills and increasing mathematical problem-solving skills.

This research study is to investigate the relationship between senior high school students' mathematical problem-solving skills and spatial visualization skills. And this research study is also to investigate a causal mediation relationship between the achievement of learning and practicing three-dimensional geometry with Cabri 3D and mathematical problem-solving skills mediated by spatial visualization skills.

Hence, as applied to this study, I would expect the independent variable to be students' achievement in learning and practicing three-dimensional geometry with Cabri 3D to develop and enhance spatial visualization skills, the mediator variable to be spatial visualization skills, and the dependent variable to be mathematical problem-solving skills.

Mediation Analysis

The title of this dissertation is "Increasing senior high school students' mathematical problem-solving skills through improving their spatial visualization skills by learning and practicing three-dimensional dynamic geometry with Cabri 3D." Hence, the core of this dissertation is a research study in the mediation relationship and mediation analysis with a regression-based approach.

Mediation analysis is a statistical method used to help answer the question as to how some causal agent X transmits its effect on Y mediated by M (Hayes, 2013) where X is an independent variable, Y is a dependent variable, and M is a mediator variable between X and Y.

The reasons why some students can solve math problems and some others cannot solve them are always related to their spatial visualization skills. One of the desired suggestions to develop mathematical skills is to suitably emphasize and develop primary abilities such as spatial ability instead of just teaching mathematics (Bishop, 1980; Kayhan, 2005). The suggestion needs us to find a solution to the issue through experimental research.

This research is for investigating whether or not there is a direct or indirect causal relationship between the effect of learning and practicing three-dimensional geometry with Cabri 3D (X) and mathematical problem-solving skills (Y) independent of (or mediated by) spatial visualization skills (M), and then for investigating the indirect effect of how much two cases that differ by one unit on X are estimated to differ by ab units on Y as a result of the effect of X on M, in turn, affecting Y in where a and b are regression coefficients of X and M respectively (Hayes, 2013). Furthermore, the direct effect is two cases that differ by one unit on X but are equal on M are estimated to differ by c' units on Y. Thus, the total effect is c = c' + ab, and c quantifies how much two cases that differ by one unit on X are estimated to differ on Y (Hayes, 2013).

This mediation analysis is to investigate whether or not learning and practicing three-dimensional geometry with Cabri 3D can develop and enhance senior high school students' spatial visualization skills, in turn, can increase their mathematical problem-solving skills. Mediation analysis will be further described in the next chapters

Rationale: Significances of Spatial Visualization Skills and Problem-solving Skills

This experimental research is for investigating the use of dynamic geometry software (Cabri 3D) for developing and enhancing spatial visualization skills and increasing mathematical problem-solving skills. Allam (2009) said as he quoted from (Piburn et al., 2005; Crown, 2001, Deno, 1995; Lord, 1985; Pallrand & Seeber, 1984; Peters et al., 1995; Sorby & Baartmans, 1996) that, modern studies, however, have indicated that spatial visualization skills can be improved through practice. Additionally, spatial visualization skills are strongly related to problem-solving skills (Carter, LaRussa, & Bodner, 1987).

Spatial visualization skills are important skills for mathematicians and other professionals like engineers and architects. They need the skills for making and visualizing a model of problem-solving and representation. Therefore, it is necessary to investigate the use of Cabri 3D for developing and enhancing spatial visualization skills. In the future, Cabri 3D and other DGS might help students effectively learn and practice three-dimensional geometry in the context of developing-and-enhancing spatial visualization skills and increasing mathematical problem-solving skills.

The investigation of the interaction between spatial visualization skills and dynamic geometry software (Cabri 3D) could shed light on increasing mathematical problem-solving skills. Problem-solving is the process of logical-deductive thinking that requires spatial skills. So, spatial visualization skills become a logical prerequisite for successful mathematical problem-solving. Instructions and training with Cabri 3D have to be able to motivate and trigger students' spatial and logical-deductive reasoning. Logical-deductive reasoning is the thinking rule of mathematical processes and mathematical problem-solving.

CHAPTER 2

REVIEW OF LITERATURE

There are several related sections in this chapter, namely: the historical perspective in learning theories, the significance of spatial visualization skills in problem-solving, dynamic geometry environment, developing spatial visualization skills, mathematical problem-solving skills, implications, summary, and theoretical framework. The section of the historical perspective in learning theories includes a literature review of some learning theories concerning the process of developing spatial visualization skills through learning three-dimensional geometry and the process of increasing mathematical problem-solving skills. The section about dynamic geometry environment mainly reviews Cabri 3D literature as an innovative tool for teaching and learning three-dimensional geometry, sub-sections of dynamic geometry environment are about the relationship between the significance of spatial visualization skills and mathematical problem-solving skills. And the last section of this chapter describes the development of the theoretical framework of this dissertation. Those sections and sub-sections are as follows:

Historical Perspective on Learning Theories

Constructivism is a philosophy of learning founded on the premise that, by reflecting on our experiences, we construct our understanding of the world where we live. Constructivism is a learning theory found in psychology that explains how people might acquire knowledge and learn. Therefore, constructivism has a direct application to education. The theory suggests that humans construct knowledge and meaning from their experiences. Piaget's theory of Constructivist learning has had a wide-ranging impact on learning theories and teaching methods in education and as an underlying theme of many education reform movements, such as using Cabri 3D software for teaching and learning three-dimensional geometry for enhancing spatial visualization skills and mathematical problem-solving skills.

Formalization of the theory of constructivism generally is attributed to Jean Piaget who articulated the mechanism by which knowledge is internalized by learners. He suggested that through processes of accommodation and assimilation, individuals construct new knowledge from their experiences. Assimilation is the incorporation of new experience into an already existing framework without changing that framework, and accommodation is the process of reframing mental representation of the external world to fit new experiences as the mechanism by which failure leads to learning. Constructivism is a theory of knowledge that argues that humans generate knowledge and meaning from an interaction between their experiences and their ideas. It has influenced several disciplines, including psychology, sociology, education, and the history of science (Eddy, 2004). "Constructivism has emerged in recent years as a dominant paradigm in education and has had a major intellectual impact on the development of pedagogy, rooted in the cognitive-developmental of Piaget and the sociocultural theory of Vygotsky (Aljohani, 2017, p. 97). In other words, knowledge is constructed individually and socially by experiences in the real practical world.

The development of senior high school students' spatial visualization skills can be categorized into the fourth development stage of Piaget-and-Inhelder's theory, namely the Formal Operational Stage. In this stage, the students' understandings of geometrical concepts develop in the complex formal system, especially in Euclidean geometry. Afterward, learning at higher levels can be successful after the students have passed through the four development stages.

Constructivism is the view in philosophy according to which all "knowledge is a compilation of human-made constructions" (Raskin, 2002, p. 4), and "not the neutral discovery of objective truth" (Castelló & Botella, 2016, p. 263). Hence, research support for constructivists' discovery-based teaching techniques has been mixed, with some research supporting the teaching techniques and other research contradicting the teaching techniques and results.

Brain-based learning theory is a learning theory based on the structure and function of the brain. Caine and Caine (1994)wrote that if the brain is not prohibited from fulfilling its normal processes, learning will occur. The human brain is unique. With the help of modern brain-scanning techniques, we can see how brain structure and brain activities linked to cognitive abilities (Cooper, 2015). This brain-based learning theory doesn't explicitly explain cognitive and spatial functions are unique to the brain. However, this theory mentions that spatial ability is a form of memory and the normal learning process will occur, wholes and parts, simultaneously with the brain as a parallel processor.

The relationship between the concepts of memory and intelligence is somewhat controversial. Behaviorists believe that intelligence is measured by the ability to learn or the capacity to think abstractly (Cooper, 2015). These skills are usually the focus of intelligence tests. Some researchers argue that both concepts (memory and intelligence) are identical, and others claim that both concepts are not identical, though correlated

(Cooper, 2015). In other words, if they are correlated then the relationship between spatial visualization skills as a form of memory and mathematical problem-solving skills as a form of intelligence still needs further investigation. Hence, brain-based learning theory is logically relevant to the theoretical framework of how to increase mathematical problem-solving skills through improving spatial visualization skills with Cabri 3D, because for significantly increasing mathematical problem-solving skills, one needs strong spatial memory and logical thinking as described in the theory of multiple intelligences, developed by Howard Gardner.

Multiple-intelligences theory is the theory of human intelligence, developed by psychologist Howard Gardner. The theory offers an alternative to the older conception of ability. Gardner formulated a list of nine types of intelligence as mentioned in Chapter 1. The first two types of intelligence (visual-spatial intelligence, linguistic-verbal intelligence) are typically valued in schools, the following three types of intelligence (logical-mathematical intelligence, bodily-kinesthetic intelligence, musical intelligence) are usually associated with the arts and mathematics, and the last two (interpersonal intelligence, intrapersonal intelligence) is what he calls ‘personal intelligence’ (Gardner, 1999, pp. 41-43). Furthermore, Gardner says that the seven types of intelligence (except interpersonal intelligence and intrapersonal intelligence) have allowed people to perceive and understand the world around them, and allowing them to identify and resolve the problems they face. However, only two types of intelligence from Gardner’s theory have a close relationship to the theoretical framework that I will develop in this chapter. The two types of intelligence are as follows:

- Logical-Mathematical – The capacities of inductive-and-deductive thinking and reasoning, the use of numbers, and the recognition of abstract patterns. It also consists of the capacities for analyzing problems logically, investigating issues scientifically, and carrying out mathematical operations. Intelligence is most often associated with scientific and mathematical thinking.

- Visual-Spatial – The ability and skill to visualize objects, spatial dimensions, and create internal images and pictures.

As cited in Chapter 1, Cerruti (2013) criticized Multiple Intelligences. He said that Multiple Intelligences was only an anatomical map of the mind but not a functional theory that detailed how the mind processes information. He argued that as the field of educational neuroscience grows, Multiple-intelligences theory may be a useful foundation for building the theory of neurocognitive learning processes.

Dynamic Geometry Environment

The dynamic geometry environment is the environment of teaching and learning two- and three-dimensional dynamic geometry by using software and computer. Software and computers are technological tools for assisting teachers and students in teaching and learning geometry. There are some advantages and disadvantages to learning geometry with software and computers. The advantages are: The technological tools are very appropriate for teaching and learning dynamic geometry as well as for constructing algebraic equations. The tools can also make teaching and learning three-dimensional geometry easy and interesting. The disadvantages of using the tools are: The tools will be useless if students cannot operate a computer and its software appropriately, and if students don't know algebraic equations and basic concepts of how and why their geometrical constructions show up on the computer screen in the forms of lines, curves, planes, and spaces. Ertekin (2014) said that "its equation [lines, planes, and spaces] can be algebraically identified … [with] the components of its normal vector in three-dimensional space" (p. 29).

So, teaching and learning mathematics with a computer requires strategy. For example, before using software for constructing images on a computer screen, a teacher must explain to their students concerning mathematics formulas and basic concepts of line, curve, and space, and the relationship between algebra and geometry. This teaching strategy will help students understand the relationship between algebra and geometry, and help the students visualize their abstract thinking algebraically and geometrically.

Cabri 3D, launched in 2004, is a dynamic geometry software for learning and exploring three-dimensional geometry. Cabri Geometry is a commercial interactive geometry software produced by the French company "Cabrilog" for teaching and learning geometry in high schools. It was designed with ease-of-use in mind. Accascina and Rogora (2006) made research using Cabri 3D Diagrams for teaching geometry, and they wrote that "Cabri 3D is potentially very useful software for learning and teaching geometry" (p. 11). Cabri 3D allows the user to animate geometric figures, proving a significant advantage over those drawn on a blackboard. Relationships between points on a geometric object can be easily demonstrated, which could be useful for the learning processes of geometry. As an advantageous tool for teaching and learning three-dimensional geometry, Cabri 3D can be a better tool for developing and enhancing spatial visualization skills than traditional tools because, with its animation tool, Cabri 3D does help students easily explore and animate pictures/images they have created on a computer screen.

Animation tool in Dynamic Geometry Software (DGS) is a very useful tool for mathematics education research, where “animated learning materials can prove more useful than static representations” (Taylor, Pountney, & Malabar, 2007, p. 249) such as in transformational geometry. DGS allows students to alter the original geometric objects by moving or dragging their components, such as vertices and edges, to different locations on the computer screen. As the objects are dragged or transformed, those objects are updated immediately on the screen. Students also can measure lengths, angles, and areas of the dragged objects and observe how the measures are affected as the objects are altered dynamically (Clements, Sarama, Yelland, & Glass, 2008). Again, these learning activities will help students develop and enhance their spatial visualization skills.

Developing spatial ability is very important in geometry education, because “without spatial ability, students cannot fully appreciate the natural world. Spatial ability is also very important for work in various fields such as computer graphics, engineering, architecture, and cartography” (Guven & Kosa, 2008, p. 100). Eight years later, Kosa (2016) investigated how to use dynamic geometry software on pre-service mathematics teachers that affected the teachers’ spatial visualization skills; and their investigation showed that dynamic geometry software was effective for improving spatial visualization skills. At that time, Kosa used Geogebra software for learning and investigating spatial analytic geometry. For understanding the complexity of spatial analytic geometry, which includes coordinate system, vectors, planes and lines in space, and surface in space, one needs well-developed three-dimensional geometry thinking and spatial visualization skills

Visualization skills are often indicative of academic performance in the engineering curriculum (Yue, 2002). Spatial visualization skills are also important for physics and chemistry students when they want to visualize and model concepts and theories in science, such as when students want to model and visualize the structures of molecules and explain molecular or atomic reactions. Spatial visualization skills will enhance students' understanding of scientific concepts and theories as well as promote their performance in problem-solving. Otherwise, academic failure is shown to be related to deficiencies in two- and three-dimensional perceptional abilities (Rochford, Fairall, Irvings, & Hurly, 1989). Because of that, spatial visualization skills should be applied throughout mathematics and science curriculums in schools as the representations and applications of two- and three-dimensional perceptions.

Furthermore, in this context, the use of Cabri 3D can be considered as the application of educational technology for enhancing spatial skills and helping students to understand mathematical problem-solving strategies. Zbiek and Hollebrands (2008) said that "There is ample evidence that the use of various forms of technology may enhance student understanding of mathematics. The key to enriched understanding lies not only in the skill development of the technology but also in how that technology is used by and with learners" (p. 287). "Dynamic interactive geometry software [DGS] allows students to create geometric constructions and manipulate them easily [and quickly]. Dragging, the dynamic feature of DGEs [or DGS], allows users to move certain elements of a drawing freely and to observe other elements responding to the altered conditions." (Goldenberg, Scher, & Feurzeig, 2008, p. 53). The statements above are very general. To better develop and enhance spatial visualization skills, we need a potentially useful software like Cabri 3D for learning and practicing three-dimensional geometry.

Cabri is truly interactive; "there is some form of communication between the student and the computer which enables the student to go further with mathematics than they would otherwise have been able to. The computer can only obey inputted instruction, and so the student must be precise about what he or she inputs" (Forsythe, 2007, p. 31) Students with interactive tools will easily transform, develop, and view various geometric objects on the computer screen. So, in a dynamic geometry environment, the students will have a bigger opportunity to develop and enhance their spatial visualization skills.

The National Council of Teachers of Mathematics (NCTM) recommends that "the mathematics curriculum for grade 5-8 should include the study of the geometry of one, two, and three dimensions in a variety of situations so that students can visualize and represent geometric figures with special attention to developing spatial sense" (National Council of Teachers of Mathematics, 1989); to do so, the most recent and powerful educational technology should be used. However, old geometry curricula [in the past decades] did not provide enough opportunity for the development of spatial skills (Usiskin, 1987). Therefore, now, educational technology such as DGS will be a very important tool for developing and enhancing spatial visualization skills and increasing mathematical problem-solving skills.

Dynamic Geometry Environment allows new opportunities for the teaching of geometry and descriptive geometry. "Descriptive Geometry is a method to study 3D geometry through 2D images thus offering insight into the structure and metrical properties of spatial objects, processes, and principles" (Nagy-Kondor, 2010). This environment makes it possible for us to create dynamic drawings quickly and easily. Nagy-Condor conducted a two-group experiment for two years using engineering students from a university in Germany as participants. She taught one of the groups with Dynamic Geometry Software (Cinderella and GeoGebra), and another group with paper-and-pencil tools (PPE). She used MCT (Mental Cutting Test) for evaluating the development of spatial ability in her experiment. From her experiment, she reported that learning descriptive geometry with dynamic geometry software like Cinderella and GeoGebra could be better for improving students' spatial skills than the standard PPE.

Also, Noraini (2009) surveyed the effects of using Geometer's Sketchpad (GSP) on 65 students at secondary school in Malaysia. Her survey indicated that most of the students felt that GSP was very useful for learning geometry. However, the subjects could be susceptible to the internal validity threat of selection because Noraini didn't assign the subjects randomly.

Piaget's Theory of Spatial Development

Jean Piaget (1896-1980) is renowned as the constructor of a highly influential model of spatial development stages in learning geometry. His theory is based on the development of children's spatial and mental maps for understanding and responding to physical experiences within their environment.

Research by Piaget and Inhelder (1967) suggest that early spatial conceptions in the infants are topological. The relationship between spatial and topological conceptions are very general and inclusive. The spatial ability will grow and refine naturally or educatively more-and-more complex until adulthood. The four basic topological concepts that can become the foundation of spatial visualization development are as follows:

- Proximity - the relative nearness of an object or event to any other object or event.
- Order - the sequence of objects or events (in time) according to size, color, or some other attributes.
- Separation - an object, event, or space, coming between other objects or events. It also involves distinguishing between objects and parts of objects.
- Enclosure - an object or event surrounded by other objects or events, which involves the ideas of the inside, outside, and between.

Piaget and Inhelder proposed four stages of development in spatial ability that they described through features of various geometric objects:

- *Sensorimotor Stage 0-2 years:* This stage is experienced by babies. Babies notice certain elementary features of objects, such as size and some contour in a topological way. According to Piaget and Inhelder, perceptions of surroundings will include events as well, such as a mother or father entering the room.
- *Pre-operational Stage 2-7 years:* In this stage, children begin to represent spatial features through drawing and modeling. The evidence of their topological thinking

is in their drawings. As is typical around this age, children still have not yet developed the type of thinking that encompasses projective geometry and would, therefore, be unable to imagine the other sides of the object.

- *Concrete Operational Stage 7-12 years:* Gradually, between the ages of about 4 and 9 years, children begin to perceive and represent objects from different points of view and incorporate ideas of perspective. These sorts of ideas can be classified as the type of projective geometry. In this stage, children also begin to use the ideas associated with Euclidean Geometry such as distinguishing between straight lines and curves, squares and circles, and the length and number of sides and angles. The measurement concepts will allow children to bring objects and parts of an object into relative proportions in their drawings. The development of the coordination of horizontal and vertical planes is illustrated in the sequence of drawings (McNally, 1975).

- *Formal Operational Stage 12-18 years:* In this stage, the understanding of geometric concepts continues to develop until reaching complex formal systems of plane geometry. Piaget and Inhelder suggest that the developments of perception as described by the types of geometry are sequential (i.e. Topological, Projective, and Euclidean), while other researchers believe that all types of geometric thinking continue to develop over time and become increasingly integrated.

Piaget's theories require that the stage of development precedes learning. Learning at higher levels becomes possible only if the individual has passed through the four stages of development.

Definition of Spatial Ability

Spatial ability has two principal factors namely: spatial visualization and spatial orientation. Spatial visualization refers to the ability to mentally rotate, manipulate, and twist two and three-dimensional stimulus objects (McGee, 1979). Spatial orientation involves "the comprehension of the arrangement of elements within a visual stimulus pattern, …." (McGee, 1979).

Battista and Clements (1998) stated that spatial ability is the ability to formulate mental images and to manipulate these images in the mind. In literature, spatial ability refers to skills in representing, transforming, generating, and recalling symbolic nonlinguistic information (Linn & Peterson, 1985). According to Tartre (1990) spatial ability is the mental skills concerned with understanding, manipulating, reorganizing, or interpreting relationships visually. Also, Lohman (1993) expressed that spatial ability is the ability to generate, retain, retrieve, and transform well-structured visual images. Hence, spatial ability or skills is mental ability or skills in generating, presenting, interpreting, manipulating and transforming visually well-structured images and relationships, and the ability to retain, retrieve or recall symbolic nonlinguistic information.

Significance of Spatial Visualization Skills in Problem-solving

One of the important factors that have contributed to various achievements in mathematical problem-solving is spatial skills. We may have many definitions of spatial skills, but Linn and Peterson (1985) concluded that "Spatial abilities [skills] generally refer to skill [skills] in representing, transforming, generating and recalling symbolic, nonlinguistic information" (p. 1482). On the other hand, Nuttall, Casey, and Pezaris (2005) said that: "Spatial skills involve the ability [skills] to think and reason using mental pictures rather than words" (p. 122). Also, Zhu (2007) said that MacCoby and Jacklin in 1974 identified that there are three cognitive abilities, namely visual-spatial ability, verbal ability, and quantitative ability. Hence, the spatial ability is a cognitive ability.

This research focuses on spatial visualization skills. Many studies have shown that spatial visualization skills have a positive relationship with problem-solving skills in geometry. In this context, spatial visualization skills are considered an important component of mathematical thought during mathematical problem-solving processes (Battista, 1990; Casey, 2003; Halpern, 2000). Therefore, the National Council of Teachers of Mathematics or NCTM (2000) strongly recommends that geometry instruction should include the study of three-dimensional geometry that provides students with opportunities to use spatial abilities and skills to solve problems.

Spatial skills are the factors that underlie mathematical aptitude, and the research study that investigated the relationship between spatial skills and mathematical aptitude mentioned: "hypothesizing that the effectiveness of certain instructional treatments in mathematics depends on students' spatial ability [spatial skills] …." (Battista, Wheatley, & Talsma, 1982, p. 332). In other studies, researchers also attempted to improve students' mathematical performance by training them with spatial tasks (Moses, 1977). At that time, the spatial task still used traditional tools (non-DGS).

The spatial tasks that emphasize concrete and pictorial representations are needed for motivating students' spatial reasoning, because "both of which [the representations] have spatial components …" (Battista, Wheatley, & Talsma, 1982, p. 333). One suggestion for developing mathematical skills is to suitably emphasize and develop primary abilities such as spatial ability instead of just teaching mathematics (Kayhan, 2005). Bishop (1980) developed theory and spatial training in mathematics that spatial training might improve spatial visualization ability. The training, primarily, was the study on the improvement of spatial skills and mathematics achievement through learning geometry with traditional tools. Bishop's theory also indicated that spatial training might help students organize situations with mental pictures during problem-solving in mathematics.

Furthermore, Linn and Peterson (1985) studied the emergence and characteristics of sex differences in spatial abilities and skills. They said that men and women perform equally well on spatial visualization skills. On the other hand, Ben-Chaim et al. (1988) found that there were statistically significant differences between men and women in spatial visualization among middle school students; other researchers concluded that the differences between men and women in spatial visualization skills were small or null among middle school students (Armstrong, 1980; Fennema & Sherman, 1977; 1978; Linn & Peterson, 1985; Tartre & Fennema, 1995; Voyer, Voyer, & Bryden, 1995). For those inconsistent results, Ben-Chaim et al. (1988) said that the results may be due to the changes over time affecting the experiential influence on the measures, or due to different sample sizes, or due to the instrumentation used.

Numerous research shows that spatial visualization skills are related to success in mathematics (Kosa, 2016). Many considerable efforts have been made to investigate the relationship between spatial skills and mathematics achievement, but the role of learning and practicing three-dimensional geometry with Cabri 3D for developing spatial visualization skills and problem-solving skills still has not been adequately investigated, and also, mediation relationship between the achievement of learning three-dimensional geometry with Cabri 3D and mathematical problem-solving skills that is mediated by spatial visualization skills still has not been investigated. The results of these investigations will support the general objectives of geometry education that can be summarized as "the students study geometry within the process of problem-solving, understanding and explaining the physical world around them (Guven & Kosa, 2008, p. 100).

Assessment of Spatial Visualization Skills

Spatial skills cover mental images such as rotations, developments, and views. One of the instruments used to assess the spatial skills of men and women is Purdue Spatial Visualization Tests (PSVT). Concerning the gap between male and female spatial abilities, Allam (2009) said from (Branoff, 1998; Peters et al., 1995; Sorby & Baartmans, 1996) that many studies reported the gender gap in spatial ability will typically appear during the pretests, with women more often having significantly lower scores than men. However, in the posttests, after the study subjects received some form of spatial visualization skills training, the gap between men and women narrows. These results indicate that spatial visualization skills can be improved through practice and instruction, and the process might eliminate the spatial visualization skills gap between genders.

Three-dimensional Euclidean geometry has not been popular in the last few decades. One of the reasons is that diagrams of three-dimensional objects are very difficult to represent with just paper-and-pencil media. Recently, with the use of Cabri 3D, such difficulty has been resolved. Accasina and Rogora (2006) said: “Cabri 3D is potentially very useful software for learning 3D geometry” (p. 11), and “[Cabri 3D] a potentially important tool for the development of visual teaching of solid geometry” (Ertekin, 2014, p. 29). We still need further investigation with the true-experiment method (using a random sampling system with a control group) for knowing whether or not Cabri 3D can be used for developing and improving spatial visualization skills.

The following figures show some examples of PSVT for testing spatial visualization skills in developments, rotations, and views:

- *Developments*: This section has students study the pattern of three-dimensional objects by choosing one correct answer from the five possible shapes listed below it that matches the pattern.

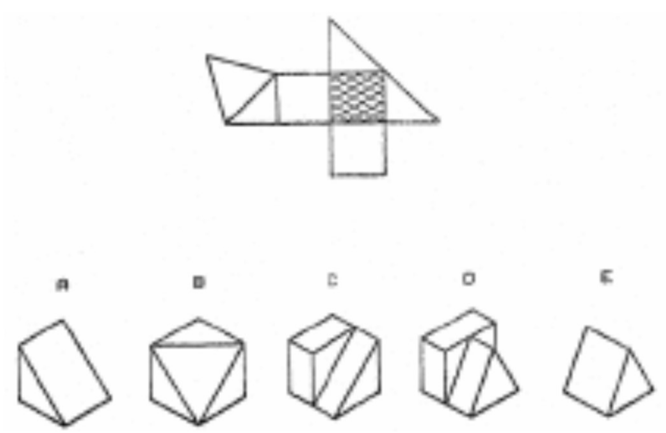

Figure 2.1 Sample question of Developments in PSVT

* *Rotations:* This section shows an object in two different positions. The first shape is rotated on the X or Y or Z axis and the second shape shows the provided rotation pattern. Students must select one object that represents the rotation as shown in the pattern.

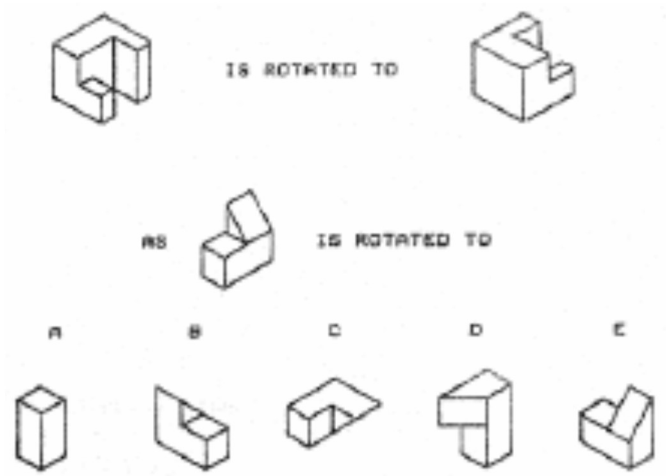

Figure 2.2 Sample question of Rotations in PSVT

* *Views:* This section tests students' ability to visualize a three-dimensional object from various perspectives. In this section, an object is placed in the center of a cube and one of the corners of the cube is marked. The students are asked to imagine the object in such a way that the marked corner will be between the object and their eyes. Students are given five possible answers to choose from, which are provided below it.

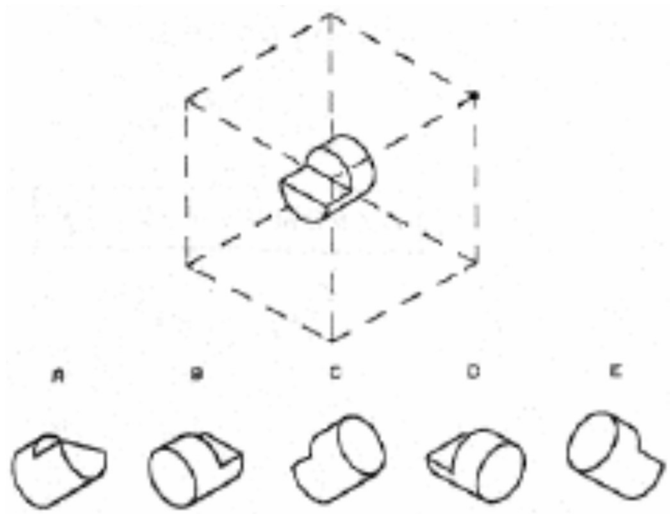

Figure 2.3 Sample question of Views in PSVT

Developing Spatial Visualization Skills

As mentioned above, spatial visualization skills are the functions of genetics and life experiences, and they can be developed and improved through instructions and practices. The simplest and oldest method to develop and improve spatial visualization skills is to use a variety of different tools, such as paper-pencils tools and wooden blocks, to represent different objects.

Dot-papers can be used for this purpose as well. Figure 2.4 is an example of the engineering drawings with PPE tools for developing and improving spatial visualization skills. Olkun (2003) said from (Baartmans & Sorby, 1996; Ben-Chaim, Lappan, Houang, 1985; Smail, 1983) that the kinds of activities used for improving spatial skills are similar to what is being used to teach engineering drawings.

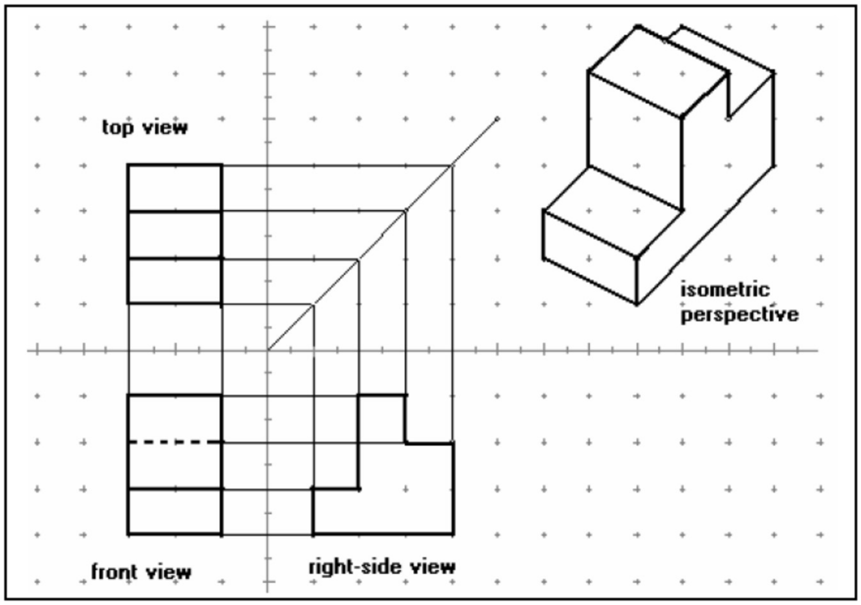

Figure 2.4 An example of the engineering drawing activities with PPT

Source from Olkun (2003)

Not much research has been conducted on the use of DGS for improving and developing spatial visualization skills in high school students, even though some research with computer-based activities have indicated that multi-media software is an effective tool for improving three-dimensional spatial skills (Gerson, Sorby, Wysocki, & Baartmans, 2001). Through quizzing, surveying, and various spatial visualization tests, they found significant improvement in students' spatial visualization skills when they used the software (a nine-piece modular/multimedia software package with a National Science Foundation grant) over those who did not.

Guven and Kosa (2008) researched for about eight weeks to know whether or not Cabri 3D could improve spatial visualization skills. The activities consisted of teaching students in basic Cabri 3D, constructing basic geometric objects (sphere, cylinder, cone, and prism), obtaining circles, ellipse, hyperbolic curves with the help of cone intersections, making vertical projection and trigonometric relations, making transformations, making object intersections, and giving students an opportunity for free exercise. From the research, they found that Cabri 3D could improve students' spatial visualization skills.

Below are two examples of Cabri 3D that Guven and Kosa used in their research.

Example 1: The screenshot of developing a 3D object with Cabri 3D.

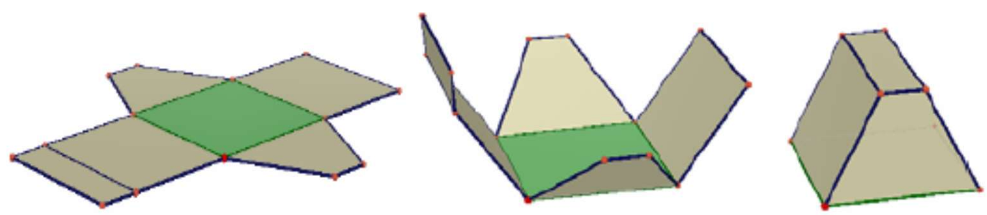

Figure 2.5 Development with Cabri 3D

Source from Guven and Kosa (2008)

Example 2: The screenshot of constructing the theorem in three perpendicular lines.

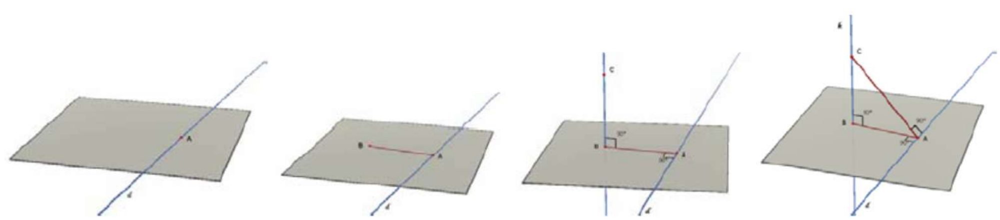

Figure 2.6 Three perpendicular lines

Source from Guven and Kosa (2008)

Hence, dynamic geometry software (DGS) and paper-and-pencil tools (non-DGS) can be used for developing and improving spatial visualization skills. As for which one is better, further research is required. There are still many DGS packages besides Cabri geometry, such as Geometer's Sketchpad (GSP) and Cinderella. One of the distinguishing features of a DGS is its ability to construct geometrical objects and verify the relationship between them. In the dynamic geometry environment, geometrical objects are constructed on the screen and can be manipulated by the mouse as its dragging tool. When the elements of construction are dragged, all the geometric properties in the figure are preserved. DGS is a potent innovative tool in geometry instruction since the Euclidean era.

Cabri 3D has unique three-dimensional visualization tools that facilitate students to visualize, animate, and represent geometric objects easily and quickly. The Cabri 3D's manual wrote that:

> Through tools that facilitate visualization, students and teachers can: Adjust viewing angles turning around the scene, appreciate the rendering of depth of the scene. Animation tools: Particularly stimulating for students, the tools enable them to simultaneously put several elements of a construction in motion. It is ideal for modeling physical phenomena. Different representations and perspectives of the same figure: The three angles of technical design, various perspectives, and Refresh-the-construction tool replaying the user's construction. (Cabri 3D's Manual, 2011)

On the other hand, Piburn et al. (2005) conducted a research study using computer software "which allows the creation of detailed and realistic, two-dimensional representations depicting three-dimensional perspectives of simple and complex geologic structures and landscapes" (p. 517) with geology college students as subjects. He used two control groups and two treatment groups, but their pretest and posttest of spatial orientation showed no significant effect. However, their spatial visualization pretest and posttest scores on the Surface Development Test showed significant improvement. This research demonstrated the potential effectiveness of two-dimensional virtual computer-based renderings of three-dimensional objects for increasing spatial visualization skills.

Problem Solving

Problem-solving is one of the five NCTM's mathematics processes standards – reasoning and proof, problem-solving, connection, communication, and representation. One of the factors that contribute to problem-solving skills is the cognitive ability that consists of spatial ability, verbal ability, and quantitative ability. "…, three-dimensional thinking is required … [to] visualize three-dimensional objects … and perform mathematical operations" (Kosa, 2016, p. 450).

The mathematical problem-solving process is a complex cognitive activity that requires three-dimensional thinking. Some mathematical literature described the mathematical problem-solving process as several separate activities such as doing word problems, creating patterns, interpreting figures, developing geometric constructions, and proving theorems (Wilson, Fernandez, & Hadaway, 1993). Understanding all mathematical complex concepts requires well-developed three-dimensional thinking skills (Kosa, 2016) because, in the process of mathematical problem-solving, problem-solvers need to deeply understand and visualize the parts of a complex structure of the problems in mind. Ertekin (2014) points out that three-dimensional thinking has special significance in spatial reasoning, whereas Polya's theory defines the mathematical problem-solving process as a process that involves several dynamic activities, namely: "understanding the problem, devise [making] a plan, carrying out the plan and looking back." (Zhu, 2007; Alacantara & Bacsa, 2017, p. 23).

As aforementioned, the mathematical problem-solving process could have a relationship with spatial skills. Numerous studies had shown that spatial skills were positively related to achievement in mathematics (Aiken, 1971; Battista, 1980; Fennema & Sherman, 1977). Referring to the process of mathematics in NCTM (2010), the use of educational technology is one of many endeavors used to support mathematical problem-solving activities.

Furthermore, the use of technology could broadly affect students' spatial reasoning and abstraction activities. In learning mathematics, a technological environment provides students with more opportunities to engage in abstraction activities because multiple actions can be easily performed using computers (Hollebrands, 2007). With educational technology, students may explore various mathematical concepts, engage in experiments and model problem settings, have access to more problem-solving tools, and make computation more efficient. Those five NCTM mathematics processes are grounded by "Reasoning and Sense-Making". The reasoning often begins with explorations, conjectures at various levels, and partial explanations before reaching a result.

Van Hiele Levels

Van Hiele levels theory was developed by two Dutch mathematics educators, Dina van Hiele-Geldof and her husband Pierre Marie van Hiele, in the late 1950s. The Van Hiele levels theory has three aspects: The existence of levels, properties of levels, and the movement from one level to the next level. It can be read in Usiskin (1982), from page 4 to page 6. This theory has been applied to explain why students have difficulty with the higher-order cognitive process and its remedy, particularly in learning proof. "Van Hiele

(1959) is more optimistic than Piaget, believing that cognitive development in geometry can be accelerated by instruction" (Usiskin, 1982, p. 5). There are five levels of the Van Hiele level theory for understanding geometry, called the Existence of levels. The brief descriptions of the levels summarized by Yazdani (2007) are as follows:

- Level 0, Visualization: Students see geometric figures as a whole, squares and rectangles seem to be different, but they do not identify the properties of figures as at the next level.

- Level 1, Analysis: Students can identify figures through their properties, their features, and characteristics like rectangles have four right angles even though they do not understand the interrelationship between different types of figures.

- Level 2, Informal Deduction (Order): Students can understand and use definitions. They can make simple deductions and to follow formal proofs but still don't understand the significance of working in an axiomatic system, and they are not able to construct proofs meaningfully.

- Level 3, Deduction: Students can construct proofs as a way of developing geometry theory. They can understand and use the interrelationship among undefined terms, definition, axiom/postulate, theorem, and proof. However, at this level, they are still unable to work in a variety of axiomatic systems.

- Level 4, Rigor: Students understand logical and geometrical methods, and can work in a variety of axiomatic systems. Students can appreciate the historical discovery of non-Euclidean geometry and understand how a multitude of distinctly different geometries can exist, be developed, investigated, and used, and they can also appreciate how a particular geometry (e.g., Euclidean) can be studied from different perspectives with different methods (synthetic, analytic, and transformational).

According to the Van Hiele levels theory, as summarized by Usiskin (1982), in understanding geometry, a person must go through the levels in order; this is known as the fixed sequence property of the levels. One of the level properties, says: “A student cannot be at Van Hiele level n without having gone through level n-1 (Usiskin, 1982, p. 5). “In this context, Van Hiele geometry thinking levels is one of the most popular theoretical frameworks to understand students’ learning process” (Karakuş & Peker, 2015, p. 338). According to Kutluca (2013), as quoted by Karukus and Peker (2015), it had been “determined that activities in which Geogebra dynamic geometry software was used were more effective for 11^{th}-grade Van Hiele thinking levels compared to traditional teaching methods.” (p. 340). It meant that using DGS (dynamic geometry software) such as Geogebra for learning geometry was better than using paper-pencil tools (traditional tools/non-DGS).

