# Modelling the Diffusion of Societal Pro-Environmental Attitudes: Discrete Choice in Continuous Time

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# 1 Introduction

We live in a society under unprecedented threat from climate change (Feldmann and Levermann, 2015). Anthropogenic emissions of carbon dioxide, methane and nitrous oxides have increased atmospheric levels of $CO_2$ to levels not seen for at least 4 million years - well before the evolution of *homo sapiens*. The resulting greenhouse effect is warming the planet, potentially faster than at any time in the last 420 million years (Foster, Royer and Lunt, 2017). Sudden increases in global mean temperatures of this degree are associated with mass extinctions, and measurements of plant, insect and animal life suggest that indeed the sixth mass extinction is well underway (Barnosky et al., 2011).

The problem has been understood for decades, and surveys now almost universally indicate that even in countries with unfavourable political leadership, there is an overwhelming majority of people who want their governments to take action (Hamilton et al., 2015; Leiserowitz et al., 2018). And yet action has not been forthcoming (den Elzen et al., 2019; Masson-Delmotte et al., 2018). The flagship for political solutions is the Paris Agreement (UNFCCC, 2015). This is a voluntary agreement for countries to commit to greenhouse gas emissions reduction targets, (so called "Nationally Determined Contributions" or NDCs) at a rate which is known to be insufficient (Tong et al., 2019), and with no penalties for countries which fail to reduce. It is clear that further pressure needs to be brought on governments (Ge et al., 2019). When confronted with the failure to act, politicians typically respond that their decisions reflect the attitudes of their electorate (see for example Marshall, 2015).

Our attitudes are therefore very important. They affect other people through the observance of social norms. They affect governments, even in countries with low levels of democracy (Roberts and Geels, 2019), and they affect companies. Companies which are able to, are very keen to supply greener versions of products, once they become aware of the demand.

The ability to predict how societal pro-environmental attitudes will change over time is therefore very important, and yet the most common assumptions made about these attitudes is that they are a linear function of education, resources and time, in spite of overwhelming evidence to the contrary (Evert and van der Doelen Frans, 2017; Kahan et al., 2012; Marshall, 2015).

This paper attempts to pull together three of the most promising approaches to modelling pro-environmental attitudes and to show how a general model can be created.

## 1.1 Modelling Framework

The base model we are going to use comes from a Synergetics approach, applied to Quantitative Sociology. Synergetics is the study of systems undergoing change from the point of view of the system as a whole. It is flexible enough to model complex dynamic systems and thus lends itself nicely to the field of modelling societal attitudes.

We take the discrete choice social norms model of Zeppini (2015) and modify it to make it amenable to the analysis performed by Weidlich and Haag (1983).

We have referred to “attitudes,” and specifically pro-environmental attitudes as the object of study, but in truth the model could equally well be applied to behaviours, or ideologies. Psychology spends a lot of time investigating individual attitudes and behaviours, and yet what is needed is an understanding of how individuals’ attitudes interact over time. This is one of the fundamental ideas behind Social Practice Theory, which, instead of starting with the individual, starts with a social practice (such as turning off light bulbs on leaving rooms) and looks at the factors which cause the practice to expand, or diminish.

It is hoped that a better theory of how social practices - or pro-environmental attitudes - spread will give policy-makers a useful tool in the fight against climate change.

Synergetics systems typically have one or more “macrovariables” (temperature, or location for example) interacting in a system involving a number of microvariables (the movement of molecules for instance). The systems typically involve nonlinear processes and can therefore give rise to interesting and chaotic dynamics. Synergetics examines the evolution of these dynamics, and is concerned with understanding how to characterise the kind of mathematical behaviour a system could be expected to exhibit.

It is not hard to see how similar dynamics can arise in systems describe the attitudes or behaviours of large groups of individuals. Attitudes are spread between individuals through seemingly insignificant encounters. Sometimes attitudes take root and grow. Othertimes they wither on the vine. A model purporting to study these effects ought at the very least be capable of reproducing them.

Typically the model used in this paper will lead either to behaviour which converges to a (consistent) mean, cyclic behaviour, or behaviour which is supercritically dependent on initial conditions. It is often interesting and informative to explore when one kind of behaviour morphs into another, and this paper will talk about bifurcation points to describe this process.

With the base model firmly in mind, we will then turn to the two other models discussed in this paper, Zeppini, 2015 and Peyton Young, 2009. In all three cases, we have adapted models, equations and variables, so as to permit a more convenient comparison.

