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ON CERTAIN TOPOLOGICAL INDICES OF THE DERIVED GRAPHS OF SUBDIVISION GRAPHS

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Abstract. The derived graph $[G]^{\dagger}$ of a graph G is the graph having the same vertex set as G, with two vertices of $[G]^{\dagger}$ being adjacent if and only if their distance in G is two. Topological indices are valuable in the study of QSAR/QSPR. There are numerous applications of graph theory in the field of structural chemistry. In this paper, we compute generalized Randić, general Zagreb, general sum-connectivity, ABC, GA, ABC_4, and GA_5 indices of the derived graphs of subdivision graphs.

Keywords: topological indices, line graph, subdivision graph, derived graph.

AMS Subject Classification: 05C90, 05C35, 05C12

1. Introduction and Preliminaries

Let G be a simple graph, with vertex set $V(G)$ and edge set $E(G)$. The degree d_u of a vertex u is the number of edges that are incident to it and $S_u = \sum_{v\in N_u} d_v$ where $N_u = \{v \in V(G)|uv \in E(G)\}$. N_u is also known as the set of neighbor vertices of u.

In structural chemistry and biology, molecular structure descriptors are utilized for modeling information of molecules, which are known as topological indices. Many topological indices are introduced to explain the physical and chemical properties of molecules (see [23]).

Li and Zhao introduced the first general Zagreb index in [16] as

$$M_\alpha(G) = \sum_{u\in V(G)} (d_u)^\alpha. \tag{1}$$

The general Randić connectivity index of G is defined as [18]

$$R_\alpha(G) = \sum_{uv\in E(G)} (d_ud_v)^\alpha. \tag{2}$$

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where α is a real number. It is easy to see that $R_1(G) = M_2(G)$.
In 2010, general sum-connectivity index $\chi_\alpha(G)$ has been introduced in [26] as

$$\chi_\alpha(G) = \sum_{uv \in E(G)} (d_u + d_v)^\alpha. \tag{3}$$

Note that $\chi_1(G) = M_1(G)$.

The atom-bond connectivity (ABC) index, introduced by Estrada et al. in [4]. The ABC index of graph G is defined as

$$ABC(G) = \sum_{uv \in E(G)} \sqrt{\frac{d_u + d_v - 2}{d_u d_v}}. \tag{4}$$

The fourth member of the class of ABC index was introduced by M. Ghorbani et al. in [6] as

$$ABC_4(G) = \sum_{uv \in E(G)} \sqrt{\frac{S_u + S_v - 2}{S_u S_v}}. \tag{5}$$

D. Vukicevic and B. Furtula introduced the geometric arithmetic (GA) index in [24]. The GA index for graph G is defined by

$$GA(G) = \sum_{uv \in E(G)} \frac{2\sqrt{d_u d_v}}{d_u + d_v}. \tag{6}$$

The 5th GA index was introduced by Graovac et al. in [7] as

$$GA_5(G) = \sum_{uv \in E(G)} \frac{2\sqrt{S_u S_v}}{S_u + S_v}. \tag{7}$$

For a collection of recent results on degree-based topological indices, we refer the interested reader to the articles [1, 2, 10, 13, 14, 15, 21].

Now we define some notions of the graph theory. The subdivision graph $S(G)$ is the graph obtained from G by replacing each of its edge by a path of length 2. The line graph $L(G)$ of graph G is the graph whose vertices are the edges of G, two vertices e and f are incident if and only if they have a common end vertex in G. The tadpole graph $T_{n,k}$ is the graph obtained by joining a cycle of n vertices with a path of length k. A ladder graph L_n is obtained by taking cartesian product of two paths $P_n \times P_2$. A wheel graph W_n of order n is composed of a vertex, which will be called the hub, adjacent to all vertices of a cycle of order n, i.e, $W_n = C_{n-1} + K_1$.

1.1. **Topological Indices of $L(S(G))$.** In 2011, Ranjini et al. calculated the explicit expressions for the Shultz index of the subdivision graphs of the tadpole graph, wheel, helm, and ladder graphs [20]. They also studied the Zagreb indices of the line graph of tadpole, wheel, and ladder graphs with subdivision in [19]. In 2015, Su and Xu calculated the general sum-connectivity index and co-index for the line graph of tadpole, wheel, and ladder graphs with subdivision [22]. In the same year(2015) M. F. Nadeem et al. computed the ABC_4 and GA_5 indices for the same graphs [17].

