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ON COINCIDENCE AND COMMON FIXED POINT OF SIX MAPS SATISFYING F-CONTRACTIONS

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Abstract. Coincidence and common fixed point of six self maps satisfying F-contractions are established via common limit in the range property without exploiting the notion of continuity or containment of range space of involved maps or completeness of space/subspace. Our results generalize, extend and improve the analogous recent results in literature.

Keywords: common fixed point, common limit range property, F-contraction, weakly compatible maps.

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1. Introduction

Fixed point theory is a useful mechanism in mathematical economics, game theory, dynamic optimization and stochastic games, functional analysis, variational calculus, defence, statistics, medicine etc. However, for many real-world situations, the conditions in the fixed point theorems are very strong and there is no assurance that a fixed point exists. Over the past few decades, a number of researchers have taken interest in generalizing, extending, and improving fixed point theorems (for instance [3], [4], [9], [11], [12] and reference therein). Recently Wardowski [14] gave a nice generalization for single valued map on complete metric space, introducing a new approach of contractive map, called F-contraction.

The aim of this paper is to establish the existence and uniqueness of coincidence and common fixed point of six self maps in non complete metric space satisfying F-contraction without exploiting the notion of continuity or containment requirement of range space of involved maps or completeness of space/subspace. In the process, we emphasize on the role of common limit range property for the existence of common fixed point under Ćirić type F-contraction and Hardy-Roger type F-contraction for six self maps, which are more general than the contraction condition introduced by Wardowski [14].

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2. Preliminaries

We denote the set of all real numbers by R, the set of all positive real numbers by R^+, the set of all natural numbers by N and a metric space by (X, d). A pair of self map A and S have a coincidence point at $x \in X$ if Ax = Sx = w. w is called a point of coincidence of A and S. Further, a point $x \in X$ is a common fixed point of A and S if Ax = Sx = x.

Definition 2.1. [4] *A self map A on X is a Ćirić type contraction for all $x, y \in X$ if*

$$d(Ax, Ay) \leq c\, max\{d(x, y), d(x, Ax), d(y, Ay), d(x, Ay), d(y, Ax)\}.$$

Definition 2.2. [7]*A self map A on X is a Hardy-Rogers type contraction for all $x, y \in X$ if*

$$d(Ax, Ay) \leq \alpha d(Ax, x) + \beta d(Ay, y) + \gamma d(x, y) + \delta d(Ax, y) + Ld(Ay, x),$$

$\alpha + \beta + \gamma + \delta + L < 1$ *and* $\alpha, \beta, \gamma, \delta, L > 0$.

Definition 2.3. [14] *A self map A on X is an F-contraction if there exist $\tau > 0$ such that*

$$\tau + F(d(Ax, Ay)) \leq F(d(x, y)) \tag{1}$$

for all $x, y \in X$ with $Ax \neq Ay$ where $F : R^+ \to R$ is a function satisfying:

(F1) *F is strictly increasing, i.e., for all $\alpha, \beta \in R$ such that $\alpha < \beta$, $F(\alpha) < F(\beta)$.*

(F2) *For each sequence $\{\alpha_n\}$ $n \in N$ of positive numbers, the following holds: $lim_{n\to\infty}\alpha_n = 0$ if and only if $lim_{n\to\infty}F(\alpha_n) = -\infty$.*

(F3) *There exist $k \in (0, 1)$ such that $lim_{\alpha\to 0^+}(\alpha^k F(\alpha)) = 0$.*

From (F1) and (1) it is easy to conclude that every F-contraction A is a contractive map and hence necessarily continuous. We denote by $\mathcal{F}$, the family of all $F : R^+ \to R$ satisfying the conditions (F1)-(F3). Taking different functions F, we obtain a variety of F-contractions, some of them being already known in the literature. Some examples of the functions belonging to $\mathcal{F}$ are:

$(1) F(\alpha) = \ln \alpha,$ $\quad$ $(2) F(\alpha) = \ln \alpha + \alpha, \alpha > 0,$

$(3) F(\alpha) = \dfrac{-1}{\sqrt{\alpha}}, \alpha > 0,$ $\quad$ $(4) F(\alpha) = \ln(\alpha^2 + \alpha), \alpha > 0.$

Every F-contraction A is a contractive map ,i.e., $d(Ax, Ay) < d(x, y)$ for all $x, y \in X$, $Ax \neq Ay$. The Banach contraction[1] is a particular case of F-contraction. Meanwhile there exist F-contractions, which are not Banach contractions (Wardowski [14]).

