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ESSAYS ON COMPETITIVE PERISHABLE FOOD SUPPLY CHAIN NETWORKS: FROM THE IMPACTS OF TARIFFS AND QUOTAS TO INTEGRATION OF QUALITY

A Dissertation Presented

by

DENIZ BESIK

Submitted to the Graduate School of the

University of Massachusetts Amherst in partial fulfillment

of the requirements for the degree of

DOCTOR OF PHILOSOPHY

May 2020

Isenberg School of Management

**ESSAYS ON COMPETITIVE PERISHABLE
FOOD SUPPLY CHAIN NETWORKS:
FROM THE IMPACTS OF TARIFFS AND QUOTAS
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DEDICATION

To Afife Buyukyilmaz Besik, Orhan Besik, and Elif Zeynep Besik.

ACKNOWLEDGMENTS

ABSTRACT

ESSAYS ON COMPETITIVE PERISHABLE
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MAY 2020

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Directed by: Professor Anna Nagurney

Food, in the form of fresh produce, meat, fish, and/or dairy, is necessary for maintaining life. In this dissertation, I focus on the modeling and analysis of some of the inherent issues in competitive perishable food supply chain networks. I investigate the impacts of trade policies such as tariffs, quotas, and their combination – tariff-rate quotas, as well as the integration of food quality deterioration into food supply chains. The research is especially timely given the prevalence of trade wars and tariffs in todays global political environment. The work is multidisciplinary with constructs from food science integrated into the economics of supply chain networks.

The first part of the dissertation overviews the methodological foundations including game theory, network and optimization theory, and variational inequality theory used for the construction and solution of the supply chain network models. In the second part of the dissertation, I first focus on perfectly competitive problems and develop a unified variational inequality framework for spatial

price network equilibrium problems with tariff-rate quotas. The accompanying case study on the dairy industry is based on trade between the United States and France. The computational results reveal that tariff-rate quotas may protect domestic producers from foreign competition, but at the expense of higher demand prices for consumers. This work is based on the paper by Nagurney, Besik, and Dong (2019). I then develop an oligopolistic supply chain network equilibrium model with differentiated products consisting of multiple firms, production sites, and demand markets, in which firms compete on product quantities and also quality. I provide a case study on soybeans, an important agricultural product, and investigate different scenarios. Insights as to firm profits and trade volumes, the average product quality, and consumer welfare, are also delineated. Specifically, I find that, although firms may benefit from the imposition of a quota or tariff, the welfare of consumers in the country imposing the quota or tariff declines. This work is based on the paper by Nagurney, Besik, and Li (2019).

In the third part of my dissertation, I demonstrate how to incorporate quality deterioration of fresh produce into perishable supply chain network models. I construct an explicit equation for fresh produce quality deterioration based on time and temperature of different pathways in supply chain networks. I first incorporate this feature into local markets in the form of farmers' markets, which serve as examples of direct to consumer channels and shorter supply chain networks. I also provide a case study of apples in western Massachusetts, under various scenarios, including production disruptions, due to negative weather conditions, resulting in an increase in apple prices at farmers' markets, a decrease in quality, and a decrease in profits for the apple orchards. These results can be used to inform food firms, policy makers, and regulators. This work is based on the paper by Besik and Nagurney (2017). Subsequently, I develop a competitive food supply chain network model in which the profit-maximizing producers decide not only as to the volume of fresh produce, but they also decide on the initial quality of fresh produce, with associated costs. I incorporate quality deterioration of the fresh produce explicitly with chemical functions depending on time, temperature, and the initial quality of the food product. I then present a case study on peaches, with supply chain disruptions to reveal valuable insights. I find that the disruptions in production result in higher demand prices, and lower initial quality. This is the first such general supply chain network model constructed to include the initial quality of fresh produce. This part of the dissertation is based on the paper by Nagurney, Besik, and Yu (2018).

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER 1

INTRODUCTION AND RESEARCH MOTIVATION

Food is essential for sustaining life. Maintaining a healthy diet requires physical and economic access to food that is nutritious, sufficient, and safe. Today the food we eat can be in various forms such as fresh produce, dairy, meat, fish, processed, etc., with each of them having different physical, chemical, and biological characteristics. Fresh produce in the form of fruits and vegetables is noted for its nutrients in terms of vitamins and minerals and is an important component of healthy diets of both children and adults alike (O'Connor (2013)). Dairy products such as milk, cheese, and yogurt contain essential nutrients, providing sufficient calcium intake for children and adults to establish healthy bones (Rozenberg et al. (2016)). Eating nutritious foods can reduce the incidence of illnesses and malnutrition and can even prolong life. Children who are well-nourished are better able to concentrate and to learn and grow. Adults who lack food security cannot prosper and neither can their families. According to Neff et al. (2009) food hardship is closely correlated with obesity and related heart diseases, diabetes, and cancers.

At present, the food and agricultural industry is one of the largest sectors in the United States, accounting for approximately 20% of its economy, comprised of an estimated 2.1 million farms, 935,000 restaurants, and with more than 200,000 registered food manufacturing, processing, and storage facilities, according to the United States Department of Homeland Security (2019). The report by the Alliance of Food Chain Workers (2016) states that there are over 21 million workers employed in the food industry, making up 14% of United States' workforce. Moreover, the agricultural sector in the United States shows a steady growth with consumer spending on food reaching over 1.6 trillion dollars annually (Ahumada and Villabos (2009), Plunkett Research (2011)). The dynamics in the food industry are very complex; the connection between various stakeholders is intertwined, and all the players work towards providing food to the consumer, starting from the farm and ending at the dinner tables of the consumers, while maximizing profits under tight competition.

The system for creating and sustaining the connection between the farm and the consumer is called a food supply chain network.

Food supply chains are very intricate local, regional or global networks, creating pathways from farms to consumers, encompassing production, processing, storage, and distribution (Yu and Nagurney (2012)). In general, food or agricultural supply chains are divided into two main types: perishable food supply chains and non-perishable food supply chains. Perishable foods include fresh produce, in the form of fruits and vegetables, dairy, meat, and fish. Fresh produce is seen as one of the most dynamic branches in the food sector with an annual consumption value of 100 billion products (Ahumada and Villabos (2009)). Population growth, coupled with the emphasis on the benefits of healthier diets, has lead to an increase in the demand for fresh produce. The Department of Agriculture (USDA) (2011) claims that the increase in the fresh produce consumption is higher than it is for traditional crops such as wheat and other grains.

In the United States, the growth in demand, and the increased expectations of the consumers for year-around availability of fresh produce, has spurred food supply chains to evolve into more sophisticated systems involving overseas production in different countries including Mexico, Argentina, Chile, and even Canada. Due to seasonality, most of the fresh produce that is sold in the grocery stores in the northeast of the United States is imported from other countries, or grown domestically in a state such as California or Florida (Cook (2002)). It is reported that two thirds of United States' vegetable imports come from Mexico, and most of the remainder arrives from Canada (Cook (2002)). Hence, the interactions between the demand and supply of food are no longer limited to a nation or a region, but have grown into a larger cross-border operation, including complex relationships and long distances (Van der Vorst (2000)).

There are multiple challenges related to food supply chain management, and it is not very straight-forward to conceive a general rule of thumb for managing food supply chains, since each and every food supply chain incorporates unique challenges pertinent to the specific product and process characteristics (Rong, Akkerman, and Grunow (2011)). One of the complexities of managing food supply chains ensues from the perishable nature of the product, making them distinct from other product supply chains (Yu and Nagurney (2012)).

Another feature associated with food supply chains is the issue of providing good quality fresh produce. Quality of perishable food products changes continuously throughout the supply chain from the farm to the fork. The quality change of food products depends on the environmental

conditions of the supply chain activities. Labuza (1982) states that keeping the quality of the perishable food products at the acceptable limits is vital for measuring the performance of food supply chains. Understanding the chemistry of quality decay, especially for fresh produce, is imperative for maintaining a better quality of the food product. In order to address this real-world supply chain issue, more explicit quality decay models, capturing time and temperature, in food supply chains are essential.

One of the other issues that is worth investigating in supply chain networks, including food supply chains, is the impact of trade policies. The increased amount of commodity flow between countries through supply chains has intensified the role of trade policies on nations' economies. The surge of globalization and the surge of global trade have induced governments to impose trade barriers in the form of tariffs, quotas, and their combination, tariff-rate quotas (TRQs), to protect their domestic markets from foreign competition. Historically, economists and policy makers believed that an increased trade volume can make a nation prosper, whereas trade restrictions may hurt its economy. Today, this belief is frequently questioned by government leaders, economists, and public officials. Hence, I attempt to provide a rigorous mathematical framework that incorporates the trade instruments used in practice today, especially in the agricultural industry, to illustrate and quantify the impacts, which, in turn, can provide valuable managerial insights for policy makers.

Additionally, considering the importance of trade instruments, and the effects of quality on supply chain network modeling, it is essential to explore the relationship between the two phenomena. This area of research, coupled with discussions on minimum quality standards, has also been garnering attention and has been the subject of debates (cf. Lutz (2005) and Nagurney and Li (2016)). In this dissertation, I also investigate quality, along with trade issues in perishable food supply chain networks.

In subsequent sections of this chapter of the dissertation I provide my motivation for studying trade and policy instruments, and product quality in perishable supply chain networks. I also provide a literature review, an outline of the dissertation, and contributions.

1.1. Trade and Policy Instruments

Trade allows countries to advance in their economic activities through the exchange of goods and services that they have an advantage of in terms of production cost (Smith (1887)). With this simple and very fundamental idea, nations engage in trade to increase their productivity levels,

employment rates, and general economic welfare. Today, products as diverse as fresh produce and other agricultural and food products such as meat, fish and seafood, cereals, including rice and wheat, and dairy products, to steel and aluminum, and a variety of other commodities, are transported across national boundaries to points of demand to satisfy the needs of the consumers.

The competition among nations, however, has become more relentless with the increase in global trade volumes, and opportunities in emerging markets. The advancements in technology and communication in the developed world created better and more efficient global supply chains, encouraged investments in the emerging markets, which, in turn, constructed a more complex and intertwined state of global trade. The spread of international trade has increased over time, fueled by globalization, reduced trade barriers, liberalized trade policies, overseas production, improved communication, and investments on technology.

A more transparent and open world economy, created by advancements in globalization, grants more opportunities for developing nations. It is reported by the World Trade Organization (WTO) that the share of developing nations in global trade has risen from a third to a half, since 1980 (WTO Report (2014)). Furthermore, China, being the top exporter today, was ranked 32nd thirty years ago (WTO Report (2014)). However, there is still an ongoing discussion about whether a more open and lenient trade environment contributes to the growth of the economy. According to a White House report in the United States in 2009, engagements on trade created more production, and also contributed to an increase in the standard of living in the United States.

Given the importance of global trade for producers and consumers alike, governments, often turn to trade policies ranging from tariffs and quotas, and their combination, in the form of tariff-rate quotas (TRQs), in order to reduce the impact of competitive foreign firms on their demand markets and to protect their less competitive domestic firms. Trade instruments can be seen as a type of tax that is imposed on goods at the time of import. For example, a tariff-rate quota (TRQ) is a two-tiered tariff, in which a lower in-quota tariff is applied to the units of imports until a quota is attained and then a higher over-quota tariff is applied to all subsequent imports (World Trade Organization (2004)). In general, tougher trade policies on imports may bolster the domestic industries of the expense of generating higher prices for domestic consumers. Skully (2001) states that the Uruguay Round in 1996 created more than 1,300 new TRQs. Furthermore, the World Trade Organization members currently have a combined total of 1,425 TRQ commitments.

In the present economic and political climate, tariffs, as well as tariff-rate quotas, are garnering prominent attention in the news on world trade with even washing machines in the United States being subject to tariff-rate quotas (cf. Office of the United States Trade Representative (2018)). The imposition of tariffs by certain countries, including the United States, is, in turn, leading to retaliation by other nations, such as China, European Union, Mexico, and Canada, with ramifications across multiple supply chains (cf. Watson (2018)). For example, in March 2018, the United States government imposed a 25 percent tariff on imported steel and a 10 percent tariff on imported aluminum (cf. Hodge (2018)). These exchanges between the world's two largest economies were seen as the beginning of the largest trade war in the economic history of trade. In fact, economists argue that the on-going trade war with China and the latest round of proposed tariffs on Chinese goods would eventually hurt the American consumers (cf. Tankersley (2018)). It was reported in 2020 that the American manufacturing activity was facing a decline, damaged by the trade war between US and China, while the dollar was steadily strong against other currencies, making American goods more expensive (cf. Swanson and Smialek (2020)). Moreover, the recent developments in the global political environment could result in the separation of world's famous conjoined twins, the United States and China, induced by the decoupling of global supply chains between the world's two largest economies (cf. Mistreanu (2019)).

While the major financial impacts on nations' economies are generated by the trade wars between the United States and China, there are other players in the most popular trade game of 2000s. It was announced in 2020 that the United States could impose 100 percent tariffs on European wine, Irish whiskey, and waffles due to European Union's financial support for the European aerospace company Airbus, a competitor of American aerospace company Boeing (cf. Lefcourt (2020), Tankersley (2019)).

As international trade continues to be a very crucial part of the global social, political, and financial environments, there are very few mathematical models for the evaluation of the impacts of tariffs, quotas, and TRQs. The development of rigorous mathematical models that can capture trade policies used in practice today and that are computable, providing both equilibrium supply market prices and demand market prices, as well as product flows, is critical. Such models should be sufficiently general to be able to handle multiple supply and demand markets in different countries, trade flows on general networks, as well as nonlinear cost and price functions that are also asymmetric and flow-dependent. With this motivation, I model TRQs, tariffs, and quotas in a supply chain equilibrium network framework, and provide a case study on dairy products in Chapter 3.

Trade and policy instruments may also generate implications on a micro level, that is, the quality of export products can change by imposition of stricter trade policies. Therefore, studying the relevance of quality of the products with trade instruments in the context of global supply chains, including food supply chains, is essential for having an understanding on the impacts of global trade. This motivation creates a foundation for the mathematical model that I present in Chapter 4, while providing a case study on soybeans.

1.2. Perishable Food Supply Chains

The transformation of global food supply chains since the early 1900s has been nothing short of remarkable. As reported in Martinez et al. (2010), in the early 1900s, much of the food bought and consumed in the United States was grown locally and about 40% of Americans resided on farms, whereas in 2000 only 1% did (cf. Pirog (2009)). Consumers over a century ago obtained information as to the quality of the foods through direct contact with farmers. Except for various food preservation activities, few foods were processed or packaged, and fresh produce, fish, and dairy products usually traveled less than 24 hours to market (see Giovannucci, Barham, and Pirog (2010)). According to a report by DeWeerdt (2016), in 1993, a Swedish researcher determined that the ingredients of a typical Swedish breakfast consisting of an apple, bread, butter, cheese, coffee, cream, orange juice, and sugar, had traveled a distance equal to the circumference of our planet before reaching the consumer.

The basic difference between food supply chains and other supply chains, and this is especially characteristic of fresh produce, is the continuous and significant change in the quality of food products throughout the entire supply chain from the points of production/harvesting to points of demand/consumption (see Sloof, Tijskens, and Wilkinson (1996), Van der Vorst (2000), Lowe and Preckel (2004), Ahumada and Villalobos (2009), Blackburn and Scudder (2009), Akkerman, Farahani, Grunow (2010), Aiello, La Scalia, and Micale (2012)).

In the next subsection, I extend the discussion on perishable food supply chains by providing more information on the ways of capturing quality of food products.

1.2.1 Quality of Food Products

The quality of food products is decreasing with time, even with the use of advanced facilities and under the best processing, handling, storage, and shipment conditions (Sloof, Tijskens, and Wilkinson (1996) and Zhang, Habenicht, and Spieß (2003)). The network topology of a fresh produce

supply chain and, in particular, the length of a path in terms of time from an origin node to a destination node can significantly influence the quality of the fresh produce that consumers purchase and consume.

Knowledgeable modern consumers are increasingly demanding high quality in their food products, including fresh produce, and, yet, they may be unaware of the great distances the food has traveled through intricate supply chains and the length of time from the initial production or "picking" of the fruits and vegetables to the ultimate delivery. Moreover, consumers, faced with information asymmetry, may not know how long the food may have been lying on the grocers' and retailers' shelves, even once delivered and unpacked. The great distances traveled create issues in terms of quality since fresh produce is perishable (see Nahmias (2011) and Nagurney et al. (2013)).

In order to capture quality degradation associated with fresh produce in a supply chain one must be aware of the various supply chain network economic activities such as production, storage, transportation, etc., as well as the duration and temperature associated with these activities. It has been discovered that the quality of fresh produce can be determined scientifically using chemical formulae, which include both time and temperature (cf. Labuza (1982), Taoukis and Labuza (1989), Tijskens and Polderdijk (1996), Rong, Akkerman, and Grunow (2011)). For example, in terms of kinetics (cf. Labuza (1982)), the quality degradation of food such as meat and fish follows first order reactions whereas that of many fresh fruits and vegetables follows zero-order reactions with the order of the reaction corresponding to the power of the differential equation for quality. Hence, studying and integrating quality decay into food supply chain networks can be seen as the pillars for my work in Chapters 5 and 6 of this dissertation.

I now review the relevant literature on: tariffs and quotas from a network equilibrium point of view, the relevance of product quality on the network equilibrium models with tariffs and quotas, and the modeling of fresh produce supply chain networks with the incorporation of fresh produce quality.

1.3. Literature Review

Through this dissertation, I contribute to the modeling and analysis of supply chain network equilibrium modeling, especially to that of perishable food supply chain networks. I contribute significantly to the literature on variational inequalities, game theory, and supply chain network equilibrium modeling by being methodologically relevant to the application and solution of such

problems. The results obtained from analyzing the mathematical models based on simulated case studies provide valuable insights and can inform policy makers.

Thus, this section discusses the relevant literature on several topics covered in this dissertation.

1.3.1 Perfectly and Imperfectly Competitive Models for Agricultural Trade Problems

To-date, the modeling of trade problems that include multiple producers in different countries as well as multiple demand markets while using supply chain network equilibrium modeling framework is limited. The studies in this area mostly consider perfect competition, in which all firms are price takers, and produce a homogeneous product. As an example of perfectly competitive models, spatial price equilibrium (SPE) models have attained prominence in the modeling, analysis, and solution of a wide spectrum of commodity trade problems. However, in many industrial sectors, the more appropriate framework for modeling is that of imperfect competition, such as oligopolistic competition, in which the market is dominated by few firms, and the decisions of one firm influencing the decisions of other firms, vice versa. In this subsection, I present a review on perfectly competitive models with a focus on SPE models, and also oligopolistic models for commodity trade problems.

1.3.1.1 Spatial Price Equilibrium Models

SPE problems are perfectly competitive models, in that therein it is assumed that there are numerous producers, and such models date to Samuelson (1952) and Takayama and Judge (1964, 1971). The need to develop extensions, over and above the original SPE models that were reformulated and solved using optimization approaches, especially quadratic programming ones, has also spearheaded advances through the use of methodologies such as complementarity theory as well as variational inequality theory.

Harker (1985) points out that SPE problems are appealing for scholars; firstly, for their application to energy markets, steel markets etc., leading researchers to reexamine the structure of the model, and, secondly, the availability of new mathematical methods and algorithms for achieving solutions of the problem. Asmuth, Eaves, and Peterson (1979) reformulate the SPE problem as a complementarity problem, and solve for an equilibrium on affine networks with Lemke's algorithm. Florian and Los (1982) propose a newer and more general formulation of SPE, providing more efficient and simpler algorithms. The authors present formulations that can be applied to general transportation networks, allowing multiple commodities on the condition of an equivalent optimization problem reformulation.

SPE models, due to their practical applications in agricultural and energy and mineral markets, have incorporated trade policies and have been constructed using variational inequality. Nagurney, Nicholson, and Bishop (1996a,b) develop an SPE model with discriminatory ad valorem tariffs. The authors provide a solution method that includes parallel programming, in which the main problem is broken down into simple subproblems, each of which is solved simultaneously. Nagurney and Zhao (1993) introduce competitive spatial market models with direct demand functions. The policy interventions in the form of price controls is permitted in their modeling framework. Nagurney, Thore, and Pan (1996) illustrate a spatial market policy modeling with goal targets, in which they use variational inequality theory for the formulation, qualitive analysis, and computation.

Complementarity theory is also widely used for the modeling of quotas and tariff schemes. Rutherford (1995) introduces new features using GAMS modeling, and solves nonlinear complementarity problems applied to various economic models. Moreover, Nolte (2008) constructs a spatial price equilibrium model of the world sugar market with various trade instruments. The authors use the mixed complementarity problem to program the SPE model, and use GAMS for the computational study. On the other hand, Grant, Hertel, and Rutherford (2009) propose a partial equilibrium model of the United States' dairy market to analyze the options of liberalization on various trade instruments including tariff quotas, and quota expansions. They conduct simulation analysis to evaluate different scenarios with various trade instruments. Recently, Johnston and van Kooten (2017) develop a spatial price equilibrium model to analyze the impacts of softwood lumber trade sanctions between Canada and the United States. The authors use a mixed complementarity problem formulated to solve a 21-region, global trade model.

1.3.1.2 Oligopolistic Models

Another framework for modeling agricultural trade is that of oligopolistic competition. The connection between spatial price equilibrium models and oligopolies are shown in the paper by Dafermos and Nagurney (1987), in which the authors present a general spatial price equilibrium model, and solve for the corresponding Cournot (1838) oligopoly equilibrium using variational inequality theory. In addition, the authors conduct a sensitivity analysis for the general spatial price equilibrium problem in their earlier work (Dafermos and Nagurney (1984)). Oligopolistic models can be very appropriate for the modeling of agricultural trade problems, since the number of firms, that is, "players" in the game may not be very large. As noted by Guyomard et al. (2005), the world banana market is dominated by a small number of firms and, hence, this raises the impor-

tance of also addressing imperfect competition. Nagurney, Besik, and Nagurney (2019) develop an global supply chain network model where firms engage in oligopolistic competition to maximize their profits in the presence of trade policy instruments in the form of tariff-rate quotas. The authors allow firms to have production sites in different countries, determining how much of the product to manufacture/produce at these production sites, along with the distribution of the product flows to the demand markets, also located in multiple countries. In Chapters 4, 5, and 6 of this dissertation, I present supply chain network models with firms competing in an oligopolistic manner.

The literature is very limited in terms of oligopoly models on trade problems. For example, Shono (2001) relaxes the assumption of perfect competition, and incorporates TRQs. The author assumes that all countries behave in the same oligopolistic manner. Moreover, Maeda, Suzuki, and Kaiser (2001, 2005) consider oligopolistic competition and TRQs with a single producer in each country and that it is faced with a separable cost function. The authors also assume that the demand function in each country is also separable, that is, the demand for a product in a country only depends on the price in that country.

For additional supply chain network equilibrium models with oligopolies the interested reader is encouraged to see Nagurney and Matsypura (2005), Zhang (2006), Daniele (2010), Qiang et al. (2013), Yu and Nagurney (2013), Li and Nagurney (2017), Nagurney, Yu, and Besik (2017), and Saberi (2018).

In the next subsection, I provide a literature review on product quality in perishable food supply chains.

1.3.2 Product Quality in Perishable Food Supply Chains

The literature on food supply chains is growing, given the great interest in this topic. The early contributions focused on perishability and, in particular, on inventory management (see Ghare and Schrader (1963), Nahmias (1982, 2011) and Silver, Pyke, and Peterson (1998) for detailed reviews). More recently, some studies have proposed integrating more than a single supply chain network activity (see, e.g., Zhang, Habenicht, and Spieß (2003), Widodo et al. (2006), Ahumada and Villalobos (2011), and Kopanos, Puigjaner, and Georgiadis (2012)) and also have emphasized the need to bring greater realism to the underlying economics and competition (cf. Yu and Nagurney (2013)). Van der Vorst (2006) noted that it is essential to analyze food supply chains within the context of the full complexity of their network structure. Monteiro (2007), further, postulated that

network economics (cf. Nagurney (1999)) provides a powerful framework in which the structure of the supply chain can be graphically captured and analyzed, and studied the traceability in food supply chains theoretically. Additional modeling and methodological contributions in the food supply chain and quality domain have been made by Blackburn and Scudder (2009) and by Rong, Akkerman, and Grunow (2011). For approaches to the quantification of quality in supply chain networks of manufactured products, including durable goods, we refer the interested reader to the book by Nagurney and Li (2016) and the references therein. For a recent book on perishable product supply chains with a variety of applications, see Nagurney et al. (2013).

Quality can be defined in multiple dimensions such as physical, emotional, and even philosophical. Therefore, it is not easy to find a proper global definition of quality which is valid for every type of industry and consumer. Reeves and Beednar (1994) present a good discussion on the evolution of quality definitions. Nagurney and Li (2014) investigate minimum quality standards in a spatial price equilibrium model and also consider information asymmetry. Nagurney and Li (2016) present a plethora of competitive supply chain network models with a focus on quality, where quality is defined as conformance to specifications as in manufactured products. Nagurney et al. (2013), in turn, focus on perishable product supply chains. Murdoch, Marsden, and Banks (2000) also present a valuable discussion on the connection between quality and nature, especially in the food sector, from the perspective of social sciences. However, there are more specific definitions of quality in the literature that are construed specifically for the food sector.

Various authors have emphasized the change in quality of food products in the supply chain until the final points of demand (see Sloof, Tijskens, and Wilkinson (1996), Van der Vorst (2000), Lowe and Preckel (2004), Ahumada and Villalobos (2009, 2011), Blackburn and Scudder (2009), Akkerman, Farahani, and Grunow (2010), and Aiello, La Scalia, and Micale (2012), Yu and Nagurney (2013)). Amorim, Costa, and Almada-Lobo (2014) utilize demand functions that depend on product quality and also price and then construct demands for different products based on age. They propose deterministic and stochastic production planning models that capture consumers' desire for fresher products. Liu, Zhang, and Tang (2015) also utilize demand functions that depend on price and quality but they depend continuously on time. The authors determine the dynamic pricing and investment strategies to reduce the deterioration rate of the quality for perishable foods.

1.3.3 Modeling of Quality in the Presence of Tariffs and Quotas

The state-of-the-art literature pertinent to the supply chain network equilibrium modeling of tariffs and quotas with the incorporation of product quality is not advanced. Much of the modeling research in this domain that includes the quality of products produced and traded has appeared in the economics literature and has focused, in terms of theoretical results, on either a monopoly or perfect competition (Falvey (1979), Krishna (1987)) or on a duopoly (Das and Donnenfeld (1989), Herguera, Kujal, and Petrakis (2000)). As for oligopolistic competition, researchers have, typically, assumed exogenously fixed product qualities or homogeneous goods (Leland (1979), Shapiro (1983), Deneckere, Kovenock, and Sohn (2000)), whereas it is clear that quality can be a strategic variable in firms' decision-making and also in terms of consumers differentiating among the firms. There have also been empirical studies conducted to assess the interrelationships between a spectrum of trade policies and product quality as in cheese (cf. Macieira and Grant (2014)), the steel industry (Boorstein and Feenstra (1991)), the footwear industry (Aw and Roberts (1986)), and the automobile industry (cf. Feenstra (1988), Goldberg (1993)). Nevertheless, the construction of a general differentiated supply chain network model with product quality and trade policies in the form of tariffs, quotas, and also minimum quality standards merits attention, especially since the theoretical literature has been limited in terms of the number of firms considered, as well as the number of demand markets, and has not included general transportation cost functions that include quality.

1.4. Dissertation Overview

The dissertation consists of seven chapters with the first chapter dealing with the research motivation and literature review. In Chapter 2, I recall the methodologies that are utilized in this dissertation, mainly variational inequality theory (Nagurney (1999)) and its relation to game theory (Nash (1950,1951)). In the next subsections, I present the contributions in Chapters 3 through 6.

1.4.1 Contributions in Chapter 3

In Chapter 3 of this dissertation, I provide a unified variational inequality framework for the modeling, analysis, and computation of solutions to a general spatial network equilibrium problem with multiple countries and regions in each country on both the production and on the consumption sides, as well as multiple routes joining the supply markets with the demand markets, in the presence of two-tiered tariffs in the form of tariff-rate quotas.

I extend the literature on spatial price equilibrium models by incorporating various policy interventions such as tariff-rate quotas. Since different supply and demand markets may have, respectively, distinct supply and demand price functions, and trade policies such as tariff-rate quotas can impose quotas over multiple countries, I believe that having a greater level of detail is meaningful. Furthermore, rather than assuming only a single path (essentially a link) between a pair of supply and demand market nodes, I allow for multiple paths, each of which is not limited to the same number of links. Different supply markets in a given country may have distinct transport mode options to demand markets in the same or other countries, and, therefore, such options can be represented as paths on the general network, with associated costs. I focus on tariff-rate quotas (TRQs) since they have been deemed challenging to formulate and only stylized examples have been reported in an SPE framework (cf. Bishop et al. (2001)). I believe that the modeling of TRQs just by itself adds a great contribution to the literature.

In addition, I construct variants of the general spatial price network equilibrium model with tariff-rate quotas, to demonstrate how the latter can be easily adapted to handle unit tariffs, ad valorem tariffs, and/or strict quotas. I also note that, through the use of multiple paths, the evaluation of avenues for transshipment, as a means to avoid tariffs, a topic that has garnered a lot of attention in the popular press recently (cf. Bradshear (2018)), is made possible. Moreover, my framework allows for the investigation of the impacts of TRQs on domestic markets, on both producers and consumers alike, that are imposed on non-domestic markets.

Another contribution of this chapter emerges from the results analyzed in the computational study section. I provide illustrative examples as well as a case study consisting of larger numerical examples for which the equilibrium path flows, and link flows, as well as the equilibrium quota rents, and the supply and demand markets prices, and path costs, are reported. The case study focuses on the dairy industry and a tariff-rate quota imposed on the US on cheese from France. Sensitivity analysis demonstrates the impacts on production outputs (both US domestic and imports from France) of a tightening of the imposed quota as well as increases in the over tariff-rate. The case study includes adding routes between producing and consuming regions in different countries. The results show that tariff-rate quotas may protect domestic producers, but at the expense of the consumers. On the other hand, adding competitive alternative transportation routes may help both domestic producers and exporters as well as consumers. These results bring great value to the supply chain network and game theory literature as well as to the literature on trade policy, since I provide

insights to policy makers for their decision making on international trade policies. Chapter 3 of this dissertation is based on the paper by Nagurney, Besik, and Dong (2019).

1.4.2 Contributions in Chapter 4

In Chapter 4, I extend the mathematical model constructed in Chapter 3 by incorporating quality. I add to the supply chain network, game theory and trade policy literature by developing an oligopolistic supply chain network equilibrium model with differentiated products in which firms have multiple production sites and multiple possible demand markets, and compete in product quantities and product quality, subject to minimum quality standards, along with upper bounds on quality. The model is then extended to include trade policy instruments in the form of a strict quota or a tariff. I identify the governing equilibrium conditions, noting that the strict quota model is characterized by a Generalized Nash Equilibrium (GNE), rather than a Nash Equilibrium. Under a strict quota, both the utility functions of the competing firms, as well as their feasible sets, will depend on the strategic variables of not only the particular firm, but also on the strategies of the others. Hence, I define a Generalized Nash Equilibrium. The GNE problem dates to Debreu (1952) and Arrow and Debreu (1954), and for background on the GNE problem, I refer the interested reader to von Heusinger (2009) and the recent review by Fischer, Herrich, and Schonefeld (2014). Moreover, I utilize the concept of a Variational Equilibrium (cf. Facchinei and Kanzow (2010), Kulkarni and Shanbhag (2012)) which is a specific kind of GNE to construct the variational inequality formulation. It is worthwhile to add that this is the first time that such concepts have been utilized in competitive supply chain network models with quality with or without trade policy instruments.

Subsequently, I introduce a differentiated product supply chain network equilibrium model with a tariff and prove that this model coincides with the model with a strict quota where the Lagrange multiplier associated with the latter, if the strict quota constraint is tight, is precisely the imposed tariff. In the economics literature, Bhagwati (1965) considered perfect competition (whereas we consider imperfect competition) and demonstrated that the tariff-quota equivalence occurs when such competition prevails in all markets. Fung (1989), in turn, considered a stylized oligopoly consisting of two countries with a single firm in each country and Cournot-Nash competition and also found an equivalence.

Moreover, I demonstrate that, the underlying equilibrium conditions for all three models can be formulated and analyzed as an appropriate variational inequality problem, for which an effective

computational scheme is also provided, which yields closed form expressions for the variables in each of the two steps of the procedure.

I also provide constructs for quantifying consumer welfare in the presence or absence of tariffs or quotas in differentiated product supply chain networks with quality. Through simple illustrative examples, I show that the imposition of a tariff or quota may adversely affect both the quality of products as well as the consumer welfare. I contribute significantly to the existing literature on the supply chain networks, and game theory. This chapter is based on the paper by Nagurney, Besik, and Li (2019).

1.4.3 Contributions in Chapter 5

In Chapter 5, I analyze fresh produce quality in a more detailed fashion in which I present a modeling and algorithmic framework for competitive farmers' markets. This work makes several contributions to the existing literature on competitive perishable supply chains. The model is network-based and the farmers engage in Cournot competition over space and time. The governing Nash equilibrium conditions are formulated as a variational inequality problem. The novelty of the framework lies in that the quality of the fresh produce product is captured as the produce propagates in the supply chain over space and time with the consumers at the markets responding both to the price and the quality of the fresh produce. Both uncapacitated and capacitated versions of the model are presented. The latter can capture limitations in supply due to harvest problems or damage during the growing season, limitations in transport and storage capabilities, and/or labor for harvesting and processing. The game theory model can address questions of farmers as to which farmers' markets they should serve; what the impact of a new competitor may be at one or more markets in terms of profits, as well as the effects of capacity disruptions (or enhancements) in their individual and others' short supply chains. In addition, the model can ascertain the impacts of changes in link parameters that capture quality. Policy makers, in turn, can also obtain useful information as to the impacts of a greater number or fewer farms represented at various markets and how reducing quality decay can affect farmers' profits.

One of my other contributions to the literature arises from handling the quality deterioration of fresh produce along supply chains. With the intent of modeling the quality of fresh produce products, I provide some preliminaries, focusing on the quality differential equations, which I then generalize to a path concept since the fresh produce will proceed on multiple supply chain links from the harvesting point to ultimate purchase at the farmers' markets. Each link has associated

with it both a time element as well as a factor such as temperature, which also affects the fresh produce quality. Moreover, I present explicit formulae for a variety of fresh produce to capture quality deterioration.

In addition, my framework considers both uncapacitated links in the supply chain network as well as capacitated ones, which may occur due to crop failures, harvesting problems, labor shortages, etc. Consequently, this is the first game theory model for farmers' markets and also the first competitive fresh produce supply chain network model in which quality deterioration of fresh produce is explicitly captured. This chapter is based on the paper Besik and Nagurney (2017).

1.4.4 Contributions in Chapter 6

In Chapter 6, I construct a competitive supply chain network model for fresh produce under oligopolistic competition among the food firms, who are profit-maximizers. The mathematical model of Chapter 5 is extended by adding a new strategic variable. Now the firms have, as their strategic variables, not only the product flows on the pathways of their supply chain networks from the production/harvesting locations to the ultimate points of demand, but also the initial quality of the produce that they grow at their production locations. The consumers at the retail outlets (demand points) differentiate the fresh produce from the distinct firms and reflect their preferences through the prices that they are willing to pay which depend on quantities of the produce as well as the average quality of the produce associated with the firm and retail outlet pair(s). Quality of the produce reaching a destination node depends on its initial quality and on the path that it took with each particular path consisting of specific links, with particular characteristics of physical features of time, temperature, etc., which are used to construct link multipliers, ultimately, yielding a path multiplier.

The governing Nash Equilibrium conditions are stated and alternative variational inequality formulations provided, along with existence results. An algorithmic scheme is outlined, which can be interpreted as a discrete-time adjustment process, and which yields closed form expressions at each iteration for the product path flows, the initial quality levels, as well as the Lagrange multipliers associated with the link capacities and the initial quality upper bounds. Stylistic examples are provided to illustrate the framework and a case study on peaches, consisting of numerical examples under status quo and disruption scenarios, is then presented, along with the computed equilibrium patterns. I also provide a few preliminaries, focusing on a review of fresh food quality deterioration and associated formulae as in Chapter 5. Moreover, I present the additional notation associated

with quality and its deterioration for the general supply chain network model. This chapter is based on the paper by Nagurney, Besik, and Yu (2018).

1.4.5   Concluding Comments

The main contributions of the methodologies and the results in this dissertation to the existing literature are summarized below.

1. The mathematical models constructed in Chapters 3, 4, 5 and 6 are the first competitive network models that capture the economics of trade and policy instruments and the food quality deterioration. The research presented in Chapters 3 and 4 of this dissertation are very timely considering the prevalence of trade wars and tariffs in todays global political environment especially in the second decade of 2000. In Chapters 5 and 6 of this dissertation, I present multidisciplinary works with constructs from food science integrated into the economics of supply chain networks.

2. The methodologies used in this dissertation are game theory and variational inequality theory. In Chapter 3, I use a spatial price network equilibrium concept, whereas, in Chapter 4, the problem is formulated as a Generalized Nash Equilibrium. In Chapters 5 and 6, governing equilibrium concept is that of Nash equilibrium. Qualitative results are provided in this dissertation along with discussion of computational procedures and results obtained from numerical cases.

3. The mathematical models constructed in Chapters 3 and 4 are general and can incorporate multiple firms, supply and demand markets in multiple countries and regions in each country. They include incorporation of various trade and policy instruments such as tariffs, quotas, and tariff-rate quotas. The modeling framework of the trade and policy instruments including the equilibrium conditions are quite unique. While a perfectly competitive model in the form of a spatial price network equilibrium problem is presented in Chapter 3, Chapter 4 provides global supply chain network model through studying a imperfectly competitive model, that is, an oligopoly.

4. In Chapter 3, a tariff-rate quota policy, consisting of an under quota tariff (applied when the imposed quota is not exceeded) and an over quota tariff (applied when the quota is exceeded), is constructed through a network equilibrium framework. The mathematical formulation of

tariff-rate quotas has been deemed challenging, making the contributions of the dissertation more significant.

5. It is shown in Chapter 4 that product supply chain network equilibrium model with a tariff coincides with the model with a strict quota.

6. In Chapters 5 and 6, I establish competitive perishable food supply chain network equilibrium models, in which the quality deterioration of the fresh produce product is captured explicitly by taking into account of the time and temperature information of fresh produce associated with supply chain network activities such as harvesting/production, processing, storage, and distribution. The quality deterioration functions related to various fresh produce are gathered from food science literature.

7. In Chapter 5 I study short food supply chains, in the form of local food supply chains, where the demand points represent the direct-to-consumer local farmers' markets, in different regions of a county or a state.

8. The mathematical model in Chapter 5 is extended to include initial quality of the fresh produce as a strategic variable in Chapter 6.

9. In the final chapter, I present my conclusions and provide further directions for research in this area.

CHAPTER 2

METHODOLOGIES

In this chapter, I provide an overview of the fundamental theories and methodologies that are utilized in this dissertation and subsequent dissertation. First, I review variational inequality theory which has been utilized throughout this dissertation as the essential methodology to analyze the equilibria of supply chain networks, especially food supply chain networks, with tariffs and quotas, and quality. Variational inequality theory is a powerful methodology with widespread use in solving supply chain network equilibrium models. Some of the relationships between variational inequality and game theory are also presented in this chapter.

In addition, I recall the theory of Generalized Nash Equilibrium which is utilized in Chapter 4 of this dissertation. Other theorems and proofs associated with finite-dimensional variational inequality theory can be found in Nagurney (1999).

Qualitative properties associated with the models in Chapters 3, 4, 5, and 6 are discussed at length in the following chapters. The quantitative results pertaining to the existence and uniqueness of solutions are also obtained using variational inequality theory.

Finally, I recall the algorithms: the Euler method and the modified projection method. The Euler method is employed to solve the variational inequality problems in Chapters 5 and 6. The modified projection method is applied to solve the variational inequality problems in Chapters 3 and 4.

2.1. Variational Inequality Theory

In this section, I provide a brief overview of the theory of variational inequalities followed by qualitative results, specifically concerning the existence and uniqueness of solutions. All definitions and theorems are taken from Nagurney (1999). All vectors are assumed to be column vectors.

- **Definition 2.1 (Finite-Dimensional Variational Inequality Problem)**

  The finite-dimensional variational inequality problem, $\mathrm{VI}(F, \mathcal{K})$, is to determine a vector $X^* \in \mathcal{K} \subset \mathcal{R}^n$, such that

  $$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{2.1a}$$

  where F is a given continuous function from $\mathcal{K}$ to $\mathcal{R}^n$, $\mathcal{K}$ is a given closed convex set, and $\langle \cdot, \cdot \rangle$ denotes the inner product in n-dimensional Euclidean space. In (2.1a), $F(X) \equiv (F_1(X), F_2(X), \ldots, F_n(X))^T$, and $X \equiv (X_1, X_2, \ldots, X_n)^T$. Recall that for two vectors $u, v \in \mathcal{R}^n$, the inner product $\langle u, v \rangle = \|u\| \|v\| cos\theta$, where θ is the angle between the vectors u and v, and (2.1a) is equivalent to

  $$\sum_{i=1}^{n} F_i(X^*) \cdot (X_i - X_i^*) \geq 0, \quad \forall X \in \mathcal{K}. \tag{2.1b}$$

The variational inequality problem is a general problem that encompasses a wide spectrum of mathematical problems, including; optimization problems, complementarity problems, and fixed point problems (see Nagurney (1999)). It has been shown that optimization problems, both constrained and unconstrained, can be reformulated as variational inequality problems. The relationship between variational inequalities and optimization problems is now briefly reviewed.

- **Proposition 2.1 (Formulation of a Constrained Optimization Problem as a Variational Inequality)**

  Let X^* be a solution to the optimization problem:

  $$Minimize \quad f(X) \tag{2.2}$$

  subject to:

  $$X \in \mathcal{K},$$

  where f is continuously differentiable and $\mathcal{K}$ is closed and convex. Then X^* is a solution of the variational inequality problem:

  $$\langle \nabla f(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{2.3}$$

  where $\nabla f(X)$ is the gradient vector of f with respect to X, where $\nabla f(X) \equiv (\frac{\partial f(X)}{\partial X_1}, \ldots, \frac{\partial f(X)}{\partial X_n})^T$.

- **Proposition 2.2 (Formulation of an Unconstrained Optimization Problem as a Variational Inequality)**

  If $f(X)$ is a convex function and X^* is a solution to $\text{VI}(\nabla f, \mathcal{K})$, then X^* is a solution to the optimization problem (2.2). In the case that the feasible set $\mathcal{K} = \mathcal{R}^n$, then the unconstrained optimization problem is also a variational inequality problem.

The variational inequality problem can be reformulated as an optimization problem under certain symmetry conditions. The definitions of positive-semidefiniteness, positive-definiteness, and strong positive-definiteness are recalled next, followed by a theorem presenting the above relationship.

- **Definition 2.2 (Positive Semi-Definiteness and Definiteness)**

  An $n \times n$ matrix $M(X)$, whose elements $m_{ij}(X); i,j = 1, ..., n$, are functions defined on the set $\mathcal{S} \subset \mathcal{R}^n$, is said to be positive-semidefinite on $\mathcal{S}$ if

  $$v^T M(X)v \geq 0, \quad \forall v \in \mathcal{R}^n, \, X \in \mathcal{S}. \tag{2.4}$$

  It is said to be positive-definite on $\mathcal{S}$ if

  $$v^T M(X)v > 0, \quad \forall v \neq 0, \, v \in \mathcal{R}^n, \, X \in \mathcal{S}. \tag{2.5}$$

  It is said to be strongly positive-definite on $\mathcal{S}$ if

  $$v^T M(X)v \geq \alpha \|v\|^2, \, for \, some \, \alpha > 0, \quad \forall v \in \mathcal{R}^n, \, X \in \mathcal{S}. \tag{2.6}$$

- **Theorem 2.1 (Formulation of an Optimization Problem from a Variational Inequality Problem Under Symmetry Assumption)**

  Assume that $F(X)$ is continuously differentiable on $\mathcal{K}$ and that the Jacobian matrix

  $$\nabla F(X) = \begin{bmatrix} \frac{\partial F_1}{\partial X_1} & \cdots & \frac{\partial F_1}{\partial X_n} \\ \vdots & \cdots & \vdots \\ \frac{\partial F_n}{\partial X_1} & \cdots & \frac{\partial F_n}{\partial X_n} \end{bmatrix} \tag{2.7}$$

is symmetric and positive-semidefinite. Then there is a real-valued convex function $f : \mathcal{K} \longmapsto$

$\mathcal{R}^1$ *satisfying*

$$\nabla f(X) = F(X) \tag{2.8}$$

with X^* the solution of $\mathrm{VI}(F, \mathcal{K})$ also being the solution of the mathematical programming

problem:

$$Minimize \quad f(X)$$

subject to:

$$X \in \mathcal{K},$$

where $f(X) = \int F(X)^T dx$, and $\int$ is a line integral.

Thus, variational inequality is a more general problem formulation than an optimization problem formulation, since it can also handle a function $F(X)$ with an asymmetric Jacobian (see Nagurney (1999)). Next, I recall the qualitative properties of variational inequality problems, especially, the conditions for existence and uniqueness of a solution.

Existence of a solution to a variational inequality problem follows from continuity of the function $F(X)$ entering the variational inequality, provided the feasible set $\mathcal{K}$ is compact as stated in Theorem 2.2.

- **Theorem 2.2 (Existence of a Solution)**

  If $\mathcal{K}$ is a compact convex set and $F(X)$ is continuous on $\mathcal{K}$, then the variational inequality problem admits at least one solution X^*.

- **Theorem 2.3 (Condition for Existence if Feasible Set is Unbounded)**

  If the feasible set $\mathcal{K}$ is unbounded, then $\mathrm{VI}(F, \mathcal{K})$ admits a solution if and only if there exists an $\mathcal{R} > 0$ and a solution of $\mathrm{VI}(F, \mathcal{S})$, X_R^*, such that $\|X_R^*\| < \mathcal{R}$, where $\mathcal{S} = \{X : \|X\| \leq \mathcal{R}\}$.

- **Theorem 2.4 (Existence Following a Coercivity Condition)**

  Suppose that $F(X)$ satisfies the coercivity condition

  $$\frac{\langle F(X) - F(X_0), X - X_0 \rangle}{\|X - X_0\|} \to \infty \tag{2.9}$$

  as $\|X\| \to \infty$ for $X \in \mathcal{K}$ and for some $X_0 \in \mathcal{K}$. Then $\mathrm{VI}(F, \mathcal{K})$ always has a solution.

According to Theorem 2.4, the existence condition of a solution to a variational inequality problem is guaranteed if the coercivity condition holds. Next, certain monotonicity conditions are utilized to discuss the qualitative properties of existence and uniqueness. Some basic definitions of monotonicity are provided first.

- **Definition 2.3 (Monotonicity)**

  $F(X)$ is monotone on $\mathcal{K}$ if

  $$\langle F(X^1) - F(X^2), X^1 - X^2 \rangle \geq 0, \quad \forall X^1, X^2 \in \mathcal{K}. \tag{2.10}$$

Next, a definition for strict monotonicity is given.

- **Definition 2.4 (Strict Monotonicity)**

  $F(X)$ is strictly monotone on $\mathcal{K}$ if

  $$\langle F(X^1) - F(X^2), X^1 - X^2 \rangle > 0, \quad \forall X^1, X^2 \in \mathcal{K}, \ X^1 \neq X^2. \tag{2.11}$$

- **Definition 2.5 (Strong Monotonicity)**

  $F(X)$ is strongly monotone on $\mathcal{K}$ if

  $$\langle F(X^1) - F(X^2), X^1 - X^2 \rangle \geq \alpha \|X^1 - X^2\|^2, \quad \forall X^1, X^2 \in \mathcal{K}, \tag{2.12}$$

  where $\alpha > 0$.

- **Definition 2.6 (Lipschitz Continuity)**

  $F(X)$ is Lipschitz continuous on $\mathcal{K}$ if there exists an $L > 0$, such that

  $$\langle F(X^1) - F(X^2), X^1 - X^2 \rangle \leq L\|X^1 - X^2\|^2, \quad \forall X^1, X^2 \in \mathcal{K}. \tag{2.13}$$

  L is called the Lipschitz constant.

- **Theorem 2.5 (Uniqueness Under Strict Monotonicity)**

  Suppose that $F(X)$ is strictly monotone on $\mathcal{K}$. Then the solution to the $\mathrm{VI}(F, \mathcal{K})$ problem is unique, if one exists.

- **Theorem 2.6 (Uniqueness Under Strong Monotonicity)**

  Suppose that $F(X)$ is strongly monotone on $\mathcal{K}$. Then there exists precisely one solution X^* to $\mathrm{VI}(F, \mathcal{K})$.

In summary of Theorems 2.2, 2.5, and 2.6, strong monotonicity of the function F guarantees both existence and uniqueness, in the case of an unbounded feasible set $\mathcal{K}$. If the feasible set $\mathcal{K}$ is compact, that is, closed and bounded, the continuity of F guarantees the existence of a solution. The strict monotonicity of F is then sufficient to guarantee its uniqueness provided its existence.

2.2. The Relationships between Variational Inequalities and Game Theory

In this section, some of the relationships between variational inequalities and game theory are briefly discussed.

Nash (1950, 1951) developed noncooperative game theory, involving multiple players, each of whom acts in his/her own interest. In particular, consider a game with m players, each player i having a strategy vector $X_i = \{X_{i1}, ..., X_{in}\}$ selected from a closed, convex set $\mathcal{K}^i \subset R^n$. Each player i seeks to maximize his/her own utility function, $U_i \colon \mathcal{K} \to R$, where $\mathcal{K} = \mathcal{K}^1 \times \mathcal{K}^2 \times \ldots \times \mathcal{K}^m \subset R^{mn}$. The utility of player i, U_i, depends not only on his/her own strategy vector, X_i, but also on the strategy vectors of all the other players, $(X_1, \ldots, X_{i-1}, X_{i+1}, \ldots, X_m)$. An equilibrium is achieved if no one can increase his/her utility by unilaterally altering the value of its strategy vector. The formal definition of the Nash equilibrium is recalled as follows.

24

- **Definition 2.7 (Nash Equilibrium)**

  A Nash equilibrium is a strategy vector

$$X^* = (X_1^*, \ldots, X_m^*) \in \mathcal{K}, \tag{2.14}$$

  such that

$$U_i(X_i^*, \hat{X}_i^*) \geq U_i(X_i, \hat{X}_i^*), \quad \forall X_i \in \mathcal{K}^i, \forall i, \tag{2.15}$$

  where $\hat{X}_i^* = (X_1^*, \ldots, X_{i-1}^*, X_{i+1}^*, \ldots, X_m^*)$.

It has been shown by Hartman and Stampacchia (1966) and Gabay and Moulin (1980) that given continuously differentiable and concave utility functions, $U_i, \forall i$, the Nash equilibrium problem can be formulated as a variational inequality problem defined on $\mathcal{K}$.

- **Theorem 2.7 (Variational Inequality Formulation of Nash Equilibrium)**

  Under the assumption that each utility function U_i is continuously differentiable and concave, X^* is a Nash equilibrium if and only if $X^* \in \mathcal{K}$ is a solution of the variational inequality

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad X \in \mathcal{K}, \tag{2.16}$$

  where $F(X) \equiv (-\nabla_{X_1} U_1(X), \ldots, -\nabla_{X_m} U_m(X))^{\mathsf{T}}$, and $\nabla_{X_i} U_i(X) = (\frac{\partial U_i(X)}{\partial X_{i1}}, \ldots, \frac{\partial U_i(X)}{\partial X_{in}})$.

The conditions for existence and uniqueness of a Nash equilibrium are now introduced. As stated in the following theorem, Rosen (1965) presented existence under the assumptions that $\mathcal{K}$ is compact and each U_i is continuously differentiable.

- **Theorem 2.8 (Existence Under Compactness and Continuous Differentiability)**

  Suppose that the feasible set $\mathcal{K}$ is compact and each U_i, is continuously differentiable $\forall i$. Then existence of a Nash equilibrium is guaranteed.

On the other hand, Gabay and Moulin (1980) relaxed the assumption of the compactness of $\mathcal{K}$ and proved existence of a Nash equilibrium after imposing a coercivity condition on $F(X)$.

- **Theorem 2.9 (Existence Under Coercivity)**

  Suppose that $F(X)$, as given in Theorem 2.7, satisfies the coercivity condition (2.9). Then there always exists a Nash equilibrium.

Furthermore, Karamardian (1969) demonstrated existence and uniqueness of a Nash equilibrium under the strong monotonicity assumption.

- **Theorem 2.10 (Existence and Uniqueness Under Strong Monotonicity)**

  Assume that $F(X)$, as given in Theorem 2.7, is strongly monotone on $\mathcal{K}$. Then there exists precisely one Nash equilibrium X^*.

Additionally, based on Theorem 2.5, uniqueness of a Nash equilibrium can be guaranteed under the assumptions that $F(X)$ is strictly monotone and an equilibrium exists.

- **Theorem 2.11 (Uniqueness Under Strict Monotonicity)**

  Suppose that $F(X)$, as given in Theorem 2.7, is strictly monotone on $\mathcal{K}$. Then the Nash equilibrium, X^*, is unique, if it exists.

2.3. Generalized Nash Equilibrium (GNE)

In this section, I present a brief discussion on Generalized Nash Equilibrium (GNE) in which the strategies of the players, defined by the underlying constraints, depend also on the strategies of their rivals. A frequently encountered class of Generalized Nash games deals with a common coupling constraint that the players' strategies are required to satisfy (Kulkarni and Shanbhag (2012)). These games are also known as Generalized Nash games with shared constraints (Facchinei and Kanow (2007), Rosen (1965)).

- **Definition 2.10 (Generalized Nash Equilibrium)**

  A strategy vector $X^* \in K \equiv \prod_{i=1}^{I} K^i, X^* \in \mathcal{S}$, constitutes a Generalized Nash Equilibrium if for each player $i; i = 1, ..., I$:

$$U_i(X_i^*, \hat{X}_i^*) \geq U_i(X_i, \hat{X}_i^*), \quad \forall X_i \in K^i, \forall X \in \mathcal{S}, \tag{2.17}$$

  where

$$\hat{X}_i^* \equiv (X_1^*, \ldots, X_{i-1}^*, X_{i+1}^*, \ldots, X_I^*),$$

26

K^i is the feasible set of individual player and $\mathcal{S}$ is the feasible set consisting of the shared constraints.

