Common Ownership of Public Goods [1]

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September 2020

[1]This paper was previously distributed under the title "Reputation and Ownership of Public Goods". We thank editor Daniela Puzzello and two anonymous referees for valuable comments. We would also like to thank Tim Besley, Paul Grout, Oliver Hart, Elisabetta Iossa, In-Uck Park and Armin Schmutzler for discussions and seminar audiences at Aberdeen, Bristol, King's College London, CEPR Conference on Partnerships between Governments and Private Sector in London, Contracts, Firms and Corporate Finance Conference in Paris-Dauphine, EARIE in Valencia and Istanbul, GW4 Game Theory Workshop in Bristol, IASC Celebrating Commons Scholarship Conference in Washington, ISNIE in Cambridge MA, PET in Galway and St Andrews Workshop in Applied Microeconomics for comments.
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Abstract

We analyze ownership of public goods in a repeated game focusing on common ownership. Under common ownership an owner's access to the public good cannot be restricted by other owners. We find that under common ownership both the value of the relationship and the gain from deviation are high. Common ownership can provide the best incentives for cooperation when the value of the public good cannot be increased much by maintenance investments or the maintenance costs are sufficiently convex. We argue that these conditions are satisfied in Ostrom's field studies of irrigation systems and common lands.

JEL classification: D23, H41, L14, L33

Keywords: public goods, common pool resources, property right theory, relational contracts, common ownership, joint ownership

1 Introduction

Ostrom (1990) challenged Hardin's (1968) argument that common pool resources (CPRs), such as grazing areas, forests and irrigation systems, are subject to overappropriation of the resource and underprovision of its maintenance. She demonstrated with field studies how common ownership has in many cases outperformed both private and public ownership – but in other cases privatization or government ownership has been successful. She referred to repeated games in explaining how common ownership can overcome overappropriation and underprovision problems.[1] However, she did not address whether and when common ownership outperforms other ownership structures in a repeated game. This is the question we address in this paper. We find that, consistent with the stylized facts, common ownership can provide the best incentives for cooperation when the value of the CPR cannot be increased much by maintenance investments. Furthermore, we show that the rationale for common ownership can also arise from sufficiently convex maintenance costs.

Irrigation systems in Nepal are an example of long-standing, successful common ownership arrangements (Ostrom and Gardner 1993). Communally owned irrigation systems cover 62% of the irrigated land and have outperformed systems managed by the national government agency. These irrigation systems typically rely on low-tech construction techniques such as nonpermanent headworks from mud, trees and stones and unlined canals. Continual maintenance is therefore required. Every spring before the monsoon season the headworks and the canals are repaired. During the monsoon rains the canals are patrolled daily, small leaks are repaired and the community is alerted for any major damage, such as a landslide (Martin and Yoder 1988). Considerable investments are therefore required in maintaining the irrigation systems. Our focus is on finding when common ownership provides the best incentives for such repeated maintenance investments. Maintenance is a public good because it is not possible to exclude any user from the improvement to the irrigation system. Given our focus on maintenance (rather than appropriation of the resource), we can apply a public good model (Ostrom 1990, p. 32).

Property rights theory of Grossman and Hart (1986) and Hart and Moore (1990) – and its extension to public goods by Besley and Ghatak (2001) – is the natural framework for analyzing the optimal ownership structure.[2] We consider two agents

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[1] See also Ostrom, Gardner and Walker (1994).

[2] Property rights, particularly in the context of natural resources, have also been analyzed by

who make repeated maintenance investments in the public good but differ in their valuation of it. The valuation difference can arise e.g. from the size of the landholding to be irrigated. In the main model we focus on two shared ownership structures, common ownership and joint ownership, and examine single ownership – such as privatization or government ownership – in an extension.

In field studies, failing to contribute to the maintenance of a CPR is typically punished by a fine paid to the community. Fines are also included in Ostrom's design principles which characterize successful communally owned CPRs (Ostrom 1990, p. 90). We therefore introduce fines in the repeated game and allow for renegotiation back to the Pareto frontier after deviation. High fines relax the incentive compatibility constraint for maintenance investments but the fines themselves have to be incentive compatible. The maximal fines depend on the value of the relationship governed by the relational contract as compared to punishment payoffs. If an agent were not to pay the fine after deviation, the other agent would choose punishment investment for one period. We consider different strengths of punishment to be feasible. Strong punishment implies that punishment investment is close to zero, while mild punishment investment is close to Nash investment of the static game. It is reasonable to assume that punishment by investment is relatively mild for a fragile, low-tech irrigation system which the community's livelihood depends on.

Let us initially consider the mildest punishment and compare Nash investments of the static game under different ownership structures. Under common ownership, according to Ostrom and Hess (2010), an owner's access to the public good cannot be restricted by other owners (but access can be denied for non-owners). Therefore in the static game there is nothing to bargain about ex post and each agent receives his full individual valuation of the public good. Under other ownership structures the agents share the joint surplus in bargaining. Under joint ownership they split the surplus 50:50 as they have to reach a unanimous decision. Under single ownership they bargain over the non-owning agent's contribution to the public good. For the investment incentives in the static game, this implies that under common ownership each agent maintains the public good according to his own valuation, ignoring the benefit to the other agent. Common ownership is then dominated by ownership by the high-valuation agent in the static game. This is because the low-valuation agent's incentives are improved when the owner has to share his higher valuation in bargaining

Libecap (1989). However, his focus is on the formation of property rights.

while under common ownership he only obtains his own low valuation.[3]

For the repeated game with the mildest punishment investments, this implies that the value of the relationship is higher under common ownership thus reducing the incentive to underinvest in maintenance (since higher fines are incentive compatible). However, also the gain from deviation is higher under common ownership. If the high-valuation agent underinvests, he obtains the maximal deviation payoff under common ownership as he gets his full valuation of the inefficiently maintained public good, while under other ownership structures he has to share his high valuation in bargaining.[4] Optimal ownership structure is then a tradeoff between increasing the value of the relationship and reducing the gain from deviation.

For common ownership to provide the best incentives for cooperation, the investment costs have to be sufficiently convex so that also the marginal cost is convex. This implies that the investment is inelastic to surplus share. Then, the mildest punishment investment is not significantly lower than the first-best investment, implying that the value of the relationship is low and only low fines are incentive compatible. Cooperation incentives can then be improved by an ownership structure that maximizes the value of the relationship. The choice is between common ownership and ownership by the low-valuation agent, another ownership structure that performs poorly in a static game. Ownership by the low-valuation agent provides the best incentives if his valuation of the public good is not too low compared to the high-valuation agent, while common ownership performs better if the valuations are not too homogeneous.

Alternatively, if the marginal cost is concave – and the investment is elastic to surplus share – even the mildest punishment investment is much lower than the first-best investment and the value of the relationship is high under any ownership structure. Then, fines can be high under any ownership structure and minimizing the gain from deviation provides the best cooperation incentives. Then, either joint ownership or ownership by the high-valuation agent is optimal. The above discussion focuses on mild punishment. However, if punishment by investment is strong enough, the value of relationship is high even when the investment is inelastic to surplus share. Also then joint ownership or ownership by the high-valuation agent provides the best incentives for cooperation.

We argue that the communally owned irrigation systems in Nepal and the Philip-

[3] High-valuation agent's incentives do not change as he obtains his full high valuation for his *own* investment both under common ownership and when he is the single owner.

[4] For the low-valuation agent there is the opposite effect but it is of a smaller magnitude.

pines can be characterized by convex marginal cost. The marginal cost of maintenance investments arises from the time and effort taken away from cultivating privately owned fields. With a decreasing returns to scale Cobb-Douglas production function, the marginal product of private cultivation is decreasing and convex. As the marginal cost of increasing maintenance investment equals the marginal product of reducing private cultivation, convexity of marginal cost arises from convex marginal product. Furthermore, if we shift the curvature to the value of the investment, our result is consistent with a stylized fact of common ownership: the value of the CPR cannot be increased much by investment (Netting 1976; Ostrom 1990, p. 63). Communal grazing lands in the Swiss Alps are an example of such common ownership. Additionally, valuations of the CPR are not too homogeneous, as the value of the CPR to a community member depends on their private holdings of cultivated land or the number of livestock and there is heterogeneity in private wealth.

Our contribution is to show that common ownership can provide the best incentives for cooperation when the agents make repeated investments in maintaining the public good. Our work is related to the extensive literature on governance of CPRs. Hardin (1968) argues that an unregulated CPR would be overused and eventually exhausted. To solve this tragedy of the commons, he proposes that the CPRs should be either privatized or kept as public property. Ostrom (1990) challenges this view by demonstrating how in numerous field studies local communities overcome the potential tragedy by creating rules to govern shared resources sustainably. From these rules she identifies design principles that characterize successful communally owned CPRs. Furthermore, while Ostrom uses insights from repeated games to explain how common ownership can overcome potential overuse and underprovision of maintenance, she does not address whether and when common ownership outperforms other ownership structures in a repeated game.

There is an extensive literature analyzing formal models of CPRs in a dynamic game, e.g. Benhabib and Radner (1992) and Copeland and Taylor (2009). Benhabib and Radner (1992) show that the potential overappropriation of the CPR can be overcome in a dynamic game if the discount factor and the stock of the CPR are high enough. In Copeland and Taylor (2009) efficient appropriation of the CPR depends also on the strength of property rights. However, this literature takes it as given that the resource is commonly owned while we examine the optimal ownership structure. Furthermore, we focus on the provision of CPRs by introducing maintenance investments. Also Leonard and Libecap (2019) analyze provision but focus on one-off construction of infrastructure and the effect of different property rights regimes in a static framework.

Our paper is closely related to Besley and Ghatak (2001) who examine optimal ownership of public goods in a static game. As our focus is on maintenance, a public good model is applicable to CPR provision as well. Besley and Ghatak (2001) build on Grossman and Hart (1986) and Hart and Moore (1990) to analyze government versus private ownership of public goods. They find that the party with the highest valuation for the public good should be the owner. This is in contrast to private goods where ownership is optimally concentrated in the hands of the agent with an important investment. We analyze Besley and Ghatak (2001) in a repeated game and furthermore introduce the concept of common ownership.

Our paper also contributes to the relational contracts literature.[5] Baker, Gibbons and Murphy (2002) and Halonen (2002) examine ownership of private goods when the agents make repeated specific investments. They show that ownership structure plays an important role in whether the holdup problem can be overcome in a repeated game – similar to our analysis of public goods.[6] The structure of our model is similar to Halonen (2002) where both parties invest and the investments are observable – and different from Baker, Gibbons and Murphy (2002) where the agent makes multiple investments that are unobservable to the principal. Our assumptions fit CPRs as typically the whole community gathers to maintain the CPR in specific days and attendance is observable.

Ramey and Watson (1997) and Halac (2015) analyze the effect of an up-front specific investment on a relational employment contract while we examine repeated investments. Finally, in Harstad, Lancia and Russo (2017) countries make repeated technology investments that affect the sustainability of a relational environmental treaty. It depends on the properties of the technology whether overinvestment or underinvestment reduces the gain from freeriding in emissions. In our model it is the ownership structure that is chosen to reduce incentives to freeride in maintenance investments and the optimal ownership structure depends on the maintenance technology.

Rosenkranz and Schmitz (2003) and Niedermayer (2013) also analyze common

[5] See Malcomson (2013) for an excellent survey on relational incentive contracts.

[6] We will compare the public and private goods cases in detail in Section 4.

ownership and joint ownership although they use different terminology and do not consider repeated game.[7] The R&D context of Rosenkranz and Schmitz (2003) differs from ours by including two sequential projects and the possibility of know-how disclosure. In their model common ownership provides good investment incentives while joint ownership induces know-how disclosure. In Niedermayer (2013) one agent invests in platform (public good) and both agents invest in applications (private goods) complementary to the platform. In his framework common ownership provides good incentives to invest in applications at the cost of a lower investment in the platform. In our model both agents invest (only) in the public good.

The rest of the paper is organized as follows. Section 2 presents the static game and Section 3 analyzes the repeated game. Section 4 extends the analysis to single ownership and Section 5 shows that our results are robust to introducing asymmetric costs. Section 6 applies our results to well-known field studies of CPRs and Section 7 concludes.

2 Static game

There are two agents, ℓ and h. Ex ante each agent makes an investment, denoted by y_ℓ and y_h, in a public good, e.g. maintaining an irrigation system.[8] Ex post the agents produce the public good. The agents value the public good differently: the low-valuation agent's utility is $\theta_\ell(y_\ell + y_h)$ and the high-valuation agent's utility is $\theta_h(y_\ell + y_h)$ where $\theta_\ell < \theta_h$.[9] The valuation difference can arise e.g. from the size of the landholding to be irrigated. Investment costs are given by $c(y_i) = (y_i)^\gamma$ for $i \in \{\ell, h\}$ where $\gamma > 1$. The investments are observable to the agents but not verifiable to a third party.

Joint surplus equals $S = (\theta_\ell + \theta_h)(y_\ell + y_h) - c(y_\ell) - c(y_h)$. The first-best invest-

[7] Our joint and common ownership are equivalent to joint ownership with and without bilateral veto power in Rosenkranz and Schmitz (2003) and to standardized and open source platform in Niedermayer (2013).

[8] In the main model the investment can be in either physical or human capital. In Section 4 the nature of the investment will matter.

[9] Assuming that investments are additive is reasonable in CPR context as maintenance typically does not require complementary skills. Maintenance duties are generally assigned to households who can send any able-bodied adult to contribute to maintenance activities.

ments are then given by the following first-order conditions.

$$(\theta_\ell + \theta_h) = c'(y_i^*) \text{ for } i \in \{\ell, h\} \tag{1}$$

We denote $y^* \equiv y_h^* = y_\ell^*$.

Contracts are incomplete and, consequently, ex ante contracts can only be written on the ownership of the project. Following the property rights theory of Grossman and Hart (1986) and Hart and Moore (1990), we define ownership in terms of residual control rights over the project. If agent i is the single owner of the project, he has the authority to exclude agent j from working on the project if they do not reach an agreement in bargaining. We defer analysis of single ownership to Section 4 and concentrate on two types of shared ownership, *common ownership* and *joint ownership*, in the main model. Joint ownership has often been analyzed in the Grossman-Hart-Moore property rights framework.[10] Under joint ownership both owners have a veto power and therefore the project cannot go ahead if they do not reach a unanimous agreement. Therefore the disagreement payoffs equal zero under joint ownership. Common ownership is a less familiar concept in the Grossman-Hart-Moore property rights theory.[11] Under common ownership, according to Ostrom and Hess (2010), an owner's access to the project cannot be restricted by other owners although access can be denied for non-owners. Therefore each agent's utility is $\theta_i(y_\ell + y_h)$ for $i \in \{\ell, h\}$ even if they disagree.

The timing is the following:

Stage 1. ℓ and h contract on the ownership of the project. We analyze joint ownership and common ownership.

Stage 2. ℓ and h invest in project-specific capital.

Stage 3. ℓ and h bargain over the completion of the project, make transfers and produce the public good.

