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# DIFFERENTIAL SUBORDINATIONS USING RUSCHEWEYH DERIVATIVE AND SĂLĂGEAN OPERATOR

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ABSTRACT. In the present paper we study the operator defined by using the Ruscheweyh derivative $R^m f(z)$ and the Sălăgean operator $S^m f(z)$, denoted $L_\alpha^m : \mathcal{A}_n \to \mathcal{A}_n$, $L_\alpha^m f(z) = (1-\alpha)R^m f(z) + \alpha S^m f(z)$, $z \in U$, where $\mathcal{A}_n = \{f \in \mathcal{H}(U) : f(z) = z + a_{n+1}z^{n+1} + \ldots,\ z \in U\}$ is the class of normalized analytic functions. We obtain several differential subordinations regarding the operator $L_\alpha^m$.

Keywords: differential subordination, convex function, best dominant, differential operator, Sălăgean operator, Ruscheweyh derivative.

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## 1. Introduction

Denote by $U$ the unit disc of the complex plane, $U = \{z \in \mathbb{C} : |z| < 1\}$ and $\mathcal{H}(U)$ the space of holomorphic functions in $U$. Let

$$\mathcal{A}_n = \{f \in \mathcal{H}(U) : f(z) = z + a_{n+1}z^{n+1} + \ldots,\ z \in U\}$$

and

$$\mathcal{H}[a, n] = \{f \in \mathcal{H}(U) : f(z) = a + a_n z^n + a_{n+1}z^{n+1} + \ldots,\ z \in U\}$$

for $a \in \mathbb{C}$ and $n \in \mathbb{N}$. Denote by $K = \left\{ f \in \mathcal{A}_n : \operatorname{Re} \frac{zf''(z)}{f'(z)} + 1 > 0,\ z \in U \right\}$, the class of normalized convex functions in $U$.

If $f$ and $g$ are analytic functions in $U$, we say that $f$ is subordinate to $g$, written $f \prec g$, if there is a function $w$ analytic in $U$, with $w(0) = 0$, $|w(z)| < 1$, for all $z \in U$, such that $f(z) = g(w(z))$ for all $z \in U$. If $g$ is univalent, then $f \prec g$ if and only if $f(0) = g(0)$ and $f(U) \subseteq g(U)$. Let $\psi : \mathbb{C}^3 \times U \to \mathbb{C}$ and $h$ be a univalent function in $U$. If $p$ is analytic in $U$ and satisfies the (second-order) differential subordination

$$\psi(p(z), zp'(z), z^2p''(z); z) \prec h(z), \qquad z \in U, \tag{1}$$

then $p$ is called a solution of the differential subordination. The univalent function $q$ is called a dominant of the solutions of the differential subordination, or more simply a dominant, if $p \prec q$ for all $p$ satisfying (1). A dominant $\widetilde{q}$ that satisfies $\widetilde{q} \prec q$ for all dominants $q$ of (1) is said to be the best dominant of (1). The best dominant is unique up to a rotation of $U$.

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**Definition 1.1.** *(Sălăgean [6]) For $f \in \mathcal{A}_n$, $n, m \in \mathbb{N}$, the operator $S^m$ is defined by $S^m : \mathcal{A}_n \to \mathcal{A}_n$,*

$$S^0 f(z) = f(z), \ S^1 f(z) = zf'(z), ...$$
$$S^{m+1} f(z) = z\left(S^m f(z)\right)', \quad z \in U.$$

**Remark 1.1.** *If $f \in \mathcal{A}_n$, $f(z) = z + \sum_{j=n+1}^{\infty} a_j z^j$, then*

$$S^m f(z) = z + \sum_{j=n+1}^{\infty} j^m a_j z^j, z \in U$$

.