The Peyton Young paper does a very good job of collating and clarifying the work done on innovation diffusion but it has two limitations which we try to address here. Firstly, it is focused on innovation *diffusion*. Yes, we can treat pro-environmental attitudes as a social meme which can spread through a population, but a commonly used modelling technique in the papers Peyton Young assesses is to model social influence via threshold (or cut-off) controls. Not all innovation diffusion can be usefully explained that way and we attempt to find away around the issue later in this paper.

The other issue has to do with complexity and stochastic processes. It is easy to make assumptions on (for example) the shape or scope of a particular function when this may not be justified. For example, we know that many models possess chaotic properties than cannot be easily studied by looking at mean behaviours. Models which include a stochastic process clearly have the advantage here. (The Zeppini paper is clearly a good example of this.)

That paper is set in a discrete time framework, and for good reasons, as that is where a lot of the discrete choice arguments can be applied. There is however, much to be gained by transposing it into continuous time, and that is the approach we take here.

The rest of the paper is structured as follows. We first examine (in the next section) a general model of individuals' attitudes in continuous time, looking at what can be said about the general dynamics before motivation in the form of utility functions is introduced.

## 2 Base Model

Suppose we have a society of $N$ individuals, with $n_g$ being the number holding a "green" attitude towards the environment and $n_b$ correspondingly the number holding a non-green (or "brown") attitude.

Define

$$n \equiv n_g = N - n_b$$

Let $p_{g,b}(n,t)$ and $p_{b,g}(n,t)$ be individual transition probabilities per unit time ($p_{g,b}$ for moving to green from brown, $p_{b,g}$ for moving to brown from green) so that for example $\int_0^t p_{g,b}(n,\tau)d\tau$ is the probability that an individual "transitions" from the brown to the green state within the period $[0,t]$. These transition probabilities are not assumed to be static, and will in general also depend on $n$. (When we add utility to the model in section 3, we will have a direct mechanism for influencing transition probabilities based on changes in $n$.)

Consider first an ensemble of such societies, and define $p(n,t)$ as the probability that a given society has state $n$ at time $t$. Whereas the individual's state is one of $\{g,b\}$, the state of each society is given by our variable $n \in [0,N]$, the number of individuals holding the "green" attitude.

So $\sum_{n=0}^{N} p(n,t) = 1$ for all $t$.

Now define $w_{j,i}$ as the transition probability per unit time for a society for moving to state $j$ from state $i$.

We can write down the general master equation:

$$\frac{dp(j,t)}{dt} = \sum_i [w_{j,i}\, p(i,t) \ - w_{i,j}\, p(j,t)] \tag{1}$$

In our model however, all societal state changes occur via “nearest neighbour” transitions, so $w_{j,i} = 0$ for all $i \neq j \pm 1$ and we can define

$$w_\uparrow(n) \equiv w_{(n+1),n} = n_b.p_{g,b} = (N-n).p_{g,b}(n) \tag{2}$$

$$w_\downarrow(n) \equiv w_{(n-1),n} = n_g.p_{b,g} = n.p_{b,g}(n) \tag{3}$$

The probability that a society goes to state $n+1$ from state $n$ in the period $[0,t]$ is $\int_0^t w_\uparrow(n)d\tau$. Note that the society can also go to state $n-1$, or stay at state $n$ in this period. Define also the probability per unit time of no transition:

$$w_=(n) \equiv 1 - [w_\uparrow(n) + w_\downarrow(n)] \tag{4}$$

We can now reformulate the master equation (1) as:

$$\begin{aligned}\frac{dp(n,t)}{dt} = & \, [w_\downarrow(n+1)p(n+1,t) - w_\downarrow(n)p(n,t)] \\ & + [w_\uparrow(n-1)p(n-1,t) - w_\uparrow(n)p(n,t)]\end{aligned} \tag{5}$$

$p(n,t)$ can therefore be written as the sum of net probability fluxes over time:

Define

$$j_\uparrow(n,t) \equiv w_\uparrow(n).p(n,t) \tag{6}$$

$$j_\downarrow(n,t) \equiv w_\downarrow(n).p(n,t) \tag{7}$$

and

$$k(n,t) = \text{net flux} \equiv j_{\uparrow}(n,t) - j_{\downarrow}(n,t) \tag{8}$$

then equation 5 can be more concisely expressed as

$$\frac{dp}{dt} = k(n-1,t) - k(n,t) \tag{9}$$

If the four components of equation 5 are in balance, then there is no net change to the system. In fact equation 9 is a continuity equation: if there is no change in the state of society (the probability that society has any given number of individuals with a "green" attitude), then $\frac{dp}{dt}$ is zero and $k(n-1,t) = k(n,t)$ for all $n$; conversely, if $k(m,t) \neq k(n,t)$ for some $(n,m)$, then the state of society must be in dynamic change.