Following Wardowski, Minak et al. [10] and Wardowski and Dung[15] independently introduced the concept of Ćirić type F-contraction. Minak et al. called it Ćirić type generalized F-contraction and Wardowski and Dung called it F-weak contraction. Later Cosentino and Vetro [6] introduced Hardy-Rogers type F-contraction. If there exists $F \in \mathcal{F}$ and $\tau > 0$ such that, for all $x, y \in X$,

$$d(Ax, Ay) > 0 \Rightarrow \tau + F(d(Ax, Ay)) \leq F(m(x, y)),$$

where $m(x, y) = max\{d(x, y), d(x, Ax), d(y, Ay), [d(x, Ay) + d(y, Ax)]/2\}$, is the Ciric type F-contraction[10, 15] and if

$m(x, y) = \alpha d(x, y) + \beta d(x, Ax) + \gamma d(y, Ay) + \delta d(x, Ay) + Ld(y, Ax),$

where $\alpha + \beta + \gamma + 2\delta = 1$, $\gamma \neq 1$ and $L \geq 0$, is the Hardy-Rogers type F-contraction[6].

On the other hand, Batra et al.[2] introduced a F-g-contraction. If there exists $F \in \mathcal{F}$ and $\tau > 0$ such that $\tau + F(d(Ax, Ay)) \leq F(d(gx, gy))$ for all $x, y \in X$ with $gx \neq gy$ and $Ax \neq Ay$.

Note that every F-contraction A is an C̀iriȅ type F-contraction [10, 15], Hardy-Rogers type F-contraction [6] but the reverse implication does not hold. On substituting g = I in F-g-contraction we get F-contraction [15].

Definition 2.4. [8] *A pair of self-maps (A,S) is weakly compatible if the pair commute on the set of their coincidence points ,i.e., for $x \in X$, $Ax = Sx$ implies $ASx = SAx$.*

Definition 2.5. [13] *A pair of self-maps (A, S) on X satisfies the common limit range property with respect to S denoted by CLR_S if there exist a sequence $\{x_n\}$ in X such that $lim_{n\to\infty}Ax_n = lim_{n\to\infty}Sx_n = t$ where $t \in SX$.*

Definition 2.6. [5] *Two pairs of self maps (A, S) and (B, T) on X satisfy the common limit range property with respect to S and T denoted by CLR_{ST} if there exist two sequences $\{x_n\}$ and $\{y_n\}$ in X such that $lim_{n\to\infty}Ax_n = lim_{n\to\infty}Sx_n = lim_{n\to\infty}By_n = lim_{n\to\infty}Ty_n = t$ where $t \in SX \cap TX$.*

3. Main Results

Definition 3.1. *Six self maps A, B, P, Q, S and T of a metric space (X, d) satisfy C̀irić type F-contraction if there exist $F \in \mathcal{F}$ and $\tau > 0$ such that for all $x, y \in X$ satisfying $d(Ax, By) > 0$ the following holds:*

$$\tau + F(d(Ax, By)) \leq F(max\{d(SPx, TQy), d(SPx, Ax), d(TQy, By), d(SPx, By), d(TQy, Ax)\}) \tag{2}$$

Theorem 3.1. *Let (A, SP) and (B, TQ) be the pairs of self maps of a metric space (X, d) satisfying $CLR_{(SP)(TQ)}$ property and C̀irić type F-contraction (2). Then the pairs (A, SP) and (B, TQ) have coincidence points in X if F is continuous. Moreover, the self maps A, B, P, Q, S and T have a unique common fixed point in X provided that the pairs (A, S), (A, P), (S, P), (B, T), (B, Q) and (T, Q) are weakly compatible.*

