TWMS J. App. Eng. Math. V.6, N.2, 2016, pp. 315-323

SOME RESULTS ON THE DISTANCE r-b-COLORING IN GRAPHS

G. JOTHILAKSHMI [1], A. P. PUSHPALATHA [1], S. SUGANTHI [2], V. SWAMINATHAN [3], §

Abstract. Given a positive integer r, two vertices $u, v \in V(G)$ are r- independent if $d(u, v) > r$. A partition of $V(G)$ into r-independent sets is called a distance r-coloring. A study of distance r-coloring and distance r-b-coloring concepts are studied in this paper.

Keywords: distance r-chromatic number, distance r-independent color partition, distance r-independent set, distance r-b-coloring number.

AMS Subject Classification: 05C69

1. Introduction

Consider a network and a coloring scheme for the nodes. Two nodes are compatible if they receive the same color. The usual coloring scheme stipulates that adjacent nodes should not receive the same color. Such a scheme is helpful in storage problems of chemicals where two noncompatible chemicals (two chemicals which, when placed nearby, will cause danger) cannot be stored in the same room. The chromatic number of such a scheme will give the minimum number of storage spaces required for keeping all the chemicals without any problem.

Two important aspects of graphs are partitions of vertex and edge sets into sets with prescribed properties. The first gives rise to different types of colorings and the second leads to decomposition in graphs. Various colorings starting from proper coloring have been defined and studied.
In this paper, we introduce a new coloring based on the distance. Given a positive integer r , two vertices $u, v \in V(G)$ are r-adjacent if u, v are adjacent in G^r and are r-independent if they are independent in G^r. A partition of $V(G)$ into r-independent sets is called a distance r-coloring. These are same as proper coloring in G^r. The chromatic number of G^r will coincide with the distance r-chromatic number of G. Variations of distance r-coloring like distance r-dominator coloring and distance $r - b$-coloring are also discussed.

Definition 1.1. *For any integer $r \geq 1$, a graph $G = (V, E)$ is said to be r-complete if every vertex in $V(G)$ is r-adjacent to every other vertex in $V(G)$. The maximum cardinality of a subset S of $V(G)$ such that $\langle S \rangle$ is r-complete is called r-clique number of G and is denoted by $\omega_r(G)$.*

[1] Department of Mathematics, Thiagarajar College of Engineering, Madurai, India.
e-mail: gjlmat@tce.edu, appmat@tce.edu;
[2] e-mail: ss_revathi@yahoo.co.in;
[3] Ramanujan Research Centre, Saraswathy Narayanan College, Madurai, India.
e-mail: swaminathan.sulanesri@gmail.com;
§ Manuscript received: February 01, 2016; accepted: March 12, 2016.

Definition 1.2. *Let $r \geq 1$. Let $u, v \in V(G)$. A vertex u distance r-dominates a vertex v if $d(u, v) \leq r$.*

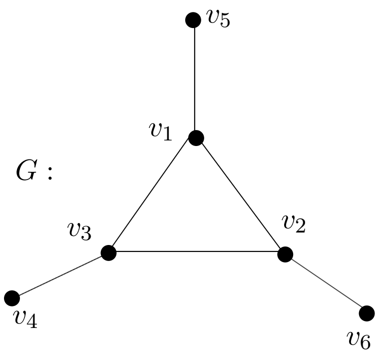
Definition 1.3. *A subset I of $V(G)$ is distance r-independent set if for any $u, v \in I$, $d(u, v) > r$. The maximum cardinality of a distance r-independent set of G is called the distance r-independence number of G and is denoted by $\beta_r(G)$.*

Definition 1.4. [14] *A partition of $V(G)$ is called distance r-independent color partition of G if each element of the partition is distance r-independent. The minimum cardinality of a distance r-independent color partition of G is called distance r-chromatic number and is denoted by $\chi_r(G)$.*

Remark 1.1. *Let G be a simple graph. Let $V(G) = \{v_1, v_2, \ldots v_n\}$. Let $\pi = \{\{v_1\}, \{v_2\}, \cdots, \{v_n\}\}$ be a distance r-color partition of a graph G. Therefore existence of distance r-color partition of any graph is guaranteed.*

Remark 1.2. *Any distance r-color partition of G is a proper color partition of G but not the other way.*

For example,



$\{v_1, v_6\}$, $\{v_3, v_5\}$,$\{v_2, v_4\}$ is a proper color partition of G, but it is not a distance 2-color partition of G.