Motivated by the results in [20, 19, 22, 17], we computed generalized Randić, general Zagreb, general sum-connectivity, ABC, GA, ABC_4, and GA_5 indices of the derived graphs of subdivision graphs.

The following straightforward, previously known, auxiliary results are important for us.

Lemma 1.1. [12] *For any graph G with n vertices and m edges, the subdivision graph $S(G)$ of G is a graph with $n+m$ vertices and $2m$ edges.*

Lemma 1.2. [12] *Let G be a graph with n vertices and m edges, then the line graph $L(G)$ of G is a graph with m vertices and $\frac{1}{2}M_1(G)-m$ edges.*

1.2. **TOPOLOGICAL INDICES OF THE DERIVED GRAPH OF SUBDIVISION GRAPH OF TADPOLE GRAPH.** The derived graph $[G]^\dagger$ of a graph G is the graph having the same vertex set as G, two vertices of $[G]^\dagger$ being adjacent if and only if their distance in G is two [11].

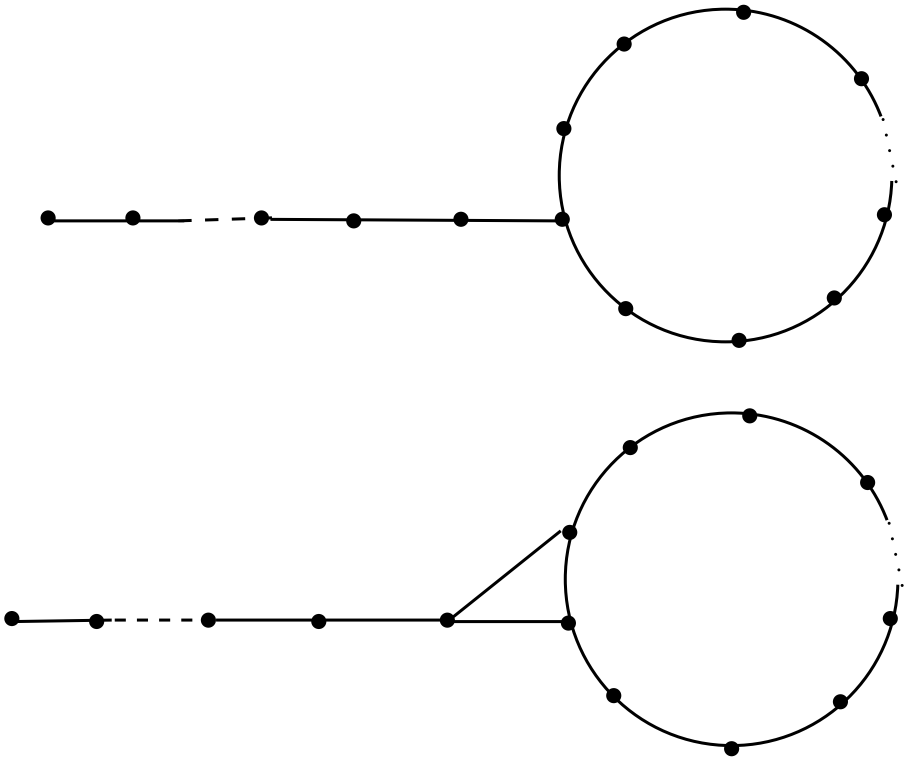

Fig.1. (a) The tadpole graph $T_{n,k}$; (b) the line graph $L(T_{n,k})$ of tadpole graph

Table 1. The edge partition of tadpole graph $T_{n,k}$.

(S_u, S_v) where $uv \in E(G)$	$(4,4)$	$(5,6)$	$(3,2)$
Number of edges	$n+k-4$	3	1

Table 2. The edge partition of line graph of tadpole graph $T_{n,k}$.