Proof. Let (A, SP) and (B, TQ) be the pairs of self maps satisfying $CLR_{(SP)(TQ)}$ property then there exist two sequences $\{x_n\}$ and $\{y_n\}$ in X such that
$lim_{n\to\infty}Ax_n = lim_{n\to\infty}SPx_n = lim_{n\to\infty}By_n = lim_{n\to\infty}TQy_n = t$,
where $t \in SPX \cap TQX$. Since $t \in SPX$ there exist $v \in X$ such that $SPv = t$. Also since $t \in TQX$ there exist $u \in X$ such that $TQu = t$.
Using (2),

$$F(d(Av, By_n)) \leq F(max\{d(SPv, TQy_n), d(SPv, Av), d(TQy_n, By_n), d(SPv, By_n), d(TQy_n, Av)\}) - \tau$$

As $n \to \infty$ and since F is continuous,

$$\begin{aligned} F(d(Av, t)) &\leq F(max\{d(t, t), d(t, Av), d(t, t), d(t, t), d(t, Av)\}) - \tau \\ &= F(d(Av, t)) - \tau \\ &< F(d(Av, t)) \end{aligned}$$

which is a contradiction. Hence $Av = SPv = t$, i.e., the pair (A, SP) have a coincidence point in X. Now, using (2),

$$\begin{aligned}F(d(t, Bu)) &= F(d(Av, Bu))\\ &\leq F(max\{d(SPv, TQu), d(SPv, Av), d(TQu, Bu), d(SPv, Bu), d(TQu, Av)\}) - \tau\\ &= F(max\{d(t,t), d(t,t), d(t, Bu), d(t, Bu), d(t,t)\}) - \tau\\ &= F(d(Bu, t)) - \tau\\ &< F(d(Bu, t)),\end{aligned}$$

which is a contradiction. Hence $Bu = TQu = t$, i.e., the pair (B, TQ) have a coincidence point in X. Now, since the pairs (A, S), (A, P) and (S, P) are weakly compatible, $At = ASPv = APSv = PASv = PSAv = PSt = SPt$ which implies $SPt = At$. Similarly, we can show that, $TQt = Bt$.

Now, to show that $At = t$. Let $At \neq t$ then using (2)

$$\begin{aligned}F(d(At, t)) &= F(d(At, Bu))\\ &\leq F(max\{d(SPt, TQu), d(SPt, At), d(TQu, Bu), d(SPt, Bu), d(TQu, At\}) - \tau\\ &= F(max\{d(At, t), d(At, At), d(t,t), d(At, t), d(t, At)\}) - \tau\\ &= F(d(At, t)) - \tau\\ &< F(d(At, t)),\end{aligned}$$

which is a contradiction. Hence $At = SPt = t$. Similarly we can show that $Bt = TQt = t$. Thus we have $At = Bt = t$, i.e., t is the common fixed point of A and B.

Now to show that this common fixed point is unique. Let $w \neq t$ be also a common fixed point of A and B, then $Aw = w = SPw$ and $Bw = w = TQw$.

$$\begin{aligned}F(d(t, w)) &= F(d(At, Bw))\\ &\leq F(max\{d(SPt, TQw), d(SPt, At), d(TQw, Bw), d(SPt, Bw), d(TQw, At)\}) - \tau\\ &= F(max\{d(t, w), d(t,t), d(w, w), d(t, w), d(w, t)\}) - \tau\\ &= F(d(t, w)) - \tau\\ &< F(d(t, w)),\end{aligned}$$

which is a contradiction. Hence $t = w$, i.e., A and B have a unique common fixed point.

By weak compatibility of the pairs $(A, S), (A, P), (S, P), (B, T), (B, Q)$ and (T, Q), we have $ASt = SAt = St$, $APt = PAt = Pt$, $BTt = TBt = Tt$, $BQt = QBt = Qt$, which gives that St and Pt are also fixed point of A. Tt and Qt are also fixed point of B. By uniqueness of the common fixed point of A and B, $St = Tt = t = Pt = Qt$. Therefore, $At = Bt = Pt = Qt = St = Tt = t$. Hence t is the unique common fixed point of A, B, P, Q, S and T. □

If we put $Q = I$ in theorem 3.1 we obtain the following definition and corollary for five maps.

