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CERTAIN INTEGRAL TRANSFORMS AND FRACTIONAL INTEGRAL FORMULAS FOR THE EXTENDED HYPERGEOMETRIC FUNCTIONS

R.K. PARMAR[1], S.D. PUROHIT[2], §

Abstract. In this present paper, we derive various integral transforms, including Euler, Varma, Laplace, and Whittaker integral transforms for the extended hypergeometric functions which has recently been introduced by Choi et al.[3]. Further, we also apply Saigo's fractional integral operators for this extended hypergeometric function. Some interesting special cases of our main results are also considered.

Keywords: Extended beta function, extended Gauss hypergeometric functions, integral transforms, fractional integral operators.

AMS Subject Classification: 26A33, 33B15, 33C05, 33C99, 44A10.

1. Introduction and preliminaries

Extensions, generalizations, and unifications of Euler's Beta function together with related higher transcendent hypergeometric type special functions were investigated recently by several authors, consult for instance (see, *e.g.*, [1], [2], [6], [9] and for a very recent work, see also [10], [11]). In particular, Chaudhry et al. [2, p. 20, Equation (1.7)] presented the following extension of the Beta function as:

$$\mathrm{B}(x,y;p)=\int_0^1 t^{x-1}\,(1-t)^{y-1}\,\mathrm{e}^{-\frac{p}{t(1-t)}}\,\mathrm{d}t\,,\qquad (\Re(p)>0\,); \tag{1}$$

where for $p=0$, $\min\{\Re(x),\,\Re(y)\}>0$. They obtained related connections of $\mathrm{B}(x,y;p)$ with Macdonald (or modified Bessel function of the second kind), error and Whittaker functions. Further, Chaudhry et al. [1] used $\mathrm{B}(x,y;p)$ to extend the Gaussian hypergeometric function in the following manner

$$F_p(a,b;c;z)=\sum_{n\geq 0}^{\infty}(a)_n\,\frac{\mathrm{B}(b+n,\,c-b\,;\,p)}{\mathrm{B}(b,\,c-b)}\,\frac{z^n}{n!}\qquad (p\geq 0,\,|z|<1\,;\,\Re(c)>\Re(b)>0\,). \tag{2}$$

Recently, Choi et al. [3] introduce *further* extension of $\mathrm{B}(x,y;p)$ and $F_p(a,b;c;z)$ in the following manner:

$$B(x,\,y\,;\,p,\,q):=\int_0^1 t^{x-1}(1-t)^{y-1}\,\exp\left(-\frac{p}{t}-\frac{q}{1-t}\right)dt \tag{3}$$

[1] Department of Mathematics, College of Engineering and Technology, Bikaner, India.
e-mail: rakeshparmar27@gmail.com;

[2] Department of HEAS(Mathematics), Rajasthan Technical University, Kota, India.
e-mail: sunil_a_purohit@yahoo.com;

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$$\left(\min\{\Re(x), \Re(y)\} > 0;\ \min\{\Re(p), \Re(q)\} \geq 0 \right)$$

and

$$F_{p,q}\,(a,\, b;\, c;\, z) := {}_2F_1\,(a,\, b;\, c;\, z\; p, q) = \sum_{n=0}^{\infty} (a)_n \,\frac{B(b+n,\, c-b\,;\, p, q)}{B(b,\, c-b)}\,\frac{z^n}{n!} \tag{4}$$

$$\left(p \geq 0,\ q \geq 0;\ |z| < 1;\ \Re(c) > \Re(b) > 0\right).$$

The present investigation requires the concept of Hadamard product which can be used to decompose a newly-emerged function into two known functions. Let $f\,(z) := \sum_{n=0}^{\infty} a_n z^n$ and $g\,(z) := \sum_{n=0}^{\infty} b_n z^n$ be two power series whose radius of convergence are given by R_f and R_g, respectively. Then their Hadamard product (see [12]) is the power series defined by

$$(f * g)\,(z) := \sum_{n=0}^{\infty} a_n b_n z^n. \tag{5}$$

The radius of convergence R of the Hadamard product series $(f * g)\,(z)$ satisfies $R_f \cdot R_g \leq R$. If, in particular, one of the power series defines an entire function, then the Hadamard product series defines an entire function, too.