We also have the following boundary conditions:

$$\begin{aligned} j_{\uparrow}(N,t) &= j_{\downarrow}(0,t) = 0 \\ k(N,t) &= k(-1,t) = 0 \end{aligned} \tag{10}$$

## 2.1 General Dynamics

Following Weidlich and Haag, it is possible to obtain general expressions for the mean and variance of the proportion $x \equiv \frac{n}{N}$ of individuals with green attitudes, expressed in terms of the transition probabilities $w_{\uparrow}$ and $w_{\downarrow}$. To do this we need to treat $x$ and $n$ as a continuous variables - an approximation which is increasingly accurate as $N$ becomes large and as long as $p(n,t)$, $w_{\uparrow}(n)$ and $w_{\downarrow}(n)$ are all well-behaved.

Writing

$$\Delta x = \frac{\Delta n}{N} = \frac{1}{N}$$

we define

$$\begin{aligned} P(x,t) &\equiv Np(n,t) = Np(Nx,t) \\ W_{\uparrow}(x) &\equiv \frac{1}{N} w_{\uparrow}(n) = (1-x)p_{g,b}(Nx) \\ W_{\downarrow}(x) &\equiv \frac{1}{N} w_{\downarrow}(n) = xp_{b,g}(Nx) \\ W_{=}(x) &\equiv \frac{1}{N} - [W_{\uparrow}(x) + W_{\downarrow}(x)] \end{aligned} \tag{11}$$

Then

$$\int_0^1 P(x,t)dx \approx \sum_{x=0}^{1} P(x,t)\Delta x = \sum_{n=0}^{N} p(n,t) = 1 \tag{12}$$

Take equation 2 and expand it using the Taylor series:

$$\begin{aligned} \frac{\partial p(n,t)}{\partial t} &= \Delta n \frac{\partial}{\partial n}[w_{\downarrow}(n)p(n,t)] + \frac{(\Delta n)^2}{2}\frac{\partial^2}{\partial n^2}[w_{\downarrow}(n)p(n,t)] \\ &\quad - \Delta n \frac{\partial}{\partial n}[w_{\uparrow}(n)p(n,t)] + \frac{(\Delta n)^2}{2}\frac{\partial^2}{\partial n^2}[w_{\uparrow}(n)p(n,t)] \end{aligned} \tag{13}$$

Setting $\Delta n = 1$,

$$\begin{aligned} \frac{\partial p(n,t)}{\partial t} &= -\frac{\partial}{\partial n}\{[w_{\uparrow}(n) - w_{\downarrow}(n)]p(n,t)\} \\ &\quad + \frac{1}{2}\frac{\partial^2}{\partial n^2}\{[w_{\uparrow}(n) + w_{\downarrow}(n)]p(n,t)\} \end{aligned} \tag{14}$$

From equation 11,

$$[w_{\uparrow}(n) - w_{\downarrow}(n)]p(n,t) = [W_{\uparrow}(n) - W_{\downarrow}(n)]P(x,t) \tag{15}$$

So

$$\frac{\partial P(x,t)}{\partial t} = -\frac{\partial}{\partial n}\{[W_\uparrow(n) - W_\downarrow(n)]P(x,t)\} + \frac{1}{2}\frac{\partial^2}{\partial n^2}\{[W_\uparrow(n) + W_\downarrow(n)]P(x,t)\} \quad (16)$$

It is useful to define a “drift coefficient”

$$K(x) = W_\uparrow(x) - W_\downarrow(x) \quad (17)$$

and a “fluctuation coefficient”

$$Q(x) = W_\uparrow(x) + W_\downarrow(x) \quad (18)$$

which allows us to write

$$\begin{aligned}\frac{\partial P(x,t)}{\partial t} &= -\frac{\partial}{\partial x}[K(x)P(x,t)] + \frac{1}{2N}\frac{\partial^2}{\partial x^2}[Q(x)P(x,t)] \\ &= -\frac{\partial}{\partial x}H(x,t)\end{aligned} \quad (19)$$

