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A MULTI-LAYER TEMPORAL NETWORK MODEL OF THE SPACE ENVIRONMENT

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Abstract

With the advent of the New Space era and the increase in the population of resident objects in Earth orbit, there is a compelling need to adopt new tools to study the complexity of the space environment. In particular, there is a need to consider the different layers of functionalities and services in an integrated and consistent framework that allows a global analysis of the evolution of the space environment. In the past two decades, there has been intense research to describe and model physical, engineering, information, social and biological systems using network theory. Most recently, multilayer networks, or networks of networks, have demonstrated a higher capability of describing failures, relationships, connectivity, and patterns, with respect to their single-layer counterpart.

This paper presents a representation of the space environment as a dynamic multilayer network, where space objects are nodes and their relationships are captured through dynamic links; each layer represents a different type of interaction. In this paper, in particular, we consider two layers: the physical and the information layer. The former models the collision between pairs of objects and how disruptions tend to propagate in the network, while the latter models the exchange of information among satellites via telecommunication. Links are probabilistic in that they model the probability of an interaction between two nodes. Moreover, the spreading dynamics of disruptions among nodes is mathematically described with a susceptible-infectious-susceptible epidemiological model.

By using a bottom-up approach, where we stochastically simulate the spreading of a disruptive event in the network, we show how it is possible to investigate different spreading scenarios and analyze the network weak links and nodes, which can then be targeted for improving the space environment resilience.

Keywords: Multilayer Temporal Network, Network Theory, Space Environment, Space Traffic Management, Space Debris.

1. Introduction

The space environment around Earth is characterized by thousands of operational and non-operational objects orbiting in different planes and distances. In such a complex and ever-more-changing scenario [1], it is fundamental to understand the interaction among these objects in order to assess the resilience of the entire dynamical population. In fact, in the NewSpace era, with many constellations being launched in the next few years, it is crucial to develop tools that can help space operators to safeguard satellite operations [2]. Having a tool that allows for a holistic view of the space traffic could help operators not only to assess the current situation more efficiently but also to identify long or short-term patterns and forecast possible scenarios, depending on whether actions are undertaken (e.g., a maneuver is executed) or not.

A collision among space objects could generate an amount of space debris that can trigger a cascade of collisions that could make the use of space dangerous and impractical for future missions [3], [4]. An effective Space Traffic Management (STM) is pivotal to expand the population of space objects safely and to reduce the burden on space operators [5]. This has to be done by handling all the various sources of disruptions and malfunctions for satellites (such as the risk of collisions, the risk of explosion due to lack of passivation, the threats posed by space weather, and others) [6], [7].

In the latest years, the resilience of many networked systems, from biology and ecology to economics and social sciences, has been investigated through the use of network system theory [8], [9], [10]. Multilayer networks (or networks of networks) have become increasingly popular due to their ability to describe connectivity patterns, relationships, and stability of complex systems made of multiple networks that interact across multiple layers [11]. Surprisingly, to our knowledge, such tools have never been exploited for the assessment and study of the space environment and its predictability and resilience. Therefore, we have decided to introduce a framework for such an investigation, which might foster a new line of research devoted to the study of the space population evolution as a complex dynamical network. In Section 2, we introduce the dynamical model that we employ to describe the long-term evolution of space objects. Then, in Section 3, we discuss the network model and we characterize the design, topology, and dynamics of the chosen network. Whereas in Section 4, we present some test cases, where we study the resilience, dynamics, and stability of the space environment through the lenses of network theory. Finally, in Section 5, we conclude with some remarks and recommendations for future work.

2. Orbital Dynamical Model

The long-term behavior of space objects is often modeled by first splitting the disturbing forces into constant components, short period, and long period variations. By doing this, one can then integrate the equations of motion with respect to their fast variable (usually over one satellite revolution) to get rid of the short period terms. In this way, one obtains averaged equations of motion, which describe the mean behavior of satellites' orbital parameters over long periods of time (i.e., typically, a one-day step-size is chosen in the numerical integrator). Usually, for doing this, the satellite state is expressed in terms of the nonsingular equinoctial elements, and the averaged equations of motion are written with respect to those elements [12]. These include the semi-major axis (a), the mean longitude (λ) and other four elements (P_1, P_2, Q_1, Q_2), which can be related to the Keplerian orbital elements as follows:

$$\begin{aligned} a &= a \\ P_1 &= e\sin(\omega+\Omega) \\ P_2 &= e\cos(\omega+\Omega) \\ Q_1 &= \tan(i/2)\sin\Omega \\ Q_2 &= \tan(i/2)\cos\Omega \\ \lambda &= M+\omega+\Omega, \end{aligned} \tag{1}$$

where e is the eccentricity, i the inclination, ω the argument of perigee, M the mean anomaly, and Ω the right ascension of the ascending node. As perturbations, in the averaged equations of motion, besides the central gravitational parameter, we have included both the J_2 gravitational term and the exponential drag perturbation [13].