Bensoussan (1974) formulated the GNE problem as a quasivariational inequality. However, GNE problems are challenging to solve when formulated as quasivariational inequality problems for which the state-of-the-art in terms of algorithmic procedures is not as advanced as that for variational inequality problems. In Kulkarni and Shanbhag (2012), the authors provide sufficient conditions to establish the theory of Variational Equilibrium as a refinement of the GNE which is utilized in Chapter 4 of this dissertation.

- **Definition 2.11 (Variational Equilibrium)**

  A strategy vector X^* is said to be a variational equilibrium of the above Generalized Nash Equilibrium game if $X^* \in K, X^* \in \mathcal{S}$ is a solution of the variational inequality:

$$-\sum_{i=1}^{I}\langle \nabla_{X_i}\hat{U}_i(X^*), X_i - X_i^* \rangle \geq 0, \quad \forall X \in K, \forall X \in \mathcal{S}. \tag{2.18}$$

Next, I present the algorithms that are used in this dissertation.

2.4.   Algorithms

In this section, I review the algorithms that are used in this dissertation. The Euler method, which is based on the general iterative scheme of Dupuis and Nagurney (1993), and the modified projection method of Korpelevich (1977) are presented.

2.4.1   The Euler Method

The Euler-type method algorithm and it's convergence conditions are given below. At an iteration τ of the Euler method (see also Nagurney and Zhang (1996)), where τ denotes an iteration counter, one computes:

$$X^{\tau+1} = P_{\mathcal{K}}(X^\tau - \alpha_\tau F(X^\tau)), \tag{2.19}$$

where F is the function in (2.1a), and $P_{\mathcal{K}}$ is the projection on the feasible set $\mathcal{K}$, defined by

$$P_{\mathcal{K}}(X) = \text{argmin}_{X' \in \mathcal{K}} \|X' - X\|. \tag{2.20}$$

I now provide the complete statement of the Euler method.

Step 0: Initialization

Set $X^0 \in \mathcal{K}$.

Let $\tau = 1$ and set the sequence $\{\alpha_\tau\}$ so that $\sum_{\tau=1}^{\infty} \alpha_\tau = \infty$, $\alpha_\tau > 0$ for all τ, and $\alpha_\tau \to 0$ as $\tau \to \infty$.

Step 1: Computation

Compute $X^\tau \in \mathcal{K}$ by solving the variational inequality subproblem:

$$\langle X^\tau + \alpha_\tau F(X^{\tau-1}) - X^{\tau-1}, X - X^\tau \rangle \geq 0, \qquad \forall X \in \mathcal{K}. \tag{2.21}$$

Step 2: Convergence Verification

If $\max |X_l^\tau - X_l^{\tau-1}| \leq \epsilon$, for all l, with $\epsilon > 0$, a pre-specified tolerance, then stop; otherwise, set $\tau := \tau + 1$, and go to Step 1.

An assumption is recalled, followed by the convergence conditions of the Euler method in Theorem 2.12 and Corollary 2.1.

Assumption 2.1

Suppose we fix an initial condition $X_0 \in \mathcal{K}$ and define the sequence $\{X_\tau, \tau \in N\}$ by (2.19). I assume the following conditions:

1. *$\sum_{i=0}^{\infty} a_i = \infty$, $a_i > 0$, $a_i \to 0$ as $i \to \infty$.*

2. *$d(F_\tau(x), \overline{F}(x)) \to 0$ uniformly on compact subsets of $\mathcal{K}$ as $\tau \to \infty$.*

3. *The sequence $\{X_\tau, \tau \in N\}$ is bounded.*

- **Theorem 2.12 (Convergence of the General Iterative Scheme)**

  Let S denote the set of stationary point of the projected dynamical system (2.19), equivalently, the set of solutions to the variational inequality problem (2.1a). Assume Assuption 2.1. Suppose $\{X_\tau, \tau \in N\}$ is the scheme generated by (2.19). Then $d(X_\tau, S) \to 0$ as $\tau \to \infty$, where $d(X_\tau, S) \to 0 = inf_{X \in S} \|X_\tau - X\|$.

- **Corollary 2.1 (Existence of a Solution Under the General Iterative Scheme)**

  Assume the conditions of Theorem 2.12, and also that S consists of a finite set of points. Then $\lim_{\tau \to \infty} X_\tau$ exists and equals to a solution to the variational inequality.

In the subsequent chapters, for each model, I derive the explicit formulae for all the strategy vectors in the respective variational inequalities formulated.

2.4.2 The Modified Projection Method

The modified projection method of Korpelevich (1977) can be utilized to solve a variational inequality problem in standard form (cf. (2.1a)). This method is guaranteed to converge if the monotonicity (cf. (2.10)) and Lipschitz continuity (cf. (2.13)) of the function F that enters the variational inequality (cf. (2.1a)) hold, and a solution to the variational inequality exists.

I now recall the modified projection method, and let τ denote an iteration counter.

Step 0: Initialization

Set $X^0 \in \mathcal{K}$. Let $\tau = 1$ and let α be a scalar such that $0 < \alpha \leq \frac{1}{L}$, where L is the Lipschitz continuity constant (cf. (2.13)).

Step 1: Computation

Compute $\bar{X}^\tau$ by solving the variational inequality subproblem:

$$\langle \bar{X}^\tau + \alpha F(X^{\tau-1}) - X^{\tau-1}, X - \bar{X}^\tau \rangle \geq 0, \quad \forall X \in \mathcal{K}. \tag{2.22}$$

Step 2: Adaptation

Compute X^τ by solving the variational inequality subproblem:

$$\langle X^\tau + \alpha F(\bar{X}^\tau) - X^{\tau-1}, X - X^\tau \rangle \geq 0, \quad \forall X \in \mathcal{K}. \tag{2.23}$$

Step 3: Convergence Verification

If $\max |X_l^\tau - X_l^{\tau-1}| \leq \epsilon$, for all l, with $\epsilon > 0$, a prespecified tolerance, then stop; else, set $\tau := \tau + 1$, and go to Step 1.

- **Theorem 2.13 (Convergence of the Modified Projection Method)**
  If $F(X)$ is monotone and Lipschitz continuous (and a solution exists), the modified projection algorithm converges to a solution of variational inequality (2.1a).

In the following chapters, I derive the variational inequality formulations of the competitive food supply chain network models. The computational algorithms reviewed in this chapter, which are the

Euler method and the modified projection method, are also adapted accordingly to solve a number of simulated case studies.

CHAPTER 3

TARIFFS AND QUOTAS IN WORLD TRADE: A UNIFIED VARIATIONAL INEQUALITY FRAMEWORK

This chapter is based on the paper by Nagurney, Besik, and Dong (2019). Here, I construct a general spatial price network equilibrium model consisting of countries and multiple supply markets and demand markets in each country under a tariff-rate quota regime. Chapter 3 is organized as follows. In Section 3.1, I first present the general spatial price network equilibrium model with tariff-rate quotas, and derive a variational inequality formulation of the governing equilibrium conditions. I then demonstrate in Section 3.2 how the model can be easily adapted to also handle unit tariffs, ad valorem tariffs, and/or quotas, using a variational inequality framework. In addition, I present several numerical examples for illustrative purposes. In Section 3.3, I provide qualitative properties and propose the computational scheme in Section 3.4, which yields closed form expressions at each iteration. A case study in Section 3.5 on the dairy industry, with a focus on cheese, is then constructed to illustrate the effectiveness of the modeling and computational approach and the type of insights that can be gained. A summary of results, along with conclusions, are provided in Section 3.6.

3.1. The Spatial Price Network Equilibrium Model with Tariff Rate Quotas

I consider a spatial price equilibrium problem on a general network consisting of the set of nodes N and the set of links L for providing a mathematical framework of the model. In classical spatial price equilibrium problems, the commodity supply prices, trade flows, and demand prices are achieved when the equilibrium conditions stating that: if there is a positive volume of trade of the commodity between a supply market and a demand market, then the commodity price at the demand market is equal to the commodity price at the supply market plus the unit cost of transportation between the two (Nagurney (1999)). One crucial feature of spatial price equilibrium

problems is the acknowledgment of space, in that the supply and demand markets are spatially separated.

In this chapter, I extend this idea further by considering multiple countries, each of which can have multiple supply markets and demand markets associated with a homogeneous commodity. Here, I assume that there are m countries with supply markets and n countries with demand markets. A typical country with supply markets is denoted by i and its set of supply markets by $k \in O_i$, whereas a typical country with demand markets is denoted by j with its demand markets denoted by $l \in D_j$. There are total of k_i supply markets in country i; $i = 1, \ldots, m$, and a total of l_j demand markets in country j; $j = 1, \ldots, n$. As an illustration, and in order to fix ideas, I present an example of a spatial price network in Figure 3.1. A top-tiered node in the spatial network, denoted by (i, k), corresponds to supply market k in country i, whereas a bottom-tiered node (j, l) corresponds to demand market l in country j. I consider both domestic and/or international trade of a commodity, meaning that the country and supply market node, i.e., (i, k_i), can be the same or distinct from the country and demand market node, say, (j, l_j).

In addition, I define groups G_g; $g = 1, \ldots, h$, with group G_g consisting of all the countries i and j comprising the group and their supply and demand markets. For typical groups, denoted by G_s and G_r, I have that $G_s \cap G_r = \emptyset, \forall s \neq r, \forall s, r$. Furthermore, for each group G_g there is an associated quota $\bar{Q}_{G_g}$ on the homogeneous product.

Let $P^{(i,k)}_{(j,l)}$ denote the set of paths connecting supply market k of country i with demand market l of country j. A path is denoted by p and the flow of the product on path p by x_p. Also, let $P^{(i,k)}$ denote the set of paths from top-tiered node (i, k) to all the demand markets in all countries (the bottom-tiered nodes); similarly, let $P_{(j,l)}$ denote the set of paths from all the supply markets in all the countries (the top-tiered nodes) to demand market l of country j, that is, the bottom-tiered node (j, l). Paths can correspond to different transport routes as well as mode options. The network topology can take on any form, as mandated by the specific application.

Let f_a denote the flow on link a in the network, $\forall a \in L$. All the link flows grouped into the vector $f \in R^{n_L}_+$, where n_L is the number of links in the network. All vectors are column vectors.

The path flows must be nonnegative, that is,

$$x_p \geq 0, \quad \forall p \in P. \tag{3.1}$$

The Supply Markets in the Countries

Transshipment Nodes

Transport Mode Options

The Demand Markets in the Countries
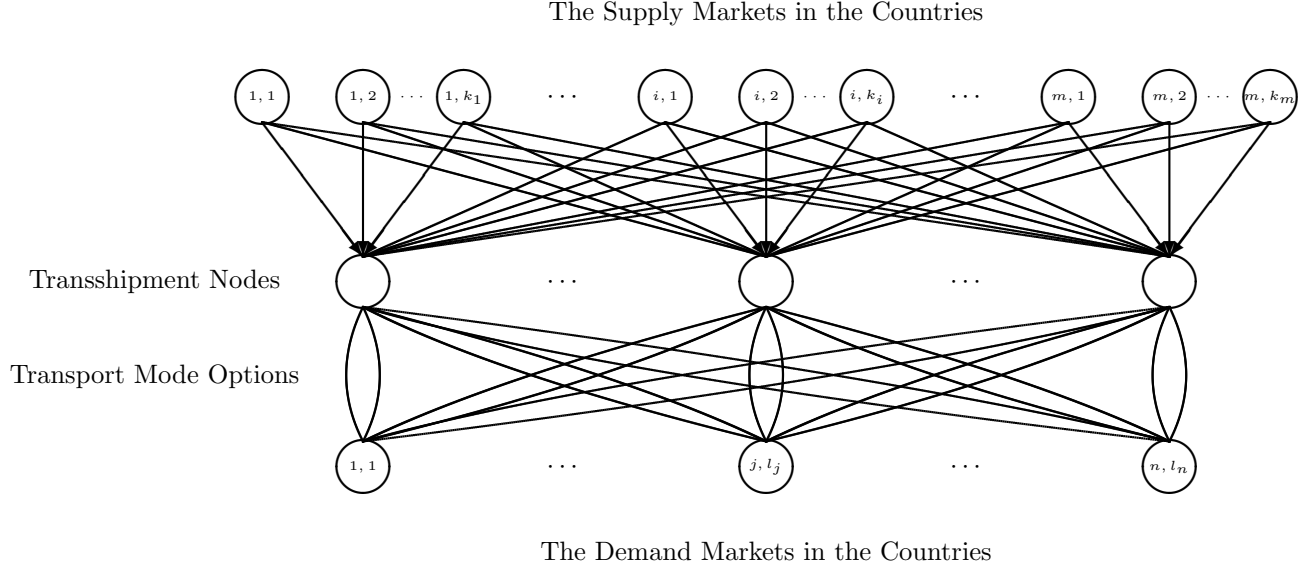
Figure 3.1. An Example of a Multicountry, Multiple Supply Market and Multiple Demand Market Spatial Price Network

The path flows are grouped into the vector $x \in R_+^{n_P}$, where n_P is the number of paths in the network.

The link flows are related to the path flows as follows:

$$f_a = \sum_{p \in P} x_p \delta_{ap}, \quad \forall a \in L, \tag{3.2}$$

that is, the flow on a link is equal to the sum of flows on paths that contain that link.

Let s_{ik} denote the supply of the product produced at supply market k of country i and let d_{jl} denote the demand for the product in demand market l of country j. I group the supplies into the vector $s \in R_+^{\sum_{i=1}^m k_i}$ and the demands into the vector $d \in R_+^{\sum_{j=1}^n l_j}$. The supplies and demands must satisfy, respectively, the following conservation of flow equations:

$$s_{ik} = \sum_{p \in P^{(i,k)}} x_p, \quad \forall i, k, \tag{3.3}$$

$$d_{jl} = \sum_{p \in P_{(j,l)}} x_p, \quad \forall j, l. \tag{3.4}$$

Associated with each supply market k of country i, (i, k), is a supply price function $\pi^{(i,k)}$ and with each demand market l of country j, (j, l), a demand price function $\rho_{(j,l)}$. It is assumed that, in

general, the supply price at a supply market in a country can depend on the vector of supplies and that the demand price at a demand market in a country can depend on the vector of demands so that:

$$\pi^{(i,k)} = \pi^{(i,k)}(s), \quad \forall i, k, \tag{3.5}$$

$$\rho_{(j,l)} = \rho_{(j,l)}(d), \quad \forall j, l. \tag{3.6}$$

These functions are assumed to be continuous with the supply price functions being monotonically increasing and the demand functions, on the other hand, monotonically decreasing.

Furthermore, a unit cost c_a associated with each link a in the network, which includes the unit transportation cost, and, in general, can depend on the vector of link flows f, so that:

$$c_a = c_a(f), \quad \forall a \in L. \tag{3.7}$$

The link cost functions are also assumed to be continuous and monotonically increasing in order to capture congestion.

The unit cost on a path p, C_p, joining a supply market in a country with a demand market in a country is then given by:

$$C_p = \sum_{a \in L} c_a(f)\delta_{ap}, \quad \forall p \in P. \tag{3.8}$$

Hence, the cost on a path is equal to the sum of the costs on the links that make up the path.

I now introduce the additional notation for the tariff-rate quotas. Recall that a tariff quota is a two-tiered tariff. Let $\tau^u_{G_g}$ denote the tariff, which is fixed and preassigned to group G_g, when the sum of the path flows of the product in the group is less than (under) the quota $\bar{Q}_{G_g}$, for g. Also, let $\tau^o_{G_g}$ denote the fixed and preassigned tariff, which is assigned to group G_g if the sum of the path flows of the product in the group is greater than $\bar{Q}_{G_g}$, for all g, and $\tau^o_{G_g} > \tau^u_{G_g}$. In addition, it is worth mentioning that the under tariffs can be set to zero, depending on the application. Let λ_{G_g}, for all G_g, denote the quota rent equivalent (see, e.g., Skully (2001)), which can have an interpretation of a Lagrange multiplier. Additionally, the quota rent equivalents are grouped into the vector $\lambda \in R^h$.

The costs, prices, and tariffs are all in a common currency.

- **Definition 3.1: Spatial Price Network Equilibrium Conditions Under Tariff Rate Quotas**

  A product flow pattern $x^* \in R_+^{n_P}$ and quota rent equivalent $\lambda^* \in R^h$ is a spatial price network equilibrium under a tariff-rate quota regime if the following conditions hold: For all groups G_g, for all g, and for all pairs of supply and demand markets in the countries: $(i,j),(k,l) \in G_g$, and all paths $p \in P_{(j,l)}^{(i,k)}$:

$$\pi^{(i,k)} + C_p + \tau_{G_g}^u + \lambda_{G_g}^* \begin{cases} = \rho_{(j,l)}, & if \quad x_p^* > 0, \\ \geq \rho_{(j,l)}, & if \quad x_p^* = 0, \end{cases} \tag{3.9}$$

  and for all g:

$$\lambda_{G_g}^* \begin{cases} = \tau_{G_g}^o - \tau_{G_g}^u, & if \quad \sum_{p \in P_{G_g}} x_p^* > \bar{Q}_{G_g}, \\ \leq \tau_{G_g}^o - \tau_{G_g}^u, & if \quad \sum_{p \in P_{G_g}} x_p^* = \bar{Q}_{G_g}, \\ = 0, & if \quad \sum_{p \in P_{G_g}} x_p^* < \bar{Q}_{G_g}. \end{cases} \tag{3.10}$$

Equilibrium conditions (3.9) are an expansion of the Samuelson (1952) and Takayama and Judge (1971) spatial price equilibrium conditions to include, in a novel manner, the incorporation of two-tiered tariffs, depending on the volumes of product flows and quotas in country groups, as stated in (3.10). See also path formulations of classical spatial price equilibrium problems using path flows by Florian and Los (1982) and Dafermos and Nagurney (1983). Note that, according to the above definition, if the sum of the product path flows is less than the imposed quota for the particular group of countries, then the assigned tariff on those paths is just equal to the "under" quota tariff $\tau_{G_g}^u$ since in this case $\lambda_{G_g}^* = 0$. On the other hand, if the sum of the path flows exceeds the quota (which is allowed in the case of tariff quotas), then the incurred tariff on the associated paths is equal to the "over" quota tariff $\tau_{G_g}^o$. If the sum of the path flows is precisely equal to the quota for the group then the incurred additional payment on the corresponding paths is less than $\tau_{G_g}^u$.

- **Remark 3.1**

In the case of a domestic group G_g, the above conditions (3.9) and (3.10) are also applicable (and this feature is illustrated in simple numerical examples in Section 3.2.1), where I set $\tau_{G_g}^u = 0$, and $\tau_{G_g}^o > \tau_{G_g}^u$, with $\bar{Q}_{G_g}$ very large, so that it is never achievable.

In view of (3.3) and (3.5), the supply price functions can be redefined as:

$$\hat{\pi}^{(i,k)}(x) \equiv \pi^{(i,k)}(s), \quad \forall i,k, \tag{3.11}$$

the demand price functions, in view of (3.4) and (3.6), as:

$$\hat{\rho}_{(j,l)}(x) \equiv \rho_{(j,l)}(d), \quad \forall j,l, \tag{3.12}$$

and, in view of (3.2), (3.7), and (3.8), the path and link cost functions:

$$\hat{C}_p(x) \equiv \sum_{a \in L} \hat{c}_a(x)\delta_{ap}, \quad \forall p \in P, \tag{3.13}$$

where

$$\hat{c}_a(x) \equiv c_a(f), \quad \forall a \in L. \tag{3.14}$$

The above definitions allow us to construct a variational inequality formulation of the governing spatial price network equilibrium conditions under a tariff-rate quota regime in path flows and quota rents in the Theorem below. The feasible set is defined as $K \equiv \{(x,\lambda)|x \in R_+^{n_P}, \lambda \in R_+^h | 0 \le \lambda_{G_g} \le \tau_{G_g}^o - \tau_{G_g}^u, \forall g\}$.

- **Theorem 3.1: Variational Inequality Formulation of Spatial Price Network Equilibrium Conditions Under Tariff Rate Quotas**

  A path flow and quota rent equivalent pattern $(x^*, \lambda^*) \in K$ is a spatial price network equilibrium under a tariff quota regime according to Definition 3.1 if and only if it satisfies the variational inequality:

$$\sum_g \sum_{(i,j) \in G_g} \sum_{k \in O_i} \sum_{l \in D_j} \sum_{p \in P_{(j,l)}^{(i,k)}} \left[\hat{\pi}^{(i,k)}(x^*) + \hat{C}_p(x^*) + \tau_{G_g}^u + \lambda_{G_g}^* - \hat{\rho}_{(j,l)}(x^*) \right] \times \left[x_p - x_p^* \right]$$

$$+ \sum_g \left[\bar{Q}_{G_g} - \sum_{p \in P_{G_g}} x_p^* \right] \times \left[\lambda_{G_g} - \lambda_{G_g}^* \right] \ge 0, \quad \forall (x,\lambda) \in K. \tag{3.15}$$

  Proof: First necessity conditions are established, that is, I prove that if a path flow and quota rent equivalent pattern $(x^*, \lambda^*) \in K$ satisfies Definition 3.1 then it also satisfies the variational inequality (3.15). Note that equilibrium conditions (3.9), in view of (3.11) − (3.14), can be

rewritten as: For all groups G_g, for all g, and for all pairs of supply and demand markets: $(i,j), (k,l) \in G_g$, and all paths $p \in P^{(i,k)}_{(j,l)}$:

$$\hat{\pi}^{(i,k)}(x^*) + \hat{C}_p(x^*) + \tau^u_{G_g} + \lambda^*_{G_g} \begin{cases} = \hat{\rho}_{(j,l)}(x^*), & \text{if } x^*_p > 0, \\[2mm] \geq \hat{\rho}_{(j,l)}(x^*), & \text{if } x^*_p = 0. \end{cases} \qquad (3.16)$$

Clearly, for a fixed path $p \in P^{(i,k)}_{(j,l)}$ with $(i,k), (j,l) \in G_g$, and group G_g, (3.16) implies that

$$\left[\hat{\pi}^{(i,k)}(x^*) + \hat{C}_p(x^*) + \tau^u_{G_g} + \lambda^*_{G_g} - \hat{\rho}_{(j,l)}(x^*)\right] \times \left[x_p - x^*_p\right] \geq 0, \quad \forall x_p \geq 0, \qquad (3.17)$$

since, according to Definition 3.1, if $x^*_p > 0$, then $[\hat{\pi}^{(i,k)}(x^*) + \hat{C}_p(x^*) + \tau^u_{G_g} + G_g^* - \hat{\rho}_{(j,l)}(x^*)] = 0$, and (3.17) holds; on the other hand, if $x^*_p = 0$, then $[\hat{\pi}^{(i,k)}(x^*) + \hat{C}_p(x^*) + \tau^u_{G_g} + \lambda^*_{G_g} - \hat{\rho}_{(j,l)}(x^*)] \geq 0$, and since, due to the nonnegativity assumption on the path flows: $[x_p - x^*_p] \geq 0$; hence, the product of these two terms is also ≥ 0, so (3.17) also holds in this case. Inequality (3.17) is independent of path p and, therefore, I can sum (3.17) over all paths, obtaining:

$$\sum_g \sum_{(i,j) \in G_g} \sum_{k \in O_i} \sum_{l \in D_j} \sum_{p \in P^{(i,k)}_{(j,l)}} \left[\hat{\pi}^{(i,k)}(x^*) + \hat{C}_p(x^*) + \tau^u_{G_g} + \lambda^*_{G_g} - \hat{\rho}_{(j,l)}(x^*)\right] \times \left[x_p - x^*_p\right] \geq 0,$$

$$\forall x \in K. \qquad (3.18)$$

Also, for a fixed g, (3.10) in Definition 3. 1 implies that:

$$\left[\bar{Q}_{G_g} - \sum_{p \in P_{G_g}} x^*_p\right] \times \left[\lambda_{G_g} - \lambda^*_{G_g}\right] \geq 0, \quad 0 \leq \lambda_{G_g} \leq \tau^o_{G_g} - \tau^u_{G_g}. \qquad (3.19)$$

Indeed, according to (3.10), if $\lambda^*_{G_g} = \tau^o_{G_g} - \tau^u_{G_g}$, then the expression before the multiplication sign in (3.19) is negative, whereas that after the multiplication sign is less than equal to zero and, thus, (3.19) holds. If $\lambda^*_{G_g} = 0$, then according to (3.10) the first term in (3.19) is nonnegative and, due to feasibility, so is the second term; therefore, their product is nonnegative and (3.19) also holds. Finally, if $\sum_{p \in P_{G_g}} x^*_p = \bar{Q}_g$, then, according to (3.10), the term to the right-hand-side of the multiplication sign in (3.19) can be positive or negative, but the multiplication of both terms is still equal to zero and (3.19) holds.

The inequality (3.19) holds for any g, and, therefore, I can conclude that summation over all g yields:

$$\sum_g \left[\bar{Q}_{G_g} - \sum_{p \in P_{G_g}} x_p^* \right] \times \left[\lambda_{G_g} - \lambda_{G_g}^* \right] \geq 0, \quad \forall \lambda \in K. \tag{3.20}$$

Summation of (3.18) and (3.20) gives us variational inequality (3.15) and necessity is established.

I now prove sufficiency, that is, a solution $(x^*, \lambda^*) \in K$ of variational inequality (3.15) also satisfies the supply chain network equilibrium conditions (3.9) and (3.10) of Definition 3.1.

Assume that a pattern $(x^*, \lambda^*) \in K$ satisfies variational inequality (3.15). Let $\lambda_{G_g} = \lambda_{G_g}^*$ for all g and let $x_q = x_q^*$ for all paths q except for path $p \in P_{(j,l)}^{(i,k)}$. Substitution of these into (3.15) then yields:

$$\left[\hat{\pi}^{(i,k)}(x^*) + \hat{C}_p(x^*) + \tau_{G_g}^u + \lambda_{G_g}^* - \hat{\rho}_{(j,l)}(x^*) \right] \times \left[x_p - x_p^* \right] \geq 0, \quad \forall x_p \geq 0. \tag{3.21}$$

But (3.21) implies that equilibrium conditions (3.9) must hold for path p and, hence, for any path.

Similarly, setting $x_p = x_p^*$, $\forall p \in P$, and $\lambda_{G_g} = \lambda_{G_g}^*$ for all $g \neq r$, and substituting the resultants into variational inequality (3.15) gives us:

$$\left[\bar{Q}_{G_r} - \sum_{p \in P_{G_r}} x_p^* \right] \times \left[\lambda_{G_r} - \lambda_{G_r}^* \right] \geq 0, \quad \forall \lambda_{Gr} \text{ such that } 0 \leq \lambda_{G_r} \leq \tau_{G_r}^o - \tau_{G_r}^u. \tag{3.22}$$

And (3.22) implies that equilibrium conditions (3.10) must hold for any r. The proof is complete.

□

I now put variational inequality (3.15) into standard form as shown in Chapter 2 equation 2.1a: determine $X^* \in \mathcal{K} \subset R^{\mathcal{N}}$, such that

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}. \tag{3.23}$$

where X and $F(X)$ are $\mathcal{N}$-dimensional vectors, $\mathcal{K}$ is a closed, convex set, and F is a given continuous function from $\mathcal{K}$ to $R^{\mathcal{N}}$. Indeed, I can define $X \equiv (x, \lambda)$ and $F(X) \equiv (F_1(X), F_2(X))$, where $F_1(X)$ consists of n_P elements: $\hat{\pi}^{(i,k)}(x) + \hat{C}_p(x) + \tau_{G_g}^u + \lambda_{G_g} - \hat{\rho}_{(j,l)}(x)$, for all paths $p \in P$, and $F_2(X)$ consists of h elements, with the g-th element given by: $\bar{Q}_{G_g} - \sum_{p \in P_{G_g}} x_p$. Also, here $\mathcal{N} = n_P + h$ and $\mathcal{K} \equiv K$.

3.2. Variants of the Spatial Price Network Equilibrium Model

In this section, I present several variants of the model constructed in Section 3.1. This is being done in order to demonstrate the flexibility of the variational inequality formalism for policy modeling in the context of world trade.

First, an extension of the classical spatial price network equilibrium model is given, and from this model policies in the form of ad valorem tariffs and then quotas incorporated.

Without loss of generality, let the set of paths P correspond to all the paths joining the country supply market nodes with the country demand market nodes with the number of paths being n_P. In the absence of a tariff-rate quota regime, the following definition is obtained, with reference to (3.16):

- **Definition 3.2: (Classical) Spatial Price Network Equilibrium Conditions**

  A product flow pattern $x^* \in R_+^{n_P}$ is a spatial price network equilibrium if the following conditions hold: For all pairs of supply and demand markets $(i,k), (j,l)$ and all paths $p \in P_{(j,l)}^{(i,k)}$:

  $$\hat{\pi}^{(i,k)}(x^*) + \hat{C}_p(x^*) \begin{cases} = \hat{\rho}_{(j,l)}(x^*), & if \quad x_p^* > 0, \\ \geq \hat{\rho}_{(j,l)}(x^*), & if \quad x_p^* = 0. \end{cases} \tag{3.24}$$

The following variational inequality is immediate using similar arguments as in the proof of Theorem 3.1.

- **Corollary 3.1: Variational Inequality Formulation of a Spatial Price Network Equilibrium**

  A product flow pattern $x^* \in R_+^{n_P}$ is a spatial price network equilibrium according to Definition 3.2 if and only if it satisfies the variational inequality:

  $$\sum_{(i,k)} \sum_{(j,l)} \sum_{p \in P_{(j,l)}^{(i,k)}} \left[\hat{\pi}^{(i,k)}(x^*) + \hat{C}_p(x^*) - \hat{\rho}_{(j,l)}(x^*) \right] \times \left[x_p - x_p^* \right] \geq 0, \quad \forall x^* \in R_+^{n_P}. \tag{3.25}$$

- **Remark 3.2**

There are alternative variational inequalities to (3.25). The first alternative would utilize the conservation of flow equations and vectors s and d and retain the path flows; the second alternative

formulation would then further utilize the link/path conservation of flow equations and have the supplies, demands, and link flows be the variables. For an example of a model with the latter, but not including explicit countries and regions, see Nagurney (1999).

Also, unit tariffs can be easily incorporated into the variational inequality (3.25) by simply adding the tariff on the appropriate link(s) of the appropriate paths.

Ad valorem tariffs are now considered, with $1 + \tau_{ij}$; $i = 1, \ldots, m$; $j = 1, \ldots, n$, denoting the ad valorem tariff associated with product flows from country i to j as in the modification of Definition 3.2 to yield Definition 3.3 below. Typically, the τ_{ij}s are in the range $0 < \tau_{ij} \leq 1$.

- **Definition 3.3: Spatial Price Network Equilibrium Conditions with Ad Valorem Tariffs**

  A product flow pattern $x^* \in R_+^{n_P}$ is a spatial price network equilibrium under an ad valorem tariff regime if the following conditions hold: For all pairs of supply and demand markets $(i, k), (j, l)$ and all paths $p \in P_{(j,l)}^{(i,k)}$:

$$(\hat{\pi}^{(i,k)}(x^*) + \hat{C}_p(x^*))(1 + \tau_{ij}) \begin{cases} = \hat{\rho}_{(j,l)}(x^*), & \text{if} \quad x_p^* > 0, \\ \geq \hat{\rho}_{(j,l)}(x^*), & \text{if} \quad x_p^* = 0. \end{cases} \tag{3.26}$$

The following corollary generalizes results of Nagurney, Nicholson, and Bishop (1996a,b) to multiple countries and multiple paths connecting each pair of supply and demand markets as well as non-fixed transportation costs.

- **Corollary 3.2: Variational Inequality Formulation of a Spatial Price Network Equilibrium with Ad Valorem Tariffs**

  A product flow pattern $x^* \in R_+^{n_P}$ is a spatial price network equilibrium under an ad valorem tariff regime according to Definition 3.3 if and only if it satisfies the variational inequality:

$$\sum_{(i,k)} \sum_{(j,l)} \sum_{p \in P_{(j,l)}^{(i,k)}} \left[(\hat{\pi}^{(i,k)}(x^*) + \hat{C}_p(x^*))(1 + \tau_{ij}) - \hat{\rho}_{(j,l)}(x^*) \right] \times \left[x_p - x_p^* \right] \geq 0, \quad \forall x^* \in R_+^{n_P}.$$

$$\tag{3.27}$$

I now consider strict quotas. Note that, in contrast, under a tariff-rate quota regime, the tariffs are two-tiered, and the quota can be exceeded, but then there is a higher associated tariff. In practice, these can be set very high.

Specifically, let $\hat{\bar{Q}}_g$ denote the strict quota for g; $g = 1, \ldots, \hat{h}$ and let $\lambda_{\hat{G}_g}$ denote the tariff/tax, which is, in effect, a Lagrange multiplier, and I refer to it as such, if the quota is reached for group $\hat{G}_g$. Then I have the following spatial price network equilibrium conditions:

- **Definition 3.4: Spatial Price Network Equilibrium Conditions Under Strict Quotas**

  A product flow pattern $x^* \in R_+^{n_P}$ and Lagrange multiplier pattern $\hat{\lambda}^* \in R_+^{\hat{h}}$ is a spatial price network equilibrium under strict quotas if the following conditions hold: For all pairs of supply and demand markets $(i,k), (j,l)$ and all paths $p \in P_{(j,l)}^{(i,k)}$:

$$
\hat{\pi}^{(i,k)}(x^*) + \hat{C}_p(x^*) + \hat{\lambda}^*_{\hat{G}_g}
\begin{cases}
= \hat{\rho}_{(j,l)}(x^*), & \text{if } x_p^* > 0, \\
\geq \hat{\rho}_{(j,l)}(x^*), & \text{if } x_p^* = 0,
\end{cases}
\tag{3.28}
$$

  and for all g:

$$
\hat{\lambda}^*_{\hat{G}_g}
\begin{cases}
\geq 0, & \text{if } \sum_{p \in P_{\hat{G}_g}} x_p^* = \hat{\bar{Q}}_{\hat{G}_g}, \\
= 0, & \text{if } \sum_{p \in P_{\hat{G}_g}} x_p^* < \hat{\bar{Q}}_{\hat{G}_g}.
\end{cases}
\tag{3.29}
$$

The following corollary follows using similar arguments as in the proof of Theorem 3.1.

- **Corollary 3.3: Variational Inequality Formulation of a Spatial Price Network Equilibrium with Strict Quotas**

  A product flow pattern $x^* \in R_+^{n_P}$ and Lagrange multiplier pattern $\hat{\lambda}^* \in R_+^{\hat{h}}$ is a spatial price network equilibrium under strict quotas according to Definition 3.4 if and only if it satisfies the variational inequality:

$$
\sum_{(i,k)} \sum_{(j,l)} \sum_{p \in P_{(j,l)}^{(i,k)}} \left[\hat{\pi}^{(i,k)}(x^*) + \hat{C}_p(x^*) + \hat{\lambda}^*_{\hat{G}_g} - \hat{\rho}_{(j,l)}(x^*) \right] \times \left[x_p - x_p^* \right]
$$

$$
+ \sum_g \left[\hat{\bar{Q}}_{\hat{G}_g} - \sum_{p \in P_{\hat{G}_g}} x_p^* \right] \times \left[\hat{\lambda}_{G_g} - \hat{\lambda}^*_{\hat{G}_g} \right] \geq 0, \quad \forall x^* \in R_+^{n_P}, \forall \hat{\lambda}^* \in R_+^{\hat{h}}.
\tag{3.30}
$$

3.2.1 Illustrative Examples

For definiteness, and in order to illustrate the model, with the focus on the TRQ one, now several illustrative examples are provided. I consider two countries with a single supply market in each country of a commodity, denoted by $(1,1)$ and $(2,1)$, respectively. There is a single demand market in country 1, $(1,1)$, which is spatially separated from its domestic supply market $(1,1)$. The network topology for the examples is as in Figure 3.2.

Country Supply Markets

$(1,1)$ ◯   ◯ $(2,1)$

◯

Country Demand Market $(1,1)$
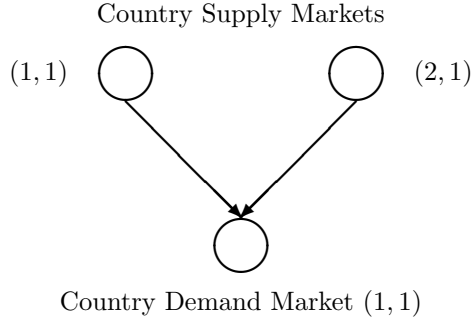
Figure 3.2. Network Topology for the Illustrative Example

Path p_1 consists of the link joining top-tiered node $(1,1)$ with bottom-tiered node $(1,1)$ and path p_2 consists of the link joining top-tiered node $(2,1)$ with bottom-tiered node $(1,1)$. There are two groups, with group G_1 consisting of the domestic production and consumption sites: top-tiered node $(1,1)$ and bottom-tiered node $(1,1)$, and group G_2 consisting of top-tiered node $(2,1)$, which is not domestic, and bottom-tiered node $(1,1)$.

3.2.1.1 Illustrative Example 1

I am interested in evaluating the impact of a tariff-rate quota (TRQ), but I first present a classical SPE example and then build the TRQ one from it.

The supply price functions are:

$$\hat{\pi}^{(1,1)}(x) = 5x_{p_1} + 5, \quad \hat{\pi}^{(2,1)}(x) = x_{p_2} + 2,$$

the unit path cost functions are:

$$\hat{C}_{p_1}(x) = x_{p_1} + 2, \quad \hat{C}_{p_2}(x) = x_{p_2} + 3,$$

42

and the demand price function is:

$$\hat{\rho}_{(1,1)}(x) = -(x_{p_1} + x_{p_2}) + 18.$$

Observe that the non-domestic supply market has a lower supply market price function but a higher unit transportation cost function than the domestic supply and demand markets.

In the classical model, without any imposed TRQs, the spatial price equilibrium conditions, according to (3.24), are, with the realization that $x_{p_1}^* > 0$ and $x_{p_2}^* > 0$:

$$\hat{\pi}^{(1,1)}(x^*) + \hat{C}_{p_1}(x^*) = \hat{\rho}_{(1,1)}(x^*),$$

$$\hat{\pi}^{(2,1)}(x^*) + \hat{C}_{p_2}(x^*) = \hat{\rho}_{(1,1)}(x^*),$$

and, hence, the following is obtained, at equilibrium:

$$5x_{p_1}^* + 5 + x_{p_1}^* + 2 = -(x_{p_1}^* + x_{p_2}^*) + 18,$$

$$x_{p_2}^* + 2 + x_{p_2}^* + 3 = -(x_{p_1}^* + x_{p_2}^*) + 18,$$

which, after algebraic simplification, results in the following system of equations:

$$7x_{p_1}^* + x_{p_2}^* = 11,$$

$$x_{p_1}^* + 3x_{p_2}^* = 13,$$

with solution: $x_{p_1}^* = 1$ and $x_{p_2}^* = 4$, with $\hat{\pi}^{(1,1)} = 10$, $\hat{C}_{p_1} = 3$, and $\hat{\rho}_{(1,1)} = 13$, and $\hat{\pi}^{(2,1)} = 6$, $\hat{C}_{p_2} = 7$, so, indeed, the classical spatial price equilibrium conditions are satisfied (cf. (3.24)).

The following tariff-rate quota is now imposed, which allows, once the problem is solved, to investigate the impacts on trade flows and incurred prices.

For the domestic pair, there is no under tariff, and the over tariff, which will never be applied, since I impose a very high quota, is as follows:

$$\tau_{G_1}^u = 0, \quad \tau_{G_1}^o = 1, \quad \bar{Q}_{G_1} = 100,$$

and for the non-domestic, domestic pair of nodes, the tariffs and associated quota are:

$$\tau^u_{G_2} = 2, \quad \tau^o_{G_2} = 4, \quad \bar{Q}_{G_2} = 3.$$

Returning to the governing equilibrium conditions for the SPE model with TRQs, given by (3.9) and (3.10), and knowing that $\lambda^*_{G_1} = 0$, now, the following equations must be satisfied:

$$5x^*_{p_1} + 5 + x^*_{p_1} + 2 = -(x^*_{p_1} + x^*_{p_2}) + 18,$$

$$x^*_{p_2} + 2 + x^*_{p_2} + 3 + \tau^u_{G_2} + \lambda^*_{G_2} = -(x^*_{p_1} + x^*_{p_2}) + 18.$$

Moreover, if $x^*_{p_2} = 3$, then $\lambda^*_{G_2} = \tau^o_{G_2} - \tau^u_{G_2} \leq 2$. Substituting $x^*_{p_2} = 3$ into the above system of equations, the following is obtained:

$$7x^*_{p_1} + 3 = 11,$$

$$x^*_{p_1} + 9 + \lambda^*_{G_2} = 11.$$

Solving now the above simplified system:

$$x^*_{p_1} = 1\frac{1}{7}, \quad \lambda^*_{G_2} = \frac{6}{7},$$

since $x^*_{p_2} = 3$.

Observe that, by imposing the TRQ, as above, the domestic supply market now produces more than in the classical SPE numerical example above, whereas the non-domestic supply market exports less. The consumers, nevertheless, now pay a higher unit price of $\hat{\rho}_{(1,1)} = 13\frac{6}{7}$ as opposed to 13. Also, now have: $\hat{\pi}^{(1,1)} = 10\frac{5}{7}$, $\hat{C}_{p_1} = 3\frac{1}{7}$, and $\hat{\pi}^{(2,1)} = 5$, $\hat{C}_{p_2} = 6$.

3.2.1.2 Illustrative Example 2

I construct another TRQ example and the network topology is as in Figure 3.2. The same markets are domestic as in Illustrative Example 3.1.

The supply price functions are now:

$$\hat{\pi}^{(1,1)}(x) = 3x_{p_1} + 1, \quad \hat{\pi}^{(2,1)}(x) = x_{p_2} + 1,$$

the unit path cost functions are:

$$\hat{C}_{p_1}(x) = 2x_{p_1} + 1, \quad \hat{C}_{p_2}(x) = x_{p_2} + 1,$$

and the demand price function is:

$$\hat{\rho}_{(1,1)}(x) = -2(x_{p_1} + x_{p_2}) + 26.$$

Proceeding as in Example 3.1, and realizing that $x_{p_1}^* > 0$ and $x_{p_2}^* > 0$, the classical SPE conditions (3.24) yield a system of equations, which, when solved algebraically, yield the following solution: $x_{p_1}^* = 2$ and $x_{p_2}^* = 5$, with the following incurred supply prices, unit path transportation costs, and demand price: $\hat{\pi}^{(1,1)} = 7$, $\hat{C}_{p_1} = 5$, and $\hat{\rho}_{(1,1)} = 12$, and $\hat{\pi}^{(2,1)} = 6$, $\hat{C}_{p_2} = 6$. Clearly, the classical spatial price equilibrium conditions are satisfied (cf. (3.24)).

Now the following tariff-rate quota is imposed, with: $\tau_{G_2}^u = 3$, $\tau_{G_2}^o = 6$, and $\bar{Q}_{G_2} = 2$.

Utilizing TRQ equilibrium conditions (3.9) and (3.10), the solution is: $x_{p_1}^* = 2\frac{1}{2}$, $x_{p_2}^* = 3\frac{1}{4} > 2$, and $\lambda_{G_2}^* = 3$. Observe that, in this example, the imports from the second supply market, which is not domestic, exceeds the imposed quota and, therefore, the higher tariff is in effect. The incurred prices and costs are: $\hat{\pi}^{(1,1)} = 8\frac{1}{2}$, $\hat{C}_{p_1} = 6$, and $\hat{\rho}_{(1,1)} = 14\frac{1}{2}$, and $\hat{\pi}^{(2,1)} = 4\frac{1}{4}$, $\hat{C}_{p_2} = 4\frac{1}{4}$. The supply price at the domestic supply market has increased, whereas that of the non-domestic one has decreased. More of the domestic product is consumed than previously without the TRQ and less of the non-domestic one. Again, the consumers experience a higher demand market price.

3.3. Qualitative Properties

In this section, I provide qualitative properties of the function F (cf. (3.23)) corresponding to the SPE model with TRQs, which are needed for convergence of the algorithmic scheme in Section 3.4. In addition, existence results for a solution X^* of the variational inequality (3.23) are provided.

In the following proposition I establish that if the supply price functions, the link cost functions, and minus the demand price functions are monotone in their respective vectors of variables, then

the function F as in (3.23) is monotone in (X, λ). Conditions that guarantee monotonicity of these functions is that their respective Jacobian matrices are positive semidefinite over the feasible set. In the proof of Proposition 3.1 I also derive an alternative variational inequality to (3.15), which includes link flows. Hence, the feasible set i defined as $K^1 \equiv \{(s, f, d, x, \lambda) | x \in R_+^{n_P}, \lambda \in R_+^h, 0 \leq \lambda_{G_g} \leq \tau_{G_g}^o - \tau_{G_g}^u, \forall g, \text{ and } (3.2), (3.3), (3.4) \text{ hold}\}$.

- **Proposition 3.1: Monotonicity of $F(X)$ in (3.23)**

  Assume that the vector of supply price functions, $\pi(s)$, the vector of link transportation cost functions, $c(f)$, and the vector of minus the demand price functions $-\rho(d)$, are all monotone (cf. Definition 2.3) as follows:

$$\langle \pi(s^1) - \pi(s^2), s^1 - s^2 \rangle \geq 0, \quad \forall (s^1, s^2) \in K^1, \tag{3.31a}$$

$$\langle c(f^1) - c(f^2), f^1 - f^2 \rangle \geq 0, \quad \forall (f^1, f^2) \in K^1, \tag{3.31b}$$

$$-\langle \rho(d^1) - \rho(d^2), d^1 - d^2 \rangle \geq 0, \quad \forall (d^1, d^2) \in K^1. \tag{3.31c}$$

  Then the function that enters the VI (3.15), as in standard form $F(X)$ in (3.23), is monotone, with respect to the path flows x and the Lagrange multipliers λ, $X = (x, \lambda)$.

Proof: Using (3.2), (3.3), (3.4), (3.7), and (3.8), and recalling that $G_g \cap G_h = \emptyset, \forall g \neq h, \forall g, h$, and the following equations:

$$s_{ik} = \sum_{p \in P_{(i,k)}} x_p = \sum_{j=1}^n \sum_{l=1}^{l_j} \sum_{p \in P_{(j,l)}^{(i,k)}} x_p = \sum_g \sum_{(i,j) \in G_g} \sum_{l \in D_j} \sum_{p \in P_{(j,l)}^{(i,k)}} x_p, \tag{3.32}$$

$$d_{jl} = \sum_{p \in P_{(j,l)}} x_p = \sum_{i=1}^m \sum_{k=1}^{k_i} \sum_{p \in P_{(j,l)}^{(i,k)}} x_p = \sum_g \sum_{(i,j) \in G_g} \sum_{k \in O_i} \sum_{p \in P_{(j,l)}^{(i,k)}} x_p, \tag{3.33}$$

and

$$\sum_g \sum_{(i,j) \in G_g} \sum_{k \in O_i} \sum_{l \in D_j} \sum_{p \in P_{(j,l)}^{(i,k)}} \hat{C}_p(x^*) \times [x_p - x_p^*] = \sum_{p \in P} \hat{C}_p(x) \times [x_p - x_p^*]$$

$$= \sum_{p \in P} \left[\sum_{a \in L} c_a(f) \delta_{ap} \right] \times [x_p - x_p^*] = \sum_{a \in L} c_a(f) \times \left[\sum_{p \in P} \delta_{ap} x_p - \sum_{p \in P} \delta_{ap} x_p^* \right] = \sum_{a \in L} c_a(f) \times [f_a - f_a^*]. \tag{3.34}$$

Using now (3.32), (3.33), and (3.34), and also (3.11) and (3.12), VI (3.15) is rewritten as follows:

$$\sum_g \sum_{(i,j)\in G_g} \sum_{k\in O_i} \sum_{l\in D_j} \sum_{p\in P^{(i,k)}_{(j,l)}} \hat{\pi}^{(i,k)}(x^*) \times \left[x_p - x_p^*\right]$$

$$+\sum_g \sum_{(i,j)\in G_g} \sum_{k\in O_i} \sum_{l\in D_j} \sum_{p\in P^{(i,k)}_{(j,l)}} \hat{C}_p(x^*) \times \left[x_p - x_p^*\right]$$

$$+\sum_g \sum_{(i,j)\in G_g} \sum_{k\in O_i} \sum_{l\in D_j} \sum_{p\in P^{(i,k)}_{(j,l)}} \left[\tau^u_{G_g} + \lambda^*_{G_g}\right] \times \left[x_p - x_p^*\right]$$

$$-\sum_g \sum_{(i,j)\in G_g} \sum_{k\in O_i} \sum_{l\in D_j} \sum_{p\in P^{(i,k)}_{(j,l)}} \hat{\rho}_{(j,l)}(x^*) \times \left[x_p - x_p^*\right]$$

$$+\sum_g \left[\bar{Q}_{G_g} - \sum_{p\in P_{G_g}} x_p^*\right] \times \left[\lambda_{G_g} - \lambda^*_{G_g}\right]$$

$$=\sum_{i,k} \pi^{(i,k)}(s^*) \times \left[s_{ik} - s_{ik}^*\right] + \sum_{a\in L} c_a(f^*) \times \left[f_a - f_a^*\right]$$

$$+\sum_g \sum_{p\in G_g} \left[\tau^u_{G_g} + \lambda^*_{G_g}\right] \times \left[x_p - x_p^*\right] - \sum_{j,l} \rho_{(j,l)}(d^*) \times \left[d_{jl} - d_{jl}^*\right]$$

$$+\sum_g \left[\bar{Q}_{G_g} - \sum_{p\in P_{G_g}} x_p^*\right] \times \left[\lambda_{G_g} - \lambda^*_{G_g}\right] \geq 0, \quad \forall(s,f,d,x,\lambda) \in K^1. \tag{3.35}$$

Now I can prove that $F(X)$ is monotone. For any $X^1 = (x^1, \lambda^1), X^2 = (x^2, \lambda^2) \in K$,

$$\langle F(X)^1 - F(X^2), X^1 - X^2 \rangle = \left\langle (F(x^1, \lambda^1) - F(x^2, \lambda^2)), \begin{bmatrix} x^1 - x^2 \\ \lambda^1 - \lambda^2 \end{bmatrix} \right\rangle$$

$$=\sum_{i,k} \left(\left[\pi^{(i,k)}(s^1) - \pi^{(i,k)}(s^2)\right] \times \left[s_{ik}^1 - s_{ik}^2\right] + \sum_{a\in L} \left[c_a(f^1) - c_a(f^2)\right] \times \left[f_a^1 - f_a^2\right]\right.$$

$$+\sum_g \left[\lambda_{G_g}^1 - \lambda_{G_g}^2\right] \times \left[\sum_{p\in P_{G_g}} x_p^1 - \sum_{p\in P_{G_g}} x_p^2\right] - \sum_{j,l} \left[\rho_{(j,l)}(d^1) - \rho_{(j,l)}(d^2)\right] \times \left[d_{jl}^1 - d_{jl}^2\right]$$

$$+\sum_g \left[-\sum_{p\in P_{G_g}} x_p^1 + \sum_{p\in P_{G_g}} x_p^2\right] \times \left[\lambda_{G_g}^1 - \lambda_{G_g}^2\right]$$

$$=\sum_{i,k} \left[\pi^{(i,k)}(s^1) - \pi^{(i,k)}(s^2)\right] \times \left[s_{ik}^1 - s_{ik}^2\right] + \sum_{a\in L} \left[c_a(f^1) - c_a(f^2)\right] \times \left[f_a^1 - f_a^2\right]$$

$$-\sum_{j,l} \left[\rho_{(j,l)}(d^1) - \rho_{(j,l)}(d^2)\right] \times \left[d_{jl}^1 - d_{jl}^2\right]. \tag{3.36}$$

With the assumptions (3.31a, b, c) on the functions in the statement of Proposition 3.1, I can conclude that expression (3.36) is greater or equal to zero. Thus, $F(X)$ is monotone. □

In addition, I can expect that $F(X)$ for the model is Lipschitz continuous under assumptions that the underlying supply price, demand price, and link cost functions are continuous and have bounded second order derivatives (cf. Definition 2.6). The definition of Lipschitz continuity is given below for easy reference.

- **Definition 3.5: Lipschitz Continuity**

  A function $F(X)$ is Lipschitz continuous if the following condition holds:

$$\|F(X') - F(X'')\| \le L\|X' - X''\|, \quad \forall X', X'' \in K, \tag{3.37}$$

  where $L > 0$ is known as the Lipschitz constant.

For convergence of the algorithmic scheme described in the next Section, which I utilize for computational purposes in the case study, only monotonicity and Lipschitz continuity of $F(X)$ are required, provided that a solution exists.

I now turn to establishing the existence of a solution X^* to VI (3.15); equivalently, (3.23). Note that the feasible set K is not compact and, hence, existence of a solution would not immediately follow from the classical theory of variational inequalities (see Kinderlehrer and Stampacchia (1980)) although the supply price, demand price, and link cost functions are assumed to be continuous.

- **Theorem 3.5: Existence of a Solution**

  Existence of a solution X^* to the VI for the model given by (3.15); equivalently, (3.23), is guaranteed.

Proof: I know that the quota rent equivalents comprising the vector λ are bounded because of the definition of the feasible set K. The demands, in turn, are bounded since the demand prices are monotonically decreasing and negative prices are not relevant. Furthermore, although demands may be large they will not be infinite since the population and, hence, demand, is bounded at each demand market. Therefore, the product path flows are also bounded and, consequently, existence of a solution X^* is guaranteed.

3.4. The Algorithm

In this section, the realization of the modified projected method (cf. Section 2.4.2) for the computation of the variational inequality (3.23), equivalently (3.15), governing the spatial equilib-

rium model with tariff-rate quotas and its variants, is described. Below, I present each step of the computation with respect to this model and provide the explicit formulae.

Step 0: Initialization

Set $(x^0, \lambda^0) \in K$. Let $t = 1$ and set φ so that $0 < \varphi \leq \frac{1}{L}$.

