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INEXTENSIBLE FLOWS OF CURVES IN THE EQUIFORM GEOMETRY OF THE PSEUDO-GALILEAN SPACE G_3^1

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Abstract. In this paper, we study inextensible flows of curves in 3-dimensional pseudo-Galilean space. We give necessary and sufficient conditions for inextensible flows of curves according to equiform geometry in pseudo-Galilean space.

Keywords: inextensible flows, equiform differential geometry, Galilean space, pseudo-Galilean space.

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1. Introduction

The flow of a curve is said to be inextensible if its arclength is preserved. Physically, the inextensible curve flows give rise to motions in which no strain energy is induced. The flows of inextensible curve and surface are used to solve many problems in computer vision [8], [12], computer animation [1] and even structural mechanics [17]. Especially the methods used in this paper developed by Gage and Hamilton [6] and Grayson [7]. The differentiation between heat flows and inextensible flows of planar curves were elaborated in detail, and some examples of the latter were given by [10]. Also, a general formulation for inextensible flows of curves and developable surfaces in $\mathbb{R}^3$ are exposed by [9]. Latifi et al.[11] studied inextensible flows of curves in Minkowski 3-space. Ogrenmis et al.[13] studied inextensible curves in the Galilean space G_3 , moreover inelastic flows of curves according to equiform in Galilean space given in [18].

The curves and the surfaces in G_3^1 are described in [14, 2, 3, 4]. Theory of curves and the curves of constant curvature in the equiform differential geometry of the isotropic space I_3^1 and I_3^2 and the Galilean space G_3 are described in [15] and [16], respectively. Also, Divjak et al.[5] studied the equiform differential geometry of curves in the pseudo-Galilean space .

In this paper, we investigate inextensible flows of curves in the equiform geometry of the pseudo-Galilean space G_3^1. Then we obtain partial differential equations in terms of inextensible flows of curves according to equiform in G_3^1.

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2. Preliminaries

The pseudo-Galilean geometry is one of the real Cayley-Klein geometries of projective signature (0,0,+,-), explained in [3]. As in [3], pseudo-Galilean inner product can be written as

$$\langle v_1, v_2\rangle = \begin{cases} x_1x_2 & , if\ x_1 \neq 0 \vee x_2 \neq 0 \\ y_1y_2 - z_1z_2 & , if\ x_1 = 0 \wedge x_2 = 0 \end{cases}$$

where $v_1 = (x_1, y_1, z_1)$ and $v_2 = (x_2, y_2, z_2)$. The pseudo-Galilean norm of the vector $v = (x, y, z)$ defined by

$$\|v\| = \begin{cases} x & , if\, x \neq 0 \\ \sqrt{|y^2 - z^2|} & , if\, x = 0 \end{cases}$$

In pseudo-Galilean space a curve is given by $\alpha : I \to G_3^1$

$$\alpha(t) = (x(t), y(t), z(t)) \tag{1}$$

where $I \subseteq \mathbb{R}$ and $x(t), y(t), z(t) \in C^3$. A curve α given by (1) is admissible if $x'(t) \neq 0$ [3].

The curves in pseudo-Galilean space are characterized as follows [2]

An admissible curve in G_3^1 can be parametrized by arclength $t = s$, given in coordinate form

$$\alpha(s) = (s, y(s), z(s)) \ . \tag{2}$$

For an admissible curve $\alpha : I \subseteq \mathbb{R} \to G_3^1$, the curvature $\kappa(s)$ and the torsion $\tau(s)$ are defined by

$$\kappa(x) = \sqrt{|y''^2 - z''^2|}, \tag{3}$$

$$\tau(s) = \frac{1}{\kappa^2(s)} \det\left(\alpha'(s), \alpha''(s), \alpha'''(s)\right). \tag{4}$$

The associated trihedron is given by

$$\begin{aligned} t(s) &= \alpha'(s) = \left(1, y'(s), z'(s)\right) \\ n(s) &= \frac{1}{\kappa(s)}\alpha''(s) = \frac{1}{\kappa(s)}\left(0, y''(s), z''(s)\right) \\ b(s) &= \frac{1}{\kappa(s)}\left(0, z''(s), y''(s)\right) \end{aligned} \tag{5}$$

[4].