Lemma 1.1. *For any graph G, $\chi(G) \leq \chi_r(G)$.*

$\chi_r(G)$ for standard graphs:

(1) $\chi_r(\overline{K_n}) = 1$, for all r.
(2) $\chi_r(K_n) = n$, for all r.
(3) $\chi_r(K_{1,n}) = \begin{cases} 2, \text{ if } r = 1 \\ n+1, \text{ if } r \geq 2. \end{cases}$
(4) $\chi_r(W_n) = \begin{cases} 3, & \text{if } r = 1 \text{ and } n \text{ is odd} \\ 2, & \text{if } r = 1 \text{ and } n \text{ is even} \\ n, \text{ if } r \geq 2. \end{cases}$
(5) $\chi_r(K_{m,n}) = \begin{cases} 2, \text{ if } r = 1 \\ m+n, \text{ if } r \geq 2. \end{cases}$
(6) $\chi_r(P_n) = \begin{cases} r+1, \text{ if } 1 \leq r < n-1 \\ n, \text{ if } r \geq n-1. \end{cases}$

(7) $\chi_r(C_n) = \begin{cases} If\ r < \lfloor n/2 \rfloor \\ r+1, \text{ if } n \equiv 0\ mod(r+1) \\ r+2,\ otherwise \\ n, \text{if } r \geq \lfloor n/2 \rfloor \end{cases}$

(8) Let $D_(m,n)$ be a double star (with $m < n$).

$$\chi_r(D_{m,n}) = \begin{cases} 2, \text{ if } r = 1 \\ m+n+2, \text{ if } r \geq 3 \\ n+2, \text{ if } r \geq 2. \end{cases}$$

Lemma 1.2. *If G has diameter $\leq$ 2, then* $\chi_r(G) = \begin{cases} n,\ if\ \ r \geq 2 \\ \chi(G),\ if\ \ r = 1 \end{cases}$

Lemma 1.3. *For any graph G, 1 $\leq$ $\chi_r(G)$ $\leq$ n.*

Theorem 1.1. $\chi_r(G)$ *= 1 if and only if* $G = \overline{K_n}$.

Proof:

Let $\chi_r(G) = 1$. Suppose G is connected. Since $\chi_r(G) = 1$, any two vertices of G are at a distance $>\ r \geq 1$. Therefore any two vertices of G are at a distance > 1, which is a contradiction. Therefore G is disconnected. Suppose $G_1, G_2, G_3, \cdots, G_t$ are the connected components of G.

Suppose $|V(G_i)| \geq 2$. Arguing as before, we get a contradiction. Therefore $|V(G_i)| = 1$, for all i. Therefore G is totally disconnected. The Converse is obvious.

Theorem 1.2. *For any graph G, $\chi_r(G) = n$ if and only if $r \geq diam(G)$.*

Proof:

If $r \geq diam(G)$, then $\chi_r(G) = n$. Suppose $\chi_r(G) = n$ and $r < diam(G)$. Let u, v be two vertices of G such that $d(u, v) = diam(G) >\ r$. Then $\{u, v\}$ is a distance-r-independent set and so $\chi_r(G) \leq n - 1$, a contradiction. Therefore $r \geq$ diam(G).

Theorem 1.3. *Let G be a graph of order n.Then* $\frac{n}{\beta_r(G)} \leq \chi_r(G) \leq n - \beta_r(G) + 1$.

Proof:

Let $\chi_r(G) = s$. Let $\pi = \{V_1, V_2, \ldots V_s\}$ be a χ_r-partition of $V(G)$. Therefore $\beta_r(G) \geq |V_i|$, for all i,$(1 \leq i \leq s)$. Now $n = |V_1| + |V_2| + \ldots + |V_s| \leq s\beta_r(G)$. Hence $\beta_r(G)\chi_r(G) \geq n$. Let D be a β_r-set of G. Let $D = \{u_1, u_2, \ldots, u_{\beta_r(G)}\}$. Let $\pi = \{D, \{u_{\beta_r+1}\}, \ldots, \{u_n\}\}$. Then π is a distance r-color partition of G. Therefore $|\pi| \geq \chi_r(G)$.