**Definition 1.2.** *([5]) For $f \in \mathcal{A}_n$, $n, m \in \mathbb{N}$, the operator $R^m$ is defined by $R^m : \mathcal{A}_n \to \mathcal{A}_n$,*

$$\begin{aligned} R^0 f(z) &= f(z), \ R^1 f(z) = zf'(z), ... \\ (m+1) R^{m+1} f(z) &= z\left(R^m f(z)\right)' + mR^m f(z), \quad z \in U. \end{aligned}$$

**Remark 1.2.** *If $f \in \mathcal{A}_n$, $f(z) = z + \sum_{j=n+1}^{\infty} a_j z^j$, then $R^m f(z) = z + \sum_{j=n+1}^{\infty} C_{m+j-1}^m a_j z^j$, $z \in U$.*

**Definition 1.3.** *([1]) Let $\alpha \geq 0$, $n, m \in \mathbb{N}$. Denote by $L_\alpha^m$ the operator given by $L_\alpha^m : \mathcal{A}_n \to \mathcal{A}_n$,*

$$L_\alpha^m f(z) = (1-\alpha) R^m f(z) + \alpha S^m f(z), \quad z \in U.$$

**Remark 1.3.** *If $f \in \mathcal{A}_n$, $f(z) = z + \sum_{j=n+1}^{\infty} a_j z^j$, then*
*$L_\alpha^m f(z) = z + \sum_{j=n+1}^{\infty} \left(\alpha j^m + (1-\alpha) C_{m+j-1}^m\right) a_j z^j$, $z \in U$.*
*This operator was studied also in [1], [2].*

**Lemma 1.1.** *(Hallenbeck and Ruscheweyh [4, Th. 3.1.6, p. 71]) Let $h$ be a convex function with $h(0) = a$, and let $\gamma \in \mathbb{C} \backslash \{0\}$ be a complex number with Re $\gamma \geq 0$. If $p \in \mathcal{H}[a, n]$ and $p(z) + \frac{1}{\gamma} zp'(z) \prec h(z)$, $z \in U$, then $p(z) \prec g(z) \prec h(z)$, $z \in U$, where $g(z) = \frac{\gamma}{nz^{\gamma/n}} \int_0^z h(t) t^{\gamma/n-1} dt$, $z \in U$.*

**Lemma 1.2.** *(Miller and Mocanu [4]) Let $g$ be a convex function in $U$ and let $h(z) = g(z) + n\alpha zg'(z)$, for $z \in U$, where $\alpha > 0$ and $n$ is a positive integer.*

*If $p(z) = g(0) + p_n z^n + p_{n+1} z^{n+1} + \ldots$, $z \in U$, is holomorphic in $U$ and $p(z) + \alpha zp'(z) \prec h(z)$, $z \in U$, then $p(z) \prec g(z)$, $z \in U$, and this result is sharp.*

## 2. Main results

**Theorem 2.1.** *Let $g$ be a convex function, $g(0) = 1$ and let $h$ be the function $h(z) = g(z) + \frac{nz}{\delta} g'(z)$, $z \in U$. If $\alpha, \delta \geq 0$, $n, m \in \mathbb{N}$, $f \in \mathcal{A}_n$ and satisfies the differential subordination*

$$\left(\frac{L_\alpha^m f(z)}{z}\right)^{\delta-1} \left(L_\alpha^m f(z)\right)' \prec h(z), \quad z \in U, \tag{2}$$

*then $\left(\frac{L_\alpha^m f(z)}{z}\right)^{\delta} \prec g(z)$, $z \in U$, and this result is sharp.*

**Theorem 2.2.** *Let $h$ be an holomorphic function which satisfies the inequality* Re $\left(1 + \frac{zh''(z)}{h'(z)}\right) > -\frac{1}{2}$, *$z \in U$, and $h(0) = 1$. If $\alpha, \delta \geq 0$, $n, m \in \mathbb{N}$, $f \in \mathcal{A}_n$ and satisfies the differential subordination*

$$\left(\frac{L_\alpha^m f(z)}{z}\right)^{\delta-1} \left(L_\alpha^m f(z)\right)' \prec h(z), \quad z \in U, \tag{3}$$

*then* $\left(\frac{L_{\alpha}^{m}f(z)}{z}\right)^{\delta} \prec q(z)$, $z \in U$, *where* $q(z) = \frac{\delta}{nz^{\frac{\delta}{n}}}\int_0^z h(t)t^{\frac{\delta}{n}-1}dt$. *The function q is convex and it is the best dominant.*

**Corollary 2.1.** *Let* $h(z) = \frac{1+(2\beta-1)z}{1+z}$ *be a convex function in U, where* $0 \leq \beta < 1$. *If* $\alpha, \delta \geq 0$, $n, m \in \mathbb{N}$, $f \in \mathcal{A}_n$ *and satisfies the differential subordination*

$$\left(\frac{L_{\alpha}^{m}f(z)}{z}\right)^{\delta-1}(L_{\alpha}^{m}f(z))' \prec h(z), \qquad z \in U, \tag{4}$$