Here, we aim to establish certain new integral transforms as Euler, Varma, Laplace, and Whittaker and Saigo fractional integral operators involving the extended hypergeometric functions $F_{p,q}(z)$ given by Choi et al. [3], which include special cases as Riemann-Liouville and Erdélyi-Kober fractional integrals operators. This work is motivated by the authors recent work [5].

2. Certain integral transforms

Below, we present certain integral transforms, as Euler, Varma, Laplace, and Whittaker of extended hypergeometric function $F_{p,q}(z)$ defined by (4). To do this, we begin by recalling the following beta transform of a function $f(z)$ (see [13]):

$$B\{f(z) : a, b\} = \int_0^1 z^{a-1}(1-z)^{b-1} f(z)\, dz. \tag{6}$$

Theorem 2.1. *Let* $\min\{\Re(l), \Re(m), \Re(p), \Re(q)\} > 0$ *and* $\Re(c) > \Re(b) > 0$. *Then the following beta transform formula holds true:*

$$B\left\{F_{p,q}\left[\begin{array}{c} l+m, b \\ c \end{array};\, yz\right] : l, m\right\} = B(l,m)\, F_{p,q}\left[\begin{array}{c} l, b \\ c \end{array};\, y;\right]\ (|y| < 1), \tag{7}$$

where B *is the beta transform in* (6) *and the beta transform of* $F_{p,q}(\cdot)$ *is assumed to exist.*

Proof. Let $\mathcal{L}$ be the left-hand side of (7). Applying the beta transform (6) to the function (4), we get

$$\mathcal{L} = \int_0^1 z^{l-1}(1-z)^{m-1} \sum_{n=0}^{\infty} (l+m)_n \frac{\mathrm{B}_{p,q}(b+n, c-b)}{B(b, c-b)} \frac{(yz)^n}{n!}\, dz. \tag{8}$$

By changing the order of integration and summation which may be verified under the conditions, and using the classical beta function $B(\alpha, \beta)$ (see, *e.g.*, [14]), we obtain

$$\mathcal{L} = \sum_{n=0}^{\infty} (l+m)_n \frac{\mathrm{B}_{p,q}(b+n, c-b)}{B(b, c-b)} \frac{\Gamma(l+n)\Gamma(m)}{\Gamma(l+m+n)} \frac{y^n}{n!}, \tag{9}$$

which, in view of (4), is seen to lead to the right-hand side of (7). □

The Varma transform of a function $f(z)$ is defined by the following integral equation (see Mathai et al. [8, p. 55]):

$$V(f,k,m;s)=\int_0^\infty (sz)^{m-\frac{1}{2}}\exp\left(-\frac{1}{2}sz\right)W_{k,m}(sz)f(z)\,dz \quad (\Re(s)>0), \tag{10}$$

where $W_{k,m}$ is the Whittaker function defined by (Mathai et al. [8, p. 55])

$$W_{k,m}(z)=\sum_{m,-m}\frac{\Gamma(-2m)}{\Gamma(\frac{1}{2}-k-m)}M_{k,m}(z) \tag{11}$$

where the summation symbol indicates that the expression following it, a similar expression with m replaced by $-m$ is to be added and

$$M_{k,m}(z)=z^{m+\frac{1}{2}}e^{-\frac{z}{2}}{}_1F_1\left(\frac{1}{2}-k+m;2m+1;z\right). \tag{12}$$

The following formula (see Mathai et al. [8, p. 56]) will be used:

$$\int_0^\infty z^{\rho-1}\exp\left(-\frac{1}{2}sz\right)W_{k,\nu}(sz)\,dz=s^{-\rho}\frac{\Gamma(\rho+\nu+\frac{1}{2})\Gamma(\rho-\nu+\frac{1}{2})}{\Gamma(1-k+\rho)} \tag{13}$$
$$(\Re(s)>0,\ \Re(\rho\pm\nu)>-1/2).$$

Theorem 2.2. *Let $y\geq 0$, $\Re(s)\geq 0$, $\min\{\Re(p),\Re(q)\}>0$, $\Re(c)>\Re(b)>0$, and $|\frac{y}{s}|<1$. Then the following Varma transform formula holds true:*

$$\begin{aligned} V\left\{z^{l-1}\,F_{p,q}\left[\begin{matrix}a,b\\c\end{matrix};yz\right]\right\}&=\frac{1}{s^l}\frac{\Gamma(l)\Gamma(2m+l)}{\Gamma(m+l-k+\frac{1}{2})}\\ &\times F_{p,q}\left[\begin{matrix}a,b\\c\end{matrix};\frac{y}{s}\right]*_3F_1\left[\begin{matrix}1,2m+l,l;\\m+l-k+\frac{1}{2};\end{matrix}\ \frac{y}{s}\right], \end{aligned} \tag{14}$$

where V is the Varma transform in (10) *and both sides of* (14) *are assumed to exist.*