(S_u, S_v) where $uv \in E(G)$	$(4,4)$	$(5,8)$	$(8,8)$	$(2,3)$
Number of edges	$n+k-6$	3	3	1

Theorem 1.1. *Let $G = T_{n,k}$ be the tadpole graph. Then*

(1) $R_\alpha(G) = (n+k-4)\cdot 4^\alpha + 3\cdot 6^\alpha + 2^\alpha$;

(2) $\chi_\alpha(G) = (n+k-4)\cdot 4^\alpha + 3\cdot 5^\alpha + 3^\alpha$;
(3) $M_\alpha(G) = (n+k-2)\cdot 2^\alpha + 3^\alpha + 1^\alpha$;
(4) $ABC(G) = (n+k)\frac{1}{\sqrt{2}}$;
(5) $GA(G) = n+k+\frac{2\sqrt{6}}{5}+\frac{2\sqrt{2}}{3}-4$;
(6) $ABC_4(G) = (n+k-4)\sqrt{\frac{3}{8}}+3\sqrt{\frac{3}{10}}+\frac{1}{\sqrt{2}}$;
(7) $GA_5(G) = n+k+\frac{6\sqrt{30}}{11}+\frac{2\sqrt{6}}{5}-4$.

Proof. The tadpole graph $G = T_{n,k}$ is shown in Fig.1. In G there are total $n+k$ vertices and edges among which $(n+k-2)$ vertices are of degree 2 and one vertex is of degree one and three respectively. Therefore we get the edge partition, based on the degrees of vertices are shown in Table 1. We apply formulas (1)-(7) to the information in Table 1 and obtain the required result. □

Theorem 1.2. *Let $H = L(T_{n,k})$ be the line graph of tadpole graph $T_{n,k}$. Then*
(1) $R_\alpha(H) = (n+k-6)\cdot 4^\alpha + 3\cdot 6^\alpha + 3\cdot 9^\alpha + 2^\alpha$;
(2) $\chi_\alpha(H) = (n+k-6)\cdot 4^\alpha + 3\cdot 5^\alpha + 3\cdot 6^\alpha + 3^\alpha$;
(3) $M_\alpha(H) = (n+k-5)\cdot 2^\alpha + 3\cdot 3^\alpha + 1^\alpha$;
(4) $ABC(H) = (n+k-2)\frac{1}{\sqrt{2}}+2$;
(5) $GA(H) = n+k+\frac{6\sqrt{6}}{5}+\frac{2\sqrt{2}}{3}-3$;
(6) $ABC_4(H) = (n+k-6)\sqrt{\frac{3}{8}}+3\sqrt{\frac{11}{40}}+3\sqrt{\frac{7}{32}}+\frac{1}{\sqrt{2}}$;
(7) $GA_5(H) = n+k+\frac{6\sqrt{40}}{13}+\frac{2\sqrt{6}}{5}-3$.

Proof. The tadpole graph $T_{n,k}$ and the line graph of tadpole graph $H = L(T_{n,k})$ are shown in Fig.1. In H there are total $n+k$ vertices and by Lemma 1.2, there exists $(n+k+1)$ edges among which $(n+k-5)$ vertices are of degree 2 and one vertex is of degree one and three respectively. Therefore we get the edge partition, based on the degrees of vertices are shown in Table 2. We apply formulas (1)-(7) to the information in Table 2 and obtain the required result. □

Theorem 1.3. *Let $[S(G)]^\dagger$ be the derived graph of subdivision graph of tadpole graph $T_{n,k}$. Then*
(1) $R_\alpha([S(G)]^\dagger) = 2(n+k-5)\cdot 4^\alpha + 3(6^\alpha+9^\alpha) + 3\cdot 6^\alpha + 2\cdot 2^\alpha$;
(2) $\chi_\alpha([S(G)]^\dagger) = 2(n+k-5)\cdot 4^\alpha + 6\cdot 5^\alpha + 3\cdot 6^\alpha + 2\cdot 3^\alpha$;
(3) $M_\alpha([S(G)]^\dagger) = (2n+2k-7)\cdot 2^\alpha + 4\cdot 3^\alpha + 2\cdot 1^\alpha$;
(4) $ABC([S(G)]^\dagger) = \frac{1}{\sqrt{2}}(2n+2k-2)+2$;
(5) $GA([S(G)]^\dagger) = 2(n+k)+\frac{8\sqrt{6}}{5}+\frac{4\sqrt{2}}{3}-7$;
(6) $ABC_4([S(G)]^\dagger) = 2(n+k-5)\sqrt{\frac{3}{8}}+\frac{2}{\sqrt{2}}+3\sqrt{\frac{11}{40}}+3\sqrt{\frac{3}{10}}+3\sqrt{\frac{7}{32}}$;
(7) $GA_5([S(G)]^\dagger) = 2(n+k)+\frac{4\sqrt{6}}{5}+\frac{6\sqrt{40}}{13}+\frac{6\sqrt{30}}{11}-7$.