The public good model is appropriate for analyzing CPRs as we focus on maintenance rather than appropriation of the resource (Ostrom 1990, p. 32). Maintenance is a public good because no user can be excluded from the improvement to the CPR.

[10] For example in Besley and Ghatak (2001), Halonen (2002), Cai (2003) and Rosenkranz and Schmitz (2003).

[11] Rosenkranz and Schmitz (2003) call a similar arrangment joint ownership with no veto right. Niedermayer (2003) examines an open source platform where no one can be excluded from the public good. Open source and common ownership are equivalent in a 2-agent setup.

Under joint ownership the disagreement payoffs are zero as each agent has veto power. Therefore the agents split the ex post surplus 50:50 in Nash bargaining resulting in the following payoffs

$$u_i^J = \frac{1}{2}(\theta_\ell + \theta_h)(y_h + y_\ell) - c(y_i) \qquad \text{for } i \in \{\ell, h\} \tag{2}$$

where superscript J denotes joint ownership. Under joint ownership the investment incentives are

$$\frac{1}{2}(\theta_\ell + \theta_h) = c'\left(y_i^J\right) \quad i \in \{\ell, h\}. \tag{3}$$

We denote $y^J \equiv y_h^J = y_\ell^J$. The familiar holdup problem arises because the agents share the value of investment in bargaining but pay the full marginal cost.

Under common ownership neither consumption of the public good nor participation in its production can be restricted for the owners. Therefore there is nothing to bargain about ex post. Consequently, the payoffs are

$$u_i^C = \theta_i(y_\ell + y_h) - c(y_i) \quad i \in \{\ell, h\}. \tag{4}$$

Common ownership is denoted by superscript C. Optimal investments under common ownership, denoted by y_i^C, are given by

$$\theta_i = c'\left(y_i^C\right) \quad i \in \{\ell, h\}. \tag{5}$$

Each agent chooses investment according to his own valuation, ignoring the benefit of the public good to the other agent.

According to (3) and (5), agent i's investment depends on his surplus share σ_i, where $\sigma_i \in \left\{\theta_i, \frac{1}{2}(\theta_\ell + \theta_h)\right\}$. The investment is increasing in the surplus share so that $y_\ell^C < y^J < y_h^C < y^*$. Solving for the explicit form of the investment, $y_i = (\sigma_i/\gamma)^{1/(\gamma-1)}$, we can show that the investment is inelastic to surplus share if and only if $\gamma > 2$.[12] This elasticity will play a key role in our analysis.

Proposition 1 compares the joint surplus under common ownership and joint ownership. At stage 1 the agents contract on the joint surplus maximizing ownership structure and make any necessary lump sum transfers to achieve it.

[12] The elasticity of agent i's investment to surplus share is given by $(\partial y_i/\partial \sigma_i)(\sigma_i/y_i) = 1/(\gamma - 1)$.

Proposition 1 *In the static game joint ownership dominates common ownership if and only if* $\gamma > 1.5$.

All the proofs are in the Appendix.

Under joint ownership both agents have equal, intermediate incentives while under common ownership the high-valuation agent has strong incentives and the low-valuation agent has weak incentives. Now if the cost function is sufficiently convex – so that the investments are not very elastic to surplus share – two intermediate investments are more cost-effective than one high and one low investment and joint ownership is optimal.

3 Repeated game

In the repeated game the agents first contract on the ownership structure. Then, stages 2 and 3 are played in periods $t \in \{1, 2, ..., \infty\}$, that is, in each period the agents first invest and then make transfers and produce the public good. In a subgame-perfect equilibrium (SPE) of the repeated game, the agents may be able to sustain higher levels of investments than in the static game. While there can be multiple SPE in the repeated game, our focus is on the Pareto-optimal SPE where the agents choose the highest sustainable investments.[13]

We denote the highest sustainable investment by ηy^*, where η is an endogenous parameter such that $\eta \in [\underline{\eta}, 1]$ and $\underline{\eta} > (1/2)^{1/(\gamma-1)}$.[14] If $\delta \to 1$, the first-best investments can be sustained in SPE and thus $\eta = 1$. For lower values of δ the target investment has to be reduced and $\eta < 1$. It depends on the ownership structure how much the target investment has to be reduced. Our aim is to characterize the ownership structure that maximizes the joint surplus, that is, supports the highest ηy^* in SPE.

In the repeated game the agents promise to invest ηy^*, agent h promises to pay agent ℓ a transfer T^* and agent ℓ promises to accept it.[15] The agents continue to invest ηy^* and to agree to T^* as long as the other agent keeps his promises. The strategies are the following.

[13] In CPR context it is reasonable to assume that the community can coordinate on a Pareto-optimal SPE in their meetings.

[14] Note that $\eta y^* = y^J$ if $\eta = (1/2)^{1/(\gamma-1)}$.

[15] T^* is not restricted to be positive.

Phase 1: Agent h : invest ηy^* and pay T^* to agent ℓ. Agent ℓ : invest ηy^* and accept T^* from agent h. If agent i deviates, start Phase 2.

Phase 2:

Agent $j \neq i$: If agent i pays fine F_i^ω for $\omega \in \{C, J\}$ at the beginning of the period, go back to Phase 1. Otherwise, invest p_j^ω and start Phase 2 in the next period.

Agent i : Pay F_i^ω to agent j and go back to Phase 1.

If any player deviates in Phase 2, re-start Phase 2 against that player.

After any deviation the agents can renegotiate back to the Pareto frontier. If agent i deviates, he can restore cooperation by paying a fine F_i^ω to agent j at the beginning of the following period.[16] This is a natural way to model renegotiation as in CPR context failing to participate in maintenance activities is typically punished by a fine paid to the community. If deviator i does not pay the fine, agent j chooses punishment investment p_j^ω in that period. We allow for different strengths of punishment by investment to be feasible so that ρy_j^ω is the lowerbound for p_j^ω, where $\rho \in [0, 1]$.[17] ρ is an exogenous parameter that depends on the type of the public good. If $\rho = 0$, zero punishment investment is feasible as in e.g. a research joint venture where it implies delaying the project. The mildest punishment, $\rho = 1$, implies that punishment investment equals Nash investment of the static game. It is reasonable to assume that ρ is relatively large for a CPR such as a fragile low-tech irrigation system which the community's livelihood depends on.[18]

The incentive compatibility constraints (ICs) for each agent to invest ηy^* are

$$u_i^* \geq (1 - \delta) D_i^\omega + \delta [-(1 - \delta) F_i^\omega + u_i^*] \quad \text{for } i \in \{\ell, h\}, \ \omega \in \{C, J\} \tag{6}$$

where $\delta \in [0, 1)$ is the common discount factor. We have multiplied the payoffs by $(1 - \delta)$ to express them as per-period averages. D_i^ω is agent i's one-shot deviation payoff under ownership structure ω and u_i^* is agent i's payoff under cooperation, where $u_h^* = 2\theta_h \eta y^* - c(\eta y^*) - T^*$ and $u_\ell^* = 2\theta_\ell \eta y^* - c(\eta y^*) + T^*$.

[16] This approach draws from the private goods case of Blonski and Spagnolo (2007).

[17] See also Harstad, Lancia and Russo (2017), where in an environmental context punishment investments in compliance technology are not minimal.

[18] Alternatively, ρy_j^ω could be the lowest expected punishment investment. Suppose that in state s_1, with probability ρ, the public good is fragile and lack of maintenance would result in a very large cost to both agents. Therefore agent j would not invest less than y_j^ω in maintenance. In state s_2, with probability $(1 - \rho)$, $y_j = 0$ is feasible.

Summing up (6) and rearranging, we obtain the aggregate IC

$$\delta\left(F_h^\omega + F_\ell^\omega\right) \geq \left(D_h^\omega + D_\ell^\omega\right) - S^*\left(\eta\right) \equiv G^\omega \tag{7}$$

where $S^*(\eta)$ is the joint surplus when both agents invest ηy^*. We denote the aggregate one-shot gain from deviation by G^ω. G^ω measures how tempting deviation is to the agents and how this immedate temptation depends on the ownership structure.[19] If the discounted aggregate fines outweigh the aggregate gain from deviation, the agents can find a suitable T^* that satisfies both agents' ICs. We will first examine the deviation payoffs, $D_\ell^\omega + D_h^\omega$, and then determine the fines, $F_h^\omega + F_\ell^\omega$.

The agents can deviate by investing less than ηy^* or they can invest ηy^* but then deviate by not agreeing to transfer T^*. Both forms of deviation are observable to the other agent.[20] We will show that deviation in investment dominates deviation in transfer. If agent i deviates in investment, agent j observes it before the transfer stage and will not agree to T^* but will engage in bargaining. Agent i therefore chooses deviation investment equal to y_i^ω to maximize his bargaining payoff.[21] His deviation payoff under joint ownership is then

$$D_i^J = \frac{1}{2}\left(\theta_\ell + \theta_h\right)\left(\eta y^* + y^J\right) - c\left(y^J\right) \text{ for } i \in \{\ell, h\}, \tag{8}$$

while under common ownership

$$D_i^C = \theta_i\left(\eta y^* + y_i^C\right) - c\left(y_i^C\right) \text{ for } i \in \{\ell, h\}. \tag{9}$$

Alternatively, agent i can first invest ηy^* and then deviate by not agreeing to T^*. Also then the payoffs are determined by bargaining and are equal to (8) and (9) where y^J and y_i^C are replaced by ηy^*. Agent i must be strictly better off by adjusting his investment to y_i^ω to maximize his bargaining payoff than by choosing ηy^* and, consequently, deviation in investment dominates deviation in transfer T^*.

[19] See Halonen (2002).

[20] Observability of investment is an important difference to much of the literature on relational contracts, e.g. Baker, Gibbons and Murphy (2002). Our assumption fits CPRs where typically the whole community gathers to maintain the CPR on specific days and attendance of community members is observable. Furthermore, the work is often organized so that the work teams can monitor the progress of other teams who have been given a task of a similar size (Ostrom 1990, p. 85).

[21] We continue to apply Nash bargaining payoffs also in the repeated game. These payoffs can arise e.g. when each party can make a take-it-or-leave-it offer with probability $\frac{1}{2}$, as in Halac (2012) and (2015). See Miller and Watson (2013) for a richer analysis of bargaining in repeated games.

Next we turn our attention to the fines. Suppose agent i has deviated in investment. The IC for agent i to pay the fine F_i^ω is

$$-\left(1-\delta\right)F_i^\omega + u_i^* \geq \left(1-\delta\right)P_i^\omega + \delta\left[-\left(1-\delta\right)F_i^\omega + u_i^*\right] \text{ for } i \in \{\ell, h\}\,,\ \omega \in \{C, J\}\,. \tag{10}$$

If agent i pays F_i^ω, cooperation is restored immediately. Otherwise, he earns punishment payoff P_i^ω in that period and, using the one-shot deviation principle, the restoration of cooperation is postponed by one period. The punishment payoff under joint ownership is given by

$$P_i^J = \frac{1}{2}\left(\theta_\ell + \theta_h\right)\left(p_j^J + y^J\right) - c\left(y^J\right) \text{ for } i, j \in \{\ell, h\}\,,\ i \neq j, \tag{11}$$

and under common ownership

$$P_i^C = \theta_i\left(p_j^C + y_i^C\right) - c\left(y_i^C\right) \text{ for } i, j \in \{\ell, h\}\,,\ i \neq j. \tag{12}$$

From (10) we can solve for the maximal fine that agent i would be willing to pay.[22]

$$F_i^\omega = \frac{u_i^* - P_i^\omega}{\left(1-\delta\right)} \text{ for } i \in \{\ell, h\}\,,\ \omega \in \{C, J\} \tag{13}$$

The aggregate fines are then

$$F_h^\omega + F_\ell^\omega = \frac{S^*\left(\eta\right) - \left(P_h^\omega + P_\ell^\omega\right)}{\left(1-\delta\right)} = \frac{V^\omega}{\left(1-\delta\right)}. \tag{14}$$

We define $V^\omega \equiv \left(S^*\left(\eta\right) - P_\ell^\omega - P_h^\omega\right)$ as the value of the relationship governed by a relational contract as compared to punishment payoffs.[23] The higher is the value of the relationship, the larger fines are incentive compatible and therefore the less incentive there is to underinvest. The maximal fine transfers all the surplus from renegotiation

[22] We analyze maximal fines while there is a lowerbound ρy_j^ω for the punishment investment. High fine hurts only the deviator and benefits the other agent. Low punishment investment hurts both agents and may risk the fragile public good. Furthermore, in the context of CPRs, graduated sanctions have been used to deal with the issue of overappropriation (Ostrom, 1990, p. 94-100). Evidence suggests that sanctions for underprovision typically take the form of a fine that is equivalent to the daily wage of a labourer. See Berg (2008) for evidence from Nepal.

[23] We adopt Halac's (2012) terminology but define the value of the relationship as the difference between trade governed by relational contract and punishment payoffs. In Halac (2012) the comparison is to outside opportunities. While separation is an applicable punishment in business relationships, it is almost nonexisting in the stable communities that manage CPRs.

to the non-defecting party. Therefore the maximal aggregate fines are equal to the discounted value of the relationship.

The final IC is for executing the punishment investment.

$$(1-\delta)\widehat{P}_i^{\omega}+\delta\left[(1-\delta)F_j^{\omega}+u_i^*\right]\geq(1-\delta)u_i^{\omega}+\delta\left[-(1-\delta)F_i^{\omega}+u_i^*\right]\text{ for }i\in\{\ell,h\},\ \omega\in\{C,J\} \tag{15}$$

where $\widehat{P}_i^{\omega}$ is the punisher's payoff and u_i^{ω} is agent i's payoff if he deviates from punishment (equal to his payoff in the static game). Note that the punisher's payoff $\widehat{P}_i^{\omega}$ includes $y_i = p_i^{\omega}$, while in the punishment payoff P_i^{ω} it is the other agent that chooses punishment investment and $y_j = p_j^{\omega}$ is substituted in. If agent i chooses the punishment investment p_i^{ω} in this period, cooperation is restored in the following period by agent j paying F_j^{ω} to agent i. If agent i chooses to deviate from punishment, he would choose y_i^{ω} and, by the one-shot deviation principle, cooperation is restored in the following period by him paying F_i^{ω} to agent j. (15) simplifies to

$$\delta\left(F_h^{\omega}+F_{\ell}^{\omega}\right)\geq u_i^{\omega}-\widehat{P}_i^{\omega}\text{ for }i\in\{\ell,h\},\ \omega\in\{C,J\}. \tag{16}$$

The consequence to agent i of deviating from the punishment investment is that not only he has to pay F_i^{ω} to agent j in the next period, but he also loses F_j^{ω} agent j would have paid him. That is why the aggregate fines appear in agent i's IC. The IC is satisfied if the discounted aggregate fines outweigh agent i's one-shot gain from deviating from the punishment investment.