We evolved more than 18,000 resident space objects [14] (among which around 10% are operational satellites and the rest is made of non-operational satellites, rocket bodies, fragments of satellites, and meteoroids) for 15 years, with an integration step of one day. In the following sections, we will explain how we have handled these propagated orbits for constructing a time-varying network.

3. Network Model

A single-layer network is a tuple $G = (V, E)$ where V is the set of nodes whereas $E \subseteq V \times V$ is the set of edges that connect the pairs of nodes [15]. The matrix that represents the connections between vertices (and their strength) is called the adjacency matrix (A).

In this study, we first treat a network system made of N nodes, each of which is characterized by a certain time-varying state $x_i(t)$ that describes the value of the ith node at time t. The time-varying dynamics of each node is influenced by both the status of the node and that of its neighbors. A time-varying adjacency matrix $A(t)$ is used to describe the interactions among the nodes. In practice, this means that the adjacency matrix terms: $a_{ij}(t)$, describe the time-varying weights between node i and j. We further assume that these weights assume values between 0 and 1 (i.e., $a_{ij}(t) \in [0,1], \forall i,j$) and that the adjacency matrix is undirected (i.e., $a_{ij}(t) = a_{ji}(t)\ \forall i,j$). In this framework, the differential equations that describe the dynamics of the nodes can be written as:

$$\frac{dx_i}{dt} = f(x_i) + \sum_{j}^{N} a_{ij}(t) g(x_i, x_j), \tag{2}$$

where each node behavior is composed of a self-dynamic (i.e., f) and coupling (i.e., g) dynamic term. For modeling a dynamical system with a dynamic network of this kind, several aspects shall be taken into account. First of all, it is important to know what are links and nodes in the studied case. Secondly, a strategy for investigating the time-varying links among nodes needs to be defined. Finally, the dynamical system shall also be characterized (i.e., the two f and g functions shall be modeled). In our case, since we would first like to model the space population of objects, we will assume that each node is an object or a cluster of objects (e.g., a cloud of space debris or a satellite formation flying).In Section 3.1, we will first show how the adjacency matrix of this model is constructed. Then, in Section 3.2, we will discuss the node dynamics, by showing how the spread of malfunctions (i.e., collisions) is modeled within the network. Finally, in Section 3.3, we will discuss how the abovementioned concepts are generalized for a multilayer network where an information layer is added.

3.1 Link Dynamics

In this section, we will deal with the modeling of the adjacency matrix (i.e., the $a_{ij}(t)$ terms in Equation (2)). Since we want to describe the long-term behavior of the space environment in terms of collisions, as already introduced in Section 2, we have first propagated the averaged equations of motion of 18,995 satellites for 150 years, with a time-step of one day. Then, at each time, we have checked each pair of satellites and we have stored the average difference of their perigee and apogee. Finally, we have mapped these values in the $[0, 1]$ range in order to build the adjacency matrix terms. This was achieved with the following mapping:

$$a_{ij}(t) = e^{-(|r_{p,i}(t) - r_{p,j}(t)| + |r_{a,i}(t) + r_{a,j}(t)|)/2}, \tag{3}$$

where $r_a(t), r_p(t)$ are the apogee and perigee values of the ith and jth satellites. In this way, we can transform orbital information into a time-varying weighted adjacency matrix. In Fig. 1, we show the histogram plot of the resulting weights, taking into account we only mapped the values that were below a 5 km threshold (this results in an adjacency matrix value of 0.0067). However, this would cause a highly sparse matrix (since most of the space objects have a distance higher than 5 km), which would not be able to account for the cascade effect that might be triggered when a collision among resident space objects happens. Therefore, we have established an arbitrary minimum adjacency matrix value for all the satellites of about 2.25×10^{-5} (which corresponds to 11 km distance), for modeling the collision cascade effect. As a consequence, the resulting adjacency matrix is a symmetric matrix made of 18995×18995 elements that range from 2.25×10^{-5} to 1 (with only around 0.027% of the pairs of satellites having values below the 5 km distance threshold). Furthermore, the diagonal elements of the matrix are zero, as we do not consider self-linked nodes.