Proof. Let $[S(G)]^\dagger$ be the derived graph of the subdivision graph of tadpole graph $S(T_{n,k})$. Then by Theorem 2.1[11], $[S(G)]^\dagger \cong L(G)\cup G$. Hence using the information in Theorem 1.1 and Theorem 1.2, we get the required result. □

1.3. TOPOLOGICAL INDICES OF THE DERIVED GRAPH OF SUBDIVISION GRAPH OF WHEEL GRAPH.

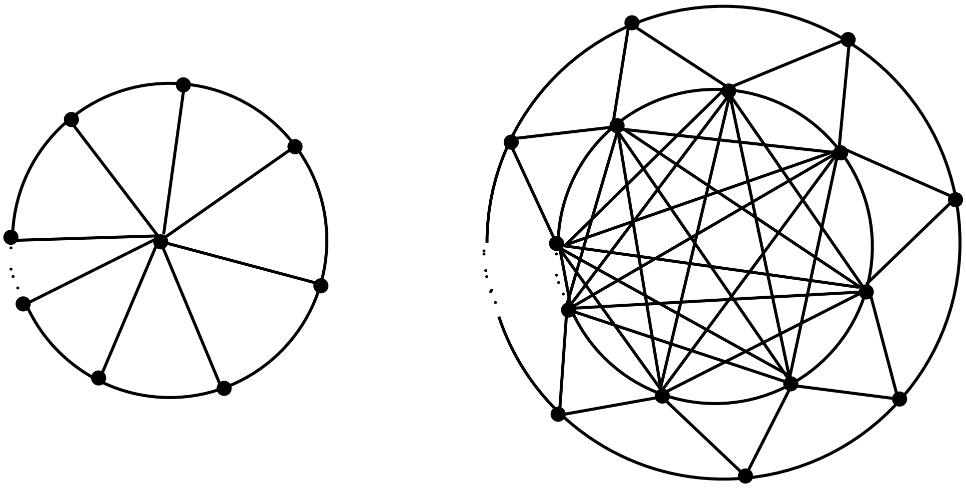

Fig.2. The Wheel graph W_n and its line graph $L(W_n)$

Table 3. The edge partition wheel graph W_n.

(S_u, S_v) where $uv \in E(G)$	$(n+5, n+5)$	$(n+5, 3(n-1))$
Number of edges	$n-1$	$n-1$

Table 4. The edge partition of line graph of wheel graph W_n.

(S_u, S_v) where $uv \in E(G)$	$(2n+8, 2n+8)$	$(2n+8, n(n-2)+8)$	$(n(n-2)+8, n(n-2)+8)$
Number of edges	$n-1$	$2(n-1)$	$\frac{(n-1)(n-2)}{2}$

Theorem 1.4. *Let $G = W_n$ be the wheel graph. Then*

(1) $R_\alpha(G) = (n-1) \cdot 9^\alpha + (n-1)(3(n-1))^\alpha$;
(2) $\chi_\alpha(G) = (n-1) \cdot 6^\alpha + (n-1)(n+2)^\alpha$;
(3) $M_\alpha(G) = (n-1)^\alpha + (n-1) \cdot 3^\alpha$;
(4) $ABC(G) = \frac{2(n-1)}{3} + (n-1)\sqrt{\frac{n}{3(n-1)}}$;
(5) $GA(G) = (n-1) + (n-1)\left(\frac{2\sqrt{3(n-1)}}{n+2}\right)$;
(6) $ABC_4(G) = \frac{(n-1)\sqrt{2n+8}}{n+5} + 2(n-1)\sqrt{\frac{n}{3(n-1)(n+5)}}$;
(7) $GA_5(G) = (n-1) + (n-1)\left(\frac{\sqrt{3(n-1)(n+5)}}{2n+1}\right)$.