The Van Hiele levels theory explicitly explains that teaching and learning mathematics must use visualization ability, analysis, and deductive reasoning. Because of that, the theory can be used for developing the theoretical framework of the relationship among learning three-dimensional geometry, development of spatial visualization skills, and mathematical problem-solving skills. It seems that the Van Hiele levels theory focuses on logical-and-deductive reasoning for achieving a higher level of mathematical thinking that accords to the NCTM's standards. The reasoning is a foundation of mathematical competence, and so reasoning habits can be a productive way of thinking mathematically.

Mathematical problem-solving will be impossible to do without reasoning and sense-making. "Research and evaluation show that technology tools … support the development of higher-order thinking. The findings hold … when students are taught to apply the processes of problem-solving and then are allowed opportunities to apply technology tools to develop solutions" (Cradler, McNabb, Freeman, & Burchett, 2002, p. 48). Also, "research results suggest that elementary and middle school students can improve their understandings of geometry concepts while using DGS such as Cabri geometry (IMAG-CNRS Universite Joseph Fourier, 1998) and Geometer's Sketchpad (Jackiw, 1991; 2002)." (Clements, Sarama, Yelland, & Glass, 2008, p. 127). According to Ertekin (2014), Karakus and Peker (2015), and Kosa (2016), three-dimensional thinking including Van Hiele levels have special significance in spatial reasoning, mathematics performance, and the theoretical framework used to understand the learning process.

Therefore, through the development of geometrical reasoning with Cabri 3D, students' spatial skills could be developed and improved, and then the spatial skills could enhance the students' problem-solving skills. The geometrical approach using Cabri 3D could be one viable way to bring mathematic processes to life. Mathematical problem-solving consists of some steps e. g., understanding the problem, identifying the known and the unknown in a problem, and translating the known and the unknown information to mathematical language. For measuring mathematical problem-solving skills in this experimental research, we used the TEA test. TEA is a mathematics standard test from the Texas Education Agency.

Reasoning as the foundation of the mathematical problem-solving process is deductive reasoning. Deductive reasoning is a logically connected sequence of accepted truths such as axioms, theorems, definitions, and mathematical conventions against a mathematical statement. On the other side, the problem-solving process in the context of a dynamic geometry environment is just an empirical justification. Empirical justification is inductive, non-conclusive evidence for the truth of a mathematical statement (Stylianides & Silver, 2004). However, empirical justification and learning mathematics inductively by explorations and making conjectures are important activities in experimental mathematics that can make mathematics more tangible, lively, and interesting.

Learning mathematics such as learning three-dimensional geometry inductively with Cabri 3D could help students deeply understand the mathematical problem before formal problem-solving with deductive reasoning is done. Therefore, problem-solving skills are the skills used for deductive reasoning and visualizing problem formulation to solve any mathematical problem. Madden and Diaz (2008) said that: "The visualization problem … requires bridging the span from the knowledge that is intuited to the knowledge that must be obtained through logical reasoning" (p. 130). Intuition is what humans use in their life to make a decision, and logic can help humans to make the right decisions and answers in the process of problem-solving.

Visualization problems in high school students should be solved by using educational technology such as a computer with DGS. In a technological environment, students will have a great opportunity to visualize abstraction activities because multiple actions can be easily performed when using a computer (Hollebrands, 2007).

Implications

Problem-solving is one of NCTM's mathematics processes. Because of that, high school students must be able to independently or with the help of their teacher remedy and hone their mathematical problem-solving ability or skills through systematically learning three-dimensional geometry with DGS.

Learning transformational three-dimensional geometry might help students hone and enhance their spatial visualization skills. The necessity of this study program will make three-dimensional geometry the most important subject in schools and necessitates the school administrator to prepare potent learning technology like Cabri 3D.

In the context of learning and practicing three-dimensional geometry with potent educational technology, Cabri 3D has been equipped with tools for constructing, rotating, and viewing objects in three-dimensional space that are impossible to manipulate with traditional tools (non-DGS). As a consequence, the expectation of a high increase in students' mathematical problem-solving skills must be focused on systematically developing and enhancing spatial visualization skills with Cabri 3D.

Summary

Piaget's theory of Constructivism has impacted learning theories and teaching methods in education and many education reform movements. Piaget-Inhelder's theory states that the stages of geometrical development can be related to students' spatial development. The brain-based theory is a learning theory based on the structure and function of the brain, and the Multiple-Intelligences theory states that people understand the world and resolve any problems they face with intelligence.

Then, according to the Van Hiele levels theory, high school students have developed levels of visualizations, analysis, and informal deduction in where students can construct proofs as a way of developing geometric theory. They can understand and use the interrelationship among undefined terms, definitions, axioms/postulates, theorems, and proofs. However, at this level, they are still unable to work in a variety of axiomatic systems. The Van Hiele level theory covers the developments of spatial visualization skills, geometrical reasoning, and mathematical problem-solving skills that are needed for developing the theoretical framework of this dissertation research.

All learning theories above mention that the learning process and mathematical problem-solving need the development of spatial visualization and logical reasoning. In education, the levels of development are commonly classified according to students' ages. Therefore, further investigations are necessary to determine the impact of using the most recent technological tools, like computers and software, on the development levels. Many types of research on spatial visualization and mathematical problem-solving only say that spatial visualization is an important factor in solving daily problems.

So far, the researchers still have not investigated how to increase mathematical problem-solving skills through enhancing spatial visualization skills in dynamic geometry environment, and also the research did not investigate the relationship between the effects of learning three-dimensional dynamic geometry with Cabri 3D and the increase of mathematical problem-solving skills that are mediated by spatial visualization skills.

Therefore, there is a reason to logically develop a theoretical framework for investigating the interaction among three components, namely the enhancement of spatial visualization skills, the use of Cabri 3D in learning and practicing three-dimensional geometry, and the increase of mathematical problem-solving skills. Spatial visualization skills will be a logical prerequisite to successful mathematical problem-solving because problem-solving is the process of logical-deductive thinking that requires spatial skills as a component of cognitive ability. Instructions and training with Cabri 3D on a three-dimensional geometry course ought to motivate and trigger students' spatial reasoning in the context of logical-deductive reasoning. Logical-deductive reasoning is the standard of thinking rule in mathematics processes including mathematical problem-solving.

Hence, the implications as written above will allow us to investigate the indirect relationship between achievement in learning three-dimensional geometry with Cabri 3D and mathematical problem-solving skills mediated by spatial visualization skills. In other words, the implications will allow us to investigate the increase of mathematical problem-solving skills through developing and enhancing spatial visualization skills with DGS and non-DGS.

Theoretical Framework

The theoretical framework protocol of this research study is shown in Figure 2.7, namely the protocol of SVS and MPS development. The framework for increasing the

SVS-MPS and mediation relationship can also be seen in Figure 3.2 in the research design section in Chapter 3.

Spatial visualization ability and mathematical problem-solving ability are considered to be a product of genetic potency and aptitude from birth, and they can develop through daily activities in the geometrical world. However, simultaneously, spatial visualization skills and mathematical problem-solving skills can also be enhanced and increased by systematically learning and practicing three-dimensional geometry and professional experiences in geometrical development. The protocol for developing spatial visualization skills and mathematical problem-solving skills in this research study is derived by doing explorations, making conjectures, and verifications.

Through explorations, students will get more experience and knowledge. The protocol, as shown in Figure 2.7, explains that the skill box and ability box are connected with the learning theory of enhancing and increasing SVS and MPS in the ellipse box. The interconnections between the two boxes (ability and skill) occur in activities of exploration, making conjectures, and verifications in the spatial visualization and mathematical problem-solving processes.

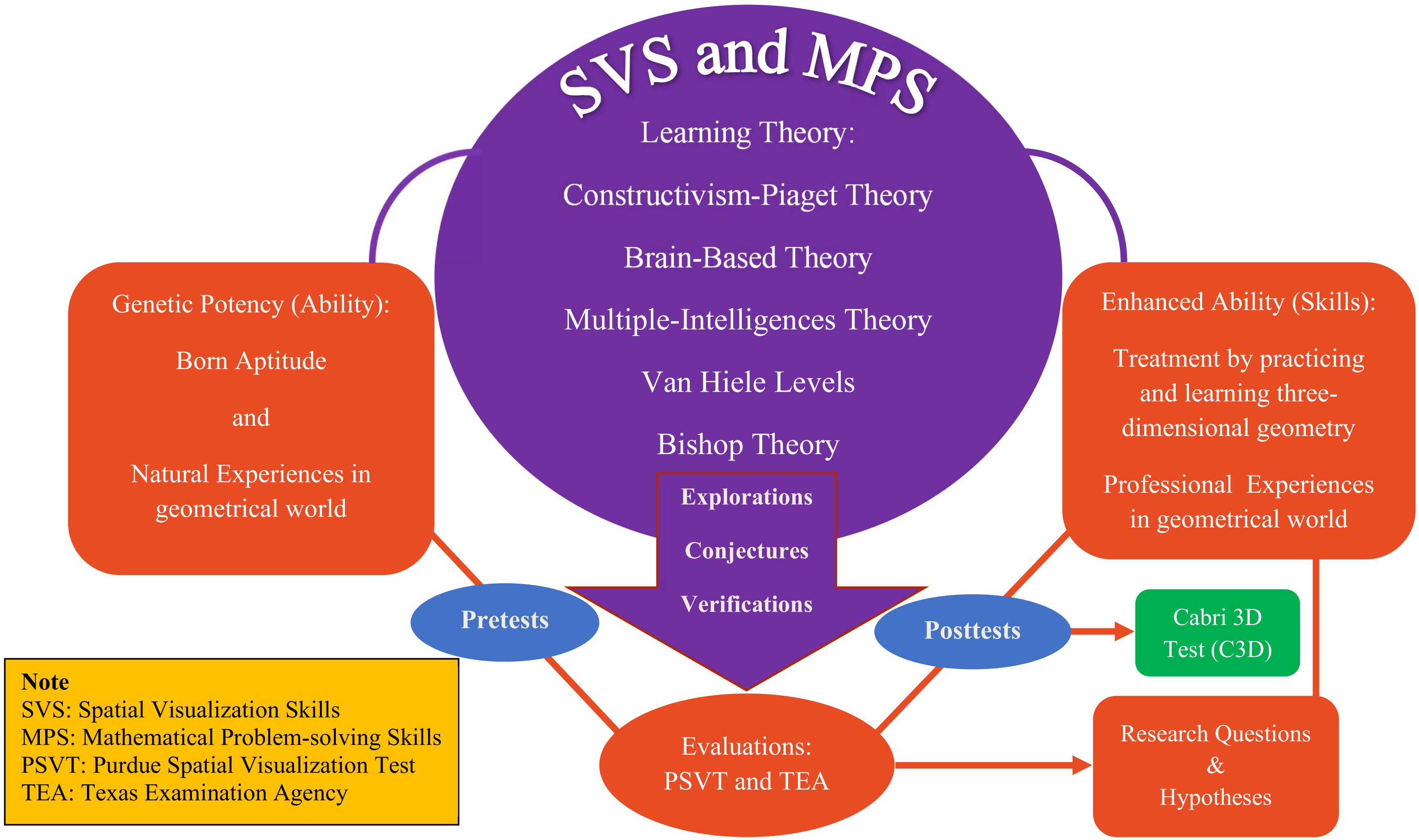

Figure 2.7 The protocol of SVS and MPS developments

The logical foundation of this experimental research is to solve the problem by using the steps of the scientific method:

- Finding a problem
- Asking questions
- Forming hypotheses
- Conducting an experimental research
- Analyzing data
- Drawing an inference

The problem in this research study is how to effectively increase senior high school students' mathematical problem-solving skills through developing and enhancing their spatial visualization skills by learning and practicing three-dimensional geometry. And which tool (Cabri 3D or traditional tools) is more effective for developing and enhancing spatial visualization skills. The primary research question of this study is to what extent does the development of spatial visualization skills by learning and practicing three-dimensional geometry with Cabri 3D affect the increase of mathematical problem-solving skills? In this context, there are three related variables namely:

- Achievement of learning and practicing three-dimensional geometry with Cabri 3D
- Development and enhancement of spatial visualization skills
- Increase of mathematical problem-solving skills.

Assumption and expectation:

- Spatial visualization skills could be a mediator variable (an intervening variable) between the causal relationship of achievement in learning-and-practicing three-dimensional geometry and mathematical problem-solving skills.

- The relationship between spatial visualization skills and mathematical problem-solving skills.

- Cabri 3D will be an effective tool for learning and practicing three-dimensional geometry in the context of developing and enhancing spatial visualization skills.

Logical structure:

- Experimental research with a control group and a treatment group for investigating which tool (Cabri 3D or traditional tools) is more effective for learning and practicing three-dimensional geometry in the context of developing and enhancing spatial visualization skills.

- Regression analysis of the relationship between spatial visualization skills and mathematical problem-solving skills after treatment.

- Causal mediation relationship between the achievement of learning and practicing three-dimensional geometry with Cabri 3D and the increase of mathematical problem-solving skills mediated by spatial visualization skills after treatment.

Key hypotheses are the relationship between spatial visualization skills and mathematical problem-solving skills, and the achievement of learning and practicing three-dimensional geometry with Cabri 3D indirectly affect the increase of mathematical problem-solving skills mediated by spatial visualization skills. The results of the tests of hypotheses are for answering the research question and sub-questions.

The key concepts of this research study are the relationship between spatial visualization skills and mathematical problem-solving skills, Cabri 3D is an effective tool for learning and practicing three-dimensional geometry in the context of developing and enhancing spatial visualization skills. The relationship between the concepts of spatial training and mathematics achievement in Bishop’s theory (1980). Referring to the process of mathematics in NCTM (2009) concerning the use of educational technology for supporting mathematical problem-solving activities. The definition of the key concepts is the increase of mathematical problem-solving skills requires well-developed spatial visualization skills, and spatial visualization skills might be developed and enhanced by learning and practicing three-dimensional geometry with Cabri 3D.

In the process of mathematical problem-solving, problem-solvers need to have three-dimensional thinking skills, and understanding all mathematical complex concepts requires well-developed three-dimensional thinking skills (Kosa, 2016). To deeply understand the complex structure of problems, problem-solvers need to visualize the parts and structures of the problems in mind. Ertekin (2014) points out that three-dimensional thinking has special significance in spatial reasoning, and Polya’s theory defines that the process of mathematical problem-solving as follows: “understanding the problem, devise [making] a plan, carrying out the plan and looking back.” (Zhu, 2007; Alacantara & Bacsa, 2017, p. 23).

The theoretical relevance of this research study has been made in the learning theories as described below:

- Constructivism Learning Theory. Constructivism is a theory of learning that argues that humans generate knowledge and meaning from the interaction between their experiences and ideas. Cabri 3D is an interactive learning tool that can give students a wider range of opportunities and learning activities for making explorations, conjectures, and verifications. These activities will generate new experiences for students and will result in an interaction between students' experiences and ideas, primarily in developing and enhancing spatial visualization skills. Knowledge from the interaction between experiences and ideas, sometimes, doesn't represent the facts, then for accepting and developing the knowledge logically, we need brain-based learning theory, multiple intelligences theory, and Van Hiele level theory.

- Brain-Based Learning Theory. Brain-based learning theory mentions that spatial ability is a form of memory and learning will occur with the brain as a parallel processor. Someone needs strong spatial reasoning and logical thinking to solve mathematical problems. Spatial reasoning and logical thinking relate to the theory of multiple intelligences developed by Howard Gardner.

- Multiple-Intelligences Theory. Two types of intelligence from the theory of multiple-intelligences that relate to this experimental research are visual-spatial intelligence and logical-mathematical intelligence. Hence, the constructivism theory with explorations can be the base of this research study in developing and enhancing spatial visualization skills. Brain-based theory and the two types of multiple-intelligence theory can support learning and practicing three-dimensional geometry with Cabri 3D in the context of developing and enhancing spatial visualization skills and increasing mathematical problem-solving skills.

- Van Hiele Levels Theory. Senior high school students' mathematical thinking can be classified at level 2. At this level, students have developed levels of visualizations, analysis, and informal deduction. Students can construct proofs as a way of developing geometric theory as the foundation for developing and increasing mathematical problem-solving ability and skills.

Technically, Cabri 3D is considered a more potent tool than traditional tools because Cabri 3D has dynamic tools that allow for extensive explorations; traditional tools do not include these dynamic tools. By doing explorations with Cabri 3D, students can find new experiences and knowledge, and they can also develop and enhance their spatial visualization skills through the exploration process.

The activities of making explorations, conjectures, and verifications in three-dimensional geometry can develop a relationship between the learning theories above. After doing explorations, students must make conjectures. In making conjectures, the students must apply logical-mathematical thinking as in Brain-based learning theory and Multiple-intelligences theory. In this step, students learn to make a formulation of mathematical problem-solving.

Afterward, the students must verify their conjectures. In this process, students develop informal deduction as in Van Hiele level 2. This process will increase their mathematical problem-solving skills. Based on Bishop's theory, if the Treatment Group gets a significantly better result in the SVS test than the Control Group, then the Treatment Group will get a significantly better result in the MPS test than the Control Group, or the outcome will at least be the same.

There is no one learning theory concerning the direct relationship between both the achievements of learning geometry with Cabri 3D and mathematical problem-solving skills. Bishop's theory just stated that spatial training with geometry might improve spatial visualization and mathematics achievement. Thus, it will be impossible to theorize that Cabri 3D will directly affect mathematical problem-solving skills without an intervening variable.

Therefore, it will be necessary to find its intervening variable by investigating the indirect relationship between both components. In this research study, I would expect that the independent variable is students' achievement in learning and practicing three-dimensional dynamic geometry with Cabri 3D, the dependent variable is mathematical problem-solving skills, and spatial visualization skills as its intervening variable.

Theoretical Implications

Developing mathematical skills should emphasize and develop primary abilities such as spatial ability instead of just teaching mathematics (Kayhan, 2005; Bishop, 1980). To increase students' mathematical problem-solving skills, teachers need to focus on enhancing their students' spatial visualization skills by motivating them to practice and learn three-dimensional geometry with Cabri 3D.

Logically, if students' achievement in learning Cabri 3D can develop and enhance their spatial visualization skills, and if spatial visualization skills can carry effects to mathematical problem-solving skills, then we could conclude that students' achievement in learning and practicing Cabri 3D can indirectly affect the increase of mathematical problem-solving skills that are mediated by their spatial visualization skills

Emphasizing the learning of three-dimensional geometry with Cabri 3D will help senior high school students easily make developments, rotations, and views in three-dimensional space. "These viewing exercises also allow for students in the Piagetian hypothetical-deductive fourth stage of cognitive development to gain what Piaget terms logical-mathematical experience in the exercise of making the mental transformations required to understand the provided external representation, …." (Allam, 2009, p. 19)

Those transformations continue affecting students in their final cognitive stage of development, as well as in their projective spatial abilities. Generally, spatial visualization ability is determined by genetics, life experiences, and developmental activity. Spatial visualization abilities can be improved through practice or further experiences in transforming object orientations in space, and it will be more effective to use educational technology like Cabri 3D.

Cabri 3D will be helpful for students in doing explorations, making conjectures, and doing verifications in terms of enhancing spatial visualization skills and increasing mathematical problem-solving skills.

Van Hiele's theory has five levels of geometrical development, namely level 0 to level 4. In this theory, senior high school students considered to be at level 2 (Informal deduction level). At this level, students still cannot construct proof meaningfully when the proof process demands a higher level of cognitive learning and mathematical-deductive reasoning as the formal mathematical thinking rule. However, the students can follow and understand their teacher's explanation of the mathematical proof, but they still cannot do the process without a teacher's guidance.

One of the fixed sequence properties of Van Hiele's theory is "Separation" that says that persons at a different level cannot understand each other. So, in making the teaching plans, mathematics teachers must know at what Van Hiele level their students now. If now the students at level n, then they must have passed through their level (n-1). This property warns mathematics teachers to know their students' current cognitive level at Van Hiele level so that their students can understand their teacher's explanation and mathematical problem-solving process.

In this research study, students must have passed through level 1 (Analysis) of the Van Hiele level and understand transformational concepts as well as basic operations of Cabri 3D, before they can use Cabri 3D for learning and practicing object transformation on a computer screen.

CHAPTER 3

METHODOLOGY, DATA COLLECTION, AND ANALYSES

Spatial visualization skills can be related to mathematical problem-solving skills. This relationship emerges as the hypothesis of students' mathematical problem-solving skills might be increased by increasing students' spatial visualization skills. From there, mathematical problem-solving skills could be directly or indirectly increased by learning and practicing three-dimensional geometry with dynamic geometry software (DGS) in the context of developing and enhancing spatial visualization skills. The name of the dynamic geometry software used in this study was Cabri 3D.

Cabri 3D is a potent tool that can help students learn and practice three-dimensional geometry. Students can animate, manipulate, and create objects using Cabri 3D. These tasks are difficult to do with traditional tools like paper-pencil, rulers, protractors, and compass. Transformational geometry such as rotations, reflections, translations, and dilations will also be easier to do using Cabri 3D versus traditional tools. The ability to learn and practice transformational geometry is the key to successful training in developing and increasing spatial visualization skills. Training in this study was experimental research with pretests and posttests in the within-subjects research design.

Pretests and posttests scores from this experimental research will be used in data analyses because the test scores can be indicative of the success in developing and increasing both spatial visualization skills and mathematical problem-solving skills. Furthermore, mediation analysis will be applied for testing mediation relationships between students' achievement in learning and practicing Cabri 3D (C3D) and mathematical problem-solving skills (MPS) that are mediated by spatial visualization skills (SVS).

Participants

Participants for this experimental research consisted of two groups that randomly selected from the population of 300 eleventh-grade students at one of the public high schools in Jombang, Jawa Timur, Indonesia. Generally, the students in this school still use paper-and-pencil tools for learning three-dimensional geometry.

The total number of samples or participants in this experimental research determined by GPower's a priori calculation. This calculation considered the following cases:

- Level of statistical significance or alpha.
- Amount of power desired in the experiment for statistical test on the null hypothesis.
- Effect size and expected differences (in the means) between the control group and the treatment group was expressed in the standard deviation unit.

For this experimental research, researcher set the values for the three factors as follows: alpha = .05, power = .80, and effect size = .50. "In this way, the experiment is planned so that the size of each treatment group provides the greatest sensitivity that the effect on the outcome … is due to the experimental manipulation in the study" (Creswell, 2009, p. 157). According to GPower's a priori calculation, for finding the values of the three factors, the minimum total number of participants for this experiment was 36, namely: 18 participants for the Control Group and 18 participants for the Treatment Group.

When randomly selecting participants from a population of 300 eleventh grade students, the researcher got a total of 87 voluntary participants, namely 28 participants for Control Group (Group 1) and 28 participants for Treatment Group (Group 2). In total, 56 participants voluntarily joined in the experimentation. The remainder of 31 participants voluntarily joined in the pilot project for testing the reliability and validity of instruments (PSVT and TEA/MPS test), and 2 (two) of them (from the remainder) are also willing to join in the pilot project (total: 30 or 28+2 participants) for testing the reliability and validity of the C3D test as explained below.

The 28 participants in Group 2 also joined in the pilot project for testing the reliability and validity of the C3D (Cabri 3D test). The C3D test was administered after the participants got a short-term basic course of Cabri 3D. The basic course of Cabri 3D took place about one month before they got a tutorial program or treatment for developing spatial visualization skills with Cabri 3D.

Variables

There are three variables in this research study namely: independent variable, dependent variable, and mediator or intervening variable that were related to the three models of statistical analyses as written below.

Model 1: Repeated measure and mixed repeated measure of ANOVA

- *Independent variable:* Groups (Group 1 and Group 2)

- *Dependent variable:* Pretest and posttest of SVS/PSVT (Purdue Spatial Visualization Tests)

Model 2: Multiple-regression based mediation analysis

- *Independent variables (X)*: C3D (Achievement of learning and practicing three-dimensional geometry with Cabri 3D)
- *Dependent variable (Y)*: MPS (Mathematical Problem-Solving skills)
- *Mediator variable (M)*: SVS (Spatial Visualization Skills)

Model 3: Simple regression analysis

- *Independent variable:* SVS (Spatial Visualization Skills)
- *Dependent variable:* MPS (Mathematical Problem-Solving Skills)

Instruments

Instruments in this experimental research are the testing tools for measuring spatial visualization skills, mathematical problem-solving skills, and achievement of learning and practicing three-dimensional geometry with Cabri 3D. The tests of PSVT, TEA-MPS, and C3D were as shown respectively in Appendix A, Appendix B, and Appendix C.

PSVT (Purdue Spatial Visualization Test) was a testing tool for measuring spatial visualization skills. Roland Guay designed this test in 1976 and still popular up now. This test consisted of three parts, namely: developments, rotations, and views.

> Developments test consists of 12 questions designed to see how well subjects can visualize the folding of 2D objects into 3D objects. Rotations test consists of 12 questions designed to see how well subjects can visualize the rotation of 3D objects, and views test consists of 12 questions designed to see how well subjects can

visualize what 3D objects look like from various viewing positions. There are also three separate 30-item test booklets: one each for developments, rotations, and views. The tests are suitable for use with subjects ages 13 or older (Guay, 1976, p. 1).

Thus, PSVT consisted of 12 questions of developments, 12 questions of rotations, and 12 questions of views. The time allotted for solving a problem was 1 minute (60 seconds).

TEA-2003 (Texas Education Agency – 2003) was a testing tool for measuring students' mathematical problem-solving skills. The TEA test consisted of 20 problems. Because the TEA test focused on measuring students' skills and strategy in solving mathematical problems, the time allotted for answering these problems was strictly limited – approximately one minute (60 seconds) per problem.

Then, C3D was a testing tool for measuring students' achievement in learning and practicing three-dimensional geometry with Cabri 3D. In this test, students had to solve 11 problems. The C3D test adapted from the Cabri 3D User's Manual, and it needed a computer with installed Cabri 3D software. The maximum time allotted for solving the 12 problems in the C3D test was one hour (60 minutes).

All the tests above were the multiple-choices tests with no penalty for wrong answers. The scores range was from 0 to 100, and students got scores from the calculation of correct answers only. The formula for calculating the total scores of each test was as follows:

Total scores = (Correct answers : Total problems) x 100

Total scores = Correct answers divided by total problems, multiplied by 100.

Although the instruments (testing tools) originated from the standard tests, the researcher still needed to retest the reliability and validity of each tool as a pilot project in the population area where the researcher would select the participants. The retesting results of the reliability and validity of the instruments (testing tools) will be presented in the next sections.

Reliability

Reliability refers to the consistency of a measure. Internal consistency reliability is used to judge the consistency of results across items on the same test. Essentially, we are comparing test items that measure the same construct to determine the tests internal consistency

PSVT has been well-known and has high reliability for measuring spatial ability. The PSVT has the K-R_{20} reliability of 0.80 and is a valid measure of spatial ability as well as is deemed a good test (Yue & Chen, 2001). Although PSVT had high reliability, the researcher still needed to retest its reliability in the population where the experimental research would be conducted. In this pilot project, 31 participants followed the reliability test for PSVT, and the other 30 participants followed the reliability tests for TEA and C3D.

As mentioned above, participants in the C3D reliability test had to take a one-month basic course of using Cabri 3D for learning and practicing three-dimensional geometry before administration of the test, and then if they wanted to continue study in the experimental research, they were only permitted to join as participants in the Treatment Group.

Internal Reliability Test

Results of the internal reliability tests for PSVT, TEA, and C3D determined by the Cronbach alpha for each test variable. The variable considered to be reliable if the variable has Cronbach alpha greater than 0.70 (Ghozali, 2011, pp. 47-51). For finding these results, these reliability test results were calculated by SPSS. Table 3.1 below presents a summary of these reliability test results.

Table 3.1 Results of Reliability Tests

Variable	*Alpha*	Criteria
Data-C3D_Pilot	0,770	Reliable
Data-MPS_Pilot	0,859	Reliable
Data-PSVT_Pilot	0,918	Reliable

The reliability test results in the table above indicate that the three alphas of all variables are greater than 0.70. Hence, we conclude that the three instruments (testing tools) are reliable and appropriate for research instruments in this experimental research.

Validity

Validity is the extent to which a test measures what it claims to measure. A test needs to be valid for the results to be accurately applied and interpreted. Validity isn't determined by a single statistic, but by a body of research that demonstrates the relationship between the test and the behavior it is intended to measure. When a test has content validity, the items on the test represent the entire range of possible items the test should cover.

Participants for these validity tests were the same as participants for the reliability tests above.

Validity Tests

Validity tests applied here were for knowing an instrument validity in acquiring data. The formulas for deciding this validity tests are:

1. If $r_{calculation} > r_{table}$ at the significance level of 5%, then items in the instruments are valid.
2. If $r_{calculation} < r_{table}$ at the significance level of 5%, then items in the instruments are not valid (Widiyanto, 2012, pp. 34-37). These validity tests used the Pearson correlation formula in SPSS. Table 3.2, Table 3.3, and Table 3.4 below present the summaries of these validity test results.

Table 3.2 Validity Tests for Variables of Data-C3D_Pilot _Pilot

Items	$r_{calculation}$	r_{table} 5% (30)	Criteria
1	0,570	0,361	Valid
2	0,571	0,361	Valid
3	0,516	0,361	Valid
4	0,545	0,361	Valid
5	0,509	0,361	Valid
6	0,569	0,361	Valid
7	0,546	0,361	Valid
8	0,571	0,361	Valid
9	0,535	0,361	Valid
10	0,635	0,361	Valid
11	0,520	0,361	Valid

Table 3.3 Validity Tests for Variables of Data-MPS_Pilot

Items	$r_{calculation}$	r_{table} 5% (30)	Criteria
1	0,497	0,361	Valid
2	0,438	0,361	Valid
3	0,529	0,361	Valid
4	0,578	0,361	Valid
5	0,575	0,361	Valid
6	0,491	0,361	Valid
7	0,528	0,361	Valid
8	0,593	0,361	Valid
9	0,461	0,361	Valid
10	0,685	0,361	Valid
11	0,520	0,361	Valid
12	0,530	0,361	Valid
13	0,506	0,361	Valid
14	0,716	0,361	Valid
15	0,454	0,361	Valid
16	0,497	0,361	Valid
17	0,528	0,361	Valid
18	0,478	0,361	Valid
19	0,515	0,361	Valid
20	0,538	0,361	Valid

Table 3.4 Validity Tests for Variables of Data-PSVT_Pilot

Items	objects	r_{table} 5% (31)	Criteria
1	0,419	0,355	Valid
2	0,440	0,355	Valid
3	0,506	0,355	Valid
4	0,469	0,355	Valid

5	0,537	0,355	Valid
6	0,684	0,355	Valid
7	0,561	0,355	Valid
8	0,437	0,355	Valid
9	0,701	0,355	Valid
10	0,431	0,355	Valid
11	0,490	0,355	Valid
12	0,425	0,355	Valid
13	0,406	0,355	Valid
14	0,631	0,355	Valid
15	0,418	0,355	Valid
16	0,622	0,355	Valid
17	0,672	0,355	Valid
18	0,492	0,355	Valid
19	0,567	0,355	Valid
20	0,422	0,355	Valid
21	0,464	0,355	Valid
22	0,447	0,355	Valid
23	0,504	0,355	Valid
24	0,442	0,355	Valid
25	0,663	0,355	Valid
26	0,417	0,355	Valid
27	0,477	0,355	Valid
28	0,412	0,355	Valid
29	0,513	0,355	Valid
30	0,693	0,355	Valid
31	0,595	0,355	Valid
32	0,511	0,355	Valid
33	0,486	0,355	Valid
34	0,507	0,355	Valid

35	0,449	0,355	Valid
36	0,435	0,355	Valid

All results of validity tests as presented in the tables above indicate that $r_{calculation}$ is greater than r_{tabel} at a significance level of 5%. Thus, we can conclude that all items in these research instruments (testing tools) are valid and appropriate for collecting data, especially in this population.

Treatments

There were two groups in this experimental research, namely: Group 1 (Control Group) and Group 2 (Treatment Group). Group 1 used traditional tools (non-DGS) in learning and practicing three-dimensional geometry and Group 2 used Cabri 3D (DGS) in learning and practicing three-dimensional geometry.

Control Group's Tutorial Program

Tutorial objective: Students can develop, improve, and enhance their spatial visualization skills by learning and practicing three-dimensional geometry with paper-pencil tools (non-DGS)

Activity objectives: Students can construct and develop various 3D objects, and views; students can make basic transformational geometry, such as rotations, reflections, translations, and dilations; students can make the intersection of objects; students can calculate distance and length, area, and volume; students can make animation and trajectory, and explorations of 3D objects in the XYZ-axes coordinate system on the paper.

Time: 90 minutes for each meeting.

Tools: paper, pencil, ruler, compass, and protractor.

Table 3.5 Schedule of Activities for Control Group

MEETING	COURSE CONTENTS
	Pretest administration.
1st	Introduction to paper-pencil tools and the use of them for forming basic 3D geometry objects such as sphere, cylinder, cube, box, pyramid, prism, cone; and views.
2nd	Naming objects; construction of an articulated dodecahedron (i.e. one that folds and unfolds), and activities with a cube net.
3rd	Cube intersection by a plane perpendicular to one of the cube diagonals and make the same plane as a plane shown on the diagonal.
4th	Making circles and ellipses, and obtaining a circle and an ellipse with the help of cone intersections.
5th	Learning transformational geometry in the 3D environment e.g., rotations, translations, dilations, reflections, and symmetry.
6th	Calculating the length, area, and volume of objects.
7th	Forming various objects by cutting prism on various surfaces.
8th	Making 3D objects intersected with their surfaces and obtaining intersection curves.
9th	Problem-solving in 3D objects using Pythagorean theorems such as measuring edges, diagonals, areas (surfaces), and volumes.
10th	Making animation and trajectory*

11th	Visualization of developments, rotations, and views with the help of paper-pencil tools*
12th	Free exercise (developments, rotations, and views). Posttest administration.

*Cannot be done effectively due to the limitations of the tools' capabilities

The Control Group got the pretest and posttest of PSVT and TEA. The pretest of PSVT and TEA was for getting data of students' spatial visualization skills and mathematical problem-solving skills before treatment with traditional tools (non-DGS), and the posttest of PSVT and TEA were for getting data of students' spatial visualization skills and mathematical problem-solving skills after treatment. During the treatment, the Control Group received the learning and practicing of three-dimensional geometry with traditional tools (non-DGS). Because of the limitations of features of the tools compared to DGS, students in this group had difficulty in making animation, trajectory, rotations, and explorations of 3D objects in the XYZ-axes coordinate system on the papers.

Treatment Group's Tutorial Program

Tutorial objectives: Students can develop, improve, and enhance their spatial visualization skills by learning and practicing three-dimensional geometry with Cabri 3D.

Activity objectives: Students can construct and develop various 3D objects as well as views; students can make basic transformational geometry, such as rotations, reflections, translations, and dilations; students can make the intersection of objects; students can calculate distance and length, area, and volume; students can make animation and trajectory, and explorations in the XYZ-axes coordinate system on a computer screen.

Time**:** 90 minutes for each meeting.

Tools: Computers and Cabri 3D software.

Table 3.6 Schedule of Activities for Treatment Group

MEETING	COURSE CONTENTS
	Pretest administration.
1st	Introduction to the tools of Cabri 3D and forming basic geometry objects such as sphere, cylinder, cube, box, pyramid, prism, cone; and views.
2nd	Naming objects and creating labels; construction of an articulated dodecahedron (i.e. one that folds and unfolds), and activities with a cube net.
3rd	Cube intersection by a plane perpendicular to one of the cube diagonals; slide the plane along the diagonal.
4th	Making circles and ellipses and obtaining circle and ellipse with the help of cone intersections.
5th	Learning transformational geometry in the 3D environment e.g., rotations, translations, dilations, reflections, and symmetry.
6th	Measuring length, area, and volume of objects including advanced use of calculator in Cabri 3D.
7th	Forming various objects by cutting prism on various surfaces.
8th	Making the objects intersected with surfaces and obtaining intersection curves.

9th	Using measure tool and Pythagorean theorem for problem-solving in 3D-objects such as measuring diagonals, angles, areas (surfaces), and volume.
10th	Animation and trajectory*
11th	Advanced use of trajectory tool and visualization with the help of Cabri 3D*
12th	Free exercise (developments, rotations, and views)
	Posttest administration

*Can be done effectively

Treatment Group got the same pretests and posttests as Control Group but plus the C3D test. The pretests of PSVT and TEA were for getting data of students' spatial visualization skills and mathematical problem-solving skills before treatment with Cabri 3D (DGS), and the posttests of PSVT and TEA were for getting data of students' spatial visualization skills and mathematical problem-solving skills after treatment. The third posttest (C3D) for the Treatment Group was for getting data of students' achievements in the use of Cabri 3D for learning and practicing transformational geometry, developments, animations and trajectory, and explorations in the XYZ-axes coordinate system on the computer screen.

Experimental Procedure

This experimental research could be considered as a true experimental design, a traditional or classical experimental design, in which the procedure involved the random assignment of one class (28 students) as participants to a control group said as Group 1 and the random assignment of another class (28 students) as participants to a treatment group said as Group 2. Both groups received pretests and posttests as scheduled.

The illustration of the experimental procedure was as follows:

Pretest-Posttest Control Group Design

Group-1 R-------------O----------------------------O

Group-2 R-------------O--------------X-------------O

R: Random assignment.

O: Observation or measurement

X: Treatment.

Threats to Validity

There are two types of potential threats to validity, namely: internal threats to validity and external threats to validity. Internal validity is an inductive estimate of the degree to which conclusions about cause-effect relationships are likely to be true, because of the measures used, the research setting, and the whole research design. Then, external validity measures to what extent one may safely generalize the internally valid cause-effect inference from the sample studied to the defined target population and (or) to other populations.

Internal threats to validity are "experimental procedure, treatments, or experiences of the participants that threaten the researcher's ability to draw correct inferences from the data about the population in an experiment" (Creswell, 2009, p. 162). These threats can be seen in Creswell's table (Table 3.7) which provides a description of the threats and offers suggestions for the researcher in response to the threats so that the threats are less likely to occur.

Table 3.7 Threats to Internal Validity and Actions to Minimize the Threats

Type of Threat to Internal Validity	Description of Threat	In Response, Actions the Researcher Can Take
History	During an experiment, events can occur that unduly influence the outcome beyond the experimental treatment.	Experimenters must have both the treatment and control groups experience the same external events.
Maturation	Participants in an experiment may mature or change during the experiment. Thus, influencing the results.	Experimenters will choose participants from the same level (ages) namely the eleventh-grade students (at the high school).
Regression	Participants with extreme scores are selected for the experiment. Naturally, their scores will probably change during the experiment. Scores, over time, regress toward the mean.	Random assignments to a treatment group and a control group from many students (classes) at the same grade will reduce the threat to this validity.
Selection	Participants can be selected who have certain characteristics that predispose them to have	Experimenters will choose participants randomly so that participants' characteristics have the

	certain outcomes (e.g., they are brighter).	same probability distribution among the experimental groups.
Mortality	Participants drop out during an experiment due to many possible reasons. The outcomes are thus unknown to these individuals.	Experimenters will make a large sample (120 participants/four classes) to account for dropouts or compare those who drop out with those who continue, in terms of the outcome.
Diffusion of treatments	Participants in the control and treatment groups communicate with each other. This communication can influence how both groups score on the outcomes.	Experimenters will keep the two groups as separate as possible during the experiment and forbid the control group from using the research facilities.
Compensatory/resentful demoralization	The benefits of an experiment may be unequal or resented when only the treatment group receives the treatment (e.g., treatment group receives therapy and the control group receives nothing).	Experimenters will provide benefits to both groups that are to give the control group the same treatment (learning Cabri 3D) after the experiment ends.
Compensatory rivalry	Participants in the control group feel that they are being devalued, as	Experimenters will take steps to reduce the expectation of the control

	compared to the treatment group, because they don't experience the treatment.	group, and will create equal conditions between the two groups, e.g.: learning 3D geometry with PPE
Testing	Participants become familiar with the outcome measure and remember responses for later testing.	The researcher won't permit participants to have copies of tests and will give them a long interval between the pretest and posttest.
Instrumentation	The instrument changes between a pretest and a posttest, thus impacting the scores on the outcome.	Experimenters will use the same instrument for the pretest and posttest.