Note that the left-hand-sides of (7) and (16) are equal. Therefore, if $u_i^{\omega}-\widehat{P}_i^{\omega}\leq G^{\omega}$, (16) is satisfied whenever (7) holds. However, if

$$u_i^{\omega}-\widehat{P}_i^{\omega}>G^{\omega}, \tag{17}$$

it is more tempting to deviate from the punishment investment than it is to deviate from ηy^*. In that case there are values of δ for which (7) is satisfied but (16) is not. Then it is optimal to increase the punishment investment to relax (16) since the closer the punishment investment is to the Nash investment, the smaller is the gain from deviating from it. Denote by $\underline{\rho}^{\omega}y_i^{\omega}$ the lowest punishment investment that is incentive compatible given (7) is satisfied. Optimal punishment investment that is both feasible and incentive compatible is then given by $p_i^{\omega}=\max\left\{\rho,\underline{\rho}^{\omega}\right\}y_i^{\omega}$.

Lemma 1 *Suppose $\eta \to 1$. There exist values of δ for which optimal punishment investment is strictly greater than its lowest feasible level, ρy_i^{ω}, if and only if $\rho < \underline{\rho}^{\omega}$ and $\gamma > \hat{\gamma}^{\omega}$ where $\hat{\gamma}^{\omega} \in (2,3)$ for $\omega \in \{C, J\}$.*

Lemma 1 shows that (17) can be satisfied and thus for some values of δ it can be optimal to increase the punishment investment from its lowest feasible level. First, when ρ is small, the gain from deviating from the punishment investment, the left-hand-side of (17), is large. Second, the gain from deviating from the target investment, G^{ω}, is small when the difference between y^* and Nash investment of the static game is small. This is the case when the investment is quite inelastic to surplus share ($\gamma > \hat{\gamma}^{\omega}$). If the investment is elastic to surplus share ($\gamma < 2$), (17) cannot be satisfied and even zero punishment investment is incentive compatible given (7) holds.

Furthermore, IC for punishment investment is not a binding constraint when ρ is not too small. Lemma 2 examines this case.

Lemma 2 *Suppose $\rho \geq \underline{\rho}^{\omega}$. Both agents investing ηy^* is SPE under ownership structure ω if and only if $\delta \geq \frac{G^{\omega}}{G^{\omega}+V^{\omega}}$.*

If $\rho \geq \underline{\rho}^{\omega}$, (16) holds as long as (7) is satisfied. We can then substitute the maximal fines from (14) in the critical IC (7) and both agents investing ηy^* is SPE if and only if

$$\delta \geq \frac{G^{\omega}}{G^{\omega} + V^{\omega}} \equiv \underline{\delta}^{\omega}(\eta). \tag{18}$$

The following Lemma shows that, given $\rho \geq \underline{\rho}^{\omega}$, characterizing the optimal ownership structure boils down to minimizing $\underline{\delta}^{\omega}(\eta)$.

Lemma 3 *(i) Suppose $\underline{\delta}^{\omega_1}(\eta) < \underline{\delta}^{\omega_2}(\eta)$ for all $\eta \in [\underline{\eta}, 1]$ and $\partial \underline{\delta}^{\omega_i}(\eta)/\partial \eta > 0$ for $i = 1, 2$. Then joint surplus under ω_1 is (weakly) greater than under ω_2 for any $\delta > 0$.*

(ii) Suppose $\underline{\delta}^{\omega_1}(\eta) < \underline{\delta}^{\omega_2}(\eta)$ if and only if $\eta > \tilde{\eta}$ where $\tilde{\eta} \in (\underline{\eta}, 1)$ and $\partial \underline{\delta}^{\omega_i}(\eta)/\partial \eta > 0$ for $i = 1, 2$. Then joint surplus under ω_1 is (weakly) greater than under ω_2 if and only if $\delta > \underline{\delta}^{\omega_1}(\tilde{\eta})$.

According to Lemma 3(i), the ownership structure that minimizes $\underline{\delta}^{\omega}(\eta)$ maximizes the joint surplus. First, if $\delta \geq \underline{\delta}^{\omega_1}(1)$, the first best investments are sustainable under ω_1 and the joint surplus is greater than under ω_2, strictly greater if $\delta < \underline{\delta}^{\omega_2}(1)$. Second, if $\delta < \underline{\delta}^{\omega_1}(1)$, the target investment has to be lowered. Denote by $\eta^{\omega_i} y^*$ the highest investment that is incentive compatible under ω_i, that is, $\delta = \underline{\delta}^{\omega_i}(\eta^{\omega_i})$. Strictly higher investments are sustainable under ω_1 than under ω_2 since $\delta = \underline{\delta}^{\omega_1}(\eta^{\omega_1}) = \underline{\delta}^{\omega_2}(\eta^{\omega_2}) < \underline{\delta}^{\omega_2}(\eta^{\omega_1})$ and $\partial\underline{\delta}^{\omega_i}(\eta)/\partial\eta > 0$. Lemma 3(*ii*) extends the result to the case where the ranking of the critical discount factors depends on η.

Characterizing the optimal ownership structure is then, according to (18), equivalent to minimizing the gain from deviation relative to the value of the relationship. It is useful to first examine the gain from deviation and the value of the relationship separately.

Lemma 4 (*i*) $G^J < G^C$,
(*ii*) $V^J \leq V^C$ *if* $\gamma > 1.5$ *and* $\rho \geq \max\left\{\widehat{\rho}, \underline{\rho}^C, \underline{\rho}^J\right\}$ *where* $\widehat{\rho} \in (0,1)$,
(*iii*) $V^J > V^C$ *if* $\gamma < 1.5$ *or* $\max\left\{\underline{\rho}^C, \underline{\rho}^J\right\} \leq \rho < \widehat{\rho}$.

There are two sources to the gain from deviation. The first part comes from the ability to expropriate from the other agent's investment. Under joint ownership, according to (8), the agent can expropriate half of the *joint* value of the other agent's investment while under common ownership, according to (9), he can expropriate his full *individual* value. However, adding these up amounts to $(\theta_\ell + \theta_h)\,\eta y^*$ under both ownership structures. Therefore the difference in the gain from deviation comes from the second source, the payoff earned from the agent's own second-best investment, as can be seen from the following equation (which is derived in (44) in the Appendix).

$$\begin{aligned} G^C - G^J &= \left(D_\ell^C + D_h^C\right) - \left(D_\ell^J + D_h^J\right) \\ &= \left[\theta_h y_h^C - c\left(y_h^C\right)\right] + \left[\theta_\ell y_\ell^C - c\left(y_\ell^C\right)\right] - 2\left[\frac{1}{2}\left(\theta_\ell + \theta_h\right) y^J - c\left(y^J\right)\right] \end{aligned} \tag{19}$$

When $\theta_\ell = \theta_h$, the ownership structures are equivalent (see (3) and (5)) and $G^C = G^J$. Using the envelope theorem, we can show that

$$\frac{\partial\left(G^C - G^J\right)}{\partial\theta_h} = y_h^C - y^J > 0 \tag{20}$$

and therefore $G^C > G^J$ for any $\theta_\ell < \theta_h$. The gain from deviation is higher under common ownership because agent h gets his full valuation from his contribution to the public good while under joint ownership he has to share his high valuation in bargaining. There is the opposite effect for agent ℓ but it is of a smaller magnitude.[24]

[24] This is because agent i's payoff from his own investment is convex in the surplus share. Agent

The difference in the value of the relationship is given by

$$\begin{aligned} V^C - V^J &= \left(P_\ell^J + P_h^J\right) - \left(P_\ell^C + P_h^C\right) \\ &= 2\left[\frac{1}{2}\left(\theta_\ell + \theta_h\right)\left(y^J + \rho y^J\right) - c\left(y^J\right)\right] \\ &\quad - \left[\theta_h\left(y_h^C + \rho y_\ell^C\right) - c\left(y_h^C\right)\right] - \left[\theta_\ell\left(y_\ell^C + \rho y_h^C\right) - c\left(y_\ell^C\right)\right]. \end{aligned}$$

If the punishment investment is zero ($\rho = 0$), each agent's punishment payoff arises only from his own investment. By (19) above this payoff is lower under joint ownership implying that the value of the relationship is higher under joint ownership. If punishment investment equals Nash investment of the static game ($\rho = 1$), aggregate punishment payoffs equal the joint surplus in the static game. Then, according to Proposition 1 common ownership maximizes the value of the relationship if $\gamma > 1.5$; this holds also for sufficiently high ρ. While for $\gamma < 1.5$ joint ownership maximizes value of the relationship for all values of ρ.

According to Lemma 4, joint ownership provides both the maximal value of the relationship and the minimal gain from deviation if either $\gamma < 1.5$ or $\rho < \widehat{\rho}$. Then, unambiguously, $\underline{\delta}^J(\eta) < \underline{\delta}^C(\eta)$. For higher values of γ and ρ there is a trade-off: common ownership maximizes the value of the relationship but also maximizes the gain from deviation.

Proposition 2 characterizes the joint surplus maximizing ownership structure.

Proposition 2 *(i) Joint surplus is (weakly) greater under common ownership than under joint ownership if* $\gamma > 2$, $\rho > \max\left\{\widetilde{\rho}, \underline{\rho}^C, \underline{\rho}^J\right\}$ *and* $\delta > \underline{\delta}^C(\widehat{\eta})$ *where* $\widetilde{\rho} \in (\widehat{\rho}, 1)$ *and* $\widehat{\eta} \in \left(\underline{\eta}, 1\right)$.

(ii) Joint surplus is (weakly) greater under joint ownership than under common ownership if $\gamma < 2$, $\max\ \underline{\rho}^C, \underline{\rho}^J\ < \rho < \widetilde{\rho}$ *or* $\delta < \underline{\delta}^C(\widehat{\eta})$.

The first-best investments can be sustained for a lower δ under common ownership when $\gamma > 2$ and ρ is sufficiently large. In this parameter range both the value of the relationship and the gain from deviation are larger under common ownership. However, the value of the relationship is relatively low even under common ownership. If punishment is mild ($\rho = 1$), the value of the relationship equals the reduction in surplus when the agents choose Nash investments rather than the first-best investments.

i's payoff from his own investment is $\sigma_i y_i(\sigma_i) - c(y_i(\sigma_i)) = \sigma_i(\sigma_i/\gamma)^{1/(\gamma-1)} - (\sigma_i/\gamma)^{\gamma/(\gamma-1)} = (\gamma-1)(\sigma_i/\gamma)^{\gamma/(\gamma-1)}$. It is straigtforward to verify that this payoff is convex in σ_i.

This reduction is small when $\gamma > 2$ because investment is inelastic to the surplus share. Furthermore, the value of the relationship remains low when ρ is relatively high. Low value of the relationship limits the effectiveness of fines in disciplining behavior. Therefore maximizing the value of the relationship provides the best incentives for cooperation – and common ownership is optimal.

If δ is not high enough for the first-best investments to be supported in SPE, the agents can lower the target investment and still achieve higher joint surplus than in the static game. Strictly higher investments can be supported under common ownership than under joint ownership as long as $\gamma > 2$ and ρ is high. However, if δ is sufficiently low so that the target investment has to be reduced below $\widehat{\eta} y^*$, common ownership no longer generates the highest joint surplus. Joint ownership becomes optimal because lower target investment reduces the gain from deviation relatively more under joint ownership. The absolute reduction in the gain is equal (by (19) $G^C - G^J$ does not depend on η) while the relative reduction is greater under joint ownership because $G^J < G^C$.

Joint ownership is optimal also when $\gamma < 2$. The value of the relationship is then high under any ownership structure even when punishment is mild because there is a large difference between the first-best and Nash investment as investment is elastic to surplus share. Alternatively, the value of the relationship is large when punishment by investment is strong ($\rho < \widetilde{\rho}$). Then minimizing the gain from deviation – and joint ownership – maximizes the joint surplus.

Finally, note that γ has a different role in the static and in the repeated game. In the static game joint ownership is optimal for high γ while in the repeated game low γ favours joint ownership. The parameter ranges interestingly overlap and joint ownership provides the best incentives in both static and repeated games if $1.5 < \gamma < 2$.

3.1 Nonlinear value of investment

Our model puts all the curvature in the cost function as that gives a clear intuition of the results. We now consider the possibility that also the value of the public good is nonlinear: $\theta_i \left[(y_\ell)^{\gamma_v} + (y_h)^{\gamma_v} \right]$, where $\gamma_v \leq 1$. The investment costs are $c(y_i) = (y_i)^{\gamma_c}$, where $\gamma_c \geq 1$ and $\frac{\gamma_c}{\gamma_v} > 1$. It is straightforward to show that this formulation is isomorphic to our main model when $\frac{\gamma_c}{\gamma_v} = \gamma$.[25]

In this formulation $\frac{\gamma_c}{\gamma_v} > 2$ has to hold for common ownership to be optimal – in addition to sufficiently high ρ and δ. Note that $\gamma_v < 0.5$ is a sufficient condition for $\gamma > 2$ since $\gamma_c \geq 1$. This is consistent with a stylized fact of common ownership: the value of the CPR cannot be increased much by investment (Netting 1976; Ostrom 1990, p. 63). The use of Alpine hillsides for communal grazing lands in Switzerland is an example of such common ownership.

$\gamma_c > 2$ is another sufficient condition for $\frac{\gamma_c}{\gamma_v} > 2$ since $\gamma_v \leq 1$. $\gamma_c > 2$ implies that the cost function is sufficiently convex so that also the marginal cost is convex. In Section 6 we argue that convex the marginal costs can explain communally owned irrigation systems in Nepal and the Philippines. Marginal cost of maintenance arises from time and effort taken away from cultivating privately owned fields and increasing maintenance activities increasingly lowers the private yield.

Conversely, concavity of marginal cost ($\gamma_c < 2$) and relatively high γ_v ($\gamma_v > 0.5$) are necessary conditions for $\frac{\gamma_c}{\gamma_v} < 2$ and can give rise to joint ownership. Horizontal research joint venture is a leading example of joint ownership of a public good.[26] Concavity of marginal cost can arise from learning effects which are milder than in learning-by-doing models. The marginal cost is still increasing but at a decreasing rate. Additionally, the investment has to have a relatively large effect on the value of the public good. Both conditions are reasonable for R&D activities.

4 Single ownership

In the main model we have compared shared ownership structures. We now turn our attention to an ownership structure where either agent ℓ or h is the single owner. Single ownership can be equivalent to either private or government ownership. This is where we build on Besley and Ghatak (2001) (BG) and their analysis of the role of ownership in public good provision. In their development context, private NGO has a higher valuation for the public good than government. In other applications the

[25] In this alternative formulation, under common ownership we have $y_i^C = \left(\frac{\theta_i}{\gamma}\right)^{\frac{1}{\gamma_v(\gamma-1)}}$ and joint surplus equals $S^C = (\theta_\ell + \theta_h)\left[\left(\frac{\theta_\ell}{\gamma}\right)^{\frac{1}{\gamma-1}} + \left(\frac{\theta_h}{\gamma}\right)^{\frac{1}{\gamma-1}}\right] - \left(\frac{\theta_\ell}{\gamma}\right)^{\frac{\gamma}{\gamma-1}} - \left(\frac{\theta_h}{\gamma}\right)^{\frac{\gamma}{\gamma-1}}$ as in the main model. Furthermore, the value of the investment, $\left(y_i^C\right)^{\gamma_v}$, is elastic to the surplus share if and only if $\gamma_c/\gamma_v = \gamma < 2$ as in the main model.