where

$$H(x,t) = K(x)P(x,t) - \frac{1}{2N}\frac{\partial}{\partial x}[Q(x)P(x,t)] \quad (20)$$

is a probability flux with the following boundary conditions:

$$H(0,t) = H(1,t) = 0 \quad (21)$$

At time $t = 0$ the probability distribution function $P(x,t)$ has a discontinuous gradient:

$$P(x,0) = \delta(x - x_0)$$

(where $\delta$ here is the Dirac delta function, with infinite height but unitary integral). We can however use a definition of gradient from "above" the discontinuity:

$$\frac{\partial P(x,0)}{\partial t} = \lim_{\Delta t \to 0^+} \frac{P(x,\Delta t) - P(x,0)}{\Delta t} \tag{22}$$

Writing $\langle f(x) \rangle_t$ for the mean value (over $x$) of a function $f(x)$ at time $t$, we define it as

$$\langle f(x) \rangle_t = \int_0^1 P(x,t) f(x) dx \tag{23}$$

And equation 16 gives us:

$$\begin{aligned} \int_0^1 \frac{\partial P(x,0)}{\partial t} x dx &= \lim_{\Delta t \to 0^+} \frac{\langle x \rangle_t - \langle x \rangle_0}{\Delta t} \\ &= \int_0^1 x\{-\frac{\partial}{\partial x}[K(x)P(x,0)] + \frac{1}{2N}\frac{\partial^2}{\partial x^2}[Q(x)P(x,0)]\}dx \\ &= K(x_0) \end{aligned} \tag{24}$$

But

$$\langle x \rangle_0 = \int \delta(x - x_0) x dx = x_0 = \int P(x,\Delta t) x_0 dx = \langle x_0 \rangle_{\Delta t} \tag{25}$$

So we can also write

$$K(x_0) = \lim_{\Delta t \to 0^+} \frac{\langle (x - x_0) \rangle_\Delta t}{\Delta t} \tag{26}$$

For the variance,

$$\begin{aligned} \int_0^1 \frac{\partial P(x,0)}{\partial t} x^2 dx &= \lim_{\Delta t \to 0^+} \frac{\langle x^2 \rangle_{\Delta t} - \langle x^2 \rangle_0}{\Delta t} \\ &= \int_0^1 x^2\{-\frac{\partial}{\partial x}[K(x)P(x,0)] + \frac{1}{2N}\frac{\partial^2}{\partial x^2}[Q(x)P(x,0)]\}dx \\ &= 2x_0\, K(x_0) + \frac{1}{N} Q(x_0) \end{aligned} \tag{27}$$

But $x_0^2 = \langle x^2 \rangle_0 = \langle x_0^2 \rangle_{\Delta t}$ so

$$\frac{1}{N}Q(x_0) = \lim_{\Delta t \to 0^+} \frac{[\langle x^2 \rangle_{\Delta t} - 2x_0 \langle x \rangle_{\Delta t} + x_0^2]}{\Delta t} = \lim_{\Delta t \to 0^+} \frac{\langle (x - x_0)^2 \rangle_{\Delta t}}{\Delta t} \tag{28}$$

So the drift and fluctuation coefficients provide respectively the mean and variance of the deviation of $x$ from its initial value, at least for small values of $t$.

Weidlich and Haag show how to obtain complete equations of motion for $\langle x \rangle$ and $\langle x^2 \rangle$. Taking into account the boundary conditions (21) this gives us

$$\frac{d\langle x \rangle_t}{dt} = \langle K(x) \rangle_t - \frac{1}{2N}[Q(x)P(x,t)]_{x=0}^{x=1} \tag{29}$$

and

$$\frac{d\langle x^2 \rangle_t}{dt} = 2\langle xK(x) \rangle_t + \frac{1}{N}\langle Q(x) \rangle_t - \frac{1}{N}[xQ(x)P(x,t)]_{x=0}^{x=1} \tag{30}$$

Writing $\sigma(t)$ for the variance

$$\sigma(t) = \langle (x - \langle x \rangle_t)^2 \rangle_t = \langle x^2 \rangle_t - \langle x \rangle_t^2 \geq 0 \tag{31}$$

We can simplify equations 29 and 30 if we are able to make the assumption that the probability distribution function $P(x,t)$ has very small values at the extremes $x = 0$ and $x = 1$. Then

$$\frac{d\langle x \rangle_t}{dt} = \langle K(x) \rangle_t \tag{32}$$

$$\frac{d\sigma(t)}{dt} = 2[\langle xK(x) \rangle_t - \langle x \rangle_t \langle K(x) \rangle_t] + \frac{1}{N}\langle Q(x) \rangle_t \tag{33}$$

This rather surprising result suggests (providing the assumption that $P(x,t)$ is negligible at $x = 0$ and $x = 1$ holds) that empirical methods could be used to produce a solution. Determining the sample mean and variance of a process assumed to be following this model would allow us to estimate $K(x)$ and $Q(x)$ and hence $W_\uparrow$, $W_\downarrow$, $w_\uparrow$ and $w_\downarrow$.