Definition 3.2. *Five self maps A, B, P, S and T of a metric space (X, d) satisfy Ćirić type F-contraction if there exist* $F \in \mathcal{F}$ *and* $\tau > 0$ *such that for all* $x, y \in X$ *satisfying* $d(Ax, By) > 0$ *the following holds:*

$$\tau + F(d(Ax, By)) \leq F(max\{d(SPx, Ty), d(SPx, Ax), d(Ty, By), d(SPx, By), d(Ty, Ax)\}) \quad (3)$$

Corollary 3.1. *Let (A, SP) and (B,T) be the pairs of self maps of a metric space (X, d) satisfying $CLR_{(SP)(T)}$ property and Ćirić type F-contraction(3). Then the pairs (A, SP) and (B, T) have a coincidence point in X if F is continuous. Moreover, the self maps A, B, P, S and T have a unique common fixed point in X provided that the pairs (A, S), (A, P), (S, P), (B, T) are weakly compatible.*

Now we furnish two examples in support of our main result.

Example 3.1. *Let $X = [1, 16)$ and d be the usual metric on X.*
Define $A, B, P, Q, S, T : X \to X$ by

$$Ax = \begin{cases} 3 & if\ x = 1,2,3\ and\ x \geq 6 \\ 7 & if\ x \in [1,6) - \{1,2,3\} \end{cases} \qquad Bx = \begin{cases} 3 & if\ x = 1,2,3\ and\ x \geq 6 \\ 8 & if\ x \in [1,6) - \{1,2,3\} \end{cases}$$

$$Px = \begin{cases} 3 & if\ x = 1,2,3 \\ 11 & if\ x \in [1,6) - \{1,2,3\} \\ 1 & if\ x \geq 6 \end{cases} \qquad Qx = \begin{cases} 3 & if\ x = 1,2,3 \\ 10 & if\ x \in [1,6) - \{1,2,3\} \\ 2 & if\ x \geq 6 \end{cases}$$

$$Sx = \begin{cases} 3 & if\ x = 1,2,3 \\ 12 & if\ x \in [1,6) - \{1,2,3\} \\ 1 & if\ x \geq 6 \end{cases} \qquad Tx = \begin{cases} 3 & if\ x = 1,2,3 \\ 9 & if\ x \in [1,6) - \{1,2,3\} \\ 2 & if\ x \geq 6 \end{cases}$$

Let $\{x_n\}$ and $\{y_n\}$ be two sequences in X such that $x_n = 6 + \frac{1}{n}$ and $y_n = 3$ then $lim_{n\to\infty}Ax_n = lim_{n\to\infty}A(6 + \frac{1}{n}) = 3, lim_{n\to\infty}SPx_n = lim_{n\to\infty}SP(6 + \frac{1}{n}) = 3$ and $lim_{n\to\infty}By_n = 3, lim_{n\to\infty}TQy_n = 3$. Hence $lim_{n\to\infty}Ax_n = lim_{n\to\infty}SPx_n = lim_{n\to\infty}By_n = lim_{n\to\infty}TQy_n = 3$, $3 \in SPX \cap TQX$, i.e., (A, SP) and (B, TQ) satisfy the $CLR_{(SP)(TQ)}$ property. Also $Ax = SPx = 3$, $Bx = TQx = 3$, where $x \in \{1, 2, 3\}$ and $x \geq 6$, i.e., pairs (A, SP) and (B, TQ) have coincidence points in X. Also $AS = SA, BT = TB, SP = PS, TQ = QT, AP = PA, BQ = QB$ at the coincidence points, ,i.e., pairs $(A, S), (A, P), (S, P), (B, T), (B, Q)$ and (T, Q) are weakly compatible. Further A, B, P, Q, S and T satisfy Ćirić type F-contraction condition (2) for $\tau = \frac{1}{200}$ and $F(\alpha) = -\frac{1}{\sqrt{\alpha}}$. Hence all the conditions of theorem 3.1 are satisfied and $x = 3$ is a unique common fixed point of A, B, P, Q, S and T. Moreover all the self maps are discontinuous at common fixed point.