Step 1: Construction and Computation

Compute $(\bar{x}^t, \bar{\lambda}^t) \in K$ by solving the variational inequality subproblem:

$$\sum_g \sum_{(i,j) \in G_g} \sum_{k \in O_i} \sum_{l \in D_j} \sum_{p \in P^{(i,k)}_{(j,l)}} [\, \bar{x}^t_p + \varphi(\hat{\pi}^{(i,k)}(x^{t-1}) + \hat{C}_p(x^{t-1}) + \tau^u_{G_g} + \lambda^{t-1}_{G_g} - \hat{\rho}_{(j,l)}(x^{t-1})) - x^{t-1}_p \,] \times [\, x_p - \bar{x}^t_p \,]$$

$$+ \sum_g \left[\bar{\lambda}^t_{G_g} + \varphi(\bar{Q}_{G_g} - \sum_{p \in P_{G_g}} x^{t-1}_p) - \lambda^{t-1}_{G_g} \right] \times \left[\lambda_{G_g} - \bar{\lambda}^t_{G_g} \right] \geq 0, \quad \forall (x, \lambda) \in K. \qquad (3.38)$$

Step 2: Adaptation

Compute $(x^t, \lambda^t) \in K$ by solving the variational inequality subproblem:

$$\sum_g \sum_{(i,j) \in G_g} \sum_{k \in O_i} \sum_{l \in D_j} \sum_{p \in P^{(i,k)}_{(j,l)}} [\, x^t_p + \varphi(\hat{\pi}^{(i,k)}(\bar{x}^t) + \hat{C}_p(\bar{x}^t) + \tau^u_{G_g} + \bar{\lambda}^t_{G_g} - \hat{\rho}_{(j,l)}(\bar{x}^t)) - x^{t-1}_p \,] \times [\, x_p - x^t_p \,]$$

$$+ \sum_g \left[\lambda^t_{G_g} + \varphi(\bar{Q}_{G_g} - \sum_{p \in P_{G_g}} \bar{x}^t_p) - \lambda^{t-1}_{G_g} \right] \times \left[\lambda_{G_g} - \lambda^t_{G_g} \right] \geq 0, \quad \forall (x, \lambda) \in K. \qquad (3.39)$$

Step 3: Convergence Verification

If $|x^t_p - x_p^{t-1}| \leq \epsilon$, for all $p \in P^{(i,k)}_{(j,l)}$, $\forall i, k, j, l$, and $|\lambda^t_{G_g} - \lambda_{G_g}^{t-1}| \leq \epsilon$, for all $g = 1, \ldots, h$, with $\epsilon > 0$, a specified tolerance, then, stop; otherwise, set $t := t + 1$ and go to Step 1.

3.4.1 Closed Form Expressions

I present the closed form expressions for the solution of problems (3.38). Analogous ones for (3.39) can be determined accordingly.

The closed form expressions for the path flows at iteration t are as follows.

For each path $p \in P_{(j,l)}^{(i,k)}$, $\forall i, k, j, l$, compute:

$$\bar{x}_p^t = \max\{0, \varphi(\hat{\rho}_{(j,l)}(x_p^{t-1}) - \hat{\pi}^{(i,k)}(x^{t-1}) - \hat{C}_p(x^{t-1}) - \tau_{G_g}^u - \lambda_{G_g}^{t-1}) + x_p^{t-1}\}, \qquad (3.40)$$

and the following is the closed form expression for all the quota rent equivalents for group G_g; $g = 1, \ldots, h$:

$$\bar{\lambda}_{G_g}^t = \max\{0, \min\{\varphi(\sum_{p \in P_{G_g}} x_p^{t-1} - \bar{Q}_{G_g} + \lambda_{G_g}^{t-1}), \tau_{G_g}^o - \tau_{G_g}^u\}\}. \qquad (3.41)$$

I now provide the convergence result for the modified projection method.

- **Theorem 3.6: Convergence**

  Assume that the function $F(X)$ that enters the variational inequality (3.23) (or (3.15)) satisfies the conditions in Proposition 3.1 and following, that is, monotonicity and Lipschitz continuity. Then, the modified projection method described above converges to a solution of the variational inequality (3.23), or equivalently (3.15).

Proof: As stated in Korpelevich (1977), the modified projection method converges to the solution of the variational inequality problem (3.23) (or (3.15)), provided that the function F that enters the variational inequality is monotone and Lipschitz continuous and that a solution exists. $\square$

3.5. A Case Study on the Dairy Industry

In this section, I focus on a case study based on the dairy industry in the United States. In 2002, the dairy industry in the United States generated 20 billion dollars in sales value (Hadjigeorgalis (2005)). In recent years, the dairy industry in the United States has experienced a shift towards larger operations with more than 500 cows, in which the dairy farmers can hold more inventory and increase their production of milk. The large size operations accounted for nearly 60% of all milk produced in 2009, a substantial increase from 39% in 2001 (USDA (2018)). The dairy production in the US occurs primarily in the states of: California, Wisconsin, New York, Pennsylvania, Idaho, Minnesota, New Mexico, Michigan, Texas, and Washington (NASS (2004), USDA (2018)). Since 2002, the US is a net importer of dairy products, especially of cheese, according to Hadjigeorgalis (2005). The international trade in the dairy industry is governed by the 1994 Uruguay Round of Multilateral Trade Negotiations (cf. United States. Office of the U.S. Trade Representative (1994)). Furthermore, in the United States tariff-rate quotas are imposed on most of the dairy products. The

US is an exporter of some dairy products such as dry nonfat milk products, with the major export market for the United States of nonfat dry milk being Mexico.

For the case study, I focus on the dairy industry, specifically, on cheese in the United States. Cheese is usually contained in a preserved form in which the main protein and milk fat are not exposed to rapid deterioration from microorganisms. According to Hadjigeorgalis (2005), the European Union dominates the cheese market in the United States by having an import value of 69%, where the closest export country is New Zealand with only 10%. Therefore, it is clear that the European Union and the United States engage in a crucial amount of agricultural trade and it is worthwhile to analyze this relationship in this case study. Especially, in 2018, with the ongoing trade wars (see Tankersley (2018)) and disputes between many countries all over the world, I believe that a case study on the dairy industry and the evaluation of the impact of tariff-rate quotas are worth investigating (cf. Paquette, Lynch, and Rauhala (2018)). Furthermore, according to Chatellier (2016), the Netherlands, Germany, and France are the top exporters of dairy products in the European Union, and cheese is counted as the top dairy product exported from France. Therefore, for this case study, I focus on France as a producing country from the European Union.

I implemented the modified projection method in MATLAB on an OS X 10.12.6 system. The code in Matlab is executed on a Macbook Pro laptop with a 2.8 GHz Intel Core i5 processor and 8GB 1600 MHz DDR3 memory. The parameter φ is set to 0.3 with the convergence tolerance being 10^{-6}, that is, the modified projection method is deemed to have converged if the absolute value of the difference of each successive variable iterate differs by no more than this value. The algorithm was initialized with each path flow set equal to 1, and with each quota rent equivalent (the Lagrange multiplier) for the variational inequality (3.15) set equal to 0. In the following examples I modify the spatial price network and also the associated tariff values to generate insights.

3.5.1 Baseline Example

The first example serves as a baseline example. The computations are made utilizing the closed form expressions provided for the modified projection method in Section 3.4. The spatial price network structure for the Baseline Example is given in Figure 3.3. For this example, the producing countries are the United States and France, which belongs to the European Union, with each having two supply markets. Furthermore, The United States has supply markets: Southwest and Midwest, represented in Figure 3 by the top-tiered nodes $(1, 1)$ and $(1, 2)$, respectively, whereas France has

supply markets: South and North, represented, respectively, by the top-tiered nodes $(2, 1)$ and $(2, 2)$ in Figure 3.3.

The transshipment node is assumed to be located in the East region of the United States, with the cheese from the South and the North regions of France arriving in the East region of the United States, and then being transported to the demand markets. Furthermore, I assume that cheese produced at a dairy farm in the Southwest and the Midwest of the United States is sent to the East region of the United States to be distributed to the demand markets. The United States has three demand markets, depicted by the bottom-tiered nodes in Figure 3.3: $(1, 2)$, $(1, 3)$, and $(1, 4)$, corresponding, without loss of generality to: the Midwest, the Northeast, and the Southeast of the United States, respectively.

There are two groups, G_1 and G_2. The first group, G_1, contains the domestic supply markets and demand markets in the United States, and includes the nodes $(1, 1)$, $(1, 2)$ (both supply and demand ones), $(1, 3)$, and $(1, 4)$. The group G_2 represents the supply markets in France, specifically, the South and the North, and the demand markets in the US, represented by the bottom-tiered nodes in Figure 3.3: $(1, 2)$, $(1, 3)$, and $(1, 4)$. The examples are constructed from the data collected from the website of the European Commission (2018). I also retrieved information from the United Nations (M49) area tool (cf. United Nations (1999)), in which they define countries and regions for regional and preferential trade agreements.
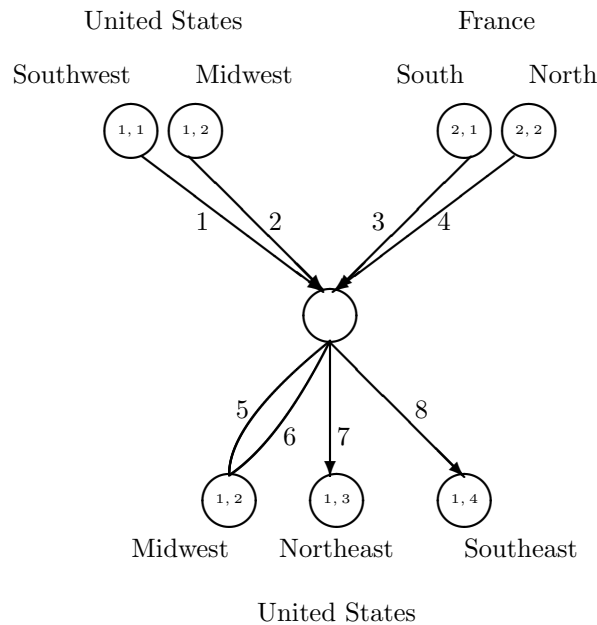
Figure 3.3. Spatial Network Structure of the Baseline Example

In the spatial price network in Figure 3.3, there are eight links in the set $L = (1, 2, 3, 4, 5, 6, 7, 8)$. Link 1 depicts the transportation of cheese from the Southwest of the United States to the transshipment point, located in the United States, such as Philadelphia, by rail transport. Link 2 represents the railway transport from the Midwest of the United States to the transshipment point. Links 3 and 4 represent, respectively, sea transportation and air transportation to the transshipment point from the South and North of France. I assume that link 5 represents the air transportation mode, whereas links 6, 7, and 8 are for the truck transportation. Having air transportation from the transshipment node to the Midwest region of the United States is acceptable, since cheese is perishable. Furthermore, there are sixteen paths in the spatial price network, and they are as follows: path $p_1 = (1, 5)$, $p_2 = (1, 7)$, $p_3 = (1, 8)$, $p_4 = (2, 5)$, $p_5 = (2, 7)$, $p_6 = (2, 8)$, $p_7 = (3, 5)$, $p_8 = (3, 7)$, $p_9 = (3, 8)$, $p_{10} = (4, 5)$, $p_{11} = (4, 7)$, $p_{12} = (4, 8)$, $p_{13} = (1, 6)$, $p_{14} = (2, 6)$, $p_{15} = (3, 6)$, and $p_{16} = (4, 6)$.

The data for the supply price functions, link cost functions, and the demand price functions are now given. The supply price functions are:

$$\pi^{(1,1)}(s) = .03s_{11} + .02s_{12} + .01s_{21} + .01s_{22} + 3,$$

$$\pi^{(1,2)}(s) = .03s_{12} + .02s_{11} + .01s_{21} + .01s_{22} + 4,$$

$$\pi^{(2,1)}(s) = .02s_{21} + .01s_{11} + .01s_{12} + .01s_{22} + 1,$$

$$\pi^{(2,2)}(s) = .02s_{22} + .01s_{11} + .01s_{12} + .01s_{21} + 2.$$

I assume that the fixed supply price term for the cheese produced in the United States at nodes $(1, 1)$ and $(1, 2)$ is higher than those supply price function terms for the cheese produced in France, at nodes $(2, 1)$ and $(2, 2)$. The reasoning behind this assumption is that the European Union has a major role in milk production, responsible for 20.1% of the world production, followed by the United States with 12% (Chatellier (2016)). Since cheese is produced from milk, it is reasonable to assume for this case study that France, as a part of the European Union, has an advantage over production of cheese against its competitor, the United States. Hence, the fixed supply price term for the cheese produced in the South and North of France is lower at nodes (2,1) and (2,2).

The link cost functions, in which the link flows are converted to path flows for the algorithm according to (3.13) and (3.14), are:

$$c_1(f) = .001f_1^2 + .01f_1, \quad c_2(f) = .001f_2^2 + .005f_2, \quad c_3(f) = .001_3^2 + .06f_3,$$

$$c_4(f) = .002f_4^2 + 0.1f_4, \ c_5(f) = .002f_5^2 + 0.1f_5, \ c_6(f) = .001_6^2 + .06f_6,$$

$$c_7(f) = .001_7^2 + .06f_7, \ c_8(f) = .001_8^2 + .06f_8.$$

The cost functions are constructed, according to the transportation modes. Links that represent truck, railway, or sea transportation are cheaper than air transportation, represented by links 4 and 5.

Furthermore, the demand price functions are as follows:

$$\rho_{(1,2)}(d) = -0.01d_{12} - 0.0075d_{13} - 0.005d_{14} + 9,$$

$$\rho_{(1,3)}(d) = -0.01d_{13} - 0.0075d_{12} - 0.005d_{14} + 10,$$

$$\rho_{(1,4)}(d) = -0.0075d_{14} - 0.005d_{12} - 0.0025d_{13} + 11.$$

For the Baseline Example, I assume similar demand price functions for cheese to be sold in the Midwest, the Southeast, and the Northeast of the United States, with the cheese demand price function fixed term in the Midwest being lower than those in the Southeast and the Northeast. Furthermore, the cheese price fixed terms in the Northeast are higher than in the Southeast, resulting from a different consumer profile. Additionally, the price for cheese in the Southeast region is less sensitive to the demand than in the other demand market regions, due to the consumer profile and the larger population. Under these assumptions, I constructed the above demand price functions.

Next, I define the necessary parameters for the tariff-rate quota regime. Recall that the two-tiered tariff requires an under quota tariff, $\tau_{G_g}^u$, and an over quota tariff, $\tau_{G_g}^o$, for each group. Since I include the domestic production in the United States for Group G_1, $\tau_{G_1}^u = 0$ and $\tau_{G_1}^o = 0$. According to the World Trade Organization (2018), the United States imposes the in quota fresh cheese tariff as 1.128 dollars per kg of cheese and the over quota tariff as 2.126 dollars per kg of cheese for the European Union, including the cheese exported from France. I assumed that $\tau_{G_2}^u = 1$ dollar per kilogram and $\tau_{G_2}^o = 2$ dollars per kilogram for Group G_2. I analyze a short time period such as a week. The quotas for the groups are assigned as $\bar{Q}_{G_1} = 10000$, which is a large number, since Group G_1 includes the domestic production and consumption, and the quota for Group G_2 is assumed to be $\bar{Q}_{G_2} = 100$ kilograms, imposed on France by the United States for this group. The data is collected from the report by USDA (2018).

The computed equilibrium cheese product path flows in kilograms for the Baseline Example and the equilibrium path costs are reported in Table 3.1.

Path	Path Links	x_p^*	$\hat{C}_p(x^*)$
p_1	(1,5)	5.30	3.25
p_2	(1,7)	9.50	4.26
p_3	(1,8)	11.92	5.52
p_4	(2,5)	2.40	2.36
p_5	(2,7)	6.60	3.38
p_6	(2,8)	9.02	4.64
p_7	(3,5)	5.06	5.82
p_8	(3,7)	9.26	6.83
p_9	(3,8)	11.67	8.10
p_{10}	(4,5)	0.99	4.98
p_{11}	(4,7)	5.19	6.00
p_{12}	(4,8)	7.60	7.26
p_{13}	(1,6)	7.25	3.25
p_{14}	(2,6)	4.34	2.36
p_{15}	(3,6)	7.00	5.82
p_{16}	(4,6)	2.93	4.98

Table 3.1. Equilibrium Path Flows and Incurred Path Costs for the Baseline Example

Furthermore, the equilibrium link flows and the incurred link costs for the Baseline Example are given in Table 3.2.

Link a	f_a^*	$c_a(f^*)$
1	33.99	1.49
2	22.37	0.61
3	33.00	3.06
4	16.72	2.23
5	13.76	1.75
6	21.53	1.75
7	30.56	2.76
8	40.23	4.03

Table 3.2. Equilibrium Link Flows and Incurred Link Costs for the Baseline Example

The computed equilibrium quota rent equivalents are: $\lambda_{G_1}^* = 0$ and $\lambda_{G_2}^* = 0$.

Notice that, for this example, I assumed a large quota for France and, therefore, the quota rent equivalent is zero. This is modified in the next example.

Also, the computed equilibrium cheese supply and demand, in kilograms, are:

$$s_{11}^* = 33.99, \ s_{12}^* = 22.37, \ s_{21}^* = 33.00, \ s_{22}^* = 16.72,$$

$$d_{12}^* = 35.30, \quad d_{13}^* = 30.56, \quad d_{14}^* = 40.23.$$

The total amount of cheese shipment from the Southwest and Midwest regions of the United States is larger than the cheese shipments from the South and North of France.

Furthermore, the incurred supply prices per kilogram of cheese in dollars are:

$$\pi^{(1,1)} = 4.96, \ \pi^{(1,2)} = 5.84, \ \pi^{(2,1)} = 2.39, \ \pi^{(2,2)} = 3.22.$$

Observe that the supply prices of cheese are higher in the Southwest and the Midwest regions of the United States than in the South and North regions of France. This should have given an advantage to France to export their cheese in a larger amount to the United States; however, since the transportation costs are larger for France, and their cheese is also subject to an under tariff of $\tau_{G_2}^u = 1$ (cf. (3.9) and (3.10)) large export amounts are not achieved.

Next, I report the incurred demand prices, per kilogram of cheese, at the equilibrium, in dollars:

$$\rho_{(1,2)} = 8.21, \quad \rho_{(1,3)} = 9.22, \quad \rho_{(1,4)} = 10.49.$$

The demand price for cheese in the Southeast of the US is higher than it is in the Northeast. This is reasonable due to the demographic differences in these demand regions.

The equilibrium conditions (3.9) and (3.10) are satisfied with excellent accuracy.

3.5.2 Change in Quotas Example

This example is constructed from the Baseline Example and has the spatial price network topology in Figure 3.3 as well as the same data, except for the following. In this example, I decrease the quota of Group G_2. In 2018, the turmoil in world trade generated an environment in which the import countries started to modify their tariffs and tariff-rate quotas to maintain their political agendas (cf. Paquette, Lynch, and Rauhala (2018)). In a broader sense, import countries can impose stricter quotas and higher under quota and over quota tariffs on their trade partners to support their domestic production. I, hence, modify the tariffs and quotas to investigate the associated impacts on the economy.

For the Change in Quotas Example, I consider the situation in which the United States imposes a stricter quota on cheese that is exported from France's supply regions. Therefore, the quota on

Group G_2, $\bar{Q}_{G_2}$, is tightened to 35 kilograms. The supply price functions, link cost functions, and the demand price functions remain as in the Baseline Example.

The computed equilibrium cheese product path flows, in kilograms, are reported in Table 3.3.

Path	Path Links	x_p^*	$\hat{C}_p(x^*)$
p_1	(1,5)	5.88	3.26
p_2	(1,7)	10.04	4.27
p_3	(1,8)	12.48	5.53
p_4	(2,5)	3.21	2.37
p_5	(2,7)	7.36	3.38
p_6	(2,8)	9.80	4.63
p_7	(3,5)	3.65	5.97
p_8	(3,7)	7.80	6.98
p_9	(3,8)	10.24	8.23
p_{10}	(4,5)	0.00	5.12
p_{11}	(4,7)	4.03	6.13
p_{12}	(4,8)	6.47	7.38
p_{13}	(1,6)	7.73	3.06
p_{14}	(2,6)	5.05	2.37
p_{15}	(3,6)	5.49	5.97
p_{16}	(4,6)	1.72	5.12

Table 3.3. Equilibrium Path Flows and Incurred Path Costs for the Change in Quotas Example

When the quota for Group G_2 is decreased, the path flows are adjusted accordingly. Notice that the flow on path p_{10} is equal to 0.00.

The equilibrium link flows and associated incurred link costs for this example are given in Table 3.4.

Link a	f_a^*	$c_a(f^*)$
1	36.14	1.66
2	25.45	0.77
3	27.21	2.37
4	12.23	1.52
5	12.76	1.60
6	20.01	1.60
7	29.25	2.61
8	39.01	3.86

Table 3.4. Equilibrium Link Flows and Incurred Link Costs for the Change in Quotas Example

The computed equilibrium quota rent equivalents are: $\lambda_{G_1}^* = 0$ and $\lambda_{G_2}^* = 1$.

Furthermore, the computed equilibrium supplies and demands, in kilograms, are:

$$s_{11}^* = 36.14, \ s_{12}^* = 25.45, \ s_{21}^* = 27.21, \ s_{22}^* = 12.23,$$

$$d_{12}^* = 32.78, \quad d_{13}^* = 29.25, \quad d_{14}^* = 39.01.$$

The equilibrium quota rent equivalent for Group G_2 is positive and at its maximum value, $\tau_{G_2}^o - \tau_{G_2}^u = 2 - 1 = 1$. The reason for having a positive quota rent equivalent for Group G_2 is that the summation of path flows originating from the South and the North of France; equivalently, the sum of the supplies from France, which is equal to 39.45, is over the imposed quota of 35.

It is clear that the equilibrium supply of cheese from the South and the North of France decreases from the respective values in the Baseline Example, due to the stricter quota on Group G_2 as well as the tariffs. The equilibrium domestic supply in the United States, represented by s_{11}^* and s_{12}^*, increases. Additionally, the equilibrium demand values decrease.

The incurred supply prices per kilogram of cheese in dollars at the equilibrium are:

$$\pi^{(1,1)} = 4.98, \ \pi^{(1,2)} = 5.88, \ \pi^{(2,1)} = 2.28, \ \pi^{(2,2)} = 3.13.$$

The supply prices of cheese in the Southwest and the Midwest of the United States increase from their values in the Baseline Example. On the other hand, the supply prices for the South and North of France decrease to try to recover any competitive advantage.

The incurred equilibrium demand prices, per kilogram of cheese, in dollars, are:

$$\rho_{(1,2)} = 8.25, \quad \rho_{(1,3)} = 9.26, \quad \rho_{(1,4)} = 10.51.$$

The demand prices increase in all three demand markets in the United States from their previous values, meaning that the cheese supply in the United States from the Southwest and the Midwest is not sufficient to keep the demand prices as equal or lower than they were in the Baseline Example. This shows that having a trade quota could result in a demand price increase, affecting negatively the consumers in the economy. This was also observed in the illustrative examples in Section 3.2.1.

I now proceed to conduct sensitivity analysis for this example. In particular, I investigate the impact of an increase in the over quota tariff $\tau_{G_2}^o$ on the production outputs of both France and the United States.

The algorithm is re-run for three different over quota tariffs, $\tau_{G_2}^o = 3$ dollars, $\tau_{G_2}^o = 5$ dollars, and $\tau_{G_2}^o = 8$ dollars for Group G_2. The equilibrium link flows for the production links $(1, 2, 3, 4)$, under different over quota tariffs are reported in Table 3.5.

Over Quota Tariffs	s_{11}^*	s_{12}^*	s_{21}^*	s_{22}^*
$\tau_{G_2}^o = 2$	36.14	25.45	27.21	12.23
$\tau_{G_2}^o = 3$	38.43	28.57	21.04	7.34
$\tau_{G_2}^o = 5$	42.89	34.38	6.18	0.00
$\tau_{G_2}^o = 8$	44.10	35.90	0.00	0.00

Table 3.5. Equilibrium Supplies under Different Over Quota Tariffs on France for Example 3.5.2

Observe from Table 3.5 that, when the over quota tariff increases, the supply of cheese from France decreases, represented by s_{21}^* and s_{22}^*. However, the supply of cheese produced in the United States in the Southwest and Midwest regions experiences a steady increase, demonstrating vividly the positive impact on producers in the US. Furthermore, notice that the equilibrium supplies decrease, as the over quota tariffs increase for the supply of cheese from North and the South regions of France. When the over quota tariff is high for Group G_2, the supply of cheese from the North and the South regions of France stay under their quota and the algorithm converges to lower supply amounts for Group G_2.

Over Quota Tariffs	$\rho_{(1,2)}$	$\rho_{(1,3)}$	$\rho_{(1,4)}$
$\tau_{G_2}^o = 2$	8.25	9.26	10.51
$\tau_{G_2}^o = 3$	8.30	9.30	10.54
$\tau_{G_2}^o = 5$	8.40	9.39	10.60
$\tau_{G_2}^o = 8$	8.42	9.42	10.62

Table 3.6. Demand Prices under Different Over Quota Tariffs on France for Example 3.5.2

Additionally, I report the demand prices under different over quota tariffs in Table 3.6. Observe that the demand price increases for the cases in which the over quota tariffs are larger, meaning that the consumers are getting affected negatively from the increase in tariffs.

3.5.3 Increase in the Number of Paths Example

In this example I explore the impact of additional paths. In particular, I use the same data as in the preceding example but now I introduce two new paths, paths p_{17} and p_{18}, with path p_{17} connecting the Midwest supply node with the Midwest demand node and with path p_{18} connecting the producing South region of France with the demand region of the Southeast of the US. The supply chain network topology is now as given in Figure 3.4. Path p_{17} consists of the new link 9 and path p_{18} is comprised of the new link 10. These paths represent direct routes without transshipment

from production sites to the consumption sites. The link cost functions on the new links are: $c_9(f) = .001f_9^2 + .001f_9$ and $c_{10}(f) = .001f_{10}^2 + .01f_{10}$.

United States   France

Southwest   Midwest   South   North
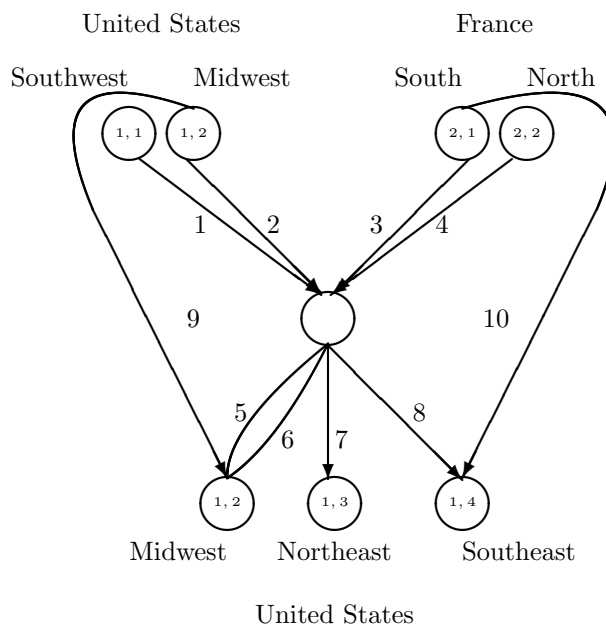
Figure 3.4. Supply Chain Network Structure of the Increase in the Number of Paths Example

The computed equilibrium path flows and path costs are reported in Table 3.7 and the computed equilibrium link flows and link costs in Table 3.8.

The computed equilibrium quota rent equivalents are now: $\lambda_{G_1}^* = 0$ and $\lambda_{G_2}^* = 1$, whereas the computed equilibrium supplies and demands, in kilograms, are:

$$s_{11}^* = 31.77, \ \ s_{12}^* = 43.12, \ \ s_{21}^* = 84.05, \ \ s_{22}^* = 10.62,$$

$$d_{12}^* = 48.33, \quad d_{13}^* = 22.25, \quad d_{14}^* = 98.97.$$

Observe that, the equilibrium supply from the Midwest region of the United States and the equilibrium supply from the South region of France increased substantially from their values in Example 3.5.2. The reason is that the cost of the additional paths p_{17} and p_{18} is smaller than the other paths in the network. Hence, even though the cheese supplied from the South of France imposed to an over-quota tariff, it can still export its cheese to the demand market region of Southwest in the United States. Furthermore, notice that, the total equilibrium supply from France, which is 89.19, is strictly over the quota, that is 35; therefore the equilibrium quota rent for Group 2, $\lambda_{G_2}^*$, is positive.

60

Path	Path Links	x_p^*	$\hat{C}_p(x^*)$
p_1	(1,5)	4.87	2.09
p_2	(1,7)	8.90	3.15
p_3	(1,8)	11.95	4.58
p_4	(2,5)	0.01	0.97
p_5	(2,7)	4.04	2.04
p_6	(2,8)	7.09	3.46
p_7	(3,5)	1.81	4.31
p_8	(3,7)	0.01	5.38
p_9	(3,8)	8.89	6.80
p_{10}	(4,5)	0.04	4.05
p_{11}	(4,7)	3.46	5.11
p_{12}	(4,8)	6.51	6.54
p_{13}	(1,6)	6.04	2.09
p_{14}	(2,6)	1.18	0.97
p_{15}	(3,6)	2.98	4.31
p_{16}	(4,6)	0.60	4.05
p_{17}	(9)	30.78	0.97
p_{18}	(10)	64.51	6.80

Table 3.7. Equilibrium Path Flows and Path Costs for the Increase in the Number of Paths Example

Link a	f_a^*	$c_a(f^*)$
1	31.77	1.32
2	12.33	0.21
3	19.53	1.55
4	10.62	1.28
5	6.74	0.76
6	10.80	0.76
7	22.25	1.83
8	34.45	3.25
9	30.78	0.97
10	64.51	4.80

Table 3.8. Equilibrium Link Flows and Link Costs for the Increase in the Number of Paths Example

The incurred supply prices per kilogram of cheese in dollars at the equilibrium are now:

$$\pi^{(1,1)} = 5.76, \ \pi^{(1,2)} = 6.87, \ \pi^{(2,1)} = 3.53, \ \pi^{(2,2)} = 3.80,$$

and the incurred equilibrium demand prices, per kilogram of cheese, in dollars, are:

$$\rho_{(1,2)} = 7.85, \quad \rho_{(1,3)} = 8.92, \quad \rho_{(1,4)} = 10.34.$$

The addition of new paths in this example results in an increase in the equilibrium supply prices for all of the regions and countries in the supply chain network as compared to their values in Example 3.5.2. Conversely, the equilibrium demand prices for all the demand regions in the United States decrease from their values under Example 3.5.2. Hence, both producers and consumers gain when there are alternative competitive transportation routes.

I now present a similar sensitivity analysis for this example, as was done for Example 3.5.2, in which I compute the solutions to variants of Example 3.5.3 for different over quota tariffs: $\tau_{G_2}^o = 3$ dollars, $\tau_{G_2}^o = 5$ dollars, and $\tau_{G_2}^o = 8$ dollars for Group G_2. In Table 3.8, I report the results of the sensitivity analysis.

Over Quota Tariffs	s_{11}^*	s_{12}^*	s_{21}^*	s_{22}^*
$\tau_{G_2}^o = 2$	31.77	43.12	84.05	10.62
$\tau_{G_2}^o = 3$	34.14	48.15	71.95	5.54
$\tau_{G_2}^o = 5$	38.21	56.27	46.01	0.00
$\tau_{G_2}^o = 8$	40.08	62.02	11.34	0.00

Table 3.9. Equilibrium Supplies under Different Over Quota Tariffs on France for Example 3.5.3

Observe from Table 3.9 that the equilibrium supply from the countries and their supply regions change drastically under different over quota tariffs. It is interesting to observe, for example, that when the imposed over quota tariff, $\tau_{G_2}^o = 5$ dollars on France, the equilibrium supply, denoted by s_{22}^*, from the North region of France drops to 0. The cheese supplied from the South of France is still positive under different over quota tariff values; however, it shows a steady decline with the increase in over quota tariffs. The reason for this is that the cost of the path, p_{18} is lower. Notice that, for the case $\tau_{G_2}^o = 8$ dollars, the total equilibrium supply for France is 38.50, which is closer to their quota, meaning that, even with the lower path cost, the over quota tariff becomes too expensive for the South of France to export its cheese to the United States. Similar to the sensitivity analysis in Example 3.5.2, the equilibrium domestic supply (domestic production) in the United States, through regions Southwest and Midwest, represented by s_{11}^* and s_{12}^*, respectively, demonstrates an increase under the increase in over quota tariffs on France. This is also an interesting result, since countries usually impose tariff-rate quotas on certain export products to increase their domestic production. However, notice also that, when the over quota tariff changes from $\tau_{G_2}^o = 5$ to $\tau_{G_2}^o = 8$, the change in the domestic production is very small. That means that the countries imposing tariff-rate quotas on products, should be careful as to how high the over quota tariff. Having a computationally tractable model such as the one presented here allows for the evaluation of the impacts of changes to quotas,

under and over quota tariffs, changes in the supply chain network topology, as well as the underlying functions.

I report the demand prices in Table 3.10, under different over quota tariffs. Notice that the demand price increases, as expected, when the over quota tariffs increase.

Over Quota Tariffs	$\rho_{(1,2)}$	$\rho_{(1,3)}$	$\rho_{(1,4)}$
$\tau_{G_2}^o = 2$	7.85	8.92	10.34
$\tau_{G_2}^o = 3$	7.91	8.97	10.37
$\tau_{G_2}^o = 5$	8.02	9.09	10.44
$\tau_{G_2}^o = 8$	8.13	9.20	10.48

Table 3.10. Demand Prices under Different Over Quota Tariffs on France for Example 3.5.3

3.6.   Summary and Conclusions

With numerous commodities from fresh produce to metals such as aluminum and steel, utilized in many product supply chains, criss-crossing the globe from points of production to locations of demand, world trade is essential for producers and consumers alike. Policy makers, as well as governments, in turn, are increasingly utilizing policy trade instruments in an attempt to protect domestic producers from competition. In fact, in the past year or so, the discussions of various trade policies, including tariffs, as well as their potential impacts, have garnered wide global news attention and coverage.

In this chapter, I provided a unified variational inequality framework for a general spatial network equilibrium problem with multiple countries and regions in each country and tariff-rate quotas. A tariff-rate quota policy consists of an under quota tariff (applied when the imposed quota is not exceeded) and an over quota tariff (applied when the quota is exceeded). I presented the governing equilibrium conditions, derived the variational inequality formulation, and also provided variational inequality formulations of the analogues of the spatial price network equilibrium models with ad valorem tariffs and with strict quotas, respectively. Qualitative properties were also obtained and an effective computational algorithm, along with conditions for convergence, outlined.

Illustrative examples were given as well as a case study consisting of larger numerical examples for which the equilibrium path flows, and link flows, as well as the equilibrium quota rents, and the supply and demand markets prices, and path costs, reported. A case study on the dairy industry is provided, in which a tariff rate quota is imposed on cheese from France by the United States. Sensitivity analysis demonstrated the impacts on production outputs (both US domestic and imports

from France) of a tightening of the imposed quota as well as increases in the over tariff rate. The results showed that while domestic producers are protected by tariff rate quotas, the consumers suffer.

CHAPTER 4

STRICT QUOTAS OR TARIFFS? IMPLICATIONS FOR PRODUCT QUALITY AND CONSUMER WELFARE IN DIFFERENTIATED PRODUCT SUPPLY CHAINS

In this chapter, I present a differentiated product supply chain network equilibrium model with quality and trade instruments, which is based on the paper by Nagurney, Besik, and Li (2019). Chapter 4 is organized as follows. In Section 4.1, I first present the differentiated product supply chain network equilibrium model with quality, but without trade policy instruments in the form of a tariff or quota. The model generalizes the model introduced in Nagurney and Li (2014) in that the demand price functions are differentiated by producing firm. I state the governing Nash equilibrium conditions and provide the variational inequality formulation.

I also provide constructs for quantifying consumer welfare in the presence or absence of tariffs or quotas in differentiated product supply chain networks with quality. Through simple illustrative examples in Section 4.1, I show that the imposition of a tariff or quota may adversely affect both the quality of products as well as the consumer welfare.

In Section 4.2, I propose the algorithm, which is then applied in Section 4.3 to a series of numerical examples, accompanied by sensitivity analysis, comprising a case study focused on the agricultural product of soybeans. The case study explores the impacts of the imposition of a strict quota or a tariff by China on soybeans that are produced in the United States on equilibrium soybean flows, product quality, prices, firm profits, and consumer welfare. Multiple scenarios are generated, including a disruption at a production side, as well as the addition of a demand market. The case study yields interesting results for decision-makers and policy makers.

In Sections 4.4 and 4.5, I summarize the results, and present conclusions.

4.1. The Differentiated Product Supply Chain Network Equilibrium Models with Quality

In this section, I construct the differentiated product supply chain network equilibrium models in which the firms compete in product quantities and quality levels. In Section 4.1.1, I consider the case without trade interventions in the form of strict quotas or tariffs. Then, in Section 4.1.2, the model is extended to include trade instruments and establish their equivalence in Section 4.1.3. The consumer welfare formula is presented in Section 4.1.4, with illustrative examples given in Section 4.1.5.

4.1.1 The Differentiated Product Supply Chain Network Equilibrium Model without Trade Interventions

The firms produce a product, which is substitutable, but differentiated by firm. In the supply chain network economy (cf. Figure 4.1), there are I firms, corresponding to the top-tier nodes, with a typical firm denoted by i, which compete with one another in a noncooperative manner in the production and distribution of the products, and on quality.
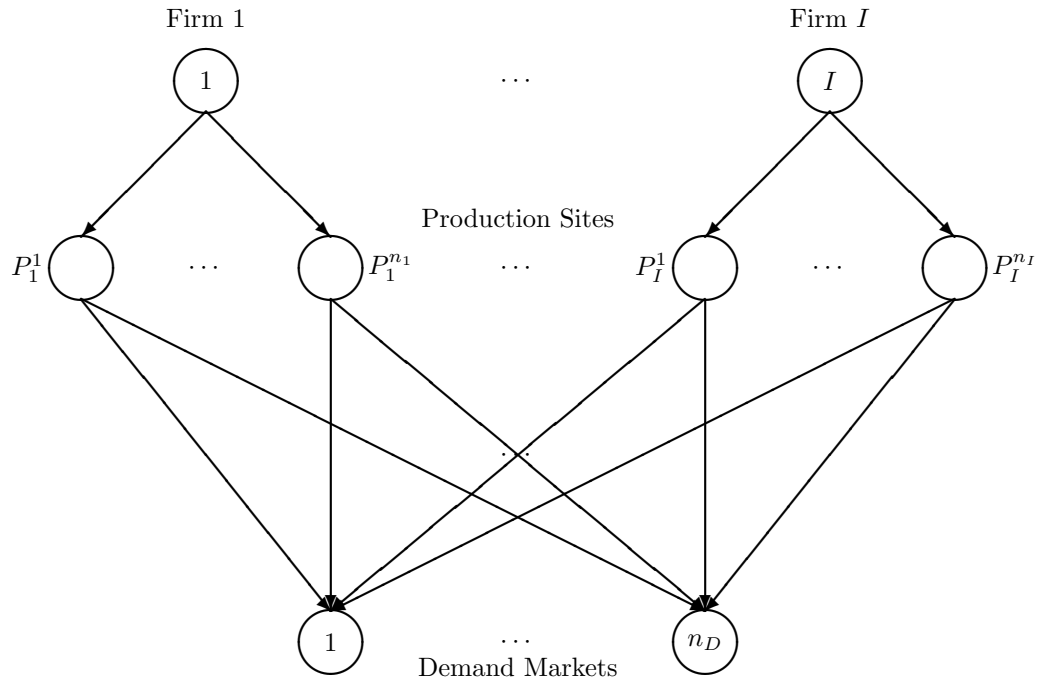
Figure 4.1. The Differentiated Product Supply Chain Network Topology

Each firm i has, at its disposal, n_i production sites, corresponding to the middle tier nodes in Figure 4.1, which can be located in the same country as the firm or be in different countries. The

production sites of firm i at the middle tier are denoted by $P_i^1, \ldots, P_i^{n_i}$, respectively, with a typical such site denoted by P_i^j. The firms determine the quantities to produce at each of their sites which are then transported to the n_D demand markets, corresponding to the bottom nodes in Figure 4.1. A typical demand market is denoted, without loss of generality, by k; $k = 1, \ldots, n_D$. In addition, the firms must determine the quality level of the product at each of their production sites, which can differ from site to site. Consumers, located at the demand markets, signal their preferences through the prices that they are willing to pay for the products, which are differentiated by firm (although they are substitutes), and, since a firm's product may be produced at one or more sites, a firm's product at a demand market is characterized by its average quality. It is assumed that the quality is preserved (with an associated cost) in the distribution process.

A link in Figure 4.1 joining a firm node with one of its production site nodes corresponds to the production/manufacturing activity, whereas a link joining a production site node with a demand market node corresponds to the activity of distribution. Note that, in the case of agricultural production, production sites would correspond to farms.

The notation for the model is given in Table 4.1.

The production output at firm i's production site P_i^j and the demand for the product at each demand market k must satisfy, respectively, the conservation of flow equations (4.1) and (4.2):

$$s_{ij} = \sum_{k=1}^{n_D} Q_{ijk}, \quad i = 1, \ldots, I; j = 1, \ldots, n_i, \tag{4.1}$$

$$d_{ik} = \sum_{j=1}^{n_i} Q_{ijk}, \quad k = 1, \ldots, n_D. \tag{4.2}$$

According to (4.1), the output produced at a firm's production site is equal to the sum of the product amounts distributed to the demand markets from that site, and, according to (4.2), the quantity of a product produced by a firm and consumed at a demand market is equal to the sum of the amounts shipped by the firm from its production sites to that demand market.

In addition, the product shipments must be nonnegative, that is:

$$Q_{ijk} \geq 0, \quad i = 1, \ldots, I; j = 1, \ldots, n_i; k = 1, \ldots, n_D. \tag{4.3}$$

The quality levels, in turn, must meet or exceed the nonnegative minimum quality standards (MQSs) at the production sites, but they cannot exceed their respective upper bounds of quality:

Notation	Definition
Q_{ijk}	the nonnegative amount of firm i's product produced at production site P_i^j and shipped to demand market k. The $\{Q_{ijk}\}$ elements for all j and k are grouped into the vector $Q_i \in R_+^{n_i n_D}$. The Q_i; $i = 1, \ldots, I$ are further grouped into the vector $Q \in R_+^{\sum_{i=1}^I n_i n_D}$.
s_{ij}	the nonnegative product output produced by firm i at its site P_i^j. The production outputs are grouped for each i; $i = 1, \ldots, I$, into the vector $s_i \in R_+^{n_i}$. Then further all such vectors are grouped into the vector $s \in R_+^{\sum_{i=1}^I n_i}$.
q_{ij}	the quality level, or, simply, the quality, of product i, which is produced by firm i at its site P_i^j. The quality levels of each firm i; $i = 1, \ldots, I$, the $\{q_{ij}\}$, are grouped into the vector $q_i \in R_+^{n_i}$. Then the quality levels of all firms are grouped into the vector $q \in R_+^{\sum_{i=1}^I n_i}$.
$\bar{q}_{ij}$	the upper bound on the quality of firm i's product produced at site P_i^j, with $i = 1, \ldots, I$; $j = 1, \ldots, n_i, \forall i$.
$\underline{q}_{ij}$	the minimum quality standard at production site P_i^j; $i = 1, \ldots, I$; $j = 1, \ldots, n_i, \forall i$, assumed to be nonnegative.
d_{ik}	the demand for firm i's product at demand market k, with d_{ik} assumed to be greater than zero. I group the demands for firm i's product for each $i = 1, \ldots, I$, into the vector $d_i \in R_+^{n_D}$ and then group the demands for all i into the vector $d \in R_+^{I n_D}$.
$\hat{q}_{ik}$	the average quality of firm i's product at demand market k; $i = 1, \ldots, I$; $k = 1, \ldots, n_D$, where $\hat{q}_{ik} = \frac{\sum_{j=1}^{n_i} q_{ij} Q_{ijk}}{d_{ik}}$. The average quality levels of all firms at all the demand markets are grouped into the vector $\hat{q} \in R_+^{I n_D}$.
Q	the strict quota defined for production sites in a particular country over which the quota is imposed for the product by another country to its demand markets.
λ	the Lagrange multiplier associated with the quota constraint.
$f_{ij}(s, q)$	the production cost at firm i's site P_i^j.
$\hat{c}_{ijk}(Q, q)$	the total transportation cost associated with distributing firm i's product, produced at site P_i^j, to demand market k.
$\rho_{ik}(d, \hat{q})$	the demand price function for firm i's product at demand market k.

Table 4.1. Notation for the Supply Chain Network Models with Product Differentiation (with and without Tariffs or Quotas)

$$\bar{q}_{ij} \geq q_{ij} \geq \underline{q}_{ij}, \quad i = 1, \ldots, I; j = 1, \ldots, n_i. \tag{4.4}$$

It is reasonable to assume upper bounds on the quality levels due to physical/technological limitations. The MQSs are imposed by regulators (or can even be self-selected by producers) and, if there are no such MQSs that the corresponding quality lower bound is set equal to zero.

In view of (4.1), the production cost functions (cf. Table 4.1) can be redefined in terms of product shipments and quality, that is,

$$\hat{f}_{ij} = \hat{f}_{ij}(Q, q) \equiv f_{ij}(s, q), \quad i = 1, \ldots, I; j = 1, \ldots, n_i, \tag{4.5}$$

and, in view of (4.2), and the definition of the average quality in Table 4.1, I redefine the demand price functions in terms of quantities and average qualities, as follows:

$$\hat{\rho}_{ik} = \hat{\rho}_{ik}(Q, q) \equiv \rho_{ik}(d, \hat{q}), \quad k = 1, \ldots, n_D. \tag{4.6}$$

It is assumed that the production cost and the transportation cost functions are convex and continuously differentiable and that the demand price functions are monotonically decreasing in demands, monotonically increasing in average quality, and continuously differentiable.

The strategic variables of firm i are its product shipments Q_i and its quality levels q_i, with the profit/utility U_i of firm i; $i = 1, \ldots, I$, given by the difference between its total revenue and its total costs:

$$U_i = \sum_{k=1}^{n_D} \hat{\rho}_{ik}(Q, q) \sum_{j=1}^{n_i} Q_{ijk} - \sum_{j=1}^{n_i} \hat{f}_{ij}(Q, q) - \sum_{k=1}^{n_D} \sum_{j=1}^{n_i} \hat{c}_{ijk}(Q, q). \tag{4.7}$$

The first term in (4.7) represents the revenue of firm i; the second term represents its total production cost and the last term in (4.7) represents the firm's total transportation costs. The transportation cost functions depend on both quantities and quality levels and were also utilized previously by Nagurney and Wolf (2014), but for Internet applications and not supply chains. Nagurney and Li (2014) did utilize such transportation cost functions in supply chains but not in a differentiated model as it is done here. Such transportation cost functions imply that the quality is preserved during the transportation process in contrast to supply chain network models in which there can be quality deterioration associated with movement down paths of a supply chain as in, for example,

Nagurney et al. (2013), Yu and Nagurney (2013), and in Nagurney, Besik, and Yu (2018). Hence, all functions in the objective function (4.7) depend on both product quantities as well as product quality levels. Moreover, the functions corresponding to a particular firm can also, hence, in general, depend on the product shipments and quality levels of the other firms. This feature enhances the modeling of competition in that firms may also compete for resources in production and distribution and, of course, compete for consumers on the demand side.

In view of (4.7), the profit can be written as a function of the product shipment pattern and quality levels, that is,

$$U = U(Q, q), \tag{4.8}$$

where U is the I-dimensional vector with components: $\{U_1, \ldots, U_I\}$.

Let K^i denote the feasible set corresponding to firm i, where $K^i \equiv \{(Q_i, q_i) | (4.3) \text{ and } (4.4) \text{ hold}\}$ and define $K \equiv \prod_{i=1}^{I} K^i$.

I consider Cournot (1838) - Nash (1950, 1951) competition, in which the I firms produce and distribute their product in a noncooperative manner, each one trying to maximize its own profit. A product shipment and quality level pattern $(Q^*, q^*) \in K$ are determined for which the I firms will be in a state of equilibrium as defined below.

- **Definition 4.1: A Differentiated Product Supply Chain Network Equilibrium with Quality**

  A product shipment and quality level pattern $(Q^*, q^*) \in K$ is said to constitute a differentiated product supply chain network equilibrium with quality if for each firm i; $i = 1, \ldots, I$,

$$U_i(Q_i^*, Q_{-i}^*, q_i^*, q_{-i}^*) \geq U_i(Q_i, Q_{-i}^*, q_i, q_{-i}^*), \quad \forall (Q_i, q_i) \in K^i, \tag{4.9}$$

  where

$$Q_{-i}^* \equiv (Q_1^*, \ldots, Q_{i-1}^*, Q_{i+1}^*, \ldots, Q_I^*) \quad and \quad q_{-i}^* \equiv (q_1^*, \ldots, q_{i-1}^*, q_{i+1}^*, \ldots, q_I^*).$$

According to (4.9), a differentiated product supply chain equilibrium is established if no firm can unilaterally improve upon its profits by choosing an alternative vector of product shipments and quality levels of its product.

In the following theorem, variational inequality (VI) formulation of the above differentiated product supply chain network equilibrium is presented.

- **Theorem 4.1: Variational Inequality Formulation of the Differentiated Product Supply Chain Network Equilibrium Model with Quality**

  Assume that for each firm i; $i = 1, \ldots, I$, the profit function $U_i(Q, q)$ is concave with respect to the variables in Q_i and q_i, and is continuous and continuously differentiable. Then the product shipment and quality pattern $(Q^*, q^*) \in K$ is a differentiated product supply chain network equilibrium with quality according to Definition 4.1 if and only if it satisfies the variational inequality

$$-\sum_{i=1}^{I}\sum_{j=1}^{n_i}\sum_{k=1}^{n_D} \frac{\partial U_i(Q^*, q^*)}{\partial Q_{ijk}} \times (Q_{ijk} - Q_{ijk}^*) - \sum_{i=1}^{I}\sum_{j=1}^{n_i} \frac{\partial U_i(Q^*, q^*)}{\partial q_{ij}} \times (q_{ij} - q_{ij}^*) \geq 0, \quad \forall (Q, q) \in K,$$

$$(4.10)$$

  that is,

$$\sum_{i=1}^{I}\sum_{j=1}^{n_i}\sum_{k=1}^{n_D} \left[-\hat{\rho}_{ik}(Q^*, q^*) - \sum_{l=1}^{n_D} \frac{\partial \hat{\rho}_{il}(Q^*, q^*)}{\partial Q_{ijk}} \sum_{h=1}^{n_i} Q_{ihl}^* + \sum_{h=1}^{n_i} \frac{\partial \hat{f}_{ih}(Q^*, q^*)}{\partial Q_{ijk}} + \sum_{h=1}^{n_i}\sum_{l=1}^{n_D} \frac{\partial \hat{c}_{ihl}(Q^*, q^*)}{\partial Q_{ijk}} \right]$$

$$\times (Q_{ijk} - Q_{ijk}^*)$$

$$+ \sum_{i=1}^{I}\sum_{j=1}^{n_i} \left[-\sum_{k=1}^{n_D} \frac{\partial \hat{\rho}_{ik}(Q^*, q^*)}{\partial q_{ij}} \sum_{h=1}^{n_i} Q_{ihk}^* + \sum_{h=1}^{n_i} \frac{\partial \hat{f}_{ih}(Q^*, q^*)}{\partial q_{ij}} + \sum_{h=1}^{n_i}\sum_{k=1}^{n_D} \frac{\partial \hat{c}_{ihk}(Q^*, q^*)}{\partial q_{ij}} \right]$$

$$\times (q_{ij} - q_{ij}^*) \geq 0, \quad \forall (Q, q) \in K. \qquad (4.11)$$

Proof: Follows using the same arguments as in the the proof of Theorem 4.1 in Nagurney and Li (2014) for a supply chain network model without product differentiation. □

Variational inequality (4.11) (cf. (4.10)) is now put into standard form given by (2.1a) in Chapter 2: determine $X^* \in \mathcal{K} \subset R^N$ such that:

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K},$$

where $\langle \cdot, \cdot \rangle$ denotes the inner product in N-dimensional Euclidean space. $F(X)$ is a given continuous function such that $F(X) : X \to \mathcal{K} \subset R^N$. $\mathcal{K}$ is a closed and convex set.

For the differentiated product supply chain network equilibrium model without trade interventions, I define $X \equiv (Q, q)$ and $F(X) \equiv (F^1(X), F^2(X))$ with $F^1_{ijk}(X) \equiv -\frac{\partial U_i(Q,q)}{\partial Q_{ijk}}; i = 1, \ldots, I; j = 1, \ldots, n_i; k = 1, \ldots, n_D$, and $F^2_{ij}(X) \equiv -\frac{\partial U_i(Q,q)}{\partial q_{ij}}; i = 1, \ldots, I; j = 1, \ldots, n_i$. Also, I define the feasible set $\mathcal{K} \equiv K$, and let $N = In_i n_D + In_i$. Then, variational inequality (4.11) (cf. (4.10)) can be put into the above standard form.

For further background on the variational inequality problem and supply chain network problems, I refer the reader to the books by Nagurney (2006) and Nagurney and Li (2016). I emphasize the above model (even without the inclusion of any trade policies) generalizes the model of Nagurney and Li (2014) in that the demand price functions are differentiated by producing firm; that is, the consumers at a demand market display their preferences accordingly.

Using the supply chain network model in Section 4.1.1 as a basis, I now construct extensions that incorporate trade instruments such as a strict quota or tariff.

4.1.2 The Differentiated Product Supply Chain Network Equilibrium Model with a Strict Quota

I first consider the inclusion of a strict quota. Specifically, I consider a country imposing a strict quota on the product flows from another country. The quota is denoted by $\bar{Q}$ and the production sites $\{j\}$ are grouped in the country on which the quota is imposed on into the set O and the demand markets $\{k\}$ in the country imposing the quota into the set D. I then define the group $\mathcal{G}$ consisting of all these production site and demand market pairs (j, k).

Under a strict quota regime, the following additional constraint must be satisfied:

$$\sum_{i=1}^{I} \sum_{(j,k) \in \mathcal{G}} Q_{ijk} \leq \bar{Q}. \tag{4.12}$$

Observe that constraint (4.12) is a common or *shared* constraint associated with firms having production sites in the country on which the quota is imposed. Hence, the original feasible set K applied in the model in Section 4.1.1 should be expanded. Hence, the set $\mathcal{S}$ is defined as:

$$\mathcal{S} \equiv \{Q | (4.12) \text{ holds}\}. \tag{4.13}$$

Note that the utility function (4.7) of a firm i depends on not only its own strategies, but also on those of the other firms, and now their feasible sets do as well since the new feasible set will correspond to $K \cap S$. Therefore, the governing equilibrium concept is no longer that of Nash Equilibrium, as was the case for the model in Section 4.1.1, but, rather, is that of a Generalized Nash Equilibrium (GNE).

- **Definition 4.2: A Differentiated Product Supply Chain Generalized Nash Network Equilibrium with Quality**

  A product shipment and quality level pattern $(Q^*, q^*) \in K \cap S$ is a differentiated product supply chain Generalized Nash Network Equilibrium with quality if for each firm i; $i = 1, \ldots, I$,

  $$U_i(Q_i^*, Q_{-i}^*, q_i^*, q_{-i}^*) \geq U_i(Q_i, Q_{-i}^*, q_i, q_{-i}^*), \quad \forall (Q_i, q_i) \in K^i \cap S. \tag{4.14}$$

As noted in Nagurney et al. (2017), for a supply chain network problem but without quality and in the absence of any trade instruments, a refinement of the Generalized Nash Equilibrium is a *variational equilibrium* and it is a specific type of GNE (see Facchinei and Kanzow (2010) and Kulkarni and Shanbhag (2012)). In particular, in a GNE defined by a variational equilibrium, the Lagrange multipliers associated with the shared/coupling constraints are all the same. This implies that the firms whose production sites are affected by the quota share a common perception of the strict quota in order to respect it.