That is $n - \beta_r(G) + 1 \geq \chi_r(G)$.

Corollary 1.1. *Let G be a graph of order n. Then* $2\sqrt{n} \leq \beta_r(G) + \chi_r(G) \leq n + 1$.

Remark 1.3. *For any graph G,* $\omega_r(G) \leq \chi_r(G) \leq 1 + \Delta_r(G)$.

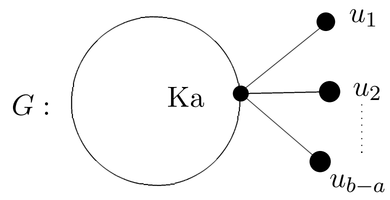
Remark 1.4. *For any graph G, if $r = diam(G)$, then* $\chi_r(G) = 1 + \Delta_r(G)$.

Proof: If $r = diam(G)$, then every vertex of G is r-adjacent to every other vertex of G. Hence $\chi_r(G) = n = 1 + \Delta_r(G)$.

Proposition 1.1. *Given positive integers a, b, and r with $a \leq b$, there exists a connected graph G such that $\chi(G) = a$, $\chi_r(G) = b$.*

Proof:

Case 1: $a = b$. For any $r \geq 1$. Then $\chi(K_a) = \chi_r(K_a) = a = b$.



Cases 2: $a < b$. Then $r \geq 2$. Attach $(b-a)$ pendent vertices at a vertex of K_a. Let G be the resulting graph. Then $\chi(G) = a$ and $\chi_r(G) = b$.

Proposition 1.2. *Let G be a connected graph of order ≥ 3 which is non complete. Let G have a full degree vertex. Then $\chi(G) \neq \chi_r(G)$, for all $r \geq 2$.*

Proof:
Let G be a connected graph of order ≥ 3 which is non complete. Let G have a full degree vertex. Then $diam(G) \leq 2$ and hence $\chi_r(G) = |V(G)|$ and $\chi(G) < n$. Therefore $\chi(G) < \chi_r(G)$. That is, $\chi(G) \neq \chi_r(G)$, for all $r \geq 2$. Suppose G has no full degree vertex.

Theorem 1.4. *For any spanning subgraph H of G, $\chi_r(H) \leq \chi_r(G)$.*

Theorem 1.5. *Suppose G is disconnected. Let $r \geq 2$. Then $\chi_r(G) = \chi(G)$ if and only if there exists a component of G which is complete and whose distance r-chromatic number is $\chi_r(G)$.*

Proof:
Let G be disconnected. Let $G_1, G_2, \ldots, G_k$ be the components of G. Let $r \geq 2$ and $\chi(G) = \chi_r(G)$. $\chi(G) = \max\limits_{1 \leq i \leq k}\{\chi(G_i)\} = \chi(G)$ say $\chi_r(G) = \max\limits_{1 \leq i \leq k}\{\chi_r(G_i)\} = \chi(G_1)$. Without loss of generality,
let $\max\limits_{1 \leq i \leq k}\{\chi_r(G_i)\} = \chi_r(G_s)$. Then $\chi_r(G_s) = \chi(G_1)$. But, $\chi_r(G_1) \geq \chi(G_1) = \chi_r(G_s) \geq \chi_r(G_1)$. Therefore $\chi_r(G_1) = \chi_r(G_s)$ and hence $\chi_r(G_1) = \chi(G_1)$. Therefore G_1 is complete and $\chi(G_1) = \chi(G) = \chi_r(G)$. Conversely, let G be disconnected and $r \geq 2$.

Let G contain a component, say G_1 which is complete and $\chi_r(G_1) = \chi_r(G)$. Since G_1 is complete, $\chi(G_1) = \chi_r(G_1) = \chi_r(G)$. Suppose $\chi(G_1) < \chi(G_s) = \chi(G)$. Then $\chi_r(G) < \chi(G_s) = \chi(G)$, a contradiction. Thus $\chi(G_1) \geq \chi(G_s)$, for all s and so $\chi(G_1) = \chi(G)$.