Example 3.2. *Let $X = (2, 20)$ and d be the usual metric on X.*
Define $A, B, P, Q, S, T : X \to X$ by

$$Ax = \begin{cases} 5 & if\ x \in (2,5] \cup [8,20) \\ 14 & if\ x \in (5,8) \end{cases} \qquad Bx = \begin{cases} 5 & if\ x \in (2,5] \cup [8,20) \\ 10 & if\ x \in (5,8) \end{cases}$$

$$Px = \begin{cases} 5 & if\ x \in (2,5] \\ 11 & if\ x \in (5,8) \\ 3 & if\ x \in [8,20) \end{cases} \qquad Qx = \begin{cases} 5 & if\ x \in (2,5] \\ 12 & if\ x \in (5,8) \\ 3 & if\ x \in [8,20) \end{cases}$$

$$Sx = \begin{cases} 5 & if\ x \in (2,5] \\ 15 & if\ x \in (5,8) \\ 3 & if\ x \in [8,20) \end{cases} \qquad Tx = \begin{cases} 5 & if\ x \in (2,5] \\ 16 & if\ x \in (5,8) \\ 3 & if\ x \in [8,20) \end{cases}$$

Let $\{x_n\}$ and $\{y_n\}$ be two sequences in X such that $x_n = 8 + \frac{1}{n}$ and $y_n = 5$ then $lim_{n\to\infty}Ax_n = lim_{n\to\infty}A(8 + \frac{1}{n}) = 5, lim_{n\to\infty}SPx_n = lim_{n\to\infty}SP(8 + \frac{1}{n}) = 5$ and $lim_{n\to\infty}By_n = 5, lim_{n\to\infty}TQy_n = 5$. Hence $lim_{n\to\infty}Ax_n = lim_{n\to\infty}SPx_n = lim_{n\to\infty}By_n$

$= lim_{n\to\infty}TQy_n = 5, 5 \in SPX \cap TQX$, i.e., (A, SP) and (B, TQ) satisfy the $CLR_{(SP)(TQ)}$ property. Also $Ax = SPx = 5$, $Bx = TQx = 5$, where $x \in (2, 5] \cup [8, 20)$, i.e., pairs (A, SP) and (B, TQ) have coincidence points in X. Also $AS = SA, BT = TB, SP = PS, TQ = QT, AP = PA, BQ = QB$ at the coincidence points, i.e., pairs $(A, S), (A, P), (S, P), (B, T), (B, Q)$ and (T, Q) are weakly compatible. Further A, B, P, Q, S and T satisfy Ćirić type F-contraction condition (2) for $\tau = 0.01$ and $F(\alpha) = log\alpha$. Hence all the conditions of theorem 3.1 are satisfied and $x = 5$ is a unique common fixed point of A, B, P, Q, S and T. Moreover all the self maps are discontinuous at common fixed point.

Definition 3.3. *Six self maps A, B, P, Q, S and T of a metric space (X, d) satisfy Hardy-Rogers type F-contraction if there exist $F \in \mathcal{F}$ and $\tau > 0$ such that for all $x, y \in X$ satisfying $d(Ax, By) > 0$ the following holds:*

$$\tau + F(d(Ax, By)) \leq F(\alpha d(SPx, TQy) + \beta d(SPx, Ax) + \gamma d(TQy, By) + \delta d(SPx, By) + Ld(TQy, Ax)) \tag{4}$$

where $\alpha + \beta + \gamma + \delta + L < 1$ and $\alpha, \beta, \gamma, \delta, L > 0$

Theorem 3.2. *Let (A, SP) and (B,TQ) be the pairs of self maps of a metric space (X, d) satisfying $CLR_{(SP)(TQ)}$ property and Hardy-Rogers type F- contraction(4). Then the pairs (A, SP) and (B, TQ) have coincidence points in X if F is continuous. Moreover, the self maps A, B, P, Q, S and T have unique common fixed point in X provided that the pairs (A, S), (A, P), (S, P), (B, T), (B, Q) and (T, Q) are weakly compatible.*

Proof. Proof follows on the same lines of Theorem 3.1 □

Now an Example in support of Theorem 3.2 is furnished.