*then* $\left(\frac{L_{\alpha}^{m}f(z)}{z}\right)^{\delta} \prec q(z)$, $z \in U$, *where q is given by* $q(z) = (2\beta-1) + \frac{2(1-\beta)\delta}{nz^{\frac{\delta}{n}}}\int_0^z \frac{t^{\frac{\delta}{n}-1}}{1+t}dt$, $z \in U$. *The function q is convex and it is the best dominant.*

**Remark 2.1.** *For* $n = 1$, $m = 1$, $\alpha = 2$, $\delta = 1$ *we obtain the same example as in* [3, Example 2.2.1, p. 26].

**Theorem 2.3.** *Let g be a convex function such that* $g(0) = 1$ *and let h be the function* $h(z) = g(z) + \frac{nz}{\gamma}g'(z)$, $z \in U$, *where* $\gamma > 0$. *If* $\alpha \geq 0$, $n, m \in \mathbb{N}$, $f \in \mathcal{A}_n$ *and the differential subordination*

$$\frac{(\gamma+1)z}{\gamma}\frac{L_{\alpha}^{m}f(z)}{\left(L_{\alpha}^{m+1}f(z)\right)^2} + \frac{z^2}{\gamma}\frac{L_{\alpha}^{m}f(z)}{\left(L_{\alpha}^{m+1}f(z)\right)^2}\left[\frac{(L_{\alpha}^{m}f(z))'}{L_{\alpha}^{m}f(z)} - 2\frac{\left(L_{\alpha}^{m+1}f(z)\right)'}{L_{\alpha}^{m+1}f(z)}\right] \prec h(z), \quad z \in U \tag{5}$$

*holds, then* $z\frac{L_{\alpha}^{m}f(z)}{\left(L_{\alpha}^{m+1}f(z)\right)^2} \prec g(z)$, $z \in U$, *and this result is sharp.*

**Theorem 2.4.** *Let h be an holomorphic function which satisfies the inequality* Re $\left(1 + \frac{zh''(z)}{h'(z)}\right) > -\frac{1}{2}$, $z \in U$, *and* $h(0) = 1$. *If* $\alpha \geq 0$, $\gamma \in \mathbb{C}\backslash\{0\}$ *be a complex number with Re* $\gamma \geq 0$, $n, m \in \mathbb{N}$, $f \in \mathcal{A}_n$ *and satisfies the differential subordination*

$$\frac{(\gamma+1)z}{\gamma}\frac{L_{\alpha}^{m}f(z)}{\left(L_{\alpha}^{m+1}f(z)\right)^2} + \frac{z^2}{\gamma}\frac{L_{\alpha}^{m}f(z)}{\left(L_{\alpha}^{m+1}f(z)\right)^2}\left[\frac{(L_{\alpha}^{m}f(z))'}{L_{\alpha}^{m}f(z)} - 2\frac{\left(L_{\alpha}^{m+1}f(z)\right)'}{L_{\alpha}^{m+1}f(z)}\right] \prec h(z), \qquad z \in U, \tag{6}$$

*then* $z\frac{L_{\alpha}^{m}f(z)}{\left(L_{\alpha}^{m+1}f(z)\right)^2} \prec q(z)$, $z \in U$, *where* $q(z) = \frac{\gamma}{nz^{\gamma/n}}\int_0^z h(t)t^{\gamma/n-1}dt$. *The function q is convex and it is the best dominant.*

**Theorem 2.5.** *Let g be a convex function such that* $g(0) = 1$ *and let h be the function* $h(z) = g(z) + \frac{nz}{\gamma}g'(z)$, $z \in U$, *where* $\gamma > 0$. *If* $\alpha \geq 0$, $n, m \in \mathbb{N}$, $f \in \mathcal{A}_n$ *and the differential subordination*

$$\frac{(\gamma+2)z^2}{\gamma}\frac{(L_{\alpha}^{m}f(z))'}{L_{\alpha}^{m}f(z)} + \frac{z^3}{\gamma}\left[\frac{(L_{\alpha}^{m}f(z))''}{L_{\alpha}^{m}f(z)} - \left(\frac{(L_{\alpha}^{m}f(z))'}{L_{\alpha}^{m}f(z)}\right)^2\right] \prec h(z), \qquad z \in U \tag{7}$$