[26] In a horizontal research joint venture innovation becomes a public good for the partners.

government may well be the high-valuation agent.[27] We differ from BG by introducing a repeated game and the concept of common ownership.

4.1 Static game

Our static model of single ownership is a simplified version of BG. We start by defining the disagreement payoffs. If bargaining breaks down, the non-owning agent leaves the project and the owner completes it without his contribution. However, some of his investment may have spilled over to the project. Denote by μ the degree of spillover, where $\mu \in [0, 1)$.[28] Then agent k's disagreement payoff under ownership by i is $\theta_k (y_i + \mu y_j)$, where $k, i, j \in \{\ell, h\}$ and $i \neq j$. The owning agent's investment contributes fully to the project while only the spillover from the non-owning agent's investment remains in the project. Note that the non-owning agent is not excluded from the *consumption* of the public good although he does not participate in its *production*.[29]

Given these disagreement payoffs, the Nash bargaining payoffs under ownership by agent i are

$$u_i^i = \theta_i (y_i + \mu y_j) + \frac{1}{2} (\theta_\ell + \theta_h) [(y_i + y_j) - (y_i + \mu y_j)] - c(y_i)$$

$$= \frac{1}{2} (\theta_\ell + \theta_h) (y_i + y_j) + \frac{1}{2} (\theta_i - \theta_j) (y_i + \mu y_j) - c(y_i), \tag{21}$$

$$u_j^i = \frac{1}{2} (\theta_\ell + \theta_h) (y_i + y_j) + \frac{1}{2} (\theta_j - \theta_i) (y_i + \mu y_j) - c(y_j). \tag{22}$$

[27] Note that here – and in BG – privatization means involvement of a private party as the owner in public good provision. Privatization can also mean transforming a public good into a private good. This type of privatization will be discussed later in this section.

[28] Now, unlike in the main model, it matters whether the investment is in physical or human capital. Investment in physical capital remains in the project if the investing agent leaves and therefore $\mu = 1$. Investment in human capital is embedded in the agent. If the agent leaves, so does the human capital unless some of it has spilled over to the project, therefore $\mu \in [0, 1)$. Project-specific human capital includes e.g. engineering skills that are specific to the project and relationships in the community.

[29] In the public goods context Nash bargaining has a credibility issue regarding ex post disagreement payoffs that we share with BG. If bargaining breaks down, the owner would benefit from giving access to the other agent so that also his investment contributes to the public good. The theory of contracts as reference points (Hart and Moore 2008; Hart 2009) provides a possible avenue to explain this ex post inefficiency arising due to the souring of the relationship.

Therefore the investment incentives in the static game are

$$\frac{1}{2}(\theta_\ell + \theta_h) + \frac{1}{2}(\theta_i - \theta_j) = c'\left(y_i^i\right), \tag{23}$$

$$\frac{1}{2}(\theta_\ell + \theta_h) + \frac{1}{2}(\theta_j - \theta_i)\,\mu = c'\left(y_j^i\right), \tag{24}$$

where superscript i denotes ownership by agent i.

Comparing h- and ℓ-ownership amounts to comparing the second terms in (23) and (24). The second term is positive for the high-valuation agent and negative for the low-valuation agent. Higher investment by either agent increases both agents' disagreement payoffs since they can consume the public good even under disagreement. Agent h's disagreement payoff increases more given his higher valuation of the public good. In other words, higher investment by either agent improves h's bargaining position relative to ℓ, explaining why the second term is positive for agent h and negative for agent ℓ.

The positive effect on agent h's incentives can be increased if his investment contributes fully to the disagreement payoff, that is, if he owns the public good. While the negative effect on agent ℓ's incentives can be weakened if only part of his investment spills over to the project, that is, if he does not own the public good. Accordingly, ownership by the high-valuation agent improves both agents' incentives. This is the main result of BG.

Comparing the incentives also to common ownership and joint ownership, (3) and (5), it follows that

$$y_\ell^C = y_\ell^\ell < y_\ell^h \leq y^J < y^*, \tag{25}$$

$$y^J \leq y_h^\ell < y_h^C = y_h^h < y^*. \tag{26}$$

Accordingly, h-ownership dominates also common ownership and therefore a repeated game is needed to provide a rationale for common ownership. Finally, comparing joint ownership and h-ownership, there is a tradeoff as ℓ's investment is higher under joint ownership but h's investment is higher under h-ownership.[30]

[30] It is easy to show that joint ownership is optimal in the static game if and only if $\gamma > 1.5$ and $\mu \geq \widetilde{\mu}$, where $0 < \widetilde{\mu} < 1$, and h-ownership is optimal otherwise.

4.2 Repeated game

As in Section 3, in the repeated game we focus on characterizing the ownership structure that minimizes the critical discount factor. In this Section, we focus our analysis on the critical discount factor above which the agents can support the first best investments.

As previously, we start by analyzing the gain from deviation and the value of the relationship.

Lemma 5 *(i)* $\max\left\{G^J, G^h\right\} < G^C < G^\ell$,
(ii) $\max\left\{V^J, V^h\right\} < V^C < V^\ell$ *if* $\gamma > 1.5$ *and* $\rho = 1$.

Lemma 5 shows that there are broadly two classes of ownership structures. First, the ownership structures that can be optimal in the static game – joint ownership and h-ownership – minimize the gain from deviation. Second, the ownership structures that are dominated in the static game – common ownership and ℓ-ownership – maximize the value of the relationship if γ and ρ are large. In line with the previous results, we will show that common ownership or ℓ-ownership provides the best incentives for cooperation when $\gamma > 2$ and ρ is sufficiently high while $\gamma < 2$ and low values of ρ favour joint ownership or h-ownership.

Proposition 3 characterizes the ownership structure that provides the best incentives to cooperate in terms of minimizing the critical discount factor above which the first best investments are sustainable in SPE.

Proposition 3 *Suppose* $\rho \geq \underline{\rho}^\omega$ *for* $\omega \in \{C, J, h, \ell\}$*. The best incentives for cooperation are provided by*

(i) joint ownership if (a) $\gamma < 2$ *or* $\rho < \tilde{\rho}$ *and (b)* $\mu \to 1$*,*
(ii) ownership by agent h *if* $\gamma < 2$*,* $\rho \to 1$ *and* $\mu \to 0$*,*
(iii) common ownership if $\gamma > 2$*,* $\rho > \rho'$ *and* $\frac{\theta_\ell}{\theta_h} \to 0$*,*
(iv) ownership by agent ℓ *if* $\gamma > 2$*,* $\rho \to 1$*,* $\frac{\theta_\ell}{\theta_h} \to 1$ *and* $\mu \to 0$*.*

Proposition 3 shows that even when we include single ownership in the analysis, joint ownership continues to provide the best cooperation incentives for similar parameter values as in Proposition 2 as long as $\mu \to 1$, while h-ownership provides stronger incentives if $\mu \to 0$. Figure 1(a) presents simulation results which show that this result holds also for intermediate values of μ.[31]

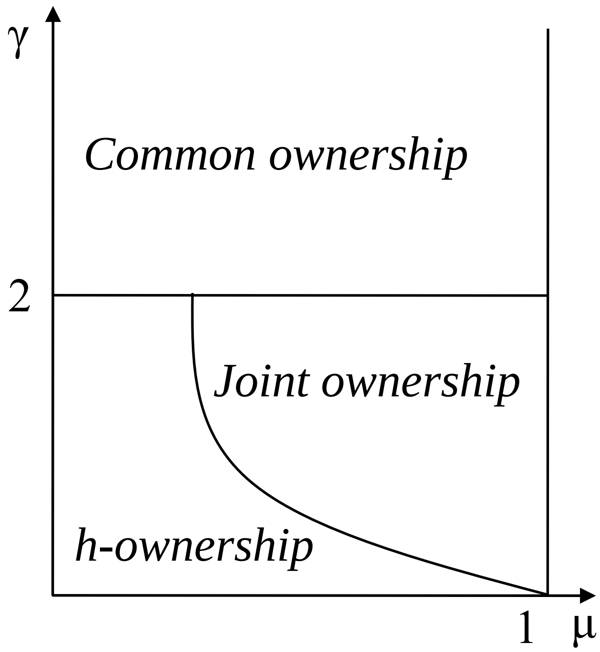

Figure 1(a)

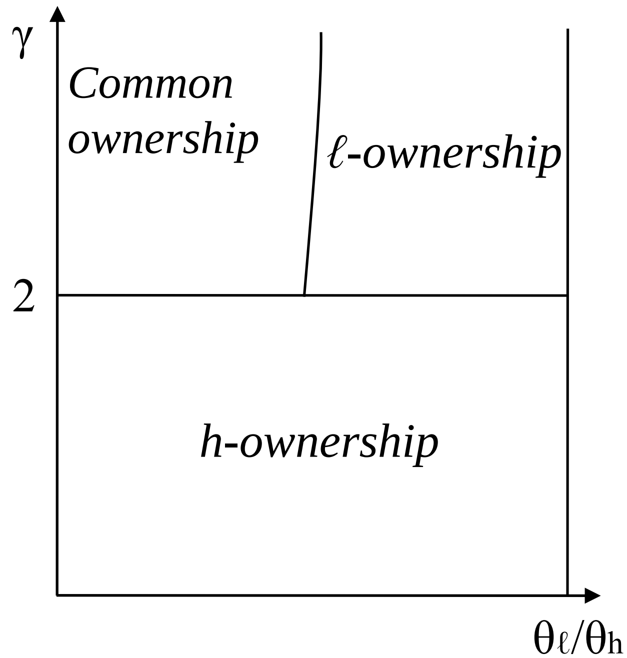

Figure 1(b)

This is the parameter range where the gain effect is important ($\gamma < 2$). Ownership by agent h provides the best cooperation incentives for small μ because the gain from deviation is minimized. Agent h's deviation payoff under h-ownership is

$$D_h^h = \frac{1}{2}(\theta_h + \theta_\ell)\left(y_h^h + y^*\right) + \frac{1}{2}(\theta_h - \theta_\ell)\left(y_h^h + \mu y^*\right) - c\left(y_h^h\right). \tag{27}$$

When agent ℓ's first-best investment is largely embedded in himself ($\mu \to 0$), h's ability to extract from it is limited.[32] While for high values of μ the gain from deviation under joint ownership (which does not depend on μ) is lower than the gain from deviation under h-ownership (which is increasing in μ) and joint ownership provides the best cooperation incentives.

The additional requirement for joint ownership is that the investments are largely sunk in the project (large μ). This matches well with R&D since innovation can typically be commercialized without the presence of the other party of the horizontal

[31] Figure 1(a) is drawn for $\rho = 1$. Furthermore, $\frac{\theta_\ell}{\theta_h}$ is sufficiently small so that common ownership is optimal for all μ if $\gamma > 2$..

[32] Note that for agent ℓ there is the opposite effect, $\partial D_\ell^h / \partial\mu < 0$, but it is of a smaller magnitude so that $\partial G^h / \partial\mu > 0$ as verified by (58).

research joint venture.

Common ownership provides the best cooperation incentives for similar parameter values as in Proposition 2 as long as $\frac{\theta_\ell}{\theta_h} \to 0$, while ownership by agent ℓ performs better if $\frac{\theta_\ell}{\theta_h} \to 1$. Simulations presented in Figure 1(b) confirm also this result for intermediate values of $\frac{\theta_\ell}{\theta_h}$.[33]

Here the effects are more subtle as the best ownership structure does not simply maximize the value of the relationship although $\gamma > 2$ ($V^\ell > V^C$ and $G^\ell > G^C$ for all parameter values). Ownership by the low-valuation agent provides the best cooperation incentives only if θ_ℓ is not too low compared to θ_h. The difference between agent ℓ's deviation payoffs, taking into account that $y_\ell^\ell = y_\ell^C$, is equal to

$$D_\ell^\ell - D_\ell^C = \frac{1}{2}\left(\theta_h - \theta_\ell\right)\left(1-\mu\right)y^* > 0.$$

Agent ℓ can extract more from h's first-best investment under ℓ-ownership since in bargaining $\theta_h y^*$ is shared while under common ownership ℓ just gets his own low valuation, $\theta_\ell y^*$. For a similar reason ℓ's deviation payoff is more responsive to a change in θ_ℓ under common ownership, $\partial\left(D_\ell^\ell - D_\ell^C\right)/\partial\theta_\ell < 0$, and therefore higher θ_ℓ increases ℓ's deviation payoff – and the aggregate gain from deviation – more under common ownership favouring ℓ-ownership. That is why cooperation incentives are strongest under ℓ-ownership for large $\frac{\theta_\ell}{\theta_h}$.

Common ownership provides the best incentives when the (marginal) effect on the value of the relationship is dominant. The difference between agent ℓ's punishment payoffs is equal to

$$P_\ell^\ell - P_\ell^C = \frac{1}{2}\left[\left(\theta_h + \theta_\ell\right) + \left(\theta_\ell - \theta_h\right)\mu\right]\rho y_h^\ell - \theta_\ell \rho y_h^C.$$

As above, $\partial\left(P_\ell^\ell - P_\ell^C\right)/\partial\theta_\ell < 0$ since the change in θ_ℓ is shared in bargaining under ℓ-ownership while it has the full effect on ℓ's payoff under common ownership. Therefore lower θ_ℓ decreases ℓ's punishment payoff more under common ownership than under ℓ-ownership.[34] Consequently, the value of the relationship increases more under common

[33] Figure 1(b) is drawn for $\rho = 1$. Furthermore, $\mu = 0$ so that h-ownership is optimal for all $\frac{\theta_\ell}{\theta_h}$ if $\gamma < 2$. Simulation results also show that $\mu \to 0$ is not a necessary condition for ℓ -ownership to be optimal (Proposition 3(iv)).

[34] Note also that $\partial^2\left(P_\ell^\ell - P_\ell^C\right)/\partial\left(\theta_\ell\right)^2 > 0$ and $\partial^2\left(D_\ell^\ell - D_\ell^C\right)/\partial\left(\theta_\ell\right)^2 < 0$ which explains why the marginal effect on the value of the relationship is dominant for low θ_ℓ and the marginal gain effect is dominant for high θ_ℓ.

ownership when $\frac{\theta_\ell}{\theta_h}$ is small and common ownership provides the best cooperation incentives.