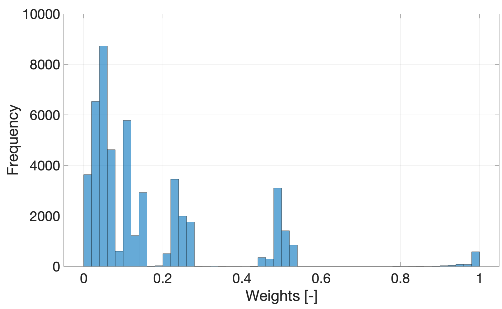

Fig. 1: adjacency matrix weight distribution (only the values with a distance threshold below 5 km are displayed).

For simplicity, since we noticed that the weight distribution does not vary significantly in the 150 years of integration, we have decided to consider a constant adjacency matrix (equal to the initial one). To corroborate this, as we show in Fig. 2, we measure the Kullback–Leibler divergence (a mathematical tool for analyzing the divergence between two distributions) [16] between the weight distribution at the initial time versus the weight distributions at future times, and we observe that it increases over time, but always maintaining values below 10^{-2}. Therefore, we considered these values small enough to neglect the time-varying nature of the adjacency matrix.

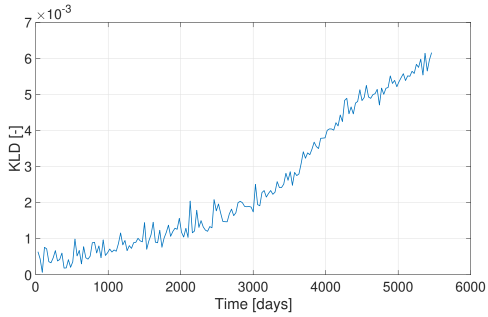

Fig. 2: KL-divergence of the weight distribution as a function of the integration time in days.

It is important to point out that we observe such small changes in the adjacency matrix weight distribution over long periods of time because we have not incorporated any model that takes into account neither the launch of megaconstellations nor the yearly launch rate of satellites, and their expected increase over time. Probably, by modeling these aspects one would discover more prominent differences among weight distributions as time progresses: in such cases, it might be pivotal to consider the time-varying nature of the adjacency matrix.

3.2 Node Dynamics

Having defined the adjacency matrix, it is now fundamental to formulate the f and g functions that describe the self and interactive dynamics among nodes (see Equation (2)). Several existing dynamical models on complex networks have been treated in the literature, these include but are not limited to epidemic processes (where x_i represents the probability of infection), biochemical dynamics (where x_i represents the concentration of reactant), birth-death processes (where x_i represents the population size at site i) and regulatory dynamics (where x_i is the expression level of a gene) [17]. In our case, we would like to model the functional status of each space object or cluster of objects. This means that we only have two possible states (i.e., functional, F, and defective, D), and we would like to know the interaction dynamics of these states. As one can already hint, some satellites might have the capability of turning from defective to functional (e.g., if a space weather phenomenon interrupts the communication of a satellite, it is possible that it will be restored after some time), while others will permanently stay defective (e.g., dead satellites or space debris). Moreover, a pair of interacting space objects can trigger collateral consequences for other satellites (e.g., when two objects collide, the debris generated from the collision might disrupt other objects). For these aspects, the space environment seems to be suitable for being modeled with a binary network dynamics (like the susceptible-infectious-susceptible, SIS, model in epidemic processes). This type of model can be described as a continuous Markov chain [18], where there are 2^N possible states that the network can assume (where N is the number of nodes). The dynamics of such systems is not deterministic but is defined by stochastic rules. Therefore, the most precise characterization of this type of models can be achieved through the master equation (also known as Kolmogorov equation) [19]. This is a set of first-order differential equations that describe the time evolution of the probability of a certain set of discrete states of the system. In its matrix form, this equation can be written as:

$$\frac{d\mathbf{X}}{dt} = P\mathbf{X}, \tag{4}$$

where P is a matrix that describes the transitions from one state to the other. For continuous Markov chains with finite state space, assuming that we would like to characterize N nodes, each of them having m possible values for the state, then Equation (4) would result in an m^N system of ordinary differential equations (i.e., $\mathbf{X} \in \mathbb{R}^{m^N}$). It is clear that even in the case of only two possible states (i.e., functional and defective) the system grows with the number of nodes as 2^N, which results to be quickly computationally intractable even for small networks. Furthermore, the transition rates are not only dependent on the dynamics, but also on the topology of the network, and their modeling may be a considerable burden for the researcher. Although it is possible sometimes to reduce the number of equations (e.g., when there are symmetries in the network topology), this is, however, not possible in general. For these reasons, rather than using a top-down approach that aims at describing the probability of each possible state of the network through the modeling and integration of the master equation, a bottom-up approach might be preferred. Such an approach starts with the description of the probability of each node being in a certain state. These probabilities are related to the states of pairs of nodes, which in turn depend on triples, and so on. Even in this case, the full system is too large to be tractable and some approximations (called closures) are needed for reducing the number of equations. A particularly interesting example for our case is related to the field of epidemic modeling and, in particular, the SIS dynamics. In this case, the differential equations that regulate the time-varying probability of each node i being in state I (where I stands for infected and S for susceptible) and the probabilities of the pair of nodes i, j being in state SI, SS or II can be written as [20]:

$$\begin{aligned}
\langle \dot{I_i} \rangle =& \tau \sum_{j=1}^{N} a_{ij} \langle S_i I_j \rangle - \gamma_i \langle I_i \rangle \\
\langle \dot{S_i I_j} \rangle =& \tau \sum_{k=1, k\neq i}^{N} a_{jk} \langle S_i S_j I_k \rangle \\
& - \tau \sum_{k=1, k\neq j}^{N} a_{ik} \langle I_k S_i I_j \rangle \\
& - \tau a_{ij} \langle S_i I_j \rangle - \gamma_j \langle S_i I_j \rangle + \gamma_i \langle I_i I_j \rangle \\
\langle \dot{I_i I_j} \rangle =& \tau \sum_{k=1, k\neq i}^{N} a_{jk} \langle I_i S_j I_k \rangle \\
& + \tau \sum_{k=1, k\neq j} a_{ik} \langle I_k S_i I_j \rangle - (\gamma_i + \gamma_j) \langle I_i I_j \rangle \\
& + \tau a_{ij} \langle S_i I_j \rangle + \tau a_{ij} \langle I_i S_j \rangle \\
\langle \dot{S_i S_j} \rangle =& - \tau \sum_{k=1, k\neq j}^{N} a_{ik} \langle I_k S_i S_j \rangle \\
& - \tau \sum_{k=1, k\neq i}^{N} a_{jk} \langle S_i S_j I_k \rangle,
\end{aligned} \tag{5}$$

where $\langle S_i \rangle = 1 - \langle I_i \rangle$, $i, j = 1, .., N$ and $i \neq j$. Furthermore, the average number of infected nodes at each time can be computed as:

$$[I] = \sum_{i=1}^{N} \langle I_i \rangle. \tag{6}$$

The a_{ij} elements (for $i, j = 1, .., N$) are the weights of the adjacency matrix, whereas τ is the infection rate (i.e., the rate at which the disease is transmitted from an infected to a susceptible individual) and γ_i is the recovery rate (each individual can, in principle, have its own). Equation (5) represents an exact set of ordinary differential equations and by solving these equations the correct values of how these probabilities vary can be found. However, as it can be seen, it is not closed: its integration requires the knowledge of triples. Hence, further differential equations would be needed to study their time evolution: these will depend on quadruples, and so on. This means that this system can soon become intractable due to the many equations to be solved. Therefore, closures approximations (e.g., some approximation to express triples as a function of pairs) are typically employed to close the system [21]. For instance, if the system is closed at the level of pairs, then only N equations are necessary, whereas if it is closed at the level of triples, $N + 3 * N_{edges}$ equations are required. In this paper, we will close the system at the level of triples with Kirkwood-type closures [22]. Each triple will be approximated as follows:

$$\langle S_i S_j I_k \rangle \approx \frac{\langle S_i S_j \rangle \langle S_j I_k \rangle}{\langle S_j \rangle} \quad (7)$$

Furthermore, in our case, we model the system as a SIS model where an infected individual is a resident space object that has lost its functions (e.g., it can be a disrupted satellite due to a collision event, but also a satellite subject to a cyberattack). The recovery rate is representative of the recovery actions that can be undertaken by operators (e.g., by maneuvering the satellites for mitigating the risk of a collision event). The adjacency matrix terms are essential for regulating the infection rate influence on each satellite, depending on the strength of the links among them. Therefore, we constructed a theoretical framework for studying the space environment as a SIS stochastic model.