*holds, then* $z^2\frac{(L_{\alpha}^{m}f(z))'}{L_{\alpha}^{m}f(z)} \prec g(z)$, $z \in U$. *This result is sharp.*

**Theorem 2.6.** *Let h be an holomorphic function which satisfies the inequality* Re $\left(1 + \frac{zh''(z)}{h'(z)}\right) > -\frac{1}{2}$, $z \in U$, *and* $h(0) = 1$. *If* $\alpha \geq 0$, $\gamma \in \mathbb{C}\backslash\{0\}$ *be a complex number with Re* $\gamma \geq 0$, $n, m \in \mathbb{N}$, $f \in \mathcal{A}_n$ *and satisfies the differential subordination*

$$\frac{(\gamma+2)z^2}{\gamma}\frac{(L_{\alpha}^{m}f(z))'}{L_{\alpha}^{m}f(z)} + \frac{z^3}{\gamma}\left[\frac{(L_{\alpha}^{m}f(z))''}{L_{\alpha}^{m}f(z)} - \left(\frac{(L_{\alpha}^{m}f(z))'}{L_{\alpha}^{m}f(z)}\right)^2\right] \prec h(z), \qquad z \in U, \tag{8}$$

*then* $z^2\frac{(L_{\alpha}^{m}f(z))'}{L_{\alpha}^{m}f(z)} \prec q(z)$, $z \in U$, *where* $q(z) = \frac{\gamma}{nz^{\gamma/n}}\int_0^z h(t)t^{\gamma/n-1}dt$. *The function q is convex and it is the best dominant.*

**Theorem 2.7.** *Let $g$ be a convex function such that $g(0) = 1$ and let $h$ be the function $h(z) = g(z)+nzg'(z)$, $z \in U$. If $\alpha \geq 0$, $n, m \in \mathbb{N}$, $f \in \mathcal{A}_n$ and the differential subordination*

$$1 - \frac{L_\alpha^m f(z) \cdot (L_\alpha^m f(z))''}{\left[(L_\alpha^m f(z))'\right]^2} \prec h(z), \quad z \in U \tag{9}$$

*holds, then $\frac{L_\alpha^m f(z)}{z(L_\alpha^m f(z))'} \prec g(z)$, $z \in U$. This result is sharp.*

**Theorem 2.8.** *Let $h$ be an holomorphic function which satisfies the inequality* $\mathrm{Re}\left(1 + \frac{zh''(z)}{h'(z)}\right) > -\frac{1}{2}$, *$z \in U$, and $h(0) = 1$. If $\alpha \geq 0$, $n, m \in \mathbb{N}$, $f \in \mathcal{A}_n$ and satisfies the differential subordination*

$$1 - \frac{L_\alpha^m f(z) \cdot (L_\alpha^m f(z))''}{\left[(L_\alpha^m f(z))'\right]^2} \prec h(z), \quad z \in U, \tag{10}$$

*then $\frac{L_\alpha^m f(z)}{z(L_\alpha^m f(z))'} \prec q(z)$, $z \in U$, where $q(z) = \frac{1}{nz^{\frac{1}{n}}} \int_0^z h(t)t^{\frac{1}{n}-1}dt$. The function $q$ is convex and it is the best dominant.*

**Corollary 2.2.** *Let $h(z) = \frac{1+(2\beta-1)z}{1+z}$ be a convex function in $U$, where $0 \leq \beta < 1$. If $\alpha \geq 0$, $n, m \in \mathbb{N}$, $f \in \mathcal{A}_n$ and satisfies the differential subordination*

$$1 - \frac{L_\alpha^m f(z) \cdot (L_\alpha^m f(z))''}{\left[(L_\alpha^m f(z))'\right]^2} \prec h(z), \quad z \in U, \tag{11}$$

*then $\frac{L_\alpha^m f(z)}{z(L_\alpha^m f(z))'} \prec q(z)$, $z \in U$, where $q$ is given by $q(z) = (2\beta - 1) + \frac{2(1-\beta)}{nz^{\frac{1}{n}}} \int_0^z \frac{t^{\frac{1}{n}-1}}{1+t}dt$, $z \in U$. The function $q$ is convex and it is the best dominant.*