Example 3.3. *Let $X = [1, 13)$ and d be the usual metric on X.*
Define $A, B, P, Q, S, T : X \to X$ by

$$Ax = \begin{cases} 2 & if\, x \in [1, 2] \cup [5, 13) \\ 3 & if\, x \in (2, 5) \end{cases} \qquad Bx = \begin{cases} 2 & if\, x \in [1, 2] \cup [5, 13) \\ 4 & if\, x \in (2, 5) \end{cases}$$

$$Px = \begin{cases} 2 & if\, x \in [1, 2] \\ 9 & if\, x \in (2, 5) \\ 7 & if\, x \in [5, 13) \end{cases} \qquad Qx = \begin{cases} 2 & if\, x \in [1, 2] \\ 8 & if\, x \in (2, 5) \\ 6 & if\, x \in [5, 13) \end{cases}$$

$$Sx = \begin{cases} 2 & if\, x \in [1, 2] \\ 12 & if\, x \in (2, 5) \\ 11 & if\, x \in [5, 13) \end{cases} \qquad Tx = \begin{cases} 2 & if\, x \in [1, 2] \\ 10 & if\, x \in (2, 5) \\ 11 & if\, x \in [5, 13) \end{cases}$$

Let $\{x_n\}$ and $\{y_n\}$ be two sequences in X such that $x_n = 1 + \frac{1}{n}$ and $y_n = 2$ then $lim_{n\to\infty}Ax_n = lim_{n\to\infty}A(1 + \frac{1}{n}) = 2, lim_{n\to\infty}SPx_n = lim_{n\to\infty}SP(1 + \frac{1}{n}) = 2$ and $lim_{n\to\infty}By_n = 2, lim_{n\to\infty}TQy_n = 2$. Hence $lim_{n\to\infty}Ax_n = lim_{n\to\infty}SPx_n = lim_{n\to\infty}By_n = lim_{n\to\infty}TQy_n = 2, 2 \in SPX \cap TQX$, i.e., (A, SP) and (B, TQ) satisfy the $CLR_{(SP)(TQ)}$ property. Also $Ax = SPx = 2$, $Bx = TQx = 2$, where $x \in [1, 2]$, i.e., pairs (A, SP) and (B, TQ) have coincidence points in X. Also $AS = SA, BT = TB, SP = PS, TQ = QT, AP = PA, BQ = QB$ at the coincidence points ,i.e., pairs $(A, S), (A, P), (S, P), (B, T), (B, Q)$ and (T, Q) are weakly compatible. Further A, B, P, Q, S and T satisfy Hardy - Rogers type F-contraction condition (4) for $\tau = \frac{1}{10}$ and $F(x) = logx$, $\alpha = \frac{1}{5}$,$\beta = \frac{1}{6}$,$\gamma = \frac{1}{10}$, $\delta = \frac{1}{6}$,$L = \frac{1}{10}$. Hence all the conditions of theorem 3.2 are satisfied and $x = 2$ is a unique common fixed point of A, B, P, Q, S and T. Moreover all the self maps are discontinuous at common fixed point.

Remark 3.1. (i) *If we put $Q = I$ in theorem 3.2, we obtain the corollary for five maps satisfying Hardy Rogers type F- contraction.*
(ii) *By choosing A, B, P, Q, S and T suitably, we get corollaries for two, three and four self maps both for Ćirić type and Hardy Rogers type F-contractions.*
(iii) *It is known that some commutative conditions for self maps are essential to establish common fixed point for more than four maps satisfying contractive type condition. In this paper we have replaced commutative conditions by weak compatibility.*

Remark 3.2. *In all the above results we have neither assumed continuity nor containment of the range space of the involved maps nor completeness of space/subspace. Moreover maps are discontinuous even at the common fixed point. Whereas Batra et al.*[2] *established coincidence point of a pair of self maps by taking containment of range space of involved maps, completeness of space along with continuity and commutativity of both the maps. Weak compatibility used is indeed a weaker than commutativity of a pair of map.*

Remark 3.3. *Since F-contraction is proper generalization of ordinary contraction hence our results generalize, extend and improve the results of Wardowski* [14] *and others existing in literature (for instance Minak et al.*[10]*, Wardowski and Dung*[15]*, Cosentino and Vetro* [6]*, Ćirić*[4]*, Hardy-Rogers*[7]*, Kannan*[9]*, Chatterjee*[3]*, Reich*[12]*, Batra et al.*[2]*) without exploiting the notion of continuity, containment of range space of involved maps and completeness of space/subspace.*

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