Specifically, the following definition is necessary:

- **Definition 4.3: Variational Equilibrium**

  A vector $(Q^*, q^*) \in K \cap S$ is said to be a variational equilibrium of the above Generalized Nash Network Equilibrium if it is a solution of the variational inequality

  $$-\sum_{i=1}^{I} \sum_{j=1}^{n_i} \sum_{k=1}^{n_D} \frac{\partial U_i(Q^*, q^*)}{\partial Q_{ijk}} \times (Q_{ijk} - Q_{ijk}^*) - \sum_{i=1}^{I} \sum_{j=1}^{n_i} \frac{\partial U_i(Q^*, q^*)}{\partial q_{ij}} \times (q_{ij} - q_{ij}^*) \geq 0,$$

  $$\forall (Q, q) \in K \cap S. \tag{4.15}$$

A solution to variational inequality (4.15) is guaranteed to exist, under the assumption that the demand for products would be finite, since then the feasible set $K \cap S$ is compact, and the function that enters the variational inequality (4.15) is continuous (cf. Chapter 2).

Moreover, with a variational inequality formulation of the supply chain network model with strict quotas, one can avail themselves of algorithmic schemes which are more highly developed than those for quasivariational inequalities (cf. Bensoussan and Lions (1978) and Baiocchi and Capelo (1984)), which have been used as a formalism for GNEs (Facchinei and Kanzow (2010)).

I define a new feasible set $\mathcal{K}$ which consists of $(Q, q) \in K$ and $\lambda \in R^1_+$. Now, a variational inequality which utilizes a Lagrange multiplier λ associated with the strict quota constraint (4.12) are provided. Utilizing the results in Nagurney (2018) and Gossler et al. (2019), the following corollary is immediate.

- **Corollary 4.1: Alternative Variational Inequality Formulation of the Differentiated Product Supply Chain Network Equilibrium Model with Quality and a Strict Quota**

  An equivalent variational inequality to (4.15) is: determine $(Q^*, q^*, \lambda^*) \in \mathcal{K}$ such that

$$-\sum_{i=1}^{I} \sum_{(j,k) \notin \mathcal{G}} \frac{\partial U_i(Q^*, q^*)}{\partial Q_{ijk}} \times (Q_{ijk} - Q^*_{ijk}) + \sum_{i=1}^{I} \sum_{(j,k) \in \mathcal{G}} (-\frac{\partial U_i(Q^*, q^*)}{\partial Q_{ijk}} + \lambda^*) \times (Q_{ijk} - Q^*_{ijk})$$

$$-\sum_{i=1}^{I} \sum_{j=1}^{n_i} \frac{\partial U_i(Q^*, q^*)}{\partial q_{ij}} \times (q_{ij} - q^*_{ij})$$

$$+(\bar{Q} - \sum_{i=1}^{I} \sum_{(j,k) \in \mathcal{G}} Q^*_{ijk}) \times (\lambda - \lambda^*) \geq 0, \quad \forall (Q, q, \lambda) \in \mathcal{K}. \tag{4.16}$$

Observe that the variational inequality (4.16) is defined on a feasible set with special structure, including box-type constraints, a feature that will yield closed form expressions for the product flows, the quality levels, and the Lagrange multiplier, at each iteration of the algorithmic scheme that I propose in the next section.

Now, variational inequality (4.16) is put into standard form (cf. Equation (2.1a)). For the differentiated product supply chain network equilibrium model with a strict quota, define $X \equiv (Q, q, \lambda)$ and $F(X) \equiv (F^3(X), F^4(X), F^5(X), F^6(X))$ with $F^3_{ijk} \equiv -\frac{\partial U_i(Q,q)}{\partial Q_{ijk}}; i = 1, \ldots, I; (j, k) \notin \mathcal{G}$, $F^4_{ijk}(X) \equiv -\frac{\partial U_i(Q,q)}{\partial Q_{ijk}} + \lambda; i = 1, \ldots, I; (j, k) \in \mathcal{G}$, $F^5_{ij}(X) \equiv -\frac{\partial U_i(Q,q)}{\partial q_{ij}}; i = 1, \ldots, I; j = 1, \ldots, n_i$, and $F^6(X) \equiv \bar{Q} - \sum_{i=1}^{I} \sum_{(j,k) \in \mathcal{G}} Q_{ijk}$. Also, I define the feasible set $\mathcal{K} \equiv \mathcal{K}$, and let $N = In_i n_D + In_i + 1$. Then, variational inequality (4.16) can be put into the standard form (cf. Equation (2.1a))).

4.1.3 The Differentiated Product Supply Chain Network Equilibrium Model with a Tariff

I now introduce the tariff model based on the model in Section 4.1.1. A group $\mathcal{G}$ is considered as in the strict quota model in Section 4.1.2, but, rather than imposing a strict quota, a tariff τ^* is imposed on the production sites in the country subject to the trade policy instrument.

The utility functions $\hat{U}_i$; $i = 1, \ldots, I$, then take the form

$$\hat{U}_i = U_i - \sum_{i=1}^{I} \sum_{(j,k) \in \mathcal{G}} \tau^* Q_{ijk}, \tag{4.17}$$

with U_i; $i = 1, \ldots, I$, as in (4.7).

The definition of an equilibrium then follows according to (4.9) but with $\hat{U}_i$ substituted for the U_i; $i = 1, \ldots, I$. The following Theorem is immediate from Theorem 4.1:

- **Theorem 4.2: Variational Inequality Formulation of the Differentiated Product Model Supply Chain Network Equilibrium Model with Quality and a Tariff**

  Under the same assumptions as in Theorem 4.1, a product shipment and quality pattern $(Q^*, q^*) \in K$ is a differentiated product supply chain network equilibrium according to Definition 4.1 with $\hat{U}_i$ replacing U_i for $i = 1, \ldots, I$, if and only if it satisfies the variational inequality:

$$-\sum_{i=1}^{I} \sum_{(j,k) \notin \mathcal{G}} \frac{\partial U_i(Q^*, q^*)}{\partial Q_{ijk}} \times (Q_{ijk} - Q_{ijk}^*) + \sum_{i=1}^{I} \sum_{(j,k) \in \mathcal{G}} \left(-\frac{\partial U_i(Q^*, q^*)}{\partial Q_{ijk}} + \tau^*\right) \times (Q_{ijk} - Q_{ijk}^*)$$

$$-\sum_{i=1}^{I} \sum_{j=1}^{n_i} \frac{\partial U_i(Q^*, q^*)}{\partial q_{ij}} \times (q_{ij} - q_{ij}^*) \geq 0, \quad \forall (Q, q) \in K. \tag{4.18}$$

I now put variational inequality (4.18) into standard form (cf. Equation (2.1a)). For the differentiated product supply chain network equilibrium model with a tariff, define $X \equiv (Q, q)$ and $F(X) \equiv (F^7(X), F^8(X), F^9(X))$ with $F_{ijk}^7(X) \equiv -\frac{\partial U_i(Q, q)}{\partial Q_{ijk}}$; $i = 1, \ldots, I$; $(j, k) \notin \mathcal{G}$, $F_{ijk}^8(X) \equiv -\frac{\partial U_i(Q, q)}{\partial Q_{ijk}} + \tau^*$; $i = 1, \ldots, I$; $(j, k) \in \mathcal{G}$, and $F_{ij}^9(X) \equiv -\frac{\partial U_i(Q, q)}{\partial q_{ij}}$; $i = 1, \ldots, I$; $j = 1, \ldots, n_i$. I also define the feasible set $\mathcal{K} \equiv K$, and let $N = In_i n_D + In_i$. Then, variational inequality (4.18) can be put into the standard form (cf. Equation (2.1a)).

4.1.4 Relationships Between the Model with a Strict Quota and the Model with a Tariff

In this section, I explore the relationships between the above two models with trade policy instruments.

Below, I first establish that a solution to the variational inequality (4.16) governing the strict quota model also satisfies the variational inequality (4.18) governing the model with a tariff where the tariff $\tau^* = \lambda^*$.

From variational inequality (4.16) its is known that for $(Q^*, q^*, \lambda^*) \in \mathcal{K}$:

$$-\sum_{i=1}^{I} \sum_{(j,k) \notin \mathcal{G}} \frac{\partial U_i(Q^*, q^*)}{\partial Q_{ijk}} \times (Q_{ijk} - Q_{ijk}^*) + \sum_{i=1}^{I} \sum_{(j,k) \in \mathcal{G}} \left(-\frac{\partial U_i(Q^*, q^*)}{\partial Q_{ijk}} + \lambda^*\right) \times (Q_{ijk} - Q_{ijk}^*)$$

$$-\sum_{i=1}^{I} \sum_{j=1}^{n_i} \frac{\partial U_i(Q^*, q^*)}{\partial q_{ij}} \times (q_{ij} - q_{ij}^*) \geq$$

$$(\bar{Q} - \sum_{i=1}^{I} \sum_{(j,k) \in \mathcal{G}} Q_{ijk}^*) \times (\lambda^* - \lambda). \tag{4.19}$$

Setting $\lambda = \lambda^*$ in (4.19) and then $\tau^* = \lambda^*$ yields:

$$-\sum_{i=1}^{I} \sum_{(j,k) \notin \mathcal{G}} \frac{\partial U_i(Q^*, q^*)}{\partial Q_{ijk}} \times (Q_{ijk} - Q_{ijk}^*) + \sum_{i=1}^{I} \sum_{(j,k) \in \mathcal{G}} \left(-\frac{\partial U_i(Q^*, q^*)}{\partial Q_{ijk}} + \tau^*\right) \times (Q_{ijk} - Q_{ijk}^*)$$

$$-\sum_{i=1}^{I} \sum_{j=1}^{n_i} \frac{\partial U_i(Q^*, q^*)}{\partial q_{ij}} \times (q_{ij} - q_{ij}^*) \geq 0, \quad \forall (Q, q) \in K, \tag{4.20}$$

which is precisely variational inequality (4.18) governing the tariff model with product differentiation and quality.

Hence, it is established that, upon the imposition of a strict quota regime, if the strict quota constraint is tight, the equilibrium product flow and equilibrium pattern that satisfies the VI with the strict quota also satisfies the variational inequality governing a tariff on the same group if the tariff is set to the equilibrium Lagrange multiplier associated with the strict quota constraint.

The above relationship also provides a nice interpretation for the Lagrange multiplier associated with the strict quota in that it is a price or, in effect, a tariff.

I now investigate whether a solution to VI (4.18) governing the model with a tariff will also solve VI (4.16) governing the strict quota model.

76

Set

$$\bar{Q} \equiv \sum_{(j,k)\in\mathcal{K}} Q_{ijk}^* \tag{4.21}$$

with the Q_{ijk}^* in (4.21) as in (4.18), and to each side of (18) then add the term:

$$(\bar{Q} - \sum_{(j,k)\in\mathcal{G}} Q_{ijk}^*) \times (\tau - \tau^*). \tag{4.22}$$

This can be done since the expression in (4.22) is equal to 0.00.

The above results in:

$$-\sum_{i=1}^{I} \sum_{(j,k)\notin\mathcal{G}} \frac{\partial U_i(Q^*,q^*)}{\partial Q_{ijk}} \times (Q_{ijk} - Q_{ijk}^*) + \sum_{i=1}^{I} \sum_{(j,k)\in\mathcal{G}} (-\frac{\partial U_i(Q^*,q^*)}{\partial Q_{ijk}} + \tau^*) \times (Q_{ijk} - Q_{ijk}^*)$$

$$-\sum_{i=1}^{I} \sum_{j=1}^{n_i} \frac{\partial U_i(Q^*,q^*)}{\partial q_{ij}} \times (q_{ij} - q_{ij}^*) + (\bar{Q} - \sum_{(j,k)\in\mathcal{G}} Q_{ijk}^*) \times (\tau - \tau^*)$$

$$\geq (\bar{Q} - \sum_{(j,k)\in\mathcal{G}} Q_{ijk}^*) \times (\tau - \tau^*) = 0, \quad \forall(Q,q) \in K \text{ and } \tau \geq 0. \tag{4.23}$$

But (4.23) is precisely VI (4.16) if the notation $\lambda = \tau$ is used.

4.1.5 Consumer Welfare with or without Tariffs or Quotas

A measure of the consumer welfare with or without tariffs or quotas is now provided. The consumer welfare associated with product i at demand market k at equilibrium with or without a strict quota, CW_{ik}, is

$$CW_{ik} = \int_0^{d_{ik}^*} \rho_{ik}(d_{-ik}^*, d_{ik}, \hat{q}^*) \, \mathrm{d}(d_{ik}) - \rho_{ik}(d^*, \hat{q}^*) d_{ik}^*, \quad i = 1, \dots, I; k = 1, \dots, n_D, \tag{4.24}$$

where $d_{-ij}^* \equiv (d_{11}^*, \dots, d_{i,j-1}^*, d_{i,j+1}^*, \dots, d_{mn}^*)$ (cf. Spence (1975) and Wildman (1984)).

4.1.6 Illustrative Examples

In this section, illustrative examples are presented to demonstrate the relevance of the models. In Section 4.2, I provide a more detailed numerical analysis through a case study on an agricultural product - that of soybeans.

The supply chain network topology of the illustrative examples is depicted in Figure 4.2. There are two firms, Firm 1 and Firm 2, competing to sell their products at a single demand market,

Demand Market 1. Firm 1 has available the production site P_1^1, whereas Firm 2 has the production site P_2^1. The production sites, P_1^1 and P_2^1, are located in different countries, with Demand Market 1 located in the same country as P_1^1. I refer to P_1^1 and the demand market as being *domestic*.
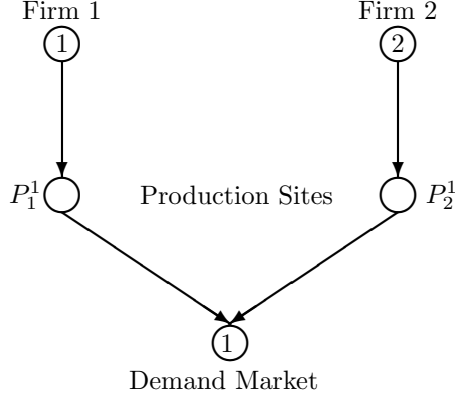
Figure 4.2. The Supply Chain Network Topology for the Illustrative Examples

The simplicity of the supply chain network topology allows us to immediately write down the conservation of flow equations (4.1) and (4.2) as:

$$s_{11} = d_{11} = Q_{111}, \quad s_{21} = d_{21} = Q_{211}.$$

I assume that the cost of production at a production site depends on the product flow and on the quality level. The production costs of Firm 1 and Firm 2 are:

$$\hat{f}_{11}(Q, q) = Q_{111}^2 + 3Q_{111} + q_{11},$$
$$\hat{f}_{21}(Q, q) = Q_{211}^2 + Q_{211} + 0.5q_{21}^2.$$

The total transportation cost associated with distributing a firm's product also depends on the product flow and the quality level of the products. As mentioned previously, the total transportation cost of Firm 2 is higher, since the distance between the demand market and the site P_2^1 is greater than that for the site P_1^1 and the demand market. In particular, the transportation cost functions are:

$$\hat{c}_{111}(Q, q) = Q_{111}^2 + 0.5Q_{111} + q_{11}, \quad \hat{c}_{211}(Q, q) = Q_{211}^2 + Q_{211} + 2q_{21}.$$

The average quality level expressions are:

$$\hat{q}_{11} = \frac{q_{11}Q_{111}}{Q_{111}} = q_{11}, \quad \hat{q}_{21} = \frac{q_{21}Q_{211}}{Q_{211}} = q_{21}.$$

The quality upper bounds are: $\bar{q}_{11} = \bar{q}_{21} = 100$. The minimum quality standards are: $\underline{q}_{11} = \underline{q}_{21} = 0.8$.

The demand price functions for the products of Firm 1 and Firm 2 at the demand market, are, in turn, functions of the average quality levels, $\hat{q}_{11}$, $\hat{q}_{22}$, and the product flows, as follows:

$$\hat{\rho}_{11}(Q,q) = -(Q_{111} + Q_{211}) + 0.5q_{11} + 20,$$

$$\hat{\rho}_{21}(Q,q) = -(Q_{211} + Q_{111}) + q_{21} + 25.$$

In the following subsections, I first solve variational inequality (4.11) governing the differentiated supply chain network equilibrium model with quality but without any trade policy instruments. Then, I incorporate a strict quota, and further demonstrate the equivalence to the model with a tariff, with the tariff corresponding to the equilibrium Lagrange multiplier associated with the solution of the former model.

The parameters in the cost and demand functions are reasonable for the size and the location of the hypothetical firms.

4.1.6.1   Illustrative Example without Trade Interventions

In this example, a strict quota or tariff is not considered, and I solve variational inequality (4.11). According to (4.11), it is reasonable to make the assumption that $Q_{111}^* > 0$, $Q_{211}^* > 0$, $\bar{q}_{11} > q_{11}^* > \underline{q}_{11}$, and $\bar{q}_{21} > q_{21}^* > \underline{q}_{21}$. Therefore, the following expressions are written, all of which are equal to 0:

$$\frac{\partial \hat{f}_{11}(Q^*,q^*)}{\partial Q_{111}} + \frac{\partial \hat{c}_{111}(Q^*,q^*)}{\partial Q_{111}} - \hat{\rho}_{11}(Q^*,q^*) - \frac{\partial \hat{\rho}_{11}(Q^*,q^*)}{\partial Q_{111}} Q_{111}^* = 0, \tag{4.25}$$

$$\frac{\partial \hat{f}_{11}(Q^*,q^*)}{\partial q_{11}} + \frac{\partial \hat{c}_{111}(Q^*,q^*)}{\partial q_{11}} - \frac{\partial \hat{\rho}_{11}(Q^*,q)}{\partial q_{11}} Q_{111}^* = 0, \tag{4.26}$$

$$\frac{\partial \hat{f}_{21}(Q^*,q^*)}{\partial Q_{211}} + \frac{\partial \hat{c}_{211}(Q^*,q^*)}{\partial Q_{211}} - \hat{\rho}_{21}(Q^*,q^*) - \frac{\partial \hat{\rho}_{21}(Q^*,q^*)}{\partial Q_{211}} Q_{211}^* = 0, \tag{4.27}$$

$$\frac{\partial \hat{f}_{21}(Q^*,q^*)}{\partial q_{21}} + \frac{\partial \hat{c}_{211}(Q^*,q^*)}{\partial q_{21}} - \frac{\partial \hat{\rho}_{21}(Q^*,q^*)}{\partial q_{21}} Q_{211}^* = 0. \tag{4.28}$$

Inserting the corresponding functions into the above equations (4.25) – (4.28), the following system of equations are obtained:

$$6Q_{111}^* + Q_{211}^* - 0.5q_{11}^* = 16.5,$$

$$0.5Q_{111}^* = 2,$$

$$6Q_{211}^* + Q_{111}^* - q_{21}^* = 23,$$

$$Q_{211}^* - q_{21}^* = 2,$$

with solution:

$$Q_{111}^* = 4.00, \quad Q_{211}^* = 3.40, \quad q_{11}^* = \hat{q}_{11} = 21.80, \qquad q_{21}^* = \hat{q}_{21} = 1.40.$$

Furthermore, the demand prices at equilibrium, in dollars, are: $\rho_{11} = 23.50$ and $\rho_{21} = 19.00$. The profits of the firms, in dollars, are: $U_1 = 4.40$ and $U_2 = 30.90$. It is evident that, even though the quality of Firm 2 is lower and it sells less at the demand market, it enjoys a higher profit than Firm 1. This is caused by the fact that the price of Firm 2's product is lower. The consumer welfare associated with the two firms' products is, respectively, $CW_{11} = 8.00$ and $CW_{21} = 5.78$.

4.1.6.2 An Illustrative Example with a Strict Quota and Tariff Equivalence

I now use, as a baseline, the illustrative example in Subsection 4.1.6.1, and impose a strict quota on the product flow from the nondomestic production site. Subsequently, I demonstrate the theoretical result of Section 4.1.4 of the equivalence of the equilibrium solutions to the model with a strict quota and the model with a tariff, over the same group, through this example, with the tariff for the latter being set to the equilibrium Lagrange multiplier of the former. The group consists of the production site P_2^1 of Firm 2 and Demand Market 1, since the country that the demand market is located in imposes a quota on the products from the country that the production site P_2^1 of Firm 2 is located in. Hence, the strict quota $\bar{Q}$ is imposed only on the product flow in this group. The strict quota $\bar{Q} = 3$ is set and, consequently, the strict quota constraint in (4.12) becomes:

$$Q_{211} \leq 3, \tag{4.29}$$

Similar to the expressions in $(4.25) - (4.28)$, the variational inequality formulation in (4.16) yields the following expressions, which are all equal 0, since it is reasonable that the equilibrium product flows will be positive and the quality levels will not be at the boundaries:

$$\frac{\partial \hat{f}_{11}(Q^*, q^*)}{\partial Q_{111}} + \frac{\partial \hat{c}_{111}(Q^*, q^*)}{\partial Q_{111}} - \hat{\rho}_{11}(Q^*, q^*) - \frac{\partial \hat{\rho}_{11}(Q^*, q^*)}{\partial Q_{111}} Q_{111}^* = 0, \tag{4.30}$$

$$\frac{\partial \hat{f}_{11}(Q^*, q^*)}{\partial q_{11}} + \frac{\partial \hat{c}_{111}(Q^*, q^*)}{\partial q_{11}} - \frac{\partial \hat{\rho}_{11}(Q^*, q^*)}{\partial q_{11}} Q^*_{111} = 0, \tag{4.31}$$

$$\frac{\partial \hat{f}_{21}(Q^*, q^*)}{\partial Q_{211}} + \frac{\partial \hat{c}_{211}(Q^*, q^*)}{\partial Q_{211}} - \hat{\rho}_{21}(Q^*, q^*) - \frac{\partial \hat{\rho}_{21}(Q^*, q^*)}{\partial Q_{211}} Q^*_{211} + \lambda^* = 0, \tag{4.32}$$

$$\frac{\partial \hat{f}_{21}(Q^*, q^*)}{\partial q_{21}} + \frac{\partial \hat{c}_{211}(Q^*, q^*)}{\partial q_{21}} - \frac{\partial \hat{\rho}_{21}(Q^*, q^*)}{\partial q_{21}} Q^*_{211} = 0, \tag{4.33}$$

$$\bar{Q} - Q^*_{211} = 0. \tag{4.34}$$

Notice that the expressions are very similar to (4.25) - (4.28). However, now, in (4.32), the Lagrange multiplier associated with the strict quota constraint (4.29) is added and (4.34) is a new linear equation.

By inserting the appropriate functions into (4.30) − (4.33), with the strict quota $\bar{Q} = 3$, the following system of equations are obtained:

$$6Q^*_{111} + Q^*_{211} - 0.5q^*_{11} = 16.5,$$

$$0.5Q^*_{111} = 2,$$

$$6Q^*_{211} + Q^*_{111} - q^*_{21} + \lambda^* = 23,$$

$$Q^*_{211} - q^*_{21} = 2,$$

$$Q^*_{211} = 3.$$

The equilibrium product flows, quality levels, and the Lagrange multiplier are, hence:

$$Q^*_{111} = 4.00, \quad Q^*_{211} = 3.00, \quad q^*_{11} = \hat{q}_{11} = 21.00, \quad q^*_{21} = \hat{q}_{21} = 1.00,$$

$$\lambda^* = 2.00.$$

The demand prices at equilibrium of Firm 1 and Firm 2 are: $\rho_{11} = 23.50$ and $\rho_{21} = 19.00$ and are the same as in the example without the strict quota. The profits of the firms are now: $U_1 = 6.00$ and $U_2 = 24.50$. With the strict quota imposed, the quality levels of the products, as well as the average quality at the demand market, decrease from their respective values when there is no imposed quota. In addition, the consumer welfare associated with the firms' products are now: $CW_{11} = 8.00$ and $CW_{21} = 4.50$. The value of CW_{21} is lower than the corresponding one for the example without a strict quota.

Further, by using the result obtained in Section 4.1.4 on the equivalence of the equilibrium solution to the model with a strict quota and that of the model with a tariff (over the same group),

provided that the equilibrium Lagrange multiplier obtained in the solution of the model with a strict quota is the assigned tariff for the group in the tariff model, the equilibrium tariff $\tau^* = \lambda^* = 2.00$ satisfies the variational inequality formulation (4.18). Clearly, the above equilibrium product flows and equilibrium quality levels then solve VI (4.18).

Hence, the above numerical example demonstrates that a strict quota or tariff may adversely affect both the quality of the products and the consumer welfare, which is clearly not good for consumers. However, the profit of the domestic firm increases, whereas that of the firm with the nondomestic production site decreases.

4.2. The Algorithm

The explicit formulae of the modified projection method (cf. Section 2.4.2) for the solution of variational inequality variational inequality (4.23), equivalently (4.15), are provided below, where t is the iteration counter.

4.2.1 Explicit Formulae for the Differentiated Product Supply Chain Network Equilibrium Model Variables without Trade Interventions in Step 1 of the Modified Projection Method

$$\bar{Q}_{ijk}^{t+1} = \max\{0, Q_{ijk}^t + \beta(\hat{\rho}_{ik}(Q^t, q^t) + \sum_{l=1}^{n_D} \frac{\partial \hat{\rho}_{il}(Q^t, q^t)}{\partial Q_{ijk}} \sum_{h=1}^{n_i} Q_{ihl}^t - \sum_{h=1}^{n_i} \frac{\partial \hat{f}_{ih}(Q^t, q^t)}{\partial Q_{ijk}}$$

$$- \sum_{h=1}^{n_i} \sum_{l=1}^{n_D} \frac{\partial \hat{c}_{ihl}(Q^t, q^t)}{\partial Q_{ijk}})\}, \quad i = 1, \ldots, I; j = 1, \ldots, n_i; k = 1, \ldots, n_D, \qquad (4.35)$$

$$\bar{q}_{ij}^{t+1} = \max\{ \underline{q}_{ij}, \min\{\bar{q}_{ij}, q_{ij}^t + \beta(\sum_{k=1}^{n_D} \frac{\partial \hat{\rho}_{ik}(Q^t, q^t)}{\partial q_{ij}} \sum_{h=1}^{n_i} Q_{ihk}^t - \sum_{h=1}^{n_i} \frac{\partial \hat{f}_{ih}(Q^t, q^t)}{\partial q_{ij}}$$

$$- \sum_{h=1}^{n_i} \sum_{k=1}^{n_D} \frac{\partial \hat{c}_{ihk}(Q^t, q^t)}{\partial q_{ij}})\}\}, \quad i = 1, \ldots, I; j = 1, \ldots, n_i. \qquad (4.36)$$

4.2.2 Explicit Formulae for the Differentiated Product Supply Chain Network Equilibrium Model Variables with a Strict Quota in Step 1 of the Modified Projection Method

$$\bar{Q}_{ijk}^{t+1} = \max\{0, Q_{ijk}^t + \beta(\hat{\rho}_{ik}(Q^t, q^t) + \sum_{l=1}^{n_D} \frac{\partial \hat{\rho}_{il}(Q^t, q^t)}{\partial Q_{ijk}} \sum_{h=1}^{n_i} Q_{ihl}^t - \sum_{h=1}^{n_i} \frac{\partial \hat{f}_{ih}(Q^t, q^t)}{\partial Q_{ijk}}$$

$$-\sum_{h=1}^{n_i}\sum_{l=1}^{n_D}\frac{\partial\hat{c}_{ihl}(Q^t,q^t)}{\partial Q_{ijk}}-\lambda^t)\}, \quad i=1,\ldots,I; j=1,\ldots,n_i; k=1,\ldots,n_D; (j,k)\in\mathcal{G}, \qquad (4.37)$$

$$\bar{\lambda}^{t+1}=\max\{0,\lambda^t+\beta(\sum_{i=1}^{I}\sum_{(j,k)\in G}Q_{ijk}^t-\bar{Q})\}. \qquad (4.38)$$

The explicit formulae for the product shipments, $\bar{Q}_{ijk}^{t+1}; i=1,\ldots,I; j=1,\ldots,n_i; k=1,\ldots,n_D;$ $(j,k)\notin\mathcal{G}$, are the same as in (4.37), with the explicit formulae for product quality, $\bar{q}_{ij}^{t+1}; i=1,\ldots,I; j=1,\ldots,n_i$, the same as in (4.38).

4.2.3   Explicit Formulae for the Differentiated Product Supply Chain Network Equilibrium Model Variables with a Tariff in Step 1 of the Modified Projection Method

$$\bar{Q}_{ijk}^{t+1}=\max\{0, Q_{ijk}^t+\beta(\hat{\rho}_{ik}(Q^t,q^t)+\sum_{l=1}^{n_D}\frac{\partial\hat{\rho}_{il}(Q^t,q^t)}{\partial Q_{ijk}}\sum_{h=1}^{n_i}Q_{ihl}^t-\sum_{h=1}^{n_i}\frac{\partial\hat{f}_{ih}(Q^t,q^t)}{\partial Q_{ijk}}$$

$$-\sum_{h=1}^{n_i}\sum_{l=1}^{n_D}\frac{\partial\hat{c}_{ihl}(Q^t,q^t)}{\partial Q_{ijk}}-\tau^*)\}, \quad i=1,\ldots,I; j=1,\ldots,n_i; k=1,\ldots,n_D; (j,k)\in\mathcal{G}. \qquad (4.39)$$

The explicit formulae for the product shipments, $\bar{Q}_{ijk}^{t+1}; i=1,\ldots,I; j=1,\ldots,n_i; k=1,\ldots,n_D;$ $(j,k)\notin\mathcal{G}$, and for product quality, $\bar{q}_{ij}^{t+1}; i=1,\ldots,I; j=1,\ldots,n_i$, are the same as in (4.35) and (4.36), respectively.

4.3.   Numerical Examples

In this section, I focus on the supply chain network of soybeans, an agricultural product that has a great impact on today's agricultural trade. Soybeans were discovered and domesticated in China over 3000 years ago, with the United States being a leader in producing, consuming, and exporting soybeans globally (Song, Xu, and Marchant (2004)). In the United States, soybean production and export have become essential parts of the agricultural economy, with soybeans ranked second among crops in farm value in 2005 (Ash, Livesey, and Dohlman (2006)). Lundgren (2018) reports that, in 2018, soybean production in the United States reached 5.11 billion bushels with an export of 2.13 billion bushels.

China, in turn, is the largest importer of soybeans due to its rapidly increasing population size (Brown (2012)). The consumption of soybeans in China, in 2017, was reported to be 112.18 million tons, but the domestic production volume was only 13 million tons (Wood (2018)). Due to this

huge gap, China has to rely heavily on soybeans imported from foreign countries, such as the United States, Brazil, and Argentina.

In 2018, the trade war between China and the United States escalated, with the Chinese government imposing quotas and tariffs on the soybeans exported from the United States in retaliation (Wong and Koty (2019)). According to Appelbaum (2018), this created an opportunity for other large soybean exporters, such as Brazil and Argentina. In 2017, Brazil exported 53.8 million tons of soybeans to China, corresponding to 75% of its production volume (Zhou et al. (2018)). Shane (2018) claims that, the Chinese importer, Hebei Power Sea Feed Technology, bought thousands of tons of soybeans for animal feed from Brazil instead of the United States in 2018.

In this section, I present a series of numerical examples comprising a case study for the differentiated product supply chain network equilibrium models for soybeans, and examine the effects of quotas and tariffs on soybean trade and quality. The models in the case study were solved via the the modified projection method, which was implemented in Matlab on an OS X 10.14.1 system. The convergence tolerance was: 10^{-6}, so that the algorithm was deemed to have converged when the absolute value of the difference between each successive product shipment, quality level, and the Lagrange multiplier was less than or equal to 10^{-6}. I set β in the algorithm to .15 and initialized the algorithm with the product shipments equal to 100 and with the quality levels and the Lagrange multiplier equal to 0.

The baseline example, Example 4.1, has no quotas or tariffs imposed. In subsequent examples, I add strict quotas, tariffs, and then also consider the addition of a demand market.

4.3.1 Example 4.1: 2 Firms, with 3 Production Sites for the First Firm, Two Production Sites for the Second, and a Single Demand Market

The supply chain network for soybeans for Example 4.1 is depicted in Figure 4.3. There are two producing firms, Firm 1 and Firm 2, which are located in the United States. The firms are assumed to be Cargill and Archer Daniels Midland Company (ADM), two of the largest agricultural companies in the United States with production sites in the United States and overseas (Zhou et al. (2018)).

Recall that the middle nodes represent production sites. Cargill owns three production sites, P_1^1, P_2^1, and P_3^1, which are located in the United States, Brazil, and Argentina, respectively. ADM's production sites are denoted by P_1^2 and P_2^2, and they are located, respectively, in the United States

and Brazil. There is a single demand market, Demand Market 1, located in China. I consider a time horizon of a month and the currency is in US dollars.
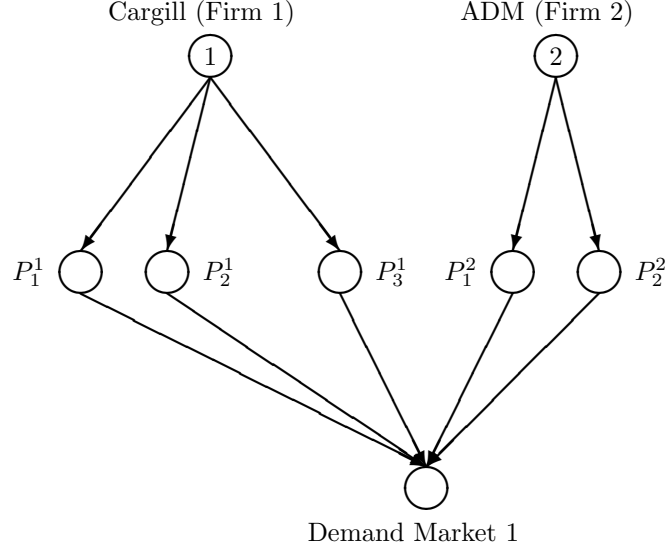
Figure 4.3. The Supply Chain Network Topology for Example 4.1

The production cost functions of Cargill at its production sites, P_1^1, P_2^1, and P_3^1 are:

$$\hat{f}_{11}(Q_{111}, q_{11}) = 0.04Q_{111}^2 + 0.35Q_{111} + 0.4Q_{111}q_{11} + 0.6q_{11}^2,$$

$$\hat{f}_{12}(Q_{121}, q_{12}) = 0.05Q_{121}^2 + 0.35Q_{121} + 0.4Q_{121}q_{12} + 0.4q_{12}^2,$$

$$\hat{f}_{13}(Q_{131}, q_{13}) = 0.05Q_{131}^2 + 0.8Q_{131} + 0.4Q_{131}q_{13} + 0.4q_{13}^2.$$

The production cost functions faced by ADM at its production sites, P_1^2 and P_2^2, are:

$$\hat{f}_{21}(Q_{211}, q_{21}) = 0.06Q_{211}^2 + 0.5Q_{211} + 1.2Q_{211}q_{21} + q_{21}^2,$$

$$\hat{f}_{22}(Q_{221}, q_{22}) = 0.07Q_{221}^2 + 0.3Q_{221} + 1.3Q_{221}q_{22} + 1.5q_{22}^2.$$

The total transportation cost functions associated with Cargill for shipping its soybeans to Demand Market 1 are:

$$\hat{c}_{111}(Q_{111}, q_{11}) = 0.02Q_{111}^2 + 0.2Q_{111} + 0.5q_{11}^2, \quad \hat{c}_{121}(Q_{121}, q_{12}) = 0.02Q_{121}^2 + 0.4Q_{121} + 0.8q_{12}^2,$$

$$\hat{c}_{131}(Q_{131}, q_{13}) = 0.02Q_{131}^2 + 0.5Q_{131} + 0.8q_{13}^2,$$

and ADM's total transportation cost functions are:

$$\hat{c}_{211}(Q_{211}, q_{21}) = 0.02Q_{211}^2 + 0.5Q_{211} + 0.6q_{21}^2, \quad \hat{c}_{221}(Q_{221}, q_{22}) = 0.02Q_{221}^2 + 0.4Q_{221} + 0.8q_{22}^2.$$

The demand price functions for the soybeans of Cargill and ADM at Demand Market 1 are:

$$\rho_{11}(d, \hat{q}) = 1500 - (0.3d_{11} + 0.2d_{21}) + 0.7\hat{q}_{11},$$

$$\rho_{21}(d, \hat{q}) = 1600 - (0.35d_{21} + 0.3d_{11}) + 2\hat{q}_{21},$$

with the average quality $\hat{q}_{11}$ and $\hat{q}_{21}$ being:

$$\hat{q}_{11} = \frac{Q_{111}q_{11} + Q_{121}q_{12} + Q_{131}q_{13}}{Q_{111} + Q_{121} + Q_{131}}, \quad \hat{q}_{21} = \frac{Q_{211}q_{21} + Q_{221}q_{22}}{Q_{211} + Q_{221}}.$$

Furthermore, the upper and lower bounds of quality levels are:

$$\bar{q}_{11} = \bar{q}_{12} = \bar{q}_{13} = \bar{q}_{21} = \bar{q}_{22} = 100,$$

$$\underline{q}_{11} = \underline{q}_{12} = \underline{q}_{13} = \underline{q}_{21} = \underline{q}_{22} = 10.$$

The modified projection method yielded the following equilibrium product flows of soybeans, in tons, as well as the equilibrium quality levels in Table 4.2. The demands for soybeans at equilibrium and the average quality are reported in Table 4.3.

Equilibrium Flows	Results	Equilibrium Quality	Results
Q^*_{111}	756.70	q^*_{11}	100.00
Q^*_{121}	591.26	q^*_{12}	73.91
Q^*_{131}	585.90	q^*_{13}	73.24
Q^*_{211}	779.32	q^*_{21}	100.00
Q^*_{221}	612.32	q^*_{22}	93.18

Table 4.2. Equilibrium Product Flows and Equilibrium Quality Levels in Example 4.1

Demand	Results	Average Quality	Results
d^*_{11}	1,933.86	$\hat{q}_{11}$	83.91
d^*_{21}	1,391.64	$\hat{q}_{21}$	97.00

Table 4.3. Equilibrium Demands and Average Quality in Example 4.1

The average quality of Cargill's soybeans is lower than that of soybeans produced by ADM. This is because, as indicated in the demand price functions, the price of ADM's soybeans is more sensitive to higher average quality at the demand market than the price of Cargill's soybeans. Therefore, ADM has a stronger incentive to provide higher quality than Cargill in Demand Market 1.

I also report the equilibrium demand prices per ton of soybeans of Cargill and ADM at Demand Market 1, in dollars, the consumer welfare associated with the two firms at the demand market, and the profits of Cargill and ADM, in dollars, in Table 4.4.

It is seen from Table 4.3 and Table 4.4 that Cargill's soybeans have an associated higher demand, a lower price, a higher associated consumer welfare, and Cargill enjoys a higher profit than does

Demand Price	Results	CW	Results	Profits	Results
ρ_{11}	700.25	CW_{11}	560,973.35	U_1	1,180,812.05
ρ_{21}	726.76	CW_{21}	338,916.81	U_2	724,196.08

Table 4.4. Equilibrium Demand Prices, Consumer Welfare, and Profits in Example 4.1

ADM, but with a lower average quality of its soybeans at Demand Market 1. For both firms, the production site in the United States produces the greatest amount of soybeans with also the highest quality as compared to the overseas sites. The reported demand prices of soybeans are close to the actual soybean price per ton, which was between 550-600 dollars in China in 2018 (Gu and Mason (2018)).

4.3.2   Example 4.2: Example 4.1 with a Strict Quota and Sensitivity Analysis

This example considers the same differentiated product supply chain network problem as in Example 4.1, but with the imposition of a strict quota of $\bar{Q} = 1200$ by the Chinese government on imports from US production sites, that is, the production site P_1^1 of Cargill and the production site P_1^2 of ADM. I report the equilibrium soybean flows, equilibrium quality levels, equilibrium demands, and average quality results in Table 4.5 and Table 4.6.

Equilibrium Flows	Results	Equilibrium Quality	Results
Q_{111}^*	528.96	q_{11}^*	72.13
Q_{121}^*	697.99	q_{12}^*	87.25
Q_{131}^*	692.63	q_{13}^*	86.58
Q_{211}^*	671.04	q_{21}^*	100.00
Q_{221}^*	708.75	q_{22}^*	100.00

Table 4.5. Equilibrium Product Flows and Equilibrium Quality Levels in Example 4.2

Demand	Results	Average Quality	Results
d_{11}^*	1,919.58	$\hat{q}_{11}$	82.84
d_{21}^*	1,379.79	$\hat{q}_{21}$	100.00

Table 4.6. Equilibrium Demands and Average Quality in Example 4.2

Notice that the equilibrium soybean flows, Q_{111}^* and Q_{211}^*, decrease from their values in Example 4.1, due to the strict quota imposed on the soybeans from the United States. Meanwhile, the soybean exports from other countries, Q_{121}^*, Q_{131}^*, and Q_{221}^*, increase. This results from the fact that firms now produce more in countries that are not restricted by the quota, that is, in Brazil and Argentina.

Furthermore, the equilibrium Lagrange multiplier, which is the equivalent tariff, is:

$$\lambda^* = 29.91.$$

Note that the Lagrange multiplier is positive since the volume of equilibrium soybean flows to China from the US production sites is equal to the imposed quota $\bar{Q} = 1200$.

When the above strict quota is imposed on United States' imports into China, the quality level associated with Cargill's production site located in the United States, q_{11}^*, decreases from its value in Example 4.1. Interestingly, the quality level of ADM's soybeans produced in the United States, q_{21}^*, does not show a change. However, q_{12}^* and q_{13}^*, denoting the quality of Cargill's soybeans produced in Brazil and Argentina, increase. Similarly, the quality of ADM's soybeans produced in Brazil, q_{22}^*, increases to its upper bound. These results indicate that, when a strict quota is imposed, the quality levels of the products produced under the quota are affected negatively or stay the same, whereas the quality of the products that are produced at sites not subject to the quota increases. Producers may wish to sustain prices as high as feasible and that can be accomplished through higher product quality.

Next, I report the equilibrium incurred demand prices per ton of soybeans of Cargill and ADM at Demand Market 1, in dollars, and the consumer welfare and the profits achieved by Cargill and ADM, in dollars, in Table 4.7.

Demand Price	Results	CW	Results	Profits	Results
ρ_{11}	706.16	CW_{11}	552,718.33	U_1	1,181,876.72
ρ_{21}	741.20	CW_{21}	333,167.37	U_2	728,637.06

Table 4.7. Equilibrium Demand Prices, Consumer Welfare, and Profits in Example 4.2

Notice that, at the demand market, which is in China, the prices of soybeans increase as compared to their values in Example 4.1. The consumer welfare associated with both firms' soybeans at the demand market decreases. Hence, consumers in China are negatively impacted by the strict quota.

Interestingly, due to increases in prices, the profits of both firms increase, as compared to Example 4.1 without the quota. The imposed quota does create an advantage for firms in this example. The two firms, by having soybean production sites in countries not under the strict quota, are able to expand their production at those sites. Hence, in a sense, they are more resilient to the imposition

of a quota than they might be otherwise. In a later example, Example 4.4, I consider the same problem as in Example 4.2 but with Cargill's production site in the United States shut down.

4.3.3 Sensitivity Analysis: Impacts of a Quality Coefficient Change in a Cost Functions

I now test the robustness of the managerial insights achieved in Example 4.2. Specifically, I test whether the following four insights obtained from the solution of Example 4.2 also hold when varying the quality coefficient in a production cost function, $\hat{f}_{12}$, not under a strict quota.

Recall that, in Example 4.2, with the imposition of a strict quota, $\bar{Q} = 1200$, by China on imports from the United States, the following results were observed:

1. The soybean flows from the United States production sites, Q_{111}^* and Q_{211}^*, decreased. Meanwhile, the soybean exports from other countries, Q_{121}^*, Q_{131}^*, and Q_{221}^*, increased.

2. The quality levels of the soybeans produced in the United States, q_{11}^* and q_{21}^*, decreased or stayed the same. However, q_{12}^*, q_{13}^*, and q_{22}^*, denoting the quality of the soybeans produced elsewhere, increased.

3. At the demand market, the prices of soybeans increased.

4. The consumer welfare associated with both firms' soybeans decreased, but both firms made higher profits.

In Tables 4.8 and 4.9 I provide sensitivity analysis results for Example 4.2 by varying the coefficient of q_{12}^2 in $\hat{f}_{12}$, with and without the strict quota. The values of the coefficient are: 0.2, 0.4, 0.6, 0.8, and 1, respectively. "Coeff." in Tables 4.8 and 4.9 denotes "Coefficient."

As can be observed from Tables 4.8 and 4.9, points 1, 2, 3, and 4 above hold even with a varying quality coefficient in $\hat{f}_{12}$. The generality of the model and computational procedure allows for such flexibility. Of course, sensitivity analyses can be conducted by varying other coefficients in the other production cost functions, as well as in the transportation cost functions, both individually and jointly.

4.3.4 Sensitivity Analysis: Impacts of Changes in the Strict Quota

I now provide a sensitivity analysis examining the effects of the value of the strict quota on the equilibrium product flows, the quality levels, and the average quality levels, demands, demand prices,

Coeff. of q_{12}^2 in $\hat{f}_{12}$	Quota	Q_{111}^*	Q_{121}^*	Q_{131}^*	Q_{211}^*	Q_{221}^*	q_{11}^*	q_{12}^*	q_{13}^*	q_{21}^*	q_{22}^*
0.2	with	525.90	716.00	679.90	674.10	705.56	71.71	100.00	84.99	100.00	100.00
0.2	without	743.97	621.85	570.98	778.96	611.51	100	93.28	71.37	100.00	93.06
0.4	with	528.96	697.99	692.63	671.04	708.75	72.13	87.25	86.58	100.00	100.00
0.4	without	756.70	591.26	585.9	779.32	612.32	100.00	73.91	73.24	100.00	93.18
0.6	with	532.48	677.26	707.29	667.52	712.42	72.61	72.56	88.41	100.00	100.00
0.6	without	765.06	571.19	595.68	779.57	612.85	100.00	61.2	74.46	100.00	93.26
0.8	with	534.99	662.5	717.72	665.01	715.03	72.95	62.11	89.72	100.00	100.00
0.8	without	770.96	557.01	602.59	779.74	613.22	100.00	52.22	75.32	100.00	93.32
1.0	with	536.86	651.45	725.53	663.14	716.99	73.21	54.29	90.69	100.00	100.00
1.0	without	775.36	546.46	607.74	779.87	613.5	100.00	45.54	75.97	10.000	93.36

Table 4.8. Equilibrium Flows, Equilibrium Quality Levels with a Varying Quality Coefficient in $\hat{f}_{12}$ with and without the Strict Quota

Coeff. of q_{12}^2 in $\hat{f}_{12}$	Quota	ρ_{11}	ρ_{21}	CW_{11}	CW_{21}	U_1	U_2
0.2	with	708.39	740.58	553996.44	333104.16	1183676.53	727808.73
0.2	without	703.45	726.19	562679.86	338344.67	1182645.7	722969.50
0.4	with	706.16	741.20	552718.33	333167.37	1181876.72	728637.06
0.4	without	700.25	726.76	560973.35	338916.81	1180812.05	724196.08
0.6	with	703.80	741.91	551248.08	333240.18	1180660.12	729587.16
0.6	without	698.39	727.14	559855.12	339292.45	1179609.38	725001.41
0.8	with	702.27	742.42	550202.67	333292.02	1179796.75	730260.94
0.8	without	697.20	727.41	559065.70	339557.98	1178759.8	725570.68
1.0	with	701.20	742.80	549421.21	333330.80	1179152.28	730763.64
1.0	without	696.36	727.61	558478.65	339755.64	1178127.73	725994.42

Table 4.9. Demand Prices, Consumer Welfare, and Profits with a Varying Quality Coefficient in $\hat{f}_{12}$ with and without the Strict Quota

profits, consumer welfare, and the Lagrange multiplier (i.e., the equivalent tariff) at equilibrium. The strict quota, $\bar{Q}$, varies from 1600 to 1200, 800, 400, and 0 tons. The results are shown in Figures 4.4 and 4.5.

As revealed in Figures 4.4.a and 4.4.b, as the strict quota imposed by China on the US production sites becomes tighter, the soybean flows from the United States, Q_{111}^* and Q_{211}^*, decrease to 0, whereas Q_{121}^*, Q_{131}^*, and Q_{221}^*, the soybean flows from production sites in countries not under the quota, increase.

Furthermore, as can be seen in Figures 4.4.c and 4.4.d, the quality levels at Cargill's and ADM's non-US sites, q_{12}^*, q_{13}^*, and q_{22}^*, increase. This result further indicates that tighter (lower) quotas may lead to an increase in both the product flows and the quality levels at the production sites where the quota is not imposed.

However, the product quality at Cargill's production site in the United States, q_{11}^*, decreases as the associated soybean flow declines when the strict quota tightens (cf. Figure 4.4.c). There's no value of it when $\bar{Q}$ becomes 400 or 0 due to no associated flows of soybeans; that is, the same holds with no value of q_{21}^* when $\bar{Q}$ is 0. Interestingly, q_{21}^*, the quality of ADM's soybeans at its United States site, remains the same (at the maximum value) as $\bar{Q}$ decreases from 1600 to 400 (cf. Figure 4.4.d). This is due to the high coefficient of the average quality, $\hat{q}_{21}$, in the demand price function of ADM, $\rho_{21}(d, \hat{q})$, which indicates a high correlation of $\hat{q}_{21}$ with respect to ρ_{21}, as compared to the other coefficients in the demand price functions. Thus, q_{21}^* needs to be at the maximum value (the upper bound) in order to maintain high prices. To illustrate this point, in Table 4.10, I report the values of q_{21}^*, for $\bar{Q} = 1200$ and 800, as the coefficient of $\hat{q}_{21}$ in $\rho_{21}(d, \hat{q})$ ranges from 1, to 1.20, 1.50, 1.80, and to 2.00. The coefficient of $\hat{q}_{11}$ in $\rho_{21}(d, \hat{q})$ remains constant at 0.7.
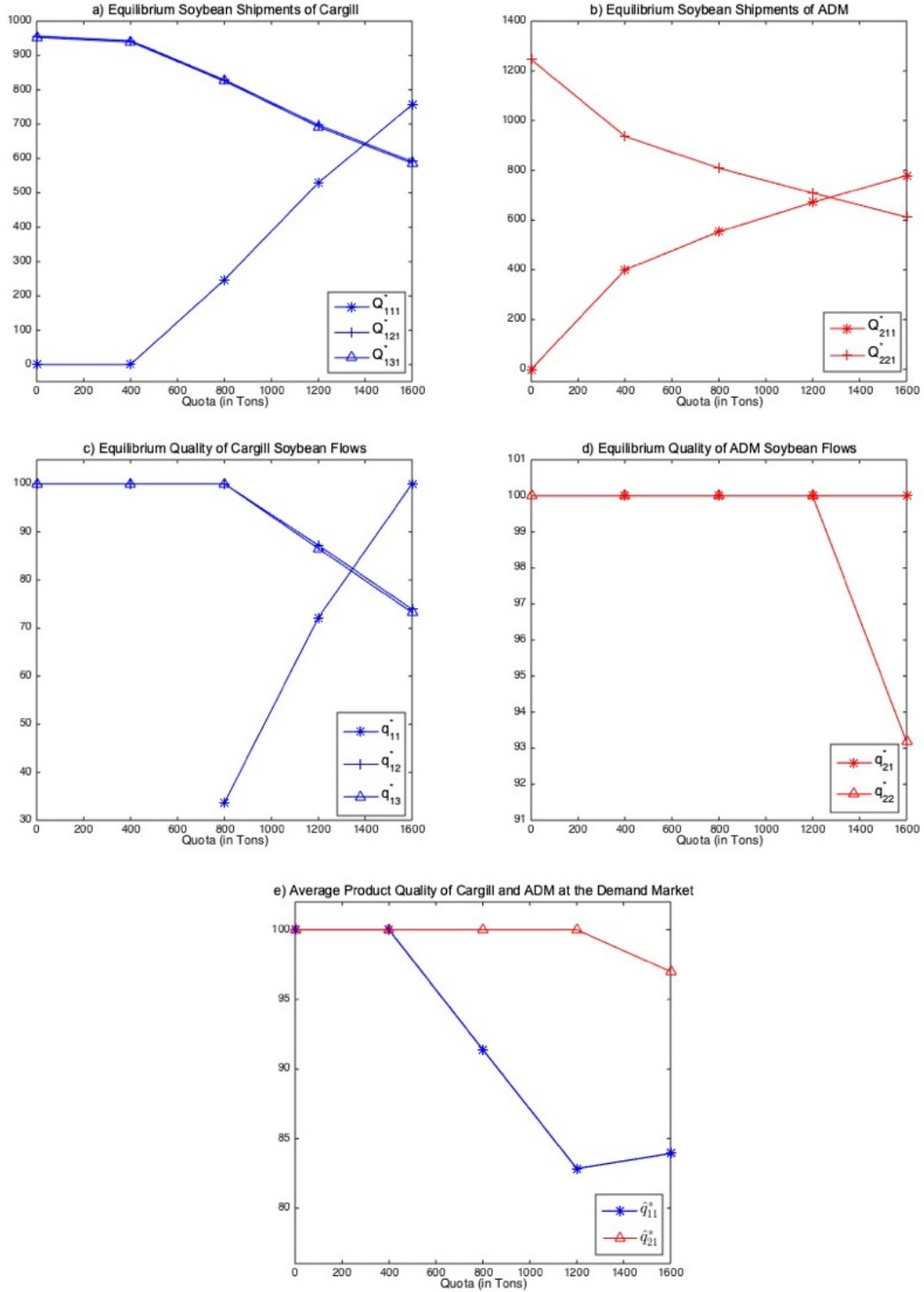
Figure 4.4. Equilibrium Product Shipments, Equilibrium Quality Levels, and Average Quality Levels as Quota Decreases for Example 4.2
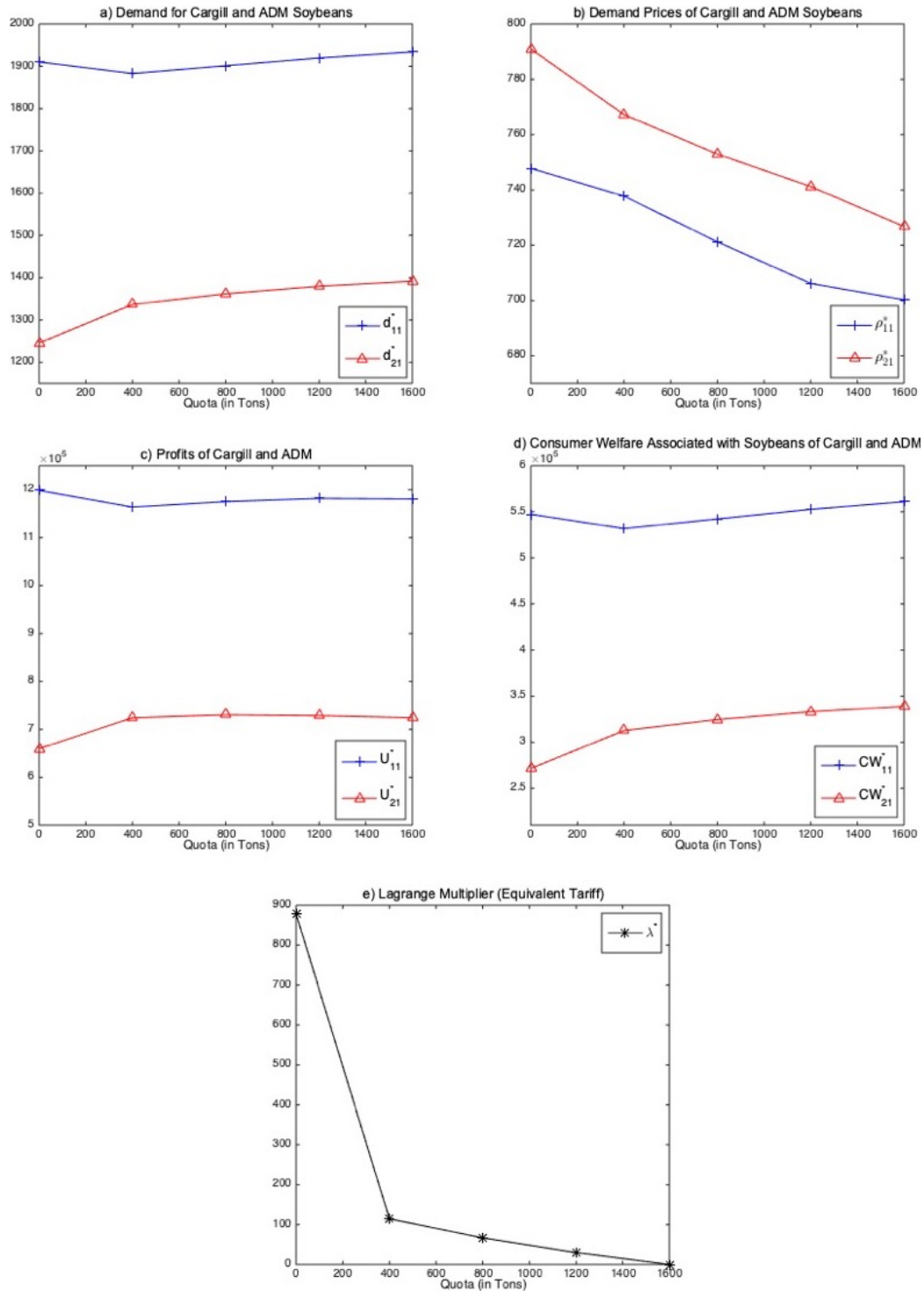
Figure 4.5. Demands, Demand Prices, Profits, Consumer Welfare, and Lagrange Multipliers at Equilibrium as Quota Decreases for Example 4.2

Coefficient of $\hat{q}_{21}$ in $\rho_{21}(d, \hat{q})$	q_{21}^* under $\bar{Q} = 1200$	q_{21}^* under $\bar{Q} = 800$
1.00	10.00	10.00
1.20	10.00	10.00
1.50	59.82	47.84
1.80	100.00	100.00
2.00	100.00	100.00

Table 4.10. Values of q_{21}^* with Varying Coefficients

The results in Table 4.10 reveal that q_{21}^* increases to its maximum value (upper bound) as the coefficient of $\hat{q}_{21}$ in $\rho_{21}(d, \hat{q})$ becomes higher; nevertheless, at the same coefficient, q_{21}^* decreases or stays the same as $\bar{Q}$ tightens. Hence, I can draw the conclusion that a tighter strict quota may negatively impact the quality of the product at the production sites where the quota is imposed. This result adds new insights on the effects of quotas on product quality to the literature.