That is , $\chi_r(G) = \chi(G)$.

2. Distance $r-b$-Coloring in Graphs

Definition 2.1. *A distance-r-color partition is called a distance-r-b-color partition if for every color class contains at least one vertex which is at a distance less than or equal to r from each of the remaining color classes. Any χ_r-partition is a distance-r-b-color partition. The maximum cardinality of a distance-r-b-color partition is called* **distance-r-b-chromatic number** *and is denoted by $\phi_{\mathbf{r}}(\mathbf{G})$. Clearly, $\chi_r(G) \leq \phi_r(G)$.*

Definition 2.2. $\Delta_r(G) = max\{deg_r(v)\}$, *where $deg_r(v) = |\{v \in V(G) : d(u,v) \leq r\}|$. Let $v_1, v_2, \cdots, v_n$ be arranged such that $deg_r(v_1) \geq deg_r(v_2) \geq \cdots \geq deg_r(v_n)$. Let $m_r(G) = max\ \{i : deg_r(v_i) \geq i-1\}$.*

Theorem 2.1. *For any graph G, $\chi_r(G) \leq \phi_r(G) \leq m_r(G)$.*

Proof:
Let $v_1, v_2, \cdots, v_n$ be arranged such that $deg_r(v_1) \geq deg_r(v_2) \geq \cdots \geq deg_r(v_n)$. Let $m_r(G) = \max\ \{i : d_r(v_i) \geq i-1\}$. Then there exist a set of $m_r(G)$ vertices of (G), each

with r-degree $\geq m_r(G)-1$ and the remaining vertices have r-degree $< m_r(G)-1$. Suppose $\phi_r(G) > m_r(G)$. Let $\pi = \{V_1, V_2, \cdots, V_{\phi_r(G)}\}$ be a color partition of G. Then there exist a color class in which the r-degree of each vertex is $\leq m_r(G)-1$. For: Suppose not, In each color class, there exist a vertex whose r-degree is $> m_r(G)-1$. Therefore there exist at least $\phi_r(G)$ vertices whose r-degree is $> m_r(G)-1$. That is there exist more than $m_r(G)$ vertices whose r-degree is $> m_r(G)-1$, a contradiction. Hence there exist a color class say, V_j in which the r-degree of each vertex is $\leq m_r(G)-1$, which implies that there exist a vertex
$u \in V_j$ which r-dominates at least one vertex from each of the remaining color classes. Therefore $deg_r(u) \geq \phi_r(G)-1 \geq m_r(G)$, a contradiction. Therefore $\phi_r(G) \leq m_r(G)$. Therefore $\chi_r(G) \leq \phi_r(G) \leq m_r(G)$.

Lemma 2.1. *For any graph G, $m_r(G) \leq 1+\Delta_r(G)$.*

Proof: Suppose $m_r(G) > 1+\Delta_r(G)$. Then there exist a set of $m_r(G)$ vertices each with r-degree $\geq m_r(G)-1 > \Delta_r(G)$, a contradiction. Therefore $m_r(G) \leq 1+\Delta_r(G)$.

Lemma 2.2. *$\chi_r(G) \leq \phi_r(G) \leq m_r(G) \leq 1+\Delta_r(G)$. Let $G = K_{m,n}$ and $r = 2$. Then $\chi_2(K_{m,n}) = m+n$. $1+\Delta_2(G) = m+n$. $\phi_2(G) = m+n$. Therefore $\phi_2(G) = 1+\Delta_2(G)$.*

$\phi_r(G)$ for standard graphs

(1) $\phi_r(K_n) = n \quad for\ all\ r$

(2) $\phi_r(K_{m,n}) = \begin{cases} 2, \text{if } \ r=1 \\ m+n, \text{if } \ r \geq 2 \end{cases}$

(3) $\phi_r(K_{1,n}) = \begin{cases} 2, \quad\quad \text{if } \ r=1 \\ n+1, \text{if } \ r \geq 2 \end{cases}$