The additional requirement for common ownership is that the valuations are not too homogeneous (small $\frac{\theta_\ell}{\theta_h}$). This condition requires further discussion. In the empirical literature there is no consensus on the role of heterogeneity in the management of CPRs (see e.g. Poteete and Ostrom 2004; Ruttan 2006, 2008). Different types of heterogeneity can have different effects. For example, economic heterogeneity can have a positive effect[35] while sociocultural heterogeneity is likely to have a negative effect. In our model the valuation difference arises from economic heterogeneity, e.g. heterogeneity in the private holdings of cultivated land to be irrigated or in the number of livestock requiring grazing. There is typically heterogeneity in private wealth in communities managing CPRs. For example, Netting (1981, p. 27) documents heterogeneity in the number of cows owned in Swiss villages.[36] Heterogeneity favours common ownership in our model because it improves the incentives to cooperate under common ownership as compared to ℓ-ownership. Ownership by the low-valuation agent provides the best cooperation incentives only if his valuation is not too low compared to the high-valuation agent, while common ownership performs better if valuations are not too homogeneous. In Section 5 we examine heterogeneity in investment costs and show that homogeneous costs favour common ownership confirming the view that different types of heterogeneity have different effects.

We have analyzed single ownership of public goods which can be equivalent to privatization. However, privatization can also take the form of transforming a public good into a private good. For example, land can remain as a public good (under common or government ownership) or it can be parceled to be a private good. In Switzerland it is the more productive arable lands in the mountain valleys that are privately owned (Netting 1976). This is consistent with Halonen (2002) where single ownership provides the best cooperation incentives for private goods when $\gamma < 2$.

We conclude this section with a comparison of our results to the private goods case analyzed by Halonen (2002). With private goods the worst ownership structure of the static game (joint ownership) provides the best cooperation incentives for $\gamma > 2$ because it maximizes the value of the relationship. While the optimal ownership

[35] See also Baland and Platteau (2003) for theoretical analysis where the effect of economic heterogeneity depends on the model setup.

[36] As the general practise is to distribute the benefits from the CPR in direct proportion to private holdings, wealthier individuals indeed have a higher valuation for the CPR (McKean 1992).

structure of the static game (single ownership) provides the best incentives when $\gamma < 2$ because it minimizes the gain from deviation. With public goods the results no longer depend on the tradeoff between the best and the worst ownership structure of the static game. First, common ownership can provide the best cooperation incentives for $\gamma > 2$ even when ℓ-ownership is the worst ownership structure in the static game. Second, joint ownership can minimize the gain from deviation even when it is not optimal in the static game. Furthermore, with private goods the results of the static game hold also in the repeated game for $\gamma < 2$ – as the gain from deviation is minimized. With public goods also the optimal ownership structure of the static game depends on γ and therefore – as discussed in Section 3 – there is overlap only for $1.5 < \gamma < 2$. Despite these differences, the critical value for γ remains the same.

5 Asymmetric costs

In the main model, the agents only differ in their valuations. In this Section, we allow the agents to differ also in terms of their investment costs.[37] Suppose the costs are given by $c_i(y_i) = \frac{1}{\varsigma_i}(y_i)^\gamma$ for $i = \ell, h$.

Simulation results show that Proposition 3 is robust to cost asymmetry. Common ownership or ℓ-ownership continue to provide the best cooperation incentives for $\gamma > 2$ and joint ownership or h-ownership for $\gamma < 2$. Cost asymmetry, however, shifts the boundary of common ownership vs. ℓ-ownership and joint ownership vs. h-ownership. Figure 2 shows the boundary for identical costs by a solid line and agent h's cost advantage ($\varsigma_h > \varsigma_\ell$) by a broken line.[38]

[37] We focus our analysis on the case of $\rho = 1$.

[38] Both figures are drawn for the same parameter values as figures 1(a) and 1(b) and moderate $\varsigma_h/\varsigma_\ell$.

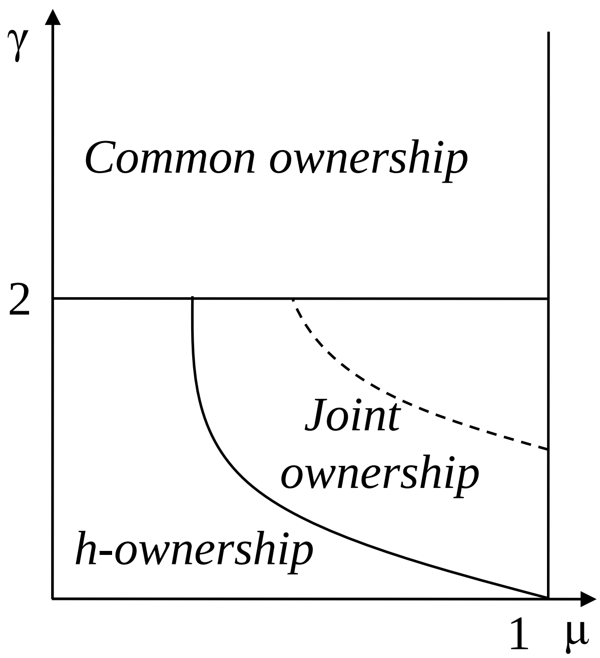

Figure 2(a)

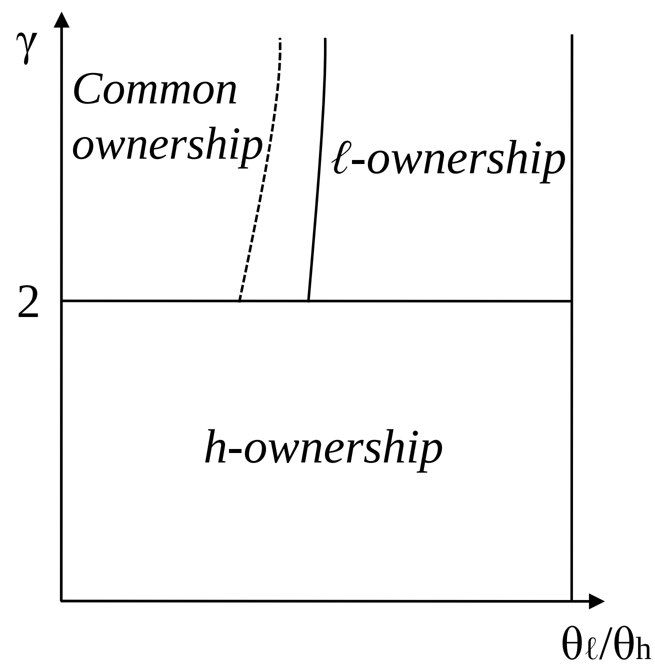

Figure 2(b)

Figure 2(a) shows that for $\gamma < 2$ the boundary shifts in favour of h-ownership. As discussed in Section 4, this boundary depends on which ownership structure minimizes the gain from deviation. The gain from deviation increases under both ownership structures when ς_h increases because agent h's first best investment increases and deviation becomes more tempting for agent ℓ. Agent ℓ's deviation payoff increases more under joint ownership because agent h's higher valuation is shared in bargaining, while under h-ownership agent ℓ cannot obtain any of the owner's high valuation. Therefore the gain minimizing ownership structure shifts in favour of h-ownership.

Figure 2(b) shows that for $\gamma > 2$, the boundary shifts in favour of ℓ-ownership. As shown in Section 4, ℓ-ownership provides the best cooperation incentives when the marginal effect on the gain from deviation is dominant, which is the case when θ_ℓ is not much lower than θ_h. A higher θ_ℓ increases agent ℓ's deviation payoff – and the aggregate gain from deviation – more under common ownership since ℓ's valuation for h's first best investment, $\theta_\ell y_h^*$, is not shared in bargaining. Therefore θ_ℓ closer to θ_h favours ℓ-ownership. When we introduce cost asymmetry, higher ς_h makes this effect even stronger because it increases y_h^*. Therefore the boundary shifts in favour of ℓ-ownership.[39]

[39] Even when agent h's cost advantage is significant, common ownership can still provide the best incentives for cooperation as long as the valuation difference is significant.

In sum, the above analysis shows that shared ownership is less likely when h has a cost advantage.

6 Common pool resources

In this Section we re-examine well documented field studies in Ostrom (1990) in the light of our analysis. We have selected CPRs which require maintenance investments: irrigation systems and common lands. Maintenance is provided communally in these cases and the communities have been successful in mobilizing a significant amount of labor regularly. For example, in the Zanjera irrigation communities in the Philippines, compliance rate was 94% (Ostrom 1990, p. 86).

Nonattendance in maintenance activities is punished by fines paid to the community (Netting 1972; McKean 1982; Ostrom 1990, p. 86) unless an acceptable excuse is provided. For example, in the villages of Japan studied by McKean (1982) the only acceptable excuses were illness, family tragedy or the absence of able-bodied adults. We examined fines in the repeated game.

We have shown that common ownership provides the best incentives for cooperation if $\gamma > 2$, punishment by investment is mild, the discount factor is sufficiently high and the valuations of the CPR are not too homogeneous. We now argue that these conditions are satisfied in Ostrom's field studies.[40] Let us start the discussion by examining whether $\gamma > 2$. It is helpful to frame the discussion around the isomorphic version of the model where both the value of the public good and the investment costs can be nonlinear. As presented in Section 3.1, the value of the public good is $\theta_i \left[(y_l)^{\gamma_v} + (y_h)^{\gamma_v}\right]$, where $\gamma_v \leq 1$, and the investment costs are $c\,(y_i) = (y_i)^{\gamma_c}$, where $\gamma_c \geq 1$. Then $\gamma = \frac{\gamma_c}{\gamma_v}$. As explained in Section 3.1, there are two sufficient conditions for $\gamma > 2$. First, $\gamma_v < 0.5$ implies that $\gamma > 2$ since $\gamma_c \geq 1$. Second, $\gamma_c > 2$ is a sufficient condition for $\gamma > 2$ since $\gamma_v \leq 1$. Next we will relate these sufficient conditions to Ostrom's field studies.

Most of the Alpine pasture in Switzerland has been under common ownership for centuries while arable land in the mountain valleys is typically privately owned.[41] Netting (1976) identifies stylized facts of Alpine land ownership (see also Ostrom 1990, p. 63). He observes that while the yield of privately owned arable land can be

[40] Heterogeneity of valuations was already discussed in Section 4.2.

[41] In Valais, Switzerland, 95% of Alpine pasture has been under common ownership (Netting 1976). Also in Italy most of the Alpine pasture has been commonly owned (Casari 2007).

increased by "*irrigation, manuring, erosion control, crop rotation, and careful horticulture*" (Netting 1976, p. 143), the possibility of improving the commonly owned Alpine grazing lands is low due to altitude, limited growing season and thin soils.[42] This stylized fact of common ownership is consistent with our model where the value of the CPR cannot be increased much by investment when γ_v is low. Extensive common lands have existed for centuries also in Japan (McKean 1982; Ostrom 1990, p. 65). Also there the maintenance investments, including the annual burning of the grasslands, have a limited impact on the yield.

According to our results, the rationale for common ownership can arise also from sufficiently convex maintenance costs. Then, in contrast to the above stylized fact, common ownership can provide the best cooperation incentives even when it is possible to increase the value of the CPR significantly by investment. Communally owned irrigation systems in Nepal and the Philippines are examples of such CPRs. The irrigation systems are very valuable to the local communities as their livelihood depends on irrigation. Furthermore, their value can be increased significantly by investments.[43] However, common ownership can be explained by the maintenance costs. For example, in one of the Zanjera irrigation communities in the Philippines, hundreds of people are involved in constructing and maintaining a 100-meter-long dam that spans the Bacarra-Vintar River. Maintenance is costly for the community members: in 1980 the average contribution was 37 days of work per person (Coward and Siy 1983; Ostrom 1990, p. 83).[44] The marginal cost of maintenance investment arises from the time and effort taken away from cultivating privately owned fields. We argue that this marginal cost is convex ($\gamma_c > 2$). Cobb-Douglas production function (with decreasing returns to scale) is often used for agricultural production. With this function, the marginal product of private cultivation is decreasing and convex.[45] Since the marginal cost of increasing maintenance investment equals the marginal product of *reducing* private cultivation, convexity of marginal product implies that the marginal cost is convex. Therefore common ownership of these irrigation systems is consistent with

[42]Maintenance of the grazing lands includes distributing manure, renewing avalanche-damaged corrals and clearing access paths and roads.

[43]For more details see e.g. Ostrom (1990, p. 82-88) for the Philippines and Yoder (1994) for Nepal.

[44]Another example comes from the Chhatis Mauja irrigation system in Nepal where in 1981 over 60,520 man-days were devoted from at least 3,000 farmers on desilting the main canal and other arduous tasks (Pradhan 1984; Ostrom 1990, p. 229).

[45]Suppose the production function is given by $q = l^{\alpha}x^{\beta}$ where q is production of private fields, l is labor input and x is other inputs. Then marginal product of labor is $MP_l = \alpha l^{\alpha-1}\ x^{\beta}$ and $\partial^2 MP_l/\partial l^2 = \alpha(\alpha-1)(\alpha-2)l^{\alpha-3}x^{\beta} > 0$ since $\alpha < 1$.

our model.[46]

The second condition for common ownership in our model is that punishment by investment is mild (ρ is sufficiently high). We argue that this is reasonable for common property arrangements which typically involve subsistence communities dependent on the CPR. For example, in the above Zanjera irrigation communities dams are constructed using poor quality materials such as bamboo poles, banana leaves, sand and rock (Ostrom 1990, p. 83). Continual maintenance is therefore crucial for the survival of the community.[47]

The final condition for common ownership is sufficiently high discount factor. Ostrom herself argued that the discount factor is high (the discount rate is low) in these communities. "*These individuals live side by side and farm the same plots year after year. They expect their children and their grandchildren to inherit their land. In other words, their discount rates are low.*" (Ostrom 1990, p. 88)

Our analysis differs from Ostrom (1990) who examined numerous field studies of successful and unsuccessful CPRs and identified design principles that characterize the successful ones. These design principles – such as well-defined boundaries, graduated sanctions and low-cost conflict-resolution mechanisms[48] – are largely endogenous. By contrast, our interest is in finding the characteristics of exogenous maintenance technology for which common ownership provides the best cooperation incentives. We show that the often cited examples of successful CPRs – irrigation systems in Nepal and the Philippines and common lands in Switzerland and Japan – do not only share similar design principles but can also be characterized by the same exogenous factor: high $\frac{\gamma_c}{\gamma_v}$ arising from either sufficiently convex cost function or value relatively unresponsive to maintenance investments.

[46] Ostrom and Gardner (1993) examine the interaction of maintenance and appropriation in a communally owned irrigation system. They find that costly maintenance is helpful for reaching equitable access to water.

[47] In the case of the common lands in Japan, timely maintenance investments, such as the annual burning of the grasslands in early spring, are essential in improving the yield of the natural products which are very important in the daily life of the local community (McKean 1982, p. 70).

[48] Other design principles are congruence between appropriation and provision rules and local conditions, collective-choice arrangements, monitoring, minimal recognition of rights to organize and nested enterprises (Ostrom 1990, p. 90).

7 Conclusions

In this paper we have analyzed ownership of public goods, in particular common ownership. From the static point of view, common ownership gives poor incentives to maintain the public good. However, building on the literature which has used insights from repeated games to understand successful management of CPRs (Ostrom 1990; Ostrom, Gardner and Walker 1994), we show that a repeated game can provide a rationale for common ownership.