Moreover, as shown in Figure 4.4.e, as the strict quota becomes tighter, the average quality at the demand market, $\hat{q}_{21}$ increases; $\hat{q}_{11}$ drops first and then improves due to the enhancement in quality in Brazil and Argentina.

Figures 4.5.a, 4.5.c, and 4.5.d show similar patterns. It is worth noting that, when the strict quota is 0, the demand for Cargill's soybeans is around 1,910 tons due to more exports from Brazil and Argentina (cf. Figure 5.a). This value is even higher than its demand when the quota is looser. Similar results are obtained for profits and consumer welfare. The profit of Cargill, U_1, with the strict quota being 0, is approximately 1.16 million dollars more than the profit when the quota is 400 tons (cf. Figure 4.5.c).

Last, but not least, as expected, the demand prices increase as the strict quota becomes tighter (cf. Figure 4.5.b). CW_{11} and CW_{21}, the consumer welfare, achieve their highest values when the quota is the greatest (cf. Figure 4.5.d). Finally, as the quota tightens, the Lagrange multiplier, i.e., the equivalent tariff (cf. Section 4.3), increases, reflecting the more restrictive trade policy interventions.

Next, another detailed sensitivity analysis by changing the value of the tariff and report the associated results is provided.

4.3.5 Example 4.3: Example 4.1 with Tariffs on Soybeans from the United States and Sensitivity Analysis

In Example 4.3, I investigate numerically the impacts of tariffs imposed by China on soybeans from the United States. I consider the same supply chain topology and the same data as in Example 4.1. The tariff is $\tau^* = 10.00$ dollars on imports of soybeans from the United States. I report the computed results for the equilibrium soybean flows, in tons, the equilibrium equilibrium quality levels, the equilibrium demands, and the average quality in Table 4.11 and Table 4.12.

Equilibrium Flows	Results	Equilibrium Quality	Results
Q^*_{111}	685.38	q^*_{11}	93.46
Q^*_{121}	624.46	q^*_{12}	78.06
Q^*_{131}	619.09	q^*_{13}	77.39
Q^*_{211}	735.79	q^*_{21}	100.00
Q^*_{221}	653.62	q^*_{22}	99.46

Table 4.11. Equilibrium Product Flows and Equilibrium Quality Levels in Example 4.3

Demand	Results	Average Quality	Results
d^*_{11}	1,928.93	$\hat{q}_{11}$	83.32
d^*_{21}	1,389.42	$\hat{q}_{21}$	99.75

Table 4.12. Equilibrium Demands and Average Quality in Example 4.3

Notice that the equilibrium demands for soybeans of both Cargill and ADM are lower than their values in Example 4.1. Similar to the discussion of the strict quota in Example 5.2, tariffs create a negative impact on the quality levels of the products. Observe that the quality level q^*_{11} decreases from its value in Example 4.1, when a tariff is imposed for the soybeans produced at P^1_1 and at P^2_1.

The incurred equilibrium demand prices per ton of soybeans of Cargill and ADM at Demand Market 1, in dollars, the consumer welfare associated with the soybeans of the two firms at the demand market, and the profits achieved by Cargill and ADM, in dollars, are reported in Table 4.13.

Demand Price	Results	CW	Results	Profits	Results
ρ_{11}	701.76	CW_{11}	558,116.66	U_1	1,174,437.42
ρ_{21}	734.52	CW_{21}	337,834.91	U_2	718,677.19

Table 4.13. Equilibrium Demand Prices, Consumer Welfare, and Profits in Example 4.3

Observe, from Table 4.13, that, after the imposition of the tariff, the equilibrium soybean demand prices associated with Cargill and ADM at Demand Market 1 are higher than their values in Example

4.1. Moreover, the consumer welfare associated with both firms decreases from that obtained in Example 4.1. This means that the introduction of tariffs creates a negative impact on Chinese consumers. As consumers at Demand Market 1 suffer from tariffs, the firms, Cargill and ADM, are faced with a profit decrease as compared to the profits enjoyed in Example 4.1, in which there were no trade policy instruments in the form of quotas or tariffs imposed.

4.3.6 Sensitivity Analysis: Impact of Changes in Tariffs

In this sensitivity analysis, the equilibrium soybean flows, the equilibrium quality levels, the average quality levels, the equilibrium demands, prices, consumer welfare, and profits are computed for a range of tariffs: $\tau^* = 20.00$, $\tau^* = 29.90$ and $\tau^* = 40.00$. According to Rapoza (2019), in 2019, China imposed an 8 dollar tariff per bushel of soybeans. In this sensitivity analysis, I test the model with higher tariffs. The results are reported in Tables 4.14, 4.15, and 4.16.

τ^*	Q_{111}^*	Q_{121}^*	Q_{131}^*	Q_{211}^*	Q_{221}^*
20.00	606.79	661.38	656.02	702.79	681.92
29.90	528.96	697.99	692.63	671.04	708.75
40.00	449.69	735.28	729.91	638.70	736.07

Table 4.14. The Equilibrium Soybean Flows for Example 4.3 under Various Tariffs

τ^*	q_{11}^*	q_{12}^*	q_{13}^*	q_{21}^*	q_{22}^*	$\hat{q}_{11}$	$\hat{q}_{21}$
20.00	82.74	82.67	82.00	100.00	100.00	82.47	100.00
29.90	72.13	87.25	86.58	100.00	100.00	82.84	100.00
40.00	61.32	91.10	91.24	100.00	100.00	84.74	100.00

Table 4.15. Equilibrium Quality Levels and Average Quality Levels for Example 4.3 under Various Tariffs

Similar conclusions to those obtained for Example 5.2 can be drawn from the results in Tables 4.14, 4.15, and 4.16, further supporting the equivalence between strict quotas and tariffs derived in Section 4.3.

τ^*	d_{11}^*	d_{21}^*	ρ_{11}	ρ_{21}	CW_{11}	CW_{21}	U_1	U_2
20.00	1,924.19	1,384.71	703.53	738.09	555,378.50	335,548.02	1,169,791.64	713,460.32
29.90	1,919.58	1,379.79	706.16	741.20	552,718.33	333,167.37	1,166,059.22	708,571.37
40.00	1,914.88	1,374.78	709.71	744.36	550,015.49	330,751.37	1,163,040.11	703,900.27

Table 4.16. Equilibrium Demands, Demand Prices, Consumer Welfare, and Profits for Example 4.3 under Various Tariffs

For example, the results in Table 4.14 show that, when the tariff increases, the equilibrium flows from the production sites under the tariff decrease. This result is the same as in Example 4.2 with the strict quota. Moreover, the equilibrium quality levels for the soybeans produced in the US decrease, as the tariff increases. Additionally, the results in Table 4.16 reveal a decrease in equilibrium demands as the tariff increases.

Similar to the discussion in Section 4.2, the demand prices of the Cargill and ADM soybeans increase and the associated consumer welfare decreases, when a higher tariff is imposed. Furthermore, notice that, when the tariff $\tau^* = 29.90$, the computed equilibrium solution is equal to that reported in Example 4.2. Hence, I have numerically illustrated/verified the equivalence between strict quotas and tariffs, where $\tau^* = \lambda^* > 0$, further supporting the theoretical results obtained in Section 4.3.

4.3.7 Example 4.4: Example 4.2 with Cargill's Production Site in the United States Shut Down

In Example 4.4, the same differentiated product supply chain network problem for soybeans as in Example 4.2 is considered, but with Cargill's soybean production site in the United States, P_1^1, shut down. Thus, the corresponding node and associated links are removed from the supply chain network in Figure 4.3, yielding the supply chain network in Figure 4.6. The strict quota of 1200 is now imposed only on ADM's soybeans from the United States produced at P_1^2.
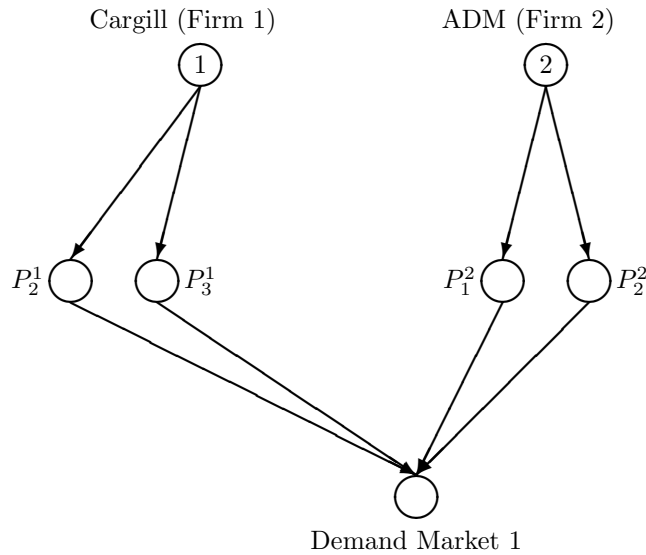
Figure 4.6. The Supply Chain Network Topology for Example 4.4

I report the equilibrium soybean flows, in tons, the equilibrium quality levels, the equilibrium demands for soybeans at Demand Market 1, and the average quality levels in Table 4.17 and Table 4.18.

Equilibrium Flows	Results	Equilibrium Quality	Results
Q^*_{111}	-	q^*_{11}	-
Q^*_{121}	930.73	q^*_{12}	100.00
Q^*_{131}	926.80	q^*_{13}	100.00
Q^*_{211}	788.93	q^*_{21}	100.00
Q^*_{221}	633.23	q^*_{22}	99.36

Table 4.17. Equilibrium Product Flows and Equilibrium Quality Levels in Example 4.4

Demand	Results	Average Quality	Results
d^*_{11}	1,857.53	$\hat{q}_{11}$	100.00
d^*_{21}	1,422.16	$\hat{q}_{21}$	98.38

Table 4.18. Equilibrium Demands and Average Quality in Example 4.4

Furthermore, the equilibrium Lagrange multiplier, which is the equivalent tariff, becomes:

$$\lambda^* = 0.00.$$

Due to the shutdown of Cargill's production site in the United States, its soybean flows from the other sites, that are located in Brazil and Argentina, increase. Meanwhile, with less competition from Cargill, ADM delivers more soybeans to China from the cheaper site, P^2_1, in the United States, and fewer soybeans from the more expensive site, P^2_2, in Brazil. Since the strict quota is only imposed on ADM in this example, there is more opportunity for ADM to increase its production in the US. Furthermore, ADM's soybean production in the US site does not meet the strict quota, resulting in a zero equilibrium Lagrange multiplier.

The results for the computed equilibrium incurred demand prices per ton of soybeans of Cargill and ADM at Demand Market 1, in dollars, the consumer welfare at the demand market, and the profits achieved by Cargill and ADM, in dollars, are provided in Table 4.19.

Demand Price	Results	CW	Results	Profits	Results
ρ_{11}	728.31	CW_{11}	517,560.59	U_1	1,131,885.86
ρ_{21}	741.75	CW_{21}	353,945.28	U_2	756,415.15

Table 4.19. Equilibrium Demand Prices, Consumer Welfare, and Profits in Example 4.4

As compared to the results in Example 4.2, the quality levels of soybeans produced by Cargill, including the average quality, increase to the maximum level, after its production site in the United

States is shut down. ADM's quality levels of its soybeans, in turn, decrease or remain the same, with its average quality decreased. Furthermore, the consumer welfare associated with Cargill's soybeans decreases due to less demand and a higher price, but the consumer welfare associated with ADM's soybeans improves. It is also worth noting that the shut down of Cargill's production site in the United States harms its profit, whereas ADM now achieves a higher profit than before.

4.3.8 Example 4.5: Example 4.1 with a New Demand Market in the United States and Additional Data

In this example, I add a new demand market, Demand Market 2, which is located in the United States, to the supply chain network in Example 4.1, as shown in Figure 4.7. The domestic consumption of soybeans in the United States is also worth analyzing, even with the United States being one of the top soybean exporters in the world. In Figure 4.7, I do not include the transportation links from Brazil and Argentina to the United States, since clearly such trade would not be cost-efficient, with the US being an exporter. The new demand market, hence, induces two additional paths and path flows, denoted by Q_{112} and Q_{212}.

The total soybean production outputs at Cargill's and ADM's production sites (cf. Figure 4.7) must satisfy the following expressions:

$$s_{11} = Q_{111} + Q_{112}, \ s_{12} = Q_{121}, \ s_{13} = Q_{131}, \ s_{21} = Q_{211} + Q_{212}, \ s_{22} = Q_{221}.$$

Furthermore, the updated production cost functions of Cargill and ADM are:

$$f_{11}(s_{11}, q_{11}) = 0.04s_{11}^2 + 0.35s_{11} + 0.4s_{11}q_{11} + 0.6q_{11}^2,$$

$$f_{12}(s_{12}, q_{12}) = 0.05s_{12}^2 + 0.35s_{12} + 0.4s_{12}q_{12} + 0.4q_{12}^2,$$

$$f_{13}(s_{13}, q_{13}) = 0.05s_{13}^2 + 0.8s_{13} + 0.4s_{13}q_{13} + 0.4q_{13}^2,$$

$$f_{21}(s_{21}, q_{21}) = 0.06s_{21}^2 + 0.5s_{21} + 1.2s_{21}q_{21} + q_{21}^2,$$

$$f_{22}(s_{22}, q_{22}) = 0.07s_{22}^2 + 0.3s_{22} + 1.3s_{22}q_{22} + 1.5q_{22}^2.$$

There are two additional total transportation cost functions associated with Cargill and ADM shipping their soybeans to Demand Market 2, given by:
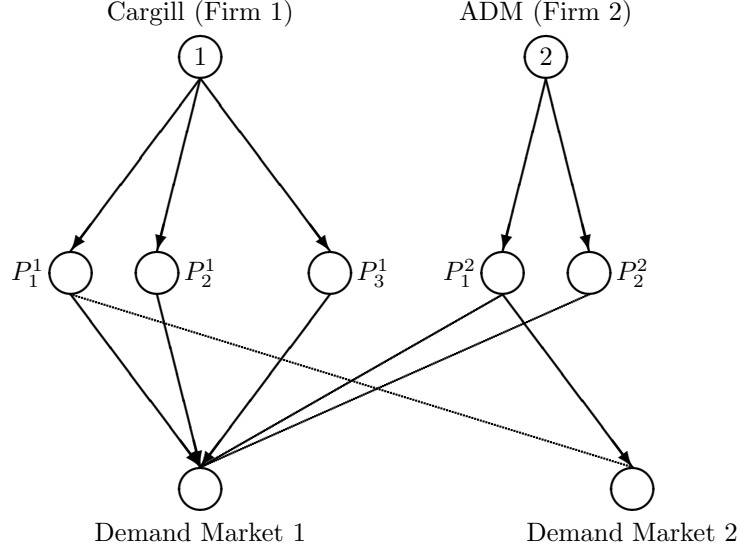
Figure 4.7. The Supply Chain Network Topology for Example 4.5

$$\hat{c}_{112}(Q_{112}, q_{11}) = 0.002Q_{112}^2 + 0.02Q_{112} + 0.8q_{11}^2, \qquad \hat{c}_{212}(Q_{212}, q_{21}) = 0.002Q_{212}^2 + 0.04Q_{212} + q_{21}^2.$$

The additional demand price functions for the soybeans of Cargill and ADM at Demand Market 2 are:

$$\rho_{12}(d, \hat{q}) = 1100 - (0.25d_{12} + 0.2d_{22}) + 0.9\hat{q}_{12},$$
$$\rho_{22}(d, \hat{q}) = 1400 - (0.3d_{22} + 0.25d_{12}) + 1.4\hat{q}_{22},$$

with the additional average quality $\hat{q}_{12}$ and $\hat{q}_{22}$ at Demand Market 2 given by:

$$\hat{q}_{12} = \frac{Q_{112}q_{11}}{Q_{112}} = q_{11}, \quad \hat{q}_{22} = \frac{Q_{212}q_{21}}{Q_{212}} = q_{21}.$$

The computed equilibrium soybean flows, in tons, equilibrium quality levels, equilibrium demands, and average quality are given in Tables 4.20 and 4.21.

Equilibrium Flows	Results	Equilibrium Quality	Results
Q_{111}^*	87.50	q_{11}^*	100.00
Q_{121}^*	913.30	q_{12}^*	100.00
Q_{131}^*	909.37	q_{13}^*	100.00
Q_{211}^*	150.11	q_{21}^*	77.79
Q_{221}^*	1,126.34	q_{22}^*	100.00
Q_{112}^*	1,469.53	-	-
Q_{212}^*	1,422.13	-	-

Table 4.20. Equilibrium Product Flows and Equilibrium Quality Levels in Example 4.5

Demand	Results	Average Quality	Results
d_{11}^*	1,910.17	$\hat{q}_{11}$	100.00
d_{21}^*	1,276.44	$\hat{q}_{21}$	97.39
d_{12}^*	1,469.53	$\hat{q}_{12}$	100.00
d_{22}^*	1,422.13	$\hat{q}_{22}$	77.79

Table 4.21. Equilibrium Demands and Average Quality in Example 4.5

Observe that the equilibrium soybean flow from Cargill's United States production site to Demand Market 1, Q_{111}^*, decreases greatly from its value reported in Example 4.1. Also, now the majority of the soybeans produced at P_1^1 are shipped to Demand Market 2 in the United States, since the transportation cost of sending soybeans domestically is cheaper. Moreover, the equilibrium soybean flows of Cargill's production sites in Brazil and Argentina increase from their values in Example 4.1 with the new demand market. A similar pattern is observed for ADM's soybean flows.

The equilibrium demands d_{11}^* and d_{21}^* decrease from their values in Example 4.1. Notice that the equilibrium demand for ADM's soybeans at Demand Market 2 in the United States is higher than that at Demand Market 1 in China. The equilibrium quality levels q_{12}^*, q_{13}^*, and q_{22}^* have higher values than in Example 4.1. Moreover, the average quality of Cargill's and ADM's soybeans at the demand markets is at the respective upper bound.

In Table 4.22, I provide the equilibrium demand prices per ton of soybeans of Cargill and ADM at Demand Market 1 and Demand Market 2, in dollars, the consumer welfare at each demand market, and the achieved profits of Cargill and ADM.

Demand Price	Results	CW	Results	Profits	Results
ρ_{11}	741.66	CW_{11}	547,310.17	U_1	1,809,217.78
ρ_{21}	774.97	CW_{21}	285,128.51	U_2	1,405,249.62
ρ_{12}	538.19	CW_{12}	269,938.75	-	-
ρ_{22}	714.89	CW_{22}	303,368.87	-	-

Table 4.22. Equilibrium Demand Prices, Consumer Welfare, and Profits in Example 4.5

With the introduction of Demand Market 2, the demand prices of the Cargill and ADM soybeans at Demand Market 1 increase from their values in Example 4.1. The consumer welfare associated with both firms' soybeans at Demand Market 1 decreases due to higher prices and lower demands. Both firms achieve a higher profit with the addition of a new demand market. ADM, in particular, enjoys a significant profit increase from that in Example 4.1.

4.3.9 Example 4.6: Example 4.5 with a Strict Quota

In Example 4.6, I consider the same supply chain topology and the data as in Example 4.5, but I assume that China now imposes a strict quota of $\bar{Q} = 100$ on its imports from the United States. Similar to the previous sections, I report the computed equilibrium soybean flows, in tons, the equilibrium quality levels, the equilibrium demands, and the average quality in Tables 4.23 and 4.24.

Equilibrium Flows	Results	Equilibrium Quality	Results
Q_{111}^*	19.30	q_{11}^*	100.00
Q_{121}^*	945.79	q_{12}^*	100.00
Q_{131}^*	941.86	q_{13}^*	100.00
Q_{211}^*	80.70	q_{21}^*	67.35
Q_{221}^*	1,182.64	q_{22}^*	100.00
Q_{112}^*	1,476.77	-	-
Q_{212}^*	1,428.25	-	-

Table 4.23. Equilibrium Product Flows and Equilibrium Quality Levels in Example 4.6

Demand	Results	Average Quality	Results
d_{11}^*	1,906.95	$\hat{q}_{11}$	100.00
d_{21}^*	1,263.34	$\hat{q}_{21}$	97.91
d_{12}^*	1,476.77	$\hat{q}_{12}$	100.00
d_{22}^*	1,428.25	$\hat{q}_{22}$	67.35

Table 4.24. Equilibrium Demands and Average Quality in Example 4.6

As expected, the equilibrium demands d_{11}^* and d_{21}^* decrease from their values in Example 4.5 with the introduction of the quota, whereas d_{12}^* and d_{22}^* increase. Notice that the equilibrium quality level of soybeans of ADM, produced in the United States, decreases from its value in Example 5.5, whereas the remaining equilibrium quality levels are the same as in Example 4.5. Interestingly, the average quality of ADM's soybeans at Demand Market 1 in China increases from its value in Example 5.5. In contrast, the average quality of ADM's soybeans at Demand Market 2 in the United States decreases.

Moreover, the equilibrium Lagrange multiplier or the equivalent tariff is:

$$\lambda^* = 12.15$$

since the volume of equilibrium soybean flows from the US production sites is at the imposed strict quota.

The equilibrium demand prices per ton of soybeans of Cargill and ADM at Demand Market 1 and Demand Market 2, in dollars, the consumer welfare at each demand market, and the achieved profits of Cargill and ADM are reported in Table 4.25.

Demand Price	Results	CW	Results	Profits	Results
ρ_{11}	745.25	CW_{11}	545,470.00	U_1	1,812,002.21
ρ_{21}	781.57	CW_{21}	279,305.28	U_2	1,403,470.22
ρ_{12}	535.16	CW_{12}	272,607.69	-	-
ρ_{22}	696.62	CW_{22}	305,984.29	-	-

Table 4.25. Equilibrium Demand Prices, Consumer Welfare, and Profits in Example 4.5

Note that the consumer welfare associated with consumers in China, represented by Demand Market 1, which has imposed the quota on production sites in the US, associated with Cargill's soybeans, CW_{11}, and with ADM's soybeans, CW_{21}, declines. This is in contrast to the values in Example 4.5, signifying a negative impact on the consumers. However, the consumer welfare of the consumers in the US (Demand Market 2), increases.

The introduction of the strict quota to Example 4.5 creates a profit increase and a profit decrease for Cargill and ADM, respectively. Having multiple production sites in Brazil and Argentina that are not under the strict quota saves Cargill from a profit drop. On the other hand, ADM is faced with a profit decrease due to having only single production site over which a strict quota is not imposed, in Brazil.

4.4. Managerial Insights

From the above numerical examples, and accompanying sensitivity analysis, a spectrum of managerial insights is revealed.

Specifically, from the consumer's perspective, the results consistently and unanimously show that consumer welfare declines for consumers in the country imposing a strict quota or tariff on an imported product. Hence, a government may wish to loosen a quota (equivalently, reduce a tariff) so as not to adversely affect its own consumers.

Producing firms, as also critical stakeholders in competitive supply chain networks, should expand their demand markets within their own countries. This allows for a basic, but, effective, redesign of the supply chain network under a tariff or quota and results in higher profits for the firms. Also, firms should expand the number of production sites to countries not under a tariff or quota to

maintain or improve upon their profits if some of their production sites are in countries subject to such trade policy instruments.

Finally, the examples numerically support the theoretical finding that a tariff has the equivalent impact on product flows and product quality as a strict quota, provided that the tariff is set to the Lagrange multiplier associated with the strict quota constraint and the constraint is tight. Hence, governments have the flexibility of imposing either a tariff or a quota to obtain equivalent trade flows and product quality levels. The imposition of a tariff may be more advisable/favored by a government, since it requires less "policing" and also yields financial rewards.

4.5. Conclusions

Supply chain networks provide the pathways for the production and distribution of products to consumers across the globe and serve as the critical infrastructure for world trade. Governments, in their desire to influence world trade, utilize trade policies, in the form of quotas or tariffs. The recent dynamism associated with the imposition of such policies worldwide merits closer attention from modeling and computational perspectives. Although such topics have a long history in the economics literature, they have been less explored in operations research / operations management, especially from a supply chain perspective. Furthermore, the identification of the impacts on product quality and how consumers are affected by such trade policy instruments, within the context of realistic supply chain networks, is relevant for producers, consumers, and policy makers alike.

In this chapter, I add to the supply chain network, game theory, and trade policy literature by constructing an oligopolistic supply chain network equilibrium model with differentiated products in which firms have multiple production sites and multiple possible demand markets. The firms compete in product quantities and product quality, subject to minimum quality standards, along with upper bounds on quality. The model is then extended to include trade policy instruments in the form of a strict quota or a tariff. I identify the governing equilibrium conditions, noting that the strict quota model is characterized by a Generalized Nash Equilibrium, rather than a Nash Equilibrium, as is the case of the other two models. I demonstrate that, nevertheless, the underlying equilibrium conditions for all three models can be formulated and analyzed as an appropriate variational inequality problem, for which an effective computational scheme is also provided, which yields closed form expressions for the variables in each of the two steps of the procedure.

I establish theoretically, and also support the results numerically, that the equilibrium Lagrange multiplier associated with the strict quota constraint in the strict quota model, if assigned as a tariff, when the strict quota is tight, yields the same equilibrium product flows and product quality levels for the tariff model as obtained for the quota model. This equivalence in a competitive supply chain network allows decision-makers the option of using either a strict quota or a tariff to obtain identical results. I also construct consumer welfare measures for the models.

In summary, the theoretical contributions in this chapter are the following:

1. The construction of the first general (not limited to a specific number of firms or demand markets or functional forms) oligopolistic supply chain network equilibrium models with strategic variables of quantities and product quality that incorporate multiple trade policy instruments;

2. The establishment of the equivalence of a tariff with that of a strict quota, provided that the strict quota constraint is tight, and the Lagrange multiplier associated with it is the set tariff. This provides decision-makers and policy makers, including governments, with the flexibility of imposing either a tariff or a strict quota in practice.

3. The construction of a formula for the determination of the consumer welfare, under a tariff or quota, in competitive supply chain networks.

4. The rigorous formulation of all the models as variational inequality problems, either with a tariff or a strict quota, and with minimum and maximum quality bounds. In the case of the model with a strict quota, the governing equilibrium conditions are those of a Generalized Nash Equilibrium, because of the shared/common constraint. There are very few GNE supply chain models in the literature to-date.

5. A proposed algorithm, with nice features for computations, for the new models, accompanied by convergence conditions.

Illustrative examples are provided, along with numerical examples inspired by an important agricultural product - that of soybeans. The numerical examples, accompanied by sensitivity analysis, reveal that a government, in imposing trade policy instruments in the form of strict quotas or tariffs, may decrease the welfare of its own consumers. The computational framework includes the quantitative measurement of the impacts of a production site disruption as well as the addition of a demand market.

In summary, the practical insights from this framework, are the following:

1. Governments should be cautious in imposing trade policy instruments in the form of tariffs or quotas on products in competitive, that is, oligopolistic, supply chain networks, since the consumer welfare of consumers in their own country can decrease as a result.

2. Governments, by imposing a tariff or quota, may help firms in their country garner enhanced profits but at the expense of consumers.

3. Producers should expand the geographic dispersion of their production sites to reduce the impact of imposed tariffs or quotas.

4. Producers should actively expand their demand markets in countries not under trade policy instrument regimes, since doing so can lead to higher profits.

CHAPTER 5

QUALITY IN COMPETITIVE FRESH PRODUCE SUPPLY CHAINS WITH APPLICATION TO FARMERS' MARKETS

According to a national survey conducted by Bond, Thilmany, and Bond (2009), four out of five people living in the United States buy local food. Thus, the demand for locally grown fresh produce is also increasing and demand and supply management becomes more crucial for farmers. A U.S. Congressionally mandated report by the U.S. Department of Agriculture (cf. Low et al. (2015)) noted that the sale of food through direct-to-consumer channels, such as through farmers' markets, and intermediated marketing channels, that is, sales to institutions, including hospitals and schools, or regional distributors, appears to be increasing. As reported therein, there were 8,268 farmers' markets in the United States in 2014, with the number having increased by 180% since 2006. This growth implies increasing consumer interest in locally sourced fresh produce due to quality and health perspectives.

The majority of research on local food markets, especially farmers' markets, pertains mostly to consumer behavior and there are not many mathematical models that consider demand or the supply side in local food supply chains. The literature on consumer behavior in farmers' markets provides valuable insights from the consumers' and farms' perspectives.

In this chapter, I develop a modeling and algorithmic framework for competitive farmers' markets. The model is network-based and the farmers engage in Cournot competition over space and time. The governing Nash equilibrium conditions are formulated as a variational inequality problem. The novelty of the framework lies in that the quality of the fresh produce product is captured as the produce propagates in the supply chain over space and time with the consumers at the markets responding both to the price and the quality of the fresh produce. Both uncapacitated and capacitated versions of the model are presented. The latter can capture limitations in supply due to harvest problems or damage during the growing season, limitations in transport and storage capabilities, and/or labor for harvesting and processing. The game theory model can address questions

of farmers as to which farmers' markets they should serve; what the impact of a new competitor may be at one or more markets in terms of profits, as well as the effects of capacity disruptions (or enhancements) in their individual and others' short supply chains. In addition, the model can ascertain the impacts of changes in link parameters that capture quality. Policy makers, in turn, can also obtain useful information as to the impacts of a greater number or fewer farms represented at various markets and how reducing quality decay can affect farmers' profits.

The chapter is organized as follows. In Section 5.1, I provide some preliminaries, focusing on the quality differential equations, which I then generalize to a path concept since the fresh produce will proceed on multiple supply chain links from the harvesting point to ultimate purchase at the farmers' markets. Each link has associated with it both a time element as well as a factor such as temperature, which also affects the fresh produce quality.

In Section 5.2, I first present the uncapacitated competitive supply chain network model for farmers' markets, give the governing Nash equilibrium conditions, and derive the variational inequality formulations in path flows and links flows. In addition, I construct the capacitated path analogue and provide its variational inequality formulation under Nash equilibrium. In Section 5.3, I present the algorithm, which resolves the variational inequality problems into subproblems in the path flows (and in the case of the capacitated model, also in terms of Lagrange multipliers) that can be solved explicitly using formulae. In Section 5.4 a case study on apples in farmers' markets is presented. I summarize and present the conclusions in Section 5.5.

This chapter is based on the paper by Besik and Nagurney (2017).

5.1. Preliminaries on Food Quality Deterioration

Before I present the fresh food supply chain network model for farmers' markets, it is important to provide some preliminaries on the definition of quality, and the kinetics behind the quality decay, since quality has major relevance for the fresh food industry. Afterwards, I provide the generalization of the quality differential equations for fresh produce supply chain networks by capturing them through the path concept.

Fresh foods are biological products, they deteriorate and, consequently, lose quality over time (Schouten et al. (2004), Singh and Anderson (2004)). Thus, an understanding of the biochemical/physicochemical reactions, which cause the deterioration, is necessary in order to be able to present an explanation of the quality loss (Singh and Cadwallader (2004)). According to Taoukis

and Labuza (1989), the rate of quality deterioration can be given as a function of its microenvironment, gas composition, relative humidity, and temperature.

In this regard, Labuza (1984) captures the quality decay of a food attribute, Q, over time t, through the differential equation:

$$\frac{\partial Q}{\partial t} = kQ^n = Ae^{(-E/RT)}Q^n. \tag{5.1}$$

Here, k is the reaction rate and is defined by the Arrhenius formula, $Ae^{(-E/RT)}$, where A is a pre-exponential constant, T is the temperature, E is the activation energy, and R is the universal gas constant (Arrhenius (1889)). Moreover, n is the reaction order, which is a non-negative integer and belongs to the set $Z^* = \{0\} \cup Z^+$. In general, the quality decay function of the food attribute can be shown in terms of its reaction order. When the reaction order n is zero, that is, $\frac{\partial Q}{\partial t} = k$, the quality decay rate of the food attribute Q can be expressed as the function:

$$f_0(Q) = -kt. \tag{5.2}$$

Furthermore, assuming that the initial quality is known and given as Q_0, the remaining quality Q_t at time t, can be defined by using the zero order quality decay function as in Tijskens and Polderdijk (1996):

$$Q_t = Q_0 + f_0(Q) = Q_0 - kt. \tag{5.3}$$

Recall that the reaction constant k can be written by the Arrhenius formula to show the relationship between temperature and quality decay explicitly. Furthermore, having the reaction order be 1 – also referred to as a first order reaction – leads to an exponential function, which is observed commonly in food quality decay (Tijskens and Polderdijk (1996)). This type of quality decay is given by the expression:

$$f_1(Q) = e^{-kt}. \tag{5.4}$$

Notice that, since this quality decay function has an exponential component, the quality Q_t at time t should be written as a multiplication of the initial quality Q_0 and the quality decay function, as shown in the following expression:

$$Q_t = Q_0 f_1(Q) = Q_0 e^{-kt}. \tag{5.5}$$

Table 5.1 shows the decay kinetics, with the related quality attributes, of some fruits and vegetables. For a detailed description of the quality decay kinetics for vegetables, see Aamir et al. (2013). Notice that the quality attributes, in Table 5.1, are related to the appearance and texture which are the two most important fresh produce characteristics for consumers.

Attribute	Fresh Produce	Reaction Order	Reference
Color Change	Peaches	First	Toralles et al. (2005)
Color Change	Raspberries	First	Ochoa et al. (2001)
Color Change	Blueberries	First	Zhang, Guo, and Ma (2012)
Nutritional (Vitamin C)	Strawberries	First	Castro et al. (2004)
Color Change	Watermelons	Zero	Dermesonlouoglou, Giannakourou, and Taoukis (2007)
Moisture Content	Tomatoes	First	Krokida et al. (2003)
Color Change	Cherries	First	Ochoa et al. (2001)
Texture Softening	Apples	First	Tijskens (1979)
Nutritional (Vitamin C)	Pears	First	Mrad et al. (2012)
Texture Softening	Avocados	First	Maftoonazad and Ramaswamy (2008)
Nutritional (Vitamin C)	Pineapples	First	Karim and Adebowale (2009)
Color Change	Spinach	Zero	Aamir et al. (2013)
Color Change	Asparagus	First	Aamir et al. (2013)
Color Change	Peas	First	Aamir et al. (2013)
Texture Softening	Beans	First	Aamir et al. (2013)
Texture Softening	Brussel Sprouts	First	Aamir et al. (2013)
Texture Softening	Carrots	First	Aamir et al. (2013)
Texture Softening	Peas	First	Aamir et al. (2013)
Color Change	Coriander Leaves	First	Aamir et al. (2013)

Table 5.1. Fresh Produce Attributes and Decay Kinetics

Now, each type of quality deterioration function can be generalized in terms of the path concept in a given fresh produce food supply chain network. I define a path p joining an origin node i with a destination node j through directed links that it is comprised of in the link set L. Furthermore, let β_a denote the quality decay incurred on link a, which depends on the reaction order n, reaction rate k_a, and time t_a on link a, according to:

$$\beta_a \equiv \begin{cases} -k_a t_a, & \text{if } n = 0, \forall a \in L, \\ \\ e^{-k_a t_a}, & \text{if } n \neq 0, \forall a \in L. \end{cases} \tag{5.6}$$

Here, k_a is the reaction constant related to the link a. Since each link on a path can have different temperature conditions, the differentiation over the temperature of the links is necessary. Thus, the reaction rate is described in the following equation for each link a by the Arrhenius formula with

the same parameters as in (5.1), except that the temperature is now denoted for each link a as T_a, where:

$$k_a = Ae^{(-E_A/RT_a)}. \tag{5.7}$$

In addition, since (5.7) contains the exponential decay as a function and the other terms are parameters, β_a is defined generally for the reaction orders greater than zero. Before I show the quality decay over a given path p, it is worthwhile to introduce the quality parameter q_{0i}, which represents the initial quality of the fresh produce, produced at the origin node i. Also, P_j^i represents the set of all paths that have origin i and destination j. I now can define the quality q_p, over a path p, joining the origin farm node, i, with a destination node farmer market, j, while incorporating the quality deterioration of the fresh produce as:

$$q_p \equiv \begin{cases} q_{0i} + \sum_{a \in p} \beta_a, & \text{if } n = 0, \forall a \in L, \ p \in P_j^i, \forall i, j, \\\\ q_{0i} \prod_{a \in p} \beta_a, & \text{if } n = 1, \forall a \in L, \ p \in P_j^i, \forall i, j. \end{cases} \tag{5.8}$$

By the above generalization, the model is differentiated from the existing works in the literature. Although I utilize Tijskens and Polderdijk (1996)'s quality decay formulations, the authors have not provided a quality deterioration formulation for a path in a food supply chain network. Rong, Akkerman, and Grunow (2011) build the food supply chain through a mixed-integer programming model and incorporate the food quality decay into this model. However, in this chapter, I present a competitive supply chain game theory model where the demand markets and the quality decay are product specific. Furthermore, Ketzenberg, Bloemhof, and Gaukler (2015) present a model to assess the value of time and temperature information of a perishable product. The authors provide a simulation study to evaluate the remaining shelf life of fresh fish with randomly realized temperature and time information. In their work, they provide a different food quality metric than the one presented in this chapter which is not based on quality kinetics and is not applied on a supply chain network.

Next, the competitive fresh produce supply chain network model for farmers' markets is presented.

5.2. The Fresh Produce Farmers' Market Supply Chain Network Models

In this section, I capture the economic behavior of farmers selling at farmers' markets and competing on quality and quantity of their fresh produce a la Cournot (1838) through both an uncapacitated and a capacitated oligopoly model and the variational inequality formulations of the governing Nash (1950, 1951) equilibrium conditions are derived. The models consist of a finite number of I farms, that are run by farmers, typically, denoted by i, and a finite number of demand points, M, which correspond to farmers' markets, and with a typical one denoted by j. Each farmers' market takes place in a region and on a given day of the week. It is assumed that the demand points correspond to farmers' markets on different days of the week since farmers may not have sufficient staff to be at multiple markets on the same day. Farmers' markets, on any given day, usually last no more than 6 hours and are repeated in the same location and day over a season, which, in the Northeast of the United States, for example, can last from May through October.

The uncapacitated model is presented in Section 5.2.1 and its capacitated analogue then outlined in Section 5.2.2.

Before introducing the variables and the notation for the models, I provide a summary of the assumptions that are made:

1. The supply chain network topology illustrates a fixed time horizon in a given season of the fresh fruit or vegetable, typically, a week.

2. The demand points represent the direct-to-consumer local farmers' markets, in different regions of a county or a state.

3. Farmers pick the harvest right before the beginning of the time horizon; therefore, a farmers' market that is available at the beginning of this time horizon sells the fresh produce without storage. Farmers' markets taking place on subsequent days store the fresh produce before transporting it to the markets.

4. Consumers can buy products that are substitutes of one other within or across the demand points. That means that the farmers' markets provide multiple options of fresh produce to consumers over space and time.

Since the local supply chains are short, the intermediate activities can be defined as: harvesting, processing and packaging, storage, and transportation. Farmers compete at the local farmers' mar-

kets noncooperatively in an oligopolistic manner. The products are differentiated at the markets according to their quality, which are presented by the quality deterioration functions described in Section 5.1. In this regard, the corresponding fresh produce food supply chain network topology is as in Figure 5.1. Based on the case study of concern here, that is, apples, the time horizon of a week and the picking of apples once a week is not unreasonable, according to experts that I have interviewed (Clements (2016a) and Drew (2016)). If more choices were to be added in terms of multiple times to pick, for example, then the network would be adapted/extended accordingly. Here the goal is to lay the foundations for the modeling of both short and long food supply chain networks under competition and with quality deterioration based on kinetics on paths consisting of distinct economic activities such as transport, storage, etc.
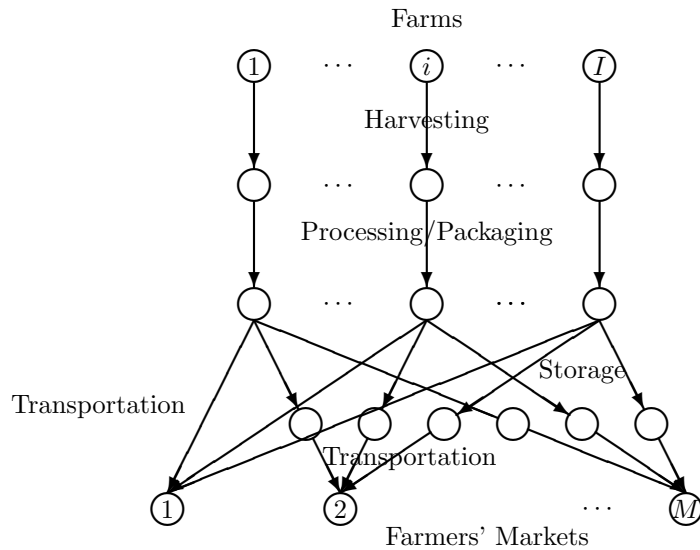
Figure 5.1. The Supply Chain Network Topology of the Farmer's Market Competitive Fresh Produce Problem

Let $G = [N, L]$ denote the graph with set of nodes N and links L in Figure 5.1. Each farm i; $i = 1, \ldots, I$, delivers fresh produce to the markets through its harvesting, processing, storage, and transportation facilities, and seeks to determine its optimal strategies in terms of how much of the produce to bring to each market in order to maximize profit. The link set L consists of the sets of links $L_1 \cup L_2 \ldots \cup L_I$, where L_i is the sequence of directed links of farm i. Moreover, the links on the network correspond to the economic activities associated with the farms.

The first set of directed links illustrates the harvesting/picking activity of each farmer i; $i = 1, \ldots, I$. The second set of directed links illustrates the processing activities such as cleaning, sorting and labeling, and also packaging/packing of the fresh produce. The last set of directed links

corresponds to the storage and transport of the fresh produce. If the demand point is a farmers' market that is open on the first day of the week, the links only capture the transportation cost. Otherwise, there is a storage link followed by a transport link each subsequent demand point, that is, farmers' market, as illustrated in Figure 5.1.

Recall that P_j^i denotes the set of paths joining farm node i with farmer market node j. In addition, P^i denotes the set of paths emanating from farm node i to all farmer market nodes j; $j = 1, \ldots, M$. P then denotes the set of all paths in the supply chain network in Figure 5.1. There are n_P paths in the network and n_L links.

5.2.1 The Uncapacitated Model

The flow on each path, joining the farmer node i to the farmers' market node j, is denoted by x_p, and all path flows must be nonnegative, that is,

$$x_p \geq 0, \quad \forall p \in P_j^i; \quad i = 1, \ldots, I; j = 1, \ldots, M. \tag{5.9}$$

Furthermore, the flow on a link a is equal to the sum of the path flows x_p, on paths that include the link a. This conservation of flow equation is expressed as:

$$f_a = \sum_{p \in P} x_p \delta_{ap}, \quad \forall a \in L, \tag{5.10}$$

where δ_{ap} is equal to 1 if the link a is included in the path p, and 0, otherwise. In the local food supply chain network, there is usually a single path between farm i and the market j; however, the path set P_j^i is introduced to present a more general definition.

Furthermore, the demand at the farmers' market j for the fresh produce product of farmer i is given by:

$$\sum_{p \in P_j^i} x_p = d_{ij}, \quad p \in P_j^i; i = 1, \ldots, I; j = 1, \ldots, M. \tag{5.11}$$

The demands between all farms and farmer market pairs (i, j) are grouped into the vector d and also the quality of their fresh produce on paths $p \in P_j^i$, $\forall i, j$, $\{q_p\}$, into the vector q. All vectors are assumed to be column vectors.

The price of the product of farm i, in turn, at farmers' market j, is denoted by ρ_{ij}, and depends not only on the demand for the farm's fresh produce, but also on the quality of that product, which

is captured by the quality decay kinetics in (5.11). Also, the price depends, in general, on the quantities of the competitors' fresh produce at the markets as well as the quality of their products.

Hence, the demand price function ρ_{ij} for farm i's product at the farmers' market j, is:

$$\rho_{ij} = \rho_{ij}(d, q), \quad i = 1, \ldots, I; \, j = 1, \ldots, M. \tag{5.12}$$

In view of (5.11) and (5.12), the demand price functions can be rewritten as:

$$\hat{\rho}_{ij} = \hat{\rho}_{ij}(x, q) \equiv \rho_{ij}(d, q), \quad i = 1, \ldots, I; \, j = 1, \ldots, M. \tag{5.13}$$

The demand price functions are assumed to be continuous, continuously differentiable, and monotone decreasing. Recall that the quality parameter vector q is defined with respect to the decay function of the fresh produce which farms sell at the farmers' markets and is time-dependent. Thus, the demand price for a farm's product is higher when the quality of its fresh produce product is high and, if the demand is high, the price is lower. Note that each farm is aware of the temperature conditions and the time associated with the links in its supply chain network.

The competition among the farms for resources is reflected in the total operational costs incurred in harvesting, processing, storage, and transportation. The quality-keeping cost is related to the food deterioration and is also considered through the transportation and the storage link costs. In general, the total operational cost of each link a, denoted by $\hat{c}_a$, depends on the flows on all the links in the fresh produce supply chain network, that is,

$$\hat{c}_a = \hat{c}_a(f), \quad \forall a \in L, \tag{5.14}$$

where f is the vector of all the link flows. The total cost on each link is assumed to be convex and continuously differentiable.

Let X_i denote the vector of path flows associated with farm i; $i = 1, \ldots, I$, where $X_i \equiv \{\{x_p\}|p \in P^i\}\} \in R_+^{n_{P^i}}$, $P^i \equiv \cup_{j=1,\ldots,M} P_j^i$, and n_{P^i} denotes the number of paths from farm i to the farmers' markets. Thus, X is the vector of all the farmers' strategies, that is, $X \equiv \{\{X_i\}|i = 1, \ldots, I\}$.

The profit function of farm i is defined as the difference between its revenue and its total costs, where the total costs are the total operational costs over L_i. The profit function is novel in terms of capturing the quality of the fresh produce in the demand price functions, which is time-dependent.

The profit/utility function of farm i, denoted by U_i, is given by:

$$U_i = \sum_{j=1}^{M} \rho_{ij}(d, q) d_{ij} - \sum_{a \in L^i} \hat{c}_a(f). \tag{5.15}$$

In lieu of the conservation of flow expressions (5.10) and (5.11), and the functional expressions (5.13) and (5.14), $\hat{U}_i(X) \equiv U_i$ is defined for each farm i; $i = 1, \ldots, I$, with the I-dimensional vector $\hat{U}$ being the vector of the profits of all the farms with respect to their earnings at the farmers' markets over a typical week:

$$\hat{U} = \hat{U}(X). \tag{5.16}$$

Next, the fresh produce supply chain network Cournot-Nash Equilibrium conditions for farmers' markets in a region are given.

- **Definition 5.1: Fresh Produce Supply Chain Network Cournot-Nash Equilibrium for Farmers' Markets in the Uncapacitated Case**

  A path flow pattern $X^* \in K = \prod_{i=1}^{I} K_i$ constitutes a fresh produce supply chain network Cournot-Nash equilibrium if for each farm i; $i = 1, \ldots, I$:

$$\hat{U}_i(X_i^*, \hat{X}_i^*) \geq \hat{U}_i(X_i, \hat{X}_i^*), \quad \forall X_i \in K_i, \tag{5.17}$$

  where $\hat{X}_i^* \equiv (X_1^*, \ldots, X_{i-1}^*, X_{i+1}^*, \ldots, X_I^*)$ and $K_i \equiv \{X_i | X_i \in R_+^{n_{Pi}}\}$. A Cournot-Nash Equilibrium is established if no farm can unilaterally improve its profit by changing its product flows throughout its supply chain network, given the product flow decisions of the other farms.

  Next, the variational inequality formulations of the Cournot-Nash equilibrium for the fresh produce supply chain network under oligopolistic competition satisfying Definition 5.1, in terms of both path flows and link flows (see, e.g., Cournot (1838), Nash (1950, 1951), Gabay and Moulin (1980), Nagurney (2006), and Nagurney et al. (2013)) are derived.

- **Theorem 5.1: Variational Inequality Formulations of the Uncapacitated Model**

  Assume that for each fresh produce farm i; $i = 1, \ldots, I$, the profit function $\hat{U}_i(X)$ is concave with respect to the variables in X_i and is continuously differentiable. Then $X^* \in K$ is a fresh produce supply chain network Cournot-Nash equilibrium for farmers' markets according to Definition 5.1 if and only if it satisfies the variational inequality:

  $$-\sum_{i=1}^{I} \langle \nabla_{X_i} \hat{U}_i(X^*), X_i - X_i^* \rangle \geq 0, \quad \forall X \in K, \tag{5.18}$$

  where $\langle \cdot, \cdot \rangle$ denotes the inner product in the corresponding Euclidean space and $\nabla_{X_i} \hat{U}_i(X)$ denotes the gradient of $\hat{U}_i(X)$ with respect to X_i. Variational inequality (5.18), in turn, for the uncapacitated model, is equivalent to the variational inequality that determines the vector of equilibrium path flows $x^* \in K^1$ such that:

  $$\sum_{i=1}^{I} \sum_{j=1}^{M} \sum_{p \in P_j^i} \left[\frac{\partial \hat{C}_p(x^*)}{\partial x_p} - \hat{\rho}_{ij}(x^*, q) - \sum_{l=1}^{M} \frac{\partial \hat{\rho}_{il}(x^*, q)}{\partial x_p} \sum_{r \in P_l^i} x_r^* \right] \times [x_p - x_p^*] \geq 0, \quad \forall x \in K^1, \tag{5.19}$$

  where $K^1 \equiv \{x | x \in R_+^{n_P}\}$, and for each path p; $p \in P_j^i$; $i = 1, \ldots, I$; $j = 1, \ldots, M$,

  $$\frac{\partial \hat{C}_p(x)}{\partial x_p} \equiv \sum_{a \in L^i} \sum_{b \in L^i} \frac{\partial \hat{c}_b(f)}{\partial f_a} \delta_{ap} \quad \text{and} \quad \frac{\partial \hat{\rho}_{il}(x, q)}{\partial x_p} \equiv \frac{\partial \rho_{il}(d, q)}{\partial d_{ij}}. \tag{5.20}$$

  Variational inequality (5.19) can also be rewritten in terms of link flows as: determine the vector of equilibrium link flows and the vector of equilibrium demands $(f^*, d^*) \in K^2$, such that:

  $$\sum_{i=1}^{I} \sum_{a \in L^i} \left[\sum_{b \in L^i} \frac{\partial \hat{c}_b(f^*)}{\partial f_a} \right] \times [f_a - f_a^*]$$

  $$+ \sum_{i=1}^{I} \sum_{j=1}^{M} \left[-\rho_{ij}(d^*, q) - \sum_{l=1}^{M} \frac{\partial \rho_{il}(d^*, q)}{\partial d_{ik}} d_{il}^* \right] \times [d_{ij} - d_{ij}^*] \geq 0, \quad \forall (f, d) \in K^2, \tag{5.21}$$

  where $K^2 \equiv \{(f, d) | x \geq 0, \text{ and } (5.10) \text{ and } (5.11) \text{ hold}\}$.