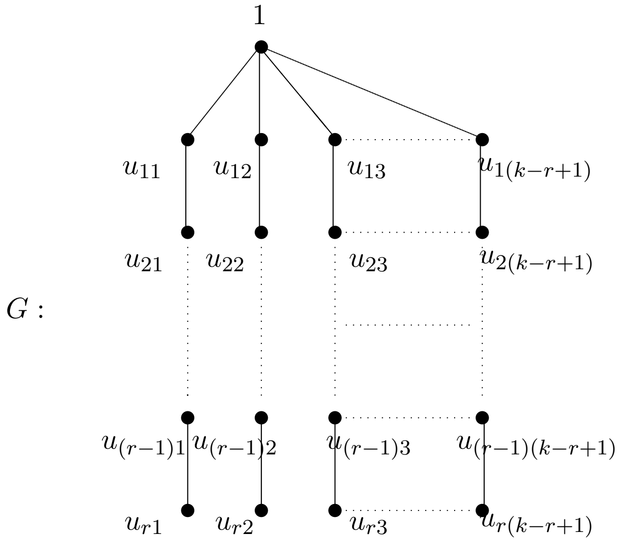
(4) $\phi_r(W_n) = \begin{cases} 3, \text{ if } \ \ n\ is\ odd\ and\ if\ r=1 \\ 4, \text{ if } \ \ n\ is\ even\ and\ if\ r=1 \\ n, \ \ \text{if } r \geq 2 \end{cases}$

(5) $\phi_r(P_n) = \begin{cases} n, \text{ if } \ \ n \ \leq r+1 \\ r+1+\lfloor (n-1)/(2r+1) \rfloor, \text{ if } \ \ r+2 \leq n \leq 4r+1 \\ r+2, \ \text{ if } n \geq 4r+2 \end{cases}$

(6) $\phi_r(C_n) = \begin{cases} n, \text{ if } \ n \leq \ 2r+1 \\ (r+1)+\left\lfloor \frac{n-2}{2r+1} \right\rfloor, \text{ if } \ \ n = 2r+2 \\ r+2, \ \text{ if } \ n \geq 2r+3 \end{cases}$

Proposition 2.1. *Given positive integers k, r with $k > r$, there exists a connected graph G with $\phi_r(G) = k$.*

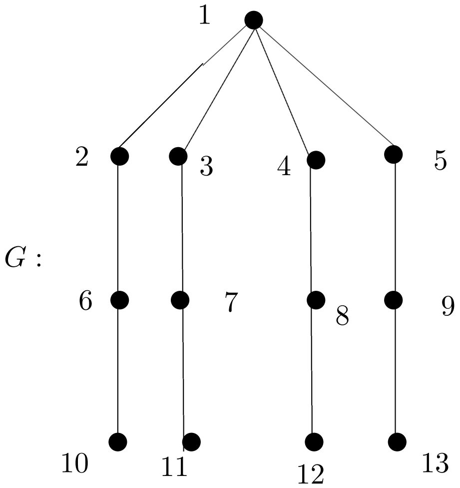
Proof:
Consider $K_{(1,k-r+1)}$. Attach a path of length $r-2$ at each of the pendent vertices of $K_{(1,k-r+1)}$. Let G be the resulting graph. Then $\phi_r(G) = (r-2)+1+(k-r+1) = k$.



$K_{(r,t)}$-free graphs

Definition 2.3. *Consider $K_{(1,t)}$. Attach a path of length $r-1$ at each of the pendant vertices of $K_{(1,t)}$. The resulting graph is called (r,t)-***claw***and is denoted by $K_{(r,t)}$.*
$G = K_{(3,4)}$ is given below.

Example 2.1.



Remark 2.1. $\beta_r(K_{(r,t)}) = t$.

Theorem 2.2. *Let G be a $K_{(r,t)}$-free graph, where $t \geq 3$. Then $\phi_r(G) \leq (t-1)(\chi_r(G)-1)+1$.*

Proof:

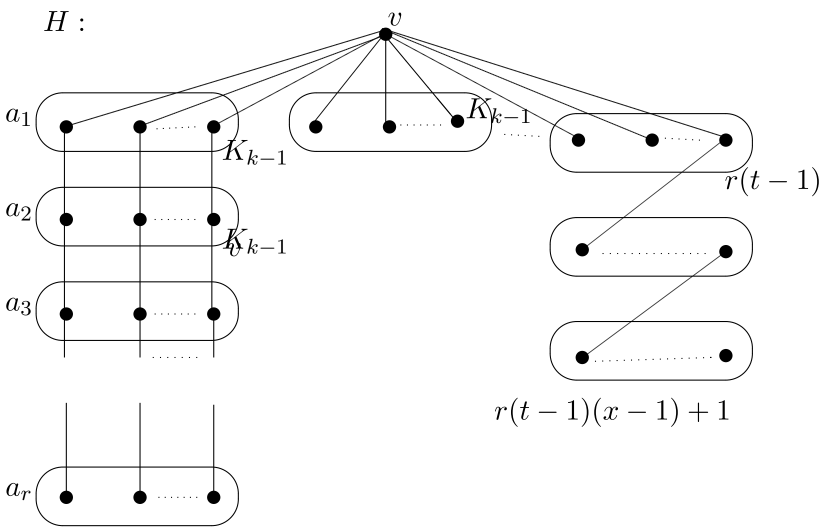
Let $\pi = \{V_1, V_2, \dots, V_{(\phi_r)}\}$ be a $\phi_r(G)$-coloring partition of G. Then each V_i contains a vertex say x_i which is at a distance $\leq r$ to each of the color classes V_j, $j \neq i$, $1 \leq j \leq \phi_r(G)$. Let $y_1, y_2, \dots, y_{(\phi_r)-1}$ be the vertices which are at a distance $\leq r$ from x_i and having $(\phi_r(G)-1)$ colors. Let $S = \{y_1, y_2, \dots, y_{(\phi_r-1)}\}$. Let $H = \langle S \rangle$. $\phi_r(H) \leq (t-1)$. Since $\frac{n}{\beta_r(G)} \leq \chi_r(G)$, $|V(H)| \leq (t-1)\chi_r(H)$. Therefore $(\phi_r(G)-1) \leq (t-1)(\chi_r(G)-1)$. Hence $\phi_r(G) \leq (t-1)(\chi_r(G)-1)+1$.

The following proposition shows that the bound is sharp for each t.

Proposition 2.2. *For any integer $t \geq 3$ and k, there exist a $K_{(r,t)}$-free graph G such that $\chi_r(G) = r(k-1)+1$ and*
$\phi_r(G) = (\chi_r(G)-1)(t-1)+1$.

Proof:

Let H be the graph constructed as follows:



Consider the vertex v. Let the neighbours of v form $r(t-1)$-mutually disjoint cliques each with $(k-1)$-vertices. Subdivide each of these arms with $r-1$-vertices such that the subdivided point at the i-th level $1 \leq i \leq r$ form a clique. Take $r(t-1)(k-1)+1$ disjoint copies of H and connect them sequentially by exactly one edge between any two consecutive copies such that these edges are not incident with v or any of the images of v. Let G be the resulting graph.

Then $\chi_r(H) = r(k-1)+1$ and $\phi_r(H) = r(k-1)(t-1)+1 = (\chi_r(H)-1)(t-1)+1$.

Proposition 2.3. *For any spanning subgraph H of G, $\phi_r(G) \geq \phi_r(H)$.*

Theorem 2.3. *For any vertex v of a graph G, $\phi_r(G-\{v\}) \leq \phi_r(G) \leq \phi_r(G-\{v\})+1$.*

Proof:

Let $\pi = \{V_1, V_2, \ldots V_k\}$ be a ϕ_r-coloring of $G-\{v\}$. If v is r-adjacent to each color class V_i, $1 \leq i \leq k$. Then $\{V_1, V_2, \ldots, V_k, \{v\}\}$ is a b-r-color partition of G, Otherwise $\pi' = \{V_1, V_2, \ldots V_i \cup \{v\}, \ldots V_k\}$ is a b-r-color partition of G, where v is not r-adjacent to V_i. Therefore $\phi_r(G-v) \leq \phi_r(G)$. Let $\pi = \{V_1, V_2, \ldots V_k\}$ be a ϕ_r-coloring of G. Let $v \in V_i$. If v is the only vertex in V_i which is r-adjacent to each color class V_i, $1 \leq i \leq k$. Then $\left\{V_1, V_2, \ldots, V_{(i-1)}, V_{(i+1)} \ldots V_k\right\}$ is a b-r-color partition of G, otherwise $\pi' = \{V_1, V_2, \ldots V_i - \{v\}, \ldots V_k\}$ is a b-r-color partition of G. Therefore $\phi_r(G) \leq \phi_r(G-v)+1$.