Proof. Let $\mathcal{L}$ be the left-hand side of (14). Then, a similar argument as in the proof of Theorem 2.1 is seen to give the following result:

$$\mathcal{L}=\frac{1}{s^l}\sum_{n=0}^\infty (a)_n\frac{\mathrm{B}_{p,q}(b+n,c-b)}{B(b,c-b)}\frac{\Gamma(l+n)\Gamma(2m+l+n)}{\Gamma(m+l-k+\frac{1}{2}+n)}\frac{(\frac{y}{s})^n}{n!}, \tag{15}$$

which, upon using the definition of Hadamard product series (5) and extended hypergeometric function (4), leads to the right-hand side of (14). □

It is interesting to observe that, for $k=m+\frac{1}{2}$ in (14), the Varma transform defined by (10) reduces to the well-known Laplace transform of a function $f(z)$ (see, *e.g.*, [13]):

$$L\{f(z):s\}=\int_0^\infty e^{-sz}f(z)\,dz. \tag{16}$$

In fact, we have an interesting Laplace transform asserted by the following corollary.

Corollary 2.1. *Let* $y \geq 0$, $\Re(s) \geq 0$, $\min\{\Re(p), \Re(q)\} > 0$, $\Re(c) > \Re(b) > 0$, *and* $|\frac{y}{s}| < 1$. *Then the following Laplace transform formula holds true:*

$$L\left\{z^{l-1}\ F_{p,q}\left[\begin{array}{c} a,b \\ c \end{array};yz\right]\right\} = \frac{\Gamma(l)}{s^l}\ F_{p,q}\left[\begin{array}{c} a,b \\ c \end{array};\frac{y}{s}\right] * {}_2F_0\left[\begin{array}{c} 1,l; \\ -; \end{array}\frac{y}{s}\right], \tag{17}$$

where L *is the Laplace transform in* (16) *and both sides of* (17) *are assumed to exist.*

Theorem 2.3. *Suppose that* $w \geq 0$, $\Re(p) \geq 0$, $\Re(q) \geq 0$ *and* ρ, $\delta \in \mathbb{C}$ *are parameters. Then the following Whittaker transform formula holds true:*

$$\int_0^\infty t^{\rho-1}e^{\frac{-\delta t}{2}}W_{\lambda,\mu}(\delta t)\ F_{p,q}\left[\begin{array}{c} a,b \\ c \end{array};wt\right]\ dt = \delta^{-\rho}\frac{\Gamma\left(\frac{1}{2}+\mu+\rho\right)\Gamma\left(\frac{1}{2}-\mu+\rho\right)}{\Gamma\left(1-\lambda+\rho\right)}$$

$$\times F_{p,q}\left[\begin{array}{c} a,b \\ c \end{array};\frac{w}{\delta}\right] * {}_3F_1\left[\begin{array}{c} 1,\frac{1}{2}+\mu+\rho,\frac{1}{2}-\mu+\rho; \\ 1-\lambda+\rho; \end{array}\frac{w}{\delta}\right], \tag{18}$$

provided that the integral transform converges.

Proof. Let $\mathcal{L}$ be the left-hand side of (18). Then, by applying (4) and setting $\delta t = \nu$, and changing the order of integration and summation, we obtain

$$\mathcal{L} = \delta^{-\rho}\sum_{n=0}^{\infty}(a)_n\frac{\mathrm{B}_{p,q}(b+n,c-b)}{B(b,c-b)}\frac{(w)^n}{\delta^n n!}\int_0^\infty \nu^{\rho+n-1}e^{\frac{-\nu}{2}}W_{\lambda,\mu}(\nu)d\nu. \tag{19}$$