## 2.2 Exact Solutions

It is possible to produce explicit solutions of these equations of motion under certain circumstances:

1. The case when there is no change over time, (see Weidlich and Haag, 1983, pp.28-30)
2. When the drift ($K(x)$) is linear in $x$ and the fluctuation ($Q(x)$) is constant (see Weidlich and Haag, 1983, p.33)
3. When a non-zero drift dominates the motion (see Van Kampen, 1976)

There are two further cases where an exact solution is not possible:

1. Fluctuation-initiated motion starting from an unstable stationary point in the path of $\langle x\rangle$
2. Fluctuation-dominated motion starting from a stable stationary point in the path of $\langle x\rangle$

# 3 Specific Cases

Let us now compare this model with the discrete choice (discrete time) model of Zeppini.

Individual $i$'s utility is given by

$$W_i = U_i - pq_i - D_i + G_i + I_i + \epsilon_i \tag{34}$$

$W_i$ is the individual's von Neumann–Morgenstern utility which is a function of $U_i$, the utility arising from the good itself, the cost of the good consumed (price $p$, quantity $q_i$) and four other terms: $D_i$ is the perceived damage resulting from the consumption (itself a function of $Q = \sum_i q_i$); $G_i$ is Andreoni's "warm-glow" (1990), which rewards individuals with a positive feeling when they take an otherwise-unmotivated altruistic decision; $I_i$ is the effect of social norms[1] on the individual, assumed therefore to be a function of $x$; lastly, $\epsilon_i$ is a stochastic element, unobservable to the modeller but known to the individual. This random component of the utility allows the individuals to make different optimisation decisions when faced with the same input variables, or, in the context of this paper, it allows us to model individuals making the decision to change their attitude at different points in time.

It is worth noting that none of the components of $W_i$ are directly dependent on $x$ with the exception of $I_i$. Thus, without the social norms, this is a straightforward demand/supply optimisation problem, solvable in the traditional way.

[1] The model is concerned only with *descriptive* social norms, that is to say, the effect on the individual caused merely by *observation* of the actions (decisions) of other individuals.

The choice of attitude in stage 1 is represented by the variable

$$\omega_i = 1 \text{ for green, else } 0 \tag{35}$$

Zeppini defines

$$\Delta W(x) \equiv W_b(x) - W_g(x) \tag{36}$$

and notes that this is the key determinant of the individual's choice. In the case where all choice is made on pure "rational" grounds, ($\epsilon_i = 0$), the path of $x(t)$ is a step function, as all individuals decide simultaneously to change state. So when the stochastic component is introduced we are interested in comparing $\Delta W$ with $\epsilon(1) - \epsilon(0)$ and it is the probability distribution of this which generates the logistic solution

$$f(x) = \frac{1}{1 + e^{-\beta \Delta W(x)}} \tag{37}$$

We address two challenges. First, a method must be chosen to introduce continuous time, and we take this first.

Let $\Delta t$ be the time interval for each iteration through the repeated game, and then examine the limit as $\Delta t \to 0$. We are interested in the probability that individual $i$ chooses $\{g\}$.

$$Pr\{\omega_i(x) = 1\} = Pr\{\epsilon_i(1) - \epsilon_i(0)\} = \frac{1}{1 + e^{-\beta \Delta W(x)}} \tag{38}$$

The second issue comes because our model has an extra degree of freedom. Specifically, the length of time the individual spends in one state before moving to another. Or to put it another way, the probability per unit time that the individual maintains the same attitude.

This is captured by $w_=$ and $W_=$ from equations 11 and 4, and makes explicit the fact that this model is a Markov process with non-zero stationary probabilities. (Recall that the states of our model is the set $\{1, ..., N\}$.)