Proof. The wheel graph $G = W_n$ is shown in Fig.2. In G there are $(n-1)$ vertices are of degree 3 and one vertex is of degree $n-1$. Therefore we get the edge partition, based on the degrees of vertices are shown in Table 3. We apply formulas (1)-(7) to the information in Table 3 and obtain the required result. □

Theorem 1.5. *Let $H = L(W_n)$ be the line graph of wheel graph W_n. Then*

(1) $R_\alpha(H) = (n-1) \cdot 6^\alpha + 2(n-1) \cdot (4n)^\alpha + \frac{(n-1)(n-2)}{2} \cdot (2n)^\alpha$;
(2) $\chi_\alpha(H) = (n-1) \cdot 8^\alpha + 2(n-1) \cdot (n+4)^\alpha + \frac{(n-1)(n-2)}{2} \cdot (2n)^\alpha$;
(3) $M_\alpha(H) = (n-1) \cdot 4^\alpha + (n-1) \cdot n^\alpha$;
(4) $ABC(H) = \frac{(n-1)\sqrt{6}}{4} + (n-1)\sqrt{\frac{n+2}{n}} + \frac{(n-1)(n-2)\sqrt{2n-2}}{2n}$;

(5) $GA(H) = n + \frac{8(n-1)\sqrt{n}}{n+4} + \frac{(n-1)(n-2)}{2} - 1$;

(6) $ABC_4(H) = (n-1)\left(\frac{\sqrt{4n+14}}{2n+8}\right) + 2(n-1)\sqrt{\frac{2n+n(n-2)+14}{(2n+8)(n(n-2)+8)}} + (n-1)(n-2)\left(\frac{\sqrt{2n(n-2)+14}}{2(n(n-2)+8)}\right)$;

(7) $GA_5(H) = n + 2(n-1)\left(\frac{2\sqrt{(2n+8)(n(n-2)+8)}}{n(n-2)+2n+16}\right) + \frac{(n-1)(n-2)}{2}$.

Proof. The wheel graph W_n and the line graph of wheel graph $H = L(W_n)$ are shown in Fig.2. In H there are total $2(n-1)$ vertices and by Lemma 1.2, there exists $\frac{(n-1)(n+4)}{2}$ edges among which $(n-1)$ vertices are of degree 4 and $(n-1)$ vertices are of degree n. Therefore we get the edge partition, based on the degrees of vertices are shown in Table 4. We apply formulas (1)-(7) to the information in Table 4 and obtain the required result. □

Theorem 1.6. *Let $[S(G)]^\dagger$ be the derived graph of subdivision graph of wheel graph W_n. Then*

(1) $R_\alpha([S(G)]^\dagger) = \frac{(n-1)(n-2)}{2}\cdot(2n)^\alpha + (n-1)(3(n-1))^\alpha + (n-1)\cdot 16^\alpha + 2(n-1)\cdot(4n)^\alpha + (n-1)\cdot 9^\alpha$;

(2) $\chi_\alpha([S(G)]^\dagger) = \frac{(n-1)(n-2)}{2}\cdot(2n)^\alpha + (n-1)[8^\alpha + 6^\alpha + (n+2)^\alpha] + 2(n-1)\cdot(n+4)^\alpha$;

(3) $M_\alpha([S(G)]^\dagger) = (n-1)^\alpha + (n-1)[n^\alpha + 4^\alpha + 3^\alpha]$;

(4) $ABC([S(G)]^\dagger) = \frac{(n-1)\sqrt{6}}{4} + (n-1)\sqrt{\frac{n+2}{n}} + \frac{(n-1)(n-2)\sqrt{2n-2}}{2n} + \frac{2(n-1)}{3} + (n-1)\sqrt{\frac{n}{3n-3}}$;

(5) $GA([S(G)]^\dagger) = 2(n-1) + \frac{2(n-1)\sqrt{3n-3}}{n+2} + \frac{8(n-1)\sqrt{n}}{n+4} + \frac{(n-1)(n-2)}{2}$;

(6) $ABC_4([S(G)]^\dagger) = \frac{(n-1)\sqrt{4n+14}}{2n+8} + 2(n-1)\sqrt{\frac{2n+n(n-2)+14}{(2n+8)(n(n-2)+8)}} + \frac{(n-1)(n-2)\sqrt{2n(n-2)+14}}{2(n(n-2)+8)} + \frac{((n-1)\sqrt{2n+8})}{(n+5)} + 2(n-1)\sqrt{\frac{n}{3(n-1)(n+5)}}$;

(7) $GA_5([S(G)]^\dagger) = 2(n-1) + \frac{4(n-1)\sqrt{(2n+8)(n(n-2)+8)}}{n(n-2)+2n+16} + \frac{(n-1)\sqrt{3(n-1)(n+5)}}{2(2n+1)} + \frac{(n-1)(n-2)}{2}$.