Here we use the following integral formula involving the Whittaker function (see Mathai et al. [8, p. 56])

$$\int_0^\infty t^{\nu-1}e^{-\frac{t}{2}}W_{\lambda,\mu}(t)\ dt = \frac{\Gamma\left(\frac{1}{2}+\mu+\nu\right)\Gamma\left(\frac{1}{2}-\mu+\nu\right)}{\Gamma\left(1-\lambda+\nu\right)} \tag{20}$$

$$\left(\Re(\nu\pm\mu) > -1/2\right).$$

Then, after a little simplification, we get

$$\mathcal{L} = \delta^{-\rho}\sum_{n=0}^{\infty}(a)_n\frac{\mathrm{B}_{p,q}(b+n,c-b)}{B(b,c-b)}\frac{(w)^n}{\delta^n n!}\ \frac{\Gamma\left(\frac{1}{2}+\mu+\rho+n\right)\Gamma\left(\frac{1}{2}-\mu+\rho+n\right)}{\Gamma\left(1-\lambda+\rho+n\right)}, \tag{21}$$

which, upon using the definition of Hadamard product series (5) and extended hypergeometric function (4), leads to the right-hand side of (18). □

3. Fractional calculus approach

Fractional integral operators involving the various special functions have been actively investigated in various mathematical tools (see, *e.g.*, [4]). Here we establish some fractional integral formulas for the extended hypergeometric function $F_{p,q}(a,b;c;z)$. To do this, we recall the following pair of Saigo hypergeometric fractional integral operators (see Mathai et al. [8, p. 104]): For $\Re(\mu) > 0$,

$$(I_{0+}^{\mu,\nu,\eta}f(t))(x) = \frac{x^{-\mu-\nu}}{\Gamma(\mu)}\int_0^x (x-t)^{\mu-1}{}_2F_1\left(\mu+\nu,-\eta;\mu;1-\frac{t}{x}\right)f(t)\ dt \tag{22}$$

and

$$(I_{-}^{\mu,\nu,\eta}f(t))(x) = \frac{1}{\Gamma(\mu)}\int_x^\infty (t-x)^{\mu-1}t^{-\mu-\nu}{}_2F_1\left(\mu+\nu,-\eta;\mu;1-\frac{x}{t}\right)f(t)\ dt, \tag{23}$$

where the function $f(t)$ is so constrained that the defining integrals in (22) and (23) exist.

The operator $I_{0+}^{\mu,\nu,\eta}(\cdot)$ contains both the Riemann-Liouville $I_{0+}^{\mu}(\cdot)$ and the Erdélyi-Kober $I_{\eta,\mu}^{+}(\cdot)$ fractional integral operators by means of the following relationships:

$$(I_{0+}^{\mu} f(t))(x) = (I_{0+}^{\mu,-\mu,\eta} f(t))(x) = \frac{1}{\Gamma(\mu)} \int_0^x (x-t)^{\mu-1} f(t)\, dt \tag{24}$$

and

$$(I_{\eta,\mu}^{+} f(t))(x) = (I_{0+}^{\mu,0,\eta} f(t))(x) = \frac{x^{-\mu-\eta}}{\Gamma(\mu)} \int_0^x (x-t)^{\mu-1} t^{\eta} f(t)\, dt. \tag{25}$$

It is noted that the operator (23) unifies the Weyl type and the Erdélyi-Kober fractional operators as follows:

$$(I_{-}^{\mu} f(t))(x) = (I_{-}^{\mu,-\mu,\eta} f(t))(x) = \frac{1}{\Gamma(\mu)} \int_x^\infty (t-x)^{\mu-1} f(t)\, dt \tag{26}$$

and

$$(K_{\eta,\mu}^{-} f(t))(x) = (I_{-}^{\mu,0,\eta} f(t))(x) = \frac{x^{\eta}}{\Gamma(\mu)} \int_x^\infty (t-x)^{\mu-1} t^{-\mu-\eta} f(t)\, dt. \tag{27}$$

We also use the following image formulas which are easy consequences of the operators (22) and (23) (see Mathai et al. [8, p. 107]):

$$\begin{gathered}(I_{0+}^{\mu,\nu,\eta} t^{\lambda-1})(x) = \frac{\Gamma(\lambda)\Gamma(\lambda-\nu+\eta)}{\Gamma(\lambda-\nu)\Gamma(\lambda+\mu+\eta)} x^{\lambda-\nu-1} \\ (\Re(\lambda)>0,\ \Re(\lambda-\nu+\eta)>0)\end{gathered} \tag{28}$$

and

$$\begin{gathered}(I_{-}^{\mu,\nu,\eta} t^{\lambda-1})(x) = \frac{\Gamma(\nu-\lambda+1)\Gamma(\eta-\lambda+1)}{\Gamma(1-\lambda)\Gamma(\nu+\mu-\lambda+\eta+1)} x^{\lambda-\nu-1} \\ (\Re(\nu-\lambda+1)>0,\ \Re(\eta-\lambda+1)>0).\end{gathered} \tag{29}$$

The Saigo fractional integrations of the generalized Gauss hypergeometric type functions $F_{p,q}(a,b;c;z)$ are given in Theorems 3.1 and 3.2.