3.3 Multilayer Network

Until now, we have discussed network models encompassing a single-layer. In particular, we have constructed a single-layer network that aims at modeling the space environment as a network of interacting objects, whose interaction is proportional to the long-term proximity of these objects. Furthermore, we have borrowed the susceptible-infectious-susceptible epidemiological model to describe the diffusion of malfunctions within the network (caused by physical collisions among resident space objects). Now, our objective is to extend the abovementioned framework to a more general model, where multiple layers of interactions can be described. In this case, each node belongs to a layer (i.e., $L^{(\alpha)}$), and the edges can connect nodes within the same layer (i.e., intra-layer edges) and also nodes across different layers (i.e., inter-layer edges). This new perspective has been given a lot of attention in recent years, due to its capability of describing relationships and interactions across multiple dimensions [23], [24], [25], [26], [27]. Furthermore, it has also been shown that these new structures have a key influence in modifying diffusion and spreading processes, with respect to their single-layer counterparts [28], [29].

Typically, multilayer networks are either described through tensors [30] or by flattening their multidimensional structure constructing a supra-adjacency matrix [31], [32]. This latter approach is often convenient as it allows us to employ several tools, methods, and theoretical results from single-layer networks to study their multilayer counterparts (since they can be described through matrices). Typically, three different matrices play a key role in this analysis: the supra-adjacency matrix, the degree matrix (i.e., a diagonal matrix in which the degree of each node is specified in the diagonal terms), and the supra-Laplacian matrix. The last one has shown to be a key ingredient for the analysis of diffusion time scales (i.e., the second smallest eigenvalue is inversely proportional to the diffusion time scale) [23]. On the other hand, for the spreading of epidemics, the spectral properties of the supra-adjacency matrix have shown to be pivotal in determining the epidemic threshold [33]. For instance, if β is the probability of an individual to get an infection and μ is the probability of an individual to recover, the epidemic threshold (i.e., β_c) for a single-layer network is then defined as:

$$\beta_c = \frac{\mu}{\lambda_1(A)}, \quad (8)$$

where $\lambda_1(A)$ is the biggest eigenvalue of the adjacency matrix. This result is extended to multilayer networks, by considering the maximum eigenvalue of the supra-adjacency matrix [34], [35], [36]. It has been shown that when single-networks are connected to build a multilayer structure, the maximum eigenvalue of the resulting supra-adjacency matrix (i.e., $\lambda_1(A_{supra})$) is higher or equal than the maximum value between the maximum eigenvalues of the adjacency matrices of every single layer (i.e., $\lambda_1(A_i)$):

$$\lambda_1(A_{supra}) \geq max(\lambda_1(A_1), ..., \lambda_1(A_M)). \quad (9)$$

This means that by connecting single-layer networks to form multilayer structures has the effect of reducing the epidemic threshold and enhancing the disease spreading [37], [38].

All these properties are a key aspect in the study of these networks since they are also strictly related to their resilience (i.e., a property related to the network dynamics that measures the ability of the system to maintain an acceptable level of service in the presence of faults or malfunctions [39]). This is a very active field of research since the different topological and dynamical structure of networks of networks make their resilience analysis very cumbersome [40].

In this study, we have employed the supra-adjacency representation for describing multilayer networks. As shown in Fig. 3, we consider two different layers: the physical layer (i.e., the network that describes the long-term behavior of collisions, as explained in Sections 3.1 and 3.2), and the information layer. This latter represents the flow of information between a portion of the resident space objects (e.g., a constellation or a cluster of objects). For this second layer, we have considered random weights for the adjacency matrix. Moreover, the inter-layer connections among nodes are described by an adjacency matrix that has either ones (when the satellite

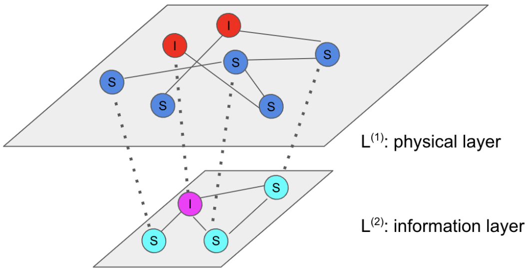

Fig. 3: Multilayer structure.

of the information layer corresponds to the satellite of the physical layer) or zeros.