Proof: (5.18) follows the discussion in Chapter 2. $\square$

Variational inequalities (5.19) and (5.21) can be put into standard form (see 2.1a): determine $X^* \in \mathcal{K}$ such that:

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \mathcal{K}, \tag{5.22}$$

where $\langle \cdot, \cdot \rangle$ denotes the inner product in N-dimensional Euclidean space. Let $X \equiv x$ and

$$F(X) \equiv \left[\frac{\partial \hat{C}_p(x)}{\partial x_p} - \hat{\rho}_{ij}(x, q) - \sum_{l=1}^{M} \frac{\partial \hat{\rho}_{il}(x, q)}{\partial x_p} \sum_{r \in P_l^i} x_r; \right.$$

$$\left. p \in P_j^i; \ i = 1, \ldots, I; \ j = 1, \ldots, M \right], \tag{5.23}$$

$\mathcal{K} \equiv K^1$, and $N = n_P$, then (5.19) can be re-expressed as (5.22). Similarly, for the variational inequality in terms of link flows, if the column vectors are defined: $X \equiv (f, d)$ and $F(X) \equiv (F_1(X), F_2(X))$, where

$$F_1(X) = \left[\sum_{b \in L^i} \frac{\partial \hat{c}_b(f)}{\partial f_a}; \ a \in L^i; \ i = 1, \ldots, I \right],$$

$$F_2(X) = \left[-\hat{\rho}_{ij}(x, q) - \sum_{l=1}^{M} \frac{\partial \hat{\rho}_{il}(x, q)}{\partial x_p} \sum_{r \in P_l^i} x_r; \ p \in P_j^i; \ i = 1, \ldots, I; \ j = 1, \ldots, M \right], \tag{5.24}$$

$\mathcal{K} \equiv K^2$, and $N = n_L + M$, then (5.21) can be re-written as (5.22).

Since the feasible set K^1 is not compact, and the same holds for K^2, the existence of a solution cannot be obtained simply based on the assumption of the continuity of F. However, the demand d_{ij} for each farm i's product; $i = 1, \ldots, I$, at every farmers' market j; $j = 1, \ldots, M$, may be assumed to be bounded, since the population requiring these products is finite (although it may be large). Consequently, in light of (5.12), the following is valid:

$$\mathcal{K}_c \equiv \{x \mid 0 \leq x \leq c, \}, \tag{5.25}$$

where $c > 0$ and $x \leq c$ means that $x_p \leq c$ for all $p \in P_j^i$; $i = 1, \ldots, I$, and $j = 1, \ldots, M$. Then $\mathcal{K}_c$ is a bounded, closed, and convex subset of K^1. Thus, the following variational inequality

$$\langle F(X^c), X - X^c \rangle \geq 0, \quad \forall X \in \mathcal{K}_c, \tag{5.26}$$

admits at least one solution $X^c \in \mathcal{K}_c$, since $\mathcal{K}_c$ is compact and F is continuous. Therefore, the following theorem for existence in Chapter 2 is immediate:

- **Theorem 5.2: Existence**

  There exists at least one solution to variational inequality (5.19) (equivalently, to (5.21)), since there exists a $c > 0$, such that variational inequality (5.29) admits a solution in $\mathcal{K}_c$ with

$$x^c \leq c. \tag{5.27}$$

In addition, I now provide a uniqueness result.

- **Theorem 5.3: Uniqueness**

  With Theorem 5.2, variational inequality (5.26) and, hence, variational inequalities (5.19) and (5.21) admit at least one solution. Moreover, if the function $F(X)$ of variational inequality (5.21), as defined in (5.22), is strictly monotone on $\mathcal{K} \equiv K^2$, that is,

$$\langle (F(X^1) - F(X^2)), X^1 - X^2 \rangle > 0, \quad \forall X^1, X^2 \in \mathcal{K}, \ X^1 \neq X^2, \tag{5.28}$$

  then the solution to variational inequality (5.21) is unique, that is, the equilibrium link flow pattern and the equilibrium demand pattern are unique.

5.2.2 The Capacitated Model

In this subsection, I provide the capacitated version of the model in Section 5.2.1.

Specifically, I retain the objective function (5.15), for each farm i; $i = 1, \ldots, I$, the nonnegativity constraints (5.9), with conservation of flow equations (5.11), as well as the previous notation, but now it becomes

$$f_a \leq u_a, \quad \forall a \in L, \tag{5.29a}$$

or, in view of (5.10):

$$\sum_{p \in P} x_p \delta_{ap} \leq u_a, \quad \forall a \in L, \tag{5.29b}$$

where u_a is the positive upper bound on the flow on link a.

The feasible set K_i^3 faced by farm i in the capacitated case is defined as: $K_i^3 \equiv \{X_i | X_i \in R_+^{n_{P^i}}$ and (5.29b) holds for $a \in L^i\}$. Also, $K^3 \equiv \prod_{i=1}^{I} K_i^3$ is given.

Hence, the following definition is immediate.

- **Definition 5.2: Fresh Produce Supply Chain Network Cournot-Nash Equilibrium for Farmers' Markets in the Capacitated Case**

  A path flow pattern $X^* \in K^3$ constitutes a fresh produce supply chain network Cournot-Nash equilibrium in the capacitated case if for each farm i; $i = 1, \ldots, I$:

  $$\hat{U}_i(X_i^*, \hat{X}_i^*) \geq \hat{U}_i(X_i, \hat{X}_i^*), \quad \forall X_i \in K_i^3, \tag{5.30}$$

  where, as before, $\hat{X}_i^* \equiv (X_1^*, \ldots, X_{i-1}^*, X_{i+1}^*, \ldots, X_I^*)$

- **Theorem 5.4: Variational Inequality Formulations of the Capacitated Model**

  Assume that for each fresh produce farm i; $i = 1, \ldots, I$, the profit function $\hat{U}_i(X)$ is concave with respect to the variables in X_i, and is continuously differentiable. Then $X^* \in K^3$ is a fresh produce supply chain network Cournot-Nash equilibrium for farmers' markets in the capacitated case according to Definition 2 if and only if it satisfies the variational inequality:

  $$-\sum_{i=1}^{I} \langle \nabla_{X_i} \hat{U}_i(X^*), X_i - X_i^* \rangle \geq 0, \quad \forall X \in K^3. \tag{5.31}$$

  Variational inequality (5.31), in turn, for the capacitated model, is equivalent to the variational inequality that determines the vector of equilibrium path flows $x^* \in K^3$ such that:

  $$\sum_{i=1}^{I} \sum_{j=1}^{M} \sum_{p \in P_j^i} \left[\frac{\partial \hat{C}_p(x^*)}{\partial x_p} - \hat{\rho}_{ij}(x^*, q) - \sum_{l=1}^{M} \frac{\partial \hat{\rho}_{il}(x^*, q)}{\partial x_p} \sum_{r \in P_l^i} x_r^* \right] \times [x_p - x_p^*] \geq 0, \quad \forall x \in K^3. \tag{5.32}$$

  Moreover, variational inequality (5.32) is equivalent to the variational inequality problem: determine $(x^*, \lambda^*) \in K^4$, where $K^4 \equiv \{x \in R_+^{n_P}, \lambda \in R_+^{n_L}\}$, such that:

  $$\sum_{i=1}^{I} \sum_{j=1}^{M} \sum_{p \in P_j^i} \left[\frac{\partial \hat{C}_p(x^*)}{\partial x_p} - \hat{\rho}_{ij}(x^*, q) - \sum_{l=1}^{M} \frac{\partial \hat{\rho}_{il}(x^*, q)}{\partial x_p} \sum_{r \in P_l^i} x_r^* + \sum_{a \in L} \lambda_a^* \delta_{ap} \right] \times [x_p - x_p^*]$$

  $$+ \sum_{a \in L} \left[u_a - \sum_{p \in P} x_p^* \delta_{ap} \right] \times [\lambda_a - \lambda_a^*] \geq 0, \quad \forall (x, \lambda) \in K^4, \tag{5.33}$$

  where $\frac{\partial \hat{C}_p(x)}{\partial x_p}$ and $\frac{\partial \hat{\rho}_{il}(x, q)}{\partial x_p}$ are as defined in (5.20).

Proof: Variational inequality (5.31) again follows directly from Gabay and Moulin (1980). Variational inequality (5.33) follows, in turn, from Bertsekas and Tsitsiklis (1989) (see also Nagurney (2010)) with notice that λ^* corresponds to the vector of optimal Lagrange multipliers associated with constraints (5.29b) and λ is the vector of Lagrange multipliers associated with the upper bound constraints on all links $a \in L$.

Variational inequality (5.33) will be used for solution of the capacitated model and note that variational inequality (5.33) can be put into the standard form (5.22) (see Nagurney (1999) and Chapter 2), if the vectors: $X \equiv (x, \lambda)$ and $F(X) \equiv (F_3(X), F_4(X))$, where

$$F_3(X) = \left[\frac{\partial \hat{C}_p(x)}{\partial x_p} - \hat{\rho}_{ij}(x, q) - \sum_{l=1}^{M} \frac{\partial \hat{\rho}_{il}(x, q)}{\partial x_p} \sum_{r \in P_l^i} x_r + \sum_{a \in L} \lambda_a \delta_{ap}, \ p \in P_j^i; \ i = 1, \ldots, I; \ j = 1, \ldots, M \right],$$
(5.34)

$$F_4(X) = \left[u_a - \sum_{p \in P} x_p \delta_{ap}, \ a \in L \right].$$
(5.35)

With $\mathcal{K} \equiv K^4$, (5.33) can be rewritten as (5.22).

The existence result below follows from the classical theory of variational inequalities (see Chapter 2).

- **Theorem 5.5: Existence of a Solution to the Capacitated Model**

  Existence of a solution X^* to variational inequality (5.31) is guaranteed since the feasible set K^3 is compact.

5.3. The Algorithm

I now present the explicit formulae for recall the Euler method to compute the solution of the capacitated and uncapacitated models in Section 5.2. The details on the Euler method are provided in Chapter 2.

5.3.1 Explicit Formulae for the Euler Method Applied to the Uncapacitated Model

The elegance of this procedure for the computation of solutions to the uncapacitated fresh produce farmers' market supply chain network problem in Section 5.2.1 can be seen in the following explicit

formulae. In particular, the following closed form expressions for the fresh produce path flows are written, for each path $p \in P_j^i$, $\forall i, j$:

$$x_p^{\tau+1} = \max\{0, x_p^\tau + a_\tau(\hat{\rho}_{ij}(x^\tau, q) + \sum_{l=1}^M \frac{\partial \hat{\rho}_{il}(x^\tau, q)}{\partial x_p} \sum_{r \in P_l^i} x_r^\tau - \frac{\partial \hat{C}_p(x^\tau)}{\partial x_p})\},$$

$$\forall p \in P_j^i; \ i = 1, \ldots, I; \ j = 1, \ldots, M. \tag{5.36}$$

Next, the closed form expression to compute the solution of the capacitated problem in Section 5.2.2 is given.

5.3.2 Explicit Formulae for the Euler Method Applied to the Capacitated Model

The explicit formulae are now given to compute the solutions to the capacitated fresh produce farmers' market supply chain problem in Section 5.2.2. The closed form expressions for the fresh produce path flows at iteration $\tau + 1$ are as follows. For each path $p \in P_j^i$, $\forall i, j$, compute:

$$x_p^{\tau+1} = \max\{0, x_p^\tau + a_\tau(\hat{\rho}_{ij}(x^\tau, q) + \sum_{l=1}^M \frac{\partial \hat{\rho}_{il}(x^\tau, q)}{\partial x_p} \sum_{r \in P_l^i} x_r^\tau - \frac{\partial \hat{C}_p(x^\tau)}{\partial x_p} - \sum_{a \in L} \lambda_a^\tau \delta_{ap})\},$$

$$\forall p \in P_j^i; \ i = 1, \ldots, I; \ j = 1, \ldots, M. \tag{5.37}$$

The Lagrange multipliers for each link $a \in L^i; i = 1, \ldots, I$, can be computed as:

$$\lambda_a^{\tau+1} = \max\{0, \lambda_a^\tau + a_\tau(\sum_{p \in P} x_p^\tau \delta_{ap} - u_a)\}, \ \forall a \in L. \tag{5.38}$$

The number of strategic variables x_p, as well as the number of the paths, in the supply chain network, for both of the uncapacitated and capacitated supply chain networks, grow linearly in terms of the number of nodes in the supply chain network, be it a farm, or a farmers' market, etc. Therefore, even a fresh produce supply chain network with hundreds of demand markets is still tractable within the proposed modeling and computational framework.

5.4. Case Study

I focus on apples for the case study. The United States holds second place in the world's apple production with 4 million metric tons of apples produced, with the leader being China, which produced 33 million metric tons of apples in 2010 (USDA (2012)). Specifically, in the case study

I consider Golden Delicious apples. This type of apple is ranked in third place among the other apple varieties in the United States with 25,000 metric tons of production value. The case study is based on farmers' markets in western Massachusetts in the United States. According to the USDA (2016) and the website of Community Involved in Sustaining Agriculture (CISA), there are 8,558 farmers markets in the United States, 312 in Massachusetts, and 40 in western Massachusetts (CISA (2016)). Also, in Massachusetts, 369 in-state apple orchards produced 34 million pounds of apples in 2010. There are approximately 27 apple orchards in western Massachusetts. For this case study, I selected three of the apple orchards located in western Massachusetts: Apex Orchards, the Park Hill Orchard, and Sentinel Farm. The locations of these orchards/farms in Massachusetts are, respectively, Shelburne, Easthampton, and Belchertown. These orchards/farms have the possibility of selling their apples at the Amherst, Northampton, South Hadley, and/or Belchertown farmers' markets, which are open on different days in a given week during the season, as shown in Figure 5.2. The Northampton Farmers' Market is open on Tuesdays, the South Hadley Farmers' Market operates on Thursdays. On Saturdays the Amherst Farmers' Market is open, and the Belchertown Farmers' Market is open on Sundays. Each orchard/farm has its own harvesting, processing, storage, and transportation units. Notice that the demand points in Figure 5.2 are sequenced depending on the day of the week. For example, since the Northampton Farmers' Market is open on Tuesdays, which is the first farmers' market of the week in this case study, it is numbered as the first demand point. The South Hadley Farmers' Market corresponds to demand point 2, and so on. Additionally, apples are picked on Tuesdays at all the orchards/farms, so that there is no storage for the Golden Delicious apples sold at the Northampton Farmers' Market. However, each farm should store its Golden Delicious apples for 2, 4, and 5 days for the South Hadley, Amherst, and the Belchertown Farmers' Markets, respectively.

The Euler method for the uncapacitated (cf. (5.36)) and the capacitated problems (cf. (5.38) and (5.39)) is implemented in MATLAB and the sequence $a_\tau = \{1, \frac{1}{2}, \frac{1}{2}, \frac{1}{3}, \ldots\}$, with the convergence tolerance being 10^{-6}, that is, the Euler method is deemed to have converged if the absolute value of the difference of each successive variable iterate differs by no more than this value. The code in Matlab is executed on a Macbook Pro laptop with a 2.8 GHz Intel Core i5 processor and 8GB 1600 MHz DDR3 memory.
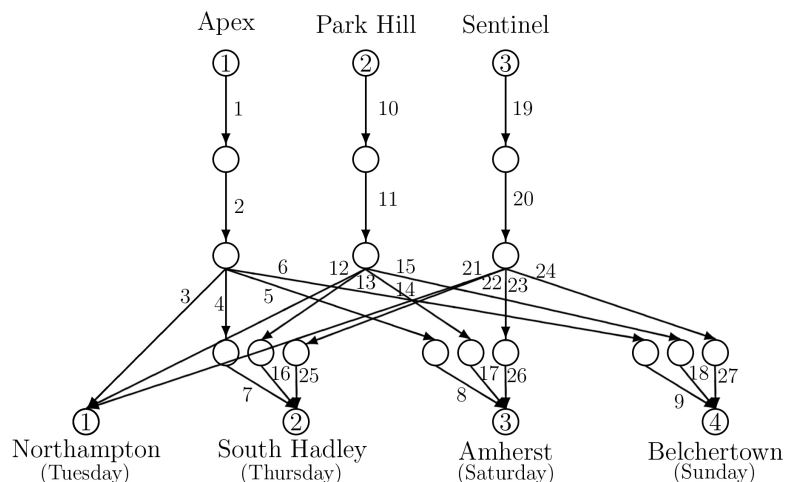
Figure 5.2. The Supply Chain Network Topology for the Apple Farmers' Markets Case Study – Scenario 1

5.4.1 Scenario 1

In Scenario 1, it is assumed that the weather conditions in Massachusetts are steady, which means that the temperature in the growing season of Golden Delicious apples is at its seasonal normal and the harvesting season is for 2015. For this scenario, I assume that orchard/farm i; $i = 1, 2, 3$, in the supply chain network has initial quality, respectively, of: $q_{01} = 1$, $q_{02} = 0.8$, and $q_{03} = 0.7$. Apex Orchards are well-known for the quality of their apples in western Massachusetts and, hence, the value for their apple initial quality is $q_{01} = 1$. In addition, it is assumed that there is no capacity limit on the links in the supply chain network. According to Table 5.1, the quality attribute, that of the texture softening, follows first order quality decay for Golden Delicious apples. The link quality decay β_a for every link a in the supply chain network and the parameters used in the calculations are provided in Table 5.2. Also, I include the quality decay at the farmers' markets by considering the selling/purchasing point as being the middle of the operation period, since quality decay is a continuous process.

The orchards/farms in this case study are of different sizes and are located at different altitudes. According to CISA, Apex, the Park Hill Orchards, and the Sentinel Farm have 170, 127, and 8 acres of land, respectively. Also, Apex Orchards are located at a higher altitude than Park Hill and Sentinel Farm, but the altitude of Park Hill and the Sentinel are similar. The harvesting season of apples is between September and October when temperatures are between 19-22 °C. In this case study, the

124

Operations	Link a	Hours	Temp (°C)	β_a
harvesting	1	4.00	19	0.992
processing	2	3.00	19	0.994
transportation	3	0.50	19	0.999
storage (2 days)	4	48.00	0	0.994
storage (4 days)	5	96.00	0	0.988
storage (5 days)	6	120.00	0	0.985
transportation	7	4.00	19	0.993
transportation	8	3.25	19	0.994
transportation	9	4.00	19	0.993
harvesting	10	3.00	22	0.992
processing	11	3.00	22	0.992
transportation	12	0.5	19	0.999
storage (2 days)	13	48.00	9	0.978
storage (4 days)	14	96.00	9	0.957
storage (5 days)	15	120.00	9	0.947
transportation	16	3.75	19	0.993
transportation	17	5.16	19	0.990
transportation	18	3.00	19	0.992
harvesting	19	5.00	22	0.986
processing	20	5.00	22	0.986
transportation	21	0.50	22	0.998
storage (2 days)	22	48.00	12	0.967
storage (4 days)	23	96.00	12	0.936
storage (5 days)	24	120.00	12	0.921
transportation	25	3.75	22	0.990
transportation	26	5.16	22	0.986
transportation	27	3.00	22	0.992

Table 5.2. Parameters for the Calculation of Quality Decay for the Case Study Scenario 1

harvesting operation is assumed to be realized at the average temperature of the season depending on the location of the orchards. Notice that Apex Orchards have the lowest harvesting temperature, since they are located at a higher altitude where the temperature is lower. Furthermore, the duration of the link operations are constructed according to the land size and the number of employees. For example, Apex has a larger land size; therefore, the harvesting operation at this orchard takes a longer time. Moreover, I assume that the larger the size of the orchard, the more employees it has. As a result, the duration of harvesting and processing operations are the longest for Sentinel Farm due to it having the lowest number of employees. The duration of transportation is calculated as the summation of the actual transportation time between the orchard and the farmers' market and half of the farmers' market's hours of operation, which the transportation link is connected to in the supply chain network. The hours of operation of the Northampton, South Hadley, Amherst, and Belchertown Farmers' Markets are, respectively: 5, 6, 5.5, and 4 hours. The hours of storage are calculated according to the days between the harvesting and when the farmers' markets open in

the week. For instance, Apple Orchards need to store their apples for 2 days to sell at the South Hadley Farmers' Market, which is open on Thursdays. It is assumed that Apex Orchards have the controlled atmospheric storage system to keep the temperature at $0°C$ which is the optimal storage temperature for apples (Iowa State University Extension (2008)). However, it is assumed that Park Hill and Sentinel Farm have regular storage which can lower the temperatures to $9°C$ and $12°C$, respectively. The link quality decay β_a is calculated by taking the universal gas constant and the activation energy as $8.314\ Jmol^{-1}K^{-1}$ and $88\ kJmol^{-1}$. The temperature and the time are converted, respectively, to Kelvin and seconds for the quality decay calculations.

The total number of paths in the supply chain network is twelve and they are as follows: path $p_1 = (1, 2, 3)$, $p_2 = (1, 2, 4, 7)$, $p_3 = (1, 2, 5, 8)$, $p_4 = (1, 2, 6, 9)$, $p_5 = (10, 11, 12)$, $p_6 = (10, 11, 13, 16)$, $p_7 = (10, 11, 14, 17)$, $p_8 = (10, 11, 15, 18)$, $p_9 = (19, 20, 21)$, $p_{10} = (19, 20, 22, 25)$, $p_{11} = (19, 20, 23, 26)$, and $p_{11} = (19, 20, 24, 27)$. Also each path has its own quality decay rate, which is calculated according to (5.11). Furthermore, the demand price functions of the orchard/farms at the farmers' markets are:

Apex Orchards:

$$\rho_{11}(d, q) = -0.04d_{11} - 0.01d_{21} - 0.01d_{31} + 8q_{p_1} - 4q_{p_5} - 3q_{p_9} + 30,$$

$$\rho_{12}(d, q) = -0.02d_{12} - 0.01d_{22} - 0.01d_{32} + 3q_{p_2} - 2q_{p_6} - 2q_{p_{10}} + 25,$$

$$\rho_{13}(d, q) = -0.04d_{13} - 0.02d_{23} - 0.01d_{33} + 8q_{p_3} - 4q_{p_7} - 3q_{p_{11}} + 30,$$

$$\rho_{14}(d, q) = -0.04d_{14} - 0.02d_{24} - 0.02d_{34} + 3q_{p_4} - q_{p_8} - 2q_{p_{12}} + 25,$$

Park Hill Orchard:

$$\rho_{21}(d, q) = -0.04d_{21} - 0.02d_{11} - 0.02d_{31} + 3q_{p_5} - 2q_{p_1} - q_{p_9} + 27,$$

$$\rho_{22}(d, q) = -0.04d_{22} - 0.01d_{12} - 0.02d_{32} + 3q_{p_6} - 2q_{p_2} - q_{p_{10}} + 28,$$

$$\rho_{23}(d, q) = -0.04d_{23} - 0.02d_{13} - 0.02d_{33} + 4q_{p_7} - 2q_{p_3} - q_{p_{11}} + 27,$$

$$\rho_{24}(d, q) = -0.02d_{24} - 0.01d_{14} - 0.01d_{34} + 2q_{p_8} - q_{p_4} - q_{p_{12}} + 28,$$

Sentinel Farm:

$$\rho_{31}(d, q) = -0.04d_{31} - 0.02d_{11} - 0.02d_{21} + 4q_{p_9} - q_{p_1} - 2q_{p_5} + 25,$$

126

$$\rho_{32}(d, q) = -0.04d_{32} - 0.01d_{12} - 0.02d_{22} + 4q_{p_{10}} - 3q_{p_2} - q_{p_6} + 28,$$

$$\rho_{33}(d, q) = -0.02d_{23} - 0.01d_{13} - 0.01d_{33} + 4q_{p_{11}} - 2q_{p_3} - q_{p_7} + 25,$$

$$\rho_{34}(d, q) = -0.04d_{34} - 0.02d_{14} - 0.02d_{24} + 3q_{p_{12}} - 2q_{p_4} - 2q_{p_8} + 28.$$

The demand price functions are constructed according to the customer and orchard characteristics. According to a former orchard owner, Colnes (2016), customers going to the Amherst Farmers' Market are more affluent and give importance to quality of the apples. The Northampton Farmers' Market is also similar to the Amherst Farmers' Market in terms of the consumers' willingness to pay for higher quality. The demand price functions at the South Hadley and Belchertown Farmers' Markets are similar to one another, with the lowest consumer willingness to pay in the supply chain network. Furthermore, some orchards are assumed to have a more positive reputation at some farmers' markets, which means that the price of the Golden Delicious apples from an orchard may be less affected by the demand for the apples of the other orchards at the same farmers' market. For example, Apex is assumed to have loyal customers at the Amherst Farmers' Market whereas the Sentinel does not; therefore, the price of Apex's Golden Delicious apples is less affected by the demand for the Sentinel's apples.

The total link cost functions and the computed equilibrium link flows are shown in Table 5.3. The cost functions are constructed based on the price data in Berkett (1994), King and Gomez (2015), and the U.S. Energy Information Administration (2016) for the fuel price. The flow unit is pecks and the total cost functions are constructed based on the dollar price per peck.

The computed equilibrium path flows and the quality decay of the paths are given in Table 5.4. As mentioned earlier, the unit of the flows is pecks of apples. Notice that, in equilibrium, there are paths and links with zero flows, which indicates the nonprofitable farmers' markets for specific orchards. For example, Apex serves the Northampton and Amherst Farmers' Markets; Park Hill serves the Northampton and South Hadley Farmers' Markets, and Sentinel serves the Northampton, South Hadley, and Belchertown Farmers' Markets. The game theory aspect of the model reveals the profitable markets and the variational inequality solution returns positive path flows for these markets and zero for the nonprofitable ones.

By substituting the equilibrium path flow and quality decay values, the demand prices of the orchard/farms at the profitable demand markets are found, in terms of dollars per peck, as:

Operations	Link a	$\hat{c}_a(f)$	f_a^*
harvesting	1	$0.02f_1^2 + 3f_1$	165.8395
processing	2	$0.015f_2^2 + 3f_2$	165.8395
transportation	3	$0.01f_3^2 + 3f_3$	111.9827
storage (2 days)	4	$0.01f_4^2 + 3f_4$	0.0000
storage (4 days)	5	$0.015f_5^2 + 4f_5$	53.8568
storage (5 days)	6	$0.03f_6^2 + 5f_6$	0.0000
transportation	7	$0.02f_7^2 + 6f_7$	0.0000
transportation	8	$0.0125f_8^2 + 4f_8$	53.8568
transportation	9	$0.02f_9^2 + 6.6f_9$	0.0000
harvesting	10	$0.0125f_{10}^2 + 6f_{10}$	94.7414
processing	11	$0.0125f_{11}^2 + 6f_{11}$	94.7414
transportation	12	$0.0045f_{12}^2 + f_{12}$	71.7812
storage (2 days)	13	$0.01f_{13}^2 + 1.67f_{13}$	22.9601
storage (4 days)	14	$0.015f_{14}^2 + 6f_{14}$	0.0000
storage (5 days)	15	$0.015f_{15}^2 + 6.6f_{15}$	0.0000
transportation	16	$0.0075f_{16}^2 + 6f_{16}$	22.9601
transportation	17	$0.01f_{17}^2 + 6f_{17}$	0.0000
transportation	18	$0.02f_{18}^2 + 4f_{18}$	0.0000
harvesting	19	$0.0125f_{19}^2 + 6f_{19}$	98.5294
processing	20	$0.015f_{20}^2 + 4f_{20}$	98.5294
transportation	21	$0.02f_{21}^2 + 4f_{21}$	17.2084
storage (2 days)	22	$0.007f_{22}^2 + 1.67f_{22}$	32.4314
storage (4 days)	23	$0.009f_{23}^2 + 6f_{23}$	0.0000
storage (5 days)	24	$0.01f_{24}^2 + 6f_{24}$	48.8896
transportation	25	$0.005f_{25}^2 + 6f_{25}$	32.4314
transportation	26	$0.005f_{26}^2 + 6f_{26}$	0.0000
transportation	27	$0.0005f_{27}^2 + 0.1f_{27}$	48.8896

Table 5.3. The Total Link Cost Functions and the Computed Equilibrium Link Flows for the Case Study Scenario 1

Path p	q_p	x_p^*
p_1	0.9851	111.9827
p_2	0.9733	0.0000
p_3	0.9684	53.8568
p_4	0.9645	0.0000
p_5	0.7864	71.7812
p_6	0.7645	22.9602
p_7	0.7458	0.0000
p_8	0.7395	0.0000
p_9	0.6791	17.2084
p_{10}	0.6514	32.4314
p_{11}	0.6280	0.0000
p_{12}	0.6217	48.8896

Table 5.4. The Path Quality Decay Rates and the Computed Equilibrium Path Flows for the Case Study Scenario 1

Apex Orchards:

$$\rho_{11} = 27.33, \quad \rho_{12} = 24.53, \quad \rho_{13} = 30.72, \quad \rho_{14} = 25.42,$$

Park Hill Orchard:

$$\rho_{21} = 21.25, \quad \rho_{22} = 26.13, \quad \rho_{23} = 26.34, \quad \rho_{24} = 27.40,$$

Sentinel Farm:

$$\rho_{31} = 20.79, \quad \rho_{32} = 25.16, \quad \rho_{33} = 24.29, \quad \rho_{34} = 24.50.$$

According to Clements (2016), who is an educator at the University of Massachusetts Amherst Extension Fruit Program, the retail price of Golden Delicious apples is usually \$2 per pound. A peck of apples is equal to 10-12 pounds which means the price of a peck can be between \$20-\$24. The results for the demand prices are close to this range and, hence, are consistent with reality. Furthermore, the profits of the orchard/farms, in dollars, at the equilibrium solution, are:

$$U_1(X^*) = 1785.40, \quad U_2(X^*) = 484.03, \quad U_3(X^*) = 460.15.$$

Apex Orchards have the largest profit in Scenario 1, followed by Park Hill Orchard and then the Sentinel Farm.

5.4.2   Scenario 2

In this scenario, it is assumed that a new orchard, which was solely selling to retailers and wholesalers previously, is attracted by the demand for apples at the farmers' markets. This hypothetical new orchard is called New Orchard, and is located in western Massachusetts and enters the local food supply chain as depicted in Figure 5.3. As in Scenario 1, the uncapacitated variational inequality problem (5.22) is solved with the Euler method, using the explicit formulae shown in (5.37).

The quality decay parameters and the link quality decay rates of New Orchard are shown in Table 5.5 and the total link cost functions are depicted in Table 5.6. The initial quality for this orchard is, $q_{04} = 1$, and the other orchard/farms have the same initial quality values as in Scenario 1. New Orchard is very similar to Apex Orchards in terms of its land size, the number of employees, and the storage technology. Therefore, the New Orchards' total link cost functions for harvesting, processing, and storage are very similar to those of Apex. It is assumed that New Orchard is located
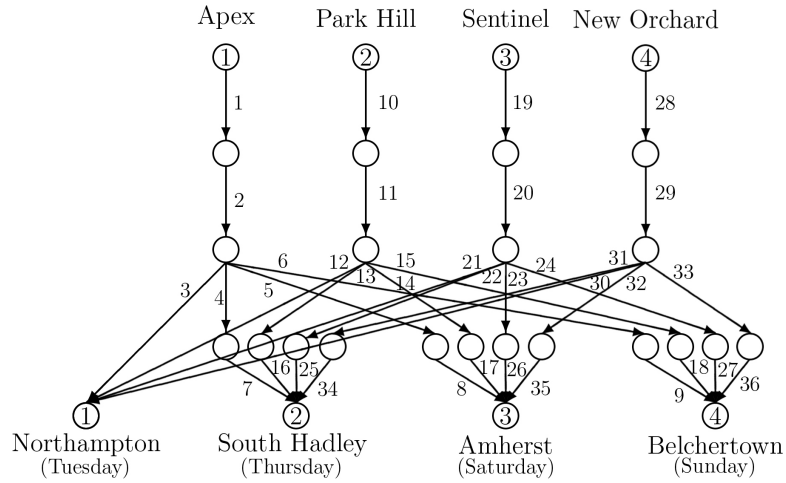
Figure 5.3. The Supply Chain Network Topology for the Apple Farmers' Markets Case Study - Scenario 2

near Sentinel Farm in Belchertown at a lower altitude than the other orchards/farms where the harvesting, processing, and transportation take place at higher temperatures. The transportation duration from the New Orchard to the farmers' markets is similar to that of the Sentinel Farm.

Operations	Link a	Hours	Temp (°C)	β_a
harvesting	28	4.00	19	0.988
processing	29	4.00	19	0.988
transportation	30	0.50	19	0.998
storage (2 days)	31	48.00	0	0.968
storage (4 days)	32	96.00	0	0.989
storage (5 days)	33	120.00	0	0.986
transportation	34	3.50	19	0.989
transportation	35	3.00	19	0.991
transportation	36	3.00	19	0.991

Table 5.5. New Orchard Parameters for the Calculation of Quality Decay for Scenario 2

Since New Orchard has the same initial quality of its Golden Delicious apples as that of the Apex Orchards, it is expected that it may lose some of its loyal customers at the farmers' markets which New Orchard is able to enter.

With the entrance of New Orchard into the supply chain, the total number of paths in the supply chain network for Scenario 2 is increased to sixteen with twelve of them being the same as in Scenario 1. The additional paths and links are: path $p_{13} = (28, 29, 30)$, $p_{14} = (28, 29, 31, 34)$, $p_{15} = (28, 29, 32, 35)$, and $p_{16} = (28, 29, 32, 36)$.

Operations	Link a	$\hat{c}_a(f)$
harvesting	28	$0.02f_{28}^2 + 3f_{28}$
processing	29	$0.015f_{29}^2 + 3f_{29}$
transportation	30	$0.01f_{30}^2 + 3f_{30}$
storage (2 days)	31	$0.01f_{31}^2 + 6f_{31}$
storage (4 days)	32	$0.015f_{32}^2 + 4f_{32}$
storage (5 days)	33	$0.035f_{33}^2 + 5f_{33}$
transportation	34	$0.025f_{34}^2 + 8f_{34}$
transportation	35	$0.015f_{35}^2 + 4f_{35}$
transportation	36	$0.025f_{36}^2 + 8f_{36}$

Table 5.6. The Total Link Cost Functions for New Orchard Under Scenario 2

Customers at the farmers' markets have enough information about New Orchard's Golden Delicious apples. This means that the demand price functions of the other orchard/farms are affected by the demand for the Golden Delicious apples of New Orchard. The new demand price functions are:

Apex Orchards:

$$\rho_{11}(d,q) = -0.053d_{11} - 0.01d_{21} - 0.01d_{31} - 0.03d_{41} + 8q_{p_1} - 2q_{p_5} - 2q_{p_9} - 4q_{p_{13}} + 30,$$

$$\rho_{12}(d,q) = -0.03d_{12} - 0.01d_{22} - 0.01d_{32} - 0.004d_{42} + 3q_{p_2} - 2q_{p_6} - 2q_{p_{10}} - q_{p_{14}} + 25,$$

$$\rho_{13}(d,q) = -0.053d_{13} - 0.01d_{23} - 0.01d_{33} - 0.03d_{43} + 8q_{p_3} - 2q_{p_7} - 2q_{p_{11}} - 4q_{p_{15}} + 30,$$

$$\rho_{14}(d,q) = -0.03d_{14} - 0.01d_{24} - 0.014d_{34} - 0.004d_{44} + 3q_{p_4} - q_{p_8} - 2q_{p_{12}} - q_{p_{15}} + 25,$$

Park Hill Orchard:

$$\rho_{21}(d,q) = -0.05d_{21} - 0.01d_{11} - 0.01d_{31} - 0.01d_{41} + 3q_{p_5} - q_{p_1} - q_{p_9} - q_{p_{13}} + 27,$$

$$\rho_{22}(d,q) = -0.04d_{22} - 0.01d_{12} - 0.02d_{32} - 0.004d_{42} + 3q_{p_6} - 2q_{p_2} - q_{p_{10}} - q_{p_{14}} + 28,$$

$$\rho_{23}(d,q) = -0.05d_{23} - 0.02d_{13} - 0.01d_{33} - 0.02d_{43} + 4q_{p_7} - 2q_{p_3} - q_{p_{11}} - 2q_{p_{15}} + 27,$$

$$\rho_{24}(d,q) = -0.04d_{24} - 0.01d_{14} - 0.02d_{34} - 0.004d_{44} + 2q_{p_8} - q_{p_4} - q_{p_{12}} - q_{p_{16}} + 28,$$

Sentinel Farm:

$$\rho_{31}(d,q) = -0.05d_{31} - 0.01d_{11} - 0.01d_{21} - 0.01d_{41} + 2q_{p_9} - q_{p_1} - q_{p_5} - q_{p_{13}} + 25,$$

$$\rho_{32}(d,q) = -0.04d_{32} - 0.01d_{12} - 0.02d_{22} - 0.004d_{42} + 4q_{p_{10}} - 3q_{p_2} - q_{p_6} - q_{p_{14}} + 28,$$

$$\rho_{33}(d,q) = -0.05d_{33} - 0.02d_{13} - 0.01d_{23} - 0.02d_{43} + 4q_{p_{11}} - 2q_{p_3} - q_{p_7} - 2q_{p_{15}} + 25,$$

$$\rho_{34}(d,q) = -0.04d_{34} - 0.01d_{14} - 0.02d_{24} - 0.004d_{44} + 3q_{p_{12}} - 2q_{p_4} - 2q_{p_8} - 2q_{p_{16}} + 28.$$

According to the demand price functions, New Orchard is not very strong in the South Hadley Farmers' Market. However, it is very effective in the Northampton, Amherst, and Belchertown Farmers' Markets where Apex and Sentinel have been market leaders. In particular, New Orchard becomes a crucial competitor for Apex Orchards. Its demand price functions are given below.

New Orchard:

$$\rho_{41}(d,q) = -0.053d_{41} - 0.03d_{11} - 0.01d_{21} - 0.01d_{31} + 5q_{p_{13}} - 2q_{p_1} - q_{p_5} - q_{p_9} + 30,$$

$$\rho_{42}(d,q) = -0.03_{42} - 0.006d_{12} - 0.01d_{22} - 0.01d_{32} + 2q_{p_{14}} - q_{p_2} - q_{p_6} - q_{p_{10}} + 25,$$

$$\rho_{43}(d,q) = -0.053d_{43} - 0.03d_{13} - 0.01d_{23} - 0.01d_{33} + 5q_{p_{15}} - 2q_{p_3} - q_{p_7} - q_{p_{11}} + 30,$$

$$\rho_{44}(d,q) = -0.03d_{44} - 0.006d_{14} - 0.01d_{24} - 0.01d_{34} + 2q_{p_{16}} - q_{p_4} - q_{p_8} - q_{p_{12}} + 25.$$

For New Orchard, there is more competition in the Amherst and the Northampton Farmers' Markets where the customers are more health conscious and pay attention to the newcomer. New Orchard's demand price is mostly affected by Apex, which is a very important competitor for New Orchard. For example, the demand price of New Orchard's Golden Delicious apples at the Northampton Farmers' Market is affected mostly by the demand for the Golden Delicious apples from Apex.

The computed equilibrium link flows, path flows, and the quality decay rates of the paths are given in Table 5.7 and Table 5.8. New Orchard enters the Northampton and Amherst Farmers' Markets, since they are the only profitable farmers' markets in the supply chain network for it. Notice that some of the equilibrium path flows are lower than in Scenario 1. For instance, the flow on the paths p_1 and p_3 are lower in Table 5.8 than they are in Table 5.4. One explanation for this result is that New Orchard is a new market player whose quality of Golden Delicious apples is very similar to that of Apex's; therefore, some of Apex's previous customers now choose to buy apples from New Orchard and cause a drop in Golden Delicious apple sales for Apex.

The demand prices of the orchard/farms, in dollars per peck, for the profitable demand markets are:

Operations	Link a	f_a^*
harvesting	1	124.0885
processing	2	124.0885
transportation	3	79.5849
storage (2 days)	4	0.0000
storage (4 days)	5	44.5036
storage (5 days)	6	0.0000
transportation	7	0.0000
transportation	8	44.5036
transportation	9	0.0000
harvesting	10	87.4808
processing	11	87.4808
transportation	12	69.2348
storage (2 days)	13	18.2459
storage (4 days)	14	0.0000
storage (5 days)	15	0.0000
transportation	16	18.2459
transportation	17	0.0000
transportation	18	0.0000
harvesting	19	86.0782
processing	20	86.0782
transportation	21	18.3520
storage (2 days)	22	30.9408
storage (4 days)	23	0.0000
storage (5 days)	24	36.7854
transportation	25	30.9408
transportation	26	0.0000
transportation	27	36.7854
harvesting	28	126.1215
processing	29	126.1215
transportation	30	82.0895
storage (2 days)	31	0.0000
storage (4 days)	32	44.0319
storage (5 days)	33	0.0000
transportation	34	0.0000
transportation	35	44.0319
transportation	36	0.0000

Table 5.7. The Computed Equilibrium Link Flows for the Case Study Scenario 2

Apex Orchards:

$$\rho_{11} = 23.49, \quad \rho_{12} = 23.66, \quad \rho_{13} = 27.49, \quad \rho_{14} = 24.44,$$

Park Hill Orchard:

$$\rho_{21} = 21.46, \quad \rho_{22} = 25.41, \quad \rho_{23} = 25.49, \quad \rho_{24} = 26.20,$$

Sentinel Farm:

$$\rho_{31} = 20.38, \quad \rho_{32} = 24.38, \quad \rho_{33} = 22.91, \quad \rho_{34} = 23.08,$$

Path p	q_p	x_p^*
p_1	0.9851	79.5849
p_2	0.9733	0.0000
p_3	0.9684	44.5036
p_4	0.9645	0.0000
p_5	0.7864	69.2348
p_6	0.7645	18.2460
p_7	0.7458	0.0000
p_8	0.7395	0.0000
p_9	0.6791	18.3520
p_{10}	0.6514	30.9408
p_{11}	0.6280	0.0000
p_{12}	0.6217	36.7854
p_{13}	0.9742	82.0895
p_{14}	0.9345	0.0000
p_{15}	0.9567	44.0319
p_{16}	0.9538	0.0000

Table 5.8. The Path Quality Decay Rates and the Computed Equilibrium Path Flows for the Case Study Scenario 2

New Orchard:

$$\rho_{41} = 23.82, \quad \rho_{42} = 23.99, \quad \rho_{43} = 27.80, \quad \rho_{44} = 24.21.$$

New Orchards' entrance causes a price decrease for the other orchard/farms at the farmers' markets due to competition in all prices except for ρ_{21}, which is almost the same as in scenario 1. For example, Apex sells its apples at \$27.33 per peck at the Northampton Farmers' Market in Scenario 1, which decreases to \$23.49 per peck.

The profits of the orchard/farms are:

$$U_1(X^*) = 1097.39, \quad U_2(X^*) = 471.71, \quad U_3(X^*) = 345.45, \quad U_4(X^*) = 1142.19.$$

The profits of Apex, Park Hill Orchards, and the Sentinel Farm decrease from their values in Scenario 1, as a result of the the entrance of New Orchard, which results in increased competition. The largest profit is gained by New Orchard, followed by Apex, Park Hill, and then Sentinel.

5.4.3 Scenario 3

This scenario is constructed to illustrate the apple shortage experienced in western Massachusetts in 2016. The volatile weather conditions of an unexpected cold snap occurred in May. According

to various news articles, the cold snap damaged the green apple buds and an apple shortage at the local markets, which includes the farmers' markets, was expected. In this scenario, I capture this apple shortage by imposing link capacities on the harvesting links of the orchard/farms. The Euler method, with the explicit formulae given in (5.37) and (5.38) for the capacitated problem, is used to compute the solution of the variational inequality formulation (5.33). The supply chain network structure for this scenario is the same as that in Figure 5.2. The total link costs functions, demand price functions of the orchard/farms, and the link quality decay rates are also the same as the corresponding ones in Scenario 1.

The link capacities (in pecks), the computed equilibrium link flows, and the computed equilibrium Lagrange multipliers for this scenario are reported in Table 5.9.

Operations	Link a	Capacity	f_a^*	λ_a^*
harvesting	1	20	20.0000	16.4077
processing	2	15000	20.0000	0.0000
transportation	3	15000	20.0000	0.0000
storage (2 days)	4	15000	0.0000	0.0000
storage (3 days)	5	15000	0.0000	0.0000
storage (4 days)	6	15000	0.0000	0.0000
transportation	7	15000	0.0000	0.0000
transportation	8	15000	0.0000	0.0000
transportation	9	15000	0.0000	0.0000
harvesting	10	50	50.0000	6.4906
processing	11	15000	50.0000	0.0000
transportation	12	15000	50.0000	0.0000
storage (2 days)	13	15000	0.0000	0.0000
storage (3 days)	14	15000	0.0000	0.0000
storage (4 days)	15	15000	0.0000	0.0000
transportation	16	15000	0.0000	0.0000
transportation	17	15000	0.0000	0.0000
transportation	18	15000	0.0000	0.0000
harvesting	19	60	60.0000	5.6685
processing	20	15000	60.0000	0.0000
transportation	21	15000	13.1918	0.0000
storage (2 days)	22	15000	18.7448	0.0000
storage (3 days)	23	15000	0.0000	0.0000
storage (4 days)	24	15000	28.0624	0.0000
transportation	25	15000	18.7448	0.0000
transportation	26	15000	0.0000	0.0000
transportation	27	15000	28.0624	0.0000

Table 5.9. Link Capacities and Computed Equilibrium Link Flows for the Case Study in Scenario 3

The capacities in Table 5.9 reflect the expected level of harvest damage at the orchard/farms. For example, Apex is assumed to experience a larger damage of its Golden Delicious apples, since it is located at a higher altitude, which causes the temperatures to drop lower than the temperatures at the other orchard/farms and damages the green buds more. Therefore, the capacity of Apex's harvesting link is assumed to be the lowest. Park Hill Orchard and Sentinel Farm are located at similar altitudes in western Massachusetts. I assume that they experience similar temperatures which result in similar capacities imposed on the harvesting links of these orchard/farms. The capacities on the other operational links remain relatively large. Furthermore, the initial quality of Apex (q_{01}), Park Hill Orchard (q_{02}), and Sentinel Farm (q_{03}) apples are assumed to be 0.4, 0.5, and 0.6, respectively. Since the most damage on the apple buds is assumed to happen at the Apex Orchards, the lowest initial quality is assigned to this orchard.

From Table 5.9 and Table 5.10 one can see that Apex and Park Hill are now only at the Northampton Farmers' Market, whereas Sentinel serves the Northampton, South Hadley, and Belchertown Farmers' Markets. Moreover, links 1, 10, and 19 are at their capacities and, hence, the associated Lagrange multipliers are positive.

The equilibrium path flows and the path quality decay values are reported in Table 5.10. Observe that the path flows of all the orchard/farms have decreased substantially with respect to the path flows in Scenario 1. The experienced shortage is especially marked for Apex since the path flows p_1, p_2, p_3, and p_4 have decreased substantially from the results reported in Scenario 1 in Table 5.4.

Path p	q_p	x_p^*
p_1	0.3940	20.0000
p_2	0.3893	0.0000
p_3	0.3873	0.0000
p_4	0.3858	0.0000
p_5	0.4915	50.0000
p_6	0.4778	0.0000
p_7	0.4662	0.0000
p_8	0.4622	0.0000
p_9	0.5821	13.1918
p_{10}	0.5584	18.7448
p_{11}	0.5383	0.0000
p_{12}	0.5329	28.0624

Table 5.10. Computed Equilibrium Path Flows for the Case Study Scenario 3

The demand prices of the orchard/farms, in dollars, are:

Apex Orchards:

$$\rho_{11} = 28.01, \quad \rho_{12} = 23.91, \quad \rho_{13} = 29.62, \quad \rho_{14} = 24.35.$$

Park Hill Orchard:

$$\rho_{21} = 24.44, \quad \rho_{22} = 27.72, \quad \rho_{23} = 27.55, \quad \rho_{24} = 27.72,$$

Sentinel Farm:

$$\rho_{31} = 24.02, \quad \rho_{32} = 27.84, \quad \rho_{33} = 25.91, \quad \rho_{34} = 26.78.$$

Since the supply is decreased, the prices of Golden Delicious apples increase in most of the farmers' markets. For example, Apex's Golden Delicious apples are now $28.01 at the Northampton Farmers' Market in this scenario whereas the price was $27.33 in Scenario 1. Additionally, since the quality of Apex's apples is worse than in Scenario 1, this causes the demand price to decrease at the Amherst Farmers' Market. In Scenario 1, the price of its apples at the Amherst Farmers' Market was $30.72; however, now it is $29.62.

The profits of the orchard/farms in this scenario, in dollars, are:

$$U_1(X^*) = 362.15, \quad U_2(X^*) = 498.28, \quad U_3(X^*) = 507.58.$$

The largest profit is achieved by Sentinel, followed by Park Hill, and Apex. In Scenario 1, Apex has the largest profit, which decreases substantially in this scenario due to it having the lowest harvesting capacity and quality.

5.5. Summary and Conclusions

Fresh produce consists of both fruits and vegetables and such supply chains are especially challenging since the quality of the product deteriorates continuously upon harvesting. At the same time, consumers are demanding fresh products and are increasingly health conscious. Farmers' markets have increased in popularity internationally and, yet, the mathematical modeling associated with such supply chains has been limited.

In this chapter, I provide explicit formulae for a variety of fresh produce to capture quality deterioration. I then identify the quality associated with different pathways in supply chain networks. Subsequently, I focus on farmers' markets, which are examples of direct to consumer supply chains and are shorter supply chains since farmers bring the product to markets at which consumers select

their purchases. Specifically, I introduce a game theory model for supply chain competition in a network framework for farmers' markets occurring within a period of time, such as a week. The farms are interested in maximizing their profits and the consumers respond to the quality of the product. I provide both qualitative properties of the equilibrium link flow pattern, propose a computational scheme, and also illustrate my framework through numerical examples focused on peaches and then in a case study for Golden Delicious apples and farmers' markets in western Massachusetts. The mathematical framework considers both uncapacitated links in the supply chain network as well as capacitated ones, which may occur due to crop failures, harvesting problems, labor shortages, etc. This is the first game theory model for farmers' markets and also the first competitive fresh produce supply chain network model in which quality deterioration of fresh produce is explicitly captured.

CHAPTER 6

DYNAMICS OF QUALITY AS A STRATEGIC VARIABLE IN COMPLEX FOOD SUPPLY CHAIN NETWORK COMPETITION: THE CASE OF FRESH PRODUCE

Fresh produce, in the form of fruits and vegetables, is essential to human health and well-being. Given the consumer demand for fresh produce, year round, global supply chains have evolved in order to satisfy customers. With fresh produce criss-crossing the globe from producers to consumers, attention to quality is essential with time and distance playing critical roles, as well as environmental factors associated with the various supply chain network activities, including processing, transportation, storage, and distribution.

In this chapter, I construct a competitive supply chain network model for fresh produce under oligopolistic competition among the food firms, who are profit-maximizers. It is an extension of the model developed in Chapter 5, where I use the same notation and definitions related to quality deterioration presented in Section 5.1. This chapter is based on the paper by Nagurney, Besik, and Yu (2018).

The chapter is organized as follows. In Section 6.1, I construct the model and derive alternative variational inequality formulations of the governing Nash (1950, 1951) equilibrium conditions. I also provide an existence result. In addition, I present several simple numerical examples for illustrative purposes.

In Section 6.2, an algorithmic scheme is outlined and its interpretation as a discrete-time tatonnement process given. It is then applied in Section 6.3 to compute solutions to numerical examples comprising a case study focusing on peaches from which managerial insights are drawn. A summary of the contributions are provided in the concluding Section 6.4.

6.1. The Competitive Fresh Produce Supply Chain Network Model with Quality and Associated Dynamics

In this section, the competitive fresh produce supply chain network model, under oligopolistic competition, and with both path flows and initial quality as strategic variables is presented. The fresh produce products are substitutable and are differentiated by food firm. Each multitiered supply chain network of firm i consists of n_M^i production/harvesting facilities: $M_1^i, \ldots, M_{n_M^i}^i$; n_C^i processors: $C_{1,1}^i, \ldots, C_{n_C^i,1}^i$, and n_D^i distribution centers, $D_{1,1}^i, \ldots, D_{n_D^i,1}^i$, and can serve the n_R retail outlets, denoted, respectively, by: $R_1, \ldots, R_{n_R}$. Different links connecting a pair of nodes correspond to distinct options. It is assumed that the time durations and temperatures of the links are fixed and known. Let $G = [N, L]$ denote the graph consisting of the set of nodes N and the set of links L in Figure 6.1. The supply chain network topology in Figure 6.1 can be adapted/modified according to the particular fresh produce product under investigation. The supply chain network topology is inspired by the one constructed in Yu and Nagurney (2013) but, here, the model is distinctly different with a focus on quality deterioration using explicit physical formulae associated with the various network economic links and associated activities of shipment, processing, storage, and distribution, as illustrated in Figure 6.1. Moreover, explicit capacities on the supply chain network links as well as an upper bound on the initial quality associated with the various production/harvesting sites are included.