Proposition 2.4. *Let G be a graph without isolates. Let $\mu(G)$ denote the Mycielski graph of G. Then*

$$\chi_r[\mu(G)] = \begin{cases} n+1, \text{if } r \geq 2 \\ \chi(G)+1, \text{if } r = 1 \end{cases}$$

Proof:

Let G be a graph without isolates. Let $\mu(G)$ denote the Mycielski graph of G. Let $V(G) = \{u_1, u_2, \ldots u_2\}$ and $V(\mu(G)) = V(G) \cup \{u'_1, u'_2, \ldots u'_n, v\}$. $N_{\mu(G)}(u'_i) = N_{(G)}(u_i) \cup$

$\{v\}$. $N_{2_{\mu(G)}}(u_i') = N_{2_{(G)}}(u_i) \cup \{u_1', \ldots, u_i', \ldots u_n', v\}$. $N_{r_{\mu(G)}}(u_i') = V[\mu(G)]$, when $r \geq 3$. $N_{\mu(G)}(v) = \{u_1', \ldots u_n'\}$. $N_{r_{\mu(G)}}(v) = V[\mu(G)]$, when $r \geq 2$. $N_{\mu(G)}(u) = N_G(u) \cup \{x' : x \in N_{(G)}(u)\}$. $N_{2_{\mu(G)}}(u) = N_{2_{(G)}}(u) \cup \{x' : x \in N_{2_G}(u)\} \cup \{v\}$. $N_{3_{\mu(G)}}(u) = N_{3_{(G)}}(u) \cup \{u_1', u_2', \ldots u_n', v\}$. $N_{r_{\mu(G)}}(u) = V[\mu(G)]$, when $r \geq 4$.

$$\chi_r[\mu(G)] = \begin{cases} n+1, \text{if } r \geq 2 \\ \chi(G)+1, \text{if } r = 1 \end{cases}.$$

Lemma 2.3.

For any connected graph G, $\phi_r(G) \leq \phi_{(r+1)}(G)$, for all $r \geq 1$ (since $\chi_r(G) \leq \chi_{(r+1)}(G)$, for all $r \geq 1$).

Lemma 2.4.

For any connected graph G, $\chi_r(G) \leq \left\lceil \frac{\Delta_r(G)\ + \omega_r(G)}{2} \right\rceil + 1$, where $\omega_r(G)$is the maximum cardinality of a maximal r-complete subgraph of G.

Lemma 2.5.

If G is a graph with $\phi_r(G) = 1 + \Delta_r(G)$. Then $\phi_r(G) + 1 \leq \phi_r(\mu(G)) \leq (r+1)\phi_r(G) - 1$.

Lemma 2.6.

$\chi_r(\mu(G)) \leq \phi_r(\mu(G)) \ \leq 1 + \Delta_r(G)$. If $r = diam(G)$, then $\phi_r(\mu(G)) = \chi_r(\mu(G)) = 1 + \Delta_r(G)$.

Conclusion: In this paper, a study of new parameter called distance $r-b$-coloring is defined and discussions about it on various dimensions are carried out. A study of these parameters with the different types of coloring such as dominator coloring and achromatic coloring is also possible. There is a good scope for further investigation on these parameters.

References

[1] R.Balakrishnan, S.Francis Raj and T.Kavaskar, *Coloring the Mycielskian*, Proceeding of International Conference -ICDM (2008), 53-57.

[2] G.Chartrand, D.Geller and S.Hedetniemi, *A generalization of chromatic number*, Proc. Cambridge Philos. Soc. 64 (1968), 265-271.

[3] M.Chundovsky and P.D.Seymour,*The structure of claw-free graphs*, manuscript 2004.

[4] Douglas B.West, *Introduction to Graph Theory*(Prentice-Hall of India), 2003.

[5] M.A.Henning, *Distance Domination in Graphs:*, *Advance Topics*(Eds:Teresa W. Haynes, Stephen T. Hedetniemi, Peter J. Slater), 321-349, Marcel Dekker, Inc., New York, 1997.

