TWMS J. App. Eng. Math. V.5, N.1, 2015, pp. 30-45.

GENERALIZED INTUITIONISTIC FUZZY LAPLACE TRANSFORM AND ITS APPLICATION IN ELECTRICAL CIRCUIT

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Abstract. In this paper we describe the generalized intuitionistic fuzzy laplace transform method for solving first order generalized intutionistic fuzzy differential equation. The procedure is applied in imprecise electrical circuit theory problem. Here the initial condition of those applications is taken as Generalized Intuitionistic triangular fuzzy numbers (GITFNs).

Keywords: Fuzzy differential equation, Generalized intuitionistic triangular fuzzy number, First order differential equation.

AMS Subject Classification: 34A07, 44A10, 34-XX

1. Introduction

Zadeh [1] and Dubois and Parade [2] , first introduced the conception of fuzzy number and fuzzy arithmetic. The generalizations of fuzzy sets theory [1] is considered to be one of Intuitionistic fuzzy set (IFS) theory. Out of various higher-order fuzzy sets, IFS was first proposed by Atanassov [3] and these is found to be suitable to deal with unexplored areas.The fuzzy set considers only the degree of belongingness and non belongingness. Fuzzy set theory does not incorporate the degree of hesitation (i.e.,degree of non-determinacy defined as, 1- sum of membership function and non-membership function.To handle such situations, Atanassov [4] explored the concept of fuzzy set theory by intuitionistic fuzzy set (IFS) theory.The degree of acceptance in Fuzzy Sets is only considered, otherwise IFS is characterized by a membership function and a non-membership function so that the sum of both values is less than one [4]. Basic arithmetic operations of TIFNs is defined by Deng-Feng Li in [5] using membership and non membership values. Basic arithmetic operations of TIFNs such as addition,subtraction and multiplication are discussed by Mahapatra and Roy in [6], by considering the six tuple number itself and division by A.Nagoorgani & K.Ponnalagu [7].

Now-a-days, IFSs are being studied widely and being used in different fields of Science and Technology. Amongst the all research works mainly on IFS we can include Atanassov [4,8-11], Atanassov and Gargov [12], Szmidt and Kacprzyk [13], Buhaescu [14], Ban [15], Deschrijver and Kerre [16], Stoyanova [17], Cornelis et al. [18], Buhaesku [19], Gerstenkorn and Manko [20], Stoyanova and Atanassov [21], Stoyanova [22], Mahapatra and Roy [23], Hajeeh [24], Persona et al. [25], Prabha et al. [26], Nikolaidis and Mourelatos [27], Kumar

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§ Manuscript received: February 01, 2014.

et al.[28] and Wang [29], Shaw and Roy [30], Adak et al.[31], A.Varghese and S.Kuriakose [32], S.P.Mondal and T.K.Roy[71].
It is seen that in current years the topic of Fuzzy Differential Equations (FDEs) has been rapidly grown. In the year 1987, the term "fuzzy differential equation" was introduced by Kandel and Byatt [33]. To study FDE there have been many conceptions for the definition of fuzzy derivative. Chang and Zadeh [34] was someone who first introduced the concept of fuzzy derivative, later on it was followed up by Dobois and Prade [35] who used the extension principle in their approach. Other methods have been discussed by Puri and Ralescu [36], Goetschel and Voxman [37], Seikkala [38] and Friedman et al. [39,40], Y. Cano, H. Flores [41], E. Hllermeier [42], H.Y. Lan, J.J. Nieto [43], J.J. Nieto, R. Lpez, D.N. Georgiou [44]. First order linear fuzzy differential equations or systems are researched under various interpretations in several papers (see [45,46,47,48,49]).There are only few papers such as [50,51,52,53,70] in which intuitionistic fuzzy number are applied in differential equation.

Laplace transform is a very useful apparatus to solve differential equation. Laplace transforms give the solution of a differential equations satisfying the initial condition directly without use the general solution of the differential equation. Fuzzy Laplace Transform (FLT) was first introduced by Allahviranloo & Ahmadi [72].Here first order fuzzy differential equation with fuzzy initial condition is solved by FLT. Tolouti & Ahmadi [73] applied the FLT in 2nd order FDE. FLT also used to solve many areas of differential equation. Salahshour et al [74] used FLT in Fuzzy fractional differential equation.Salahshour & Haghi used FLT in Fuzzy Heat Equation. Ahmad et al [75] used FLT in Fuzzy Duffing's Equation.

Fuzzy differential equations play an important preface in the field of biology, engineering, physics as well as among other field of science. For example, in population models [54], civil engineering [55], bioinformatics and computational biology [56], quantum optics and gravity [57] , modeling hydraulic [58], HIV model [59], decay model [60], predator-prey model [61], population dynamics model [62] , Friction model [63], Growth model [64], Bacteria culture model [65], bank account and drug concentration problem [66], barometric pressure problem [67]. First order linear fuzzy differential equations have many applications among all the Fuzzy differential equation.