Proof. Let $[S(G)]^\dagger$ be the derived graph of the subdivision graph of wheel graph $S(W_n)$. Then by Theorem 2.1[11], $[S(G)]^\dagger \cong L(G) \cup G$. Hence using the information in Theorem 1.4 and Theorem 1.5, we get the required result. □

1.4. **TOPOLOGICAL INDICES OF THE DERIVED GRAPH OF SUBDIVISION GRAPH OF LADDER GRAPH.**

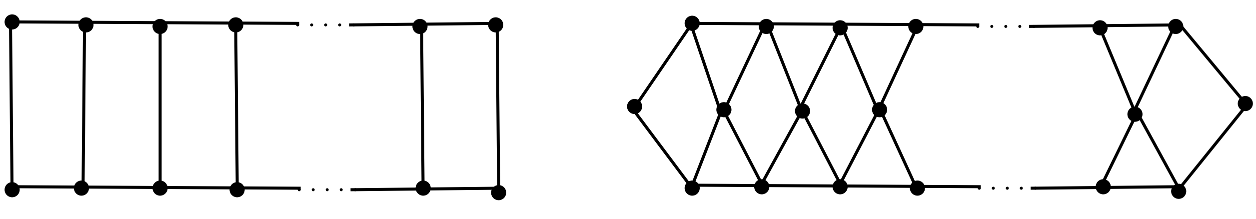

Fig. 3. The ladder graph $P_n \times P_2$ and its line graph $L(P_n \times P_2)$

Table 5. The edge partition of the ladder graph $P_n \times P_2; n \geq 4$.

(S_u, S_v) where $uv \in E(G)$	$(5,5)$	$(5,8)$	$(8,9)$	$(8,8)$	$(9,9)$
Number of edges	2	4	4	2	$3n-14$

Table 6. The edge partition of the line graph of ladder graph $P_n \times P_2; n \geq 6$.

(S_u, S_v) where $uv \in E(G)$	$(6,10)$	$(10,15)$	$(10,14)$	$(14,15)$	$(15,16)$	$(16,16)$
Number of edges	4	4	4	4	8	$6n-32$

Theorem 1.7. *Let $G = P_n \times P_2$ be the ladder graph. Then*

(1) $R_\alpha(G) = 2 \cdot 4^\alpha + 4 \cdot 6^\alpha + (3n-8) \cdot 9^\alpha$;
(2) $\chi_\alpha(G) = 2 \cdot 4^\alpha + 4 \cdot 5^\alpha + (3n-8) \cdot 6^\alpha$;
(3) $M_\alpha(G) = 4 \cdot 2^\alpha + (2n-4) \cdot 3^\alpha$;
(4) $ABC(G) = \frac{2(3n-8)}{3} + \frac{6}{\sqrt{2}}$;
(5) $GA(G) = 3n + \frac{8\sqrt{6}}{5} - 6$;
(6) $ABC_4(G) = \frac{2\sqrt{8}}{5} + 4\sqrt{\frac{11}{40}} + 4\sqrt{\frac{15}{72}} + \frac{\sqrt{14}}{4} + \frac{4(3n-14)}{9}$;
(7) $GA_5(G) = 3n + \frac{8\sqrt{40}}{13} + \frac{8\sqrt{72}}{17} - 10$.

Proof. The ladder graph $G = P_n \times P_2$ is shown in Fig.3. It is easy to calculate the topological indices value for G up to $n = 3$. Therefore we consider $n \geq 4$. In G there are total $2n$ vertices and $3n-2$ edges among which $(2n-4)$ vertices are of degree 3 and remaining vertices are of degree 2. Therefore we get the edge partition, based on the degrees of vertices are shown in Table 5. We apply formulas (1)-(7) to the information in Table 5 and obtain the required result. □

Theorem 1.8. *Let $H = L(P_n \times P_2)$ be the line graph of ladder graph $P_n \times P_2$. Then*