The vectors $t(s)$,$n(s)$ and $b(s)$ are called the vectors of tangent, principal normal and binormal line of α, respectively. The curve α given by (2) is timelike if $n(s)$ is spacelike vector. For derivatives of tangent vector $t(s)$, principal normal vector $n(s)$ and binormal vector $b(s)$, respectively, the following Frenet formulas hold

$$\begin{aligned} t'(s) &= \kappa(s)\, n(s), \\ n'(s) &= \tau(s)\, b(s), \\ b'(s) &= \tau(s)\, n(s). \end{aligned} \tag{6}$$

If the admissible curve β is given by $\beta(x) = (x, y(x), 0)$ and for this admissible curve the curvature $\kappa(s)$ and the torsion $\tau(s)$ are defined by

$$\kappa(x) = y''(x)$$

$$\tau(x) = \frac{a_2'(x)}{a_3(x)}$$

where $a(x) = (0, a_2(x), a_3(x))$. The associated trihedron is given by

$$\begin{aligned} t(x) &= \left(1, y'(x), 0\right), \\ n(x) &= (0, a_2(x), a_3(x)), \\ b(x) &= (0, a_3(x), a_2(x)). \end{aligned}$$

For derivatives of tangent vector $t(s)$, principal normal vector $n(s)$ and binormal vector $b(s)$, respectively, the following Frenet formulas hold

$$\begin{aligned} t'(x) &= \kappa(x)(\cosh\phi(x)\, n(x) - \sinh\phi(x)\, b(x)) \\ n'(x) &= \tau(x)\, b(x) \\ b'(x) &= \tau(x)\, n(x) \end{aligned} \tag{7}$$

where ϕ is the angle between $a(x)$ and the plane $z = 0$ [4].

3. Frenet Formulas in Equiform Geometry in G_3^1

Let $\alpha : I \to G_3^1$ be an admissible curve. We define the equiform parameter of α by

$$\sigma := \int \frac{ds}{\rho} = \int \kappa ds \ ,$$

for $\rho = \dfrac{1}{\kappa}$ is the radius of curvature of the curve α. Then

$$\frac{d\sigma}{ds} = \frac{1}{\rho} \quad , i.e., \quad \frac{ds}{d\sigma} = \rho \ . \tag{8}$$

Let h be a homothety with the center in the origin and the coefficient λ. If we put $\tilde{\alpha} = h(\alpha)$, we obtain

$$\tilde{s} = \lambda s \quad and \quad \tilde{\rho} = \lambda\rho$$

where $\tilde{s}$ is the arclength parameter of $\tilde{\alpha}$ and $\tilde{\rho}$ the radius of curvature of this curve. Then, σ is an equiform invariant parameter of α [5].
From now on, we define the Frenet formula of the curve α with respect to the equiform invariant parameter σ in G_3^1. The vector

$$T = \frac{d\alpha}{ds}$$

is called a tangent vector of the curve α in the equiform geometry. Using (5) and (8) we have

$$T = \frac{d\alpha}{ds} \cdot \frac{ds}{d\sigma} = \rho \cdot \frac{d\alpha}{ds} = \rho \cdot t \ . \tag{9}$$

Also, we have the principal normal vector and binormal vector by

$$N = \rho \cdot n \quad , \quad B = \rho \cdot b \ . \tag{10}$$

One can say that the trihedron $\{T, N, B\}$ is an equiform invariant trihedron of the curve α. On the other hand, the derivations of these vectors with respect to σ are given by

$$\begin{aligned} T' &= \frac{dT}{d\sigma} = \dot{\rho}T + N, \\ N' &= \frac{dN}{d\sigma} = \dot{\rho}N + \rho\tau B, \\ B' &= \frac{dB}{d\sigma} = \dot{\rho}B + \rho\tau N. \end{aligned}$$

Definition 3.1. *The function* $\mathbb{K} : I \to \mathbb{R}$ *defined by*

$$\mathbb{K} = \dot{\rho}$$

is called the equiform curvature of the curve α.