We find that common ownership can provide the best incentives for cooperation when investment is inelastic to surplus share. Mild punishment investment is then not significantly lower than the first-best investment, implying that the value of the relationship is low and fines are quite ineffective in disciplining maintenance investments. Cooperation incentives can then be improved by choosing an ownership structure that maximizes the value of the relationship. Common ownership provides the best incentive for cooperation if additionally valuations for the public good are not too homogeneous.

Inelasticity of maintenance investment can arise when the value of the public good is relatively unresponsive to investment, consistent with an important stylized fact derived from communally owned Alpine grazing lands in Switzerland (Netting 1976; Ostrom 1990, p. 63). We show that inelastic maintenance investment can also result from sufficiently convex maintenance costs, as in the irrigation systems in Nepal and the Philippines where time and effort taken away from private cultivation increasingly lowers the private yield. Our analysis shows that many successful CPRs do not only share similar (endogenous) design principles identified by Ostrom (1990) but also the same exogenous characteristics.

Our analysis of public goods applies to provision and maintenance of CPRs. An important direction for future work is to extend the analysis to the possibility of overuse of the CPR. Also, future work could explore the difference between local and global CPRs – resources that go beyond national jurisdictions (e.g. earth's oceans and global climate). Global CPRs do not only differ from local CPRs in terms of characteristics but also pose different challenges.[49]

[49] See Stern (2011) for a useful discussion of the differences between global and local CPRs and the applicability of Ostrom's design principles to global CPRs. The design principles were largely informed by local CPRs.

A Appendix

Firstly, the explicit forms of the investments derived in (1) , (3) and (5) are

$$y^{*} = \left(\frac{\theta_{\ell} + \theta_{h}}{\gamma}\right)^{\frac{1}{\gamma-1}}, \tag{28}$$

$$y^{J} = \left(\frac{\theta_{\ell} + \theta_{h}}{2\gamma}\right)^{\frac{1}{\gamma-1}}, \tag{29}$$

$$y_{\ell}^{C} = \left(\frac{\theta_{\ell}}{\gamma}\right)^{\frac{1}{\gamma-1}}, \tag{30}$$

$$y_{h}^{C} = \left(\frac{\theta_{h}}{\gamma}\right)^{\frac{1}{\gamma-1}}. \tag{31}$$

Proof of Proposition 1.

Denote by S^{ω} joint surplus under ownership structure $\omega \in \{J, C\}$. $S^{J} > S^{C}$ if and only if

$$2\left(\theta_{\ell} + \theta_{h}\right) y^{J} - 2c\left(y^{J}\right) > \left(\theta_{\ell} + \theta_{h}\right)\left(y_{\ell}^{C} + y_{h}^{C}\right) - c\left(y_{\ell}^{C}\right) - c\left(y_{h}^{C}\right). \tag{32}$$

Substituting $\theta_{h} = \alpha\theta_{\ell}$, where $\alpha > 1$, and (29) – (31) in (32) we obtain

$$2\left(\alpha+1\right)\theta_{\ell}\left(\frac{\left(\alpha+1\right)\theta_{\ell}}{2\gamma}\right)^{\frac{1}{\gamma-1}} - 2\left(\frac{\left(\alpha+1\right)\theta_{\ell}}{2\gamma}\right)^{\frac{\gamma}{\gamma-1}} >$$

$$\left(\alpha+1\right)\theta_{\ell}\left[\left(\frac{\alpha\theta_{\ell}}{\gamma}\right)^{\frac{1}{\gamma-1}} + \left(\frac{\theta_{\ell}}{\gamma}\right)^{\frac{1}{\gamma-1}}\right] - \left(\frac{\alpha\theta_{\ell}}{\gamma}\right)^{\frac{\gamma}{\gamma-1}} - \left(\frac{\theta_{\ell}}{\gamma}\right)^{\frac{\gamma}{\gamma-1}}$$

which is equivalent to

$$2\left(\alpha+1\right)\left(\frac{\alpha+1}{2\gamma}\right)^{\frac{1}{\gamma-1}} - 2\left(\frac{\alpha+1}{2\gamma}\right)^{\frac{\gamma}{\gamma-1}} >$$

$$(\alpha+1)\left[\left(\frac{\alpha}{\gamma}\right)^{\frac{1}{\gamma-1}}+\left(\frac{1}{\gamma}\right)^{\frac{1}{\gamma-1}}\right]-\left(\frac{\alpha}{\gamma}\right)^{\frac{\gamma}{\gamma-1}}-\left(\frac{1}{\gamma}\right)^{\frac{\gamma}{\gamma-1}}. \tag{33}$$

Multiplying by $\gamma^{\frac{1}{\gamma-1}}$ and rearranging, (33) is equivalent to $\psi_s(\alpha,\gamma) > \chi_s(\alpha,\gamma)$ where

$$\psi_s(\alpha,\gamma)=\frac{2\gamma-1}{\gamma}\left(\frac{1}{2}\right)^{\frac{1}{\gamma-1}}(\alpha+1)^{\frac{\gamma}{\gamma-1}}-(\alpha+1)\alpha^{\frac{1}{\gamma-1}}+\frac{1}{\gamma}\alpha^{\frac{\gamma}{\gamma-1}},$$

$$\chi_s(\alpha,\gamma)=(\alpha+1)-\frac{1}{\gamma}.$$

Note that $\psi_s(1,\gamma)=\chi_s(1,\gamma)=\frac{2\gamma-1}{\gamma}$ and $\frac{\partial\psi_s(\alpha,\gamma)}{\partial\alpha}|_{\alpha=1}=\frac{\partial\chi_s(\alpha,\gamma)}{\partial\alpha}|_{\alpha=1}=1$ given

$$\frac{\partial\psi_s(\alpha,\gamma)}{\partial\alpha}=\frac{2\gamma-1}{\gamma-1}\left(\frac{1}{2}\right)^{\frac{1}{\gamma-1}}(\alpha+1)^{\frac{1}{\gamma-1}}-\alpha^{\frac{1}{\gamma-1}}-\frac{1}{\gamma-1}\alpha^{\frac{2-\gamma}{\gamma-1}}. \tag{34}$$

In the Online Appendix we establish that $\partial^2\psi_s(\alpha,\gamma)/\partial\alpha^2>0$ for any $\alpha>1$ if and only if $\gamma>1.5$. Since $\psi_s(\alpha,\gamma)$ and $\chi_s(\alpha,\gamma)$ are tangent at $\alpha=1$ and $\chi_s(\alpha,\gamma)$ is linear and increasing in α, $\psi_s(\alpha,\gamma)>\chi_s(\alpha,\gamma)$ for any $\alpha>1$ if and only if $\gamma>1.5$. Consequently, $S^J>S^C$ if and only if $\gamma>1.5$. Q.E.D.

Proof of Lemma 1.

Examine first joint ownership. We aim to show when $u_i^J-\hat{P}_i^J>G^J$.

$$\begin{aligned} u_i^J-\hat{P}_i^J &= \left[\frac{1}{2}(\theta_\ell+\theta_h)2y^J-c\left(y^J\right)\right]-\left[\frac{1}{2}(\theta_\ell+\theta_h)(1+\rho)y^J-c\left(\rho y^J\right)\right] \\ &= \left[\frac{1}{2}(\theta_\ell+\theta_h)y^J-c\left(y^J\right)\right]-\left[\frac{1}{2}(\theta_\ell+\theta_h)\rho y^J-c\left(\rho y^J\right)\right]\geq 0 \end{aligned} \tag{35}$$

From (35), $u_i^J-\hat{P}_i^J>0$ if $\rho=0$, $u_i^J-\hat{P}_i^J=0$ if $\rho=1$ and $\frac{\partial\left(u_i^J-\hat{P}_i^J\right)}{\partial\rho}<0$ for $\rho\in[0,1)$.

$$\begin{aligned} G^J &= \left(D_h^J+D_\ell^J\right)-S^*(\eta)=2\left[\frac{1}{2}(\theta_\ell+\theta_h)\left(y^J+\eta y^*\right)-c\left(y^J\right)\right]-\left[2(\theta_\ell+\theta_h)\eta y^*-2c(\eta y^*)\right] \\ &= 2\left[\frac{1}{2}(\theta_\ell+\theta_h)y^J-c\left(y^J\right)\right]-2\left[\frac{1}{2}(\theta_\ell+\theta_h)\eta y^*-c(\eta y^*)\right]>0 \end{aligned} \tag{36}$$

$G^J>0$ for any $\eta y^*>y^J$ since y^J maximizes the first term in square brackets.

Suppose $\rho=1$. $G^J>u_i^J-\hat{P}_i^J$ since $u_i^J-\hat{P}_i^J=0$ if $\rho=1$.

Suppose $\rho = 0$. From (35) and (36) $G^J \geq u_i^J - \hat{P}_i^J$ if and only if

$$\left[\frac{1}{2}(\theta_\ell + \theta_h) y^J - c\left(y^J\right)\right] - 2\left[\frac{1}{2}(\theta_\ell + \theta_h)\eta y^* - c(\eta y^*)\right] \geq 0 \tag{37}$$

Substituting y^J and y^* from (28) and (29) in (37), we obtain

$$\frac{1}{2}\left(\frac{\theta_\ell + \theta_h}{\gamma}\right)^{\frac{\gamma}{\gamma-1}}\left[(\gamma - 1)\left(\frac{1}{2}\right)^{\frac{1}{\gamma-1}} - 2(\gamma\eta - 2\eta^\gamma)\right] \geq 0. \tag{38}$$

In the Online Appendix we prove that (38) is satisfied for $\eta \to 1$ if and only if $\gamma \leq \hat{\gamma}^J$ where $\hat{\gamma}^J \in (2, 3)$.

Therefore, if $\gamma \leq \hat{\gamma}^J$, $u_i^J - \hat{P}_i^J \leq G^J$ for all $\rho \in [0, 1]$ since $u_i^J - \hat{P}_i^J \leq G^J$ for both $\rho = 0$ and $\rho = 1$, $\frac{\partial\left(u_i^J - \hat{P}_i^J\right)}{\partial \rho} < 0$ and $\frac{\partial G^J}{\partial \rho} = 0$. Therefore IC for punishment investment (16) is satisfied whenever IC for target investment ηy^* (7) holds. Accordingly, the optimal punishment investment equals the lowest feasible punishment investment, $p_i^J = \rho y^J$.

If $\gamma > \hat{\gamma}^J$, there exists $\underline{\rho}^J \in (0, 1)$ such that $u_i^J - \hat{P}_i^J \leq G^J$ if and only if $\rho \geq \underline{\rho}^J$. This follows since $u_i^J - \hat{P}_i^J > G^J$ if $\rho = 0$, $u_i^J - \hat{P}_i^J < G^J$ if $\rho = 1$, $\frac{\partial\left(u_i^J - \hat{P}_i^J\right)}{\partial \rho} < 0$ and $\frac{\partial G^J}{\partial \rho} = 0$. Therefore, if $p_i^J = \rho y^J$ and $\rho < \underline{\rho}^J$, there are values of δ for which (7) is satisfied but (16) is not. Then it is optimal to increase p_i^J so that $u_i^J - \hat{P}_i^J = G^J$. That is, optimal punishment investment equals $p_i^J = \underline{\rho}^J y^J > \rho y^J$ if and only if $\gamma > \hat{\gamma}^J$ and $\rho < \underline{\rho}^J$ proving the statement in Lemma 1 for joint ownership.

Under common ownership the ICs for punishment investments are

$$\delta\left(F_\ell^C + F_h^C\right) \geq u_i^C - \widehat{P}_i^C \text{ for } i \in \{\ell, h\}. \tag{39}$$

Examine the right-hand-side of (39).

$$\begin{aligned} u_i^C - \widehat{P}_i^C &= \left[\theta_i\left(y_i^C + y_j^C\right) - c\left(y_i^C\right)\right] - \left[\theta_i\left(\rho y_i^C + y_j^C\right) - c\left(\rho y_i^C\right)\right] \\ &= \left[\theta_i y_i^C - c\left(y_i^C\right)\right] - \left[\theta_i \rho y_i^C - c\left(\rho y_i^C\right)\right] \\ &= \left(\frac{\theta_i}{\gamma}\right)^{\frac{\gamma}{\gamma-1}}\left[(\gamma - 1) - (\gamma\rho - \rho^\gamma)\right] \geq 0 \end{aligned} \tag{40}$$

It follows from (40) that $u_h^C - \hat{P}_h^C > u_\ell^C - \hat{P}_\ell^C$ and therefore the IC for agent ℓ's punishment investment is slack.

As above, we derive G^C.

$$G^C = \left(D_h^C + D_\ell^C\right) - S^*(\eta)$$
$$= \left[\theta_h y_h^C - c\left(y_h^C\right)\right] - \left[\theta_h \eta y^* - c(\eta y^*)\right] + \left[\theta_\ell y_\ell^C - c\left(y_\ell^C\right)\right] - \left[\theta_\ell \eta y^* - c(\eta y^*)\right] > 0. \quad (41)$$

$G^C > 0$ for any $\eta y^* > y_h^C$ since y_h^C maximizes the first term in square brackets and y_ℓ^C maximizes the third term in square brackets.

$u_h^C - \widehat{P}_h^C \leq G^C$ is equivalent to

$$\left[\theta_h \rho y_h^C - c\left(\rho y_h^C\right)\right] + \left[\theta_\ell y_\ell^C - c\left(y_\ell^C\right)\right] - \left[(\theta_\ell + \theta_h)\eta y^* - 2c(\eta y^*)\right] \geq 0 \qquad (42)$$

Substitute $\rho = 0$ in (42).

$$\left(\frac{\theta_\ell}{\gamma}\right)^{\frac{\gamma}{\gamma-1}} \left[(\gamma - 1) - (1+\alpha)^{\frac{\gamma}{\gamma-1}} (\gamma\eta - 2\eta^\gamma)\right] \geq 0 \qquad (43)$$

In the Online Appendix we prove that (43) is satisfied if and only if $\gamma \leq \hat{\gamma}^C$ where $\hat{\gamma}^C \in (2, 3)$.

As above, if $\gamma \leq \hat{\gamma}^C$, $u_h^C - \hat{P}_h^C \leq G^C$ for all $\rho \in [0, 1]$ since $u_h^C - \hat{P}_h^C \leq G^C$ for both $\rho = 0$ and $\rho = 1$, $\frac{\partial\left(u_h^C - \hat{P}_h^C\right)}{\partial\rho} < 0$ and $\frac{\partial G^C}{\partial\rho} = 0$. Accordingly, $p_i^C = \rho y_i^C$ for $i \in \{\ell, h\}$.

If $\gamma > \hat{\gamma}^C$, there exists $\underline{\rho}^C$ such that $u_h^C - \hat{P}_h^C \leq G^C$ if and only if $\rho \geq \underline{\rho}^C$. This follows since $u_h^C - \hat{P}_h^C > G^C$ if $\rho = 0$, $u_h^C - \hat{P}_h^C < G^C$ if $\rho = 1$, $\frac{\partial\left(u_h^C - \hat{P}_h^C\right)}{\partial\rho} < 0$ and $\frac{\partial G^C}{\partial\rho} = 0$. Therefore $p_h^C = \underline{\rho}^C y_h^C > \rho y_h^C$.