[6] R.W.Irving and D.F.Manlove. *The b-chromatic number of a graph*, Discrete Appl. Math., **91** (1999) 127-141.

[7] G.Jothilakshmi, " Studies in domination in graphs with special reference to (k, r)-domination and weak convexity", Madurai kamaraj University Ph.D Thesis, 2009.

[8] G.Jothilakshmi, A.P.PushpaLatha, S.Suganthi and V.Swaminathan, *(k, r)-coloring, International Journal of Mathematics, Computer science and Information Technology.,* **Vol-I** December 2008, 211-219.

[9] M.Kouider and M.Maheo, *some bounds for the b-chromatic number of a graph*, Discrete Math., **256**(2002), 267-277.

[10] J.Kratochv'il, Z.Tuza and M.Voigt, *On the b-chromatic number of graphs*, Proceedings WG02 - 28th International Workshop on Graph-Theoretic Concepts in Computer Science, Cesky Krumlov, Czech Republic, Volume 2573 of Lecture Notes in Computer Science. Springer Verlag 2002.

[11] E.Sampath Kumar and L.Pushpa Latha, *Semi strong chromatic number of a graph*, Indian Journal of Pure and Applied Mathematics., **26**(1): 35-40, January 1995.

[12] E.Sampath Kumar and C.V.Venatachalam, *Chromatic partitions of a graph*, Discrete Math.**74** (1989), 227-239.

[13] H.B.Walikar, B. D. Acharya and E. Sampathkumar, *Recent developments in the theory of domination in graphs*, MRI Lecture Notes in Math. **1** (1976).

[14] F.Kramer and H.Kramer,*Un probleme de coloration des sommets dun graphe*, C.R. Acad. Sci. Paris A 268 (1969) 4648.

[15] G.Jothilakshmi, A.P.Pushpalatha, G.Sudhalakshmi, S.Suganthi and V.Swaminathan, *Distance r-Coloring and Distance r-Dominator Coloring number of a graph* , International Journal of Mathematics Trends and Technology, Volume 5 Number 3, January 2014, Page 242-246.

[16] G.Jothilakshmi, A.P.Pushpalatha, S.Suganthi and V.Swaminathan,(k, r)-*Semi Strong Chromatic Number of a Graph*, International Journal of Computer Applications, (09758887),Volume 21 No.2, May 2011.

[17] G.Jothilakshmi, A.P.Pushpalatha, S.Suganthi and V.Swaminathan,(k, r)-*domination and* (k, r)-*independence number of a graph*, International Journal of Computing Technology, Volume 1, No.2, March 2011.

[18] G.Jothilakshmi, A.P.Pushpalatha, S.Suganthi and V.Swaminathan, *Distance r-coloring and distance -r chromatic free, fixed and totally free vertices in a graph*, Global journal of Pure and Applied Mathematics, Volume 10, Number 1,2014, pp 53-62.

Dr.G. Jothilakshmi received her master degree in mathematics in 1990 and received Bakthavathchalam award for getting first rank in M.Sc. at the university. She received her Ph.D degree in 2010 from Madurai Kamaraj University, Madurai, India. She is currently working as associate professor of mathematics in Thiagarajar College of Engineering, Madurai, India. She has published nearly 26 research articles. She has 25 years of teaching experience and 12 years of research experience. She acquired ASDF Best Professor award in 2015. Her area of interest includes graph theory, linear algebra, and analysis.

Dr.A.P. Pushpalatha is an associate professor in Department of Mathematics at Thiagarajar College of Engineering, Madurai, India. She obtained her M.Sc. (1995) and Ph.D. (2011) degree from Madurai Kamaraj University, Madurai, India. She has published nearly 20 research articles. She has 21 years of teaching experience and 12 years of research experience. Her area of interest includes graph theory, discrete mathematics, and numerical methods.

Dr.S. Suganthi, got her M.Sc (1998) and Ph.D(2011) degree in mathematics from Madurai Kamaraj University, Madurai, India. She has 10 years of teaching experience and 8 years of research experience. Her area of interest includes graph theory, discrete mathematics, and analysis.

V.Swaminathan, for the photograph and short biography, see TWMS J. Appl. and Eng. Math., V.5, No.1, 2015.