2. Preliminary concept

Definition 2.1:Fuzzy Set: A fuzzy set $\tilde{A}$ is defined by $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x\epsilon A, \mu_{\tilde{A}}(x)\epsilon[0, 1]\}$. In the pair $(x, \mu_{\tilde{A}}(x))$ the first element belongs to the classical set A, the second element $\mu_{\tilde{A}}(x)$, belongs to the interval $[0, 1]$, called membership function.

Definition 2.2:Height: The height $h(\tilde{A})$, of a fuzzy set $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x\epsilon X\}$, is the largest membership grade obtained by any element in that set i.e., $h(\tilde{A}) = sup\mu_{\tilde{A}}(x)$.

Definition 2.3:Convex Fuzzy sets: A fuzzy set $\tilde{A} = \{(x, \mu_{\tilde{A}}(x))$ is called convex fuzzy set if all A_α for every $\alpha\epsilon[0, 1]$ are convex sets i.e., for every element $x_1\epsilon A_\alpha$ and $x_2\epsilon A_\alpha$ and $\lambda x_1 + (1 - \lambda)\lambda x_2\epsilon A_\alpha\forall\lambda\epsilon[0, 1]$. Otherwise the fuzzy set is called non-convex fuzzy sets.

Definition 2.4:Fuzzy Number: A fuzzy number is an extension of a regular number in the sense that it does not refer to one single value but rather to a connected set of possible values, where each possible value has its own weight between 0 and 1.This weight is called membership function. Thus a fuzzy number is a convex and normal fuzzy set.

Definition 2.5:Intuitionistic Fuzzy set: Let a set X be fixed. An IFS $\tilde{A}^i$ in X is an object having the form $\tilde{A}^i = \{< x, \mu_{\tilde{A}^i}(x), \vartheta_{\tilde{A}^i}(x) > 0 : x\epsilon X\}$, where the $\mu_{\tilde{A}^i}(x) : X \to [0,1]$ and $\vartheta_{\tilde{A}^i}(x) : X \to [0,1]$ define the degree of membership and degree of non-membership respectively, of the element $x\epsilon X$ to the set $\tilde{A}^i$, which is a subset of X, for every element of $x\epsilon X, 0 \leq \mu_{\tilde{A}^i}(x) + \vartheta_{\tilde{A}^i}(x) \leq 1$.

Definition 2.6:(α, β)-level Interval or (α, β)-cuts: A set of (α, β)-cuts, generated by an IFS $\tilde{A}^i$, where $\alpha, \beta\epsilon[0,1]$ are fixed number such that $\alpha + \beta \leq 1$ is defined as
$A_{\alpha,\beta} = \{(x, \mu_{\tilde{A}^i}(x), \vartheta_{\tilde{A}^i}(x)) : x\epsilon X, \mu_{\tilde{A}^i}(x) \geq \alpha, \vartheta_{\tilde{A}^i}(x) \leq \beta, \alpha, \beta\epsilon[0,1]\}$
we define (α, β)-level Interval or (α, β)-cut, denoted by $A_{\alpha,\beta}$, as the crisp of elements x which is belongs to $\tilde{A}^i$ at the least to the degree α and which belong to $\tilde{A}^i$ at most to the degree β.

Definition 2.7:Intuitionistic Fuzzy number: An IFN $\tilde{A}^i$ is defined as follows
(i) an intuitionistic fuzzy subject of real line.
(ii) normal. i.e., there is any $x_0\epsilon R$ such that $\mu_{\tilde{A}^i}(x_0) = 1$(so $\vartheta_{\tilde{A}^i}(x_0) = 0$)
(iii) a convex set for the membership function $\mu_{\tilde{A}^i}(x)$, i.e.,
$\mu_{\tilde{A}^i}(\lambda x_1 + (1-\lambda)x_2) \geq min(\mu_{\tilde{A}^i}(x_1), \mu_{\tilde{A}^i}(x_2))\forall x_1, x_2\epsilon R, \lambda\epsilon[0,1]$
(iv)a concave set for the non-membership function $\vartheta_{\tilde{A}^i}(x)$, i.e.,
$\vartheta_{\tilde{A}^i}(\lambda x_1 + (1-\lambda)x_2) \leq min(\vartheta_{\tilde{A}^i}(x_1), \vartheta_{\tilde{A}^i}(x_2))\forall x_1, x_2\epsilon R, \lambda\epsilon[0,1]$

Definition 2.8: Triangular Intuitionistic Fuzzy number: A TIFN $\tilde{A}^i$ is a subset of IFN in R with following membership function and non membership function as follows:

$$\mu_{\tilde{A}_i}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1} & \text{if } a_1 \leq x \leq a_2 \\ \frac{a_3-x}{a_3-a_2} & \text{if } a_2 \leq x \leq a_3 \\ 0 & \text{otherwise} \end{cases}$$

and

$$\vartheta_{\tilde{A}_i}(x) = \begin{cases} \frac{a_2-x}{a_2-a_1'} & \text{if } a_1' \leq x \leq a_2 \\ \frac{x-a_2}{a_3'-a_2} & \text{if } a_2 \leq x \leq a_3' \\ 1 & \text{otherwise} \end{cases}$$

where $a_1' \leq a_1 \leq a_2 \leq a_3 \leq a_3'$ and TIFN is denoted by $\tilde{A}_{TIFN} = (a_1, a_2, a_3; a_1', a_2, a_3')$