SOURCE: Adapted from Creswell (2009)

Then, external threats to validity must be identified, and preventive ways must be designed or created to prevent or minimize these threats. External threats to validity "arise when experimenters draw incorrect inferences from the sample data to other persons, other settings, and past or future situations" (Creswell, 2009, p. 162). Such threats arise due to the characteristics of individuals in the sample, characteristics of the participant's setting, and the timing or history of the experiment. The Creswell's table (Table 3.8) describes the external threats to validity and offers suggestions for the researcher in response to the threats so that the threats are less likely to occur.

Table 3.8 Threats to External Validity and Actions to Minimize the Threats

Types of Threats to External Validity	Description	In Response, Actions the Researcher Can Take

Interaction of selection and treatment	Because of the narrow characteristics of participants in the experiment, the researcher cannot generalize to individuals who do not have the characteristics of participants.	Experimenters will take participants from a public high school in Indonesia because the public school has a variety of student characteristics.
Interaction of setting and treatment	Because of the characteristics of the participants setting in an experiment, a researcher cannot generalize to individuals in other settings.	Experimenters will take participants randomly from 10 classes of 11^{th} grade.
Interaction of history and treatment	Because the results of an experiment are time-bound, a researcher cannot generalize the results to future or post-situations.	Experimenters need to replicate the study at later times to determine if the same results occur as in the earlier time.

SOURCE: Adapted from Creswell (2009)

Other threats to validity are threats to statistical conclusion validity and threats to construct validity. Threats to statistical conclusion validity arise when the researcher draws an inaccurate conclusion caused by inadequate statistical power or the violation of statistical assumptions (Creswell, 2009). And the threats to construct validity will arise when the researcher uses inadequate definitions and measures of variables (Creswell, 2009).

Procedure

This experimental research can be categorized as international research because its location and participants were in Indonesia (not in the USA). Because of that, regulations and requirements in researching an institution in this country had to be firstly understood.

There were several steps that researcher did for and during research at a public senior high school in Indonesia:

Step 1: Determining research location and place, and participants at one of the senior high schools (in Jombang city, East Java, Indonesia) that had many eleventh-grade students and had not used dynamic geometry software (Cabri 3D) in their learning of 3D geometry.

Step 2: Making agreement and cooperation with the senior high school's headmaster for a research project in a dynamic geometry environment and Cabri 3D training for the eleventh-grade students at his senior high school.

Step 3: Training two mathematics teachers from the senior high school to be experimenters in the three-dimensional geometry environment, one teacher with Cabri 3D (DGS), and another teacher with traditional tools (non-DGS).

Step 4: Selecting 56 participants randomly from eleventh-grade students at the senior high school consisted of 28 students as the control group and the other 28 students as the treatment group.

Step 5: Administering the pretest of PSVT and TEA for both the control group and the treatment group.

Step 6: Making the schedules of training as shown in Table 3.5 and Table 3.6.

Step 7: Administering posttests for both the control group and the treatment group with PSVT and TEA, and C3D tests as an additional test for the treatment group.

Step 8: Processing of statistical analyses. All data acquired from the results of students' pretests and posttests were analyzed by Mixed repeated measures of ANOVA, Regression, and Hayes' PROCESS in SPSS. The Mixed repeated measures of ANOVA and Regression were for analyzing the relationships of data of PSVT and TEA. And the Hayes' PROCESS was for analyzing data of the post-tests of C3D, PSVT, and TEA in the mediation relationship between students' achievement in learning and practicing Cabri 3D and mathematical problem-solving skills mediated by spatial visualization skills.

Research Design

This research design can be categorized as a within-subjects research design and mediation relationship. The advantages of within-subjects research design among other things are the statistical power of research outcome and the possibility of a reduction in error variance associated with individual differences. In this case, the principle of fundamental inferential statistics indicates that as the number of subjects increases then its statistical power could increase, and error variance associated with individual differences could decrease. Hence, it is always better to have more subjects, and also a significance table such as the t-table that shows that as the number of subjects increases then the t-value for statistical significance decreases.

There were three steps of analyses applied for finding results about which tool was better for developing and enhancing high school students' spatial visualizations skills and increasing their mathematical problem-solving skills.

- Mixed repeated measures of ANOVA-1 and ANOVA-2.

  Mixed repeated measures of ANOVA-1 were for comparing experimental results between the control group and the treatment group. These steps were for knowing whether or not Cabri 3D was better than traditional tools for developing and enhancing senior high school students' spatial visualization skills. These analysis results were for drawing inference and answering questions about to what extent the enhancement of senior high school students' spatial visualization skills was due to treatment.

  Mixed repeated measures of ANOVA-2 were for analyzing whether there was or was not a significantly different effect in mathematical problem-solving skills between the control group and the treatment group due to treatment. These analysis results were for drawing inference and answering questions about to what extent the increase of senior high school students' mathematical problem-solving skills was due to treatment.

- Regression analysis and Mediation analysis by PROCESS.

The first analysis was for analyzing a direct relationship between senior high school students' spatial visualization skills and their mathematical problem-solving skills. And the second analysis was for analyzing a mediation relationship between the effects of senior high school students' achievement in learning-and-practicing three-dimensional geometry with Cabri 3D and their mathematical problem-solving skills mediated by spatial visualization skills. In other words, the achievement of learning and practicing three-dimensional geometry with Cabri 3D (Independent Variable) could enhance spatial visualization skills (Mediator or Intervening Variable), and then the enhanced spatial visualization skills could increase senior high school students' mathematical problem-solving skills (Dependent Variable). If this causal mediation relationship significantly occurred, then this analysis would indicate that there was an indirect relationship between C3D and MPS mediated by SVS. These analysis results were for drawing inference and answering questions about to what extent an indirect effect of C3D's senior high school students to their MPS was due to treatment.

Figure 3.1 shows an experimental research model for developing and enhancing students' SVS. Treatment on both groups means that Control Group (Group 1) used traditional tools and Treatment Group (Group 2) used Cabri 3D software.

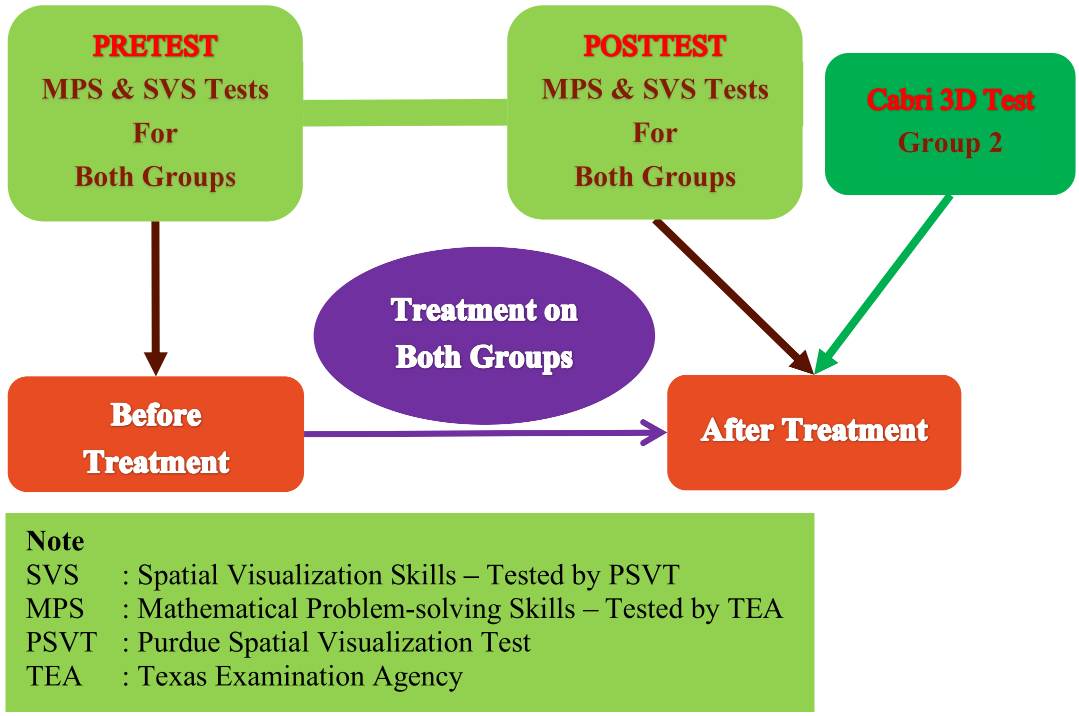

Figure 3.1 Tests of SVS and MPS before and after treatment

This research design is completed with the protocol as shown in Figure 3.2. This protocol describes the way of the solution to analyzing and testing the hypotheses of the relationship between SVS and MPS and then the mediation relationship between C3D and MPS mediated by SVS.

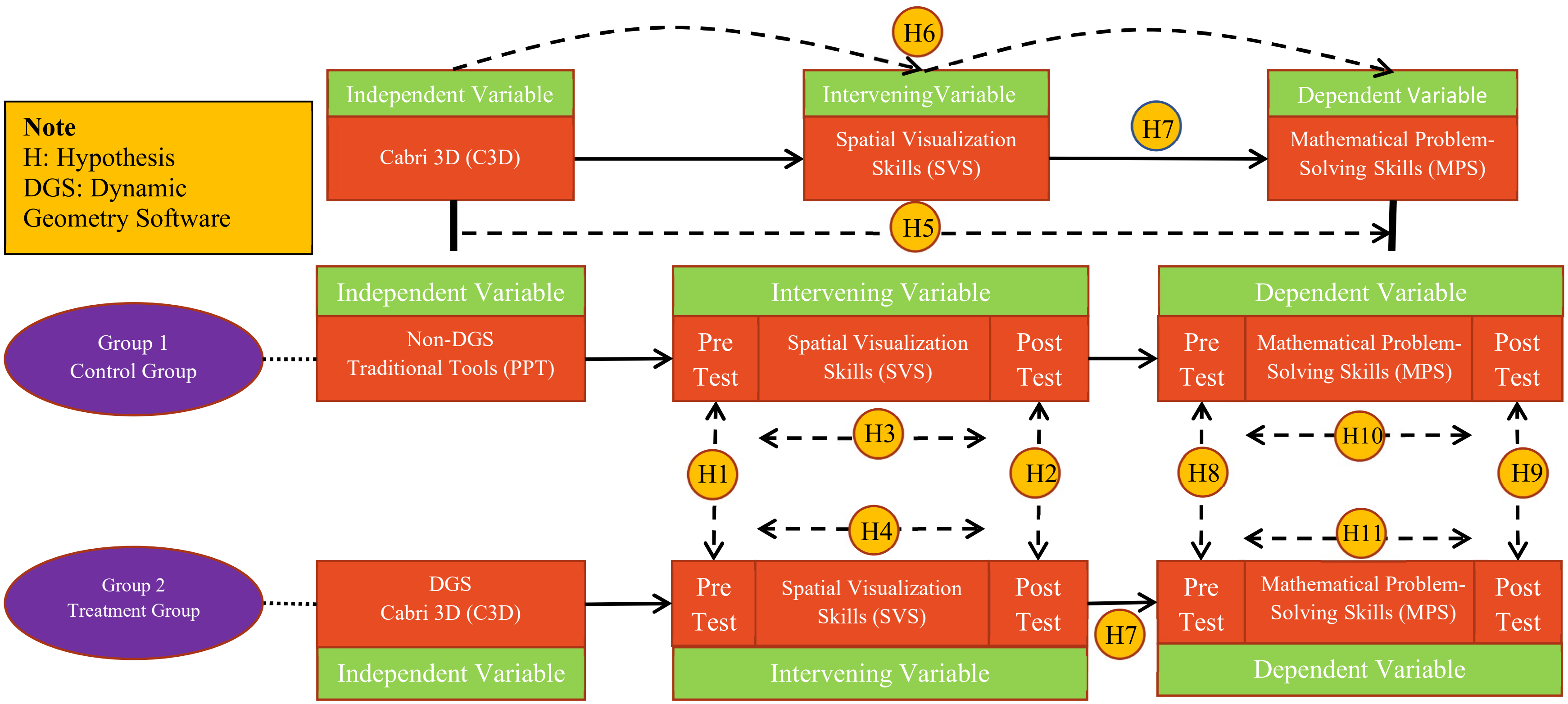

Figure 3.2 The protocol of enhancing SVS-MPS and mediation relationship

In the dynamic geometry environment, students' ability in using Cabri 3D for constructing objects, doing explorations, transformations, and animations on the computer screen is very important in the context of developing and enhancing spatial visualization skills. Hence, in this research study we needed:

- Data of Cabri 3D test from Treatment Group concerning students' achievement in using Cabri 3D (C3D) for learning and practicing three-dimensional geometry on computers.

- Data of pretests and posttests on spatial visualization skills and mathematical problem-solving skills from both Control Group and Treatment Group for analyzing and testing 8 hypotheses in the steps as follows. Analysis of these data with Mixed repeated measures of ANOVA is for analyzing and testing hypotheses H1, H2, H3, H4, H8, H9, H10, and H11. The test of hypothesis H2 is for knowing treatment effects on SVS between the Control Group and the Treatment Group. The tests of hypotheses H3 and H4 are for knowing treatment effects on SVS within the Control Group and within the Treatment Group. The test of hypothesis H9 is for knowing treatment effects on MPS between the Control Group and the Treatment Group. Then, the tests of hypotheses H10 and H11 are for knowing treatment effects on MPS within the Control Group and within the Treatment Group.

- Regression analysis of data on the Treatment Group's posttests of spatial visualization skills and mathematical problem-solving skills is for analyzing and testing hypothesis H7. The test of hypothesis H7 is for knowing there is or no relationship between SVS and MPS after treatments by Cabri 3D.

- Data on the Cabri 3D test and the posttests of spatial visualization skills and mathematical problem-solving skills from Treatment Group are for mediation analyses with PROCESS and testing hypotheses H5 and H6. The tests of H5 and H6 are for knowing whether or not there is an indirect effect after treatment between C3D and MPS mediated by SVS.

Sample and A Priori Calculations

According to Cohen: the effect size (power) of 0.02 would be small, 0.15 would be medium, and 0.40 would be large. And according to the one rule of thumb, if one wants to claim a complete mediation, the ratio of the indirect effect (ab) to the direct effect (c') or ab/c' should be at least .80.

The sample size could determine the effect size of the experiment result. In this research study, the researcher used GPower software for determining its sample size. GPower's a priori analysis as shown below indicated that for finding an effect size of 0.25, power of 0.95, and a significance level of 0.05 researcher needed a sample size of at least 18 participants for each group. Then, for being in harmony with the protocol of power analysis below, each group of this experiment consisted of 28 participants, e.g.: 28 participants were for the control group and 28 participants were for the treatment group.

The protocol of Power Analyses:

F tests - ANOVA: Repeated measures, within factors

Analysis: A priori: Compute the required sample size

Input: Effect size f = 0.25

	α err prob	=	0.05
	Power (1-β err prob)	=	0.95
	Number of groups	=	2
	Number of measurements	=	4
	Corr among rep measures	=	0.5
	Nonsphericity correction ε	=	1
Output:	Noncentrality parameter λ	=	18.0000000
	Critical F	=	2.6937209
	Numerator df	=	3.0000000
	Denominator df	=	102
	Total sample size	=	36

Actual power = 0.9517650

Central and noncentral distributions:

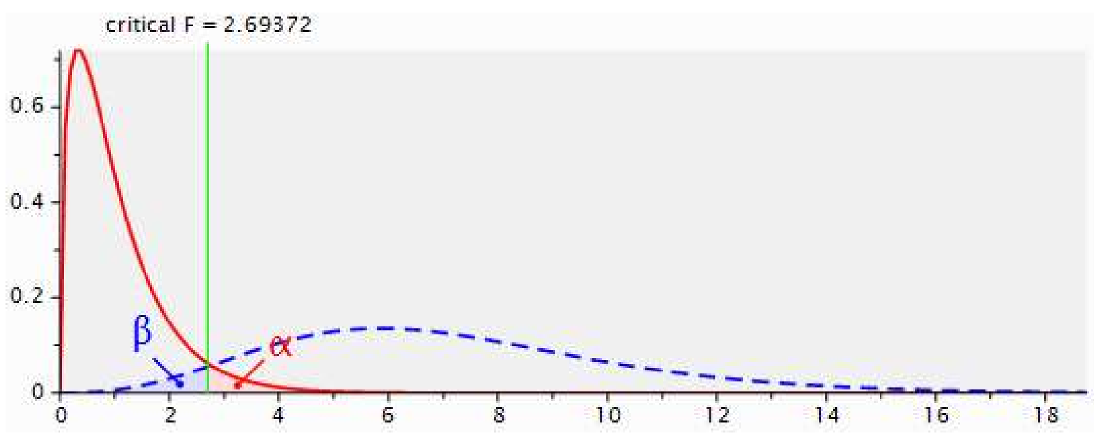

Figure 3.3 Central and noncentral distributions from GPower

Underlying Considerations

This experimental research is not only for collecting data concerning which tool is more effective for enhancing senior high school students' spatial visualization skills but also for investigating whether or not there is a causal relationship between students' achievement in learning and practicing three-dimensional geometry with Cabri 3D and mathematical problem-solving skills that are mediated by their spatial visualization skills. In other words, this study is for investigating whether or not learning and practicing three-dimensional geometry with Cabri 3D could indirectly affect the increase of students' mathematical problem-solving skills through their enhanced spatial visualization skills.

Data Collections Procedure

Primary data in this study were acquired from the results of pretests and posttests as scheduled in the program activities of the control group and the treatment group. Hence, the primary data collections were the results of pretests and posttests of PSVT (Spatial Visualization Skills), TEA (Mathematical Problem-solving Skills), and C3D (Cabri 3D achievement test). The results of pretests and posttests of PSVT from both groups used for testing which tool was better (Cabri 3D or paper-pencil tools) for developing and enhancing spatial visualization skills. And the results of pretests and posttests of TEA were for analyzing and testing the effects of treatments on the increase of mathematical problem-solving skills. Then, the results of the posttests of PSVT, TEA, and C3D were for mediation analysis.

Tool's Effects on Spatial Visualization Skills and Mathematical Problem-solving Skills

The sequence of this research study to be explained as follows: This research study investigated the direct effects of the uses of Cabri 3D and paper-pencil tools on students' spatial visualization skills. Then, this study investigated the treatment effects on the increase of mathematical problem-solving skills and the relationship between spatial visualization skills and mathematical problem-solving skills due to treatment with Cabri 3D. Furthermore, this research study focused on the investigation of the indirect effects of the use of Cabri 3D on the increase of mathematical problem-solving skills through developing and enhancing spatial visualization skills. The effects of using the tools on the control and treatment groups calculated by Mixed repeated measures of ANOVA.

The preparation for and steps in using Mixed repeated measures of ANOVA were as follows:

- Wrote the null and alternative hypotheses:

  H_0: $\mu_1 = \mu_2$

  H_1: $\mu_1 \neq \mu_2$ (where μ_1 were the means of pretest of PSVT/SVS and MPS, and μ_2 were the means of posttests of PSVT/SVS and MPS that the control and the treatment groups had).

- Determined if this test was a one-tailed or two-tailed. Because the hypotheses involved the phrase "different" and there was no ordering of the means specified, the Mixed repeated measures of ANOVA had to be a two-tailed test.

- Specified the α level: $\alpha = 0.05$

- Determined the appropriate statistical test. Since the standard deviation of the population was not known, the z-test would be inappropriate, hence we determined that the appropriate statistical test in this within-subjects design was Mixed repeated measures of ANOVA.

- Let SPSS calculated the p-value for the pretests and posttests of spatial visualization skills (PSVT/SVS) and mathematical problem-solving skills (MPS) from control and treatment groups.

Regression for Mediation Effect Model

The causal mediation relationship can be explained as follows: a variable to be considered as a mediator (M) determined by the extent to which it carries the influence of a given independent variable (X) to a given dependent variable (Y), where: X = The achievement of learning and practicing three-dimensional Geometry with Cabri 3D, M = Spatial Visualization Skills, and Y = Mathematical Problem-solving Skills.

Three conditions need to be established in this case to determine whether the mediation has occurred:

- The independent variable (X) predicts the dependent variable (Y)
- The independent variable (X) predicts the mediator (M)
- The mediator (M) predicts the dependent variable (Y)

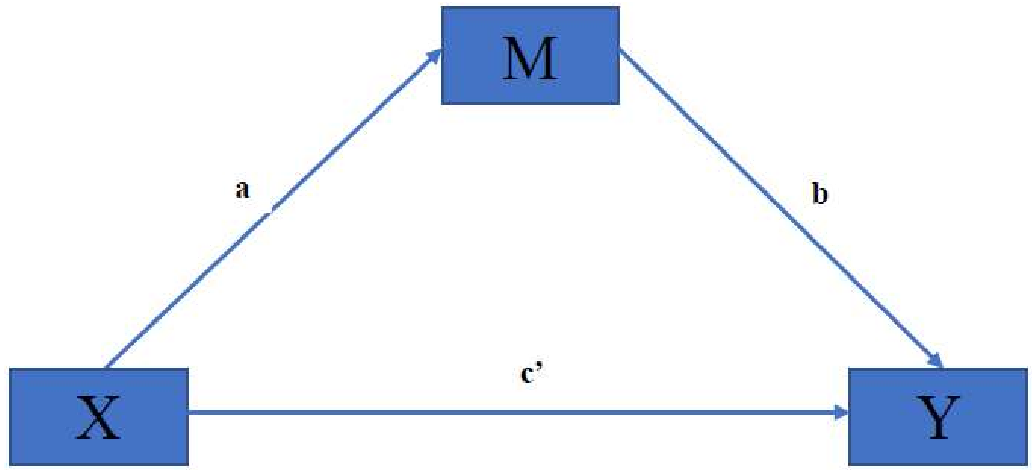

Figure 3.4 Causal mediation relationship between X and Y mediated by M

Figure 3.4 above shows that paths a and b are the regression coefficients of X and M and the indirect effect ab is the product of paths between regression coefficients of X and M. Indirect effect measures the two cases that differ by one unit on X are estimated to differ by ab units on Y as a result of the effect of X on M which, in turn, affects on Y. The direct effect is the coefficient c’, c’ estimates the direct effect of X on Y. The direct effect is that two cases that differ by one unit on X but equal on M are estimated to differ by c’ units on Y. Then, the total effect c measures how much two cases that differ by one unit on X are estimated to differ on Y. In this linear regression, the total effect is equal to the sum of the direct effect and indirect effect, namely c = c’ + ab. (Hayes, 2013, p. 92-93). The direct and indirect effects of the Cabri 3D software on the mediation effects model were calculated by the Bootstrapping method and Hayes’ PROCESS as mentioned above.

Research Questions and Hypotheses

This research study has six research questions including its sub-questions and eleven hypotheses associated with models and variables in this experimentation. The research questions and the hypotheses are as follows:

Research Questions

This research study has one primary research question and five sub-questions as described below:

Primary research question:

"To what extent does the development of spatial visualization skills by learning and practicing three-dimensional geometry with Cabri 3D affect the increase of mathematical problem-solving skills?"

Sub questions:

- Sub-question #1: Is Cabri 3D (DGS) better than traditional tools (non-DGS) for learning and practicing three-dimensional geometry in the context of developing and enhancing spatial visualization skills?
- Sub-question #2: Is there a direct relationship between the achievement of learning-and-practicing three-dimensional geometry with Cabri 3D and mathematical problem-solving skills?

- Sub-question #3: Is there an indirect relationship between the achievement of learning-and-practicing three-dimensional geometry with Cabri 3D and mathematical problem-solving skills mediated by spatial visualization skills?

- Sub-question #4: Is there a relationship between spatial visualization skills and mathematical problem-solving skills?

- Sub-question #5: Is there an increase in mathematical problem-solving skills after treatment?

Hypotheses

For investigating the effects of using traditional tools (non-DGS) in the Control Group and using Cabri 3D (DGS) in the Treatment Group on high school students' SVS and MPS, and for investigating mediation relationship between high school students' achievement in learning-and-practicing three-dimensional geometry with Cabri 3D and the students' MPS mediated by their SVS, the researcher needed to test eleven hypotheses

1. Null hypothesis H_0: $\mu_1 = \mu_2$: There is no significant difference in the results of pretests of SVS between Group 1 and Group 2; and its alternative hypothesis is H_1: $\mu_1 \neq \mu_2$ (where μ_1 and μ_2 are the means of the SVS tests that the control and treatment groups have).

2. Null hypothesis H_0: $\mu_1 = \mu_2$: There is no significant difference in the results of posttests of SVS between Group 1 and Group 2; and its alternative hypothesis is

H_1: $\mu_1 \neq \mu_2$ (where μ_1 and μ_2 are the means of the SVS tests that the control and treatment groups have).

3. Null hypothesis H_0: $\mu_1 = \mu_2$: There is no significant difference before and after treatment using traditional tools on high school students' SVS within Group 1; and its alternative hypothesis is H_1: $\mu_1 \neq \mu_2$ (where μ_1 and μ_2 are the means of the pretest and posttest of SVS that the control group has).

4. Null hypothesis H_0: $\mu_1 = \mu_2$: There is no significant difference before and after treatment using Cabri 3D on high school students' SVS within Group 2; and its alternative hypothesis is H_1: $\mu_1 \neq \mu_2$ (where μ_1 and μ_2 are the means of the pretest and posttest of SVS that the treatment group has).

5. Null hypothesis H_0: $c' = 0$

   There is no direct effect from C3D to MPS independence of SVS.

   Alternative H_a: $c' \neq 0$

6. Null hypothesis H_0: $c' = 0$

   There is no indirect effect from C3D to the MPS dependence of SVS.

   Alternative H_a: $c' \neq 0$

7. Null hypothesis H_0: There is no relationship between high school students' SVS and their MPS.

8. Null hypothesis H_0: $\mu_1 = \mu_2$: There is no significant difference in the results of pretests of MPS between Group 1 and Group 2; and its alternative hypothesis is H_1: $\mu_1 \neq \mu_2$ (where μ_1 and μ_2 are the means of MPS tests that the control and treatment groups have).

9. Null hypothesis H_0: $\mu_1 = \mu_2$: There is no significant difference in the results of posttests of MPS between Group 1 and Group 2; and its alternative hypothesis is H_1: $\mu_1 \neq \mu_2$ (where μ_1 and μ_2 are the means of MPS tests that the control and treatment groups have).

10. Null hypothesis H_0: $\mu_1 = \mu_2$: There is no significant difference before and after treatment with traditional tools on high school students' MPS within Group 1; and its alternative hypothesis is H_1: $\mu_1 \neq \mu_2$ (where μ_1 and μ_2 are the means of pretest and posttest of MPS that the control group has).

11. Null hypothesis H_0: $\mu_1 = \mu_2$: There is no significant difference before and after treatment with Cabri 3D on high school students' MPS within Group 2; and its alternative hypothesis is H_1: $\mu_1 \neq \mu_2$ (where μ_1 and μ_2 are the means of pretest and posttest of MPS that the treatment group has).

For being able to answer the primary research question, at first, we need to find the answers to its five sub-questions as formulated below:

- Sub-question #1: For answering this question, we need to know the results of the tests of hypotheses H1, H2, H3, and H4.

- Sub-question #2: For answering this question, we need to know the result of the test of hypothesis H5.

- Sub-question #3: For answering this question, we need to know the result of the test of hypothesis H6.

- Sub-question #4: For answering this question, we need to know the result of the test of hypothesis H7.

- Sub-question #5: For answering this question, we need to know the results of the tests of hypotheses H8, H9, H10, and H11.

Analyses

For analyzing data obtained from this experimental research, the researcher used Mixed repeated measures of ANOVA and PROCESS as explained in the two sections below. Mixed repeated measures of ANOVA is the name of the modified syntax of SVS and MPS for finding detailed research results in SPSS. And PROCESS is the name of Hayes's multi-regression based mediation analysis embedded in SPSS.

Test for Within-Subjects Research Design with Mixed Repeated Measures of ANOVA

Repeated measures analyses are for comparing the acquired data from pretests and posttests on the control group and the treatment group. Original Repeated measures of ANOVA cannot give detailed outcomes in the within-subjects and between-subjects research design for both groups with two pretests and two posttests. Because of that, for getting detailed outcomes, we need to modify its syntax as shown below. The modified syntax for analyses in this chapter is named Mixed Repeated Measures of ANOVA.

Original syntax
SVS

```
GLM Pretest Posttest BY Group
/WSFACTOR=Time 2 Polynomial
/MEASURE=SVS
/METHOD=SSTYPE(3)
/POSTHOC=Group(TUKEY)
/PLOT=PROFILE(Time*Group)
/EMMEANS=TABLES(Time) COMPARE ADJ(BONFERRONI)
/EMMEANS=TABLES(Group*Time)
/PRINT=DESCRIPTIVE ETASQ OPOWER HOMOGENEITY
/CRITERIA=ALPHA(.05)
/WSDESIGN=Time
/DESIGN=Group
```

Modified syntax
SVS

```
GLM Pretest Posttest BY Group
  /WSFACTOR=Time 2 Polynomial
  /MEASURE=SVS
  /METHOD=SSTYPE(3)
  /POSTHOC=Group(TUKEY)
  /PLOT=PROFILE(Time*Group)
  /EMMEANS=TABLES(Time) COMPARE ADJ(BONFERRONI)
  /EMMEANS=TABLES(Group*Time) COMPARE (Group) ADJ(BONFERRONI)
  /EMMEANS=TABLES(Group*Time) COMPARE (Time) ADJ(BONFERRONI)
  /PRINT=DESCRIPTIVE ETASQ OPOWER HOMOGENEITY
  /CRITERIA=ALPHA(.05)
  /WSDESIGN=Time
  /DESIGN=Group
```

Original syntax
MPS

```
GLM Pretest Posttest BY Group
/WSFACTOR=Time 2 Polynomial
/MEASURE=MPS
/METHOD=SSTYPE(3)
/POSTHOC=Group(TUKEY)
/PLOT=PROFILE(Time*Group)
/EMMEANS=TABLES(Time) COMPARE ADJ(BONFERRONI)
/EMMEANS=TABLES(Group*Time)
/PRINT=DESCRIPTIVE ETASQ OPOWER HOMOGENEITY
/CRITERIA=ALPHA(.05)
/WSDESIGN=Time
/DESIGN=Group
```

Modified syntax
MPS

```
GLM Pretest Posttest BY Group
  /WSFACTOR=Time 2 Polynomial
  /MEASURE=MPS
  /METHOD=SSTYPE(3)
  /POSTHOC=Group(TUKEY)
  /PLOT=PROFILE(Time*Group)
  /EMMEANS=TABLES(Time) COMPARE ADJ(BONFERRONI)
  /EMMEANS=TABLES(Group*Time) COMPARE (Group) ADJ(BONFERRONI)
  /EMMEANS=TABLES(Group*Time) COMPARE (Time) ADJ(BONFERRONI)
  /PRINT=DESCRIPTIVE ETASQ OPOWER HOMOGENEITY
  /CRITERIA=ALPHA(.05)
  /WSDESIGN=Time
  /DESIGN=Group
```

Testing for Mediation

PROCESS command embedded in SPSS is the mediation analyses for testing whether or not learning and practicing three-dimensional geometry with Cabri 3D affect Spatial Visualization Skills, and in turn, the Spatial Visualization Skills will affect Mathematical Problem-Solving Skills. PROCESS command and linear regression in SPSS then be used for further analyses in this chapter.

Sobel product of coefficients approach:

In the Sobel approach, we make two models as shown and depicted below. Model-1 depicts the multiple regression effects for M and X predicting Y, d_1 is its intercept, and c’ and b are the regression coefficients of X and M respectively. And model-2 involves the relationship between X and M, d_2 is its intercept, and a is the regression coefficient of X. A product is formed by multiplying two coefficients together

Model 1:

$$\begin{tikzcd} X \arrow[rr, bend left, "c'"] & M \arrow[r, "b"'] & Y \end{tikzcd}$$

$$Y = d_1 + c'X + bM + e_y$$

Model 2:

$$X \xrightarrow{a} M$$

$$M = d_2 + aX + e_m$$

Indirect effect = ab

Direct effect = c'

Total effect = c' + ab

- Null hypothesis H_0: c' = 0
- There is no evidence that achievement in learning and practicing three-dimensional geometry with Cabri 3D (C3D) directly affects students' mathematical problem-solving skills (MPS).

This mediation test is for testing whether or not there is a causal relationship between learning and practicing three-dimensional geometry with Cabri 3D (X) and mathematical problem-solving skills (Y) mediated by Spatial Visualization Skills (M). Mediation analysis with bootstrap confidence interval in the causal relationship is for analyzing its data and testing its H_0 (Null hypothesis). A confidence interval is an estimate of the range of upper and lower statistical values that are consistent with the observed data and are likely to contain the actual population mean (Creswell, 2009).

This test also considers its effect size. The effect size will identify the strength of conclusions about group differences or the relationship among variables in this research (Creswell, 2009). For these purposes, Hayes offers a macro (http://www.afhayes.com) that calculates bootstrapping directly within SPSS, and a computer program named PROCESS for statistical analyses. From PROCESS, we also can see p-value and a 95% confidence interval. If the direct effect is not statistically different from zero, then the Null hypothesis cannot be rejected. Also, if "this confidence interval does include zero, so zero cannot be confidently ruled out as plausible value for the direct effect." (Hayes, 2013, p. 101).

According to Cohen: the effect sizes (powers) of 0.02 would be small, 0.15 would be medium, and 0.40 would be large. And concerning the ratio of the indirect effect (ab) to the direct effect (c'), the rule of thumb states that, if one wants to claim a complete mediation, ab/c' should be at least .80. Then, another list of the effect size measures is Preacher and Kelly's Kappa Squared, called as index "Kappa-Squared" (κ^2) and defined as "the ratio of the indirect effect relative to its maximum value in the data …, κ^2 is bound between 0 and 1, with a value closer to 1 representing a bigger indirect effect." (Hayes, 2013, p. 191).

PROCESS Command must be embedded in SPSS with the following steps:
Open http://www.afhayes.com

1. Click Introduction to Mediation, Moderation, and Conditional Process
2. Scroll down to PROCESS for SPSS
3. Download "processv211.zip"
4. Extract all files

5. Follow installation process
   - Open SPSS as Administrator by right-clicking on the SPSS shortcut, then click Run as Administrator
   - Install PROCESS by clicking on Utilities -> Custom Dialogs -> Install Custom Dialog
   - Select the PROCESS Spd file and click open.

<u>Tool Effectiveness and Limitations</u>

Observations in this experimental research found that learning transformational geometry with Cabri 3D (DGS) was more effective than with traditional tools (PPE/non-DGS). The following figures are some examples of using Cabri 3D for learning transformational geometry.

Rotation in View

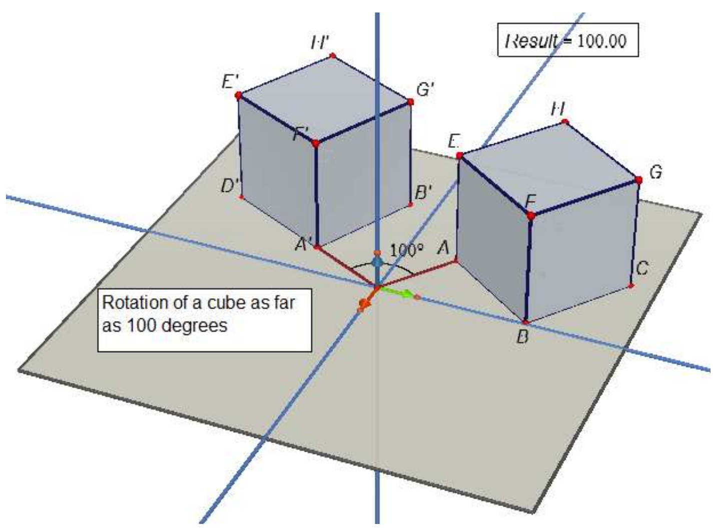

Figure 3.5 Rotation in View using Cabri 3D

Rotation in View from the top

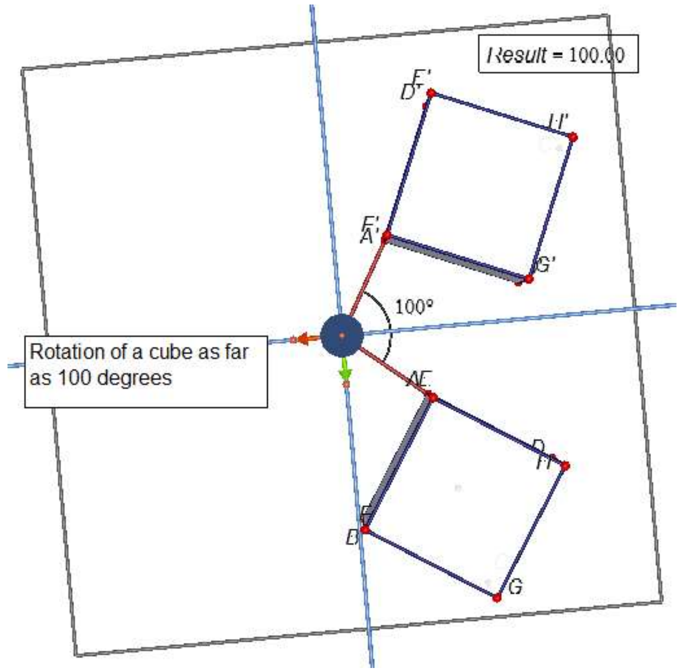

Figure 3.6 Rotation in View from the top using Cabri 3D

The figures above show how Cabri 3D can effectively rotate a cube in three-dimensional space around the z-axis as far as 100° that are very difficult to construct and rotate with traditional tools. Cabri 3D has effective tools for constructing three-dimensional objects, making animations, trajectories, reflections, translations, dilations, and rotations in space. These effective tools have made Cabri 3D powerful software for resolving the limitations of traditional tools' features and capabilities for interactively learning and practicing three-dimensional dynamic geometry.

CHAPTER 4

DATA ANALYSES AND RESULTS

This chapter will present two analyses with Mixed repeated measures of ANOVA for analyzing the effects of treatments on spatial visualization skills and the effects of treatments on mathematical problem-solving skills, and then, one mediation analysis with Hayes' PROCESS for analyzing the effects of spatial visualization skills as an intervening variable in the relationship between the achievement of learning and practicing three-dimensional geometry with Cabri 3D as the independent variable and mathematical problem-solving skills as the dependent variable.

Data Collections

There are three groups of data collections from experimental research. The first group of data as presented in Table 4.1 is for analyzing and determining which tool is significantly better for developing and enhancing high school students' spatial visualization skills, the second group of data as presented in Table 4.2 is for analyzing whether or not spatial visualization skills can significantly mediate the relationship of students' achievement in learning-and-practicing three-dimensional geometry with Cabri 3D and mathematical problem-solving skills, then the third group of data as presented in Table 4.3 is for analyzing the increase of students' mathematical problem-solving skills due to treatments.