**Example 2.1.** *Let $h(z) = \frac{1-z}{1+z}$ a convex function in $U$ with $h(0) = 1$ and $Re\left(\frac{zh''(z)}{h'(z)} + 1\right) > -\frac{1}{2}$.*

*Let $f(z) = z + z^2$, $z \in U$. For $n = 1$, $m = 1$, $\alpha = 2$, we obtain*

$$L_2^1 f(z) = -R^1 f(z) + 2S^1 f(z) = -zf'(z) + 2zf'(z) = zf'(z) = z + 2z^2.$$

*Then $\left(L_2^1 f(z)\right)' = 1 + 4z$,*

$$\frac{L_2^1 f(z)}{z\left(L_2^1 f(z)\right)'} = \frac{z + 2z^2}{z(1 + 4z)} = \frac{1 + 2z}{1 + 4z},$$

$$1 - \frac{L_2^1 f(z) \cdot \left(L_2^1 f(z)\right)''}{\left[\left(L_2^1 f(z)\right)'\right]^2} = 1 - \frac{\left(z + 2z^2\right) \cdot 4}{(1 + 4z)^2} = \frac{8z^2 + 4z + 1}{(1 + 4z)^2}.$$

*We have*

$$q(z) = \frac{1}{z} \int_0^z \frac{1 - t}{1 + t}dt = -1 + \frac{2\ln(1 + z)}{z}.$$

*Using Theorem 2.8 we obtain $\frac{8z^2+4z+1}{(1+4z)^2} \prec \frac{1-z}{1+z}$, $z \in U$, induce $\frac{1+2z}{1+4z} \prec -1 + \frac{2\ln(1+z)}{z}$, $z \in U$.*

**Theorem 2.9.** *Let $g$ be a convex function such that $g(0) = 0$ and let $h$ be the function $h(z) = g(z)+nzg'(z)$, $z \in U$. If $\alpha \geq 0$, $n, m \in \mathbb{N}$, $f \in \mathcal{A}_n$ and the differential subordination*

$$\left[(L_\alpha^m f(z))'\right]^2 + L_\alpha^m f(z) \cdot (L_\alpha^m f(z))'' \prec h(z), \quad z \in U \tag{12}$$

*holds, then $\frac{L_\alpha^m f(z) \cdot (L_\alpha^m f(z))'}{z} \prec g(z)$, $z \in U$. This result is sharp.*

**Theorem 2.10.** *Let $h$ be an holomorphic function which satisfies the inequality* $\mathrm{Re}\left(1+\frac{zh''(z)}{h'(z)}\right) > -\frac{1}{2}$, $z \in U$, *and* $h(0)=0$. *If* $\alpha \geq 0$, $n,m \in \mathbb{N}$, $f \in \mathcal{A}_n$ *and satisfies the differential subordination*

$$\left[(L_\alpha^m f(z))'\right]^2 + L_\alpha^m f(z) \cdot (L_\alpha^m f(z))'' \prec h(z), \qquad z \in U, \tag{13}$$

*then* $\frac{L_\alpha^m f(z)\cdot(L_\alpha^m f(z))'}{z} \prec q(z)$, $z \in U$,*where* $q(z) = \frac{1}{nz^{\frac{1}{n}}}\int_0^z h(t)t^{\frac{1}{n}-1}dt$. *The function $q$ is convex and it is the best dominant.*

**Corollary 2.3.** *Let* $h(z) = \frac{1+(2\beta-1)z}{1+z}$ *be a convex function in $U$, where* $0 \leq \beta < 1$. *If* $\alpha \geq 0$, $n,m \in \mathbb{N}$, $f \in \mathcal{A}_n$ *and satisfies the differential subordination*

$$\left[(L_\alpha^m f(z))'\right]^2 + L_\alpha^m f(z) \cdot (L_\alpha^m f(z))'' \prec h(z), \qquad z \in U, \tag{14}$$