I first present the variables and then the various functions associated with the supply chain network. The flow of the fresh produce product on path p joining an origin node i with a destination node k is denoted by x_p. For each path the following nonnegativity condition must hold:

$$x_p \geq 0, \quad \forall p \in P_k^i;\ i = 1, \ldots, I;\ k = R_1, \ldots, R_{n_R}. \tag{6.1}$$

Furthermore, q_{0a}^i, the initial quality of the fresh produce on the top-most links a of an origin node i, must be nonnegative, that is,

$$q_{0a}^i \geq 0, \quad \forall a \in L_1^i;\ i = 1, \ldots, I. \tag{6.2}$$

In addition, it is reasonable to assume that the quality is bounded from above by a maximum value; hence, the following can be obtained:
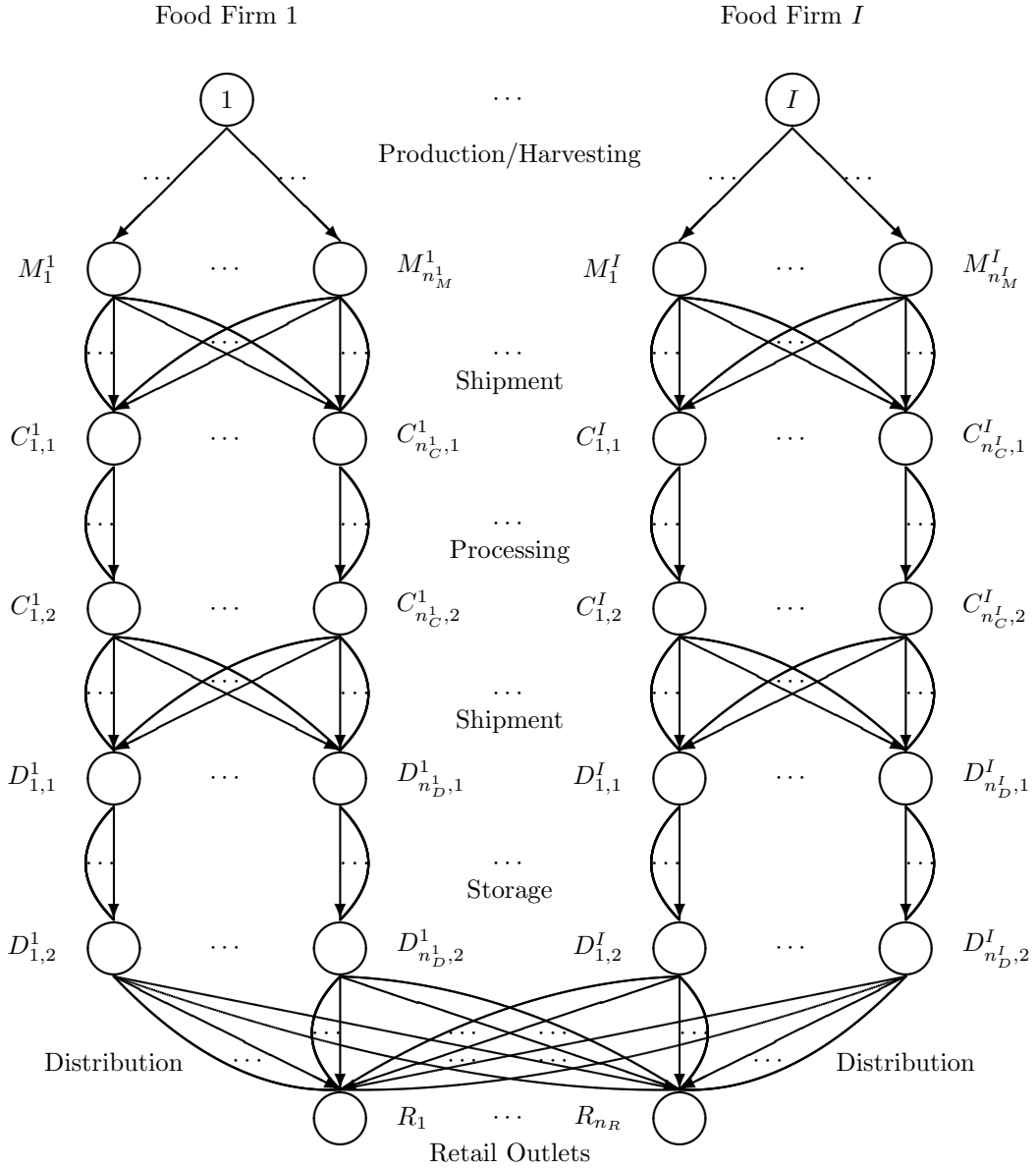
Figure 6.1. The Fresh Produce Supply Chain Network Topology with Quality Deterioration

$$q_{0a}^i \leq \bar{q}_{0a}^i, \quad \forall a \in L_1^i; \, i = 1, \ldots, I, \tag{6.3}$$

where $\bar{q}_{0a}^i$ is the positive upper bound on the quality of the produce produced/harvested on link a of food firm i.

The conservation of flow equations that relate the link flows of each food firm i; $i = 1, \ldots, I$, to the path flows are given by:

$$f_l = \sum_{p \in P} x_p \delta_{lp}, \quad \forall l \in L^i;\ i = 1, \ldots, I, \tag{6.4}$$

where f_l denotes the flow on link l, $\delta_{lp} = 1$, if link l is contained in path p, and 0, otherwise, and P denotes the set of all paths. Therefore, since the supply chain networks of the firms do not share any common links, the flow of a firm's fresh produce product on a link is equal to the sum of that product's flows on paths that contain that link. We group the link flows into the vector $f \in R_+^{n_L}$, where n_L denotes the number of links in L. All vectors in this chapter are column vectors.

In addition, since the link flows must satisfy capacity constraints, we have that:

$$f_l \leq u_l, \quad \forall l \in L, \tag{6.5}$$

where u_l denotes the positive upper bound on link l.

Also, observe that, in view of the conservation of flow equations (6.4), (6.5) can be rewritten in terms of path flows as:

$$\sum_{p \in P} x_p \delta_{lp} \leq u_l, \quad \forall l \in L. \tag{6.6}$$

In general, consumers, at the retail outlets, respond not only to the quantities available of the product but also to their average quality, where the average quality of the product at retail outlet k, associated with the fresh produce product of firm i, and denoted by $\hat{q}_{ik}$, is given by the expression:

$$\hat{q}_{ik} = \frac{\sum_{p \in P_k^i} q_p x_p}{\sum_{p \in P_k^i} x_p}, \quad i = 1, \ldots, I;\ k = R_1, \ldots, R_{n_R}, \tag{6.7}$$

where recall that q_p is fresh produce product specific with its value computed according to (5.8). We group the average product quality of all firms into the vector $\hat{q} \in R_+^{I \times n_R}$. We exclude all food firm / retail outlet pairs that do not conduct business with one another so that the denominator in (6.7) is never equal to zero.

The demand for food firm i's fresh food product at retail outlet k is denoted by d_{ik} and is equal to the sum of all the fresh produce flows on paths joining (i, k), so that:

$$\sum_{p \in P_k^i} x_p = d_{ik}, \quad i = 1, \ldots, I;\ k = R_1, \ldots, R_{n_R}. \tag{6.8}$$

The demands for the fresh food products of all firms at all retail outlets are grouped into the vector $d \in R_+^{I \times n_R}$.

I now present the underlying functions in the competitive supply chain network model with quality.

The demand price of food firm i's product at retail outlet k is denoted by ρ_{ik} and assume that

$$\rho_{ik} = \rho_{ik}(d, \hat{q}), \quad i = 1, \ldots, I; \; k = R_1, \ldots, R_{n_R}. \tag{6.9}$$

Note that the price of food firm i's product at a particular retail outlet may depend not only on the demands for and the average quality of its product, but also on the demands for and the average quality of the other substitutable food products at all the retail outlets. These demand price functions are assumed to be continuous, continuously differentiable, and monotone decreasing.

The cost of production/harvesting at firm i's production site a depends, in general, on the initial quality q_{0a}^i, and the product flow on the production/harvesting link, that is,

$$\hat{z}_a = \hat{z}_a(f_a, q_{0a}^i), \quad \forall a \in L_1^i; \; i = 1, \ldots, I. \tag{6.10}$$

Furthermore, the operational cost functions associated with the remaining links in the supply chain network are defined as:

$$\hat{c}_b = \hat{c}_b(f), \quad \forall b \in L_2^i; \; i = 1, \ldots, I. \tag{6.11}$$

The total operational cost on each such link is assumed to be convex and continuously differentiable and the same holds for each production/harvesting cost function.

Let X_i denote the vector of path flows associated with firm i; $i = 1, \ldots, I$, where $X_i \equiv \{\{x_p\}|p \in P^i\}\} \in R_+^{n_{P^i}}$, $P^i \equiv \cup_{k=R_1, \ldots, R_{n_R}} P_k^i$, and n_{P^i} denotes the number of paths from firm origin node i to the retail outlets. Then, X is the vector of all the food firms' path flow strategies, that is, $X \equiv \{\{X_i\}|i = 1, \ldots, I\}$. Similarly, the vector of initial quality levels, associated with firm i; $i = 1, \ldots, I$, is denoted by q_0^i, where $q_0^i \equiv \{\{q_{0a}^i\}|a \in L_1^i\}\} \in R_+^{n_{L_1^i}}$, where $n_{L_1^i}$ is the number of top-most links in firm i's supply chain network. Finally, q_0 is the vector of all initial quality levels of the food firms; that is, $q_0 \equiv \{\{q_0^i\}|i = 1, \ldots, I\}$.

The utility of food firm i is its profit, which is the difference between its revenue and its total costs, where the total costs are composed of the total costs on the production/harvesting links and

the total operational costs on the post-harvest links in its supply chain network. Hence, the utility of firm i; $i = 1, \ldots, I$, denoted by U_i, is expressed as:

$$U_i = \sum_{k=R_1}^{R_{n_R}} \rho_{ik}(d, \hat{q}) d_{ik} - \left(\sum_{a \in L_1^i} \hat{z}_a(f_a, q_{0a}^i) + \sum_{b \in L_2^i} \hat{c}_b(f) \right). \tag{6.12}$$

In view of (5.8), (6.5), and (6.8), (6.9) can be rewritten as:

$$\hat{\rho}_{ik}(x, q_0) \equiv \rho_{ik}(d, \hat{q}), \quad i = 1, \ldots, I; \ k = R_1, \ldots, R_{n_R}. \tag{6.13}$$

In lieu of the constraints (6.4) and (6.6), and the functional expressions (6.10), (6.11), and (6.13), $\hat{U}_i(X, q_0) \equiv U_i$ can be defined for all firms i; $i = 1, \ldots, I$, with the I-dimensional vector $\hat{U}$ being the vector of the profits of all the firms:

$$\hat{U} = \hat{U}(X, q_0). \tag{6.14}$$

In the competitive oligopolistic market framework, each firm selects its product path flows as well as its initial quality levels at its production sites in a noncooperative manner, seeking to maximize its own profit, until an equilibrium is achieved, according to the definition below:

- **Definition 6.1: Supply Chain Network Nash Equilibrium with Fresh Produce Quality**

  A fresh produce path flow pattern and initial quality level $(X^*, q_0^*) \in K = \prod_{i=1}^{I} K_i$ constitutes a supply chain network Nash Equilibrium with fresh produce quality if for each food firm i; $i = 1, \ldots, I$:

  $$\hat{U}_i(X_i^*, X_{-i}^*, q_0^{i*}, q_0^{-i*}) \geq \hat{U}_i(X_i, X_{-i}^*, q_0^i, q_0^{-i*}), \quad \forall (X_i, q_0^i) \in K_i, \tag{6.15}$$

  where $X_{-i}^* \equiv (X_1^*, \ldots, X_{i-1}^*, X_{i+1}^*, \ldots, X_I^*)$, $q_0^{-i*} \equiv (q_0^{1*}, \ldots, q_0^{i-1*}, q_0^{i+1*}, \ldots, q_0^{I*})$ and $K_i \equiv \{(X_i, q_0^i) | X_i \in R_+^{n_{P^i}}, q_0^i \in R_+^{n_{L_1^i}}, \ (6.3) \ and \ (6.6) \ hold \ for \ l \in L^i\}$.

In other words, an equilibrium is established if no food firm can unilaterally improve upon its profit by altering its product flows and initial quality at production sites in its supply chain network, given the product flows and initial quality decisions of the other firms.

Next, alternative variational inequality formulations of the Nash Equilibrium for the fresh produce supply chain network under oligopolistic competition satisfying Definition 6.1, in terms of path flows and initial quality levels are derived (see Nash (1950, 1951)).

The λ_a; $a \in L_1$ and γ_l; $l \in L$ are the Lagrange multipliers associated with constraints (6.3) and (6.6) (or (6.5)), respectively. We group these Lagrange multipliers into the n_{L_1}-dimensional vector λ and the n_L-dimensional vector γ, respectively.

- **Theorem 6.1: Variational Inequality Formulation of the Governing Equilibrium Conditions**

  Assume that, for each food firm i; $i = 1, \ldots, I$, the profit function $\hat{U}_i(X, q_0)$ is concave with respect to the variables X_i and q_0^i, and is continuously differentiable. Then $(X^*, q_0^*) \in K$ is a supply chain network Nash Equilibrium with fresh produce quality according to Definition 6.1 if and only if it satisfies the variational inequality:

  $$-\sum_{i=1}^{I} \langle \nabla_{X_i} \hat{U}_i(X^*, q_0^*), X_i - X_i^* \rangle - \sum_{i=1}^{I} \langle \nabla_{q_0^i} \hat{U}_i(X^*, q_0^*), q_0^i - q_0^{i*} \rangle \geq 0, \quad \forall (X, q_0) \in K, \quad (6.16)$$

  where $\langle \cdot, \cdot \rangle$ denotes the inner product in the corresponding Euclidean space. Furthermore, $\nabla_{X_i} \hat{U}_i(X, q_0)$ denotes the gradient of $\hat{U}_i(X, q_0)$ with respect to X_i and $\nabla_{q_0^i} \hat{U}_i(X, q_0)$ denotes the gradient of $\hat{U}_i(X, q_0)$ with respect to q_0^i. The solution of variational inequality (6.16), in turn, is equivalent to the solution of the variational inequality: determine $(x^*, q_0^*, \lambda^*, \gamma^*) \in K^1$ satisfying:

  $$\sum_{i=1}^{I} \sum_{k=R_1}^{R_{n_R}} \sum_{p \in P_k^i} \left[\frac{\partial \hat{Z}^i(x^*, q_0^{i*})}{\partial x_p} + \frac{\partial \hat{C}^i(x^*)}{\partial x_p} + \sum_{l \in L^i} \gamma_l^* \delta_{lp} - \hat{\rho}_{ik}(x^*, q_0^*) - \sum_{j=R_1}^{R_{n_R}} \frac{\partial \hat{\rho}_{ij}(x^*, q_0^*)}{\partial x_p} \sum_{r \in P_j^i} x_r^* \right]$$

  $$\times [x_p - x_p^*] + \sum_{i=1}^{I} \sum_{a \in L_1^i} \left[\frac{\partial \hat{Z}^i(x^*, q_0^{i*})}{\partial q_{0a}^i} + \lambda_a^* - \sum_{j=R_1}^{R_{n_R}} \frac{\partial \hat{\rho}_{ij}(x^*, q_0^*)}{\partial q_{0a}^i} \sum_{r \in P_j^i} x_r^* \right] \times [q_{0a}^i - q_{0a}^{i*}]$$

  $$+ \sum_{i=1}^{I} \sum_{a \in L_1^i} [\bar{q}_{0a}^i - q_{0a}^{i*}] \times [\lambda_a - \lambda_a^*] + \sum_{i=1}^{I} \sum_{l \in L^i} \left[u_l - \sum_{r \in P} x_r^* \delta_{lr} \right] \times [\gamma_l - \gamma_l^*] \geq 0, \quad \forall (x, q_0, \lambda, \gamma) \in K^1,$$

  $$(6.17)$$

  where $K^1 \equiv \{(x, q_0, \lambda, \gamma) | x \in R_+^{n_P}, q_0 \in R_+^{n_{L_1}}, \lambda \in R_+^{n_{L_1}}, \gamma \in R_+^{n_L}\}$ and for each path p; $p \in P_k^i$; $i = 1, \ldots, I$; $k = R_1, \ldots, R_{n_R}$:

$$\frac{\partial \hat{Z}^i(x, q_0^i)}{\partial x_p} \equiv \sum_{a \in L_1^i} \frac{\partial \hat{z}_a(f_a, q_{0a}^i)}{\partial f_a} \delta_{ap}, \tag{6.18a}$$

$$\frac{\partial \hat{C}^i(x)}{\partial x_p} \equiv \sum_{b \in L_2^i} \sum_{l \in L^i} \frac{\partial \hat{c}_b(f)}{\partial f_l} \delta_{lp}, \tag{6.18b}$$

$$\frac{\partial \hat{\rho}_{ij}(x, q_0)}{\partial x_p} \equiv \frac{\partial \rho_{ij}(d, \hat{q})}{\partial d_{ik}} + \frac{\partial \rho_{ij}(d, \hat{q})}{\partial \hat{q}_{ik}} \left(\frac{q_p}{\sum_{r \in P_k^i} x_r} - \frac{\sum_{r \in P_k^i} q_r x_r}{(\sum_{r \in P_k^i} x_r)^2} \right). \tag{6.18c}$$

For each a; $a \in L_1^i$; $i = 1, \ldots, I$,

$$\frac{\partial \hat{Z}^i(x, q_0^i)}{\partial q_{0a}^i} \equiv \frac{\partial \hat{z}_a(f_a, q_{0a}^i)}{\partial q_{0a}^i}, \tag{6.18d}$$

$$\frac{\partial \hat{\rho}_{ij}(x, q_0)}{\partial q_{0a}^i} \equiv \sum_{h=R_1}^{R_{n_R}} \sum_{s \in P_h^i} \frac{x_s}{\sum_{r \in P_h^i} x_r} \frac{\partial \rho_{ij}(d, \hat{q})}{\partial \hat{q}_{ih}} \frac{\partial q_s}{\partial q_{0a}^i}. \tag{6.18e}$$

In particular, if link a is not included in path s, $\frac{\partial q_s}{\partial q_{0a}^i} = 0$; if link a is included in path s, following (5.8), the following is immediate:

$$\frac{\partial q_s}{\partial q_{0a}^i} = \begin{cases} 1, & \text{if } n = 0, \\[2em] \prod_{b \in s \cap L_2^i} \beta_b, & \text{if } n = 1. \end{cases} \tag{6.18f}$$

Proof: (6.16) follows directly from Gabay and Moulin (1980); see also Dafermos and Nagurney (1987). Under the imposed assumptions, (6.16) holds if and only if (see, e.g., Bertsekas and Tsitsiklis (1989)) the following holds:

$$\sum_{i=1}^I \sum_{k=R_1}^{R_{n_R}} \sum_{p \in P_k^i} \left[-\frac{\partial \hat{U}_i}{\partial x_p} + \sum_{l \in L^i} \gamma_l^* \delta_{lp} \right] \times [x_p - x_p^*] + \sum_{i=1}^I \sum_{a \in L_1^i} \left[-\frac{\partial \hat{U}_i}{\partial q_{0a}^i} + \lambda_a^* \right] \times [q_{0a}^i - q_{0a}^{i*}]$$

$$+ \sum_{i=1}^I \sum_{a \in L_1^i} [\bar{q}_{0a}^i - q_{0a}^{i*}] \times [\lambda_a - \lambda_a^*] + \sum_{i=1}^I \sum_{l \in L^i} \left[u_l - \sum_{r \in P} x_r^* \delta_{lr} \right] \times [\gamma_l - \gamma_l^*] \geq 0, \quad \forall (x, q_0, \lambda, \gamma) \in K^1. \tag{6.19}$$

For each path p; $p \in P_k^i$, we have that

$$\frac{\partial \hat{U}_i}{\partial x_p} = \frac{\partial [\sum_{j=R_1}^{R_{n_R}} \rho_{ij}(d, \hat{q}) d_{ij} - (\sum_{e \in L_1^i} \hat{z}_e(f_e, q_{0e}^i) + \sum_{b \in L_2^i} \hat{c}_b(f))]}{\partial x_p}$$

$$= \frac{\partial [\sum_{j=R_1}^{R_{n_R}} \rho_{ij}(d, \hat{q}) d_{ij}]}{\partial x_p} - \frac{\partial [\sum_{e \in L_1^i} \hat{z}_e(f_e, q_{0e}^i)]}{\partial x_p} - \frac{\partial [\sum_{b \in L_2^i} \hat{c}_b(f)]}{\partial x_p}$$

146

$$
= \sum_{j=R_1}^{R_{n_R}} \left[\frac{\partial[\rho_{ij}(d,\hat{q})d_{ij}]}{\partial d_{ik}} \frac{\partial d_{ik}}{\partial x_p} + \frac{\partial[\rho_{ij}(d,\hat{q})d_{ij}]}{\partial \hat{q}_{ik}} \frac{\partial \hat{q}_{ik}}{\partial x_p} \right] - \sum_{a \in L_1^i} \sum_{e \in L_1^i} \frac{\partial \hat{z}_e(f_e, q_{0e}^i)}{\partial f_a} \frac{\partial f_a}{\partial x_p}
$$

$$
- \sum_{b \in L_2^i} \sum_{l \in L^i} \frac{\partial \hat{c}_b(f)}{\partial f_l} \frac{\partial f_l}{\partial x_p}
$$

$$
= \rho_{ik}(d,\hat{q}) + \sum_{j=R_1}^{R_{n_R}} \left[\frac{\partial \rho_{ij}(d,\hat{q})}{\partial d_{ik}} d_{ij} + \frac{\partial \rho_{ij}(d,\hat{q})}{\partial \hat{q}_{ik}} d_{ij} \left(\frac{\partial \hat{q}_{ik}}{\partial[\sum_{r \in P_k^i} q_r x_r]} \frac{\partial[\sum_{r \in P_k^i} q_r x_r]}{\partial x_p} \right. \right.
$$

$$
\left. \left. + \frac{\partial \hat{q}_{ik}}{\partial[\sum_{r \in P_k^i} x_r]} \frac{\partial[\sum_{r \in P_k^i} x_r]}{\partial x_p} \right) \right] - \sum_{a \in L_1^i} \frac{\partial \hat{z}_a(f_a, q_{0a}^i)}{\partial f_a} \delta_{ap} - \sum_{b \in L_2^i} \sum_{l \in L^i} \frac{\partial \hat{c}_b(f)}{\partial f_l} \delta_{lp}
$$

$$
= \rho_{ik}(d,\hat{q}) + \sum_{j=R_1}^{R_{n_R}} \left[\frac{\partial \rho_{ij}(d,\hat{q})}{\partial d_{ik}} + \frac{\partial \rho_{ij}(d,\hat{q})}{\partial \hat{q}_{ik}} \left(\frac{q_p}{\sum_{r \in P_k^i} x_r} - \frac{\sum_{r \in P_k^i} q_r x_r}{(\sum_{r \in P_k^i} x_r)^2} \right) \right] d_{ij}
$$

$$
- \sum_{a \in L_1^i} \frac{\partial \hat{z}_a(f_a, q_{0a}^i)}{\partial f_a} \delta_{ap} - \sum_{b \in L_2^i} \sum_{l \in L^i} \frac{\partial \hat{c}_b(f)}{\partial f_l} \delta_{lp}, \tag{6.20}
$$

and for each link a; $a \in L_1^i$, we know that

$$
\frac{\partial \hat{U}_i}{\partial q_{0a}^i} = \frac{\partial[\sum_{j=R_1}^{R_{n_R}} \rho_{ij}(d,\hat{q})d_{ij} - (\sum_{e \in L_1^i} \hat{z}_e(f_e, q_{0e}^i) + \sum_{b \in L_2^i} \hat{c}_b(f))]}{\partial q_{0a}^i}
$$

$$
= \frac{\partial[\sum_{j=R_1}^{R_{n_R}} \rho_{ij}(d,\hat{q})d_{ij}]}{\partial q_{0a}^i} - \frac{\partial[\sum_{e \in L_1^i} \hat{z}_e(f_e, q_{0e}^i)]}{\partial q_{0a}^i} - \frac{\partial[\sum_{b \in L_2^i} \hat{c}_b(f)]}{\partial q_{0a}^i}
$$

$$
= \sum_{h=R_1}^{R_{n_R}} \sum_{j=R_1}^{R_{n_R}} \frac{\partial[\rho_{ij}(d,\hat{q})d_{ij}]}{\partial \hat{q}_{ih}} \frac{\partial \hat{q}_{ih}}{\partial q_{0a}^i} - \frac{\partial \hat{z}_a(f_a, q_{0a}^i)}{\partial q_{0a}^i}
$$

$$
= \sum_{h=R_1}^{R_{n_R}} \sum_{j=R_1}^{R_{n_R}} \frac{\partial \rho_{ij}(d,\hat{q})}{\partial \hat{q}_{ih}} d_{ij} \frac{\partial \hat{q}_{ih}}{\partial[\sum_{r \in P_h^i} q_r x_r]} \frac{\partial[\sum_{r \in P_h^i} q_r x_r]}{\partial q_{0a}^i} - \frac{\partial \hat{z}_a(f_a, q_{0a}^i)}{\partial q_{0a}^i}
$$

$$
= \sum_{h=R_1}^{R_{n_R}} \sum_{j=R_1}^{R_{n_R}} \frac{\partial \rho_{ij}(d,\hat{q})}{\partial \hat{q}_{ih}} d_{ij} \frac{1}{\sum_{r \in P_h^i} x_r} \sum_{s \in P_h^i} \frac{\partial[\sum_{r \in P_h^i} q_r x_r]}{\partial q_s} \frac{\partial q_s}{\partial q_{0a}^i} - \frac{\partial \hat{z}_a(f_a, q_{0a}^i)}{\partial q_{0a}^i}
$$

$$
= \sum_{h=R_1}^{R_{n_R}} \sum_{j=R_1}^{R_{n_R}} \sum_{s \in P_h^i} \frac{x_s}{\sum_{r \in P_h^i} x_r} \frac{\partial \rho_{ij}(d,\hat{q})}{\partial \hat{q}_{ih}} \frac{\partial q_s}{\partial q_{0a}^i} d_{ij} - \frac{\partial \hat{z}_a(f_a, q_{0a}^i)}{\partial q_{0a}^i}. \tag{6.21}
$$

By making use of (6.8), (6.13), and the definitions in (6.18(a))-(6.18(e)), variational inequality (6.17) is immediate.

In addition, it is obvious that $\frac{\partial q_s}{\partial q_{0a}^i} = 0$ if link a is not included in path s. If link a is included in path s, $\frac{\partial q_s}{\partial q_{0a}^i}$ in (6.18f) follows directly from (5.8). $\square$

147

Existence of a solution $(X^*, q_0^*) \in K$ to variational inequality (6.16) follows from the classical theory of variational inequalities since the feasible set K is compact, that is, closed and bounded, and the components of the gradients are continuous under the imposed assumptions.

6.1.1 Simple Illustrative Examples

In this subsection, I provide illustrative numerical examples. There are two food firms, Food Firm 1 and Food Firm 2, competing in a duopolistic manner to sell their fresh produce products, which are substitutable. There is a single retail outlet R_1. The supply chain network topology is given in Figure 6.2.
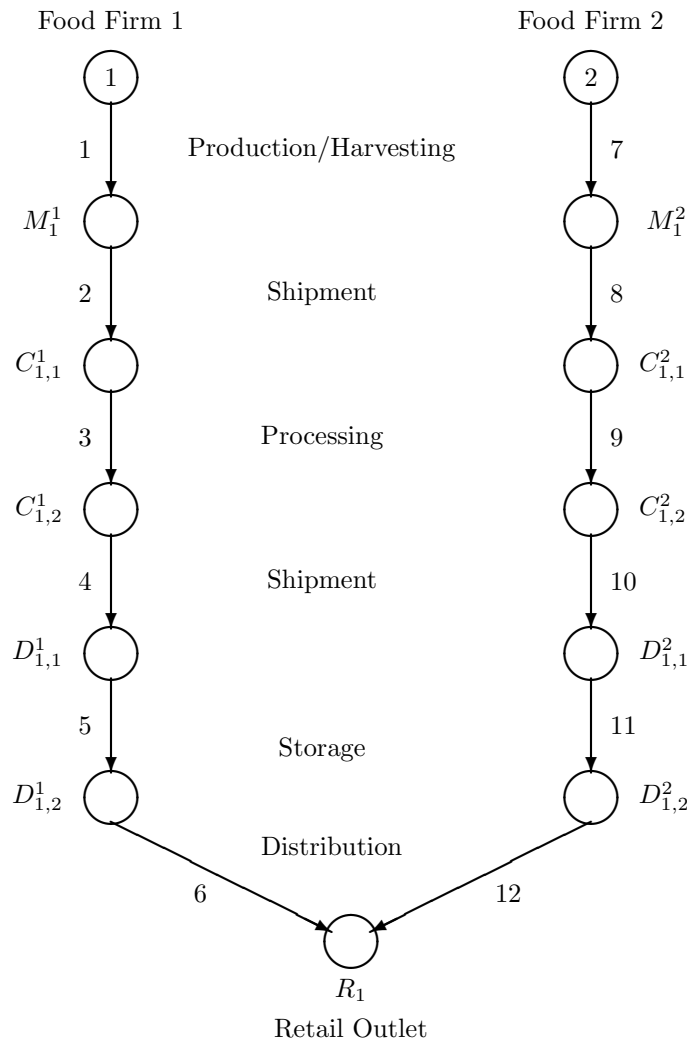
Figure 6.2. Supply Chain Network Topology for the Illustrative Examples

The cost of production/harvesting depends on the initial quality and on the product flow on the production/harvesting links. In Figure 6.2, there are two production/harvesting links that belong, respectively, to set L_1^1 and set L_1^2. The cost of production/harvesting is higher for Food Firm 1 since it uses better machinery and has invested more into the necessary chemicals to maintain the soil quality, with the top-tiered link cost functions being:

$$\hat{z}_1(f_1, q_{01}^1) = f_1^2 + 8f_1 + 3q_{01}^1, \quad \hat{z}_7(f_7, q_{07}^2) = f_7^2 + 3q_{07}^2.$$

There are ten additional links, belonging to the sets L_2^1 and L_2^2, in the supply chain network and their total link cost functions are:

$$\hat{c}_2(f_2) = 5f_2^2 + 10f_2, \ \hat{c}_3(f_3) = 2f_3^2, \ \hat{c}_4(f_4) = 2f_4^2 + f_4, \ \hat{c}_5(f_5) = 3f_5^2, \ \hat{c}_6(f_6) = f_6^2 + f_6,$$

$$\hat{c}_8(f_8) = f_8^2 + f_8, \ \hat{c}_9(f_9) = 3f_9^2 + f_9, \ \hat{c}_{10}(f_{10}) = 2f_{10}^2, \ \hat{c}_{11}(f_{11}) = 6f_{11}^2 + f_{11}, \ \hat{c}_{12}(f_{12}) = 6f_{12}^2 + f_{12}.$$

The total link cost functions are constructed according to the assumptions made for Food Firm 1, Food Firm 2, and Retail Outlet R_1.

Table 6.1 displays the quality decay β_b incurred on the links, when the reaction order $n = 0$ and when $n = 1$.

Link b	Hours	Temperature (Celsius)	β_b $(n=0)$	β_b $(n=1)$
2	48	22	-0.1784	0.8366
3	10	22	-0.0372	0.9635
4	12	10	-0.0167	0.9835
5	10	22	-0.0372	0.9635
6	10	22	-0.0372	0.9635
8	4	22	-0.0149	0.9852
9	2	22	-0.0074	0.9926
10	2	5	-0.0004	0.9995
11	8	22	-0.0297	0.9707
12	2	22	-0.0149	0.9852

Table 6.1. Parameters for the Calculation of Quality Decay for the Illustrative Examples

The reaction rate and quality decay rate on each link are calculated according to (5.6), (5.7), and (5.8), where the universal gas constant and activation energy are taken as 8.314 $Jmol^{-1}K^{-1}$ and 150 $kJmol^{-1}$, respectively.

The shipment time is longer for Food Firm 1 than for Food Firm 2 because of their respective distances to their processing facilities, which can be seen from the time difference between link 2 and link 8.

There are two paths, p_1 and p_2, in this supply chain network, defined as: $p_1 = (1, 2, 3, 4, 5, 6)$ and $p_2 = (7, 8, 9, 10, 11, 12)$. Since there exists one path for each food firm, it is known, because of conservation of flow, that: $d_{11}^* = x_{p_1}^*$ and $d_{21}^* = x_{p_2}^*$.

I now consider different quality decay functions and provide specific details in the examples below.

6.1.2 Example 6.1a: Linear Quality Decay (Zero Order Kinetics)

As mentioned in Section 5.1, in a zero order quality decay function, the reaction order $n = 0$, and the quality q_p over a path p can be determined by the appropriate formula in (5.8), for $n = 0$. The initial quality variables are q_{01}^1 and q_{07}^2.

The demand price functions are:

$$\hat{\rho}_{11}(x, q_0) \equiv \rho_{11}(d, \hat{q}) = -2x_{p_1} - x_{p_2} + \frac{q_{p_1} x_{p_1}}{x_{p_1}} + 100$$

and

$$\hat{\rho}_{21}(x, q_0) \equiv \rho_{21}(d, \hat{q}) = -3x_{p_2} - x_{p_1} + \frac{q_{p_2} x_{p_2}}{x_{p_2}} + 90,$$

with the path quality q_p for the two paths constructed according to (5.8), for $n = 0$ for $i = 1, 2$, given by:

$$q_{p_1} = q_{01}^1 + \beta_2 + \beta_3 + \beta_4 + \beta_5 + \beta_6 = q_{01}^1 - 0.3066,$$

$$q_{p_2} = q_{07}^2 + \beta_8 + \beta_9 + \beta_{10} + \beta_{11} + \beta_{12} = q_{07}^2 - 0.0674.$$

The upper bounds (capacities) on the links are set to 200. Since the capacities on the links are high, it is known that (cf. variational inequality (6.17)) the equilibrium Lagrange multipliers γ^* associated with the links will be equal to 0. Similarly, for simplicity, and exposition purposes, the initial quality bounds are set to 100, so that the corresponding equilibrium Lagrange multipliers λ^*, based on the data, will also be 0.

Given the data, it is also reasonable to expect that $x_{p_1}^* > 0$, $x_{p_2}^* > 0$, $q_{01}^{1*} > 0$, and $q_{07}^{2*} > 0$. Hence, in order to obtain the equilibrium path flows and the equilibrium initial quality levels that satisfy variational inequality (6.17), the following expressions must be equal to 0:

$$\frac{\partial \hat{Z}^1(x^*, q_{01}^{1*})}{\partial x_{p_1}} + \frac{\partial \hat{C}^1(x^*)}{\partial x_{p_1}} - \hat{\rho}_{11}(x^*, q_0^*) - \frac{\partial \hat{\rho}_{11}(x^*, q_0^*)}{\partial x_{p_1}} x_{p_1}^* = 0, \tag{6.22}$$

$$\frac{\partial \hat{Z}^1(x^*, q_{01}^{1*})}{\partial q_{01}^1} - \frac{\partial \hat{\rho}_{11}(x^*, q_0^*)}{\partial q_{01}^1} x_{p_1}^* = 0, \tag{6.23}$$

$$\frac{\partial \hat{Z}^2(x^*, q_{07}^{2*})}{\partial x_{p_2}} + \frac{\partial \hat{C}^2(x^*)}{\partial x_{p_2}} - \hat{\rho}_{21}(x^*, q_0^*) - \frac{\partial \hat{\rho}_{21}(x^*, q_0^*)}{\partial x_{p_2}} x_{p_2}^* = 0, \tag{6.24}$$

$$\frac{\partial \hat{Z}^2(x^*, q_{07}^{2*})}{\partial q_{07}^2} - \frac{\partial \hat{\rho}_{21}(x^*, q_0^*)}{\partial q_{07}^2} x_{p_2}^* = 0. \tag{6.25}$$

Utilizing the functions for this example and $(6.18a) - (6.18f)$, the following can be constructed:

$$\frac{\partial \hat{C}^1(x^*)}{\partial x_{p_1}} = 10x_{p_1}^* + 10 + 4x_{p_1}^* + 4x_{p_1}^* + 1 + 6x_{p_1}^* + 2x_{p_1}^* + 1 = 26x_{p_1}^* + 12,$$

$$\frac{\partial \hat{Z}^1(x^*, q_{01}^{1*})}{\partial x_{p_1}} = 2x_{p_1}^* + 8,$$

$$-\hat{\rho}_{11}(x^*, q_0^*) = 2x_{p_1}^* + x_{p_2}^* - \frac{(q_{01}^{1*} - 0.3066)x_{p_1}^*}{x_{p_1}^*} - 100 = 2x_{p_1}^* + x_{p_2}^* - q_{01}^{1*} - 99.6934,$$

$$-\frac{\partial \hat{\rho}_{11}(x^*, q_0^*)}{\partial x_{p_1}} x_{p_1}^* = 2x_{p_1}^*,$$

$$\frac{\partial \hat{Z}^1(x^*, q_{01}^{1*})}{\partial q_{01}^1} = 3,$$

$$-\frac{\partial \hat{\rho}_{11}(x^*, q_0^*)}{\partial q_{01}^1} x_{p_1}^* = -x_{p_1}^*.$$

Analogous expressions for path p_2 are:

$$\frac{\partial \hat{C}^2(x^*)}{\partial x_{p_2}} = 2x_{p_2}^* + 1 + 6x_{p_2}^* + 1 + 4x_{p_2}^* + 12x_{p_2}^* + 1 + 12x_{p_2}^* + 1 = 36x_{p_2}^* + 4,$$

$$\frac{\partial \hat{Z}^2(x^*, q_{07}^{2*})}{\partial x_{p_2}} = 2x_{p_2}^*,$$

$$-\hat{\rho}_{21}(x^*, q_0^*) = 3x_{p_2}^* + x_{p_1}^* - \frac{(q_{07}^{2*} - 0.0674)x_{p_2}^*}{x_{p_2}^*} - 90 = 3x_{p_2}^* + x_{p_1}^* - q_{07}^{2*} - 89.9326,$$

$$-\frac{\partial \hat{\rho}_{21}(x^*, q_0^*)}{\partial x_{p_2}} x_{p_2}^* = 3x_{p_2}^*,$$

$$\frac{\partial \hat{Z}^2(x^*, q_{07}^{2*})}{\partial q_{07}^2} = 3,$$

$$-\frac{\partial \hat{\rho}_{21}(x^*, q_0^*)}{\partial q_{07}^2} x_{p_2}^* = -x_{p_2}^*.$$

Grouping the terms above corresponding to each equation $(6.22) - (6.25)$ obtain the following system of equations are obtained:

$$32x^*_{p_1} + x^*_{p_2} - q^{1*}_{01} = 79.6934,$$

$$3 - x^*_{p_1} = 0,$$

$$x^*_{p_1} + 44x^*_{p_2} - q^{2*}_{07} = 85.9326,$$

$$3 - x^*_{p_2} = 0,$$

with solution:

$$x^*_{p_1} = 3, \ x^*_{p_2} = 3, \quad q^{1*}_{01} = 19.3066, \ q^{2*}_{07} = 49.0674.$$

Hence, the path quality levels are: $q_{p_1} = 19$ and $q_{p_2} = 49$, the demand prices are: $\rho_{11} = 110$ and $\rho_{21} = 127$, with Food Firm 1 enjoying a profit (in dollars) of $\hat{U}_1(X^*, q^*_0) = 87.0000$ and Food Firm 2 a profit of $\hat{U}_2(X^*, q^*_0) = 51.0000$.

Observe that Food Firm 1 has a higher profit than Food Firm 2, although its fresh produce is of lower quality, both at its production site and at the retail outlet.

Next, I present an example with exponential quality decay.

6.1.3 Example 6.1b: Exponential Quality Decay (First Order Kinetics)

Example 6.1b is constructed from Example 6.1a and has the same data except that the product now has an exponential quality decay with a reaction order $n = 1$. The quality levels of the paths are constructed using the appropriate expression in (5.8) and the β_b values in Table 6.1, for $n = 1$ for $i = 1, 2$, yielding:

$$q_{p_1} = q^1_{01} \times \beta_2 \times \beta_3 \times \beta_4 \times \beta_5 \times \beta_6 = (q^1_{01})(0.7359),$$

$$q_{p_2} = q^2_{07} \times \beta_8 \times \beta_9 \times \beta_{10} \times \beta_{11} \times \beta_{12} = (q^2_{07})(0.9418).$$

I now proceed to solve the equations (6.22) – (6.25) for this example, with the following terms for paths p_1 and p_2 presented for completeness and convenience:

$$-\hat{\rho}_{11}(x^*, q^*_0) = 2x^*_{p_1} + x^*_{p_2} - \frac{(q^{1*}_{01})(0.7359)(x^*_{p_1})}{x^*_{p_1}} - 100 = 2x^*_{p_1} + x^*_{p_2} - (q^{1*}_{01})(0.7359) - 100,$$

$$-\frac{\partial \hat{\rho}_{11}(x^*, q_0^*)}{\partial q_{01}^1} x_{p_1} = -0.7359 x_{p_1}^*,$$

$$-\hat{\rho}_{21}(x^*, q_0^*) = 3x_{p_2}^* + x_{p_1}^* - \frac{(q_{07}^{2*})(0.9418)(x_{p_2}^*)}{x_{p_2}^*} - 90 = 3x_{p_2}^* + x_{p_1}^* - (q_{07}^{2*})(0.9418) - 90,$$

$$-\frac{\partial \hat{\rho}_{21}(x^*, q_0^*)}{\partial q_{07}^2} x_{p_2} = -0.9418 x_{p_2}^*.$$

Grouping the terms above corresponding to each equation, I obtain the following system of equations:

$$32x_{p_1}^* + x_{p_2}^* - 0.7359 q_{01}^{1*} = 80,$$
$$3 - 0.7359 x_{p_1}^* = 0,$$

$$x_{p_1}^* + 44x_{p_2}^* - 0.9418 q_{07}^{2*} = 86,$$

$$3 - 0.9418 x_{p_2}^* = 0.$$

Straightforward calculations yield the following equilibrium path flows and equilibrium initial quality levels:

$$x_{p_1}^* = 4.0766, \ x_{p_2}^* = 3.1854, \quad q_{01}^{1*} = 72.8857, \ q_{07}^{2*} = 61.8329.$$

The path quality levels are, hence, $q_{p_1} = 53.6366$ and $q_{p_2} = 58.2342$.

Notice that, even though the initial quality of Food Firm 1's fresh produce is higher, Food Firm 2 sells its fresh produce with a higher quality at the retail outlet.

By substituting the equilibrium path flows and the equilibrium initial qualities into the demand price functions, I obtain the following equilibrium demand prices at the retail outlet for Food Firm 1 and Food Firm 2, respectively: $\rho_{11} = 142.30$ and $\rho_{21} = 134.60$. The price of Food Firm 2's fresh produce is higher than that of Food Firm 1's. Furthermore, the profits of the food firms are calculated, in dollars, as $\hat{U}_1(X^*, q_0^*) = 47.2497$ and $\hat{U}_2(X^*, q_0^*) = 37.7258$. Notice that, when the quality decay becomes exponential, the profits of the food firms decrease significantly. It would be reasonable to expect that food firms invest more to keep the initial quality high, hence, the end quality would not get too low with a faster quality degradation. This causes the total cost to become higher, therefore the the profits to decrease.

6.2. The Algorithm

Here I outline the computational procedure used to solve the examples in Section 3.3. Specifically, the variational inequality (6.26) is amenable to solution via the Euler method of Dupuis and

Nagurney (1993) which is described in Section 2.4. Below I present the closed form expression for the path flows and initial quality levels.

6.2.1 Explicit Formulae for the Euler Method Applied to the Model

The closed form expressions for the fresh produce path flows at iteration $\tau + 1$ are as follows. For each path $p \in P_k^i$, $\forall i, k$, compute:

$$x_p^{\tau+1} = \max\{0, x_p^\tau + \alpha_\tau(\hat{\rho}_{ik}(x^\tau, q_0^\tau) + \sum_{j=R_1}^{R_{n_R}} \frac{\partial \hat{\rho}_{ij}(x^\tau, q_0^\tau)}{\partial x_p} \sum_{r \in P_j^i} x_r^\tau - \frac{\partial \hat{Z}^i(x^\tau, q_0^{i\tau})}{\partial x_p} - \frac{\partial \hat{C}^i(x^\tau)}{\partial x_p} - \sum_{l \in L^i} \gamma_l^\tau \delta_{lp})\}.$$
(6.27)

For each initial quality level $a \in L_1^i$, $\forall i$, in turn, compute:

$$q_{0a}^{i\tau+1} = \max\{0, q_{0a}^{i\tau} + \alpha_\tau(\sum_{j=R_1}^{R_{n_R}} \frac{\partial \hat{\rho}_{ij}(x^\tau, q_0^\tau)}{\partial q_{0a}^i} \sum_{r \in P_j^i} x_r^\tau - \frac{\partial \hat{Z}^i(x^\tau, q_0^{i\tau})}{\partial q_{0a}^i} - \lambda_a^\tau)\}.$$
(6.28)

The Lagrange multiplier for each top-most link $a \in L_1^i$; $i = 1, \ldots, I$, associated with the initial quality bounds is computed as:

$$\lambda_a^{\tau+1} = \max\{0, \lambda_a^\tau + \alpha_\tau(q_{0a}^{i\tau} - \bar{q}_{0a}^i)\}.$$
(6.29)

Finally, the Lagrange multiplier for each link $l \in L^i$; $i = 1, \ldots, I$, associated with the link capacities is computed according to:

$$\gamma_l^{\tau+1} = \max\{0, \gamma_l^\tau + \alpha_\tau(\sum_{r \in P} x_r^\tau \delta_{lr} - u_l)\}.$$
(6.30)

The expressions (6.27) through (6.30) may be interpreted as a discrete-time adjustment or tatonnement process with the food firms updating at each discrete point in time their fresh produce path flows, their initial quality levels as well as the Lagrange multipliers associated with the initial quality levels at the production sites, and the Lagrange multipliers associated with the link capacities, until an equilibrium is achieved. Observe that these computations can all be done simultaneously and, hence, in parallel. Moreover, at each iteration, only the iterates from the preceding iteration (point in time) are needed for these computations.

Next, I present larger numerical examples comprising a case study focusing on peaches.

154

6.3.   A Case Study of Peaches

In this section, I focus on the peach market in the United States, specifically in Western Massachusetts. Peaches [*Prunus persica (L.)* Batsch] are very vital for fresh produce markets in the United States and also all over the world. It is noted that, in 2015, the United States peach production was 825,415 tons in volume, and 606 million dollars in worth (USDA NASS (2016), Zhao et al. (2017)). For the case study, I selected two orchards from Western Massachusetts: Apex Orchards and Cold Spring Orchard, located, respectively, in Shelburne, MA and Belchertown, MA. The supply chain network topology for the peach case study is illustrated in Figure 6.3. It is assumed that the Apex Orchards farm has two production sites, two processors, and two distribution centers. After production/harvesting, Apex Orchards can ship its peaches to processors $C_{1,1}^1$ or $C_{2,1}^1$, with shipment depicted via links 4, 5, 6, and 7 in Figure 3. Similarly, after processing, Apex Orchards can transfer its peaches to $D_{1,1}^1$ or $D_{2,1}^1$, as shown in Figure 6.3 via links 12, 13, 14, and 15. Cold Spring Orchard is smaller in size and, therefore, it only has only a single production site, one processor, and a single distribution center. Both of the orchards sell their peaches to two retailers, Whole Foods, located in Hadley, MA, and Formaggio Kitchen, located in Cambridge, MA. The mode of transportation for both of the orchards is trucks.

According to Toralles et. al (2015), the color change attribute of peaches, in the form of browning, follows a first-order, that is, an exponential decay function. In Table 6.2, following (5.6), I calculate the link quality degradation, β_b, when the reaction order is $n = 1$, for the supply chain network topology in Figure 6.3. The universal gas constant and activation energy are taken as 8.314 $Jmol^{-1}K^{-1}$ and 147.9 $kJmol^{-1}$, respectively (Toralles et al. (2005)). The harvesting season for peaches is usually mid-July to mid -September in Massachusetts, and the temperature reported in Table 6.3, for the processing, shipment, and distribution links, is the average temperature of these months. It is assumed that, since the Apex Orchards farm is located at a higher altitude, that the average temperature of Apex's operations is lower.

It is assumed that the shipment and distribution operations are made at an average temperature, since the orchards do not have the necessary technology in their trucks to keep the temperature at sufficient levels to deter quality degradation. Furthermore, according to Crisosto and Valero (2008) the ideal storage temperature of peaches is between -1 C$^\circ$ and 1 C$^\circ$. In this case study, the Apex Orchards owner is assumed to have the technology to keep the storage temperature at 1 C$^\circ$ and Cold Spring Orchard is assumed to decrease the storage temperature only to 18 C$^\circ$. The product

Apex Orchards

Cold Spring Orchard

Production/Harvesting

Shipment

Processing

Shipment

Storage

Distribution

Whole Foods

Formaggio Kitchen
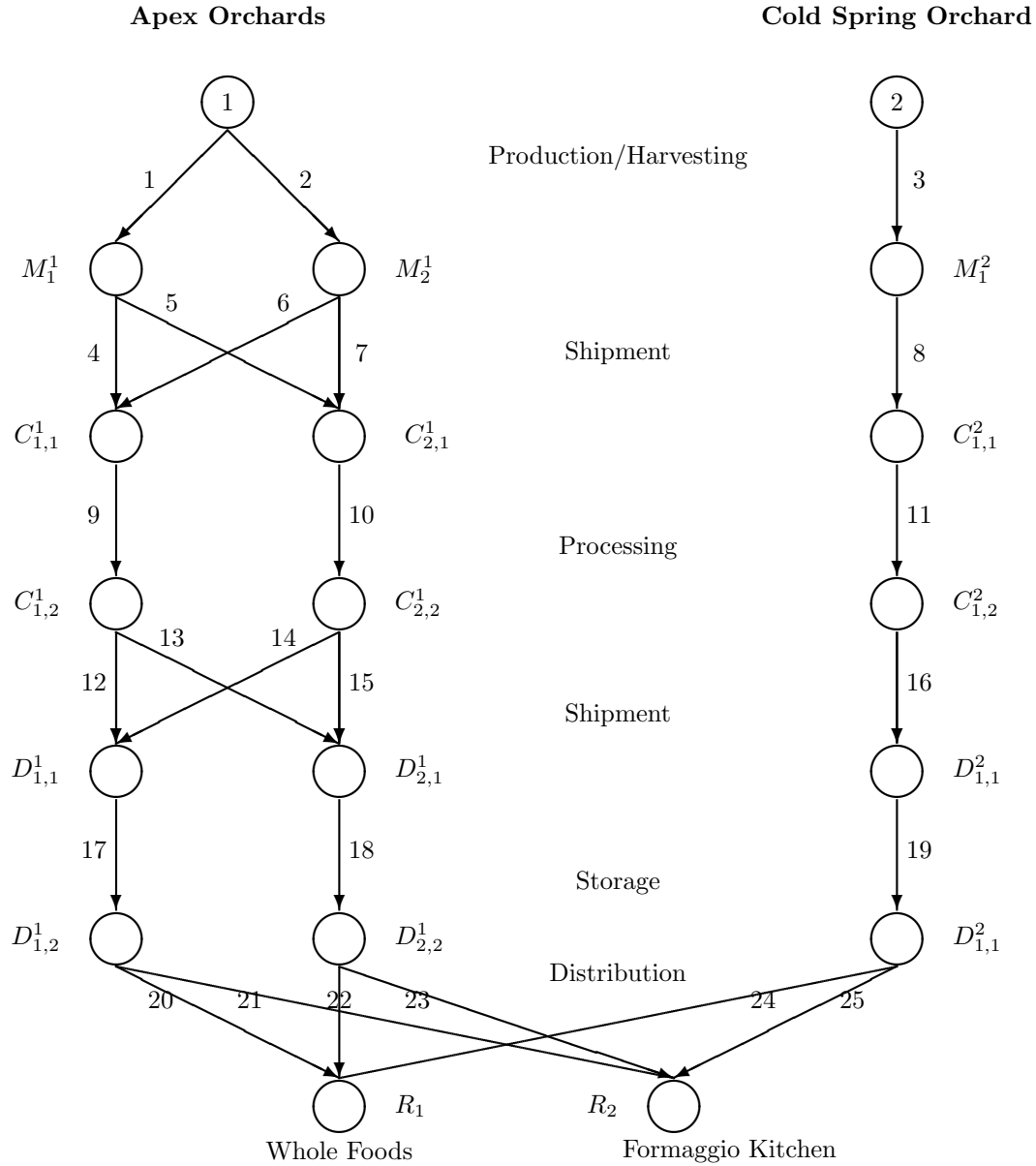
Figure 6.3. Supply Chain Network Topology of the Case Study Focusing on Peaches

flows and capacities are in pecks and I emphasize that a peck of peaches is approximately 12 pounds of peaches.

In Tables 6.3 and 6.4, I report the total production / harvesting cost functions, the upper bounds on the initial quality, the total operational cost functions, and the link flow capacities. I constructed the cost functions, in Tables 6.3 and 6.4, through the data, gathered from Sumner and Murdock (2017), in which the authors made a sample cost analysis. I also utilize from Dris and Jain (2007)

Link b	Hours	Temperature (Celsius)	β_b $(n=1)$
4	1	23	0.9961
5	2	23	0.9922
6	2	23	0.9922
7	1	23	0.9961
8	2	27	0.9913
9	3	18	0.9906
10	3	18	0.9906
11	4	25	0.9836
12	1	23	0.9961
13	2	23	0.9922
14	2	23	0.9922
15	1	23	0.9961
16	3	27	0.9870
17	48	1	1.0000
18	72	1	1.0000
19	96	18	0.7397
20	2	27	0.9913
21	4	27	0.9827
22	1	27	0.9956
23	4	27	0.9827
24	0.5	27	0.9978
25	4	27	0.9827

Table 6.2. Parameters for the Calculation of Quality Decay for the Peach Case Study

to construct the total storage link cost functions. The time horizon, under consideration, is that of a week.

Link a	$\hat{z}_a(f_a, q_{0a}^i)$	u_a	$\bar{q}_{0a}^i$
1	$.002f_1^2 + f_1 + 0.7q_{01}^1 + .01(q_{01}^1)^2$	200	98
2	$.002f_2^2 + f_2 + 0.7q_{02}^1 + .01(q_{02}^1)^2$	200	95
3	$.002f_3^2 + f_3 + 0.5q_{03}^2 + .001(q_{03}^1)^2$	150	90

Table 6.3. Total Production / Harvesting Cost Functions, Link Capacities, and Upper Bounds on Initial Quality

The Euler method (cf. (6.27) – (6.30)) is implemented in FORTRAN and a Linux system at the University of Massachusetts used for the computations. The sequence $a_\tau = \{1, \frac{1}{2}, \frac{1}{2}, \frac{1}{3}, \ldots\}$, with the convergence tolerance being 10^{-7}, that is, the Euler method is deemed to have converged if the absolute value of the difference of each successive variable iterate differs by no more than this value. The algorithm was initialized with each path flow set equal to 1, each Lagrange multiplier set equal to 0, and each initial quality level set to 50.