Q.E.D.

Proof of Lemma 3.

(i) If $\delta \geq \underline{\delta}^{\omega_2}(1)$, investing y^* is SPE under both ownership structures. If $\underline{\delta}^{\omega_1}(1) \leq \delta < \underline{\delta}^{\omega_2}(1)$, investing y^* is SPE only under ω_1.

If $\delta < \underline{\delta}^{\omega_1}(1)$, investing y^* is not a SPE under either ownership structure. Define η^{ω_i} such that $\delta = \underline{\delta}^{\omega_i}(\eta^{\omega_i})$. Accordingly, $\eta^{\omega_i} y^*$ is the highest investment sustainable in a SPE under ω_i. Therefore, $\delta = \underline{\delta}^{\omega_1}(\eta^{\omega_1}) = \underline{\delta}^{\omega_2}(\eta^{\omega_2}) < \underline{\delta}^{\omega_2}(\eta^{\omega_1})$. Since $\partial\underline{\delta}^{\omega_i}(\eta)/\partial\eta > 0$, this implies that $\eta^{\omega_1} > \eta^{\omega_2}$ and strictly higher investments are sustainable in SPE under ω_1 than under ω_2.

In sum, the joint surplus under ω_1 is (weakly) greater than under ω_2 for any $\delta > 0$.

(ii) As in (i), investing y^* is a SPE under ω_1 if $\delta \geq \underline{\delta}^{\omega_1}(1)$ and the joint surplus under ω_1 is (weakly) greater than under ω_2.

Now we have $\underline{\delta}^{\omega_1}(\tilde{\eta}) = \underline{\delta}^{\omega_2}(\tilde{\eta})$, $\delta = \underline{\delta}^{\omega_1}(\eta^{\omega_1}) = \underline{\delta}^{\omega_2}(\eta^{\omega_2}) < \underline{\delta}^{\omega_2}(\eta^{\omega_1})$ if and only if

$\eta > \tilde{\eta}$ and $\delta = \underline{\delta}^{\omega_2}(\eta^{\omega_2}) = \underline{\delta}^{\omega_1}(\eta^{\omega_1}) < \underline{\delta}^{\omega_1}(\eta^{\omega_2})$ if and only if $\eta < \tilde{\eta}$. Consequently, the joint surplus is maximized under ω_1 if and only if $\delta > \underline{\delta}^{\omega_1}(\tilde{\eta})$.

Q.E.D.

Proof of Lemma 4.

(i) From (36) and (41) we obtain

$$G^C - G^J = \left[\theta_h y_h^C - c\left(y_h^C\right)\right] + \left[\theta_\ell y_\ell^C - c\left(y_\ell^C\right)\right] - 2\left[\frac{1}{2}\left(\theta_\ell + \theta_h\right) y^J - c\left(y^J\right)\right]. \quad (44)$$

(20) in Section 4 proves that $G^C > G^J$ for any $\theta_h > \theta_\ell$.

(ii) Since $\rho \geq \max\left\{\underline{\rho}^C, \underline{\rho}^J\right\}$, $p_i^\omega = \rho y_i^\omega$ for $i \in \{h, \ell\}$ and $\omega \in \{C, J\}$.

$$V^C = S^*(\eta) - \left[\theta_h\left(y_h^C + \rho y_\ell^C\right) - c\left(y_h^C\right)\right] - \left[\theta_\ell\left(y_\ell^C + \rho y_h^C\right) - c\left(y_\ell^C\right)\right],$$

$$V^J = S^*(\eta) - 2\left[\frac{1}{2}\left(\theta_h + \theta_\ell\right)\left(y^J + \rho y^J\right) - c\left(y^J\right)\right].$$

If $\rho = 1$, $V^C = S^*(\eta) - S^C$ and $V^J = S^*(\eta) - S^J$. According to Proposition 1, $S^C < S^J$ if and only if $\gamma > 1.5$. Therefore if $\rho = 1$ $V^C > V^J$ if and only if $\gamma > 1.5$.

Furthermore, if $\rho = 0$

$$V^J - V^C = \left[\theta_h y_h^C - c\left(y_h^C\right)\right] + \left[\theta_\ell y_\ell^C - c\left(y_\ell^C\right)\right] - 2\left[\frac{1}{2}\left(\theta_h + \theta_\ell\right) y^J - c\left(y^J\right)\right] > 0. \quad (45)$$

(45) is equivalent to (19) and therefore positive.

Suppose $\gamma > 1.5$. Since $V^J > V^C$ if $\rho = 0$, $V^J < V^C$ if $\rho = 1$ and V^J and V^C are linear in ρ, there exists $\hat{\rho} \in (0, 1)$ such that $V^J \leq V^C$ if and only if $\rho \geq \hat{\rho}$.

Suppose $\gamma < 1.5$. Since $V^J > V^C$ for both $\rho = 0$ and $\rho = 1$ and V^J and V^C are linear in ρ, it follows that $V^J > V^C$ for all $\rho \in [0, 1]$.

In sum, $V^J \leq V^C$ if $\gamma > 1.5$ and $\rho \geq \max\left\{\hat{\rho}, \underline{\rho}^C, \underline{\rho}^J\right\}$. $V^J > V^C$ if $\gamma < 1.5$ or $\max\left\{\underline{\rho}^C, \underline{\rho}^J\right\} < \rho < \hat{\rho}$.

Q.E.D.

Proof of Proposition 2.

Step 1. Derive $\underline{\delta}^J$ and $\underline{\delta}^C$.

To derive the explicit form of $\underline{\delta}^J$ we first substitute (28) and (29) in (36) obtaining

$$\begin{aligned} G^J &= 2\left[\frac{1}{2}(\theta_\ell+\theta_h)\left(\frac{\theta_\ell+\theta_h}{2\gamma}\right)^{\frac{1}{\gamma-1}} - \left(\frac{\theta_\ell+\theta_h}{2\gamma}\right)^{\frac{\gamma}{\gamma-1}}\right] \\ &\quad -2\left[\frac{1}{2}(\theta_\ell+\theta_h)\,\eta\left(\frac{\theta_\ell+\theta_h}{\gamma}\right)^{\frac{1}{\gamma-1}} - \eta^\gamma\left(\frac{\theta_\ell+\theta_h}{\gamma}\right)^{\frac{\gamma}{\gamma-1}}\right] \\ &= \left(\frac{\theta_\ell+\theta_h}{\gamma}\right)^{\frac{\gamma}{\gamma-1}}\left[(\gamma-1)\left(\frac{1}{2}\right)^{\frac{1}{\gamma-1}} - (\eta\gamma - 2\eta^\gamma)\right] > 0. \qquad (46) \end{aligned}$$

(46) is positive by (36).

$G^J + V^J = \left(D_\ell^J + D_h^J\right) - \left(P_\ell^J + P_h^J\right)$ equals

$$\begin{aligned} G^J+V^J &= 2\left[\frac{1}{2}(\theta_\ell+\theta_h)\left(y^J+\eta y^*\right) - c\left(y^J\right)\right] - 2\left[\frac{1}{2}(\theta_h+\theta_\ell)\left(y^J+\rho y^J\right) - c\left(y^J\right)\right] \\ &= (\theta_\ell+\theta_h)\left(\eta y^* - \rho y^J\right) \\ &= (\theta_\ell+\theta_h)\left[\eta\left(\frac{\theta_\ell+\theta_h}{\gamma}\right)^{\frac{1}{\gamma-1}} - \rho\left(\frac{\theta_\ell+\theta_h}{2\gamma}\right)^{\frac{1}{\gamma-1}}\right] \\ &= \left(\frac{1}{\gamma}\right)^{\frac{1}{\gamma-1}}(\theta_\ell+\theta_h)^{\frac{\gamma}{\gamma-1}}\left[\eta - \rho\left(\frac{1}{2}\right)^{\frac{1}{\gamma-1}}\right]. \qquad (47) \end{aligned}$$

Using (46) and (47) we obtain

$$\underline{\delta}^J = \frac{G^J}{G^J+V^J} = \frac{(\gamma-1)\left(\frac{1}{2}\right)^{\frac{1}{\gamma-1}} - (\eta\gamma - 2\eta^\gamma)}{\gamma\left[\eta - \rho\left(\frac{1}{2}\right)^{\frac{1}{\gamma-1}}\right]}. \qquad (48)$$

Similarly, G^C from (41) and $G^C + V^C = \left(D_\ell^C + D_h^C\right) - \left(P_\ell^C + P_h^C\right)$ equal

$$\begin{aligned} G^C &= \left[\theta_h \left(\frac{\theta_h}{\gamma}\right)^{\frac{1}{\gamma-1}} - \left(\frac{\theta_h}{\gamma}\right)^{\frac{\gamma}{\gamma-1}}\right] + \left[\theta_\ell \left(\frac{\theta_\ell}{\gamma}\right)^{\frac{1}{\gamma-1}} - \left(\frac{\theta_\ell}{\gamma}\right)^{\frac{\gamma}{\gamma-1}}\right] \\ &\quad - \left[(\theta_\ell + \theta_h)\,\eta \left(\frac{\theta_\ell + \theta_h}{\gamma}\right)^{\frac{1}{\gamma-1}} - 2\eta^\gamma \left(\frac{\theta_\ell + \theta_h}{\gamma}\right)^{\frac{\gamma}{\gamma-1}}\right] \\ &= \left(\frac{\theta_\ell}{\gamma}\right)^{\frac{\gamma}{\gamma-1}} \left[(\gamma-1)\,\alpha^{\frac{\gamma}{\gamma-1}} + (\gamma-1) - (\eta\gamma - 2\eta^\gamma)(\alpha+1)^{\frac{\gamma}{\gamma-1}}\right], \end{aligned} \tag{49}$$

$$\begin{aligned} G^C + V^C &= \left[\theta_h \left(y_h^C + \eta y^*\right) - c\left(y_h^C\right)\right] + \left[\theta_\ell \left(y_\ell^C + \eta y^*\right) - c\left(y_\ell^C\right)\right] \\ &\quad - \left[\theta_h \left(y_h^C + \rho y_\ell^C\right) - c\left(y_h^C\right)\right] - \left[\theta_\ell \left(y_\ell^C + \rho y_h^C\right) - c\left(y_\ell^C\right)\right] \\ &= \theta_h \left(\eta y^* - \rho y_\ell^C\right) + \theta_\ell \left(\eta y^* - \rho y_h^C\right) \\ &= (\theta_\ell)^{\frac{\gamma}{\gamma-1}} \left(\frac{1}{\gamma}\right)^{\frac{1}{\gamma-1}} \left[\left(\eta(\alpha+1)^{\frac{1}{\gamma-1}} - \rho(\alpha)^{\frac{1}{\gamma-1}}\right) + \alpha\left((\eta\alpha+1)^{\frac{1}{\gamma-1}} - \rho\right)\right] \end{aligned} \tag{50}$$

We have substituted in $\theta_h = \alpha\theta_\ell$. Using (49) and (50) we obtain

$$\underline{\delta}^C = \frac{(\gamma-1) + (\gamma-1)\,\alpha^{\frac{\gamma}{\gamma-1}} - (\eta\gamma - 2\eta^\gamma)(\alpha+1)^{\frac{\gamma}{\gamma-1}}}{\gamma\left[\left(\eta(\alpha+1)^{\frac{1}{\gamma-1}} - \rho(\alpha)^{\frac{1}{\gamma-1}}\right) + \alpha\left(\eta(\alpha+1)^{\frac{1}{\gamma-1}} - \rho\right)\right]}. \tag{51}$$

Step 2. Proof that for $\eta = \rho = 1$ $\underline{\delta}^C < \underline{\delta}^J$ if and only if $\gamma > 2$

In the Online Appendix we establish that for $\rho = 1$ $\underline{\delta}^C < \underline{\delta}^J \Leftrightarrow \chi_r(\alpha,\gamma,\eta) < \psi_r(\alpha,\gamma,\eta)$ where

$$\chi_r(\alpha,\gamma,\eta) = (\gamma-1)\left[\eta - \left(\frac{1}{2}\right)^{\frac{1}{\gamma-1}}\right] + \alpha\left[(\gamma-1)\left(\frac{1}{2}\right)^{\frac{1}{\gamma-1}} - (\eta\gamma - 2\eta^\gamma)\right], \tag{52}$$

$$\begin{aligned} \psi_r(\alpha,\gamma,\eta) &= (2\eta^\gamma - \eta)\left(\frac{1}{2}\right)^{\frac{1}{\gamma-1}}(\alpha+1)^{\frac{\gamma}{\gamma-1}} - (\gamma-1)\,\alpha^{\frac{\gamma}{\gamma-1}}\left[\eta - \left(\frac{1}{2}\right)^{\frac{1}{\gamma-1}}\right] \\ &\quad - \alpha^{\frac{1}{\gamma-1}}\left[(\gamma-1)\left(\frac{1}{2}\right)^{\frac{1}{\gamma-1}} - (\eta\gamma - 2\eta^\gamma)\right]. \end{aligned} \tag{53}$$

Therefore, if $\eta = 1$ we have

$$\chi_r(\alpha,\gamma,1) = (\gamma-1)\left[1-\left(\frac{1}{2}\right)^{\frac{1}{\gamma-1}}\right] + \alpha\left[(\gamma-1)\left(\frac{1}{2}\right)^{\frac{1}{\gamma-1}} - (\gamma-2)\right],$$

$$\psi_r(\alpha,\gamma,1) = \left(\frac{1}{2}\right)^{\frac{1}{\gamma-1}}(\alpha+1)^{\frac{\gamma}{\gamma-1}} - (\gamma-1)\,\alpha^{\frac{\gamma}{\gamma-1}}\left[1-\left(\frac{1}{2}\right)^{\frac{1}{\gamma-1}}\right]$$
$$-\alpha^{\frac{1}{\gamma-1}}\left[(\gamma-1)\left(\frac{1}{2}\right)^{\frac{1}{\gamma-1}} - (\gamma-2)\right].$$

Differentiating with respect to α gives

$$\frac{\partial\chi_r(\alpha,\gamma,1)}{\partial\alpha} = \left[(\gamma-1)\left(\frac{1}{2}\right)^{\frac{1}{\gamma-1}} - (\gamma-2)\right] > 0, \quad (54)$$

$$\frac{\partial\psi_r(\alpha,\gamma,1)}{\partial\alpha} = \frac{\gamma}{\gamma-1}\left(\frac{1}{2}\right)^{\frac{1}{\gamma-1}}(\alpha+1)^{\frac{1}{\gamma-1}} - \gamma\alpha^{\frac{1}{\gamma-1}}\left[1-\left(\frac{1}{2}\right)^{\frac{1}{\gamma-1}}\right]$$
$$-\frac{1}{\gamma-1}\alpha^{\frac{2-\gamma}{\gamma-1}}\left[(\gamma-1)\left(\frac{1}{2}\right)^{\frac{1}{\gamma-1}} - (\gamma-2)\right]. \quad (55)$$

Note that $\chi_r(1,\gamma,1) = \psi_r(1,\gamma,1) = 1$ and $\frac{\partial\chi_r(\alpha,\gamma,1)}{\partial\alpha}|_{\alpha=1} = \frac{\partial\psi_r(\alpha,\gamma,1)}{\partial\alpha}|_{\alpha=1} = \left[(\gamma-1)\left(\frac{1}{2}\right)^{\frac{1}{\gamma-1}} - (\gamma-2)\right]$. Note also that (54) is positive by (46). In the Online Appendix we establish that $\partial^2\psi_r(\alpha,\gamma,1)/\partial\alpha^2 > 0$ for any $\alpha > 1$ if and only if $\gamma > 2$. Since $\chi_r(\alpha,\gamma,1)$ and $\psi_r(\alpha,\gamma,1)$ are tangent at $\alpha = 1$ and $\chi_r(\alpha,\gamma,1)$ is linear and increasing in α, $\psi_r(\alpha,\gamma,1) > \chi_r(\alpha,\gamma,1)$ for any $\alpha > 1$ if and only if $\gamma > 2$. Consequently, for $\eta = \rho = 1$ $\underline{\delta}^C < \underline{\delta}^J$ if and only if $\gamma > 2$.