Table 4.1 Data of Pretest-Posttest on Spatial Visualization Skills

N	Group 1 – Control Group		N	Group 2 – Treatment Group	
	Pretest of PSVT	Posttest of PSVT		Pretest of PSVT	Posttest of PSVT
1	64.00	69.00	1	55.00	92.00
2	67.00	78.00	2	58.00	69.00
3	52.00	52.00	3	27.00	52.00
4	69.00	87.00	4	62.00	80.00
5	80.00	89.00	5	23.00	56.00
6	73.00	81.00	6	64.00	89.00
7	55.00	42.00	7	36.00	78.00
8	55.00	61.00	8	61.00	84.00
9	52.00	65.00	9	47.00	58.00
10	84.00	90.00	10	30.00	61.00
11	23.00	36.00	11	50.00	73.00
12	67.00	78.00	12	67.00	74.00
13	55.00	84.00	13	36.00	59.00
14	72.00	69.00	14	36.00	53.00
15	47.00	42.00	15	50.00	64.00
16	67.00	61.00	16	28.00	66.00
17	36.00	42.00	17	25.00	28.00
18	33.00	39.00	18	61.00	75.00
19	64.00	84.00	19	95.00	95.00
20	14.00	22.00	20	47.00	75.00
21	30.00	45.00	21	20.00	20.00
22	47.00	69.00	22	33.00	42.00
23	58.00	45.00	23	45.00	67.00
24	44.00	44.00	24	69.00	70.00
25	31.00	44.00	25	58.00	81.00
26	86.00	95.00	26	58.00	69.00

27	67.00	87.00	27	41.00	47.00
28	44.00	64.00	28	58.00	64.00

N: Number
PSVT: Purdue Spatial Visualization Test

Table 4.2 Data of Posttests on Treatment Group

N	Cabri 3D	PSVT	MPS
1	73.00	92.00	70.00
2	64.00	69.00	40.00
3	73.00	52.00	55.00
4	82.00	80.00	90.00
5	55.00	56.00	70.00
6	82.00	89.00	85.00
7	73.00	78.00	70.00
8	45.00	84.00	70.00
9	64.00	58.00	50.00
10	55.00	61.00	85.00
11	55.00	73.00	75.00
12	73.00	74.00	70.00
13	64.00	59.00	80.00
14	55.00	53.00	80.00
15	45.00	64.00	60.00
16	64.00	66.00	75.00
17	64.00	28.00	35.00
18	55.00	75.00	80.00
19	73.00	95.00	90.00
20	45.00	75.00	85.00
21	55.00	20.00	65.00
22	64.00	42.00	65.00

23	73.00	67.00	70.00
24	73.00	70.00	70.00
25	73.00	81.00	75.00
26	64.00	69.00	65.00
27	45.00	47.00	85.00
28	82.00	64.00	60.00

N: Number
PSVT: Purdue Spatial Visualization Skills Test
TEA-MPS: Mathematical Problem-solving Skills Test

Table 4.3 Data of Pretest-Posttest on Mathematical Problem-solving Skills

N	Group 1 – Control Group		N	Group 2 – Treatment Group	
	Pretests of MPS	Posttests of MPS		Pretests of MPS	Posttests of MPS
1	45.00	65.00	1	50.00	70.00
2	30.00	80.00	2	55.00	40.00
3	50.00	60.00	3	60.00	55.00
4	65.00	60.00	4	40.00	90.00
5	55.00	70.00	5	70.00	70.00
6	65.00	65.00	6	55.00	85.00
7	40.00	75.00	7	65.00	70.00
8	55.00	70.00	8	35.00	70.00
9	25.00	60.00	9	70.00	50.00
10	55.00	65.00	10	70.00	85.00
11	55.00	55.00	11	45.00	75.00
12	60.00	65.00	12	50.00	70.00
13	55.00	65.00	13	35.00	80.00
14	70.00	70.00	14	40.00	80.00
15	60.00	70.00	15	85.00	60.00
16	50.00	65.00	16	35.00	75.00
17	55.00	75.00	17	60.00	35.00

18	50.00	70.00	18	45.00	80.00
19	75.00	80.00	19	65.00	90.00
20	55.00	65.00	20	45.00	85.00
21	45.00	60.00	21	30.00	65.00
22	40.00	80.00	22	70.00	65.00
23	60.00	75.00	23	70.00	70.00
24	55.00	55.00	24	15.00	70.00
25	40.00	90.00	25	50.00	75.00
26	50.00	70.00	26	60.00	65.00
27	65.00	65.00	27	35.00	85.00
28	65.00	80.00	28	55.00	60.00

N: Number
TEA-MPS: Mathematical Problem-solving Skills Test

Pre-analysis Activities

Five assumptions must be met by the data above before the analysis process with the Mixed repeated measures of ANOVA can be applied. These pre-analysis activities – as shown below – indicate that the five assumptions have been met.

- Assumption #1: The dependent variable should be measured at a continuous level. These dependent variables are the scores of two pretests and two posttests of spatial visualization skills that range from 0 to 100.

- Assumption #2: Two within-subject factors, namely two independent variables consist of at least two categorical related groups or matched pairs. The two within-subject factors also mean that the same subjects in each group have been measured on two occasions (pretest and posttest) on the same dependent variable.

- Assumption #3: There are no significant outliers in any combination of the related groups as indicated by the outliers test result below. Outliers are data points that appear to be significantly different than the majority of data points.

  According to Hoaglin and Iglewicz (1987), significant outliers can be calculated by using the following formulas:

  Lower bound (L) = $Q_1 - 2.2*(Q_3 - Q_1)$

  Upper bound (U) = $Q_3 + 2.2*(Q_3 - Q_1)$

  Pretests: PSVT

  Group 1: L = 44.0 – 2.2*(67.0 – 44.0) = -5.6 and U = 67.0 + 2.2*(67.0 – 44.0) = 117.6

  From the extreme values table: Lowest score = 14 and Highest score: 85. Hence: There are no significant outliers in the pretest data of group 1.

  Group 2: L = 33.8 – 2.2*(60.3 – 33.8) = -24.5 and U = 60.3 + 2.2*(60.3 – 33.8) = 118.6

  From the extreme values table: Lowest score = 20 and Highest score: 95. Hence: There are no significant outliers in the pretest data of group 2.

Posttests: PSVT

Group 1: L = 44.0 – 2.2*(83.3 – 44.0) = -42.5 and U = 83.3 + 2.2*(83.3 – 44.0) = 169.8

From the extreme values table: Lowest score = 22 and Highest score: 95. Hence: There are no significant outliers in the posttest data of group 1.

Group 2: L = 56.5 – 2.2*(77.3 – 56.5) = 10.7 and U = 77.3 + 2.2*(77.3 – 56.5) = 123.1

From the extreme values table: Lowest score = 20 and Highest score: 95. Hence: There are no significant outliers in the posttest data of group 2.

Table 4.4 Percentiles of PSVT

		Group	Percentiles						
			5	10	25	50	75	90	95
Weighted Average (Definition 1)	Pretest	1	18.0500	29.3000	44.0000	55.0000	67.0000	80.4000	85.1000
		2	21.3500	24.8000	33.7500	48.5000	60.2500	67.2000	83.3000
	Posttest	1	28.3000	38.7000	44.0000	64.5000	83.2500	89.1000	92.7500
		2	23.6000	40.6000	56.5000	68.0000	77.2500	89.3000	93.6500
Tukey's Hinges	Pretest	1			44.0000	55.0000	67.0000		
		2			34.5000	48.5000	59.5000		
	Posttest	1			44.0000	64.5000	82.5000		
		2			57.0000	68.0000	76.5000		

Table 4.5 Extreme Values of PSVT

Group				Case Number	Value
Pretest	1	Highest	1	26	86.00
			2	10	84.00
			3	5	80.00
			4	6	73.00
			5	14	72.00
		Lowest	1	20	14.00
			2	11	23.00
			3	21	30.00
			4	25	31.00
			5	18	33.00
	2	Highest	1	47	95.00
			2	52	69.00
			3	40	67.00
			4	34	64.00
			5	32	62.00
		Lowest	1	49	20.00
			2	33	23.00
			3	45	25.00
			4	31	27.00
			5	44	28.00
Posttest	1	Highest	1	26	95.00
			2	10	90.00
			3	5	89.00
			4	4	87.00
			5	27	87.00
		Lowest	1	20	22.00
			2	11	36.00
			3	18	39.00
			4	17	42.00

		5	15	42.00[a]
2	Highest	1	47	95.00
		2	29	92.00
		3	34	89.00
		4	36	84.00
		5	53	81.00
	Lowest	1	49	20.00
		2	45	28.00
		3	50	42.00
		4	55	47.00
		5	31	52.00

a. Only a partial list of cases with the value 42.00 are shown in the table of lower extremes.

Pretests: MPS

Group 1: L = 46.3 – 2.2*(60.0 – 46.3) = 16.2 and U = 60.0 + 2.2*(60.0 – 46.3) = 90.1

From the extreme values table: Lowest score = 25.0 and Highest score: 75.0.

Hence: There are no significant outliers in the pretest data of group 1

Group 2: L = 40.0 – 2.2*(65.0 – 40.0) = -15.0 and U = 65.0 + 2.2*(65.0 – 40.0) = 120.0

From the extreme values table: Lowest score = 15 and Highest score: 85. Hence: There are no significant outliers in the pretest data of group 2.

Posttests: MPS

Group 1: L = 65.0 – 2.2*(75.0 – 65.0) = 43 and U = 75.0 + 2.2*(75.0 – 65.0) = 97

From extreme values table: Lowest score = 55 and Highest score: 90. Hence:

There are no significant outliers in the posttest data of group 1.

Group 2: L = 65.0 – 2.2*(80.0 – 65.0) = 32.0 and U = 80.0 + 2.2*(80.0 – 65.0) = 113.0

From extreme values table: Lowest score = 35 and Highest score: 90. Hence:

There are no significant outliers in the posttest data of group 2.

Table 4.6 Percentiles of MPS Tests

		Group	Percentiles						
			5	10	25	50	75	90	95
Weighted Average (Definition 1)	Pretest	1	27.2500	39.0000	46.2500	55.0000	60.0000	65.5000	72.7500
		2	21.7500	34.5000	40.0000	52.5000	65.0000	0.07000	78.2500
	Posttest	1	55.0000	59.5000	65.0000	67.5000	75.0000	80.0000	85.5000
		2	37.2500	49.0000	65.0000	70.0000	80.0000	85.5000	90.0000
Tukey's Hinges	Pretest	1			47.5000	55.0000	60.0000		
		2			40.0000	52.5000	65.0000		
	Posttest	1			65.0000	67.5000	75.0000		
		2			65.0000	70.0000	80.0000		

Table 4.7 Extreme Values of MPS Tests

Group				Case Number	Value
Pretest	1	Highest	1	19	75.00
			2	14	70.00
			3	4	65.00
			4	6	65.00
			5	27	
		Lowest	1	9	25.00
			2	2	30.00
			3	25	40.00
			4	22	40.00
			5	7	40.00
	2	Highest	1	43	85.00
			2	33	70.00
			3	37	70.00
			4	38	70.00
			5	50	
		Lowest	1	52	15.00
			2	49	30.00
			3	55	35.00
			4	44	35.00
			5	41	
Posttest	1	Highest	1	25	90.00
			2	2	80.00
			3	19	80.00
			4	22	80.00
			5	28	80.00
		Lowest	1	24	55.00
			2	11	55.00
			3	21	60.00
			4	9	60.00

		5	4	
2	Highest	1	32	90.00
		2	47	90.00
		3	34	85.00
		4	38	85.00
		5	48	
	Lowest	1	45	35.00
		2	30	40.00
		3	37	50.00
		4	31	55.00
		5	56	

a. Only a partial list of cases with the value 65.00 are shown in the table of upper extremes.

b. Only a partial list of cases with the value 70.00 are shown in the table of upper extremes.

c. Only a partial list of cases with the value 35.00 are shown in the table of lower extremes.

d. Only a partial list of cases with the value 60.00 are shown in the table of lower extremes.

e. Only a partial list of cases with the value 85.00 are shown in the table of upper extremes.

- Assumption #4 (Normality of data): According to the analysis and test of normality in SPSS that focus on Shapiro-Wilk's significances as shown below, the assumption of normality of data has been met.

  Pretests: PSVT

  Group 1: Sig = 0.679 > alpha (0.05). The null hypothesis that says this data is normal cannot be rejected.

Group 2: Sig = 0.241 > alpha (0.05). The null hypothesis that says this data is normal cannot be rejected.

Posttests: PSVT

Group 1: Sig = 0.096 > alpha (0.05). The null hypothesis that says this data is normal cannot be rejected.

Group 2: Sig = 0.357 > alpha (0.05). The null hypothesis that says this data is normal cannot be rejected.

Table 4.8 Tests of Normality of PSVT

	Group	Kolmogorov-Smirnov[a]			Shapiro-Wilk		
		Statistic	df	Sig.	Statistic	df	Sig.
Pretest	1	.119	28	.200*	.974	28	.679
	2	.114	28	.200*	.953	28	.241
Posttest	1	.169	28	.039	.938	28	.096
	2	.103	28	.200*	.960	28	.357

a. Lilliefors Significance Correction
*. This is a lower bound of the true significance.

Pretests: MPS

Group 1: Sig = 0.227 > alpha (0.05). The null hypothesis that says this data is normal cannot be rejected.

Group 2: Sig = 0.666 > alpha (0.05). The null hypothesis that says this data is normal cannot be rejected.

Posttests: MPS

Group 1: Sig = 0.147 > alpha (0.05). The null hypothesis that says this data is normal cannot be rejected.

Group 2: Sig = 0.063 > alpha (0.05). The null hypothesis that says this data is normal cannot be rejected.

Table 4.9 Test of Normality of MPS

		Kolmogorov-Smirnov[a]			Shapiro-Wilk		
	Group	Statistic	df	Sig.	Statistic	df	Sig.
Pretest	1	.169	28	.039	.958	28	.312
	2	.093	28	.200*	.973	28	.666
Posttest	1	.173	28	.031	.945	28	.147
	2	.168	28	.041	.930	28	.063

a. Lilliefors Significance Correction

*. This is a lower bound of the true significance.

- Assumption #5: Known as sphericity, the variances of the differences between all combinations of related groups must be equal. Mauchly's test of sphericity is appropriate for analyzing data that have at least three times. Since these data only have two times (pretests and posttests), these data don't need Mauchly's test of sphericity.

Hence, data from pretests and posttests of PSVT and MPS can be analyzed with Repeated measures of ANOVA because all assumptions for running it have been met.

Then, the mediation relationship in this experiment will be analyzed with the Bootstrapping method and Hayes' PROCESS. The bootstrapping method is a non-parametric statistic. The bootstrapping method involves repeatedly randomly sampling observations with replacement from the data set to compute the desired statistic in each resample. Over hundreds or thousands of bootstrap resamples provide an approximation of the sampling distribution of the statistic of interest. This method provides point estimates and confidence intervals by which one can assess the significance or non-significance of a mediation effect. If zero does not fall between the resulting confidence intervals of the bootstrapping method, one can confidently conclude that there is a significant mediation effect to report (Preacher & Hayes, 2004).

Bootstrapping is becoming the most popular method for testing and analyzing a mediation relationship because it does not require the normality assumption to be met, and also it can be effectively utilized with smaller sample sizes ($N < 25$). For these purposes, Hayes offers a computer program named PROCESS that will be presented in the next section.

Analyzing the Effects of Treatment on Spatial Visualization Skills

Because the original Repeated measures of ANOVA cannot give detailed outcomes in the analysis results of two groups with two pretests and two posttests, then we need to modify its syntax for finding the detailed outcomes. The modified syntax as shown below is named Mixed repeated measures of ANOVA.

Original syntax:

```
GLM Pretest Posttest BY Group
/WSFACTOR=Time 2 Polynomial
/MEASURE=SVS
/METHOD=SSTYPE(3)
/POSTHOC=Group(TUKEY)
/PLOT=PROFILE(Time*Group)
/EMMEANS=TABLES(Time) COMPARE ADJ(BONFERRONI)
/EMMEANS=TABLES(Group*Time)
/PRINT=DESCRIPTIVE ETASQ OPOWER HOMOGENEITY
/CRITERIA=ALPHA(.05)
/WSDESIGN=Time
/DESIGN=Group
```

Modified syntax:

```
GLM Pretest Posttest BY Group
/WSFACTOR=Time 2 Polynomial
/MEASURE=SVS
/METHOD=SSTYPE(3)
/POSTHOC=Group(TUKEY)
/PLOT=PROFILE(Time*Group)
/EMMEANS=TABLES(Time) COMPARE ADJ(BONFERRONI)
/EMMEANS=TABLES(Group*Time) COMPARE (Group) ADJ(BONFERRONI)
/EMMEANS=TABLES(Group*Time) COMPARE (Time) ADJ(BONFERRONI)
/PRINT=DESCRIPTIVE ETASQ OPOWER HOMOGENEITY
/CRITERIA=ALPHA(.05)
/WSDESIGN=Time
/DESIGN=Group
```

Descriptive Statistics for SVS

Table 4.10 Within-Subjects Factors_SVS

Time	Dependent Variable
1	Pretest
2	Posttest

Table 4.11 Between-Subjects Factors_SVS

		N
Group	1	28
	2	28

Table 4.12 Descriptive Statistics_SVS

	Group	Mean	Std. Deviation	N
Pretest	1	54.8571	18.38017	28
	2	47.8571	17.29636	28
	Total	51.3571	18.03273	56
Posttest	1	63.0000	20.32878	28
	2	65.7500	17.58708	28
	Total	64.3750	18.88488	56

Descriptive statistics above shows Pretest of Group 1 (M = 54.9, SD = 18.4) and Pretest of Group 2 (M = 47.9, SD = 17.3), then Posttest of Group 1 (M = 63.0, SD = 20.3) and Posttest of Group2 (M = 65.8, SD = 17.6)

In this part, the researcher analyzes descriptively the increases of SVS scores on Group 1 and Group 2 for determining which group is better due to treatments. However, before calculating the increases of scores from both groups, we need to calculate Coefficients of Variation (COV) of data presented in Table 4.12 above. Coefficients of Variation (COV) is Standard Deviation (SD) divided by Mean (M). COV defined as the standard deviation divided by the mean that describes the variability of a sample relative to its mean.

This research study is within-subject design, thus the COV calculations for SVS are as follows:

Pretest of Group 1: COV = 18.4 : 54.9 = 0.34 or 34%

Posttest of Group 1: COV = 20.3 : 63.0 = 0.32 or 32%

Thus, the variability decreases = 34% - 32% = 2%

Pretest of Group 2: COV = 17.3 : 47.9 = 0.36 or 36%

Posttest of Group 2: COV = 17.6 : 65.8 = 0.27 or 27%

Thus, the variability decreases = 36% - 27% = 9%.

The calculations results above indicate that the treatment with Cabri 3D in Group 2 gave a bigger effect on SVS than the treatment with traditional tools (non-DGS) in Group 1.

Then, from Table 4.12 above, we can see that both Means increase. The increases of SVS scores in both groups can be calculated from the scores in the column "Means" of Group 1 and Group 2. The effect percentage due to the treatments can be seen in the following calculations:

Increase of score in Group 1: 63.0 – 54.9 = 8.1 points.

Percentage of the score increase in Group 1: (8.1 : 54.9) x 100% =15%.

Increase of score in Group 2: 65.8 – 47.9 = 17.9 points.

Percentage of the score increase in Group 2: (17.9 : 47.9) x 100% = 37%.

Percentage of the score increase of Group 2 to Group 1: [(37 – 15) : 15] x 100% = 147%.

The calculation results above further indicate that the result of Cabri 3D treatment on Group 2 gave a much bigger effect on SVS (about 147%) than traditional tools (non-DGS) treatment on Group 1.

Also, we need to calculate the increase of the total scores mean of the SVS pretest and posttest for comparing whether the increase of each group's scores is lower or higher than the increase of the total scores mean. As can be seen in the Table of Descriptive Statistics, the scores have increased from 51.4 to 64.4 or 13.0 points. The number of 13.0 in this study is named the scores average increase.

The calculations above show that Group 1's scores increase (8.1 points) < scores average increase (13.0 points), and Group 2's scores increase (17.9 points) > scores average increase (13.0 points). Hence, from the calculations of data in the descriptive statistics, we can conclude that treatment with Cabri 3D in this study has made Group 2 better in SVS (about 147%) than Group 1. In other words, this calculation is to verify that learning and practicing three-dimensional geometry with Cabri 3D is more effective than with traditional tools in terms of developing and enhancing senior high school students' spatial visualization skills.

Statistical Analysis for SVS

Table 4.13 Levene's Test of Equality of Error Variances[a] _SVS

	F	df1	df2	Sig.
Pretest	.058	1	54	.810
Posttest	2.196	1	54	.144

Tests the null hypothesis that the error variance of the dependent variable is equal across groups.

a. Design: Intercept + Group
Within Subjects Design: Time

Table 4.13 above shows the Levene's Test is non-significant because all significance values for Pretest and Posttest respectively $F(1,54) = .058, p > .05$ and $F(1,54) = 2.196, p > .05$. This test indicates that variances are homogeneous (more or less the same for Group 1 and Group 2) for all levels of the repeated measures variables. Hence, the assumption of homogeneity of variance has been met, and this analysis can be continued.

Estimated Marginal Means

1. Time

Table 4.14 Estimates_Measure: SVS-1

			95% Confidence Interval	
Time	Mean	Std. Error	Lower Bound	Upper Bound
1	51.357	2.385	46.576	56.138
2	64.375	2.540	59.283	69.467

Table 4.15 Pairwise Comparisons_Measure: SVS-1

(I) Time	(J) Time	Mean Difference (I-J)	Std. Error	Sig.[a]	95% Confidence Interval for Difference[a]	
					Lower Bound	Upper Bound
1	2	-13.018*	1.490	.000	-16.004	-10.031
2	1	13.018*	1.490	.000	10.031	16.004

Based on estimated marginal means

*. The mean difference is significant at the .05 level.

a. Adjustment for multiple comparisons: Bonferroni.

Table 4.16 Multivariate Tests_SVS-1

	Value	F	Hypothesis df	Error df	Sig.	Partial Eta Squared	Noncent. Parameter	Observed Power[b]
Pillai's trace	.586	76.372[a]	1.000	54.000	.000	.586	76.372	1.000
Wilks' lambda	.414	76.372[a]	1.000	54.000	.000	.586	76.372	1.000
Hotelling's trace	1.414	76.372[a]	1.000	54.000	.000	.586	76.372	1.000
Roy's largest root	1.414	76.372[a]	1.000	54.000	.000	.586	76.372	1.000

Each F tests the multivariate effect of Time. These tests are based on the linearly independent pairwise comparisons among the estimated marginal means.

a. Exact statistic

b. Computed using alpha = .05

Pretests (I) and posttests (J) of SVS as shown in the tables above, respectively indicates significant differences: (M = 51.4, SD = 2.4) and (M = 64.4, SD = 2.5), and then M(I-J) = -13.0, SD = 1.5, p < 0.05, 95% CI [-16.0, -10.0] or M(I-J) = 13.0, SD = 1.5, p < .05, 95% CI [10.0, 16.0]. Multivariate tests of SVS also indicates there is significant difference between pretest and posttest with Wilks' lamda value = .414, $F(1,54) = 76.372$, $p < .05$, partial $\eta^2 = .586$, and observed power = 1.000. In general, the multivariate tests above indicate that spatial visualization skills can be significantly developed with large effect size on both Group 1 and Group 2 through learning three-dimensional geometry with traditional tools (non-DGS/PPT) or with Cabri 3D. But for testing four hypotheses of SVS and for finding specific results as proposed in this study, we need to continue analyses procedure as described below.

2. Group * Time

Table 4.17 Estimates_Measure: SVS-2

				95% Confidence Interval	
Group	Time	Mean	Std. Error	Lower Bound	Upper Bound
1	1	54.857	3.373	48.095	61.619
	2	63.000	3.592	55.798	70.202
2	1	47.857	3.373	41.095	54.619
	2	65.750	3.592	58.548	72.952

Table 4.18 Pairwise Comparisons_Measure: SVS-2

Time	(I) Group	(J) Group	Mean Difference (I-J)	Std. Error	Sig.[a]	95% Confidence Interval for Difference[a]	
						Lower Bound	Upper Bound
1	1	2	7.000	4.770	.148	-2.563	16.563
	2	1	-7.000	4.770	.148	-16.563	2.563
2	1	2	-2.750	5.080	.590	-12.935	7.435
	2	1	2.750	5.080	.590	-7.435	12.935

Based on estimated marginal means

a. Adjustment for multiple comparisons: Bonferroni.

Table 4.19 Univariate Tests_Measure: SVS

Time		Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared	Noncent. Parameter	Observed Power[a]
1	Contrast	686.000	1	686.000	2.154	.148	.038	2.154	.302
	Error	17198.857	54	318.497					
2	Contrast	105.875	1	105.875	.293	.590	.005	.293	.083
	Error	19509.250	54	361.282					

Each F tests the simple effects of Group within each level combination of the other effects shown. These tests are based on the linearly independent pairwise comparisons among the estimated marginal means.

a. Computed using alpha = .05

The tables of Group*Time as shown above respectively indicate that there is no significant difference between the pretest of Group 1 and the pretest of Group 2, and there is no significant difference between the posttest of Group 1 and the posttest of Group 2, but with small effect sizes.

Group 1 (I) and Group 2 (J)

Pretest (I): (M = 54.9, SD = 3.4), 95% CI [48.1, 61.6],

Posttest (I): (M = 63.0, SD = 3.6), 95% CI [55.8, 70.2]

Time 1: M(I-J) = 7.0, SD = 4.8, p > .05, 95% CI [-2.6, 16.6] or M(I-J) = -7.0, SD = 4.8, p > .05, 95% CI [-16.6, 2.6]

Pretest (J): (M = 47.9, SD = 3.4, 95% CI [41.1, 54.6]

Posttest (J): (M = 65.8, SD = 3.6, 95% CI [58.5, 73.0]

Time 2: M(I-J) = -2.8, SD = 5.1, p > .05, 95% CI [-12.9, 7.4] or M(I-J) = 2.8, SD = 5.1, p > .05, 95% CI [-7.4, 12.9]

Univariate tests of Contrast indicate that

- In Time 1: $F(1,54) = 2.154$, $p > .05$, partial η^2 (effect size) = .038, observed power = .302
- In Time 2: $F(1,54) = .293$, $p > .05$, partial η^2 (effect size) = .005, observed power = .083

Results:

- Hypothesis #1 cannot be rejected, that says: There is no significant difference in the results of pretests of SVS between Group 1 and Group 2.
- Hypothesis #2 cannot be rejected, that says: There is no significant difference in the results of posttests of SVS between Group 1 and Group 2.

- Effect size (partial $\eta^2 = .038$) of Contrast in pretests between Group 1 and Group 2 and effect size (partial $\eta^2 = .005$) of Contrast in posttests between Group 1 and Group 2 are very small.

The results indicate that there is a non-significant difference in the results of pretests and posttests between Group 1 and Group 2. In the pretests, the difference is non-significant because, $F(1,54) = 2.154$, $p > .05$. In other words, the significance value (.148) is greater than alpha (.05), and in the posttests, the difference is also non-significant because, $F(1,54) = .293$, $p > .05$, the significance value (.590) is greater than alpha (.05). Due to these results, we cannot reject the null hypothesis that says that there is no significant difference in the effects of treatments on spatial visualization skills between Group 1 and Group 2.

Hence, although the effect sizes are very small, the analysis results above indicate that there is no significantly different effect between learning-and-practicing three-dimensional geometry with traditional tools and learning-and-practicing three-dimensional geometry with Cabri 3D in the context of developing and enhancing senior high school students' spatial visualization skills. Furthermore, for getting more detailed results, we need to get the analysis results of within-subjects design.

3. Group * Time

Table 4.20 Estimates_Measure: SVS-3

				95% Confidence Interval	
Group	Time	Mean	Std. Error	Lower Bound	Upper Bound
1	1	54.857	3.373	48.095	61.619
	2	63.000	3.592	55.798	70.202
2	1	47.857	3.373	41.095	54.619
	2	65.750	3.592	58.548	72.952

Table 4.21 Pairwise Comparisons_Measure: SVS-3

						95% Confidence Interval for Difference[a]	
Group	(I) Time	(J) Time	Mean Difference (I-J)	Std. Error	Sig.[a]	Lower Bound	Upper Bound
1	1	2	-8.143*	2.107	.000	-12.366	-3.919
	2	1	8.143*	2.107	.000	3.919	12.366
2	1	2	-17.893*	2.107	.000	-22.116	-13.669
	2	1	17.893*	2.107	.000	13.669	22.116

Based on estimated marginal means

*. The mean difference is significant at the .05 level.

a. Adjustment for multiple comparisons: Bonferroni.

Tables of Group*Time as shown above respectively indicate that there are significant interactions between groups and in treatments:

Group 1:

Pretest (I): ($M = 54.9$, $SD = 3.4$), 95% CI [48.1, 61.6],

Posttest (J): ($M = 63.0$, $SD = 3.6$), 95% CI [55.8, 70.2]

$M(I-J) = -8.1$, $SD = 2.1$, $p < .05$, 95% CI [-12.4, -3.9] or $M(I-J) = 8.1$, $SD = 2.1$, $p < .05$, 95% CI [3.9, 12.4]

Group 2

Pretest (I): ($M = 47.9$, $SD = 3.4$, 95% CI [41.1, 54.6]

Posttest (J): ($M = 65.8$, $SD = 3.6$, 95% CI [58.5, 73.0]

$M(I-J) = -17.9$, $SD = 2.1$, $p < .05$, 95% CI [-22.1, -13.7] or $M(I-J) = 17.9$, $SD = 2.1$, $p < .05$, 95% CI [13.7, 22.1]

Table 4.22 Multivariate Tests_SVS-2

Group		Value	F	Hypothesis df	Error df	Sig.	Partial Eta Squared	Noncent. Parameter	Observed Power[b]
1	Pillai's trace	.217	14.941[a]	1.000	54.000	.000	.217	14.941	.967
	Wilks' lambda	.783	14.941[a]	1.000	54.000	.000	.217	14.941	.967
	Hotelling's trace	.277	14.941[a]	1.000	54.000	.000	.217	14.941	.967
	Roy's largest root	.277	14.941[a]	1.000	54.000	.000	.217	14.941	.967
2	Pillai's trace	.572	72.141[a]	1.000	54.000	.000	.572	72.141	1.000
	Wilks' lambda	.428	72.141[a]	1.000	54.000	.000	.572	72.141	1.000
	Hotelling's trace	1.336	72.141[a]	1.000	54.000	.000	.572	72.141	1.000
	Roy's largest root	1.336	72.141[a]	1.000	54.000	.000	.572	72.141	1.000

Each F tests the multivariate simple effects of Time within each level combination of the other effects shown. These tests are based on the linearly independent pairwise comparisons among the estimated marginal means.

a. Exact statistic

b. Computed using alpha = .05

There is a significant effect of using traditional tools on Group 1 with Wilks' lambda value = .783, $F(1,54) = 14.941$, $p < .05$, partial η^2 (effect size) = .217, and observed power = .967; and there is a significant effect of using Cabri 3D on Group 2 with Wilk's lambda value = .428, $F(1,54) = 72.141$, $p < 0.05$, partial η^2 (effect size) = .572, and observed power = 1.000.

Results:

- Null hypothesis #3 is rejected, that says: There is no significant difference before and after treatment using traditional tools on senior high school students' SVS within Group 1.
- Null hypothesis #4 is rejected, that says: There is no significant difference before and after treatment using Cabri 3D on senior high school students' SVS within Group 2.
- Effect size (partial $\eta^2 = .572$) of treatment with Cabri 3D in Group 2 is large and even much larger than effect size (partial $\eta^2 = .217$) of using traditional tools in Group 1.

Profile Plots

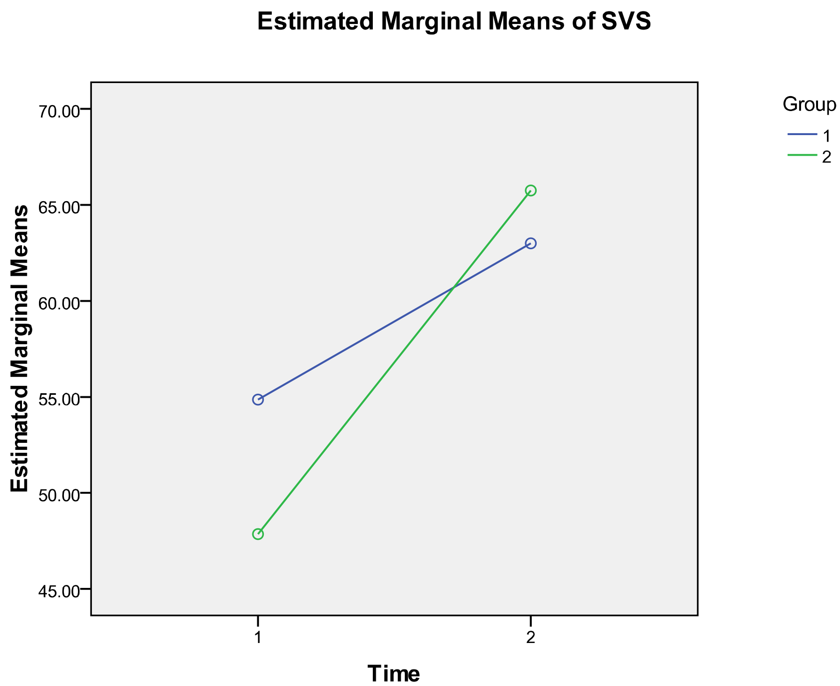

Figure 4.1 Estimated Marginal Means of SVS for Group 1 and Group 2

The design of this research study is within-subjects design, and because of that, we have to focus on the results of the testings of both hypothesis #3 and hypothesis #4 when making conclusions. Hence, we conclude that although senior high school students' spatial visualization skills can be developed and enhanced through learning and practicing three-dimensional geometry with traditional tools or with Cabri 3D, from the analyses results, treatment with Cabri 3D indicates that students' spatial visualization skills are better developed and enhanced by Cabri 3D (effect size, partial η^2 = .572 of rejection to its null hypothesis #4) than by traditional tools (effect size, partial η^2 = .217 of rejection to its null hypothesis #3). And data calculations of the descriptive statistics above also indicate that developing and enhancing senior high school students' spatial visualization skills with Cabri 3D is better (about 147 %) than with traditional tools.

Mediation Analysis

The mediation relationship in this study will be analyzed with the Bootstrapping method and Hayes' PROCESS. The bootstrapping method is a non-parametric statistic. Bootstrapping involves repeated random sampling observations with replacement from the data set to compute the desired statistic in each resample. Over hundreds or thousands of bootstrap resamples provide an approximation of the sampling distribution of the statistic of interest. This method provides point estimates and confidence intervals by which one can assess the significance or non-significance of a mediation effect. If zero does not fall between the resulting confidence intervals of the bootstrapping method, one can confidently conclude that there is a significant mediation effect to report (Preacher & Hayes, 2004).

Bootstrapping is becoming the most popular method for testing and analyzing a mediation relationship because it does not require the normality assumption to be met, and also it can be effectively utilized with smaller sample sizes ($N < 25$). For these purposes, Hayes offers a macro (http://www.afhayes.com) that calculates bootstrapping directly within SPSS, a computer program named PROCESS used for statistical analyses. "Bootstrap confidence intervals are a better approach to inference when the original data are available for analysis … and bootstrap confidence intervals tend to be more powerful than competing methods such as the normal theory approach." (Hayes, 2013, p. 139).

If there are conflicting results between bootstrap confidence intervals and normal theory approach in indirect effect in mediation analysis, then bootstrap confidence intervals will be chosen because the bootstrap confidence intervals approach is higher in power than the normal theory approach in providing an inference, and "… bootstrap confidence intervals respect to the irregularity of the sampling distribution of the indirect effect." (Hayes, 2013, p. 139). For the specific indirect effect analysis, PROCESS will generate the confidence intervals as the Bias-corrected bootstrap confidence intervals under the section of the output that reads the "Indirect effect of X on Y."

Mediation:

1) Variable X predicts variable Y – path c
2) Variable X predicts variable M – path a
3) X and M together predicting Y
    a. M predicts Y – path b
    b. X is no longer predicting Y or is lessened predicting Y – path c'

Mediation syntax

```
/* PROCESS for SPSS v2.11 */.
/* Written by Andrew F. Hayes */.
/* www.afhayes.com */.
/* Copyright 2014 */.
/* Read the documentation */.
/* available in Appendix A of */.
/* Hayes (2013) prior to use */.
/* www.guilford.com/p/hayes3 */.
/* For proper results, variable */.
/* names in data file must be distinct */.
/* in the first eight characters */.
set printback = off.
```

Table 4.23 Mediation Analysis_PROCESS

Matrix

```
[DataSet1] G:\Data_SVS-MPS-C3D.sav


Run MATRIX procedure:

***************** PROCESS Procedure for SPSS Release 2.11 *************

          Written by Andrew F. Hayes, Ph.D.       www.afhayes.com
    Documentation available in Hayes (2013). www.guilford.com/p/hayes3

**************************************************************************
Model = 4
    Y = MPS
    X = C3D
    M = SVS

Sample size
         28

**************************************************************************
Outcome: SVS

Model Summary
          R       R-sq          F        df1        df2          p
      .3244      .1052     3.0572     1.0000    26.0000      .0922

Model
              coeff         se          t          p       LLCI
ULCI
constant    33.9782    18.4513     1.8415      .0770    -3.9502
71.9066
C3D           .4975      .2846     1.7485      .0922     -.0874
1.0825

**************************************************************************
Outcome: MPS

Model Summary
          R       R-sq          F        df1        df2          p
      .4946      .2446     4.0474     2.0000    25.0000      .0300

Model
              coeff         se          t          p       LLCI
ULCI
constant    59.2792    14.3599     4.1281      .0004    29.7033
88.8551
SVS           .4073      .1436     2.8375      .0089      .1117
.7030
C3D          -.2459      .2202    -1.1169      .2747     -.6995
.2076
```

```
******************* DIRECT AND INDIRECT EFFECTS ********************

Direct effect of X on Y
     Effect         SE          t          p       LLCI       ULCI
     -.2459      .2202    -1.1169      .2747     -.6995      .2076

Indirect effect of X on Y
        Effect    Boot SE   BootLLCI   BootULCI
SVS      .2027      .1475      .0156      .6355

Partially standardized indirect effect of X on Y
        Effect    Boot SE   BootLLCI   BootULCI
SVS      .0148      .0103      .0005      .0426

Completely standardized indirect effect of X on Y
        Effect    Boot SE   BootLLCI   BootULCI
SVS      .1691      .1058      .0108      .4420

Ratio of indirect to total effect of X on Y
        Effect    Boot SE   BootLLCI   BootULCI
SVS    -4.6843 5.742E+011 -1755.3747    -1.5969

Ratio of indirect to direct effect of X on Y
        Effect    Boot SE   BootLLCI   BootULCI
SVS     -.8241    14.5234   -33.7722      .9677

R-squared mediation effect size (R-sq_med)
        Effect    Boot SE   BootLLCI   BootULCI
SVS     -.0364      .0578     -.2209      .0385

Preacher and Kelley (2011) Kappa-squared
        Effect    Boot SE   BootLLCI   BootULCI
SVS      .1833      .1086      .0212      .4560

Normal theory tests for indirect effect
     Effect         se          Z          p
      .2027      .1421     1.4258      .1539

******************* ANALYSIS NOTES AND WARNINGS ********************

Number of bootstrap samples for bias corrected bootstrap confidence
intervals:
    10000

WARNING: Bootstrap CI endpoints below not trustworthy.  Decrease
confidence or increase bootstraps
 -1755.3747

Level of confidence for all confidence intervals in output:
    95.00

------ END MATRIX -----
```

Table 4.24 ANOVA

Model		Sum of Squares	Df	Mean Square	F	Sig.
1	Regression	1054.460	1	1054.460	6.783	.015[a]
	Residual	4041.969	26	155.460		
	Total	5096.429	27			

a. Predictors: (Constant), SVS

b. Dependent Variable: MPS

In the part of TOTAL, DIRECT, AND INDIRECT EFFECTS from the PROCESS' output above, we can find:

Results:

- Null hypothesis #5 cannot be rejected, that says: There is no direct effect from C3D to MPS independence of SVS. This null hypothesis cannot be rejected because of $p > 0.05$ and this direct effect is not statistically different from zero, as revealed by its 95% CI [-.6995, .2076]. This confidence interval does include zero, so zero cannot be ruled out as a plausible value for direct effect.
- Null hypothesis #6 is rejected, that says: There are no indirect effects from C3D to MPS dependence of SVS. This null hypothesis was rejected because this indirect effect is statistically different from zero, as revealed by its 95% BootCI [.0156, .6355]. This Bootstrap Confidence Interval does not include zero, so zero can be ruled out as a plausible value for the indirect effects. Contrary to the Bootstrap Confidence Interval, the Normal theory tests for indirect effects,

as shown in the table above, cannot reject the null hypothesis of no indirect effects, $z = 1.4258$ and $p = .1539$, ($p > 0.05$).

- And its Preacher and Kelly's Kappa-Squared is 18.3%. The result means that the observed indirect effects of $ab = .203$ are about 18.3% of its maximum possible value given the association between the variables observed in the sample. So, the indirect effects = .203 of senior high school student's achievement in learning and practicing three-dimensional geometry with Cabri 3D to the increase of their mathematical problem-solving skills (mediated by the development and enhancement of their spatial visualization skills) are around 18.3% of its possible value.

- As aforementioned, when there are contrary results between Bootstrap Confidence Interval and Normal theory approach concerning indirect effect in mediation analysis, we choose Bootstrap Confidence Interval's result. Therefore, we decide that there is an indirect effect from C3D to the MPS dependence of SVS.