Definition 3.2. *The function* $\mathbb{T} : I \to \mathbb{R}$ *defined by*

$$\mathbb{T} = \rho\tau = \frac{\tau}{\kappa}$$

is called the equiform torsion of the curve α.

Then we can write the Frenet formulas in the equiform geometry of the pseudo-Galilean space as follows [5]

$$\begin{aligned} \frac{dT}{d\sigma} &= \mathbb{K}.T + N, \\ \frac{dN}{d\sigma} &= \mathbb{K}.N + \mathbb{T}.B \\ \frac{dB}{d\sigma} &= \mathbb{T}.N + \mathbb{K}.B \end{aligned} \tag{11}$$

4. Inextensible Flows of Curves According to Equiform in pseudo-Galilean Space G_3^1

Throughout this paper, we assume that $F : [0, l] \times [0, w] \to G_3^1$ is a one parameter family of smooth curves in pseudo-Galilean space G_3^1, where l is the arclength of initial curve. Let u be the curve parametrization variable, $0 \leq u \leq l$. We put $v = \left\| \frac{\partial F}{\partial u} \right\|$, from which the arclength of F is defined by $s(u) = \int_0^u v du$.

Also, the operator $\frac{\partial}{\partial s}$ is given in terms of u by $\frac{\partial}{\partial s} = \frac{1}{v}\frac{\partial}{\partial u}$, and the arclength parameter is given by $ds = vdu$.

On the equiform invariant orthonormal frame $\{T, N, B\}$ of a curve α in G_3^1 any flow of F can be written by

$$\frac{\partial F}{\partial t} = fT + gN + hB \tag{12}$$

for f, g, h are the tangential, principal normal, binormal speeds of the curve in G_3^1, respectively. We set $s(u,t) = \int_0^u vdu$, it is called the arclength variation of F. Then, the requirement that the curve not be subject to any elongation or compression can be expressed by the condition

$$\frac{\partial}{\partial s} s(u,t) = \int_0^u \frac{\partial v}{\partial t} du = 0 \ , \tag{13}$$

for all $u \in [0, l]$.

Definition 4.1. *A curve evolution* $F(u,t)$*and its flow* $\frac{\partial F}{\partial t}$ *in pseudo-Galilean space* G_3^1 *are said to be inextensible if*

$$\frac{\partial}{\partial t} \left\| \frac{\partial F}{\partial u} \right\| = 0.$$

Lemma 4.1. *Let* $\dfrac{\partial F}{\partial t} = fT + gN + hB$ *be a smooth flow of the curve* α *in pseudo-Galilean space* G_3^1*. The flow is inextensible if and only if*

$$\frac{\partial v}{\partial t} = \frac{\partial f}{\partial u} + fv\mathbb{K} \tag{14}$$

Proof. Let $\dfrac{\partial F}{\partial t}$ be a smooth flow of the curve F in pseudo-Galilean space G_3^1. From the definition of v, we obtain

$$v\frac{\partial v}{\partial t} = \left\langle \frac{\partial F}{\partial u}, \frac{\partial}{\partial u}\left(fT + gN + hB\right)\right\rangle. \tag{15}$$

From (11) we obtain

$$\frac{\partial v}{\partial t} = \left\langle T, \left(\frac{\partial f}{\partial u} + fv\mathbb{K}\right)T + \left(\frac{\partial g}{\partial u} + fv + hv\mathbb{T} + gv\mathbb{K}\right)N + \left(\frac{\partial h}{\partial u} + gv\mathbb{T} + hv\mathbb{K}\right)B\right\rangle.$$

If we make necessary calculations, we get (14). □

Theorem 4.1. *Suppose that* $\dfrac{\partial F}{\partial t} = fT + gN + hB$ *be a smooth flow of the curve* F *in pseudo-Galilean space* G_3^1*. The flow is inextensible if and only if*

$$\frac{\partial f}{\partial u} = -f\mathbb{K}.$$

Proof. Using (13), we have

$$\frac{\partial}{\partial t}s\left(u,t\right) = \int_0^u \frac{\partial v}{\partial t}du = \int_0^u \left(\frac{\partial f}{\partial u} + fv\mathbb{K}\right) = 0\ . \tag{16}$$

Substituting (14) in (16) complete the proof of the theorem. □

Suppose that, $v = 1$ and the local coordinate u corresponds to the curve arc length s.