Much effort has been put in the network theory field for studying the coupling between dynamics and topology in networked systems [41]. This has shown to be particularly cumbersome for the multilayer case since the multidimensional structure plays a pivotal role in modifying the dynamic processes happening in the network [34]. In this study, we approach the study of the space environment network and its dynamics from two perspectives: through a bottom-up agent-based stochastic model (i.e., simulating the stochastic dynamics of contagion for each agent in the network) and through the analysis of the network topology (i.e., by analyzing the spectrum and other features of the adjacency matrix).

3.4 Centrality Measures

Several methods exist to investigate the network properties from its topology. Some of these properties, which have been partially discussed in Section 3.3, tend to focus on the interplay between topology and dynamics (e.g., by analyzing the spectral properties of the Laplacian and adjacency matrices it is possible to infer the diffusion time-scale and other properties), while others attempt to characterize how important each node is. In this section, we will discuss this latter case by presenting some of the centrality measures that have been used in this study. Due to the different possible meanings of importance in this context, several definitions of centrality exist.

The simplest centrality measure is the degree centrality, which measures how many neighboring nodes each node has:

$$deg(i) = \sum_{j=1}^{N} a_{ij}. \tag{10}$$

An extension of degree centrality is eigenvector centrality. In this case, it is taken into account the fact that not all the neighbors are equivalent, but having neighbors that are more important increases the node's importance. Therefore, instead of awarding nodes importance proportionally to the number of neighbors, it also considers the importance of the neighbors. Thus, a high eigenvector score means that the node is linked with other nodes that are themselves important. Its calculation is performed by computing the eigenvalues of the adjacency matrix [42].

Furthermore, another widely used centrality measure is closeness centrality, which is computed by taking the reciprocal of the sum of the shortest path between each node and all the other nodes [43]. Moreover, two other well-known centrality measures that have been used in this study are the PageRank and the betweenness centrality measures. Their computation is widely discussed in literature [44], [45].

Several techniques exist to generalize these measures to multilayer networks: some use tensor algebra [46], [11], while others directly generalize the single-layer definitions to the multilayer case by considering the weighted sum across layers of the centrality measures of each layer [34].

4. Test Cases

In this section, we discuss some of the results that we obtained by modeling the space population as a single and multilayer network. We have first partitioned the space population of roughly 19,000 objects into a smaller population of 500 samples, in order to reduce the computational burden. Then, as we have already explained, we have modeled two layers of interaction: one, called the physical layer, where each node represents a satellite and its interaction with the other satellites aims at modeling their chance of colliding, and another one, called the information layer, where each node is still a satellite but its interaction with the other population represents the flow of information among space objects. The first network was therefore made of 500 elements (partitioned from the original population) whose adjacency matrix was built as explained in Section 3.1. Whereas the second layer was made of 5 nodes (taken from the 500 individuals of the first layer) with random weights (assuming that each node is not linked with itself). Furthermore, we assumed that, in both cases, malfunctions can be modeled with a susceptible-infectious-susceptible dynamics, where the infection represents a collision in the first case and a cyberattack or a disruption in the telecommunication system in the second case. Within this framework, we have studied the characteristics of the network and its diffusion patterns, when one of the nodes was initially disrupted due to a collision. As we will observe, several different scenarios can be triggered, depending not only on the centrality measure value of the infected node but also on the topology of the network. We first investigate the single-layer case, where we study three disruption scenarios without recovery actions. Then we analyze the role of recovery actions (e.g. maneuvers) and how they can play a pivotal role in avoiding a domino effect. Finally, we expand the single-layer network to a multilayer scenario, where the communication layer is added and its dynamics in the presence of disruption is studied and compared to the single-layer counterpart.

In Fig. 4, 5 and 6, we plot the probability of each node being disrupted (i.e., 0 means 0% and 1 means 100%) as a function of time in days. Three different scenarios are displayed: in the first figure a random node is initially infected (i.e., disrupted), in the second one, the node with the lowest centrality measure value is disrupted, and in the third one, the node with the highest centrality measure value is disrupted. The centrality measures gave similar results by indicating node 1 as the least central, and node 386 as the most central one. In all the three figures, we show the probability of each satellite being disrupted as a function of time. As can be seen, in the beginning, all the nodes have zero probability of being disrupted except for one (which is the initially disrupted one). However, as time progresses, due to the network connectivity, the probability of other satellites colliding increases. We observe that this increasing behavior varies depending on the initially infected node: for the case in which the most connected satellite first collides, two surrounding satellites are immediately affected by this and their probability soon reaches values of one (i.e., 100% probability of collision). Moreover, it can be seen that in the case in which the least important node is initially infected, after 770 days, 95% of the population is likely to have collided. However, for the most important case, this happens 100 days earlier, while the random case fits in the middle, taking 700 days for having 95% of the population likely to be disrupted.