- After powerful tools in mediation analysis are invented, for analyzing indirect effects we no longer need to meet path *a* or path *b* or both paths are significant. With PROCESS we can minimize inferential procedures in mediation analysis. "It is better to minimize the number of inferential procedures one must employ to support a claim. A single inferential test of the indirect effect is all that is needed." (Hayes, 2013, pp. 168-169).

- Null hypothesis #7 is rejected, that says: There is no relationship between high school students' SVS and their MPS due to $p < .05$ in the tables of ANOVA and PROCESS' result. Alternatively: There is a relationship between high school students' SVS and their MPS.

Hence, there is an indirect effect from high school students' achievement in learning and practicing three-dimensional geometry with Cabri 3D to their mathematical problem-solving skills mediated by their spatial visualization skills. In other words, achievement in learning and practicing three-dimensional geometry with Cabri 3D will predict mathematical problem-solving skill dependence on spatial visualization skills. And the rejection of the null hypothesis #7 above to confirm that there is a relationship between mathematical problem-solving skills and spatial visualization skills.

Furthermore, from the table of mediation analysis above, we can get the formula of its causal mediation relationship as follows:

SVS = 33.978 + 0.498 C3D

MPS = 59.279 – 0.246 C3D + 0.407 SVS

Indirect effect = 0.498 * 0.407 = 0.203 (medium)

Note:

C3D: Achievement in learning and practicing three-dimensional geometry with Cabri 3D

SVS: Spatial Visualization Skills

MPS: Mathematical Problem-Solving Skills

Analyzing the Effects of Treatment on Mathematical Problem-solving Skills

Original syntax:

```
GLM Pretest Posttest BY Group
/WSFACTOR=Time 2 Polynomial
/MEASURE=MPS
/METHOD=SSTYPE(3)
/POSTHOC=Group(TUKEY)
/PLOT=PROFILE(Time*Group)
/EMMEANS=TABLES(Time) COMPARE ADJ(BONFERRONI)
/EMMEANS=TABLES(Group*Time)
/PRINT=DESCRIPTIVE ETASQ OPOWER HOMOGENEITY
/CRITERIA=ALPHA(.05)
/WSDESIGN=Time
/DESIGN=Group
```

Modified syntax:

```
GLM Pretest Posttest BY Group
/WSFACTOR=Time 2 Polynomial
/MEASURE=MPS
/METHOD=SSTYPE(3)
/POSTHOC=Group(TUKEY)
/PLOT=PROFILE(Time*Group)
/EMMEANS=TABLES(Time) COMPARE ADJ(BONFERRONI)
/EMMEANS=TABLES(Group*Time) COMPARE (Group) ADJ(BONFERRONI)
/EMMEANS=TABLES(Group*Time) COMPARE (Time) ADJ(BONFERRONI)
/PRINT=DESCRIPTIVE ETASQ OPOWER HOMOGENEITY
/CRITERIA=ALPHA(.05)
/WSDESIGN=Time
/DESIGN=Group
```

Descriptive Statistics for MPS

Table 4.25 Within-Subjects Factors_Measure: MPS

Time	Dependent Variable
1	Pretest
2	Posttest

Table 4.26 Between-Subjects Factors_MPS

		N
Group	1	28
	2	28

Table 4.27 Descriptive Statistics_MPS

	Group	Mean	Std. Deviation	N
Pretest	1	53.3929	11.16323	28
	2	52.1429	15.77789	28
	Total	52.7679	13.55660	56
Posttest	1	68.7500	8.34721	28
	2	70.3571	13.73887	28
	Total	69.5536	11.29267	56

Descriptive statistics in Table 4.27 shows that Pretest of Group 1 (M = 53.4, SD = 11.2) and Pretest of Group 2 (M = 52.1, SD = 15.8), then Posttest of Group 1 (M = 68.8, SD = 8.3) and Posttest of Group 2 (M = 70.4, SD = 13.7)

This research study is within-subjects design, thus the COV calculations for MPS are as follows:

Pretest of Group 1: COV = 11.2 : 53.4 = 0.21 or 21%

Posttest of Group 1: COV = 8.3 : 68.8 = 0.12 or 12%

Thus, the variability decreases = 21% - 12% = 9%

Pretest of Group 2: COV = 15.8 : 52.1 = 0.30 or 30%

Posttest of Group 2: COV = 13.7 : 70.4 = 0.19 or 19%

Thus, the variability decreases = 30% - 19% = 11%

The calculations above indicate that the treatment with Cabri 3D on Group 2 gave a bigger effect on MPS than the treatment with traditional tools (non-DGS) on Group 1. Furthermore, the effect percentage due to the treatments can be seen in the following calculations:

Increase of score in Group 1: 68.8 – 53.4 = 15.4 points.

Percentage of the score increase in Group 1: (15.4 : 53.4) x 100% = 29%.

Increase of score in Group 2: 70.4 – 52.1 = 18.3 points.

Percentage of the score increase in Group 2: (18.3 : 52.1) x 100% = 35%

Percentage of the score increase of Group 2 to Group 1: [(35 – 29) : 29] x 100% = 21%

The calculations above further indicate that the treatment with Cabri 3D on Group 2 has given a bigger effect on MPS (about 21%) than the treatment with traditional tools (non-DGS) on Group 1.

Then, we need to calculate the increase of total scores mean of the MPS pretest and posttest for comparing whether each group's scores increase is lower or higher than the increase of the total scores mean. As can be seen in the Table of Descriptive Statistics, the scores have increased from 52.8 to 69.6 or 16.8 points. Then, the number of 16.8 in this study is named the scores average increase.

The calculations above show that Group 1's scores increase (15.4 points) < scores average increase (16.8 points), and Group 2's scores increase (18.3 points) > scores average increase (16.8 points). Hence, from the calculation of data in the descriptive statistics, we can also conclude that treatment with Cabri 3D in this study has indirectly affected MPS in Group 2 better (about 21%) than Group 1. In other words, this calculation is to verify that learning and practicing three-dimensional geometry with Cabri 3D is more effective than with traditional tools in terms of indirectly increasing high school students' mathematical problem-solving skills.

Statistical Analysis for MPS

Table 4.28 Levene's Test of Equality of Error Variances[a] _MPS

	F	df1	df2	Sig.
Pretest	3.882	1	54	.054
Posttest	3.064	1	54	.086

Tests the null hypothesis that the error variance of the dependent variable is equal across groups.

a. Design: Intercept + Group
Within Subjects Design: Time

Table 4.28 shows that the significant values of Levene's Test are non-significant because in the Pretest $F(1,54) = 3.882$, $p > .05$ and in the Posttest $F(1,54) = 3.068$, $p > .05$. This test also indicates that the variances are homogeneous (more or less the same for Group 1 and Group 2) for all levels of the repeated measures variables. Hence, the assumption of homogeneity of variance has been met and this analysis can be continued.

Estimated Marginal Means

1. Time

Table 4.29 Estimates_Measure: MPS-1

			95% Confidence Interval	
Time	Mean	Std. Error	Lower Bound	Upper Bound
1	52.768	1.826	49.106	56.429
2	69.554	1.519	66.508	72.599

Table 4.30 Pairwise Comparisons_Measure: MPS-1

					95% Confidence Interval for Difference[a]	
(I) Time	(J) Time	Mean Difference (I-J)	Std. Error	Sig.[a]	Lower Bound	Upper Bound
1	2	-16.786*	2.651	.000	-22.102	-11.470
2	1	16.786*	2.651	.000	11.470	22.102

Based on estimated marginal means

*. The mean difference is significant at the .05 level.

a. Adjustment for multiple comparisons: Bonferroni.

Table 4.31 Multivariate Tests_MPS-1

	Value	F	Hypothesis df	Error df	Sig.	Partial Eta Squared	Noncent. Parameter	Observed Power[b]
Pillai's trace	.426	40.079[a]	1.000	54.000	.000	.426	40.079	1.000
Wilks' lambda	.574	40.079[a]	1.000	54.000	.000	.426	40.079	1.000
Hotelling's trace	.742	40.079[a]	1.000	54.000	.000	.426	40.079	1.000
Roy's largest root	.742	40.079[a]	1.000	54.000	.000	.426	40.079	1.000

Each F tests the multivariate effect of Time. These tests are based on the linearly independent pairwise comparisons among the estimated marginal means.
a. Exact statistic
b. Computed using alpha = .05

Pretests (I) and posttests (J) of MPS as shown in the Tables 4.31 above, respectively indicates significant differences: (M = 52.8, SD = 1.8), 95% CI [49.1, 56.4] and (M = 69,6 SD = 1.5), 95% CI [66.5, 72.6], and then M(I-J) = -16,8 SD = 2.7, p < 0.05, 95% CI [-22,1 -11.5] or M(I-J) = 16.8, SD = 2.7, p < .05, 95% CI [11.5, 22.1]. Multivariate tests of MPS also indicates that there is a significant difference between pretest and posttest with Wilks' lamda value = .574, F(1,54) = 40.079, p < .05, partial η^2= .426, and observed power = 1.000.

2. Group * Time

Table 4.32 Estimates_Measure: MPS-2

Group	Time	Mean	Std. Error	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
1	1	53.393	2.583	48.215	58.571
	2	68.750	2.148	64.443	73.057
2	1	52.143	2.583	46.965	57.321
	2	70.357	2.148	66.050	74.664

Table 4.33 Pairwise Comparisons_Measure: MPS-2

Time	(I) Group	(J) Group	Mean Difference (I-J)	Std. Error	Sig.[a]	95% Confidence Interval for Difference[a] Lower Bound	95% Confidence Interval for Difference[a] Upper Bound
1	1	2	1.250	3.653	.734	-6.073	8.573
	2	1	-1.250	3.653	.734	-8.573	6.073
2	1	2	-1.607	3.038	.599	-7.698	4.484
	2	1	1.607	3.038	.599	-4.484	7.698

Based on estimated marginal means

a. Adjustment for multiple comparisons: Bonferroni.

Table 4.34 Univariate Tests_Measure: MPS

Time		Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared	Noncent. Parameter	Observed Power[a]
1	Contrast	21.875	1	21.875	.117	.734	.002	.117	.063
	Error	10086.107	54	186.780					
2	Contrast	36.161	1	36.161	.280	.599	.005	.280	.081
	Error	6977.679	54	129.216					

Each F tests the simple effects of Group within each level combination of the other effects shown. These tests are based on the linearly independent pairwise comparisons among the estimated marginal means.
a. Computed using alpha = .05

Tables of Group*Time as shown above respectively indicate that there are no significant differences between pretest of Group 1 and pretest of Group 2, and between posttest of Group 1 and posttest of Group 2, but with very small effect sizes

Group 1 (I) and Group 2 (J)

Pretest (I): (M = 53.4, SD = 2.6), 95% CI [48.2, 58.6],

Posttest (I): (M = 68.8, SD = 2.1), 95% CI [64.4, 73.1]

Time 1: M(I-J) = 1.3, SD = 3.7, $p > .05$, 95% CI [-6.1, 8.6] or M(I-J) = -1.3, SD = 3.7, $p > .05$, 95% CI [-8.6, 6.1]

Pretest (J): (M = 52.1, SD = 2.6, 95% CI [47.0, 57.3]

Posttest (J): (M = 70.3, SD = 2.1, 95% CI [66.1, 74.7]

Time 2: M(I-J) = -1.6, SD = 3.0, $p > .05$, 95% CI [-7.7, 4.5] or M(I-J) = 1.6, SD = 3.0, $p > .05$, 95% CI [-4.5, 7.7]

Univariate tests of Contrast indicate that

- In Time 1: $F(1,54) = .117$, $p > .05$, partial η^2(effect size) = .002, observed power = .063
- In Time 2: $F(1,54) = .280$, $p > .05$. partial η^2 (effect size) = .005, observed power = .081

Results:

- Hypothesis #8 cannot be rejected, that says: There is no significant difference in the results of pretests of MPS between Group 1 and Group 2 due to $F(1,54) = .117$, $p > .05$.
- Hypothesis #9 cannot be rejected, that says: There is no significant difference in the results of posttests of MPS between Group 1 and Group 2 due to $F(1,54) = .280$, $p > .05$.
- Effect size (partial $\eta^2 = .002$) of Contrast in the results of pretests between Group 1 and Group 2 and effect size (partial $\eta^2 = .005$) of Contrast in the results of posttests between Group 1 and Group 2 are very small.

3. Group * Time

Table 4.35 Estimates_Measure: MPS-3

Group	Time	Mean	Std. Error	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
1	1	53.393	2.583	48.215	58.571
	2	68.750	2.148	64.443	73.057
2	1	52.143	2.583	46.965	57.321
	2	70.357	2.148	66.050	74.664

Table 4.36 Pairwise Comparisons_Measure: MPS-3

Group	(I) Time	(J) Time	Mean Difference (I-J)	Std. Error	Sig.[a]	95% Confidence Interval for Difference[a] Lower Bound	95% Confidence Interval for Difference[a] Upper Bound
1	1	2	-15.357*	3.750	.000	-22.875	-7.839
	2	1	15.357*	3.750	.000	7.839	22.875
2	1	2	-18.214*	3.750	.000	-25.732	-10.697
	2	1	18.214*	3.750	.000	10.697	25.732

Based on estimated marginal means

*. The mean difference is significant at the .05 level.

a. Adjustment for multiple comparisons: Bonferroni.

Table 4.37 Multivariate Tests_MPS-2

Group		Value	F	Hypothesis df	Error df	Sig.	Partial Eta Squared	Noncent. Parameter	Observed Power[b]
1	Pillai's trace	.237	16.774[a]	1.000	54.000	.000	.237	16.774	.980
	Wilks' lambda	.763	16.774[a]	1.000	54.000	.000	.237	16.774	.980
	Hotelling's trace	.311	16.774[a]	1.000	54.000	.000	.237	16.774	.980
	Roy's largest root	.311	16.774[a]	1.000	54.000	.000	.237	16.774	.980
2	Pillai's trace	.304	23.596[a]	1.000	54.000	.000	.304	23.596	.998
	Wilks' lambda	.696	23.596[a]	1.000	54.000	.000	.304	23.596	.998
	Hotelling's trace	.437	23.596[a]	1.000	54.000	.000	.304	23.596	.998
	Roy's largest root	.437	23.596[a]	1.000	54.000	.000	.304	23.596	.998

Each F tests the multivariate simple effects of Time within each level combination of the other effects shown. These tests are based on the linearly independent pairwise comparisons among the estimated marginal means.

a. Exact statistic

b. Computed using alpha = .05

Tables of Group*Time as shown above respectively indicate that there are significant interactions between group and in treatments:

Group 1

Pretest (I): (M = 53.4, SD = 2.6), 95% CI [48.2, 58.6],

Posttest (J): (M = 68.8, SD = 2.1), 95% CI [64.4, 73.0]

M(I-J) = -15.4, SD = 3.8, p < .05, 95% CI [-22.9, -7.8] or M(I-J) = 15.4, SD = 3.8, p < .05, 95% CI [7.8, 22.9]

Group 2

Pretest (I): (M = 52.1, SD = 2.6, 95% CI [47.0, 57.3]

Posttest (J): (M = 70.4, SD = 2.1, 95% CI [66.1, 74.7]

M(I-J) = -18.2, SD = 3.8, p < .05, 95% CI [-25.7, -10.7] or M(I-J) = 18.2, SD = 3.8, p < .05, 95% CI [10.7, 25.7]

Related to the results of mediation analysis above, we can infer that there is an indirectly significant effect on the increase of high school students' mathematical problem-solving skills using traditional tools (non-DGS) in Group 1: Wilks' lambda value = .763, $F(1,54) = 16.774$, $p < .05$, partial η^2 (effect size) = .237, and observed power = .980. And then, there is an indirectly significant effect on the increase of high school students' mathematical problem-solving skills using Cabri 3D in Group 2: Wilk's lambda value = .696, $F(1,54) = 23.596$, $p < 0.05$, partial η^2 (effect size) = .304, and observed power = .998.

Results:

- Null hypothesis #10 is rejected, that says: There is no significant difference before and after treatment with traditional tools on the increase of high school students' MPS within Group 1.

- Null hypothesis #11 is rejected, that says: There is no significant difference before and after treatment with Cabri 3D on high school students' MPS within Group 2.

- Effect size (partial η^2 = .304) of treatment with Cabri 3D in Group 2 is larger than effect size (partial η^2 = .237) of using traditional tools in Group 1.

Profile Plots

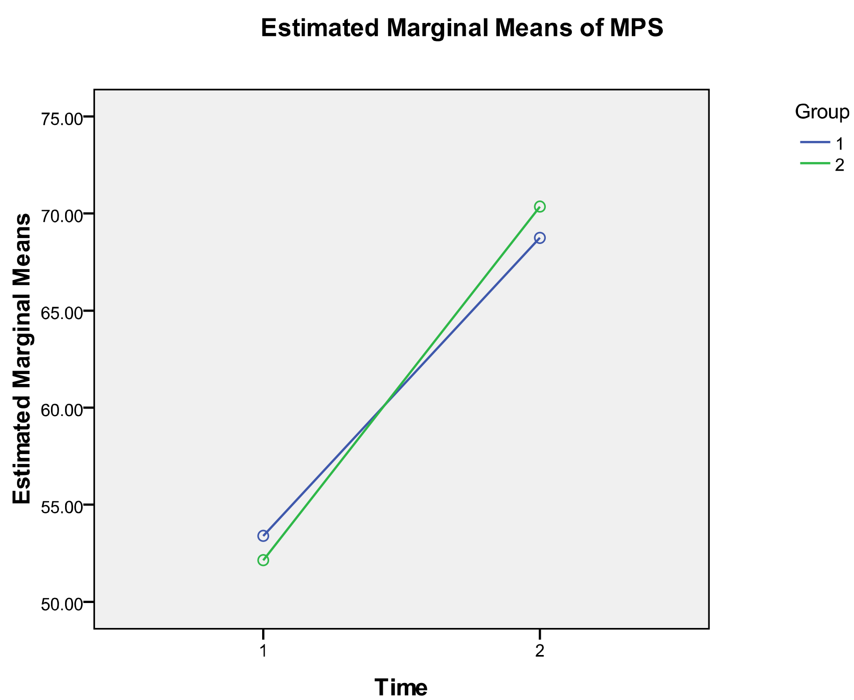

Figure 4.2 Estimated Marginal Means of MPS for Group 1 and Group 2

As aforementioned, the design of this research study is a within-subjects design, and because of that, we have to focus on the testing results of both hypothesis #10 and hypothesis #11 when making conclusions. Therefore, we can conclude that although senior high school students' mathematical problem-solving skills can be indirectly increased by learning and practicing three-dimensional geometry with traditional tools or with Cabri 3D, analyses of Group 1 and Group 2 indicate that students' mathematical problem-solving skills are better increased with Cabri 3D (effect size, partial $\eta^2 = .304$ of rejection to its null hypothesis #11) than by traditional tools (effect size, partial $\eta^2 = .237$ of rejection to its null hypothesis #10). Also, data calculations of the descriptive statistics above indicate that the increase of senior high school students' mathematical problem-solving skills with Cabri 3D is better (about 21 %) than with traditional tools.

If the analyses and testing results of the hypotheses by ANOVA combined with the testing results of PROCESS, we can further conclude that the increase of students' mathematical problem-solving skills caused by the indirect effect of the achievement of learning and practicing three-dimensional geometry with Cabri 3D mediated by the enhanced spatial visualization skills.

Students' Feedback: Questionnaires

In the final step of experimentation, the researcher sent questionnaire-1 to 28 students in the Control Group and sent questionnaire-2 to 28 students in the Treatment Group. The results are 100% of the questionnaires were successfully returned by both groups.

Scores of answers:

Strongly disagree : 1

Disagree : 2

Don't know : 3

Agree : 4

Strongly agree : 5

Questionnaire 1

Table 4.38 Control Group – Questionnaire 1

No.	Statements	Strongly disagree	Disagree	Don't know	Agree	Strongly agree
1	Learning 3D geometry with traditional tools (paper-pencil) can develop my Spatial Visualization Skills especially in developments, rotations, and views	0	4	1	20	3
2	After learning 3D geometry with traditional tools (paper-pencil) to develop Spatial Visualization Skills, I feel my Mathematical Problem-Solving skills increase	0	1	6	17	4
3	Learning 3D geometry with traditional tools (paper-pencil) is interesting	1	6	2	15	4

4	Doing transformations of 3D geometry (reflections, rotations, dilations, and translations) with traditional tools (paper-pencil) is easy	1	10	0	15	2
5	Learning 3D geometry with traditional tools (paper-pencil) helps do explorations, make conjectures, and executing the process of verifications	0	3	2	19	4

Scores of answers on Control Group:

1. Learning 3D geometry with traditional tools (paper-pencil) can develop my Spatial Visualization Skills especially in developments, rotations, and views: 0 + 8 + 3 + 80 + 15 = 106

2. After learning 3D geometry with traditional tools (paper-pencil) to develop Spatial Visualization Skills, I feel my Mathematical Problem-Solving skills increase: 0 + 2 + 18 + 68 + 20 = 108

3. Learning 3D geometry with traditional tools (paper-pencil) is interesting: 1 + 12 + 6 + 60 + 20 = 99

4. Doing transformations of 3D geometry (reflections, rotations, dilations, and translations) with traditional tools (paper-pencil) is easy: 1 + 20 + 0 + 60 + 10 = 91

5. Learning 3D geometry with traditional tools (paper-pencil) is helpful in doing explorations, making conjectures, and executing process of verifications: 0 + 6 + 6 + 76 + 20 = 108

Questionnaire 2

Table 4.39 Treatment Group – Questionnaire 2

No.	Statements	Strongly disagree	Disagree	Don't know	Agree	Strongly agree
1	Learning 3D geometry with Cabri 3D can develop my Spatial Visualization Skills especially in developments, rotations, and views	0	0	1	21	6
2	After having treatment to develop Spatial Visualization Skills with Cabri 3D, I feel my Mathematical Problem-Solving skills increase	0	1	8	10	9
3	Learning 3D geometry with Cabri 3D is interesting	0	0	2	12	14

4	Doing transformations of 3D geometry (reflections, rotations, dilations, and translations) with Cabri 3D is easy	0	0	3	10	15
5	Learning 3D geometry with Cabri 3D helps do explorations, make conjectures, and executing the process of verifications	0	0	5	12	11

Scores on Treatment Group:

1. Learning 3D geometry with Cabri 3D can develop my Spatial Visualization Skills especially in developments, rotations, and views: 0 + 0 + 3 + 84 + 30 = 117
2. After having treatment to develop Spatial Visualization Skills with Cabri 3D, I feel my Mathematical Problem-Solving skills increase: 0 + 2 + 24 + 40 + 45 = 111
3. Learning 3D geometry with Cabri 3D is interesting: 0 + 0 + 6 + 48 + 70 = 124
4. Doing transformations of 3D geometry (reflections, rotations, dilations, and translations) with Cabri 3D is easy: 0 + 0 + 9 + 40 + 75 = 124
5. Learning 3D geometry with Cabri 3D is helpful in doing explorations, making conjectures, and executing process of verifications: 0 + 0 + 15 + 48 + 55 = 118

Results of the questionnaires above indicate that all scores of all items in the questionnaire for the Treatment Group are higher than all scores of all items in the questionnaire for the Control Group. In other words, the answers to these questionnaires can be said as students' feedback and the answers show that, in this experimental research, the Treatment Group achieved skills more than the Control Group. These results also support the results of statistical analyses as written above.

CHAPTER 5

DISCUSSION, CONCLUSIONS, AND RECOMMENDATIONS

The purpose of this study is to investigate whether or not learning and practicing three-dimensional geometry with Cabri 3D contributes to the development of and enhances senior high school students' spatial visualization skills as a precursor to spatial visualization skills mediating the effect of achievement of learning three-dimensional geometry with Cabri 3D to the increase of mathematical problem-solving skills

The research design of this research study is a within-subjects design that includes an examination of mediating relationships. The presentation of the between-effects of treatment in data analyses in chapter 4 was conducted to confirm the existence of the significant effects of treatment between both groups. However, the detailed results of the significant effects of the treatment are found solely in the within-subjects analyses for each group.

The first section of this chapter is to find the answers to the research questions in chapter 3. Then discussion, conclusions, and recommendations. However, before answering the research questions and discuss the testing results of the hypotheses, the researcher needs to restate the research questions and the hypotheses as follows:

Research Questions and Hypotheses

The objectives of this experimental research are: for finding an effective way-and-tool for improving spatial visualization skills and increasing mathematical problem-solving skills, for finding intervening variable between the causal relationship of the way-and-tool and mathematical problem-solving skills, and for investigating the achievement of learning and practicing three-dimensional geometry as a way for improving spatial visualization skills and increasing mathematical problem-solving skills.

Research Questions

Primary research question and its five sub-research-questions are presented as follows:

Primary research question:

"To what extent does the development of spatial visualization skills by learning and practicing three-dimensional geometry with Cabri 3D affect the increase of mathematical problem-solving skills?"

Sub-questions:

- Sub-question #1: Is Cabri 3D (DGS) better than traditional tools (non-DGS) for learning and practicing three-dimensional geometry in the context of developing and enhancing spatial visualization skills?
- Sub-question #2: Is there a direct relationship between the achievement of learning-and-practicing three-dimensional geometry with Cabri 3D and mathematical problem-solving skills?

- Sub-question #3: Is there an indirect relationship between the achievement of learning-and-practicing three-dimensional geometry with Cabri 3D and mathematical problem-solving skills mediated by spatial visualization skills?

- Sub-question #4: Is there a relationship between spatial visualization skills and mathematical problem-solving skills?

- Sub-question #5: Is there an increase in mathematical problem-solving skills after treatment?

Hypotheses

These hypotheses formed are for investigating the effects of using traditional tools (non-DGS) and Cabri 3D (DGS) on senior high school students' SVS and MPS as well as for investigating mediation effects and relationship between senior high school students' achievement in learning and practicing three-dimensional geometry with Cabri 3D and their MPS mediated by their SVS.

Table 5.1 Hypotheses

H#	Hypotheses
H1	Null hypothesis H_0: $\mu_1 = \mu_2$: There is no significant difference in the results of pretests of SVS between Group 1 and Group 2; and its alternative hypothesis is H_1: $\mu_1 \neq \mu_2$ (where μ_1 and μ_2 are the means of the SVS tests that the control and treatment groups have).
H2	Null hypothesis H_0: $\mu_1 = \mu_2$: There is no significant difference in the results of posttests of SVS between Group 1 and Group 2; and its alternative hypothesis is

H_1: $\mu_1 \neq \mu_2$ (where μ_1 and μ_2 are the means of the SVS tests that the control and treatment groups have).

H3 Null hypothesis H_0: $\mu_1 = \mu_2$: There is no significant difference before and after treatment using traditional tools on high school students' SVS within Group 1; and its alternative hypothesis is H_1: $\mu_1 \neq \mu_2$ (where μ_1 and μ_2 are the means of the pretest and posttest of SVS that the control group has).

H4 Null hypothesis H_0: $\mu_1 = \mu_2$: There is no significant difference before and after treatment using Cabri 3D on high school students' SVS within Group 2; and its alternative hypothesis is H_1: $\mu_1 \neq \mu_2$ (where μ_1 and μ_2 are the means of the pretest and posttest of SVS that the treatment group has).

H5 Null hypothesis H_0: c' = 0

There is no direct effect from C3D to MPS independence of SVS.

Alternative H_a: c' $\neq$ 0

H6 Null hypothesis H_0: c' = 0

There is no indirect effect from C3D to the MPS dependence of SVS.

Alternative H_a: c' $\neq$ 0

H7 Null hypothesis H_0: There is no relationship between high school students' SVS and their MPS.

H8 Null hypothesis H_0: $\mu_1 = \mu_2$: There is no significant difference in the results of pretests of MPS between Group 1 and Group 2; and its alternative hypothesis is H_1: $\mu_1 \neq \mu_2$ (where μ_1 and μ_2 are the means of MPS tests that the control and treatment groups have).

H9 Null hypothesis H_0: $\mu_1 = \mu_2$: There is no significant difference in the results of posttests of MPS between Group 1 and Group 2; and its alternative hypothesis is H_1: $\mu_1 \neq \mu_2$ (where μ_1 and μ_2 are the means of MPS tests that the control and treatment groups have).

H10	Null hypothesis H_0: $\mu_1 = \mu_2$: There is no significant difference before and after treatment with traditional tools on high school students' MPS within Group 1; and its alternative hypothesis is H_1: $\mu_1 \neq \mu_2$ (where μ_1 and μ_2 are the means of pretest and posttest of MPS that the control group has).
H11	Null hypothesis H_0: $\mu_1 = \mu_2$: There is no significant difference before and after treatment with Cabri 3D on high school students' MPS within Group 2; and its alternative hypothesis is H_1: $\mu_1 \neq \mu_2$ (where μ_1 and μ_2 are the means of pretest and posttest of MPS that the treatment group has).

H#: Hypothesis number

The answer to the primary research question is the conclusion of the answers to its five sub-questions as formulated below:

- Sub-question #1: For finding the answer to this question, we need to infer the results of the tests of hypotheses H1, H2, H3, and H4 as shown in Table 5.2 below.
- Sub-question #2: For finding the answer to this question, we need to infer the result of the test of hypothesis H5 as shown in Table 5.2 below.
- Sub-question #3: For finding the answer to this question, we need to infer the result of the test of hypothesis H6 as shown in Table 5.2 below.
- Sub-question #4: For finding the answer to this question, we need to infer the result of the test of hypothesis H7 as shown in Table 5.2 below.

- Sub-question #5: For finding the answer to this question, we need to infer the results of the tests of hypotheses H8, H9, H10, and H11 as shown in Table 5.2 below.

Table 5.2 Results of the Tests of Hypotheses

H#	Hypotheses	Test Results
H1	There is no significant difference in the results of the pretests of SVS between Group 1 and Group 2	This hypothesis cannot be rejected. It means that, before treatments, spatial visualization skills of Control Group and Treatment Groups are significantly the same. These results are good for starting treatments in experimental research.
H2	There is no significant difference in the results of posttests of SVS between Group 1 and Group 2	This hypothesis cannot be rejected. It means that, after treatments, spatial visualization skills of Control Group and Treatment Group are significantly the same. In other words, traditional tools and Cabri 3D can significantly develop spatial visualization skills.
H3	There is no significant difference before and after treatment using traditional tools on high school students' SVS within Group 1	This hypothesis is rejected. It means that the posttest result after treatment with traditional tools in Control Group is different from its pretest before treatment. In other words, traditional tools can significantly develop spatial visualization skills.

H4	There is no significant difference before and after treatment using Cabri 3D on high school students' SVS within Group 2	This hypothesis is rejected. It means that Cabri 3D can significantly develop spatial visualization skills in the Treatment Group.
H5	There is no direct effect from C3D to MPS independence of SVS	This hypothesis cannot be rejected. It means that there is a significant indirect effect from C3D to MPS mediated by SVS. In other words, the treatment by Cabri 3D will significantly affect the development and enhancement of spatial visualization skills (SVS) and then the enhanced SVS will affect the increase of mathematical problem-solving skills.
H6	There is no indirect effect from C3D to MPS dependence of SVS.	This hypothesis is rejected. This result significantly confirms the result of the test of hypothesis #5 which states that there is a significant indirect effect from C3D to MPS mediated by SVS.
H7	There is no relationship between high school students' SVS and their MPS	This hypothesis is rejected. It means that there is a significant relationship between spatial visualization skills and mathematical problem-solving skills
H8	There is no significant difference in the results of the pretests of MPS between Group 1 and Group 2	This hypothesis cannot be rejected. It means that pretests of MPS in Group 1 and Group 2 have significantly the same results. These results are good for

		starting treatments in experimental research.
H9	There is no significant difference in the results of posttests of MPS between Group 1 and Group 2	This hypothesis cannot be rejected. It means that posttests of MPS in Group 1 and Group 2 have the same results. In other words, after treatments, traditional tools and Cabri 3D can significantly affect the increase of MPS in Group 1 and Group 2.
H10	There is no significant difference before and after treatment with traditional tools on high school students' MPS within Group 1	This hypothesis is rejected. It means that treatment with traditional tools gives a significant effect on the increase of students' MPS in Group 1.
H11	There is no significant difference before and after treatment with Cabri 3D on high school students' MPS within Group 2	This hypothesis is rejected. It means that treatment with Cabri 3D gives a significant effect on the increase of students' MPS in Group 2.

H#: Hypothesis number

Table 5.3 Results of the Descriptive Statistics Calculations

#	Group 1	Group 2	Results
1	Spatial Visualization Skills' (SVS') scores increase: 8.1 points	Spatial Visualization Skills' (SVS') scores increase: 17.9 points	Scores average increase: 13.0 points Group 1 < 13.0 points Group 2 > 13.0 points Group 2 is better (about 147%) than Group 1

2	Mathematical Problem-Solving Skills' (MPS') scores increase: 15.4 points	Mathematical Problem-Solving Skills' (MPS') scores increase: 18.3 points	Scores average increase: 16.8 points Group 1 < 16.8 points Group 2 > 16.8 points Group 2 is better (about 21%) than Group 1

#: Number

Revisiting the Research Questions

The primary research question is:

"To what extent does the development of spatial visualization skills by learning and practicing three-dimensional geometry with Cabri 3D affect the increase of mathematical problem-solving skills?"

As noted above, it is important to answer the five sub-questions before answering the primary research question.

The results of the analyses of the sub-questions are summarized below:

- Sub-question #1: Is Cabri 3D (DGS) better than traditional tools (non-DGS) for learning and practicing three-dimensional geometry in the context of developing and enhancing spatial visualization skills?

  Answer: The results of the tests of the hypotheses H1, H2, H3, and H4 indicate that Cabri 3D (DGS) and traditional tools (non-DGS) can significantly develop *and enhance spatial visualization skills. However, the result of the calculation of descriptive statistics in developing and enhancing spatial visualization skills in Chapter 4 indicates that the result of treatment with Cabri 3D (DGS) on Group 2 is better (about 147%) than the result of treatment with traditional tools (non-DGS) on Group 1. And also, from the results of the tests of hypotheses, the researcher finds that Cabri 3D is a more effective and better tool than traditional tools for developing and enhancing spatial visualization skills.*

- Sub-question #2: Is there a direct relationship between learning and practicing three-dimensional geometry with Cabri 3D and mathematical problem-solving skills?

  Answer: The result of the test of hypothesis H5 indicates that there is no direct relationship or direct effect between learning and practicing three-dimensional geometry with Cabri 3D and mathematical problem-solving skills. There are only two options in mediation analysis. If the result of the test of hypothesis indicates the direct relationship is refused then the indirect relationship must be accepted. Besides that, according to the result of the test of hypothesis in chapter 4, the indirect relationship is not refused. Thus, statistically, the mediation analysis implies that there is an indirect relationship between both variables. Hence, there is an indirect relationship between both learning-and-practicing three-dimensional geometry with Cabri 3D and mathematical *problem-solving skills, and the mediator of the indirect relationship is spatial visualization skills.*

- Sub-question #3: Is there an indirect relationship between learning and practicing three-dimensional geometry with Cabri 3D and mathematical problem-solving skills?

  Answer: The result of the test of hypothesis H6 is to confirm that there is an indirect relationship or an indirect effect from learning and practicing three-dimensional geometry with Cabri 3D to the increase of mathematical problem-solving skills mediated by the enhanced spatial visualization skills. From the result of the test of the hypothesis, the researcher finds the medial system with DGS for increasing mathematical problem-solving skills. DGS like Cabri 3D is a tool for better and effectively developing and enhancing spatial visualization skills, in turn, the enhanced spatial visualization skills will affect the increase of mathematical problem-solving skills.

- Sub-question #4: Is there a relationship between spatial visualization skills and mathematical problem-solving skills?

  Answer: The result of the test of hypothesis H7 indicates that there is a significant relationship between spatial visualization skills and mathematical problem-solving skills. The result of the test will further explain Bishop's theory about the relationship between spatial training and mathematics achievement. Mathematics achievement as Bishop's theory states is still general. The spatial *training or treatment will directly affect mathematical problem-solving skills or ability because mathematical problem-solving skills or ability is the processor for attaining and defending mathematics achievement. Higher mathematical problem-solving skills will result in higher mathematics achievement.*

- Sub-question #5: Is there an increase in mathematical problem-solving skills after treatment?

  Answer: The results of the tests of hypotheses H8, H9, H10, and H11 indicate that treatment with traditional tools on Group 1 and treatment with Cabri 3D on Group 2 significantly affect the increase of mathematical problem-solving skills. The effects are not direct but indirect as stated in the answers of sub-questions #2 and #3. The treatment effects need a mediator of spatial visualization skills for continuing their effects on mathematical problem-solving skills. The calculation in descriptive statistics shows that the treatment effects with Cabri 3D (DGS) in the increase of mathematical problem-solving skills are better (about 21%) than the treatment effects with traditional tools (non-DGS). The results of the tests of hypotheses also confirm the answer to sub-question #1 that mentions that Cabri 3D is a more effective and better tool for developing and enhancing spatial visualization skills.

Then, the answer to the primary research question will be extracted from the answers of its five sub-questions. Nevertheless, before answering the primary research question, the researcher needs to explain how the five sub-questions inform what comes next.

- The answer to sub-question #1 will inform to what extent Cabri 3D is effective compared to traditional tools for developing and enhancing spatial visualization skills

- The answer to sub-question#2 will reveal that the relationship between practicing three-dimensional geometry and learning achievement is as an indirect relationship that needs a mediator of spatial visualization skills.

- The answer to sub-question #3 confirms the finding of the medial system with DGS for increasing mathematical problem-solving skills. Any DGS, with at least the same capacity as Cabri 3D, can be used for the next study of developing spatial visualization skills and increasing mathematical problem-solving skills.

- The answer to sub-question #4 will elaborate on the relationship between spatial training and mathematics achievement as stated in Bishop's theory, which explains that there is a relationship between the development of spatial visualization skills and an increase in mathematical problem-solving skills.

- The answer to sub-question #5 will inform the indirect effect of treatment with Cabri 3D to the increase of mathematical problem-solving skills. Cabri 3D is a powerful tool in DGS for learning and practicing three-dimensional geometry in the context of developing spatial visualization skills and increasing mathematical-problem solving skills.

The answer to the primary research question as follows:

"To what extent does the development of spatial visualization skills by learning and practicing three-dimensional geometry with Cabri 3D affect the increase of mathematical problem-solving skills?"

Answers: The answers to this research question are based on the answers to its five sub-questions and statistical analyses in the previous chapter.

- *Using Cabri 3D software (DGS) for learning and practicing three-dimensional geometry can better develop and enhance spatial visualization skills (about 147%) than using traditional tools (non-DGS). In Chapter 2, Rogora (2006) made research using Cabri 3D for teaching geometry, and they wrote that "Cabri 3D is potentially very useful software for learning and teaching geometry" (p. 11). Cabri 3D is a software tool and the interactive spatial geometry that can overcome the difficulties of constructions in three-dimensional geometry. Cabri 3D won the BETT Award 2007. This research study used a Cabri 3D version as the winner of BETT Award 2007. (BETT: British Educational Training and Technology). Nevertheless, other DGS that at least have the same capacity as Cabri 3D can be used for such this research study as well.*

- *Achievement of learning and practicing three-dimensional geometry with Cabri 3D does indirectly affect the increase of mathematical problem-solving skills. However, the increase of mathematical problem-solving skills has a relationship with the development and enhancement of spatial visualization skills with Cabri*

3D. The increase of mathematical problem-solving skills in this study is caused by mediation effects (as proved by the tests of hypotheses).

- *Mediation analysis in chapter 4 has proved that the achievement of learning and practicing three-dimensional geometry with Cabri 3D indirectly affects mathematical problem-solving skills that are mediated by spatial visualization skills with effect size = .203 (medium size according to Cohen's category in Chapter 3). Other statistical analyses in SVS and MPS in chapter 4 also prove that Cabri 3D significantly enhance spatial visualization skills and increase mathematical problem-solving skills.*

- *The relationship between the enhanced spatial visualization skills and the increased mathematical problem-solving skills are significant as well. So, all of the statistical analyses in Chapter 4 prove that the increase of mathematical problem-solving skills has a relationship with the development and enhancement of spatial visualization skills in the context of treatment with Cabri 3D and traditional tools.*

- *Further mediation analysis shows that there is a significant mediation relationship among the three variables (C3D, SVS, and MPS) with effect size = .203 and Kappa-squared = 18.3%. The results mean that the observed indirect effect of .203 is about 18.3% of its maximum possible value, given the association between the variables observed in the sample. The results of the descriptive statistics calculations indicate that treatment with Cabri 3D (DGS) can indirectly affect the increase of mathematical problem-solving skills. The increasing effects are higher (about 21%) than treatment with traditional tools (non-DGS).*

Discussion and Conclusions

This section will describe the effects of treatment on spatial visualization skills, mediation analysis, the effects of treatment on mathematical problem-solving skills, conclusions, and the results of questionnaires.