Theorem 3.1. *Suppose $x>0$, then the following fractional integral formula holds true:*

$$\begin{aligned}\left(I_{0+}^{\mu,\nu,\eta}\left[t^{\rho-1}\, F_{p,q}\left[\begin{matrix} a,b \\ c \end{matrix}\ ;et\right]\right]\right)(x) &= x^{\rho-\nu-1}\frac{\Gamma(\rho)\Gamma(\rho-\nu+\eta)}{\Gamma(\rho+\mu+\nu)\Gamma(\rho-\nu)} \\ &\times F_{p,q}\left[\begin{matrix} a,b \\ c \end{matrix}\ ;ex\right] * {}_3F_2\left[\begin{matrix} 1,\rho,\rho-\nu+\eta; \\ \rho-\nu,\rho+\mu+\eta; \end{matrix}\ ex\right],\end{aligned} \tag{30}$$

provided $\min\{\Re(p), \Re(q), \Re(\mu), \Re(\rho)\} > 0$ *and* $\Re(\rho) > \max\{0, \Re(\nu-\eta)\}$.

Proof. Let $\mathcal{L}$ be the left-hand side of (30). Then, using (4) and changing the order of integration and summation, which is valid under the given conditions, we have

$$\mathcal{L} = \sum_{n=0}^{\infty} (a)_n \frac{\mathrm{B}_{p,q}(b+n,c-b)}{B(b,c-b)} \frac{(e)^n}{n!} \left(I_{0+}^{\mu,\nu,\eta}\{t^{\rho+n-1}\}\right)(x). \tag{31}$$

Here, making use of the result (28), we obtain

$$\mathcal{L} = x^{\rho-\nu-1}\sum_{n=0}^{\infty}(a)_n\frac{\mathrm{B}_{p,q}(b+n,c-b)}{B(b,c-b)} \times \frac{\Gamma(\rho+n)\Gamma(\rho-\nu+\eta+n)}{\Gamma(\rho-\nu+n)\Gamma(\rho+\mu+\eta+n)}\frac{(ex)^n}{n!}, \tag{32}$$

which, in view of upon using the definition of Hadamard product series (5) and extended hypergeometric function (4), gives the right-hand side of (30). □

Theorem 3.2. *Let* $x > 0$, $\min\{\Re(p), \Re(q), \Re(\mu), \Re(\rho)\} > 0$ *and* $\Re(\rho) < 1+\min\{\Re(\eta), \Re(\nu)\}$. *Then the following fractional integral formula holds true:*

$$\left(I_{-}^{\mu,\nu,\eta}\left[t^{\rho-1}\ F_{p,q}\left[\begin{array}{c} a,b \\ c \end{array};\frac{e}{t}\right]\right]\right)(x) = x^{\rho-\nu-1}\frac{\Gamma(1-\rho+\nu)\Gamma(1-\rho+\eta)}{\Gamma(1-\rho)\Gamma(1-\rho-\eta+\nu+\mu)} \times F_{p,q}\left[\begin{array}{c} a,b \\ c \end{array};\frac{e}{x}\right] * {}_3F_2\left[\begin{array}{c} 1,1-\rho+\nu,1-\rho+\eta; \\ 1-\rho,1-\rho+\mu+\nu-\eta; \end{array}\frac{e}{x}\right]. \tag{33}$$

Proof. Similarly as in the proof of Theorem 3.1, taking the operator (23) and the result (29) into account will establish the result (33). So the details of proof are omitted. □

Setting $\nu = 0$ in Theorems 3.1 and 3.2 yields certain interesting results asserted by the following corollaries.