Lemma 4.2. *Let* $\dfrac{\partial F}{\partial t} = fT + gN + hB$ *be a smooth flow of the curve* α *in pseudo-Galilean space* G_3^1*. Using (11) we have*
i) if $\langle N, N\rangle = -1$ *and* $\langle B, B\rangle = 1$ *we have*

$$\frac{\partial T}{\partial t} = \left(\frac{\partial g}{\partial s} + f + g\mathbb{K} + h\mathbb{T}\right)N + \left(\frac{\partial h}{\partial s} + g\mathbb{T} + h\mathbb{K}\right)B \tag{17}$$

$$\frac{\partial N}{\partial t} = \left(\frac{\partial g}{\partial s} + f + g\mathbb{K} + h\mathbb{T}\right)T + \varphi B \tag{18}$$

$$\frac{\partial B}{\partial t} = -\left(\frac{\partial h}{\partial s} + g\mathbb{T} + h\mathbb{K}\right)T + \varphi N \tag{19}$$

ii) if $\langle N, N\rangle = 1$ *and* $\langle B, B\rangle = -1$ *we have*

$$\frac{\partial T}{\partial t} = \left(\frac{\partial g}{\partial s} + f + g\mathbb{K} + h\mathbb{T}\right)N + \left(\frac{\partial h}{\partial s} + g\mathbb{T} + h\mathbb{K}\right)B \tag{20}$$

$$\frac{\partial N}{\partial t} = -\left(\frac{\partial g}{\partial s} + f + g\mathbb{K} + h\mathbb{T}\right)T - \varphi B \tag{21}$$

$$\frac{\partial B}{\partial t} = \left(\frac{\partial h}{\partial s} + g\mathbb{T} + h\mathbb{K}\right)T - \varphi N \tag{22}$$

where $\varphi = \left\langle \dfrac{\partial N}{\partial t}, B\right\rangle$ *provided that* $\left(\dfrac{\partial g}{\partial s} + f + g\mathbb{K} + h\mathbb{T}\right) = 0.$

Proof. Using definition of F, we have

$$\frac{\partial T}{\partial t}=\frac{\partial}{\partial t}\frac{\partial F}{\partial s}=\frac{\partial}{\partial s}\left(fT+gN+hB\right).$$

Using (11), we have

$$\frac{\partial T}{\partial t}=\left(\frac{\partial f}{\partial s}+f\mathbb{K}\right)T+\left(\frac{\partial g}{\partial s}+f+g\mathbb{K}+h\mathbb{T}\right)N+\left(\frac{\partial h}{\partial s}+g\mathbb{T}+h\mathbb{K}\right)B \tag{23}$$

On the other hand using theorem 4.1. in (23), we get

$$\frac{\partial T}{\partial t}=\left(\frac{\partial g}{\partial s}+f+g\mathbb{K}+h\mathbb{T}\right)N+\left(\frac{\partial h}{\partial s}+g\mathbb{T}+h\mathbb{K}\right)B.$$

i) Now we differentiate the Frenet frame by t:

$$\begin{aligned}
0 &= \frac{\partial}{\partial t}\langle T,N\rangle=-\left(\frac{\partial g}{\partial s}+f+g\mathbb{K}+h\mathbb{T}\right)+\left\langle T,\frac{\partial N}{\partial t}\right\rangle\\
0 &= \frac{\partial}{\partial t}\langle T,B\rangle=\left(\frac{\partial h}{\partial s}+g\mathbb{T}+h\mathbb{K}\right)+\left\langle T,\frac{\partial B}{\partial t}\right\rangle\\
0 &= \frac{\partial}{\partial t}\langle N,B\rangle=\varphi+\left\langle N,\frac{\partial B}{\partial t}\right\rangle
\end{aligned}$$