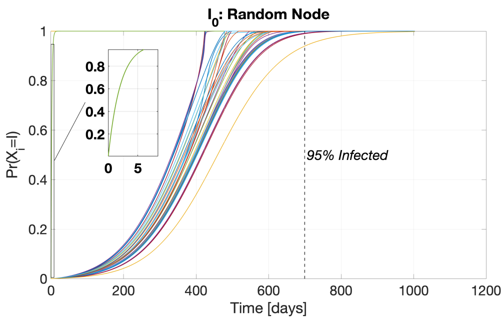

Fig. 4: Single-layer network without recovery, where a random node is initially disrupted.

While such cases can be interesting to study cascade effects of collisions among satellites when different initial nodes are first disrupted and no recovery action is taken by space operators, however, in practice, owner/operators of satellites continuously monitor active satellites and sometimes decide to perform maneuvers (e.g., recovery actions) to reduce the collision risk. In

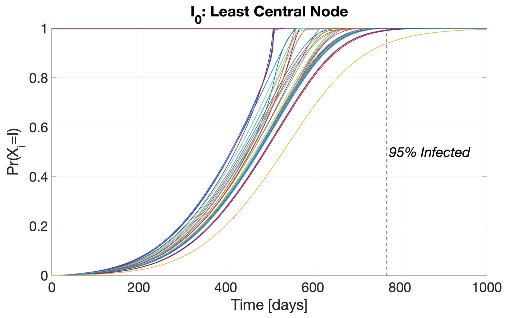

Fig. 5: Single-layer network without recovery, where the least central node is initially disrupted.

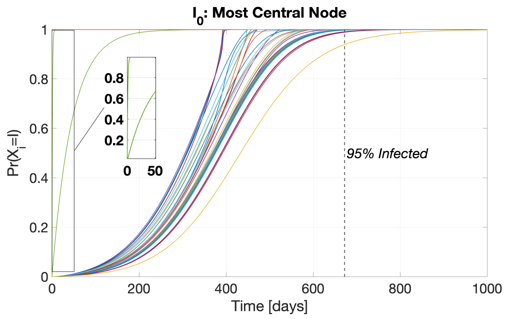

Fig. 6: Single-layer network without recovery, where the most central node is initially disrupted.

Fig. 7, we have modeled such recovery actions for the case in which the most important node is first affected by a collision. We assumed that each satellite has the same recovery rate of 2.65×10^{-2}. As we can observe, after an initial period of transition, the network finally reaches an epidemic state, where each satellite has a constant probability of being infected between zero and one. Overall, we observe that 6.8% of the population is likely to be affected by a collision after 2000 days. Therefore, by comparing this scenario with respect to the case in which no recovery action was taken, we notice that recovery actions not only reduced the number of satellites affected by a collision, but they also slowed down the domino effect (most of the satellites move from the 0% probability with less steep curves).

Then, in Fig. 8, we simulate a multilayer structure of the single-layer network with recovery, by adding the second information layer made of 5 nodes connected through inter and intra-edges. In this case, it turns out that the network is more prone to failure with respect to its single-layer counterpart. Indeed, it can be seen that more nodes are affected by the rise in the probability of disruption, and that, at the end of the propagation, 9% of the satellites are likely to have collided: 2.2% more than the single-layer case. This demonstrates that the multilayer structure has decreased the resilience of the network and its epidemic threshold. By analyzing the maximum

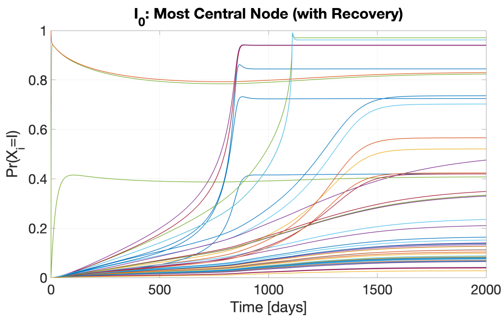

Fig. 7: Single-layer network with recovery, where the most central node is initially disrupted.

eigenvalue of the supra-adjacency matrix (which drives the epidemic threshold, as shown in Equation (8)), we observe an increase of around 6% on its maximum eigenvalue, when compared with the maximum eigenvalue of the single-layer adjacency matrix. This shows that both the topology and dynamics aspects indicate a decreased resilience when the multilayer structure is built.