(1) $R_\alpha(H) = 4 \cdot 6^\alpha + 8 \cdot 12^\alpha + (6n-20) \cdot 16^\alpha$;
(2) $\chi_\alpha(H) = 4 \cdot 5^\alpha + 8 \cdot 7^\alpha + (6n-20) \cdot 8^\alpha$;
(3) $M_\alpha(H) = 2 \cdot 2^\alpha + 4 \cdot 3^\alpha + (3n-8) \cdot 4^\alpha$;
(4) $ABC(H) = \frac{4}{\sqrt{12}} + \frac{8\sqrt{5}}{\sqrt{12}} + \frac{(6n-20)\sqrt{6}}{4}$;
(5) $GA(H) = 6n + \frac{8\sqrt{6}}{5} + \frac{16\sqrt{12}}{7} - 20$;
(6) $ABC_4(H) = \frac{4\sqrt{7}}{\sqrt{30}} + \frac{4\sqrt{23}}{\sqrt{150}} + \frac{4\sqrt{11}}{\sqrt{70}} + \frac{4\sqrt{27}}{\sqrt{210}} + \frac{8\sqrt{29}}{\sqrt{240}} + \frac{(6n-32)\sqrt{30}}{16}$;
(7) $GA_5(H) = 6n + \frac{\sqrt{60}}{2} + \frac{8\sqrt{150}}{25} + \frac{8\sqrt{140}}{24} + \frac{8\sqrt{210}}{29} + \frac{16\sqrt{240}}{31} - 32$.

Proof. The ladder graph $G = P_n \times P_2$ and the line graph of ladder graph $H = L(P_n \times P_2)$ are shown in Fig.3. It is easy to calculate the topological indices value for G up to $n = 5$. Therefore we consider $n \geq 6$. In H there are total $3n-2$ vertices and by Lemma 1.2, there exists $6n-8$ edges among which $(3n-8)$ vertices are of degree 4 and two vertices are of degree 2 and four vertices are of degree 3. Therefore we get the edge partition, based on the degrees of vertices are shown in Table 6. We apply formulas (1)-(7) to the information in Table 6 and obtain the required result. □

Theorem 1.9. *Let $[S(G)]^\dagger$ be the derived graph of subdivision graph of ladder graph $P_n \times P_2$. Then*

(1) $R_\alpha([S(G)]^\dagger) = (6n-20) \cdot 16^\alpha + (3n-8) \cdot 9^\alpha + 8 \cdot 6^\alpha + 8 \cdot 12^\alpha + 2 \cdot 4^\alpha$;
(2) $\chi_\alpha([S(G)]^\dagger) = (6n-20) \cdot 8^\alpha + (3n-8) \cdot 6^\alpha + 8 \cdot 5^\alpha + 8 \cdot 7^\alpha + 2 \cdot 4^\alpha$;
(3) $M_\alpha([S(G)]^\dagger) = (3n-8) \cdot 4^\alpha + (2n-4) \cdot 3^\alpha + 6 \cdot 2^\alpha + 4 \cdot 3^\alpha$;
(4) $ABC([S(G)]^\dagger) = \frac{2(3n-8)}{3} + \frac{(6n-20)\sqrt{6}}{4} + \frac{10}{\sqrt{2}} + 8\sqrt{\frac{5}{12}}$;
(5) $GA([S(G)]^\dagger) = 9n + \frac{16\sqrt{6}}{5} + \frac{16\sqrt{12}}{7} - 26$;
(6) $ABC_4([S(G)]^\dagger) = \frac{4\sqrt{7}}{\sqrt{30}} + \frac{4\sqrt{23}}{\sqrt{150}} + \frac{4\sqrt{11}}{\sqrt{70}} + \frac{4\sqrt{27}}{\sqrt{210}} + \frac{8\sqrt{29}}{\sqrt{240}} + \frac{(6n-32)\sqrt{30}}{16} + \frac{2\sqrt{8}}{5} + 4\sqrt{\frac{11}{40}} + 4\sqrt{\frac{15}{72}} + \frac{\sqrt{14}}{4} + \frac{4(3n-14)}{9}$;
(7) $GA_5([S(G)]^\dagger) = 9n + \frac{\sqrt{60}}{2} + \frac{8\sqrt{150}}{25} + \frac{8\sqrt{140}}{24} + \frac{8\sqrt{210}}{29} + \frac{16\sqrt{240}}{31} + \frac{8\sqrt{40}}{13} + \frac{8\sqrt{72}}{17}$.

Proof. Let $[S(G)]^{\dagger}$ be the derived graph of the subdivision graph of ladder graph $S(P_n \times P_2)$. Then by Theorem 2.1[11], $[S(G)]^{\dagger} \cong L(G) \cup G$. Hence using the information in Theorem 1.7 and Theorem 1.8, we get the required result. □

Conclusion In this paper, we have obtained the expression for certain degree based topological indices in terms of the elements of G, which gives new direction to the field of structural chemistry.

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