Then, a straight forward computation using above system gives

$$\begin{aligned}
\frac{\partial N}{\partial t} &= \left(\frac{\partial g}{\partial s}+f+g\mathbb{K}+h\mathbb{T}\right)T+\varphi B\\
\frac{\partial B}{\partial t} &= -\left(\frac{\partial h}{\partial s}+g\mathbb{T}+h\mathbb{K}\right)T+\varphi N
\end{aligned}$$

where $\varphi=\left\langle\frac{\partial N}{\partial t},B\right\rangle$ provided that $\left(\frac{\partial g}{\partial s}+f+g\mathbb{K}+h\mathbb{T}\right)=0$. So, we obtain the lemma 4.2. (i). □

ii) Now we differentiate the Frenet frame by t:

$$\begin{aligned}
0 &= \frac{\partial}{\partial t}\langle T,N\rangle=\left(\frac{\partial g}{\partial s}+f+g\mathbb{K}+h\mathbb{T}\right)+\left\langle T,\frac{\partial N}{\partial t}\right\rangle\\
0 &= \frac{\partial}{\partial t}\langle T,B\rangle=-\left(\frac{\partial h}{\partial s}+g\mathbb{T}+h\mathbb{K}\right)+\left\langle T,\frac{\partial B}{\partial t}\right\rangle\\
0 &= \frac{\partial}{\partial t}\langle N,B\rangle=\varphi+\left\langle N,\frac{\partial B}{\partial t}\right\rangle
\end{aligned}$$

Then, a straight forward computation using above system gives

$$\begin{aligned}
\frac{\partial N}{\partial t} &= -\left(\frac{\partial g}{\partial s}+f+g\mathbb{K}+h\mathbb{T}\right)T-\varphi B\\
\frac{\partial B}{\partial t} &= \left(\frac{\partial h}{\partial s}+g\mathbb{T}+h\mathbb{K}\right)T-\varphi N
\end{aligned}$$

where $\varphi=\left\langle\frac{\partial N}{\partial t},B\right\rangle$ provided that $\left(\frac{\partial g}{\partial s}+f+g\mathbb{K}+h\mathbb{T}\right)=0$. Thus, we obtain the lemma 4.2. (ii).

Theorem 4.2. *Let* $\dfrac{\partial F}{\partial t}$ *be inextensible. Then, using (11), the following system of partial differential equations holds:*
i) if $\langle N,N\rangle = -1$ *and* $\langle B,B\rangle = 1$ *we have*

$$\begin{aligned}
\frac{\partial \mathbb{K}}{\partial t} &= 0\\
\frac{\partial \mathbb{T}}{\partial t} &= -\left(\frac{\partial h}{\partial s} + g\mathbb{T} + h\mathbb{K}\right) + \frac{\partial \varphi}{\partial s}\\
\varphi &= \left(\frac{\partial^2 h}{\partial s^2} + \frac{\partial}{\partial s}(g\mathbb{T}) + \frac{\partial}{\partial s}(h\mathbb{K})\right)
\end{aligned}$$

ii) if $\langle N,N\rangle = 1$ *and* $\langle B,B\rangle = -1$ *we have*

$$\begin{aligned}
\frac{\partial \mathbb{K}}{\partial t} &= 0\\
\frac{\partial \mathbb{T}}{\partial t} &= \left(\frac{\partial h}{\partial s} + g\mathbb{T} + h\mathbb{K}\right) - \frac{\partial \varphi}{\partial s}\\
\varphi &= -\left(\frac{\partial^2 h}{\partial s^2} + \frac{\partial}{\partial s}(g\mathbb{T}) + \frac{\partial}{\partial s}(h\mathbb{K})\right)
\end{aligned}$$

where $\varphi = \left\langle \dfrac{\partial N}{\partial t}, B\right\rangle$ *provided that* $\left(\dfrac{\partial g}{\partial s} + f + g\mathbb{K} + h\mathbb{T}\right) = 0.$