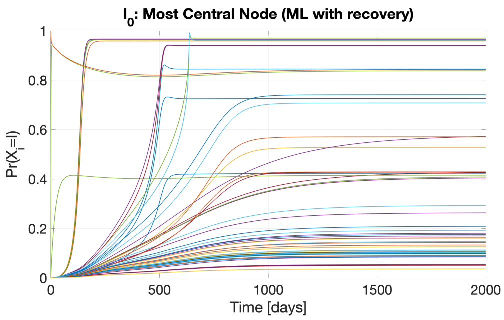

Fig. 8: Multilayer network with recovery, where the most central node is initially disrupted.

Finally, in Fig. 9, we display the average number of infected nodes as a function of time (computed as shown in Equation (6)), for all the 5 test cases that we discussed above. First of all, we can observe that the three networks without recovery rate eventually reach a state in which all the satellites are disrupted, although the rate and timescale at which they do it differs, depending on the initial disrupted node. In general, we observe that the higher the centrality value of the initially disrupted satellite, the more rapidly a domino effect is triggered. In the same figure, we also show single and multilayer networks with recovery. As we can observe, the multilayer structure not only reaches an epidemic state with more disrupted nodes, but it also does it in a faster way.

5. Conclusions and Future Work

In this paper, we have modeled the space environment as a dynamic network of satellites. In this framework, satellites are the nodes of the network, whose inter-

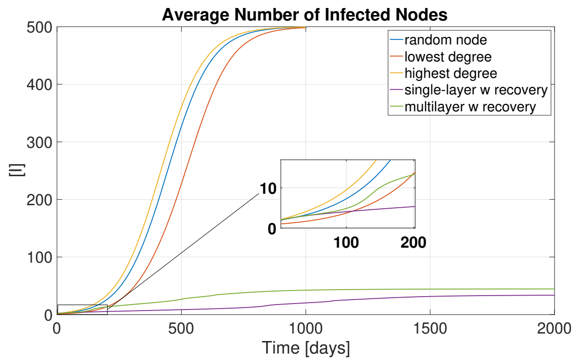

Fig. 9: Average number of infected nodes as a function of time, for the five studied test cases.

relationships are captured through dynamic links that model either the distance among satellites (for modeling their chance of colliding) or the flow of information among them. We modeled the spread of disruptive events within the network with a stochastic epidemic model, where the probability of each satellite being disrupted (in terms of either a collision or a loss of signal) as a function of time was modeled. This has led to different test cases, where we investigated the resilience and dynamics of different networks. In particular, we first studied the differences in the spreading dynamics for three cases, in which the first collision affected different satellites with different centrality (i.e., importance) values within the network. From these experiments, we verified that due to the network connectivity, a domino effect of collisions is triggered that eventually leads all the satellites to disruption, although the different centrality measures play a pivotal role in determining the speed at which the epidemic state is reached. Then, we studied two more cases, where recovery actions (e.g. maneuvers) from space operators were also considered. In the first case, a single-layer collision network was modeled, whereas in the latter a multilayer structure was built, where the communication layer was added to the former. In both cases, we demonstrate how recovery actions help the network to stabilize to an epidemic state where only a small percentage of the whole population is disrupted. Furthermore, we studied the differences between the multilayer network and its single-layer counterpart, confirming that the multilayer structure makes the network less resilient and more sensitive to failures.

To our knowledge, this is the first time that the space environment is modeled as a multilayer network and studied through the lenses of network theory. Therefore, we emphasize the introductory and experimental nature of this work. Moreover, we would like to highlight the difficulties in accurately modeling physical aspects (e.g., collisions, the flow of information, recovery actions) in the context of network theory, and stochastic epidemic models. Further work should focus on a more thorough physical and network modeling. For instance, the adjacency matrix terms of the collision layer shall be modeled directly as the probability of collisions. Furthermore, we considered only a small portion (i.e., 500 individuals) of the entire space population, in order to prove the applicability and efficacy of the proposed technique, without incurring in very high computational costs. However, to accurately studying the networked space environment and its real-world dynamics and resilience patterns, a bigger network shall be built. More accurate modeling of recovery actions shall also be considered in such a structure. Finally, future work will also address more layers, which encompass more levels of interactions among objects (e.g., economic factors, environmental aspects, etc.).

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