Link b	$\hat{c}_b(f)$	u_b
4	$.001f_4^2 + .7f_4$	150
5	$.002f_5^2 + .7f_5$	150
6	$.001_6^2 + .5f_6$	120
7	$.002f_7^2 + .5f_7$	120
8	$.002f_8^2 + .9f_8$	100
9	$.0025f_9^2 + 1.2f_9$	200
10	$.0025f_{10}^2 + 1.2_{10}$	200
11	$.0026f_{11}^2 + 1.5f_{11}$	150
12	$.001f_{12}^2 + .6f_{12}$	150
13	$.002f_{13}^2 + .6f_{13}$	150
14	$.001f_{14}^2 + .6f_{14}$	150
15	$.002f_{15}^2 + .6f_{15}$	150
16	$.002f_{16}^2 + .6f_{16}$	120
17	$.003f_{17}^2 + .5f_{17}$	150
18	$.0037f_{18}^2 + .9f_{18}$	150
19	$.002f_{19}^2 + .7f_{19}$	120
20	$.002f_{20}^2 + .6f_{20}$	150
21	$.003f_{21}^2 + .7f_{21}$	120
22	$.002f_{22}^2 + .6f_{22}$	150
23	$.003f_{23}^2 + .7f_{23}$	100
24	$.002f_{24}^2 + .6f_{24}$	100
25	$.003f_{25}^2 + .7f_{25}$	100

Table 6.4. Total Operational Link Cost Functions and Link Capacities

6.3.1 Example 6.1 - Baseline

It is assumed that the consumers at the retailers are discerning about the quality of the peaches that they are buying. The demand price functions are constructed for Apex Orchards and Cold Spring Orchard based on information from the orchards themselves and also the retailers, along with observed prices of peaches at the retail level. In the case study, both of the orchards sell their peaches to two retailers, Whole Foods, located in Hadley, MA, and Formaggio Kitchen, located in Cambridge, MA. It is known that both retailers sell high quality food products, with Formaggio Kitchen selling peaches at a higher price due to its emphasis on quality. Therefore, in the demand price functions, the coefficients of the average quality levels, representing the sensitivity to the food quality, are higher for Formaggio Kitchen than those for Whole Foods. Furthermore, through conversations at the retailers, it can be concluded that Apex Orchards sell their peaches at a higher price. Therefore, the demand price functions for Apex Orchards include higher constant terms than those in Cold Spring Orchard's demand price functions. The corresponding demand price functions for the peaches of Apex Orchards and of Cold Spring Orchard, at the retailers R_1 and R_2, are as follows:

Apex Orchards:

$$\rho_{11} = -.02d_{11} - .01d_{21} + 0.008\hat{q}_{11} + 20, \quad \rho_{12} = -.02d_{12} - .01d_{22} + 0.01\hat{q}_{12} + 22;$$

Cold Spring Orchard:

$$\rho_{21} = -.02d_{21} - .015d_{11} + 0.008\hat{q}_{21} + 18, \quad \rho_{22} = -.02d_{22} - .015d_{12} + 0.01\hat{q}_{22} + 19.$$

In Tables 6.5 and 6.6, I report the equilibrium solution. The equilibrium link flows rather than the equilibrium path flows are reported for compactness.

Link a	f_a^*	q_{0a}^{i*}	γ_a^*	λ_a^*
1	133.43	97.54	0.00	0.00
2	166.57	95.00	0.00	0.05
3	100.00	65.61	0.00	0.00

Table 6.5. Example 6.1 Equilibrium Link Flows, Equilibrium Initial Quality, and the Equilibrium Production Site Lagrange Multipliers

The demand price of the peaches, evaluated at the computed equilibrium solution, for each orchard, in dollars, per peck, is as follows:

Apex Orchards:

$$\rho_{11} = 17.67, \quad \rho_{12} = 19.00,$$

Cold Spring Orchard:

$$\rho_{21} = 15.47, \quad \rho_{22} = 15.86,$$

with the computed equilibrium demands being: $d_{11}^* = 129.95, d_{12}^* = 170.05, d_{21}^* = 47.80, d_{22}^* = 52.20$.

These prices (recall that they are per peck) are very reasonable.

The average quality of the peaches of the orchards at the retailers, at the equilibrium, is:

Apex Orchards:

$$\hat{q}_{11} = 93.40, \quad \hat{q}_{12} = 92.56,$$

Cold Spring Orchard:

$$\hat{q}_{21} = 46.60, \quad \hat{q}_{22} = 45.90.$$

The profits of the orchards, in dollars, at the equilibrium solution, are:

Link b	f_b^*	γ_b^*
4	69.31	0.00
5	64.12	0.00
6	90.33	0.00
7	76.24	0.00
8	100.00	6.53
9	159.64	0.00
10	140.36	0.00
11	100.00	0.00
12	81.96	0.00
13	77.68	0.00
14	68.04	0.00
15	72.32	0.00
16	100.00	0.00
17	150.00	6.95
18	150.00	6.19
19	100.00	0.00
20	64.84	0.00
21	85.16	0.00
22	65.10	0.00
23	84.90	0.00
24	47.80	0.00
25	52.20	0.00

Table 6.6. Example 6.1 Equilibrium Link Flows and the Equilibrium Link Lagrange Multipliers

$$U_1 = 3,302.01, \quad U_2 = 787.65.$$

Recall that the time period in question is that of a week. Notice that the Apex Orchards farm enjoys a higher profit by selling its peaches at higher prices and at a higher average quality.

Observe from Table 6.5 that the equilibrium initial quality at Apex Orchards' production site corresponding to link 1 is at its upper bound and, hence, the corresponding Lagrange multiplier λ_1^* is positive. In addition, note that the flows on both links 17 and 18 corresponding to Apex Orchards' storage facilities are at their upper bounds, and, therefore, the associated link Lagrange multipliers γ_{17}^* and γ_{18}^* are positive. Finally, the flow on link 8 associated with shipment of the peaches from Cold Spring Orchard's production site is at its capacity and, therefore, the corresponding Lagrange multiplier γ_8^* is also positive. The orchards may wish to invest in enhancing their capacity with Apex Orchards focusing on the storage facilities and Cold Spring Orchard on its freight shipment capacity.

Indeed, when u_8 is raised to 150, while keeping all the other data as above, the profit of Cold Spring orchard increased to 921.74 whereas that of Apex Orchards (because of the competition) decreased to 3,272.11.

On the other hand, when both u_{17} and u_{18} are raised to 200 and kept all the other data as in Example 6.1 above, then the profit enjoyed by Apex Orchards increased to 3,884.80 and that of Cold Spring Orchard decreased to 696.87.

Finally, $u_8 = 150$ and u_{17} and u_{18} both are set to 200 with the remainder of the data as in Example 6.1. The profit garnered by Apex Orchards was now 3,844.89 and that of Cold Spring: 815.37. Both firms gain as compared to the profit values in Example 6.1. Interestingly, the demand prices were now lower but the average quality higher with $\rho_{11} = 16.54$, $\rho_{12} = 17.94$, $\rho_{21} = 14.64$, and $\rho_{22} = 15.10$, and $\hat{q}_{11} = 93.79$, $\hat{q}_{12} = 92.92$, $\hat{q}_{21} = 63.93$, and $\hat{q}_{22} = 67.96$. Hence, by investing in supply chain infrastructure both producers and consumers gain.

Example 6.2 - Disruption Scenario 1

Example 6.2 is constructed from Example 6.1. It is now considered a disruption scenario in which a natural disaster has significantly affected the capacity of the orchard production sites of both orchards. Such an incident occurred in 2016 in the Northeast of the United States when extreme weather in terms of cold temperatures "decimated" the peach crop (cf. Tuohy (2016)).

Example 6.2 has the same data as Example 6.1 except for the following changes to capture the impacts of the natural disaster. I now have the following capacities on the production/harvesting links: $u_1 = 100$, $u_2 = 150$, and $u_3 = 80$.

The computed equilibrium solution is reported in Tables 6.7 and 6.8.

Link a	f_a^*	q_{0a}^{i*}	γ_a^*	λ_a^*
1	100.00	75.54	8.17	0.00
2	150.00	75.54	8.18	0.00
3	80.00	11.02	7.78	0.00

Table 6.7. Example 6.2 Equilibrium Link Flows, Equilibrium Initial Quality, and the Equilibrium Production Site Lagrange Multipliers

The demand price of the peaches, evaluated at the computed equilibrium solution, for each orchard, in dollars, per peck, is as follows:

Link b	f_b^*	γ_b^*
4	50.28	0.00
5	49.72	0.00
6	82.76	0.00
7	67.24	0.00
8	80.00	0.00
9	133.04	0.00
10	116.96	0.00
11	80.00	0.00
12	80.78	0.00
13	52.25	0.00
14	69.22	0.00
15	47.75	0.00
16	80.00	0.00
17	150.00	0.17
18	100.00	0.00
19	80.00	0.00
20	67.2	0.00
21	82.79	0.00
22	37.46	0.00
23	62.54	0.00
24	37.67	0.00
25	42.33	0.00

Table 6.8. Example 6.2 Equilibrium Link Flows and the Equilibrium Link Lagrange Multipliers

Apex Orchards:

$$\rho_{11} = 18.12, \quad \rho_{12} = 19.40,$$

Cold Spring Orchard:

$$\rho_{21} = 15.74, \quad \rho_{22} = 16.05,$$

with the computed equilibrium demands being: $d_{11}^* = 104.67$, $d_{12}^* = 145.33$, $d_{21}^* = 37.67$, $d_{22}^* = 42.33$.

The average quality of the peaches of the orchards at the retailers, at the equilibrium, is:

Apex Orchards:

$$\hat{q}_{11} = 73.42, \quad \hat{q}_{12} = 72.68,$$

Cold Spring Orchard:

$$\hat{q}_{21} = 7.83, \quad \hat{q}_{22} = 7.71.$$

The profits of the orchards, in dollars, at the equilibrium solution, are:

$$U_1 = 2,984.07, \quad U_2 = 675.72.$$

Observe from the above equilibrium solution that all the production sites are now at their capacities and, hence, the corresponding link Lagrange multipliers are all positive. Also, observe that the average quality of each orchard's peaches has decreased at each retailer, as compared to the results for Example 6.1. The demand prices have increased but more for the peaches of Apex Orchards than those from Cold Spring Orchard. As expected, the profit is reduced for both orchards because of the limitations on how many pecks of peaches they can produce and harvest due to the disruption caused by the natural disaster.

6.3.2 Example 6.3 - Disruption Scenario 2

Example 6.3 is also constructed from the baseline Example 6.1 but now I illustrate how another type of supply chain disruption can be analyzed within the model. In particular, I consider a disruption that affects transportation in that the links 5 and 6 associated with the supply chain network of Apex Orchards (cf. Figure 6.3) are no longer available. This can occur and has occurred in western Massachusetts as a result of flooding. In order to handle this situation, I keep the data as in Example 6.1 but the upper bounds on these links are now set to zero so that: $u_5 = 0$ and $u_6 = 0$.

The computed new equilibrium solution is reported in Tables 6.9 and 6.10.

Link a	f_a^*	q_{0a}^{i*}	γ_a^*	λ_a^*
1	150.00	84.50	0.00	0.00
2	120.00	84.50	0.00	0.00
3	100.00	65.59	0.00	0.00

Table 6.9. Example 6.3 Equilibrium Link Flows, Equilibrium Initial Quality, and the Equilibrium Production Site Lagrange Multipliers

The demand price of the peaches, evaluated at the computed equilibrium solution, for each orchard, in dollars, per peck, is:

Apex Orchards:

$$\rho_{11} = 17.89, \quad \rho_{12} = 19.19,$$

Cold Spring Orchard:

$$\rho_{21} = 15.69, \quad \rho_{22} = 16.09,$$

with the computed equilibrium demands being: $d_{11}^* = 114.74$, $d_{12}^* = 155.26$, $d_{21}^* = 47.86$, $d_{22}^* = 52.14$.

Link b	f_b^*	γ_b^*
4	150.00	6.94
5	0.00	78.33
6	0.00	79.92
7	120.00	7.27
8	100.00	6.75
9	150.00	0.00
10	120.00	0.00
11	100.00	0.00
12	85.36	0.00
13	64.64	0.00
14	64.64	0.00
15	55.36	0.00
16	100.00	0.00
17	150.00	0.40
18	120.00	6.19
19	100.00	0.00
20	66.22	0.00
21	83.78	0.00
22	48.52	0.00
23	71.48	0.00
24	47.86	0.00
25	52.14	0.00

Table 6.10. Example 6.3 Equilibrium Link Flows and the Equilibrium Link Lagrange Multipliers

The average quality of the peaches of the orchards at the retailers, at the equilibrium, is:

Apex Orchards:

$$\hat{q}_{11} = 82.32, \quad \hat{q}_{12} = 81.46,$$

Cold Spring Orchard:

$$\hat{q}_{21} = 46.59, \quad \hat{q}_{22} = 45.88.$$

The profits of the orchards, in dollars, at the equilibrium solution, are:

$$U_1 = 3,074.72, \quad U_2 = 811.35.$$

Apex Orchards farm experiences a loss in profits, whereas its competitor, Cold Spring Orchards, garners a higher profit, as compared to the baseline Example 6.1. Both orchards raise their prices and the average quality of their produce drops although much more significantly for Apex Orchards, which has suffered a supply chain disruption in terms of transportation/shipment possibilities.

I then addressed the following questions: What would be the impact on profits if only link 5 was restored to its original capacity of 150 (and link 6 remained unavailable)? What would be the

impact on profits if only link 6 was restored to its original capacity of 120 (and link 5 remained unavailable)? I found the following: The profit of Apex Orchards was 3,272.78 with link 5 restored only and that of Cold Spring Orchard was: 787.64. On the other hand, if link 6 was restored only, then Apex Orchards garnered 3,283.32 in profit and Cold Spring Orchard 787.67 in profit. Hence, given the choice, Apex Orchards should advocate for restoration of link 6 versus link 5 if only one link restoration is feasible.

6.4.   Summary and Conclusions

In this chapter, I constructed a general framework for the modeling, analysis, and computation of solutions to competitive fresh produce supply chain networks in which food firm owners seek to maximize their profits while determining both the initial quality of the fresh produce with associated costs as well as the fresh produce flows along pathways of their supply chain network through the various activities of harvesting, processing, storage, and distribution. In this framework, I utilize explicit formulae associated with quality deterioration on the supply chain network links which are a function of physical characteristics, including temperature and time. The prices at the retailers are a function not only of the demand for the produce but also of the average quality level of the produce at the retailers.

I provide the governing Nash Equilibrium conditions and the alternative variational inequality formulations. The algorithm is presented, which shows a discrete-time adjustment process, and which yields closed form expressions at each iteration for the product path flows, the initial quality levels, as well as the Lagrange multipliers associated with the link capacities and the initial quality upper bounds. I present examples to illustrate the mathematical framework and provide a case study on peaches, consisting of numerical examples under status quo and disruption scenarios.

CHAPTER 7

CONCLUSIONS AND FUTURE RESEARCH

7.1.   Conclusions

In this dissertation, my goal was to study the challenging issues on supply chains in the food industry. I focused on two main issues: impacts of trade and policy instruments, and the fresh produce quality deterioration. The mathematical models in this dissertation were based on the economics of competitive supply chain networks. The results that I observed through simulated case studies with various scenarios can inform policy makers.

In this dissertation, I contributed to the literature on variational inequalities, game theory, and supply chain network equilibrium modeling. In Chapter 3 of this dissertation, I provided a unified variational inequality framework for the modeling, analysis, and computation of solutions to a general spatial network equilibrium problem in the presence of tariff-rate quotas. The supply chain network equilibrium model considers multiple countries and regions in each country. In Chapter 4, I extended the mathematical model constructed in Chapter 3 by incorporating quality as a strategic variable. In this chapter, I also studied an imperfectly competitive model in the form of an oligopoly. In Chapter 5, I presented a modeling and algorithmic framework for competitive farmers' markets by integrating fresh produce quality deterioration explicitly by chemical formulae depending on time and temperature. I extended this model in Chapter 6 by including initial quality of the food product as a strategic variable.

7.2.   Future Research

I intend to construct a global food supply chain equilibrium model through the integration of trade instruments and exchange rates while capturing the quality deterioration of perishable food products. I will specifically focus on banana supply chain networks as an example of global food chain networks. My goal in the future is to connect the previous chapters of this dissertation, focusing both on the modeling of trade instruments and fresh produce quality deterioration, in a single unifying mathematical modeling framework.

Bananas, the most consumed fruit in the United States, greater than the amount of apples and oranges combined (Koeppel (2008)), are excellent sources for gaining necessary vitamins, and minerals such as potassium, magnesium, sodium, and iron (UN (2016)). Bananas are also relatively cheap and are available in many parts of the world. These advantages make bananas one of most frequently bought fruit, accounting for about 14.1 million metric tons export volume in 2006 (Liu (2009)).

As mentioned in Chapter 1, global food supply chain networks are very unique, since the products have a perishable nature. Banana supply chain networks are very similar to other fresh produce supply chain networks, in terms of their perishable nature, but they are also very distinctive with their high consumption levels. This creates a highly competitive environment, coupled with high quality expectations, and the logistical challenges emerging from the long distances.

The two biggest banana importers in the world are the US and the European Union, whereas the biggest exporter of bananas is Ecuador. The other major export countries of bananas are: Costa Rica, the Dominican Republic, Colombia, and Peru (Liu (2009)). Hence, in the banana supply chain network that I plan on formulating, the first tier will consist of farms located in countries such as: the Dominican Republic, Ecuador, Peru, Colombia, Costa Rica, and Honduras. Furthermore, the demand points will be the supermarkets in the countries such as: the United States, the United Kingdom, Canada, Germany, or France.

Trade instruments in the form of tariff rate quotas are being used frequently in the global banana trade. In the 2000s, the European Union and Latin American banana exporters were engaged in a dispute over banana trade, in which they reached an agreement resulting in the decrease of tariffs gradually, beginning in 2011. However, according to Vega (2011), Ecuador still faces high import duties while entering the European markets. As I also mention in Section 1.4.5, exchange rates are also affecting the fresh produce trade negatively, resulting from the currency fluctuations in Latin America.

It would be of great value to study the effects of trade instruments, and quality deterioration in global banana trade. I will also attempt to integrate exchange rates into the modeling of a banana supply chain network model. I also wish to include another tier of decision makers, in the form of wholesalers, and to investigate the impacts on quality deterioration. The results obtained from this research can inform food firms, policy makers, and regulators.

BIBLIOGRAPHY

Aamir, M., Ovissipour, M., Sablani, S.S., Rasco, B., 2013. Predicting the quality of pasteurized vegetables using kinetic models: a review. International Journal of Food Science, Article ID 271271.

Abbott, J.A., 1999. Quality measurement of fruits and vegetables. Postharvest Biology and Technology, 15(3), 207-225.

Ahumada, O., Villalobos, J.R., 2007. Application of planning models in the agri-food supply chain: A review. European Journal of Operational Research, 196(1), 1-20.

Ahumada, O., Villalobos, J.R., 2011. A tactical model for planning the production and distribution of fresh produce. Annals of Operations Research, 190(1), 339-358.

Aiello, G., La, Scalia G., Micale, R., 2012. Simulation analysis of cold chain performance based on time-temperature data. Production Planning & Control, 23(6), 468-476.

Akkerman, R., Farahani, P., Grunow, M., 2010. Quality, safety and sustainability in food distribution: A review of quantitative operations management approaches and challenges. OR Spectrum, 32(4), 863-904.

Alliance, F.C.W., 2016. No Piece of the Pie: US Food Workers. Solidarity Research Cooperative.

Amorum, P., Costa A.M., Almada-Lobo, B., 2014. Influence of consumer purchasing behaviour on the production planning of perishable food. OR Spectrum, 36(3), 669-692.

Appelbaum, B., 2018. Their soybeans piling up, farmers' hope trade war ends before beans rot. The New York Times, November 5.

Aregu, R., 2014. Market and Price Decision Enhancement Services for Farmers in Uganda. Doctoral Dissertation, University of Groningen, Groningen, The Netherlands.

Arrhenius, S., 1889. Über die reaktionsgeschwindigkeit bei der inversion von rohrzucker durch suren. Zeitschrift Für Physikalische Chemie, 4, 226-248.

Arrow, K.J., Debreu, G., 1954. Existence of an equilibrium for a competitive economy. Econometrica, 22, 265-290.

Asebo, K., Moxnes J. A., Lieblein, G., Svennerud, M., Francis, C., 2007. Farmer and consumer attitudes at farmers' markets in Norway. Journal of Sustainable Agriculture, 30(4), 67-93.

Ash, M.S., Livezey, J., Dohlman, E.N., 2006. Soybean backgrounder. US Department of Agriculture, Economic Research Service, Washington DC.

Asmuth, R., Eaves, C.B., Peterson, E.L., 1979. Computing economic equilibria on affine networks with Lemke's algorithm. Mathematics of Operations Research, 4(3), 209-214.

Aw, B.Y., Roberts, M.J., 1986. Measuring quality in quota-constrained import markets: The case of U.S. footwear. Journal of International Economics, 21(1/2), 45-60.

Babiak, L., 2013. Exploring Local Food System Practices and Perceptions: Insights from Florida's SNAP Authorized Farmers' Markets. Master of Science Thesis. Department of Geography, Environment, and Planning, College of Arts and Sciences, University of South Florida, Tampa, Florida.

Baiocchi, C., Capelo, A., 1984. Variational and Quasi-variational Inequalities J. Wiley and Sons, New York.

Barrett, D.M., Beaulieu, J.C., Shewfelt, R., 2010. Color, flavor, texture, and nutritional quality of fresh-cut fruits and vegetables: desirable levels, instrumental and sensory measurement, and the effects of processing. Critical Reviews in Food Science and Nutrition, 50(5), 369-389.

Bensoussan, A., Lions, J.L., 1978. Applications des Inequations Variationelles en Control et en Stochastiques. Dunod, Paris, France.

Berkett, L.P., 1994. Management Guide for Low-Input Sustainable Apple Production. DIANE Publishing, May 1.

Bertsekas, D.P., Tsitsiklis, J.N., 1989. Parallel and Distributed Computation. Prentice-Hall, Englewood Cliffs, New Jersey.

Besik, D., Nagurney, A., 2017. Quality in fresh produce supply chains with application to farmers' markets. Socio-Economic Planning Sciences, 60, 62-76.

Bhagwati, J., 1965. On the equivalence of tariffs and quotas. In: Trade, Growth and the Balance of Payments: Essays in Honor of G. Haberler, Baldwin, R.E., Bhagwati, J., Caves, R.E., Johnson, H.G. (Editors). Rand McNally, Chicago, Illinois.

Bishop, P.M., Nicholson, C.F., Pratt, J.E., Novakovic, A.M., 2001. Tariff-rate quotas: Difficult to model or plan simple? Contributed paper presented at the 2001 Annual Conference of the New Zealand Agricultural and Resource Economics Society, Blenheim, New Zealand, 6-7 July.

Blackburn, J., Scudder, G., 2009. Supply chain strategies for perishable products: The case of fresh produce. Production and Operations Management, 18(2), 129-137.

Bond, J.K., Thilmany, D., Bond, C., 2009. What influences consumer choice of fresh produce purchase location? Journal of Agricultural and Applied Economics, 41(1), 61-74.

Boorstein, R., Feenstra, R.C., 1991. Quality upgrading and its welfare cost in U.S. steel imports. In: International Trade and Trade Policy, Helpman, E., Razin, A. (Editors). MIT Press, Cambridge, Massachusetts.

Bosona, T., Gebresenbet, G., Nordmark, I., Ljungberg, D., 2011. Integrated logistics network for the supply chain of locally produced food, Part I: Location and route optimization analyses. Journal of Service Science and Management, 4(2), 174-183.

Bradshear, K., 2018. Tariff dodgers stand to profit off U.S.-China trade dispute. The New York Times, April 22.

Brown, L.R., 2012. Full Planet, Empty Plates: The New Geopolitics of Food Scarcity. WW Norton & Company, New York.

Castro, I., Teixeira, J.A., Salengke, S., Sastry, S.K., Vicente, A.A., 2004. Ohmic heating of strawberry products: electrical conductivity measurements and ascorbic acid degradation kinetics. Innovative Food Science & Emerging Technologies, 5(1), 27-36.

Chen, C.R., Ramaswamy, H.S., 2002. Color and texture change kinetics in ripening bananas. LWT-Food Science and Technology, 35(5), 415-419.

Chatellier, V., 2016. International, European and French Trade in Dairy Products: Trends and Competitive Dynamics. Working Papers 258070, Institut National de la recherche Agronomique (INRA), 29 (3), 143-62.

Clements, J., 2016a. Private communication, University of Massachusetts Amherst. December 27.

Clements, J., 2016b. Private communication, University of Massachusetts Amherst. June 25.

Colnes, R., 2016. Private communication, Salem, Massachusetts. February 6.

Community Involved in Sustaining Agriculture (CISA), 2016. Farmers' markets in western Massachusetts.
http://www.farmfresh.org/food/farmersmarkets.php?zip=01002

Cook, R. L., 2002. The U.S. fresh produce industry: An industry in transition. In: Postharvest Technology of Horticultural Crops, third edition, Kader, A. A. (Editor). University of California Agriculture & Natural Resources, Publication 3311, Oakland, CA, USA, 5-30.

Cournot, A.A., 1838. Researches into the Mathematical Principles of the Theory of Wealth, English translation. MacMillan, London, England.

Crisosto, C.H., Valero, D., 2008. Harvesting and postharvest handling of peaches for the fresh market. In: The Peach: Botany, Production and Uses, Layne, D.R., Bassin, D. (Editors). CAB International, Oxford, UK, 575-596.

Dafermos, S., Nagurney, A., 1984. Sensitivity analysis for the general spatial economic equilibrium problem. Operations Research, 32, 1069-1086.

Dafermos, S., Nagurney, A., 1987. Oligopolistic and competitive behavior of spatially separated markets. Regional Science and Urban Economics, 17(2), 245-254.

Daniele, P., 2004. Time-dependent spatial price equilibrium problem: Existence and stability results for the quantity formulation model. Journal of Global Optimization, 28(3-4), 283-295.

Daniele, P., 2010. Evolutionary variational inequalities and applications to complex dynamic multi-level models. Transportation Research E, 46(6), 855-880.

Das, S.P., Donnenfeld, S., 1989. Oligopolistic competition and international trade: Quantity and quality restrictions. Journal of International Economics, 27, 299-318.

Debreu, G., 1952. A social equilibrium existence theorem. Proceedings of the National Academy of Sciences, 38(10), 886-893.

Deneckere, R., Kovenock, D., Sohn, Y.Y., 2000. Quotas and tariffs with endogenous conduct. Industrial Organization, 9, 37-68.

Dermesonlouoglou, E., Giannakourou, M., Taoukis, P., 2007. Kinetic modelling of the quality degradation of frozen watermelon tissue: effect of the osmotic dehydration as a pre-treatment. International Journal of Food Science & Technology, 42(7), 790-798.

DeWeerdt, S., 2016. Is local food better? World Watch Magazine, 22(3). May/June.

Drew, B., 2016. Private communication, Apex Orchards, Shelburne, Massachusetts. June 25.

Dris, R., Jains, S. M., 2007. Quality Handling and Evaluation. Springer Science & Business Media, New York.

Dupuis, P., Nagurney, A., 1993. Dynamical systems and variational inequalities. Annals of Operations Research, 44(1), 9-42.

Enke, S., 1951. Equilibrium among spatially separated markets: solution by electric analogue. Econometrica, 19, 40-47.

European Commission, 2018. United States: Trade picture. Available at: http://ec.europa.eu/trade/policy/countries-and-regions/countries/united-states/

Fabbrizzi, S., Menghini, S., Marinelli, N., 2014. The short food supply chain: A concrete example of sustainability. A literature review. Rivista di Studi Sulla Sostenibilit, 2, 189-206.

Facchinei, F., Kanzow, C., 2010. Generalized Nash equilibrium problems. Annals of Operations Research, 175(1), 177-211.

Falvey, R.E., 1979. The composition of trade within import-restricted categories. Journal of Political Economy, 87, 1105-1114.

Feenstra, R.C., 1988. Quality change under trade restraints in Japanese autos. Quarterly Journal of Economics, 103(1), 131-146.

Fischer, A., Herrich, M. and Schnefeld, K., 2014. Generalized Nash equilibrium problems-recent advances and challenges. Pesquisa Operacional, 34(3), 521-558.

Florian, M., Los, M., 1982. A new look at static spatial price equilibrium models. Regional Science and Urban Economics, 12, 579-597.

Friesz, T.L., Harker, P.T., Tobin, R.L., 1984. Alternative algorithms for the general network spatial price equilibrium problem. Journal of Regional Science, 24, 475-507.

Fu, B., Labuza, T.P., 1993. Shelf-life prediction: Theory and application. Food Control, 4(3), 125-133.

Fung, K.C., 1989. Tariffs, quotas, and international oligopoly. Oxford Economic papers, 41(4), 749-757.

Gabay, D., Moulin, H., 1980. On the uniqueness and stability of Nash equilibria in noncooperative games. In: Applied Stochastic Control of Econometrics and Management Science, Bensoussan, A., Kleindorfer, P., Tapiero, C.S. (Editors). North-Holland, Amsterdam, The Netherlands, 271-294.

Ghare, P., Schrader, G., 1963. A model for an exponentially decaying inventory. Journal of Industrial Engineering, 14, 238-243.

Giovannucci, D., Barham, E., Pirog, R., 2010. Defining and marketing "local" foods: Geographical indications for US products. Journal of World Intellectual Property, 15(2), 94-120.

Glen, J.J., 1987. Mathematical models in farm planning: A survey. Operations Research, 35(5), 641-666.

Goldberg, P.K., 1993. Trade policies in the U.S. automobile industry. Japan and the World Economy, 6(2), 175-208.

Gossler, T., Wakolbinger, T., Nagurney, A., Daniele, P., 2019. How to increase the impact of disaster relief: A study of transportation rates, framework agreements and product distribution. European Journal of Operational Research, 274(1), 126-141.

Grant, J.H., Hertel, T.W., Rutherford, T.F., 2009. Dairy tariff quota liberalization: Contrasting bilateral and most favored nation reform options. American Journal of Agricultural Economics, 91(3), 673-684.

Greene, D.W., Clements, J.M., 2007. Consumer evaluation of new, antique, and little known apple varieties. Fruit Notes, 72, 1-13.

Gu H., Mason J., 2018. China soybean prices spike as trade war worries feed supply fears. August 8.

Guin, R.P., Barroca, M.J., 2014. Quantification of browning kinetics and colour change for quince (Cydonia oblonga Mill.) exposed to atmospheric conditions. Agricultural Engineering International: CIGR Journal, 16(4), 285-298.

Guyomard, H. Le Mouel, C., Lever F., Lombana, J., 2005. The tariff-only import regime for bananas in the European Union: Is setting the tariff at right level an impossible mission? Paper prepared for presentation at the XIth EAAE Congress (European Association of Agricultural Economists), Copenhagen, Denmark, August 24-27.

Hadjigeorgalis, E., 2005. The US dairy industry and international trade in dairy products. Department of Agricultural Economics and Agricultural Business, New Mexico State University, Las Cruces, NM.

Harker, P.T., Editor, 1985. Spatial Price Equilibrium: Advances in Theory, Computation and Application. Springer, Heidelberg, Germany.

Hendrickson, J., 2004. Energy use in the US food system: A summary of existing research and analysis. Center for Integrated Agricultural Systems, University of Wisconsin, Madison, Wisconsin.

Herguera, I., Kujal, P., Petrakis, E., 2000. Quantity restrictions and endogenous quality choice. International Journal of Industrial Organization, 18(8), 1259-1277.

Hodge, S., 2018. New Tariff Plan Could Cost States $9 Billion. Tax Foundation. March 6. Available at:
https://taxfoundation.org/new-tariffs-could-cost-states-9-billion/.

Holloway, L., Kneafsey, M., 2000. Reading the space of the farmers' market: A preliminary investigation from the UK. Sociologia Ruralis, 40(3), 285-299.

Iowa State University Extension, 2008. Harvesting and storing apples, Ames, Iowa.

Johnston, C., van Kooten, G.C., 2017. Impact of inefficient quota allocation under the Canada-U.S. softwood lumber dispute: A calibrated mixed complementarity approach. Forest Policy and Economics, 74, 71-80.

Kader, A.A., 1997. Fruit maturity, ripening, and quality relationships. In International Symposium Effect of Pre-& Postharvest Factors in Fruit Storage, 485, 203-208.

Karim, O.R., Adebowale, A.A., 2009. A dynamic method for kinetic model of ascorbic acid degradation during air dehydration of pretreated pineapple slices. International Food Research Journal, 16, 555-560.

Ketzenberg, M., Bloemhof, J. and Gaukler, G., 2015. Managing perishables with time and temperature history. Production and Operations Management, 24(1), 54-70.

Kinderlehrer, D., Stampacchia, G., 1980. An Introduction to Variational Inequalities and Their Applications. Academic Press, New York.

King, R.P., Hand, M.S., Gomez, M.I., 2015. Growing Local: Case Studies on Local Food Supply Chains. University of Nebraska Press, Lincoln, Nebraska, Feb 1.

Korpelevich. G.M., 1977. The extragradient method for finding saddle points and other problems. Matekon 13, 35-49.

Krishna, K. 1987. Tariffs vs. quotas with endogenous quality. Journal of International Economics, 23, 97-122.

Krokida, M.K., Karathanos, V.T., Maroulis, Z.B., Marinos-Kouris, D., 2003. Drying kinetics of some vegetables. Journal of Food Engineering, 59(4), 391-403.

Kopanos, G.M., Puigjaner, L., Georgiadis, M.C., 2012. Simultaneous production and logistics operations planning in semicontinuous food industries. Omega, 40, 634-650.

Kuhn, A., 2018. China sets tariffs on $60 billion in U.S. goods, retaliating against U.S. duties. NPR, September 18.

Kulkarni, A.A., Shanbhag, U.V., 2012. On the variational equilibrium as a refinement of the generalized Nash equilibrium. Automatica, 48, 45-55.

Labuza, T.P., 1982. Shelf-life Dating of Foods. Food & Nutrition Press, Westport, CT.

Labuza, T.P., 1984. Application of chemical kinetics to deterioration of foods. Journal of Chemical Education, 61(4), 348-358.

Lefcourt, J., 2020. The Insanity of Trumps Wine Tariffs. The New York Times. January 5.

Leland, H.E., 1979. Quacks, lemons, and licensing: A theory of minimum quality standards. Journal of Political Economy, 87, 1328-1346.

Li, D., Nagurney, A., Yu, M., 2018. Consumer learning of product quality with time delay: Insights from spatial price equilibrium models with differentiated products. Omega, 18, 150-168.

Liu, G., Zhang J., Tang W., 2015. Strategic transfer pricing in a marketingoperations interface with quality level and advertising dependent goodwill. Omega, 56, 1-15.

Liu, Z., Nagurney, A., 2012. Multiperiod competitive supply chain networks with inventorying and a transportation network equilibrium reformulation. Optimization and Engineering, 13(2), 471-503.

Low, S.A., Adalja, A., Beaulieu, E., Key, N., Martinez, S., Melton, A., Perez, A., Ralston, K., Stewart, H., Suttles, S., Vogel, S., Jablonski, B.B.R., 2015. Trends in U.S. local and regional food systems, AP-068, U.S. Department of Agriculture, Economic Research Service, Washington DC, January.

Lowe, T.J., Preckel, P.V., 2004. Decision technologies for agribusiness problems: A brief review of selected literature and a call for research. Manufacturing & Service Operations Management, 6(3), 201-208.

Lundgren, L., 2018. US soybean production estimated at record high: USDA. S & P Global Platts, September 12.

Lutz, S., 2005. The effect of quotas on domestic price and quality. International Advances in Economic Research, 11, 163-173.

Macieira, J., Grant. J., 2014. Hedonic approach to meausring product quality effects of ad valorem tariffs: The specialty cheese case. Virginia Tech, Blacksburg, Virginia.

Maeda, K., Suzuki, N., Kaiser, H.M., 2001. An economic evaluation of the new, agricultural trade negotiations: A nonlinear imperfectly competitive spatial equilibrium approach. Working paper, Department of Applied Economics and Management, Cornell University, Ithaca, New York, August.

Maftoonazad, N., Ramaswamy, H.S., 2008. Effect of pectin-based coating on the kinetics of quality change associated with stored avocados. Journal of Food Processing and Preservation, 32(4), 621-643.

Martinez, S., Hand, M., Da, P.M., Pollack, S., Ralston, K., Smith, T., Vogel, S., Clark, S., Lohr, L., Low, S., Newman, C., 2010. Local food systems concepts, impacts, and issues. Economic Research Report Number 97. Economic Research Service, United States Department of Agriculture, Washington DC.

Masoumi, A.H., Yu, M., Nagurney, A., 2012. A supply chain generalized network oligopoly model for pharmaceuticals under brand differentiation and perishability. Transportation Research E, 48(4), 762-780.

Meixell, M.J., Gargeya, V.B., 2005. Global supply chain design: A literature review and critique. Transportation Research E, 41(6), 531-550.

Meyer, D., 2019. Trump's steel tariffs have forced the EU to launch quotas on steel imports. Fortune, February 1.

Mistreanu, S., 2019. Beyond Decoupling: How China Will Reshape Global Trade In 2020. Forbes. December 3.

Monteiro, D. M. S., 2007. Theoretical and empirical analysis of the economics of traceability adoption in food supply chains. Ph.D. thesis. University of Massachusetts Amherst, Amherst, MA.

Mrad, N.D., Boudhrioua, N., Kechaou, N., Courtois, F., Bonazzi, C., 2012. Influence of air drying temperature on kinetics, physicochemical properties, total phenolic content and ascorbic acid of pears. Food and Bioproducts Processing, 90(3), 433-441.

Murdoch, J., Marsden, T., Banks, J., 2000. Quality, nature, and embeddedness: Some theoretical considerations in the context of the food sector. Economic Geography, 76(2), 107-125.

Nagurney, A., 1999. Network Economics: A Variational Inequality Approach, second and revised edition. Kluwer Academic Publishers, Dordrecht, The Netherlands.

Nagurney, A., 2006. Supply Chain Network Economics: Dynamics of Prices, Flows, and Profits. Edward Elgar Publishing, Cheltenham, England.

Nagurney A. 2010. Supply chain network design under profit maximization and oligopolistic competition. Transportation Research E, 46(3), 281-294.

Nagurney, A., 2018. A multitiered supply chain network equilibrium model for disaster relief with capacitated freight service provision. In: Dynamics of Disasters: Algorithmic Approaches and Applications, Kotsireas, I.S., Nagurney, A., Pardalos, P.M. (Editors), Springer International Publishing Switzerland, 85-108.

Nagurney, A., Aronson, J., 1988. A general dynamic spatial price equilibrium model: formulation, solution, and computational results. Journal of Computational and Applied Mathematics, 22(2-3), 339-357.

Nagurney, A., Besik, D., Dong, J., 2019. Tariffs and quotas in world trade: A unified variational inequality framework. European Journal of Operational Research, 275(1), 347-360.

Nagurney, A., Besik, D., Li, D., 2019. Strict quotas or tariffs? Implications for product quality and consumer welfare in differentiated product supply chains. Transportation Research Part E: Logistics and Transportation Review, 129, 136-161.

Nagurney, A., Besik, D., Nagurney L.S., 2019. Global Supply Chain Networks and Tariff Rate Quotas: Equilibrium Analysis with Application to Agricultural Products. Journal of Global Optimization, 75, 439-460.

Nagurney, A., Besik, D., Yu, M., 2018. Dynamics of quality as a strategic variable in complex food supply chain network competition: The case of fresh produce. Chaos, 28, 043124.

Nagurney, A., Dupuis, P., Zhang, D., 1994. A dynamical systems approach for network oligopolies and variational inequalities. Annals of Regional Science, 28, 263-283.

Nagurney, A., Li, D., 2014. Equilibria and dynamics of supply chain network competition with information asymmetry in quality and minimum quality standards. Computational Management Science, 11(3), 285-315.

Nagurney, A., Li, D., 2016. Competing on Supply Chain Quality: A Network Economics Perspective. Springer International Publishing, Switzerland.

Nagurney, A., Li D, Nagurney, L.S., 2014. Spatial price equilibrium with information asymmetry and minimum quality standards. International Journal of Production Economics, 158, 300-313.

Nagurney, A., Matsypura, D., 2005. Global supply chain network dynamics with multicriteria decision-making under risk and uncertainty. Transportation Research E, 41, 585-612.

Nagurney, A., Nicholson, C.F., Bishop, P.M., 1996a. Massively parallel computation of large-scale spatial price equilibrium models with discriminatory ad valorem tariffs. Annals of Operations Research, 68, 281-300.

Nagurney, A., Nicholson, C.F., Bishop, P.M., 1996b. Spatial price equilibrium models with discriminatory ad valorem tariffs: Formulation and comparative computation using variational inequalities. In Recent Advances in Spatial Equilibrium Modeling: Methodology and Applications, van den Bergh, J.C.J.M., Nijkamp, P., Rietveld, P. (Editors), New York, Springer, Heidelberg, Germany, 179-200.

Nagurney, A., Takayama, T., Zhang, D., 1995. Massively parallel computation of spatial price equilibrium problems as dynamical systems. Journal of Economic Dynamics and Control, 18, 3-37.

Nagurney, A., Thore, S., Pan, J., 1996. Spatial market models with goal targets. Operations Research, 44, 393-406.

Nagurney, A., Wolf, T., 2014. A Cournot-Nash-Bertrand game theory model of a service-oriented Internet with price and quality competition among network transport providers. Computational Management Science, 11(4), 475-502.

Nagurney, A., Zhang, D., 1996. Projected Dynamical Systems and Variational Inequalities with Applications. Kluwer Academic Publishers, Norwell, Massachusetts.

Nagurney, A., Yu, M., 2012. Sustainable fashion supply chain management under oligopolistic competition and brand differentiation. International Journal of Production Economics, 135, 532-540.

Nagurney, A., Yu, M., Besik, D., 2017. Supply chain network capacity competition with outsourcing: A variational equilibrium framework. Journal of Global Optimization, 69(1), 231-254.

Nagurney, A., Yu, M., Masoumi, A.H., Nagurney, L.S., 2013. Networks Against Time: Supply Chain Analytics for Perishable Products. Springer Science + Business Media, New York.

Nagurney, A., Zhao, L., 1993. Networks and variational inequalities in the formulation and computation of market disequilibria: the case of direct demand functions. Transportation Science, 27, 4-15.

Nahmias, S., 1982. Perishable inventory theory: A review. Operations Research, 30(4), 680-708.

Nahmias, S., 2011. Perishable Inventory Systems. Springer Science & Business Media, New York.

Nash, J.F., 1950. Equilibrium points in n-person games. Proceedings of the National Academy of Sciences, USA, 36, 48-49.

Nash, J.F., 1951. Noncooperative games. Annals of Mathematics, 54, 286-298.

National Agricultural Statistics Service (NASS), 2004. Dairy products annual survey. United States Department of Agriculture, Washington, DC.

Neff, R.A., Palmer, A.M., McKenzie, S.E., Lawrence, R.S., 2009. Food systems and public health disparities. Journal of Hunger and Environmental Nutrition, 4, 282-314.

Nolte, S., 2008. The future of the world sugar market - A spatial price equilibrium analysis.Dissertation, Humboldt-University of Berlin. Available at:
http://edoc.hu-berlin.de/docviews/ abstract.php?id=29001.

Ochoa, M.R., Kesseler, A.G., De Michelis, A., Mugridge, A., Chaves, A.R., 2001. Kinetics of colour change of raspberry, sweet (Prunus avium) and sour (Prunus cerasus) cherries preserves packed in glass containers: light and room temperature effects. Journal of Food Engineering, 49(1), 55-62.

O'Connor, A., 2013. Really? The claim: Fresh produce has more nutrients than canned. The New York Times, May 27.

Office of the United States Trade Representative, 2018. President Trump approves relief for U.S. washing machine and solar cell manufacturers. Washington DC. Available at: https://ustr.gov/about-us/policy-offices/press-office/press -releases/2018/january/president-trump-approves-relief-us

Paquette, P., Lynch, D.J., Rauhala, E., 2018. As Trump's trade war starts, China vows retaliation. Washington Post, July 6.

Pirog, R., 2009. Local foods: farm fresh and environmentally friendly. Leopold Center for Sustainable Agriculture, Ames, Iowa.

Plunkett Research, 2011. U.S. food industry overview. http://www.plun kettresearch.com/food%20beverage%20grocery%20market%20research/industry%20statistics.

Qiang, Q., Ke, K., Anderson, T., Dong, J., 2013. The closed-loop supply chain network with competition, distribution channel investment, and uncertainties. Omega, 41(2), 186-194.

Reeves, C.A., Bednar, D.A., 1994. Defining quality: alternatives and implications. Academy of Management Review, 19(3), 419-445.

Rong, A., Akkerman, R., Grunow, M., 2011. An optimization approach for managing fresh food quality throughout the supply chain. International Journal of Production Economics, 131(1), 421-429.

Rozenberg, S., Body, J.J., Bruyere, O., Bergmann, P., Brandi, M.L., Cooper, C., Devogelaer, J.P., Gielen, E., Goemaere, S., Kaufman, J.M. and Rizzoli, R., 2016. Effects of dairy products consumption on health: benefits and beliefs – a commentary from the Belgian Bone Club and the European Society for Clinical and Economic Aspects of Osteoporosis, Osteoarthritis and Musculoskeletal Diseases. Calcified Tissue International, 98(1), 1-17.

Rutherford, T.F. 1995. Extensions of GAMS for complementarity problems arising in applied economics. Journal of Economic Dynamics and Control, 19, 1299-1324.

Saberi, S., 2018. Sustainable, multiperiod supply chain network model with freight carrier through reduction in pollution stock. Transportation Research E, 118, 421-444.

Samora, R., 2018. Brazil to pass U.S. as world's largest soy producer in 2018. Reuters. May, 11.

Samuelson, P.A., 1952. Spatial price equilibrium and linear programming. American Economic Review, 42, 283-303.

Schouten, R.E., Kooten, O.V., Jalink, H., Kappers, I.F., Snel, J.F.H., Jordi, W., Steele, R., 2004. Genetic and physiological factors affecting colour and firmness. In: Understanding and Measuring the Shelf-life of Food, Steele, R. (Editor), Elsevier, Woodhead Publishing, Cambridge, United Kingdom, 69-90.

Schouten, R.E., Otma, E.C., van Kooten, O., Tijskens, L.M.M., 1997. Keeping quality of cucumber fruits predicted by biological age. Postharvest Biology and Technology, 12(2), 175-181.

Shane, D., 2018. China may soon regret slapping tariffs on US soybeans. CNN, August 30.

Shapiro, C., 1983. Premiums for high quality products as returns to reputations. Quarterly Journal of Economics, 98, 659-679.

Sheshinski, E., 1976. Price, quality, and quantity regulation in monopoly situations. Economica, 43(170), 127-137.

Shively, H., 2018. Consumers are about to feel the effect of tariffs on Chinese goods. Here's how. Dayton Daily News, October 16.

Shono, C., 2001. WTO to Kokusai-Nuseihin-Boeki (The World Dairy Trade and WTO) Norin-lbkci-Kyokai, Tokyo, Japan (in Japanese).

Silver, E.A., Pyke, D.F., Peterson, R., 1998. Inventory Management and Production Planning and Scheduling, third edition. John Wiley & Sons, New York.

Singh, R.P., Anderson, B.A., 2004. The major types of food spoilage: an overview. Understanding and Measuring the Shelf-life of Food, 3-23.

Singh, T.K., Cadwallader, K.R., 2004. Ways of measuring shelf-life and spoilage. Understanding and Measuring the Shelf-life of Food, 165-183.

Smith, A., 1887. An Inquiry Into the Nature and Causes of the Wealth of Nations. T. Nelson and Sons.

Skully, D.W., 2001. Economics of tariff-rate quota administration. USDA Economic Research Service, Technical Bulletin No 1893, April, Washington DC.

Sloof, M., Tijskens, L.M.M., Wilkinson, E.C., 1996. Concepts for modelling the quality of perishable products. Trends in Food Science & Technology, 7(5), 165-171.

Song, B., Xu, S., Marchant, M.A., 2004. Who dominates the US soybean industry: Producers, consumers, or agribusinesses? 2004 American Agricultural Economics Association Annual meeting, Denver, CO, August 1-4.

Spence, M., 1975. Monopoly, quality, and regulation. The Bell Journal of Economics, 6(2), 417-429.

Sumner, D. A., Murdock, J., 2017. University of California agriculture and natural resources sample costs for processing peaches cling and freestone extra-early harvested varieties.

Swanson, A., Smialek, J., 2020. U.S. Manufacturing Slumps as Trade War Damage Lingers. The New York Times, January 3.

Takayama, T., Judge, G.G., 1964. Spatial equilibrium and quadratic programming. Journal of Farm Economics, 46(1), 67-93.

Takayama, T., Judge, G.G., 1971. Spatial and Temporal Price and Allocation Models. North-Holland Publishing Company, Amsterdam, The Netherlands.

Tankersley, J., 2018. Trade wars with China in aisle 12. The New York Times, July 12.

Tankersley, J., 2019. Irish Whiskey, Olive Oil and Waffles Could Face Tariffs Up to 100 Percent. The New York Times, December 13.

Taoukis, P.S., Labuza, T.P., 1989. Applicability of time-temperature indicators as shelf life monitors of food products. Journal of Food Science, 54(4), 783-787.

Tellioglu, I., Bailey, K.W., Blandford, D., 2006. Tariffs on US Imports of Dairy Products: A Product Component Analysis. Doctoral Dissertation, Pennsylvania State University, University Park, PA.

The White House Report to Congress, 2009. Available at:
https://obamawhitehouse.archives.gov/briefing-room/disclosures/annual-records/2009

Thompson, J.F., 2002. Waste management and cull utilization. In: Postharvest Technology of Horticultural Crops, third edition, Kader, A.A. (Editor). University of California Agriculture & Natural Resource, Oakland, California, Publication 3311, 81-84.

Tijskens, L.M.M., Polderdijk, J.J., 1996. A generic model for keeping quality of vegetable produce during storage and distribution. Agricultural Systems, 51(4), 431-452.

Tijskens, L.M.M., 1979. Texture of Golden Delicious apples during storage. Lebensmittel-Wissenschaft und Technologie, 12, 138-142.

Toralles, R.P., Vendruscolo, J.L., Vendruscolo, C.T., del Pino. F.A.B., Antunes, PL., 2005. Properties of polyphenoloxidase and peroxidase from Granada clingstone peaches. Brazilian Journal of Food Technology, 8(3), 233-242.

Trobe, H.L., 2001. Farmers' markets: consuming local rural produce. International Journal of Consumer Studies, 25(3), 181-192.

Tuohy, L., 2016. The Northeast's peach crop was decimated by the mid-February freeze. boston.com available at:
https://www.boston.com/news/business/2016/05/07/northeasts-peach-crop-decimated-mid-february-freeze

United Nations (UN), 1999. Standard Country or Area Codes for Statistical Use (M49). Available at:
https://unstats.un.org/unsd/methodology/m49/

United States Department of Agriculture (USDA), 2018. Dairy products. May. Available at:
https://www.nass.usda.gov/Publications/Todays_Reports/reports/dary0718.pdf

United States Department of Agriculture (USDA), 2011. Fruits and vegetables (farm weight): Per capita availability, 1970-2009. Available at:
http://www.ers.usda.gov/Data/FoodConsumption/spreadsheets/fruitveg.xls.

United States Department of Agriculture (USDA), 2012. US apple statistics. Economic Research Service, Washington DC. Available at:
http://usda.mannlib.cornell.edu/MannUsda/viewDocumentInfo.do?documentID=1825

United States Department of Agriculture (USDA), 2016. Local food directories: National farmers' market directories. Agricultural Marketing Service, Washington DC.
http://www.ams.usda.gov/local-food-directories/farmersmarkets

United States Department of Agriculture (USDA), 2016. Noncitrus fruits and nuts 2015 preliminary summary. July 13. Available at:
http:// usda.mannlib.cornell.edu/usda/current/NoncFruiNu/ NoncFruiNu-07-06-2016.pdf.

United States Department of Homeland Security, 2019. A report on Food and Agriculture Sector. Available at:
https://www.dhs.gov/cisa/food-and-agriculture-sector

U.S. Energy Information Administration, June 2016. Gasoline and fuel update. Available at: http://www.eia.gov/petroleum/gasdiesel/

United States. Office of the U.S. Trade Representative, 1994. Final Act Embodying the Results of the Uruguay Round of Multilateral Trade Negotiations. Executive Office of the President, Washington, D.C.

van den Bergh, J.C., Nijkamp, P., Rietveld, P., 1996. Recent Advances in Spatial Equilibrium Modelling: Methodology and Applications. Springer, Heidelberg, Germany.

Van der Vorst, J.G.A.J, 2000. Effective food supply chains: Generating, modelling and evaluating supply chain scenarios. Doctoral Dissertation, Wageningen University, The Netherlands.

von Heusinger, A., 2009. Numerical methods for the solution of the generalized Nash equilibrium problem (Doctoral dissertation, PhD Dissertation, University of Wurtburg, Germany).

Watson, P.W., 2018. Even if Trump's tariffs are effective, there's one reason they won't work. Forbes, October 10.

Widodo, K.H., Nagasawa, H., Morizawa, K., Ota, M., 2006. A periodical flowering-harvesting model for delivering agricultural fresh products. European Journal of Operational Research, 170, 24-43.

Wildman, S.S., 1984. A note on measuring surplus attributable to differentiated products. Journal of Industrial Economics, 33(1), 123-32.

Wong, D., Koty, A.C., 2019. The US-China trade war: A timeline. China-Briefing, January 10.

Wood, L., 2018. Soybean import in China (2018-2022): The oil yields of domestic soybeans range from 16% to 17%, while those of imported transgenic ones range from 19% to 20%. Cision PR Newswire, November 19.

World Trade Organization, 2004. Market access: tariffs and tariff quotas, December 1. Available at:
https://www.wto.org/english/tratop_e/agric_e/negs_bkgrnd10_access_e.htm

World Trade Organization, 2014. World Trade Report 2014, Trade and Development: Recent Trends and the Role of the WTO. Available at:
https://www.wto.org/english/res_e/publications_e/wtr14_e.htm

World Trade Organization, 2018. Get tariff data. Available at:
https://www.wto.org/English/tratop_e/tariffs_e/tariff_data_e.htm

Yu, M., Nagurney, A., 2013. Competitive food supply chain networks with application to fresh produce. European Journal of Operational Research, 224(2), 273-282.

Zhang, D., 2006. A network economic model for supply chain versus supply chain competition. Omega 34(3), 283-295.

Zhang, N., Guo, Q., Ma, Y., 2012. Stability of anthocyanin in wild blueberry fruit and its degradation kinetics. In: Information Technology and Agricultural Engineering, Springer, Berlin, Germany, 539-546.

Zhang, G., Habenicht, W., Spieß, W.E.L., 2003. Improving the structure of deep frozen and chilled food chain with tabu search procedure. Journal of Food Engineering, 60, 67-79.

Zhao, S., Yue, C., Luby, J., Gallardo, K., McCracken, V., McFerson, J., Layne, D., 2017. U.S. peach producer preference and willingness to pay for fruit attributes. HortScience, 52(1), 116-121.

Zhou, Y., Baylis, K., Coppess, J., Xie, Q., 2018. Evaluating potential long-run impacts of Chinese tariff on US soybeans. Farmdoc Daily, 8, 179.