In order to address this issue, we need to modify the structure of the repeated game. For instance, we could introduce a stage zero, where the length of time the individual spends in a state before making their stage one decision. It is clear that this modification, along with the continuous time modification, would then enable the discrete choice model to reproduce the range of behaviours implicit in our base model. However, it would allow permit a far greater degree of variability resulting in a loss of analytical tractability. We would not, for instance, be able to predict a logistic curve with any certainty. The approach taken by Peyton Young goes some way to address this issue. See below.

# 4 Comparison with other models

In this section, we compare the synergetics model of section 2 with models of innovation diffusion, likening change in attitude to the introduction of a novel innovation.

Peyton Young (2009) classifies three kinds of innovation diffusion: Contagion, Social Influence and Social Learning. Under contagion, individuals merely have to come into contact with others with the attitude for it to "catch." Social Influence is described as adoption spread by a "conformity motive:" innovations spread when sufficient people in the group have adopted.

The last of Peyton Young's three types of innovation diffusion is Social Learning, which requires that the individuals first observe evidence that the innovation is worth copying.

A drawback of the approach that Peyton Young takes is that with innovation diffusion, the question is about when the individuals adopt the innovation, with the implicit assumption that once adopted, they cannot later unadopt.

In both models, individuals have certain probability per unit time of moving from state one to state two. With a model of attitude dynamics, they may later revert. In innovation diffusion, once at state two, the chance of reversion is zero.

This is disappointing, as it leaves a large problem space unexamined. Our base model, described in section 2, is however flexible enough to cope with this discrepancy. We simply require $p_{b,g}(n) = 0$.

As a consequence,

$$w_{\downarrow}(n) = W_{\downarrow}(x) = j_{\downarrow}(n,t) = 0 \tag{39}$$

$$k(n,t) = j_{\uparrow}(n,t) = 0 \tag{40}$$

$$K(x) = Q(x) = W_{\uparrow}(x) \tag{41}$$

and the equations of motion for the mean and variance are

$$\frac{d\langle x\rangle_t}{dt} = \langle W_{\uparrow}(x)\rangle_t \tag{42}$$

$$\frac{d\sigma(t)}{dt} = 2[\langle xW_{\uparrow}(x)\rangle_t - \langle x\rangle_t\langle W_{\uparrow}(x)\rangle_t] + \frac{1}{N}\langle W_{\uparrow}(x)\rangle_t \tag{43}$$

Let us take a closer look at the Contagion case. The definition is "a process in which people adopt a new product or practice when they come in contact with others who have adopted it." [2] The key difference between this and Social Influence, at least as defined by Peyton Young, is that his model for Social Influence requires an adoption threshold for each individual. While a worthy object of enquiry, it does not match our use of descriptive social norms in section 3.

[2] Peyton Young, 2009, p.1900

A closer match however, is the contagion case. It is a very close match if we ignore the human layers of meaning implicit in the word "influence" and focus instead purely on the mathematical definitions, as will be shown below.

Let $x(t)$ as before be the proportion of individuals with a "green" attitude. Let $\lambda(t)$ be the rate at which an individual from the "brown" subset has their attitude changed via an interaction with an individual from the "green" subset. Let $\gamma(t)$ correspondingly be the rate that a brown individual's attitude is changed (to green) by a "brown" individual. (This is by no means an unlikely scenario; consider for instance conversations about climate change between individuals who hitherto did not consider it serious enough to take action. It is certainly possible that any such interchange could result in the rethinking of premises or assumptions which may no longer hold weight, resulting in a "brown-on-brown" green conversion.)

The equation of motion for this system is

$$\frac{dx(t)}{dt} = [\lambda x(t) + \gamma][1 - x(t)] \tag{44}$$

which has the following solution:

$$x(t) = \frac{1 - \beta\gamma e^{-(\lambda+\gamma)t}}{1 + \beta\gamma e^{-(\lambda+\gamma)t}} \tag{45}$$

Encouragingly, when $\lambda = 0$, we get

$$x(t) = \frac{1}{1 + c\gamma e^{-\gamma t}} \tag{46}$$

which can be compared with equation 37.

# 5 Conclusion

Synergetics is a rich field with plenty to add to Economics. We have seen how a framework which is more commonly used to describe physico-chemical systems can nonetheless be useful in other disciplines.

Specifically, it is possible to create models which are both detailed enough to capture the complex dynamics required by an analysis of pro-environmental attitudes, while at the same time also being sufficiently analytically tractable to lend themselves to exact solutions.

Crossovers have been found between the models of Peyton Young and Zeppini, and it is possible that a future model could unite these with the Weidlich and Haag framework, if an alternative method can be found to describe the multi-state Markov process.

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