Corollary 3.1. *Let* $x > 0$, $\min\{\Re(p), \Re(q), \Re(\mu), \Re(\rho)\} > 0$ *and* $\Re(\rho) > \Re(-\eta)$. *Then the right-side Erdélyi-Kober fractional integrals of the extended hypergeometric function are given as follows:*

$$\left(I_{\eta,\mu}^{+}\left[t^{\rho-1}\ F_{p,q}\left[\begin{array}{c} a,b \\ c \end{array};et\right]\right]\right)(x) = x^{\rho-1}\frac{\Gamma(\rho+\eta)}{\Gamma(\rho+\mu)} \times F_{p,q}\left[\begin{array}{c} a,b \\ c \end{array};ex\right] * {}_2F_1\left[\begin{array}{c} 1,\rho+\eta; \\ \rho+\mu+\eta; \end{array}ex\right]. \tag{34}$$

Corollary 3.2. *Let* $x > 0$, $\min\{\Re(p), \Re(q), \Re(\mu), \Re(\rho)\} > 0$ *and* $\Re(\rho) < 1+\Re(\eta)$. *The following identity holds true:*

$$\left(K_{\eta,\mu}^{-}\left[t^{\rho-1}\ F_{p,q}\left[\begin{array}{c} a,b \\ c \end{array};\frac{e}{t}\right]\right]\right)(x) = x^{\rho-1}\frac{\Gamma(1-\rho+\eta)}{\Gamma(1-\rho-\eta+\mu)} \times F_{p,q}\left[\begin{array}{c} a,b \\ c \end{array};\frac{e}{x}\right] * {}_2F_1\left[\begin{array}{c} 1,1-\rho+\eta; \\ 1-\rho+\mu-\eta; \end{array}\frac{e}{x}\right]. \tag{35}$$

Further, replacing ν by $-\mu$ in Theorems 3.1 and 3.2 and making use of the relations (24) and (26) gives the other Riemann-Liouville and Weyl fractional integrals of the extended hypergeometric function in (4) given by the following corollaries.

Corollary 3.3. *Let* $x > 0$ *and* $\min\{\Re(p), \Re(q), \Re(\mu), \Re(\rho)\} > 0$. *Then the following formula holds true:*

$$\left(I_{0+}^{\mu}\left[t^{\rho-1}\ F_{p,q}\left[\begin{array}{c} a,b \\ c \end{array};et\right]\right]\right)(x) = x^{\rho+\mu-1}\frac{\Gamma(\rho+\mu+\eta)}{\Gamma(\rho+\mu)}$$

$$\times\, F_{p,q}\left[\begin{array}{c} a,b \\ c \end{array};ex\right] * {}_2F_1\left[\begin{array}{c} 1,\rho; \\ \rho+\mu; \end{array} ex\right]. \tag{36}$$

Corollary 3.4. *Let $x>0$ and $\min\{\Re(p),\, \Re(q),\, \Re(\mu),\, \Re(\rho)\}>0$. Then the following formula holds true:*

$$\left(I_{-}^{\mu}\left[t^{\rho-1}\, F_{p,q}\left[\begin{array}{c} a,b \\ c \end{array};\frac{e}{t}\right]\right]\right)(x) = x^{\rho+\mu-1}\frac{\Gamma(1-\rho-\mu)}{\Gamma(1-\rho)}\frac{\Gamma(1-\rho+\eta)}{\Gamma(1-\rho-\eta)}$$

$$\times\, F_{p,q}\left[\begin{array}{c} a,b \\ c \end{array};\frac{e}{x}\right] * {}_3F_2\left[\begin{array}{c} 1,1-\rho-\mu,\,1-\rho+\eta; \\ 1-\rho,\,1-\rho-\eta; \end{array} \frac{e}{x}\right]. \tag{37}$$

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Rakesh Kumar Parmar is presently working as an assistant professor in the Department of Mathematics, Govt. College of Engineering and Technology, Bikaner, Rajasthan, India and have 11 years of teaching and research experience. His research interests include special functions, fractional calculus, and integral transforms. He has published more than 35 research papers in various international journals. He has delivered talks in various national and international conferences. He is a reviewer of Mathematical Reviews, USA and various international mathematical journals. He is also a member of the Editorial Board of various international mathematical journals.

Sunil Dutt Purohit is an associate professor of mathematics in the Department of HEAS (Mathematics), Rajasthan Technology University, Kota-324010, Rajasthan, India. His research interests include special functions, fractional calculus, integral transforms, basic hypergeometric series, geometric function theory, integral inequalities, and mathematical physics. He has published more than 90 research papers in international esteemed journals. He is a reviewer for Mathematical Reviews, USA (American Mathematical Society) and Zentralblatt MATH, Berlin since last eight years. He is a member of the Editorial Board for a number of international mathematical and interdisciplinary journals.