Effects of Treatment on Spatial Visualization Skills

For investigating the effects of treatments on senior high school students' spatial visualization skills, the researcher needed to form two groups, namely the first group is Control Group (Group 1) and the second group is Treatment Group (Group 2):

1) Control Group used traditional tools (paper-pencil tools) for learning and practicing three-dimensional geometry.

2) Treatment Group used Cabri 3D for learning and practicing three-dimensional geometry.

The Control Group and the Treatment Group must have the scores of pretests and posttests of PSVT (Purdue Spatial Visualization Skills Test) that consists of three skills e.g.: developments, rotations, and views.

The experiments on the Control Group and the Treatment Group generated similar results. The result indicates that learning and practicing three-dimensional geometry with Cabri 3D and with traditional tools (paper-pencil tools) don't have different effects. In other words, Cabri 3D (DGS) and traditional tools (non-DGS) can be used for developing and enhancing senior high school students' spatial visualization skills.

Although the treatments with Cabri 3D (DGS) and traditional tools (non-DGS) have the same significant result, exactly the effects are not the same. Within-effects in the Treatment Group (effect size = .572 and observed power = 1.000) are much higher than within-effects in the Control Group (effect size = .217 and observed power = .967). Hence, in the context of developing and enhancing senior high school students' spatial visualization skills, Cabri 3D (DGS) is a better and more effective tool than traditional tools (non-DGS). The effect size of Cabri 3D treatment, as shown above, is much higher than the effect size of traditional tools. Cabri 3D is a software tool and interactive spatial geometry. Cabri 3D can overcome the difficulties of constructions in three-dimensional geometry. Further data calculations on the descriptive statistics also demonstrate that the increase of the Treatment Group's spatial visualization skills is much higher (about 147%) than the increase of the Control Group's spatial visualization skills.

Mediation Analysis

Mediation analysis in this study is used for analyzing the mediation relationship between senior high school students' achievement in learning and practicing three-dimensional geometry with Cabri 3D and the increase of their mathematical problem-solving skills mediated by their enhanced spatial visualization skills. As mentioned previously, spatial visualization skills in this study were measured by the test of PSVT; achievement of learning and practicing three-dimensional geometry with Cabri 3D was measured by the test of Cabri 3D (C3D), and mathematical problem-solving skills were measured by the test of TEA.

Regression analysis in chapter 4 indicates that there is a significant relationship between posttests of MPS and SVS in the Treatment Group. Hence, this regression analysis indicates that there is a significant relationship between enhanced spatial visualization skills and increased mathematical problem-solving skills. Also, mediation analysis in chapter 4 indicates that there is an indirectly significant mediation relationship between the achievement of learning and practicing three-dimensional geometry with Cabri 3 and the increase of mathematical problem-solving skills that is mediated by the enhanced spatial visualization skills.

Logically, **if** the relationship between the enhanced spatial visualization skills (SVS) and the increased mathematical problem problem-solving skills (MPS) are significant where the relationship is in the context of an indirectly significant relationship between the achievement of learning-and-practicing three-dimensional geometry with Cabri 3D (C3D) and the increase of mathematical problem-solving skills (MPS), **then** we can conclude that the increase of mathematical problem-solving skills (MPS) is caused by the achievement of learning and practicing three-dimensional geometry (C3D) that is mediated by the enhanced spatial visualization skills (SVS). As mentioned in the section above, the effect size of its indirect relationship is *.203 (medium effect size).*

Effect of Treatment on Mathematical Problem-solving Skills

The result of the mediation analysis of C3D, SVS, and MPS on Treatment Group indicates that learning and practicing three-dimensional geometry with Cabri 3D can indirectly improve students' mathematical problem-solving skills. And the results of analyses in SVS and MPS on Control Group and Treatment Group indicate that the improvement of students' spatial visualization skills can improve mathematical problem-solving skills.

The improvements in the mathematical problem-solving skills are caused by the treatments with traditional tools on the Control Group and the treatment with Cabri 3D on the Treatment Group. Although both treatment effects are significant, mathematically they are not the same. Their effect sizes and powers are different. Effect size and power in the Treatment Group (effect size = .304 and observed power = .998) are larger than effect size and power in the Control Group (effect size = .237 and observed power = .980). Also, data calculations of the descriptive statistics indicate that the increase of the Treatment Group's mathematical problem-solving skills is higher (about 21%) than the increase of the Control Group's mathematical problem-solving skills.

The result of the mediation analysis of C3D, SVS, and MPS on the Treatment Group (as shown in Chapter 4) indicates that the improvement of mathematical problem-solving skills in the Treatment Group is caused by the improvement of spatial visualization skills. Hence, the improvement of mathematical problem-solving skills can be categorized as indirect effects of the treatments in the context of improving spatial visualization skills and mathematical problem-solving skills.

Conclusions

This study has found that spatial visualization skills can be developed and enhanced by learning and practicing three-dimensional geometry with Cabri 3D. This finding means that Cabri 3D can directly affect spatial visualization skills. Furthermore, spatial visualization skills have a direct relationship with mathematical problem-solving skills. Achievement of learning and practicing three-dimensional geometry with Cabri 3D cannot directly affect the increase of mathematical problem-solving skills. Thus, the relationship among Cabri 3D achievement (C3D), spatial visualization skills (SVS), and mathematical problem-solving skills (MPS) can be stated as a mediation relationship.

Cabri 3D (DGS) is a powerful tool for learning and practicing three-dimensional geometry and is a more effective tool than traditional tools (non-DGS) for directly developing spatial visualization skills and indirectly increasing mathematical problem-solving skills. The effectiveness of Cabri 3D will be able to indirectly affect the increase of students'mathematical problem-solving skills if the teaching and learning of the Cabri 3D program have been specially designed for developing and enhancing the students' spatial visualization skills.

In Chapter 2, brain-based learning theory mentions that spatial ability is the form of memory, and the normal learning process will occur with the brain as a parallel processor. In the theory of multiple intelligences, Gardner asserts that through intelligence, people can perceive and understand the world which allows them to find and resolve problems. Some researchers argue that both concepts (memory and intelligence) are identical, and others claim that both concepts are not identical, though correlated (Cooper, 2015). Intelligence is an intellectual capacity for acquiring and applying knowledge, ability, and skills. Logically, if memory and intelligence are identical or not identical but correlated, then intelligence could also be indirectly increased through developing and enhancing spatial visualization skills. This study will not verify the concept because the researcher doesn't have enough data about the intelligence concept.

Hence, the findings of this research study are:

- First, the enhanced spatial visualization skills mediate the effects of learning and practicing three-dimensional geometry with Cabri 3D to the increased mathematical problem-solving skills. This finding is named "The medial system with DGS for increasing mathematical problem-solving skills."

- Second, there is a significant relationship between spatial visualization skills and mathematical problem-solving skills.

- Third, Cabri 3D is a better and more effective tool than traditional tools (non-DGS) for learning and practicing three-dimensional geometry and developing spatial visualization skills.

- Fourth, the effectiveness of Cabri 3D for learning and practicing three-dimensional geometry can largely affect the development and enhancement of spatial visualizations skills and the increase of mathematical problem-solving skills.

- Fifth, one of the endeavors of how to increase senior high school students' mathematical problem-solving skills is by designing Cabri 3D (DGS) program especially transformational geometry for learning and practicing three-dimensional dynamic geometry in the context of developing and enhancing the students' spatial visualization skills.

Feedback: Results of Questionnaires

Results of questionnaire 1 and questionnaire 2 are summarized in Table 5.4 and Table 5.5 below:

Table 5.4 Results of Questionnaire 1 for Group 1

Q#	Questions	Scores
1	Learning 3D geometry with traditional tools (paper-pencil) can develop my Spatial Visualization Skills especially in developments, rotations, and views	106
2	After learning 3D geometry with traditional tools (paper-pencil) to develop Spatial Visualization Skills, I feel my Mathematical Problem-Solving skills increased	108
3	Learning 3D geometry with traditional tools (paper-pencil) is interesting	99
4	Doing transformations of 3D geometry (reflections, rotations, dilations, and translations) with traditional tools (paper-pencil) is easy	91
5	Learning 3D geometry with traditional tools (paper-pencil) helps do explorations, make conjectures, and executing the process of verifications	108
	Total	512

Q#: Question number

Table 5.5 Results of Questionnaire 2 for Group 2

Q#	Questions	Scores
1	Learning 3D geometry with Cabri 3D can develop my Spatial Visualization Skills especially in developments, rotations, and views	117
2	After having treatment to develop Spatial Visualization Skills with Cabri 3D, I feel my Mathematical Problem-Solving skills increased	111
3	Learning 3D geometry with Cabri 3D is interesting	124
4	Doing transformations of 3D geometry (reflections, rotations, dilations, and translations) with Cabri 3D is easy	124
5	Learning 3D geometry with Cabri 3D helps do explorations, make conjectures, and executing the process of verifications	118
	Total	594

Q#: Question number

Below are two pairs of interesting results of Control Group's questionnaire and Treatment Group's questionnaire. We find two pairs of contradictive results in Control Group and Treatment Group as shown below. The lowest score of the item in Control Group's questionnaire is: Doing transformations of 3D geometry with traditional tools is easy. But conversely, the highest score of the item in Treatment Group's questionnaire is: Doing transformations of 3D geometry with Cabri 3D is easy.

Control Group's scores:

1. Learning 3D geometry with traditional tools (paper-pencil) is interesting: 99
2. Doing transformations of 3D geometry (reflections, rotations, dilations, and translations) with traditional tools (paper-pencil) is easy: 91

Treatment Group's scores:

1. Learning 3D geometry with Cabri 3D is interesting:124
2. Doing transformations of 3D geometry (reflections, rotations, dilations, and translations) with Cabri 3D is easy:124

Hence, the conclusions of the results of the two questionnaires (as shown above) state that transformational three-dimensional geometry as the foundation for developing and enhancing spatial visualization skills is much easier done by Cabri 3D than by traditional tools as well as learning and practicing three-dimensional geometry is more interesting with Cabri 3D than with traditional tools. As observed in the experimental activities and confirmed by the answers of the questionnaires, the Control Group had more difficulty than the Treatment Group in learning and practicing three-dimensional geometry, especially for executing the transformations in three-dimensional space due to the limitations of traditional tools' properties and capabilities. Taken together, the results of the experiment and questionnaires indicate that the Treatment Group outperformed the Control Group in developing and enhancing spatial visualization skills and in increasing mathematical problem-solving skills.

Implications and Recommendations

Building upon the findings from the analyses conducted for this study, there are compelling implications for research, practice, and policy. These implications are respectively addressed in the following three sub-sections.

Implications for Research

As noted in Chapter 2, skills acquired by the processes of learning and intensive training can be considered to be an advanced cognitive ability, and the development of these processes should begin no later than senior high school. Systematically learning and practicing three-dimensional dynamic geometry could develop and transform the natural abilities of spatial visualization and mathematical problem-solving to be spatial visualization skills and mathematical problem-solving skills.

This dissertation research focused on experimental research in developing and increasing spatial visualization skills and mathematical problem-solving skills by learning and practicing three-dimensional geometry with Cabri 3D. This experimental research conducted at one of the prestigious public senior high schools in Indonesia. There are two groups in this experiment, e.g.: Control Group and Treatment Group, and each group consisted of male and female students. Thus, there was no gender classification in this experimental research. So, we need further research by gender classification on senior high school students in Indonesian (and perhaps elsewhere) for investigating which group (male students or female students) has a higher pretest score in spatial visualization skills and mathematical problem-solving skills, and which group (male students or female students) has a higher posttest score or significant improvement in their spatial visualization skills and mathematical problem-solving skills after treatment.

Further research is not only needed for resolving the inconsistency of previous gender-based research results in spatial visualization skills, but also for resolving and improving men's or women's spatial thinking in the context of increasing mathematical problem-solving skills, particularly in the case of Indonesia and other similar contexts. An alternative method for investigating and resolving the inconsistent previous research results would be conducting further research using the same sample sizes, test instruments, and treatment tools as applied in this dissertation research.

Hopefully, from further research, we will acquire new knowledge concerning the differences of gender-based spatial visualization skills and problem-solving skills; if the differences or gaps exist, one may hypothesize such differences or gaps could be caused by the differences in their work and life experiences. Generally, women do much of their activities at home – in small spaces and limited areas, conversely, men do much of their activities in large and open areas, which may affect the development of their spatial visualization skills and problem-solving skills.

Implications for Practice

The findings of this study will not only inform the process of teaching and learning three-dimensional geometry as part of pure mathematics with traditional tools, but also have the potential to inform how we construct the environment for teaching and learning three-dimensional geometry, from non-dynamic geometry environment (non-DGS) to dynamic geometry environment (DGS) since dynamic geometry environment will have a better impact on the increase of spatial visualization skills and mathematical problem-solving skills.

Teaching and learning three-dimensional dynamic geometry for developing and enhancing spatial visualization skills is very important in senior high schools in Indonesia in the context of increasing students' mathematical problem-solving skills. Because of that, teachers and school leaders need to continue to offer a three-dimensional geometry course in schools, upgrade their teaching-and-learning method by using the most recent educational technology, and design a curriculum that can allow students to develop and enhance their spatial visualization skills and reasoning by three-sequential method e.g.: doing explorations, making conjectures, and doing verifications. The National Council of Teachers of Mathematics strongly recommends that geometry instruction should include the study of three-dimensional geometry that provides students with opportunities to use spatial abilities and skills to solve problems (National Council of Teachers of Mathematics, 2000).

Interactive geometry learning tools such as Cabri 3D can help students easily make difficult constructions of spatial geometry on the computer screen and these tools are still not used consistently in Indonesia, nor is there highly developed capacity among teachers to use Cabri 3D or similar digital tools. Therefore, teaching and learning mathematics with the more recent educational technology is necessary to make teaching, learning, and practicing mathematics especially three-dimensional geometry more interesting and easier for students.

Implications for Policy

The implications for policy from these findings should match with the general objective of mathematics education as mentioned in Chapter 2, namely students study geometry within the process of problem-solving, understanding, and explaining the physical world around them. Also, in teaching and learning mathematics, teachers and students should follow and apply the NCTM process standard, e.g. problem solving, reasoning and proof, connections, communications, and representations. For reaching or embodying the objective and for applying the process standard, teachers and students do need educational technology like computers and software.

The findings of this study have further implications for policy as follows:

- First, mathematics educators need to design the course of three-dimensional dynamic geometry not only as part of pure mathematics teaching in class but also as the training system for developing and enhancing spatial visualization skills. Development and enhancement of spatial visualization skills are necessary for increasing mathematical problem-solving skills. This will require national and regional policies that support new supports for pre- and in-service training of teachers.

- Second, policy-makers and school managers at multiple levels (national, regional, and local) need to provide computers and software for teaching and learning dynamic geometry. The need to provide these resources will require increases in funding for schools and teacher training institutions.

- Third, schools need to have not only mathematics teachers who have competency and skills in teaching mathematics with computers and dynamic geometry software but also mathematics teachers that have competency in selecting appropriate educational technology for senior high school students.

- Fourth, colleges of education need to have a course of dynamic geometry as part of the program of teaching and learning mathematics with computers. This program is not only for preparing mathematics teachers that will have competency and skills in teaching mathematics with computers but also for preparing teachers that can make research and access to the updated educational technology information.

The implication for policy stated in this dissertation is to help policymakers in institutions or authorities to make policies for developing and advancing mathematics education in schools and colleges. Also, the policymakers need to think about the schools' needs of the competent mathematics teachers and the most recent educational technology for teaching and learning dynamic geometry in the context of developing students' spatial visualization skills and mathematical problem-solving skills. Furthermore, policymakers should consider how the schools and colleges in their areas can have access to the most recent educational technology.

Recommendations for Practice

The foundation of learning mathematics requires spatial reasoning and the ability to solve problems mathematically. Furthermore, by periodically learning and practicing mathematical problem-solving, gradually the ability becomes skills that can be called mathematical problem-solving skills. The finding of this research study indicates that mathematical problem-solving skills can be increased by enhancing spatial visualization skills, and spatial visualization skills can be developed and enhanced by learning and practicing three-dimensional geometry in the classroom and field.

This recommendation is about developing and increasing mathematical problem-solving skills by learning and practicing three-dimensional geometry with educational technology such as computers and Cabri 3D software. In this context, learning and practicing three-dimensional geometry must consist of developing spatial visualization skills at least in developments, rotations, and views of objects. Therefore, for developing and increasing students' mathematical problem-solving skills, the curriculum of three-dimensional geometry in class must cover the topic of developments, rotations, and views with dynamic geometry software like Cabri 3D. By using the software, teachers and students can easily construct, animate, and manipulate three-dimensional objects that cannot easily be done by traditional tools.

Teaching and learning dynamic geometry should also apply the steps and processes in Van Hiele level theory, as summarized in Chapter 2, which states that in understanding geometry, a person must go through the levels in order. A student cannot be at Van Hiele level n without having gone through level n-1. The Van Hiele level geometry thinking is one of the most popular theoretical frameworks for understanding students' learning process in geometry. Besides, Piaget and Inhelder in Chapter 2 also suggested that the development of perception of geometry is sequential, e.g. from topological to projective and Euclidean geometry. Senior high school students have been in the stage of the formal operational system. In this stage, the students' understanding of geometric concepts continues to develop until they reach complex formal systems of Euclidean geometry.

The most important things to be noted for applying these recommendations at senior high schools are that senior high schools need to have competent teachers in teaching two- and three-dimensional geometry with dynamic geometry software and have enough budget for preparing appropriate educational technology and learning facilities. Conversely, the limitations for applying these recommendations will be experienced by senior high schools that have less budget for preparing such educational technology and facilities.

APPENDICES:

APPENDIX A

RELIABILITY AND VALIDITY OF INSTRUMENTS

Test for Reliability Data-Pilot

Reliability Data-Cabri3D_Pilot

[DataSet0]

Scale: ALL VARIABLES

Case Processing Summary

		N	%
Cases	Valid	30	100,0
	Excluded[a]	0	,0
	Total	30	100,0

a. Listwise deletion based on all variables in the procedure.

Reliability Statistics

Cronbach's Alpha	N of Items
,770	11

Item-Total Statistics

	Scale Mean if Item Deleted	Scale Variance if Item Deleted	Corrected Item-Total Correlation	Cronbach's Alpha if Item Deleted
P1	5,9667	6,447	,445	,750
P2	6,0333	6,378	,437	,751
P3	6,0000	6,552	,376	,758

P4	6,5000	6,603	,429	,752
P5	5,8667	6,740	,397	,756
P6	5,8333	6,695	,475	,750
P7	6,0333	6,447	,406	,755
P8	6,1333	6,326	,428	,752
P9	6,1667	6,420	,384	,758
P10	6,1000	6,162	,507	,742
P11	6,3667	6,516	,376	,759

Reliability Data-MPS_Pilot

[DataSet0]

Scale: ALL VARIABLES

Case Processing Summary

		N	%
Cases	Valid	30	100,0
	Excluded[a]	0	,0
	Total	30	100,0

a. Listwise deletion based on all variables in the procedure.

Reliability Statistics

Cronbach's Alpha	N of Items
,859	20

Item-Total Statistics

	Scale Mean if Item Deleted	Scale Variance if Item Deleted	Corrected Item-Total Correlation	Cronbach's Alpha if Item Deleted

P1	12,5333	17,982	,451	,854
P2	13,0333	17,344	,338	,858
P3	12,5667	17,702	,475	,853
P4	13,0667	16,754	,494	,851
P5	12,7000	17,045	,503	,851
P6	13,2333	17,357	,411	,854
P7	12,5333	17,913	,484	,853
P8	12,7333	16,892	,520	,850
P9	12,7667	17,357	,371	,856
P10	12,7000	16,631	,627	,846
P11	12,7000	17,252	,442	,853
P12	13,0667	16,961	,440	,854
P13	12,8000	17,131	,417	,854
P14	12,7667	16,323	,657	,844
P15	12,8333	17,316	,358	,857
P16	12,5333	17,982	,451	,854
P17	12,5333	17,913	,484	,853
P18	12,7667	17,289	,390	,855
P19	13,3000	17,459	,446	,853
P20	12,7000	17,183	,462	,852

Reliability Data-PSVT_Pilot

[DataSet0]

Scale: ALL VARIABLES

Case Processing Summary

		N	%
Cases	Valid	31	100,0

	Excluded[a]	0	,0
	Total	31	100,0

a. Listwise deletion based on all variables in the procedure.

Reliability Statistics

Cronbach's Alpha	N of Items
,918	36

Item-Total Statistics

	Scale Mean if Item Deleted	Scale Variance if Item Deleted	Corrected Item-Total Correlation	Cronbach's Alpha if Item Deleted
P1	21,8387	69,873	,394	,917
P2	21,9032	69,157	,407	,917
P3	22,1290	67,649	,462	,916
P4	22,3226	67,826	,420	,917
P5	22,0968	67,490	,495	,916
P6	22,0323	66,632	,654	,914
P7	22,2903	67,013	,518	,916
P8	22,0968	68,290	,390	,917
P9	22,1290	66,049	,670	,914
P10	21,9355	68,996	,394	,917
P11	22,1290	67,783	,445	,917
P12	22,4194	68,318	,376	,917
P13	21,9032	69,357	,371	,917
P14	22,0323	67,032	,597	,915
P15	22,0645	68,529	,372	,917
P16	22,2258	66,514	,583	,915
P17	22,2581	66,065	,637	,914

P18	22,3226	67,626	,445	,917
P19	22,2258	66,981	,525	,915
P20	22,2903	68,213	,371	,918
P21	22,1935	67,895	,415	,917
P22	22,3548	68,037	,398	,917
P23	22,0645	67,862	,461	,916
P24	22,4516	68,256	,395	,917
P25	22,1613	66,273	,628	,914
P26	22,1290	68,383	,368	,917
P27	22,0000	68,333	,436	,917
P28	22,0968	68,490	,364	,918
P29	22,4839	67,791	,471	,916
P30	22,2903	65,880	,660	,914
P31	22,0968	67,024	,557	,915
P32	22,1935	67,495	,465	,916
P33	22,0645	67,996	,443	,917
P34	22,0323	67,966	,466	,916
P35	22,0000	68,533	,407	,917
P36	21,8387	69,806	,410	,917

Test for Validity_Data-Pilot

Correlations Data-Cabri3D_Pilot

[DataSet0]

Correlations

		Score_Total
P1	Pearson Correlation	,570
	Sig. (2-tailed)	,001

	N	30
	Pearson Correlation	,571
P2	Sig. (2-tailed)	,001
	N	30
	Pearson Correlation	,516
P3	Sig. (2-tailed)	,004
	N	30
	Pearson Correlation	,545
P4	Sig. (2-tailed)	,002
	N	30
	Pearson Correlation	,509
P5	Sig. (2-tailed)	,004
	N	30
	Pearson Correlation	,569
P6	Sig. (2-tailed)	,001
	N	30
	Pearson Correlation	,546*
P7	Sig. (2-tailed)	,002
	N	30
	Pearson Correlation	,571
P8	Sig. (2-tailed)	,001
	N	30
	Pearson Correlation	,535
P9	Sig. (2-tailed)	,002
	N	30
	Pearson Correlation	,635*
P10	Sig. (2-tailed)	,000
	N	30
P11	Pearson Correlation	,520

	Sig. (2-tailed)	,003
	N	30
	Pearson Correlation	1**
Score_Total	Sig. (2-tailed)	
	N	30

*. Correlation is significant at the 0.05 level (2-tailed).
**. Correlation is significant at the 0.01 level (2-tailed).

Correlations Data-MPS_Pilot

[DataSet0]

Correlations

		Score_Total
P1	Pearson Correlation	,497**
	Sig. (2-tailed)	,005
	N	30
P2	Pearson Correlation	,438
	Sig. (2-tailed)	,015
	N	30
P3	Pearson Correlation	,529
	Sig. (2-tailed)	,003
	N	30
P4	Pearson Correlation	,578
	Sig. (2-tailed)	,001
	N	30
P5	Pearson Correlation	,575**
	Sig. (2-tailed)	,001
	N	30
P6	Pearson Correlation	,491

	Sig. (2-tailed)	,006
	N	30
	Pearson Correlation	,528
P7	Sig. (2-tailed)	,003
	N	30
	Pearson Correlation	,593
P8	Sig. (2-tailed)	,001
	N	30
	Pearson Correlation	,461
P9	Sig. (2-tailed)	,010
	N	30
	Pearson Correlation	,685
P10	Sig. (2-tailed)	,000
	N	30
	Pearson Correlation	,520
P11	Sig. (2-tailed)	,003
	N	30
	Pearson Correlation	,530
P12	Sig. (2-tailed)	,003
	N	30
	Pearson Correlation	,506*
P13	Sig. (2-tailed)	,004
	N	30
	Pearson Correlation	,716*
P14	Sig. (2-tailed)	,000
	N	30
	Pearson Correlation	,454
P15	Sig. (2-tailed)	,012
	N	30

P16	Pearson Correlation	,497**
	Sig. (2-tailed)	,005
	N	30
P17	Pearson Correlation	,528**
	Sig. (2-tailed)	,003
	N	30
P18	Pearson Correlation	,478*
	Sig. (2-tailed)	,007
	N	30
P19	Pearson Correlation	,515
	Sig. (2-tailed)	,004
	N	30
P20	Pearson Correlation	,538
	Sig. (2-tailed)	,002
	N	30
Score_Total	Pearson Correlation	1**
	Sig. (2-tailed)	
	N	30

**. Correlation is significant at the 0.01 level (2-tailed).
*. Correlation is significant at the 0.05 level (2-tailed).

Correlations Data-PSVT_Pilot

[DataSet0]

Correlations

		Score_Total
P1	Pearson Correlation	,419
	Sig. (2-tailed)	,019
	N	31

P2	Pearson Correlation	,440
	Sig. (2-tailed)	,013
	N	31
P3	Pearson Correlation	,506
	Sig. (2-tailed)	,004
	N	31
P4	Pearson Correlation	,469
	Sig. (2-tailed)	,008
	N	31
P5	Pearson Correlation	,537
	Sig. (2-tailed)	,002
	N	31
P6	Pearson Correlation	,684**
	Sig. (2-tailed)	,000
	N	31
P7	Pearson Correlation	,561
	Sig. (2-tailed)	,001
	N	31
P8	Pearson Correlation	,437
	Sig. (2-tailed)	,014
	N	31
P9	Pearson Correlation	,701*
	Sig. (2-tailed)	,000
	N	31
P10	Pearson Correlation	,431
	Sig. (2-tailed)	,016
	N	31
P11	Pearson Correlation	,490
	Sig. (2-tailed)	,005

	N	31
	Pearson Correlation	,425
P12	Sig. (2-tailed)	,017
	N	31
	Pearson Correlation	,406
P13	Sig. (2-tailed)	,024
	N	31
	Pearson Correlation	,631
P14	Sig. (2-tailed)	,000
	N	31
	Pearson Correlation	,418
P15	Sig. (2-tailed)	,019
	N	31
	Pearson Correlation	,622
P16	Sig. (2-tailed)	,000
	N	31
	Pearson Correlation	,672
P17	Sig. (2-tailed)	,000
	N	31
	Pearson Correlation	,492
P18	Sig. (2-tailed)	,005
	N	31
	Pearson Correlation	,567**
P19	Sig. (2-tailed)	,001
	N	31
	Pearson Correlation	,422
P20	Sig. (2-tailed)	,018
	N	31
P21	Pearson Correlation	,464

	Sig. (2-tailed)	,009
	N	31
	Pearson Correlation	,447
P22	Sig. (2-tailed)	,012
	N	31
	Pearson Correlation	,504
P23	Sig. (2-tailed)	,004
	N	31
	Pearson Correlation	,442
P24	Sig. (2-tailed)	,013
	N	31
	Pearson Correlation	,663
P25	Sig. (2-tailed)	,000
	N	31
	Pearson Correlation	,417
P26	Sig. (2-tailed)	,020
	N	31
	Pearson Correlation	,477
P27	Sig. (2-tailed)	,007
	N	31
	Pearson Correlation	,412
P28	Sig. (2-tailed)	,021
	N	31
	Pearson Correlation	,513
P29	Sig. (2-tailed)	,003
	N	31
	Pearson Correlation	,693
P30	Sig. (2-tailed)	,000
	N	31

	Pearson Correlation	,595**
P31	Sig. (2-tailed)	,000
	N	31
	Pearson Correlation	,511
P32	Sig. (2-tailed)	,003
	N	31
	Pearson Correlation	,486
P33	Sig. (2-tailed)	,006
	N	31
	Pearson Correlation	,507**
P34	Sig. (2-tailed)	,004
	N	31
	Pearson Correlation	,449
P35	Sig. (2-tailed)	,011
	N	31
	Pearson Correlation	,435
P36	Sig. (2-tailed)	,015
	N	31
	Pearson Correlation	1*
Score_Total	Sig. (2-tailed)	
	N	31

*. Correlation is significant at the 0.05 level (2-tailed).
**. Correlation is significant at the 0.01 level (2-tailed).

r_{tabel} Distribution for Significance 5% and 1%

N	The Level of Significance		N	The Level of Significance	
	5%	1%		5%	1%
3	0.997	0.999	38	0.320	0.413
4	0.950	0.990	39	0.316	0.408
5	0.878	0.959	40	0.312	0.403
6	0.811	0.917	41	0.308	0.398
7	0.754	0.874	42	0.304	0.393
8	0.707	0.834	43	0.301	0.389
9	0.666	0.798	44	0.297	0.384
10	0.632	0.765	45	0.294	0.380
11	0.602	0.735	46	0.291	0.376
12	0.576	0.708	47	0.288	0.372
13	0.553	0.684	48	0.284	0.368
14	0.532	0.661	49	0.281	0.364
15	0.514	0.641	50	0.279	0.361
16	0.497	0.623	55	0.266	0.345
17	0.482	0.606	60	0.254	0.330
18	0.468	0.590	65	0.244	0.317
19	0.456	0.575	70	0.235	0.306
20	0.444	0.561	75	0.227	0.296
21	0.433	0.549	80	0.220	0.286
22	0.432	0.537	85	0.213	0.278

23	0.413	0.526	90	0.207	0.267
24	0.404	0.515	95	0.202	0.263
25	0.396	0.505	100	0.195	0.256
26	0.388	0.496	125	0.176	0.230
27	0.381	0.487	150	0.159	0.210
28	0.374	0.478	175	0.148	0.194
29	0.367	0.470	200	0.138	0.181
30	0.361	0.463	300	0.113	0.148
31	0.355	0.456	400	0.098	0.128
32	0.349	0.449	500	0.088	0.115
33	0.344	0.442	600	0.080	0.105
34	0.339	0.436	700	0.074	0.097
35	0.334	0.430	800	0.070	0.091
36	0.329	0.424	900	0.065	0.086
37	0.325	0.418	1000	0.062	0.081

APPENDIX B

TEST OF PSVT – SVS

PURDUE SPATIAL VISUALIZATION TEST

Roland Guay, PhD

Do NOT open this booklet until you are instructed to do so.

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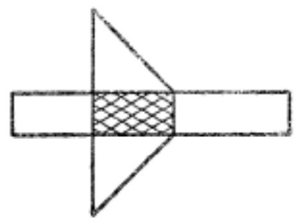
Answers A, C, D, and E are wrong. Only object B can be made by folding the given development. In all three sections of this test, each question has only one correct answer.

Now look at the next example shown below and try to select the one three-dimensional object that can be made when the given development is folded. Remember that the development shows the inside of the object and the development's shaded portion indicates the bottom of the object.



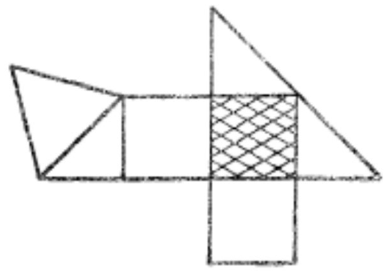
A B C D E

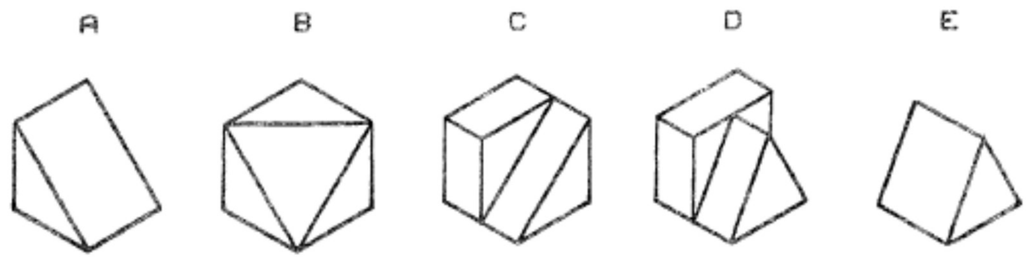
The correct answer for this example is E.

During the test you are to show your choices on the answer card by making a heavy black mark in the space with the same letter as the answer you choose.

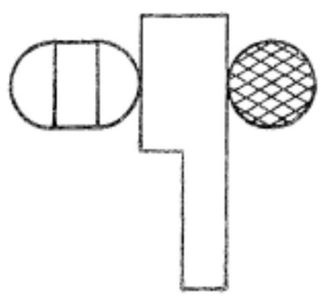
Do NOT make any marks in this booklet. Mark your answers on the separate answer card. You will be told when to begin.

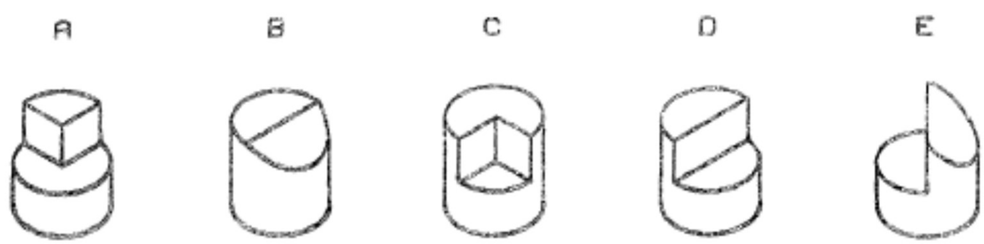
1



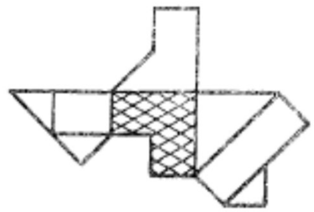


2



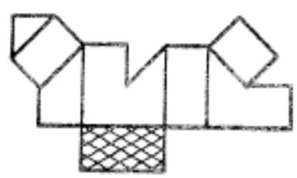


3



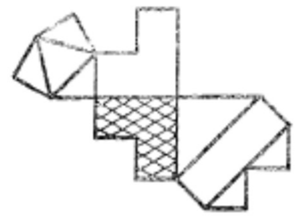
A B C D E

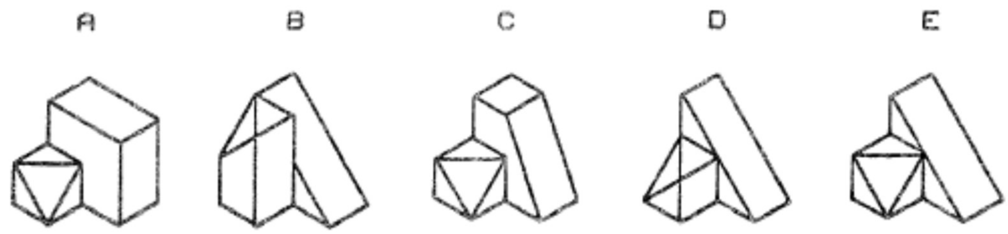
4



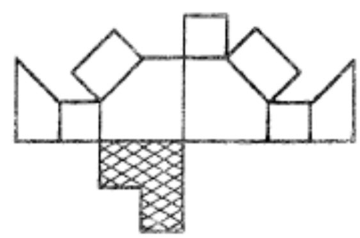
A B C D E

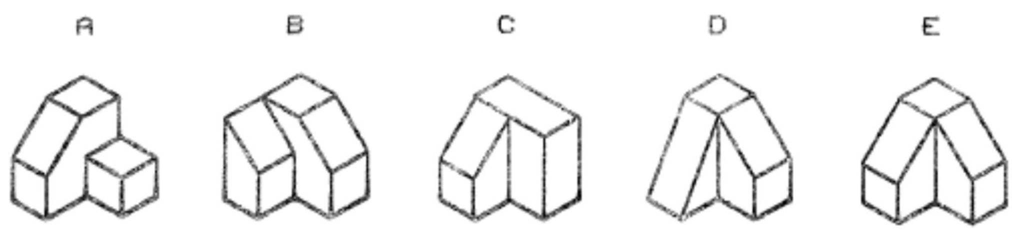
5



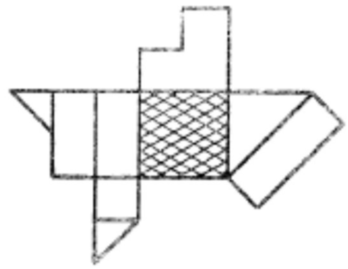


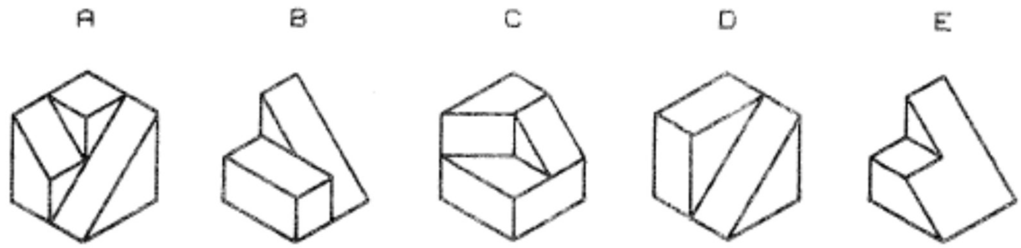
6



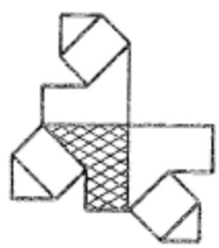


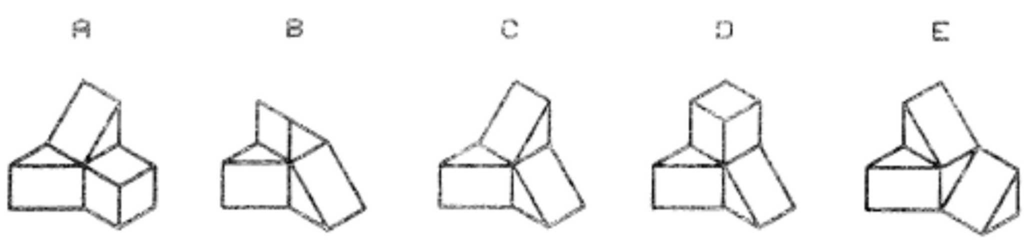
7



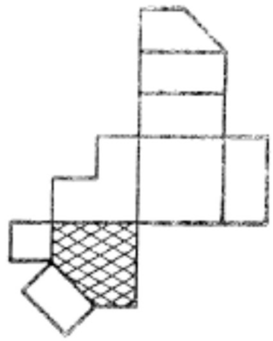


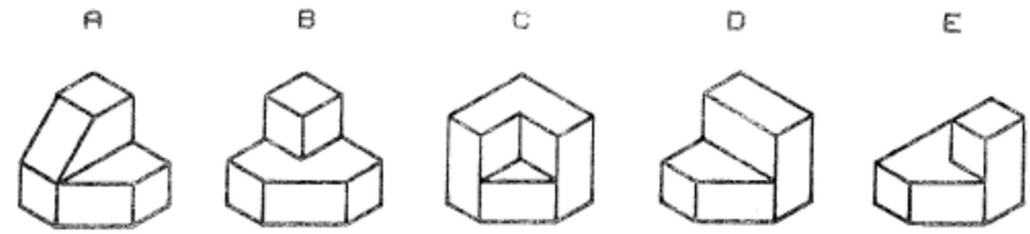
8



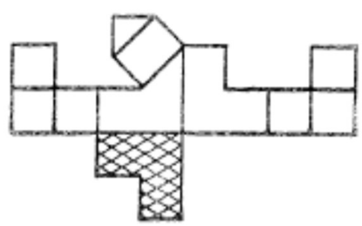


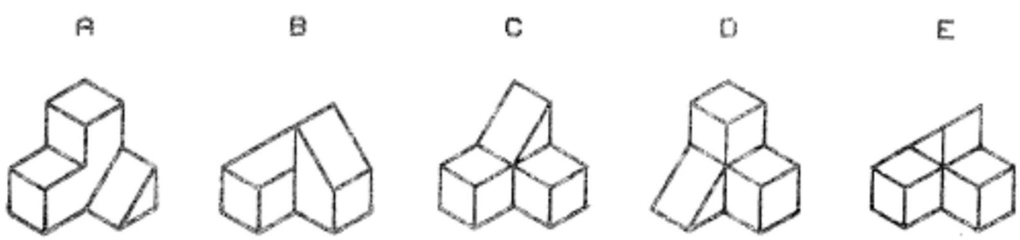
9



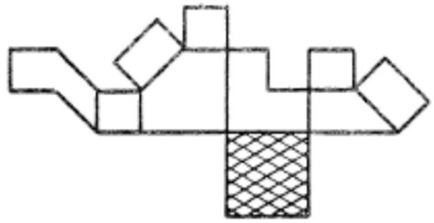


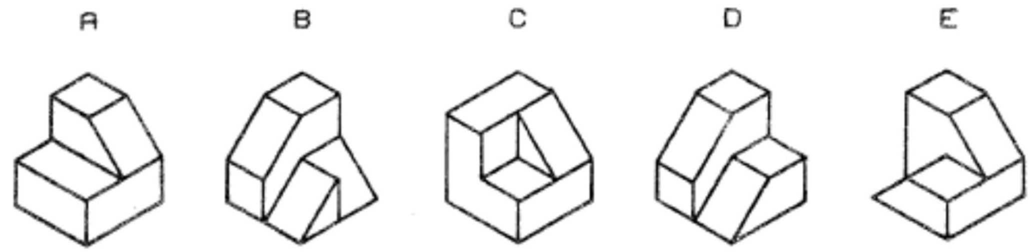
10



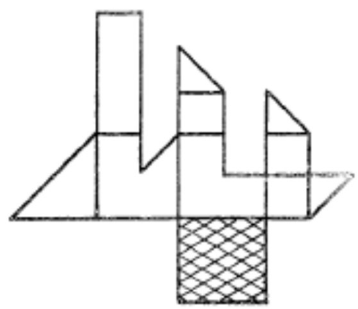


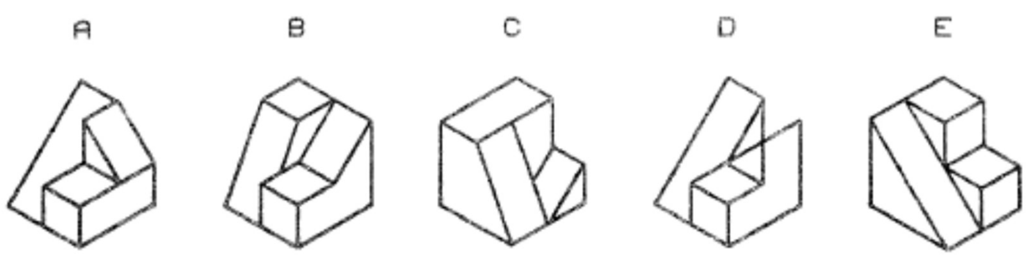
11





12





STOP !