*then* $\frac{L_\alpha^m f(z)\cdot(L_\alpha^m f(z))'}{z} \prec q(z)$, $z \in U$, *where $q$ is given by* $q(z) = (2\beta-1) + \frac{2(1-\beta)}{nz^{\frac{1}{n}}}\int_0^z \frac{t^{\frac{1}{n}-1}}{1+t}dt$, $z \in U$. *The function $q$ is convex and it is the best dominant.*

**Example 2.2.** *Let* $h(z) = \frac{1-z}{1+z}$ *a convex function in $U$ with* $h(0) = 1$ *and* $Re\left(\frac{zh''(z)}{h'(z)}+1\right) > -\frac{1}{2}$. *Let* $f(z) = z + z^2$, $z \in U$. *For* $n=1$, $m=1$, $\alpha=2$, *we obtain*

$$L_2^1 f(z) = -R^1 f(z) + 2S^1 f(z) = -zf'(z) + 2zf'(z) = zf'(z) = z + 2z^2, z \in U$$

*Then*

$$\left(L_2^1 f(z)\right)' = 1 + 4z,$$

$$\frac{L_2^1 f(z)\cdot\left(L_2^1 f(z)\right)'}{z} = \frac{\left(z+2z^2\right)(1+4z)}{z} = 8z^2 + 6z + 1,$$

$$\left[\left(L_2^1 f(z)\right)'\right]^2 + L_2^1 f(z)\cdot\left(L_2^1 f(z)\right)'' = (1+4z)^2 + \left(z+2z^2\right)\cdot 4 = 24z^2 + 12z + 1.$$

*We have*

$$q(z) = \frac{1}{z}\int_0^z \frac{1-t}{1+t}dt = -1 + \frac{2\ln(1+z)}{z}.$$

*Using Theorem 2.10 we obtain* $24z^2 + 12z + 1 \prec \frac{1-z}{1+z}$, $z \in U$, *induce*

$$8z^2 + 6z + 1 \prec -1 + \frac{2\ln(1+z)}{z}, \; z \in U.$$

**Theorem 2.11.** *Let $g$ be a convex function such that* $g(0) = 0$ *and let $h$ be the function* $h(z) = g(z) + \frac{nz}{1-\delta}g'(z)$, $z \in U$. *If* $\alpha \geq 0$, $\delta \in (0,1)$, $n,m \in \mathbb{N}$, $f \in \mathcal{A}_n$ *and the differential subordination*

$$\left(\frac{z}{L_\alpha^m f(z)}\right)^\delta \frac{L_\alpha^{m+1} f(z)}{1-\delta}\left(\frac{\left(L_\alpha^{m+1} f(z)\right)'}{L_\alpha^{m+1} f(z)} - \delta\frac{(L_\alpha^m f(z))'}{L_\alpha^m f(z)}\right) \prec h(z), \qquad z \in U \tag{15}$$

*holds, then* $\frac{L_\alpha^{m+1} f(z)}{z}\cdot\left(\frac{z}{L_\alpha^m f(z)}\right)^\delta \prec g(z)$, $z \in U$. *This result is sharp.*

**Theorem 2.12.** *Let $h$ be an holomorphic function which satisfies the inequality* $\mathrm{Re}\left(1+\frac{zh''(z)}{h'(z)}\right) > -\frac{1}{2}$, $z \in U$, *and* $h(0)=1$. *If* $\alpha \geq 0$, $\delta \in (0,1)$, $n,m \in \mathbb{N}$, $f \in \mathcal{A}_n$ *and satisfies the differential subordination*

$$\left(\frac{z}{L_\alpha^m f(z)}\right)^\delta \frac{L_\alpha^{m+1} f(z)}{1-\delta}\left(\frac{\left(L_\alpha^{m+1} f(z)\right)'}{L_\alpha^{m+1} f(z)} - \delta\frac{(L_\alpha^m f(z))'}{L_\alpha^m f(z)}\right) \prec h(z), \qquad z \in U, \tag{16}$$

*then* $\frac{L_\alpha^{m+1}f(z)}{z}\cdot\left(\frac{z}{L_\alpha^m f(z)}\right)^\delta \prec q(z)$, $z\in U$, *where* $q(z)=\frac{1-\delta}{nz^{\frac{1-\delta}{n}}}\int_0^z h(t)t^{\frac{1-\delta}{n}-1}dt$. *The function* $q$ *is convex and it is the best dominant.*

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