Proof. i) Using (17), we have

$$\begin{aligned}
\frac{\partial}{\partial s}\frac{\partial T}{\partial t} &= \frac{\partial}{\partial s}\left[\left(\frac{\partial g}{\partial s} + f + g\mathbb{K} + h\mathbb{T}\right)N + \left(\frac{\partial h}{\partial s} + g\mathbb{T} + h\mathbb{K}\right)B\right]\\
&= \left[\left(\frac{\partial^2 g}{\partial s^2} + \frac{\partial}{\partial s}(f) + \frac{\partial}{\partial s}(g\mathbb{K}) + \frac{\partial}{\partial s}(h\mathbb{T})\right)\right]N\\
&+ \left(\frac{\partial g}{\partial s} + f + g\mathbb{K} + h\mathbb{T}\right)(\mathbb{K}\mathrm{N} + \mathbb{T}\mathrm{B})\\
&+ \left(\frac{\partial^2 h}{\partial s^2} + \frac{\partial}{\partial s}(g\mathbb{T}) + \frac{\partial}{\partial s}(h\mathbb{K})\right)B + \left(\frac{\partial h}{\partial s} + g\mathbb{T} + h\mathbb{K}\right)(\mathbb{T}N + \mathbb{K}B)
\end{aligned}$$

On the other hand, from (11), we have

$$\begin{aligned}
\frac{\partial}{\partial s}\frac{\partial T}{\partial t} &= \frac{\partial}{\partial t}(\mathbb{K}T + N)\\
&= \frac{\partial \mathbb{K}}{\partial t}T + \mathbb{K}\left(\frac{\partial g}{\partial s} + f + g\mathbb{K} + h\mathbb{T}\right)N + \mathbb{K}\left(\frac{\partial h}{\partial s} + g\mathbb{T} + h\mathbb{K}\right)B\\
&= \left(\frac{\partial g}{\partial s} + f + g\mathbb{K} + h\mathbb{T}\right)T + \varphi B
\end{aligned}$$

Hence we see that

$$\frac{\partial \mathbb{K}}{\partial t} = 0\ ,$$

and

$$\varphi = \left(\frac{\partial^2 h}{\partial s^2} + \frac{\partial}{\partial s}(g\mathbb{T}) + \frac{\partial}{\partial s}(h\mathbb{K})\right)\ .$$

where $\varphi = \left\langle \dfrac{\partial N}{\partial t}, B\right\rangle$ provided that $\left(\dfrac{\partial g}{\partial s} + f + g\mathbb{K} + h\mathbb{T}\right) = 0.$

Also, we have from lemma 4.2.(i)

$$\begin{aligned}
\frac{\partial}{\partial s}\frac{\partial B}{\partial t} &= \frac{\partial}{\partial s}\left[-\left(\frac{\partial h}{\partial s}+g\mathbb{T}+h\mathbb{K}\right)T+\varphi N\right] \\
&= -\left(\frac{\partial^2 h}{\partial s^2}+\frac{\partial}{\partial s}(g\mathbb{T})+\frac{\partial}{\partial s}(h\mathbb{K})\right)T-\left(\frac{\partial h}{\partial s}+g\mathbb{T}+h\mathbb{K}\right)(\mathbb{K}T+N) \\
&+ \frac{\partial \varphi}{\partial s}N+\varphi(\mathbb{K}N+\mathbb{T}B)
\end{aligned}$$

On the other hand, from (11), we have

$$\begin{aligned}
\frac{\partial}{\partial t}\frac{\partial B}{\partial s} &= \frac{\partial}{\partial t}(\mathbb{T}N+\mathbb{K}B) \\
&= \left(\frac{\partial \mathbb{T}}{\partial t}+\mathbb{K}\varphi\right)N+\left[\mathbb{T}\left(\frac{\partial g}{\partial s}+f+g\mathbb{K}+h\mathbb{T}\right)-\mathbb{K}\left(\frac{\partial h}{\partial s}+g\mathbb{T}+h\mathbb{K}\right)\right] \\
&= \left(\frac{\partial \mathbb{K}}{\partial t}+\mathbb{T}\varphi\right)B
\end{aligned}$$