You have completed this section of the test.

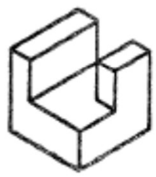
Do <u>NOT</u> go on to the next section
until you are instructed to do so.

If you have time remaining,
go back and check your answers.

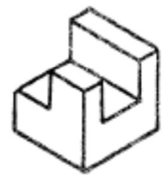
SECTION 2: ROTATIONS

Directions

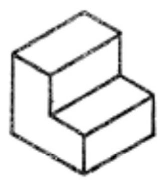
The second section consists of 12 questions designed to see how well you can visualize the rotation of three-dimensional objects. Shown below is an example of the type of question included in the second section.



IS ROTATED TO

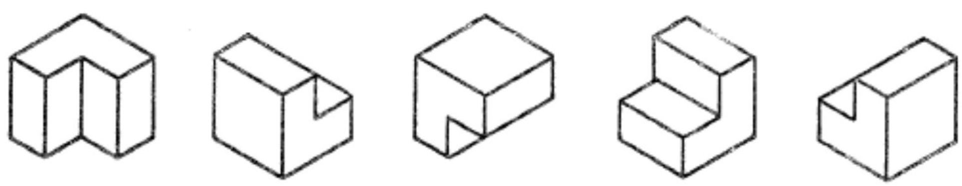


AS



IS ROTATED TO

A B C D E



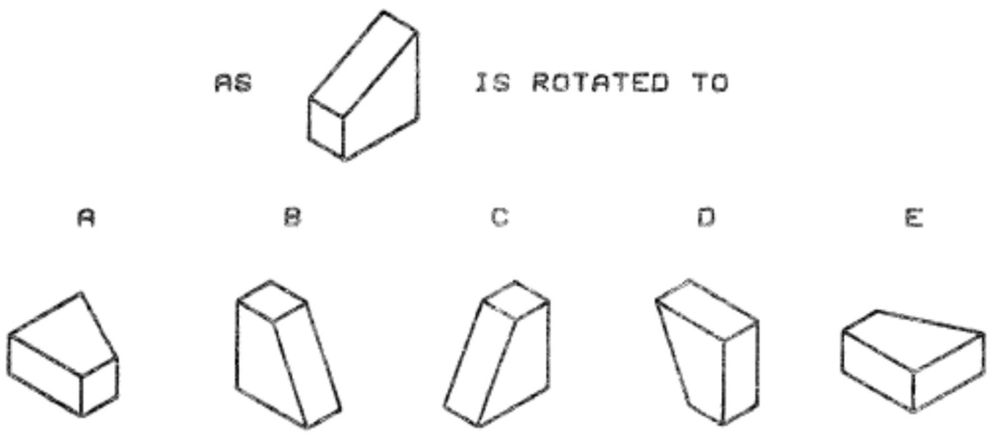
You are to:

1. study how the object in the top line of the question is rotated;
2. picture in your mind what the object shown in the middle line of the question looks like when rotated in exactly the same manner;
3. select from among the five drawings (A, B, C, D, or E) given in the bottom line of the question the one that looks like the object rotated in the correct position.

What is the correct answer to the example shown above?

Answers A, B, C, and E are wrong. Only drawing D looks like the object rotated according to the given rotation. Remember that each question has only one correct answer.

Now look at the next example shown below and try to select the drawing that looks like the object in the correct position when the given rotation is applied.



Notice that the given rotation in this example is more complex. The correct answer for this example is B.

13

IS ROTATED TO

AS

IS ROTATED TO

A B C D E

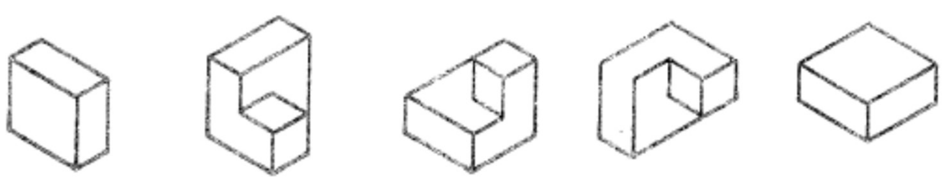
14

IS ROTATED TO

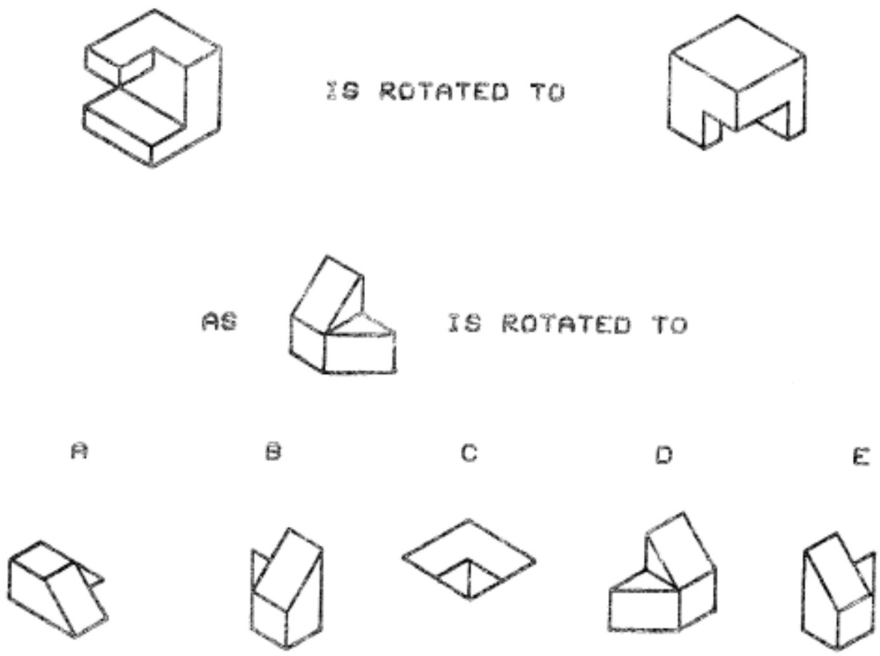
AS

IS ROTATED TO

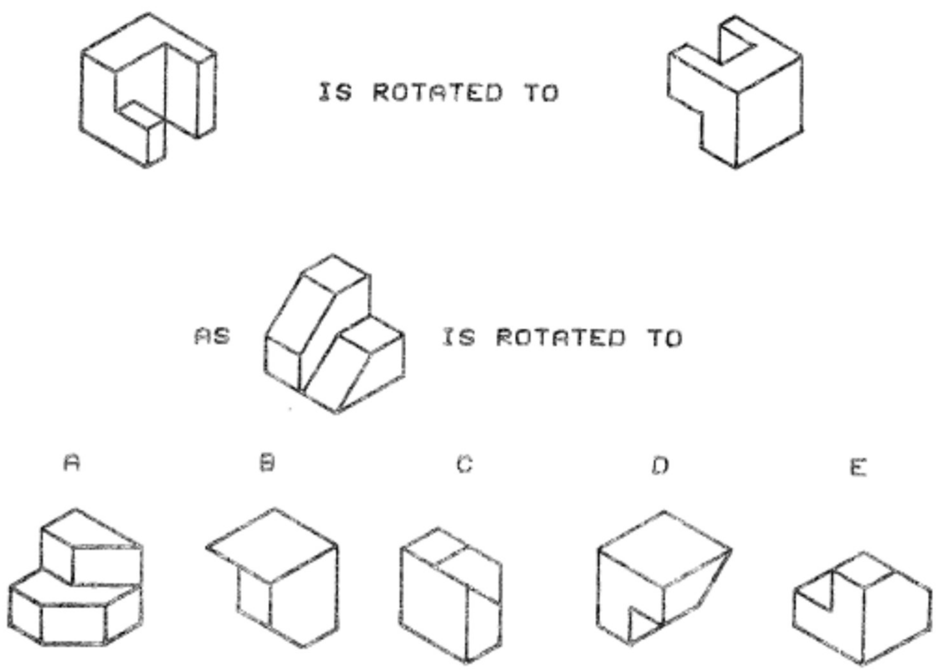
A B C D E



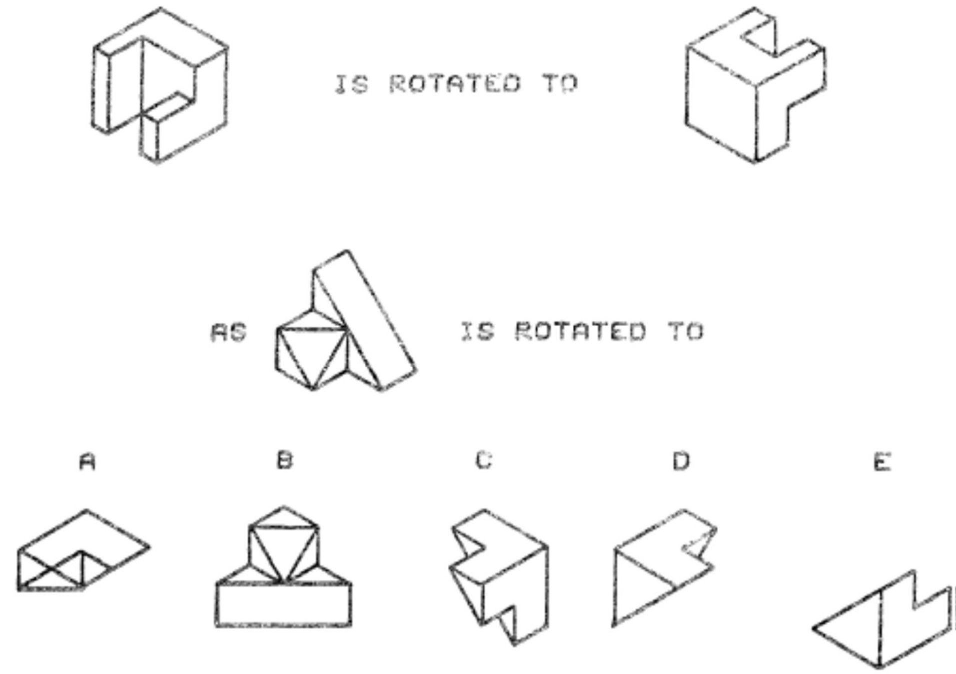
15


IS ROTATED TO
AS
IS ROTATED TO
A
B
C
D
E

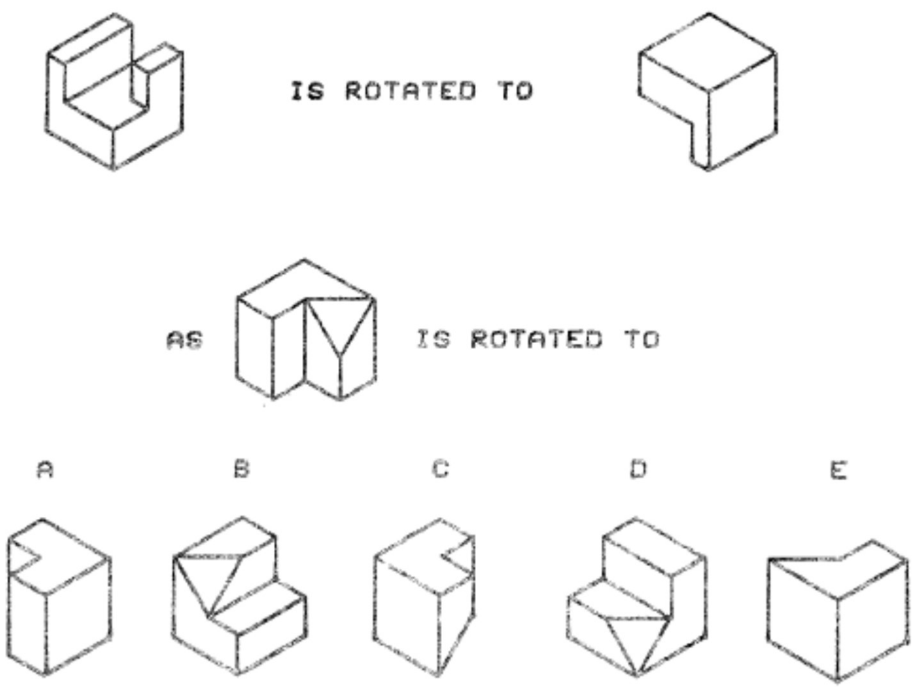
16


IS ROTATED TO
AS
IS ROTATED TO
A
B
C
D
E

17


IS ROTATED TO
AS
IS ROTATED TO
A
B
C
D
E

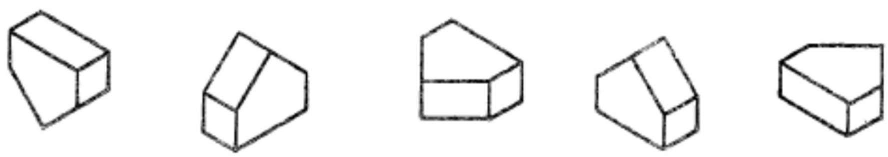
18


IS ROTATED TO
AS
IS ROTATED TO
A
B
C
D
E

19  IS ROTATED TO 

AS  IS ROTATED TO

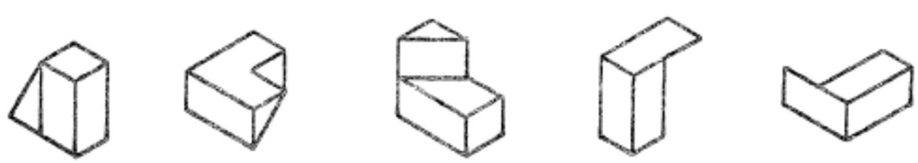
A B C D E



20  IS ROTATED TO 

AS  IS ROTATED TO

A B C D E



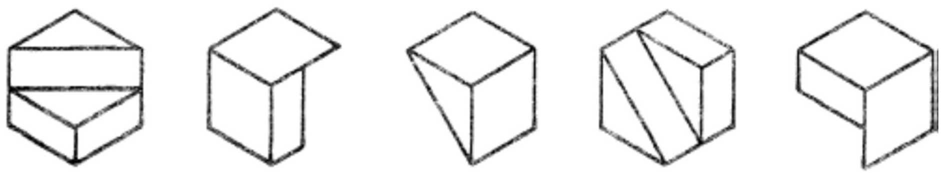
21

IS ROTATED TO

AS

IS ROTATED TO

A B C D E



22

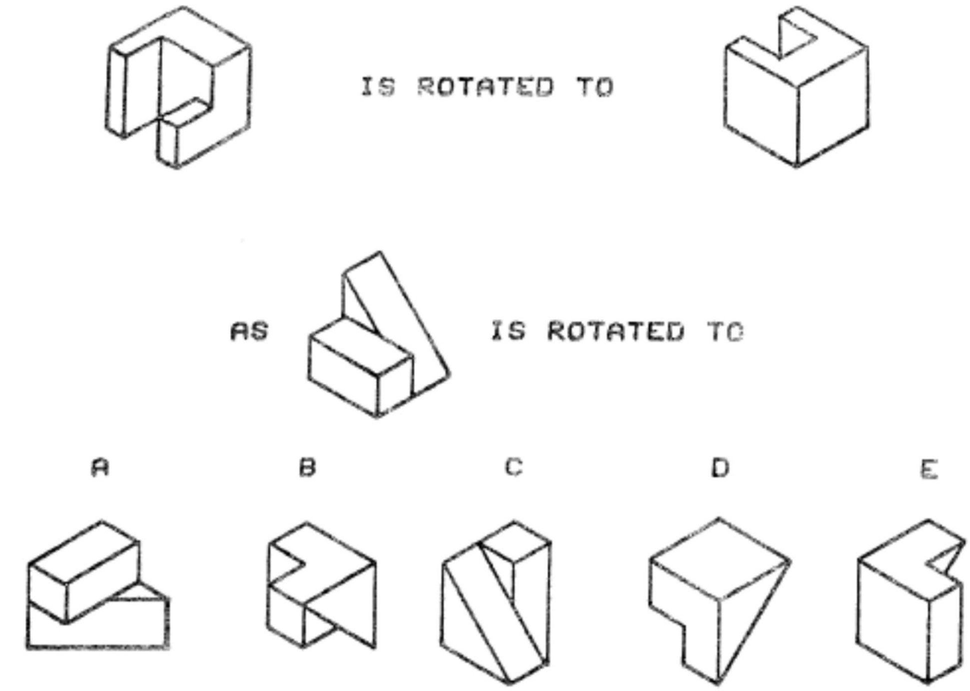
IS ROTATED TO

AS

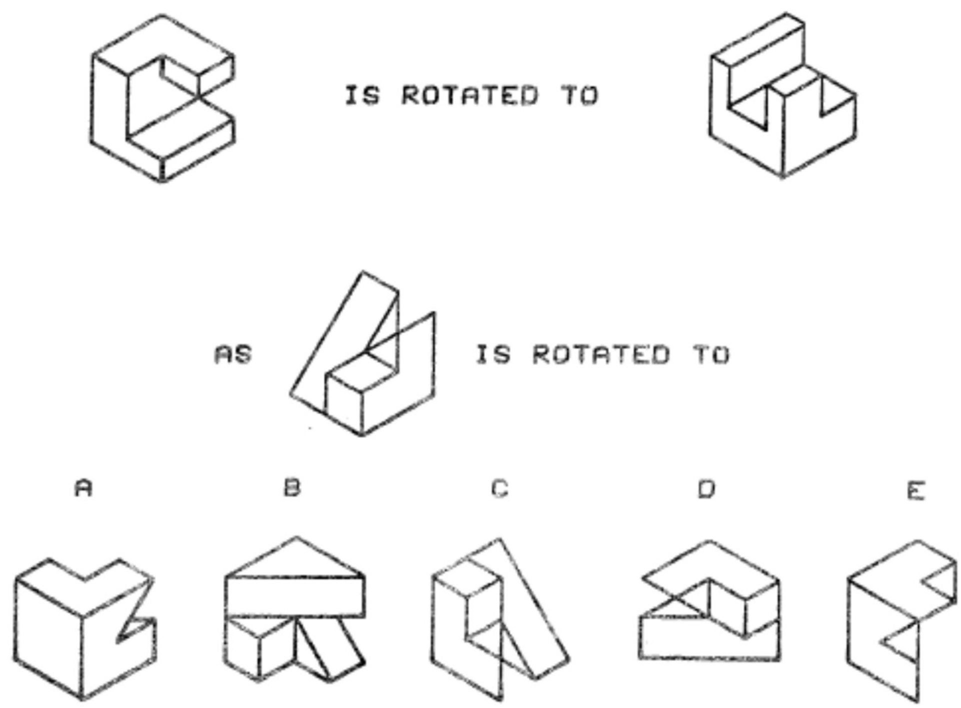
IS ROTATED TO

A B C D E

23


IS ROTATED TO
AS
IS ROTATED TO
A
B
C
D
E

24


IS ROTATED TO
AS
IS ROTATED TO
A
B
C
D
E

STOP !

You have completed this section of the test.

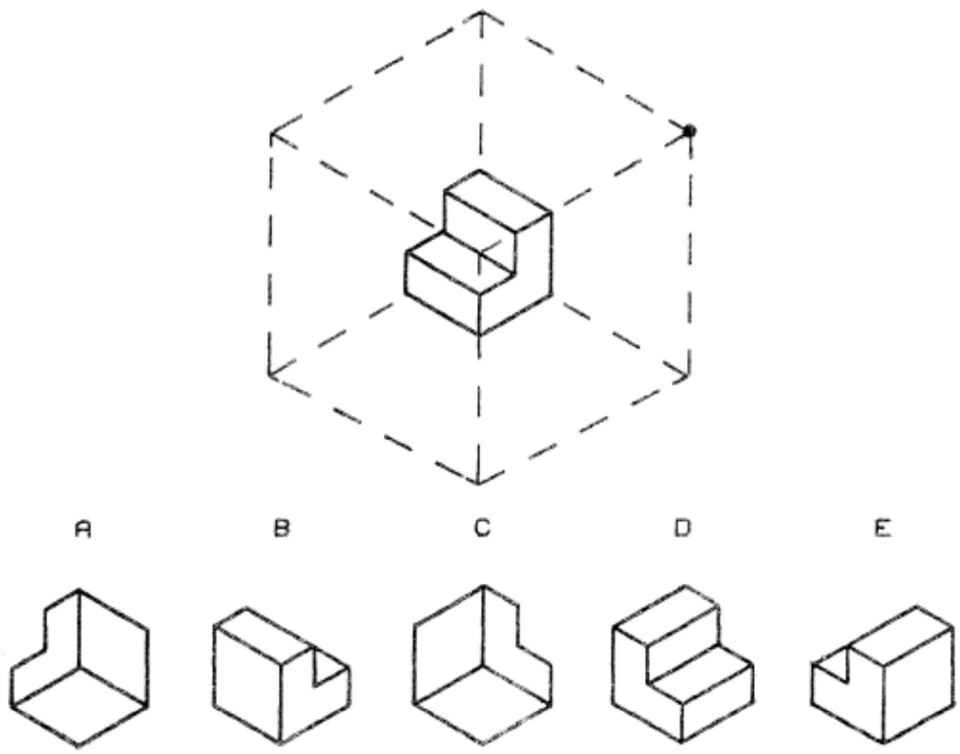
Do NOT go on to the next section
until you are instructed to do so.

If you have time remaining,
go back and check your answers.

SECTION 3: VIEWS

Directions

The third section consists of 12 questions designed to see how well you can visualize what three-dimensional objects look like from various viewing positions. Shown below is an example of the type of question included in the third section.



The example shows an object positioned in the middle of a "glass box" and five drawings representing what the same object looks like when seen from different viewing positions. The black dot in the top right corner of the "glass box" identifies the desired viewing position. You are to:

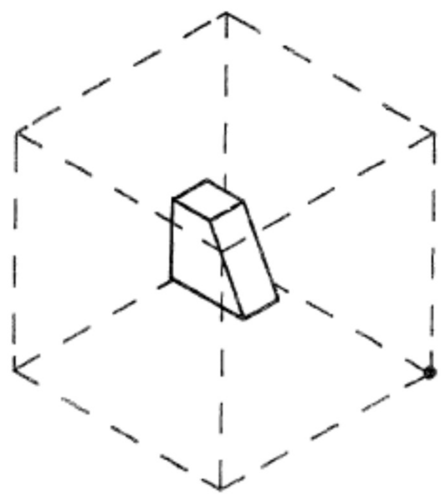
1. imagine yourself moving around the "glass box" until the black dot is located directly between you and the object;

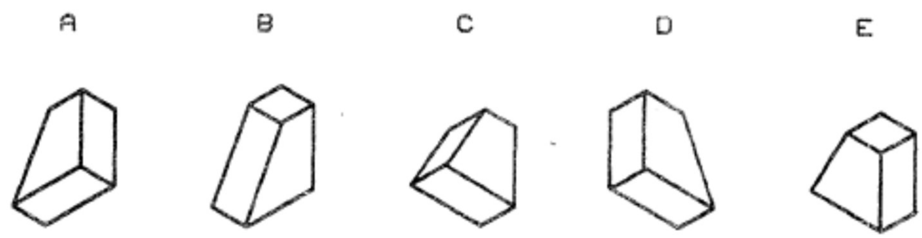
2. from this viewing position picture in your mind what the object in the "glass box" looks like;
3. select from among the five drawings (A, B, C, D, or E) the one that looks like the object as seen from the viewing position.

What is the correct answer to the example shown on the page just before this one?

Answers A, B, C, and D are wrong. Only drawing E looks like the object as seen from the given viewing position. Remember that each question has only one correct answer.

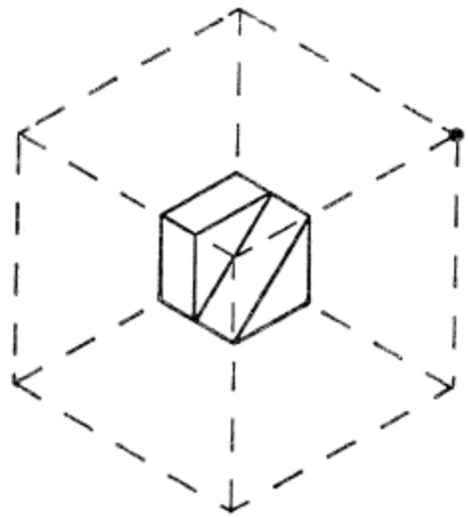
Now look at the next example shown below and try to select the one drawing that represents what the object looks like from the given viewing position. Remember that the object is located in the middle of the "glass box" and you are imagining yourself looking at the object with the black dot between you and the object.

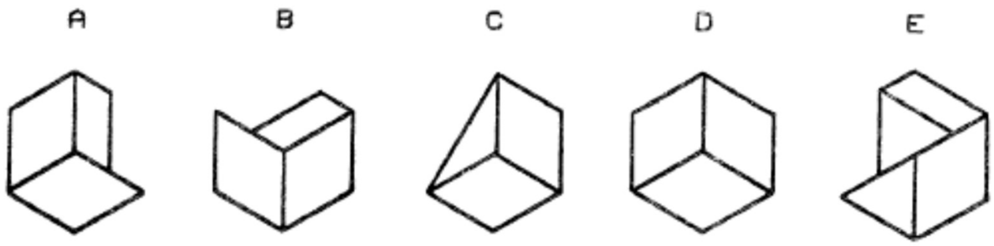




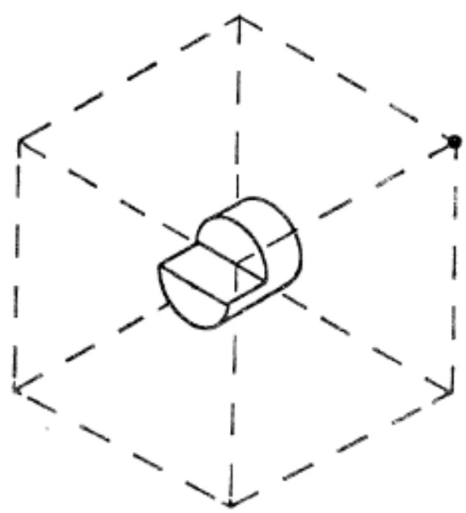
The correct answer for this example is C.

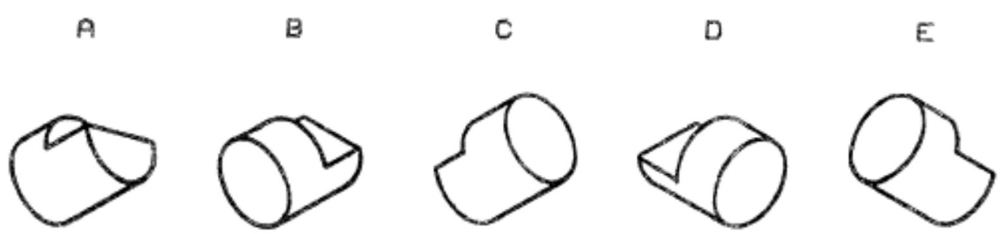
25



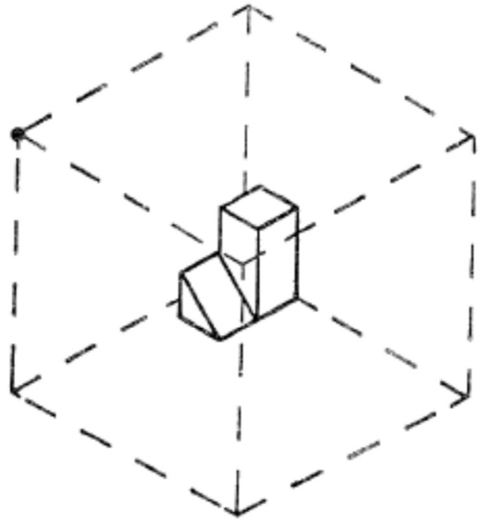


26

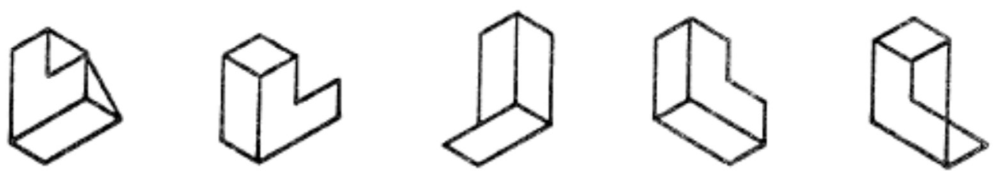




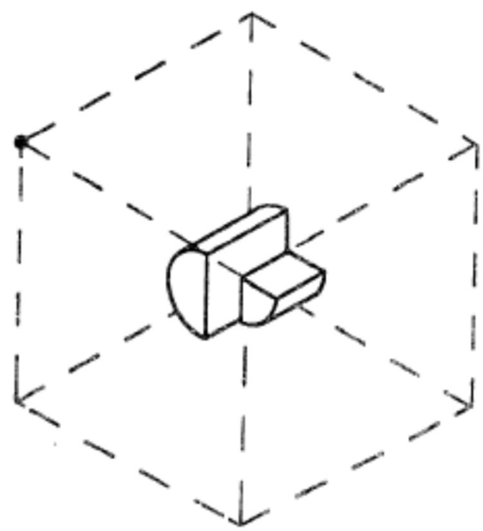
27



A B C D E

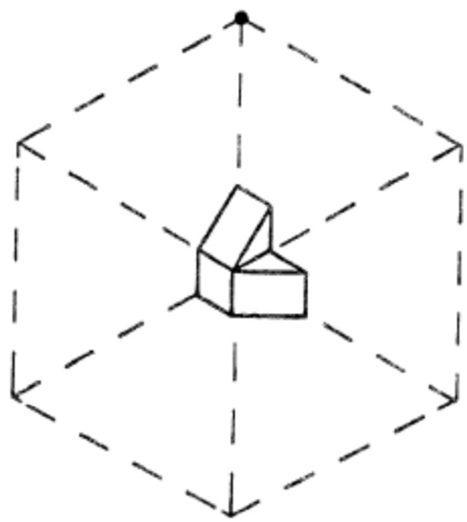


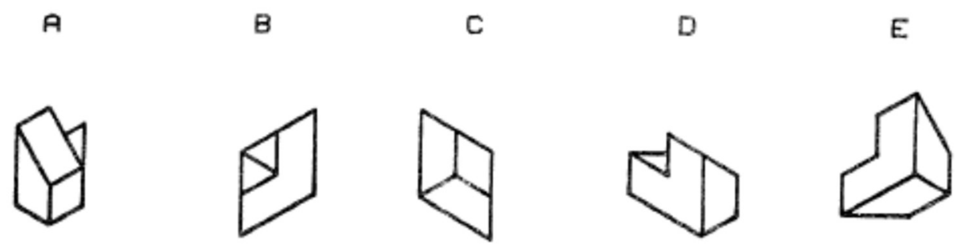
28



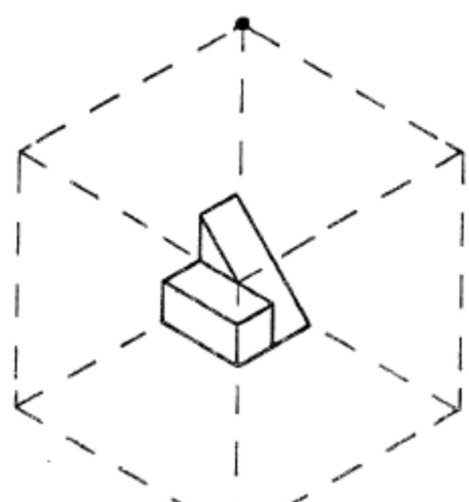
A B C D E

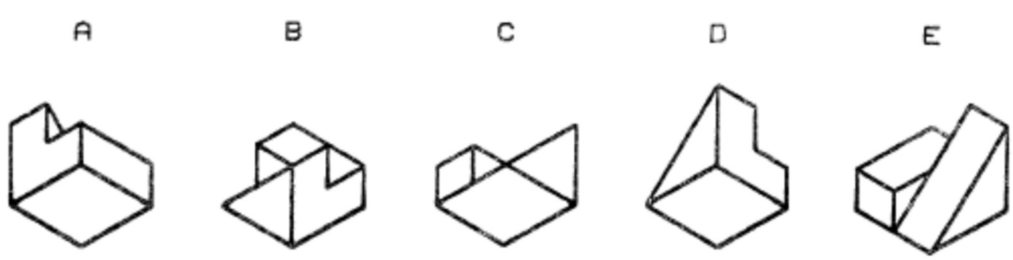
29



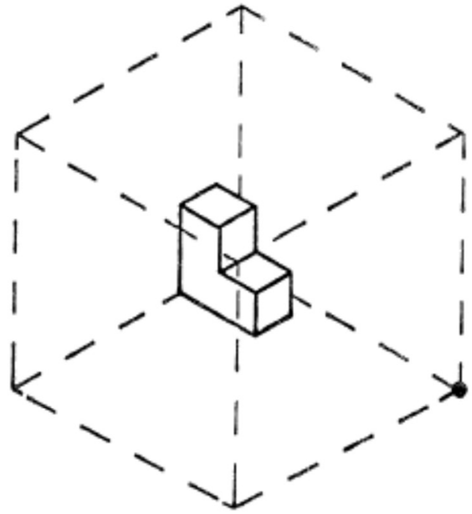


30



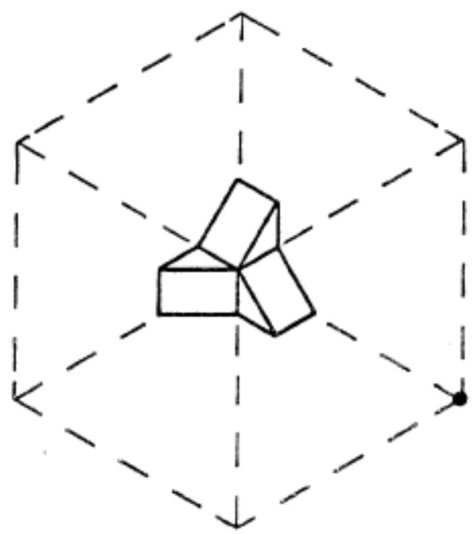


31



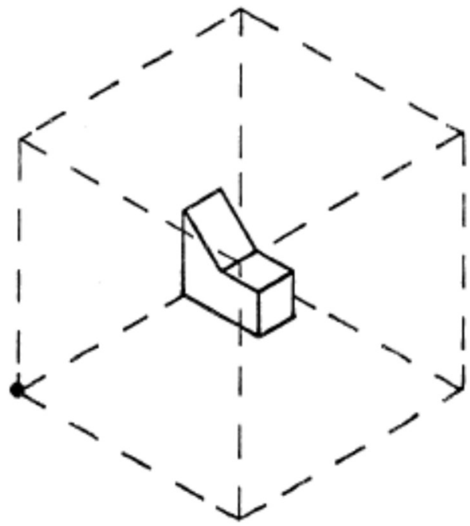
A B C D E

32



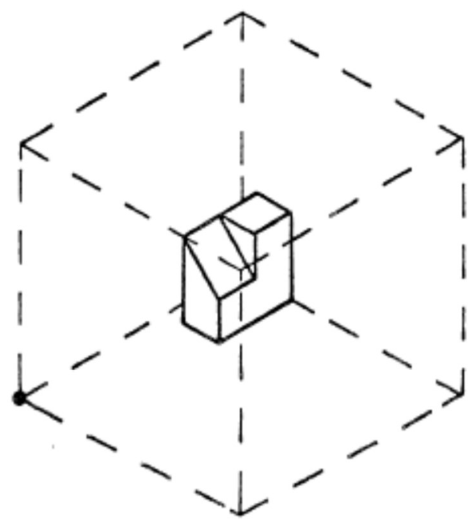
A B C D E

33



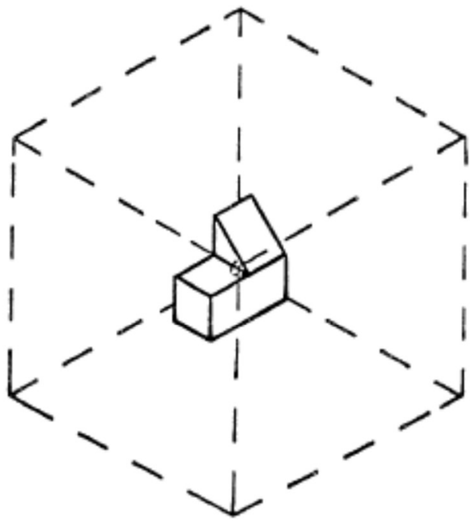
A B C D E

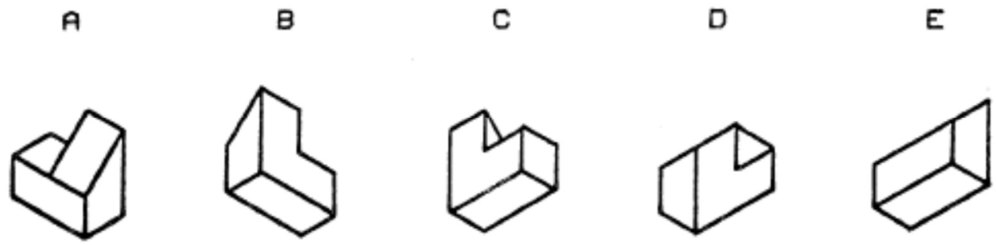
34



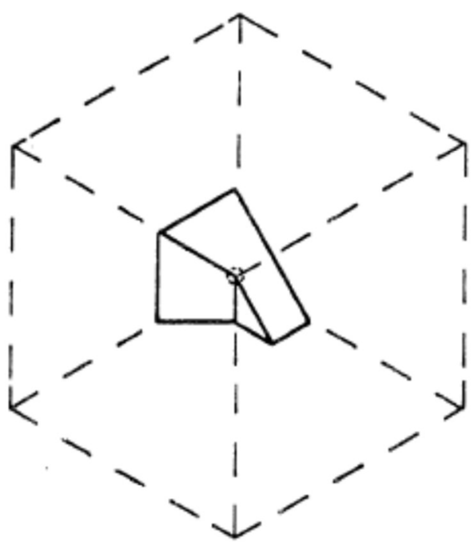
A B C D E

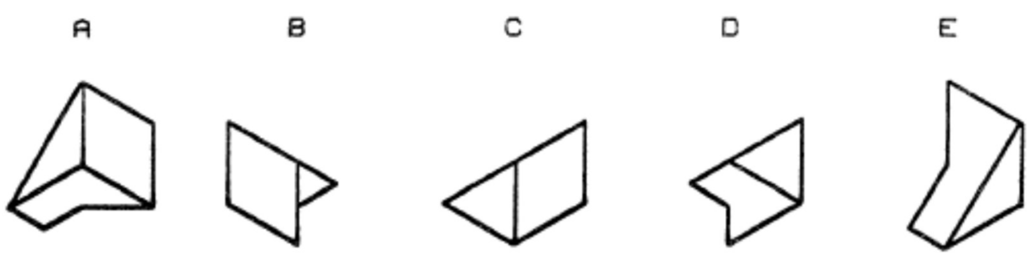
35





36





STOP !