Step 3. Proof that for $\rho = 1$ $\underline{\delta}^C \leq \underline{\delta}^J$ if and only if $\gamma > 2$ and $\eta \geq \hat{\eta}$.

Step 2 establishes that $\chi_r(\alpha,\gamma,1) < \psi_r(\alpha,\gamma,1)$ if and only if $\gamma > 2$. It is also straightforward to show that $\chi_r\left(\alpha,\gamma,\left(\frac{1}{2}\right)^{\frac{1}{\gamma-1}}\right) = \psi_r\left(\alpha,\gamma,\left(\frac{1}{2}\right)^{\frac{1}{\gamma-1}}\right)$ since $\eta\gamma - 2\eta^\gamma = (\gamma-1)\left(\frac{1}{2}\right)^{\frac{1}{\gamma-1}}$ and $2\eta^\gamma - \eta = 0$ if $\eta = \left(\frac{1}{2}\right)^{\frac{1}{\gamma-1}}$. Note that $\eta y^* = y^J$ if $\eta = \left(\frac{1}{2}\right)^{\frac{1}{\gamma-1}}$.

Rearrange terms in (52) and (53) so that $\hat{\chi}_r(\alpha,\gamma,\eta)$ is linear in η and $\hat{\psi}_r(\alpha,\gamma,\eta)$ is nonlinear in η.

$$\hat{\chi}_r(\alpha,\gamma,\eta) = (\gamma-1)\left(\frac{1}{2}\right)^{\frac{1}{\gamma-1}}\left[\alpha + \alpha^{\frac{1}{\gamma-1}} - \alpha^{\frac{\gamma}{\gamma-1}} - 1\right]$$

$$+\eta\left[(\gamma-1)\left(1+\alpha^{\frac{\gamma}{\gamma-1}}\right)-\gamma\left(\alpha+\alpha^{\frac{1}{\gamma-1}}\right)+\left(\frac{1}{2}\right)^{\frac{1}{\gamma-1}}(\alpha+1)^{\frac{\gamma}{\gamma-1}}\right]$$

$$\hat{\psi}_r(\alpha,\gamma,\eta)=2\eta^{\gamma}\left[(\alpha+1)^{\frac{\gamma}{\gamma-1}}\left(\frac{1}{2}\right)^{\frac{1}{\gamma-1}}-\alpha^{\frac{1}{\gamma-1}}+\alpha\right]$$

In the Online Appendix we show that $\frac{\partial\hat{\chi}_r}{\partial\eta}>0$ if $\gamma>2$, $\frac{\partial\hat{\psi}_r}{\partial\eta}>0$ and $\frac{\partial^2\hat{\psi}_r}{\partial\eta^2}>0$.

Suppose $\gamma>2$. We have proved that $\hat{\chi}_r\left(\alpha,\gamma,\left(\frac{1}{2}\right)^{\frac{1}{\gamma-1}}\right)=\hat{\psi}_r\left(\alpha,\gamma,\left(\frac{1}{2}\right)^{\frac{1}{\gamma-1}}\right)$ and according to Step 2 $\hat{\chi}_r(\alpha,\gamma,1)<\hat{\psi}_r(\alpha,\gamma,1)$. Since $\hat{\chi}_r(\alpha,\gamma,\eta)$ is increasing and linear in η and $\hat{\psi}_r(\alpha,\gamma,\eta)$ is increasing and convex in η, there exists $\hat{\eta}\in\left(\left(\frac{1}{2}\right)^{\frac{1}{\gamma-1}},1\right)$ such that $\hat{\chi}_r(\alpha,\gamma,\eta)\leq\hat{\psi}_r(\alpha,\gamma,\eta)$ if and only if $\eta\geq\hat{\eta}$. Accordingly, $\underline{\delta}^C\leq\underline{\delta}^J$ if $\gamma>2$, $\eta\geq\hat{\eta}$ and $\rho=1$.

Suppose $\gamma<2$. Since $\hat{\psi}_r(\alpha,\gamma,\eta)$ is increasing and convex in η and $\hat{\chi}_r(\alpha,\gamma,\eta)$ is linear in η, it must be that $\hat{\chi}_r(\alpha,\gamma,\eta)$ is increasing in η also for $\gamma<2$ since $\hat{\chi}_r\left(\alpha,\gamma,\left(\frac{1}{2}\right)^{\frac{1}{\gamma-1}}\right)=\hat{\psi}_r\left(\alpha,\gamma,\left(\frac{1}{2}\right)^{\frac{1}{\gamma-1}}\right)$ and $\hat{\chi}_r(\alpha,\gamma,1)>\hat{\psi}_r(\alpha,\gamma,1)$. Therefore $\hat{\chi}_r(\alpha,\gamma,\eta)>\hat{\psi}_r(\alpha,\gamma,\eta)$ for all $\eta>\left(\frac{1}{2}\right)^{\frac{1}{\gamma-1}}$. Accordingly, $\underline{\delta}^C>\underline{\delta}^J$ if $\gamma<2$ and $\rho=1$.

Step 4. Proof that (i) $\underline{\delta}^C<\underline{\delta}^J$ if $\gamma>2$, $\eta\geq\hat{\eta}$ and $\rho\geq\max\left\{\tilde{\rho},\underline{\rho}^C,\underline{\rho}^J\right\}$ and (ii) $\underline{\delta}^J<\underline{\delta}^C$ if $\gamma<2$, $\eta<\hat{\eta}$ or $\max\left\{\underline{\rho}^C,\underline{\rho}^J\right\}\leq\rho<\tilde{\rho}$.

Since $\rho\geq\max\left\{\underline{\rho}^C,\underline{\rho}^J\right\}$, $p_i^{\omega}=\rho y_i^{\omega}$ for $i\in\{h,\ell\}$ and $\omega\in\{C,J\}$.

$\underline{\delta}^C\leq\underline{\delta}^J$ is equivalent to

$$V^C\geq\frac{G^C}{G^J}V^J. \tag{56}$$

$\frac{G^C}{G^J}>1$ by Lemma 4(i) and does not depend on ρ.

Suppose $\gamma>2$ and $\eta\geq\hat{\eta}$. According to the proof of Lemma 4(ii) $V^C=V^J$ if $\rho=\hat{\rho}$. Thus (56) is not satisfied if $\rho=\hat{\rho}$. By step 2 of this proof (56) is satisfied if $\rho=1$ given $\gamma>2$ and $\eta\geq\hat{\eta}$. Since V^C and V^J are linear in ρ, there exists $\tilde{\rho}\in(\hat{\rho},1)$ such that $\underline{\delta}^C\leq\underline{\delta}^J$ if and only if $\rho\geq\tilde{\rho}$ given $\gamma>2$, $\eta\geq\hat{\eta}$ and $\rho\geq\max\left\{\underline{\rho}^C,\underline{\rho}^J\right\}$.

Suppose $\gamma<1.5$. According to the proof of Lemma 4(ii) $V^J>V^C$ for all $\rho\in[0,1]$. Therefore (56) cannot be satisfied.

Finally, suppose $1.5<\gamma<2$. According to the proof of Lemma 4(ii) $V^C>V^J$ if and only if $\rho>\hat{\rho}$. However, even at $\rho=1$ $\underline{\delta}^J\leq\underline{\delta}^C$ according to step 2 of this proof. Therefore (56) cannot be satisfied. Therefore $\underline{\delta}^J\leq\underline{\delta}^C$ if $\gamma<2$.

In sum, (i) $\underline{\delta}^C\leq\underline{\delta}^J$ if and only if $\gamma>2$, $\eta\geq\hat{\eta}$ and $\rho\geq\max\left\{\tilde{\rho},\underline{\rho}^C,\underline{\rho}^J\right\}$, (ii) $\underline{\delta}^C>\underline{\delta}^J$ if $\gamma<2$, $\eta<\hat{\eta}$ or $\max\left\{\underline{\rho}^C,\underline{\rho}^J\right\}\leq\rho<\tilde{\rho}$.

By Lemma 3(ii), (i) implies that the joint surplus is (weakly) greater under common ownership than under joint ownership if and only if $\gamma > 2$, $\delta \geq \delta(\hat{\eta})$ and $\rho \geq \max\left\{\tilde{\rho}, \underline{\rho}^C, \underline{\rho}^J\right\}$. Consequently, the joint surplus is greater under joint ownership than under common ownership if $\gamma < 2$, $\delta < \delta(\hat{\eta})$ or $\max\left\{\underline{\rho}^C, \underline{\rho}^J\right\} \leq \rho < \tilde{\rho}$.

Q.E.D.

Proof of Lemma 5.

(i) The gain from deviation under ownership by agent i is equal to

$$\begin{aligned} G^i &= \left[\frac{1}{2}(\theta_\ell+\theta_h)\left(y_i^i+y^*\right)+\frac{1}{2}(\theta_i-\theta_j)\left(y_i^i+\mu y^*\right)-c\left(y_i^i\right)\right]+\left[\frac{1}{2}(\theta_\ell+\theta_h)\left(y^*+y_j^i\right)\right. \\ &\quad \left.+\frac{1}{2}(\theta_j-\theta_i)\left(y^*+\mu y_j^i\right)-c\left(y_j^i\right)\right]-\left[2(\theta_\ell+\theta_h)y^*-2c(y^*)\right] \\ &= \left[\theta_i y_i^i-c\left(y_i^i\right)\right]-\left[\theta_i y^*-c(y^*)\right] \\ &\quad +\left[\frac{1}{2}(\theta_\ell+\theta_h)y_j^i+\frac{1}{2}(\theta_j-\theta_i)\mu y_j^i-c\left(y_j^i\right)\right] \\ &\quad -\left[\frac{1}{2}(\theta_\ell+\theta_h)y^*+\frac{1}{2}(\theta_j-\theta_i)\mu y^*-c(y^*)\right] \end{aligned} \tag{57}$$

Using the envelope theorem and taking into account that $\partial y^*/\partial\mu = 0$, we obtain

$$\frac{\partial G^h}{\partial \mu} = \frac{1}{2}(\theta_\ell-\theta_h)\left(y_\ell^h-y^*\right) > 0 \tag{58}$$

$$\frac{\partial G^\ell}{\partial \mu} = \frac{1}{2}(\theta_h-\theta_\ell)\left(y_h^\ell-y^*\right) < 0. \tag{59}$$

Note that common ownership is formally equivalent to single ownership when $\mu = 1$.[50] Therefore $G^h = G^\ell = G^C$ for $\mu = 1$, and (58) and (59) prove that $G^h < G^C < G^\ell$ for any $\mu < 1$. According to Lemma 4 $G^J < G^C$. Therefore $\max\left\{G^J, G^h\right\} < G^C < G^\ell$.

(ii) According to Proposition 1 $S^C < S^J$ if and only if $\gamma > 1.5$. Therefore $V^C > V^J$ if $\rho = 1$ and $\gamma > 1.5$. (25) and (26) prove that $V^h < V^C < V^\ell$ if $\rho = 1$ and $\mu < 1$ proving the statement in the Lemma.

Q.E.D.

[50] In both cases both agents' investments fully contribute to the value of the project even under disagreement – either because the access of common owners cannot be restricted or because the non-owner's investment is fully sunk in the project.

Proof of Proposition 3.

(i) According to (21) $-$ (24), common ownership is formally equivalent to single ownership if $\mu = 1$. Therefore $\underline{\delta}^C = \underline{\delta}^h = \underline{\delta}^\ell$ if $\mu = 1$. According to the proof of Proposition 2 (assuming $\eta = 1$), $\underline{\delta}^J < \underline{\delta}^C$ if $\gamma < 2$ or $\rho < \tilde{\rho}$. Therefore for $\mu = 1$ $\underline{\delta}^J < \underline{\delta}^C = \underline{\delta}^h = \underline{\delta}^\ell$ if $\gamma < 2$ or $\rho < \tilde{\rho}$. By continuity the same holds for $\mu \rightarrow 1$.

(ii) In the Online Appendix we prove that $\underline{\delta}^h < \underline{\delta}^J < \underline{\delta}^\ell$ if $\gamma < 2$, $\rho = 1$ and $\mu = 0$. According to Proposition 2, $\underline{\delta}^J < \underline{\delta}^C$ if $\gamma < 2$. Therefore $\underline{\delta}^h < \min\left\{\underline{\delta}^C, \underline{\delta}^J, \underline{\delta}^\ell\right\}$ if $\gamma < 2$, $\rho = 1$ and $\mu = 0$. By continuity the same holds for $\rho \rightarrow 1$ and $\mu \rightarrow 0$.

(iii) In the Online Appendix we prove that $\underline{\delta}^C < \min\left\{\underline{\delta}^J, \underline{\delta}^h, \underline{\delta}^\ell\right\}$ if $\gamma > 2$, $\rho > \rho'$ and $\frac{\theta_\ell}{\theta_h} \rightarrow 0$.

(iv) In the Online Appendix we prove that $\underline{\delta}^\ell < \underline{\delta}^J < \underline{\delta}^h$ if $\gamma > 2$, $\rho = 1$ and $\mu = 0$. In the Online Appendix we also prove that $\underline{\delta}^\ell < \underline{\delta}^C$ if $\gamma > 2, \rho = 1$, $\frac{\theta_\ell}{\theta_h} \rightarrow 1$ and $\mu \rightarrow 0$. Therefore $\underline{\delta}^\ell < \min\left\{\underline{\delta}^J, \underline{\delta}^C, \underline{\delta}^h\right\}$ if $\gamma > 2$, $\rho = 1$, $\frac{\theta_\ell}{\theta_h} \rightarrow 1$ and $\mu \rightarrow 0$. By continuity the same holds for $\rho \rightarrow 1$.

Q.E.D.

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