Hence we see that

$$\frac{\partial \mathbb{T}}{\partial t}=-\left(\frac{\partial h}{\partial s}+g\mathbb{T}+h\mathbb{K}\right)+\frac{\partial \varphi}{\partial s}\ .$$

ii) Using (20), we have

$$\begin{aligned}
\frac{\partial}{\partial s}\ \frac{\partial T}{\partial t} &= \frac{\partial}{\partial s}\left[\left(\frac{\partial g}{\partial s}+f+g\mathbb{K}+h\mathbb{T}\right)N+\left(\frac{\partial h}{\partial s}+g\mathbb{T}+h\mathbb{K}\right)B\right] \\
&= \left[\left(\frac{\partial^2 g}{\partial s^2}+\frac{\partial}{\partial s}(f)+\frac{\partial}{\partial s}(g\mathbb{K})+\frac{\partial}{\partial s}(h\mathbb{T})\right)\right]N+\left(\frac{\partial g}{\partial s}+f+g\mathbb{K}+h\mathbb{T}\right)(\mathbb{K}N+\mathbb{T}B) \\
&+ \left(\frac{\partial^2 h}{\partial s^2}+\frac{\partial}{\partial s}(g\mathbb{T})+\frac{\partial}{\partial s}(h\mathbb{K})\right)B+\left(\frac{\partial h}{\partial s}+g\mathbb{T}+h\mathbb{K}\right)(\mathbb{T}N+\mathbb{K}B)
\end{aligned}$$

On the other hand, from (11), we have

$$\begin{aligned}
\frac{\partial}{\partial s}\ \frac{\partial T}{\partial t} &= \frac{\partial}{\partial t}(\mathbb{K}T+N) \\
&= \frac{\partial \mathbb{K}}{\partial t}T+\mathbb{K}\left(\frac{\partial g}{\partial s}+f+g\mathbb{K}+h\mathbb{T}\right)N+\mathbb{K}\left(\frac{\partial h}{\partial s}+g\mathbb{T}+h\mathbb{K}\right)B \\
&- \left(\frac{\partial g}{\partial s}+f+g\mathbb{K}+h\mathbb{T}\right)T-\varphi B
\end{aligned}$$

Hence we see that

$$\frac{\partial \mathbb{K}}{\partial t}=0\ ,$$

and

$$\varphi=-\left(\frac{\partial^2 h}{\partial s^2}+\frac{\partial}{\partial s}(g\mathbb{T})+\frac{\partial}{\partial s}(h\mathbb{K})\right)\ .$$

where $\varphi=\left\langle\frac{\partial N}{\partial t},B\right\rangle$ provided that $\left(\frac{\partial g}{\partial s}+f+g\mathbb{K}+h\mathbb{T}\right)=0$

Also, we have from lemma 4.2.(ii)

$$\begin{aligned}\frac{\partial}{\partial s}\frac{\partial B}{\partial t} &= \frac{\partial}{\partial s}\left[\left(\frac{\partial h}{\partial s}+g\mathbb{T}+h\mathbb{K}\right)T-\varphi N\right]\\ &= \left(\frac{\partial^2 h}{\partial s^2}+\frac{\partial}{\partial s}(g\mathbb{T})+\frac{\partial}{\partial s}(h\mathbb{K})\right)T+\left(\frac{\partial h}{\partial s}+g\mathbb{T}+h\mathbb{K}\right)(\mathbb{K}T+N)\\ &- \frac{\partial \varphi}{\partial s}N-\varphi(\mathbb{K}N+\mathbb{T}B)\end{aligned}$$

On the other hand, from (11), we have

$$\begin{aligned}\frac{\partial}{\partial t}\frac{\partial B}{\partial s} &= \frac{\partial}{\partial t}(\mathbb{T}N+\mathbb{K}B)\\ &= \left(\frac{\partial \mathbb{T}}{\partial t}-\mathbb{K}\varphi\right)N+\left[-\mathbb{T}\left(\frac{\partial g}{\partial s}+f+g\mathbb{K}+h\mathbb{T}\right)+\mathbb{K}\left(\frac{\partial h}{\partial s}+g\mathbb{T}+h\mathbb{K}\right)\right]\\ &= \left(\frac{\partial \mathbb{K}}{\partial t}-\mathbb{T}\varphi\right)B\end{aligned}$$

Hence we see that

$$\frac{\partial \mathbb{T}}{\partial t}=\left(\frac{\partial h}{\partial s}+g\mathbb{T}+h\mathbb{K}\right)-\frac{\partial \varphi}{\partial s}\ .$$

Thus, we prove the theorem. □

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