You have completed this section of the test.

If you have time remaining,
go back and check your answers.

APPENDIX C

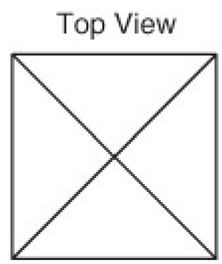
TEST OF TEA – MPS

MATHEMATICAL PROBLEM-SOLVING TEST

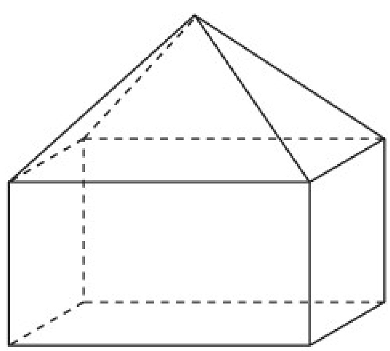
Instructions:

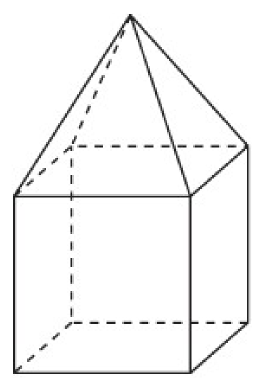
1. This test is to measure your mathematical problem-solving skills.
2. Choose one correct answer among the four options below the problems.
3. This test will take 30 minutes, without penalty given for the wrong answers.
4. This test was adopted from Texas Examination Agency.

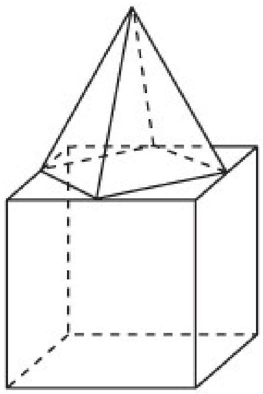
1. The drawing shows the top view of a 3-dimensional object.

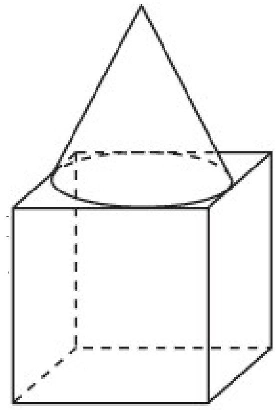


Which of the following drawings best represents this 3-dimensional object?

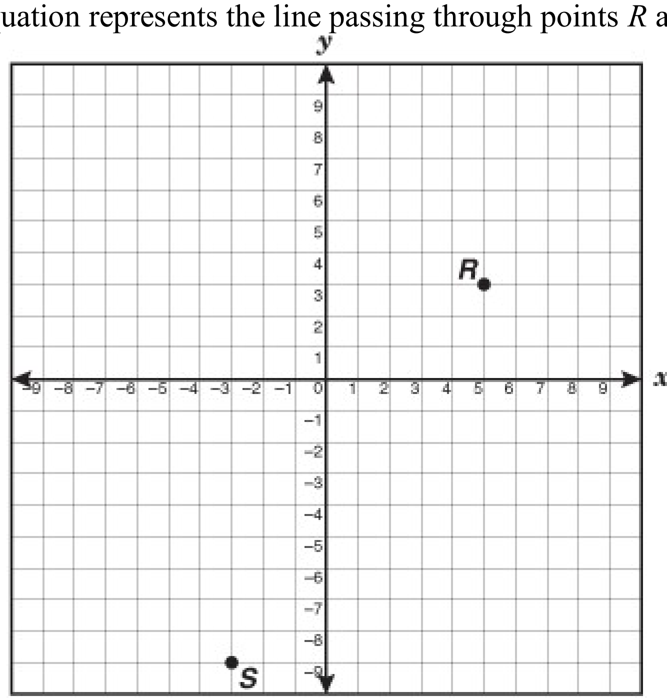
A. 

B. 

C. 

D 

2. Which linear equation represents the line passing through points *R* and *S*?



A. F$y = 1.5x - 4.5$

B. G$y = 1.5x + 4.5$

C. H$y = 0.5x - 4.5$

D. J$y = 0.5x + 4.5$

3. Lee, Kelly, Linda, and Madison all took the same math test. Linda earned a lower score than Kelly, but she did not earn the lowest score. The highest test scorer's name does not begin with an L. Madison earned a higher score than Kelly. Which person earned the lowest score on the math test?

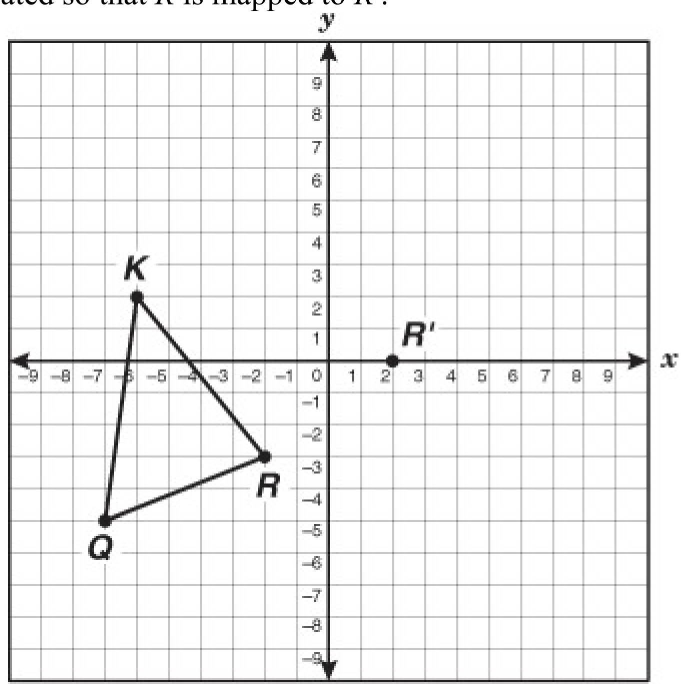
A. Kelly

B. Lee

C. Linda

D. Madison

4. ΔKQR is translated so that R is mapped to R'.



Which ordered pair best represents either point K' or point Q'?

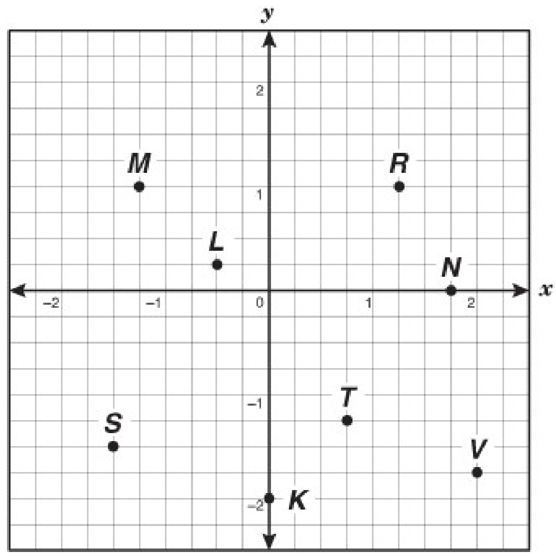
A. K' (–3, 6)

B. Q' (–4, –1)

C. K' (5, –2)

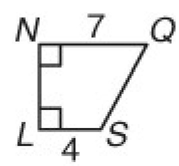
D. Q' (–3, –2)

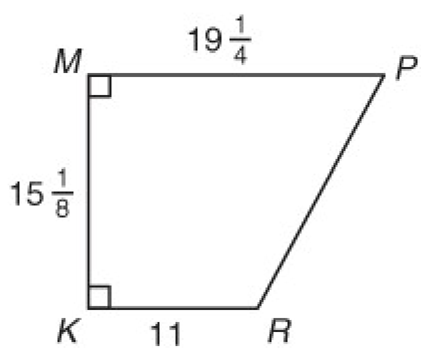
5. Which of the following ordered pairs best represents the location of point T?



- A. $(\frac{3}{2}, -\frac{5}{2})$
- B. $(\frac{3}{4}, -\frac{5}{4})$
- C. $(\frac{3}{4}, -\frac{3}{2})$
- D. $(\frac{5}{2}, -\frac{3}{2})$

6. Trapezoid *KMPR* is similar to trapezoid *LNQS*.

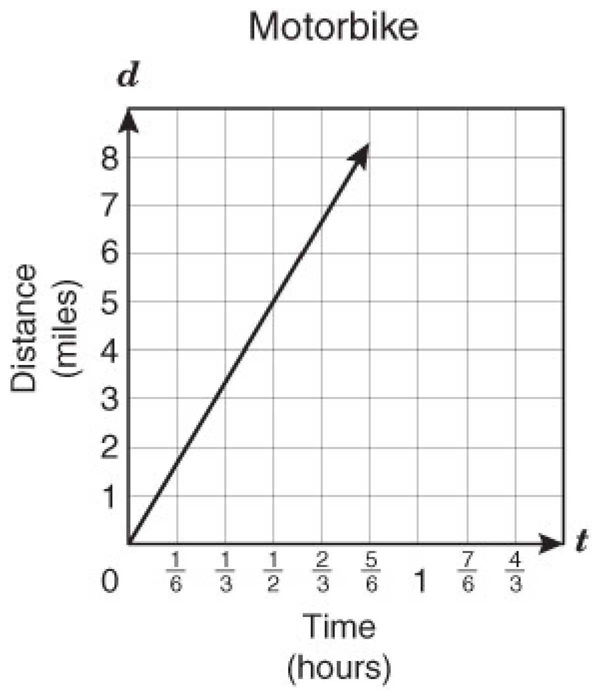




Which is closest to the perimeter of trapezoid *LNQS*?

- A. 23 units
- B. 31 units
- C. 25 units
- D. 63 units

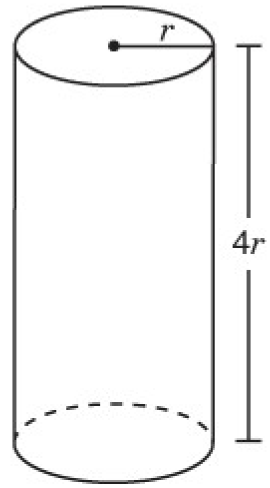
7. The graph shows the distance a certain motorbike can travel at a constant speed to time.



Which of the following best describes the meaning of the slope of the line representing this situation?

- A. The motorbike travels at a speed of about 8 miles per hour.
- B. The motorbike travels at a speed of about 2.5 miles per hour.
- C. The motorbike travels at a speed of about 5 miles per hour.
- D. The motorbike travels at a speed of about 10 miles per hour.

8. Look at the cylinder shown below.



Which equation best represents the volume, V, of this cylinder in terms of π?

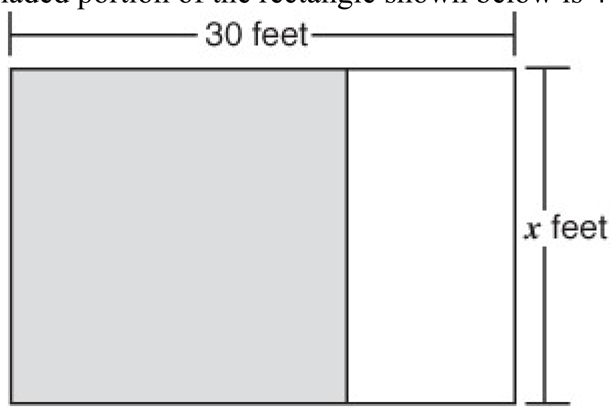
A. $V = 16\pi r^2$

B. $V = 8\pi r^3$

C. $V = 4\pi r^2$

D. $V = 4\pi r^3$

9. The area of the shaded portion of the rectangle shown below is 440 square feet.



How can the area of the unshaded portion of the rectangle be expressed in terms of x in square feet?

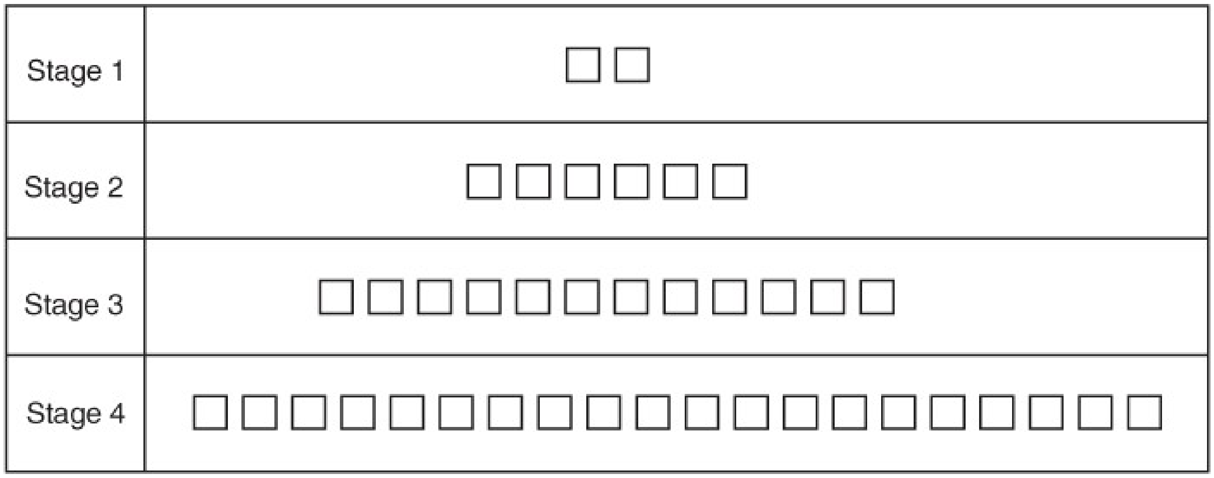
A. $440x - 30$

B. $(30 + x)440$

C. $(30 - 440)x$

D. $30x - 440$

10. The squares below show a pattern.



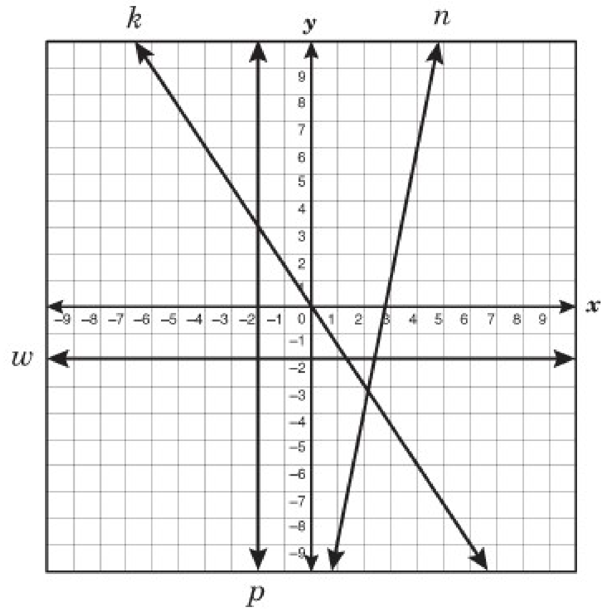
Which expression can be used to determine the number of squares at stage n?

- A. $5n - 3$
- B. $4n - 2$
- C. $2n^2$
- D. $n^2 + n$

11. The length of each leg of an isosceles triangle is 5 centimeters more than twice the length of the base. If the perimeter of this isosceles triangle is 95 centimeters, what is the length of the base?

- A. 17 cm
- B. 21 cm
- C. 30 cm
- D. 39 cm

12. Which line appears to have a slope of zero?



A. Line *n*

B. Line *k*

C. Line *w*

D. Line *p*

13. Jeremy's house is 45 feet wide. In a photograph, the width of the house was 2.5 inches, and its height was 2 inches. What is the actual height of Jeremy's house?

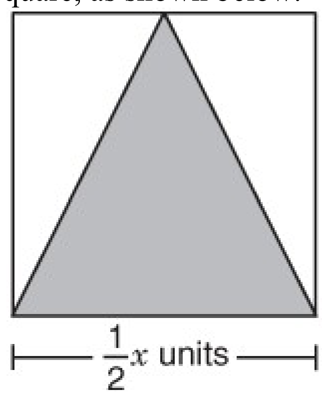
A. 18 ft

B. 26 ft

C. 32 ft

D. 36 ft

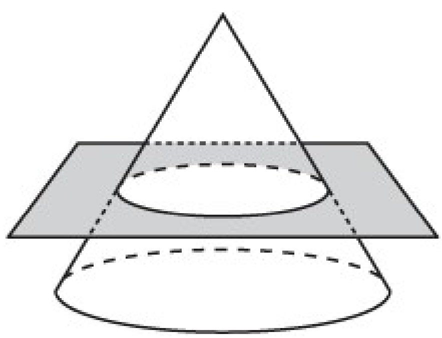
14. A triangle is inscribed in a square, as shown below.



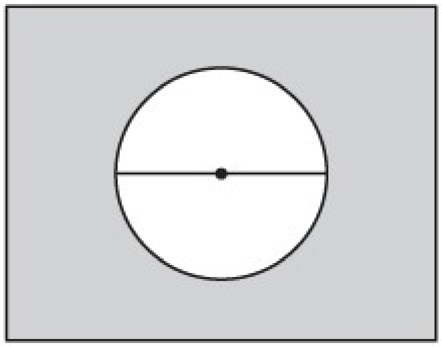
What is the area of the shaded triangle inscribed in the square?

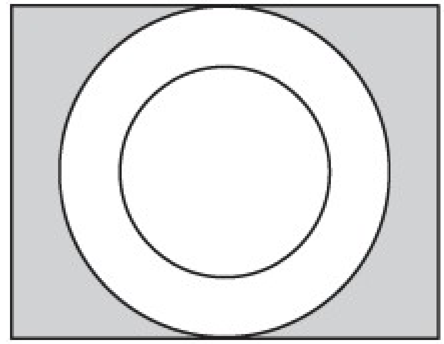
A. $\frac{1}{4}x^2$ units^2
B. $\frac{1}{2}x^2$ units^2
C. $\frac{1}{8}x^2$ units^2
D. $\frac{1}{3}x^2$ units^2

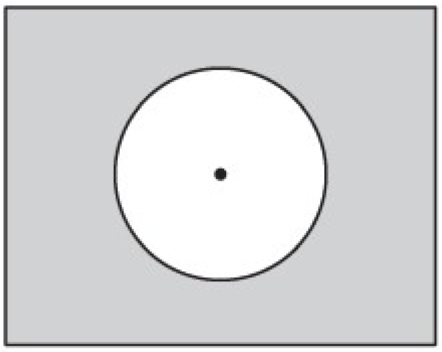
15. Lee designed a table that appears to be a cone with a wooden board passing through it, as shown below.

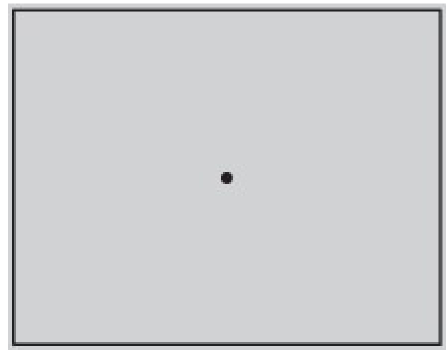


Which of the following best represents a top view of this table?

A. 

B. 

C. 

D. 

16. A rectangle has a length of $2x + 1$ and a width of $5x - 4$. Which expression best describes the area of the rectangle?

A. $7x - 3$

B. $14x - 6$

C. $10x^2 - 3x - 4$

D. $10x^2 + 13x - 4$

17. Which data set is best described by the function $y = -2x^2 + 5x$?

A.

x	y
–4	–52
–1	–3
2	2
3	9
6	–42

B.

x	y
–5	–75
–3	–33
1	7
4	52
6	–42

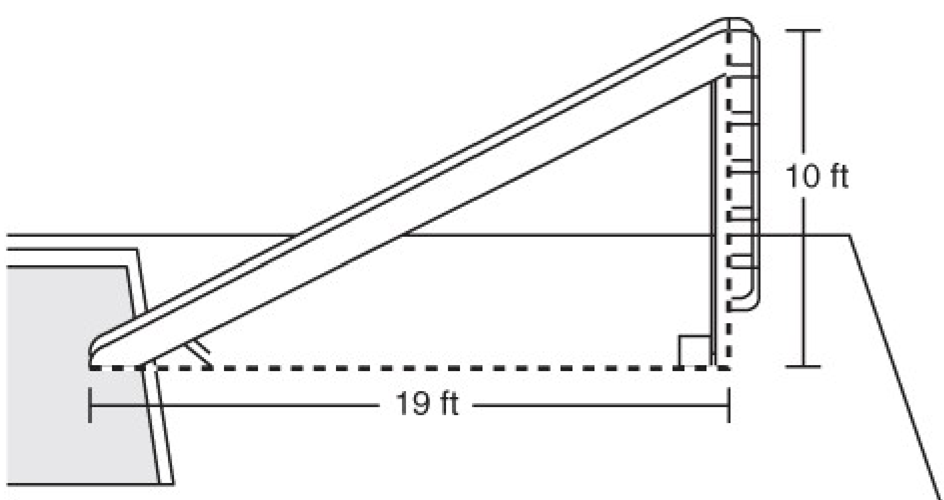
C.

x	y
–3	–33
–1	–7
2	2
3	–3
5	–25

D.

x	y
–2	–18
2	–2
5	–24
6	–42
8	–88

18. A slide was installed at the local swimming pool, as shown below.



Which is closest to the length of the slide?

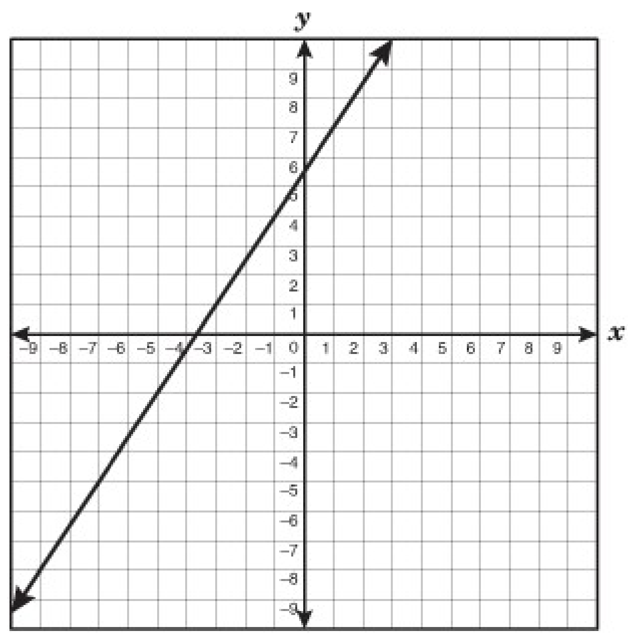
A. 29 ft

B. 16 ft

C. 21 ft

D. 81 ft

19. Which table best describes points on the line graphed below?



A.

x	y
–7	–5
–3	–1
–1	4
1	7
3	9

B.

x	y
–9	–8
–5	–2
–1	4
1	7
3	10

C.

x	y
–9	–8
–2	–5
–1	4
1	6
3	10

D.

x	y
–7	–5
–5	–2
–3	–1
7	1
10	3

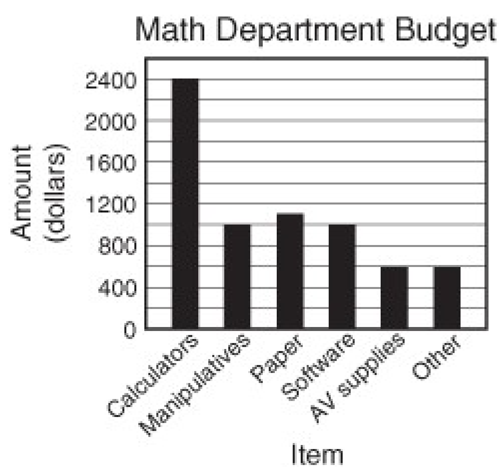
20. The table shows the math department's budget for the upcoming school year.

Math Department Budget

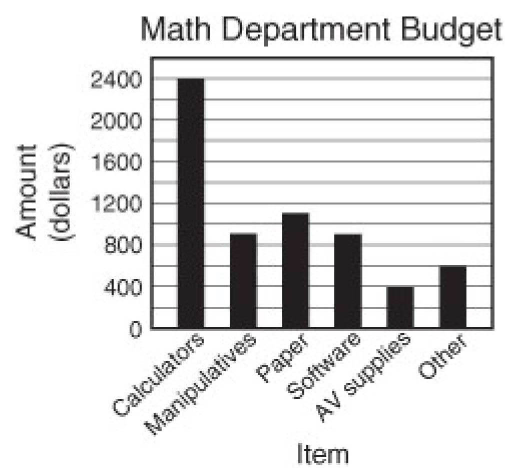
Item	Amount (dollars)
Calculators	2400
Manipulatives	900
Paper	1100
Software	900
AV supplies	500
Other	600

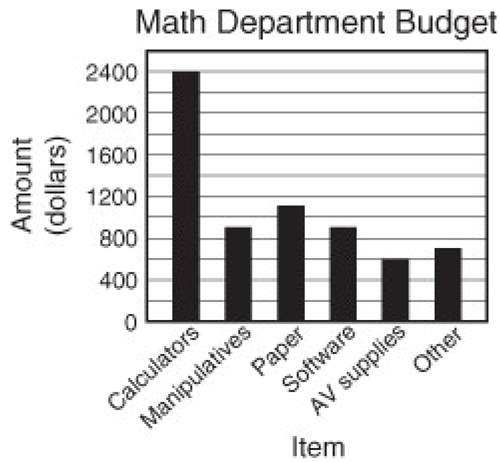
Which bar graph best represents the data given in the table?

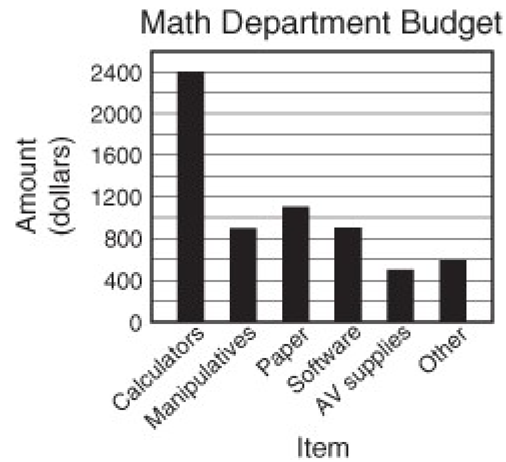
A.



B.



C. 
Math Department Budget
Amount (dollars)
2400
2000
1600
1200
800
400
0
Calculators
Manipulatives
Paper
Software
AV supplies
Other
Item

D. 
Math Department Budget
Amount (dollars)
2400
2000
1600
1200
800
400
0
Calculators
Manipulatives
Paper
Software
AV supplies
Other
Item

APPENDIX D

TEST OF CABRI 3D

Cabri 3D Test

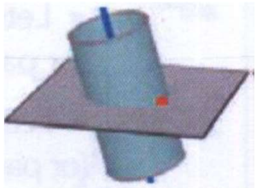
Instructions:

- This test is a computer-assisted test for measuring high school students' skills in using Cabri 3D for learning and practicing three-dimensional geometry.
- For assisting students in choosing a correct answer from the five options placed below each question, students may open the Cabri 3D program on the computer.
- Questions in this test were adapted from dynamic geometry problem-solving in the User Manual of Cabri 3D.
- This test will take 60 minutes without penalty for wrong answers.

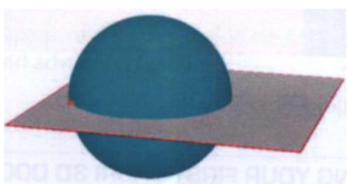
Direction:

At first, construct on your computer screen the three-dimensional objects as shown on each question. Write the step-by-step process of constructing the objects. Then, match your step-by-step process to one of the five options given below the questions for getting a true answer.

1. Construct a cylinder around a line as shown in the picture.
    a. Make a line by using the Line tool (hold down the Ctrl key – move the point up) – use the cylinder tool – click on the line (first point) – move the pointer to determine the radius and click the line (second point).
    b. Make a circle - move the point up and click – make a line – use cylinder tool.
    c. Make a line by using the Line tool (click on the plane, hold down the Shift key – move the point vertically, release, and click a point in the space – use the Cylinder tool – click on the line (first point) – move the pointer to determine the radius and click (the second point).
    d. Make two circles – make a line – use the Cylinder tool – click twice on the line.
    e. All of the answers above are false.

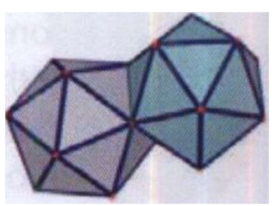


2. Construct a sphere as shown in the picture.
   a. Use Sphere tool – click on the plane twice.
   b. Use Sphere tool – click on the plane – move the pointer and hold the shift key.
   c. Use circle tool – click on the plane – use Sphere tool to determine the radius– click on the circle.
   d. Use circle tool to make two circles – use Sphere tool to make the radius– click twice on the plane.
   e. Use Sphere tool – click on the plane – move the pointer to determine the radius and click.



3. Construct a cone defined by a point as the vertex as shown in the picture.
   a. Make a circle – hold down the shift key – move the pointer up from the center of the circle and click – use the Conic tool and click any point in the space.
   b. Make a circle – hold down the shift key – move the pointer up from the center of the circle and click (a point in the space) – use the Conic tool – click the second point in the space.
   c. Make a circle on the plane – make a perpendicular line through the center of the circle – use the conic tool – click the circle and the line.
   d. Make a circle on the plane – use the Conic tool –bring the pointer in the center of the circle – hold down Shift key – move the point up vertically – click the point in the space and click on the circle.
   e. All of the answers above are false.

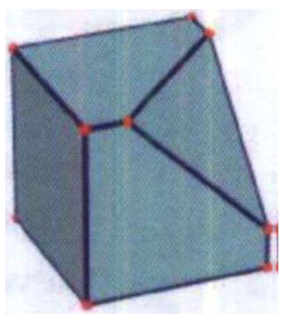
4. Construct an icosahedron.
   a. Use the appropriate Regular polyhedron tool (select and click icosahedrons) – click once on the plane – click a point as the center of a face on the plane – move the pointer and click on the appropriate place on the plane.
   b. Use the appropriate Regular polyhedron tool (select and click icosahedrons) – click twice on the plane – click a point as the center of a face on the plane – move the pointer and click once on the appropriate place on the plane.
   c. Use the appropriate Regular polyhedron tool (select and click icosahedrons) – click twice on the plane – click a point as the center of a face on the plane – move the pointer and click twice on the appropriate place on the plane.
   d. Use the appropriate Regular polyhedron tool (select and click icosahedrons) – click twice on the plane – move the pointer and click twice on the appropriate place on the plane.
   e. All of the answers above are false.



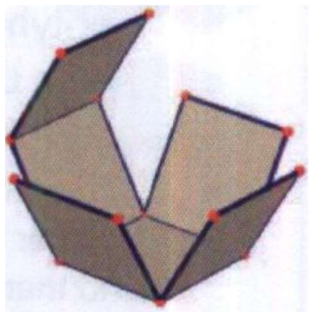
5. Use the View Angle, how many faces do icosahedrons have? It has…..
   a. 12 faces.
   b. 16 faces.
   c. 18 faces.
   d. 20 faces.
   e. 22 faces.

6. Construct the intersection of a polyhedron and the half-space delimited by a plane, and hide part of the polyhedron. Use the View Angle function to rotate the construction for seeing the appropriate hidden part.

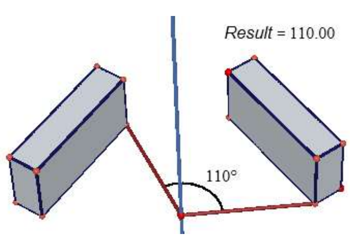
a. Construct a polyhedron – construct a plane that intersects the polyhedron – using the Cut polyhedron tool – select the polyhedron – select the intersection plane – use the Hide function for hiding the hidden part of the polyhedron.
b. Construct a polyhedron – construct a plane that intersects the polyhedron – using the Cut hide function – select the polyhedron – select the intersection plane – use the Hide function for hiding the hidden part of the polyhedron.
c. Construct a polyhedron – construct two lines that intersect the polyhedron – using the Cut polyhedron tool – select the polyhedron – select the intersection plane – use the Hide function for hiding the hidden part of the polyhedron.
d. Construct a polyhedron – construct a plane that intersects the polyhedron – using the Cut polyhedron tool – select the intersection plane – use the Hide function for hiding the hidden part of the polyhedron.
e. Construct two planes that intersect the polyhedron – using the Cut polyhedron tool – select the polyhedron – select the intersection plane – use the Hide function for hiding the hidden part of the polyhedron.



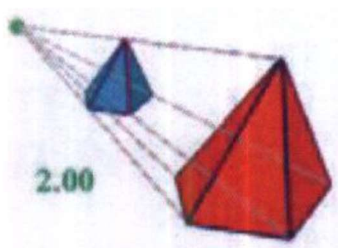
7. Open the faces of a cube (polyhedron), and then lay it flat to create a printable net.
   a. Construct a cube – with the Open Polyhedron tool, click twice on the cube – use the manipulation tool and drag one of the faces with the mouse.
   b. Construct a cube – with the Open Polyhedron tool, click on the cube – use the manipulation tool, click on one of the faces with the mouse.
   c. Construct a cube – with the Open Polyhedron tool, click on the cube – use the manipulation tool and drag one of the faces with the mouse.
   d. Construct a cube – with the Open Polyhedron tool, hold down the shift key – use the manipulation tool, and drag one of the faces with the mouse.
   e. Construct a cube – with the Open Polyhedron tool, click on the cube – Hold down Ctrl key.



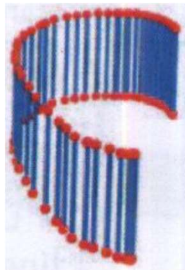
8. How do you rotate counterclockwise and as far as 110 degrees, the box in the right of the picture as shown below?
   a. Construct a box by using the XYZ box tool – make a line as the center of the rotation by using the Perpendicular line tool – using the Calculator function, type 110 and click Insert – activate rotation by clicking Rotation tool – select and click the XYZ box, the line, and the Result = 110.
   b. Construct a box by using the XYZ box tool – make two segments and a line as the center of the rotation by using the Perpendicular line tool – use the Calculator function, type 110 and click Insert – activate rotation by clicking Rotation tool – select and click the XYZ box, the line, and the Result = 110.
   c. Construct a box by using the XYZ box tool – make a line as the center of the rotation by using the Perpendicular line tool – use the Calculator function, type 110, and click Insert – select and click the XYZ box, the line, and the Result = 110.
   d. Construct a box by using the Cube tool – make a line as the center of the rotation by using the Perpendicular line tool – use the Calculator function, type 110 and click Insert – activate rotation by clicking the Rotation tool – select and click the XYZ box, the line, and the Result = 110.
   e. Construct a box by using the XYZ box tool – make a line as the center of the rotation by using the Perpendicular line tool – using the Calculator function, type 110 – activate rotation by clicking the Rotation tool – select and click the XYZ box, the line, and the Result = 110.



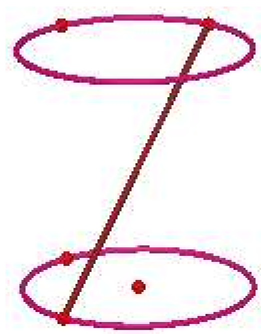
9. Dilation of a pyramid defined by a point and a numerical scale factor as shown in the picture
   a. Construct a pyramid – select a point as the center of dilation – use Calculator, fill in the scale factor, and click Insert – use Dilation tool – click on the point, the scale factor, and the pyramid.
   b. Construct a pyramid – select a point as the center of dilation – click on the point and the pyramid.
   c. Construct a pyramid – use Calculator, fill in the scale factor, and click insert – click on the scale factor and the pyramid.
   d. Construct a pyramid – select a point as the center of dilation – use calculator, fill in the scale factor, and click insert – click on the point and the scale factor.
   e. All of the answers above are false.



10. Display a trace of trajectory created by the movement of a segment as shown in the picture.
   a. Construct a segment in the space by using the Segment tool – click on the plane, hold down the Shift key, move the pointer up, and click a point in the space – use trajectory tool – click on the segment – click-hold and move the pointer to make the trace.
   b. Construct a segment in the space by using the Segment tool – click on the plane, hold down the Shift key, click two points in the space – use trajectory tool – click on the segment – click-hold and move the pointer to make the trace.
   c. Construct a segment in the space by using the Segment tool – click twice on the plane, hold down the Shift key, click twice on a point in the space – use trajectory tool –move the pointer to make the trace.
   d. Construct a segment on the plane by using the Segment tool – click on the plane, hold down the Shift key, move the pointer up, and click a point in the space – use trajectory tool.
   e. All of the answers above are false.

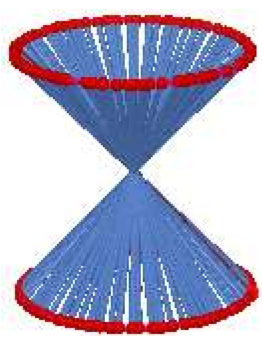


11. The trajectory tool in conjunction with Animation function can be used to create new objects as shown in the five pictures below. Animate the illustration below. View precisely the two end-points of the segment on the circles. In the animation box, adjust the speed to 4.00 cm/s for the two end-points.

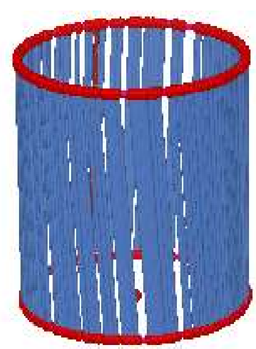


Press the Start Animation button. The segment will move between the two circles, leaving a trace that forms a hyperboloid. Among the pictures below, which picture is the best in illustrating this animation?

a.

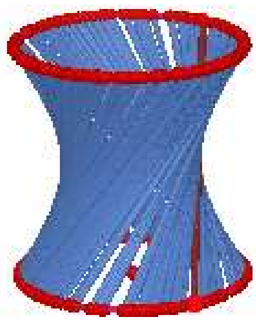


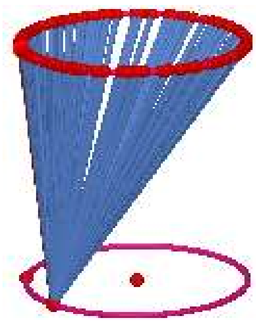
b.



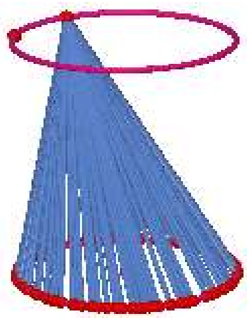
c.

d.





e.



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