Note 2.1: Here $\mu_{\tilde{A}_i}(x)$ increases with constant rate for $x\epsilon[a_1, a_2]$ and decreases with constant rate for $x\epsilon[a_2, a_3]$ but $\vartheta_{\tilde{A}_i}(x)$ decreases with constant rate for $x\epsilon[a_1', a_2]$ and increases with constant rate for $x\epsilon[a_2, a_3']$

Definition 2.9: (α, β)-level Interval or (α, β)-cuts of a TIFN: If $\tilde{A}_i$ is a TIFN, then (α, β)-level Interval or (α, β)-cuts is given by

$$A_{\alpha,\beta} = \begin{cases} [A_1(\alpha), A_2(\alpha)] & \text{for degree of acceptance } \alpha\epsilon[0,1] \\ [A_1'(\beta), A_2'(\beta)] & \text{for degree of regection } \beta\epsilon[0,1] \end{cases}$$

with $\alpha + \beta \leq 1$.
Here (i) $\frac{dA_1(\alpha)}{d\alpha} > 0$, $\frac{dA_2(\alpha)}{d\alpha} < 0$, $\forall \alpha \epsilon [0,1], A_1(1) \leq A_2(1)$
and (ii) $\frac{dA_1^{'}(\beta)}{d\beta} < 0$, $\frac{dA_2^{'}(\beta}{d\beta} > 0$, $\forall \beta \epsilon [0,1], A_1^{'}(0) \leq A_2^{'}(0)$.
It is expressed as $A_{\alpha,\beta} = \{[A_1(\alpha), A_2(\alpha)]; [A_1^{'}(\beta), A_2^{'}(\beta)]\}, \alpha + \beta \leq 1, \alpha, \beta \epsilon [0,1]$.

For instance, if $\tilde{A}_{iTIFN} = (a_1, a_2, a_3; a_1^{'}, a_2, a_3^{'})$ then (α, β)-level Interval or (α, β)-cuts is given by

$A_{\alpha,\beta} = \{[a_1 + \alpha(a_2 - a_1), a_3 - \alpha(a_3 - a_2)]; [a_2 - \beta(a_2 - a_1^{'}), a_2 + \beta(a_3^{'} - a_2)]\}$,
where $\alpha + \beta \leq 1, \alpha, \beta \epsilon [0,1]$.

Definition 2.10: Generalized Intuitionistic Fuzzy Number: An IFN $\tilde{A}^i$ is defined as follows

(i) an intuitionistic fuzzy subject of real line.

(ii) normal. i.e., there is any $x_0 \epsilon R$ such that
$\mu_{\tilde{A}^i}(x_0) = \omega$(so $\vartheta_{\tilde{A}^i}(x_0) = \sigma$) for $0 < \omega + \sigma \leq 1$.

(iii) a convex set for the membership function $\mu_{\tilde{A}^i}(x)$, i.e.,

$\mu_{\tilde{A}^i}(\lambda x_1 + (1-\lambda)x_2) \geq min(\mu_{\tilde{A}^i}(x_1), \mu_{\tilde{A}^i}(x_2)) \forall x_1, x_2 \epsilon R, \lambda \epsilon [0, \omega]$

(iv)a concave set for the non-membership function $\vartheta_{\tilde{A}^i}(x)$, i.e.,

$\vartheta_{\tilde{A}^i}(\lambda x_1 + (1-\lambda)x_2) \leq min(\vartheta_{\tilde{A}^i}(x_1), \vartheta_{\tilde{A}^i}(x_2)) \forall x_1, x_2 \epsilon R, \lambda \epsilon [\sigma, 1]$

(v) $\mu_{\tilde{A}^i}$ and $\vartheta_{\tilde{A}^i}$ is continuous mapping from R to the closed interval $[0, \omega]$ and $[\sigma, 1]$ respectively and $x_0 \epsilon R$, the relation $0 \leq \mu_{\tilde{A}^i} + \vartheta_{\tilde{A}^i} \leq 1$ holds.

Definition 2.11: Generalized Triangular Intuitionistic Fuzzy number: A GTIFN $\tilde{A}^i$ is a subset of IFN in R with following membership function and non membership function as follows:

$$\mu_{\tilde{A}_i}(x) = \begin{cases} \omega \frac{x-a_1}{a_2-a_1} & \text{if } a_1 \leq x \leq a_2 \\ \omega & \text{if } x = a_2 \\ \omega \frac{a_3-x}{a_3-a_2} & \text{if } a_2 \leq x \leq a_3 \\ 0 & \text{otherwise} \end{cases}$$

and

$$\vartheta_{\tilde{A}_i}(x) = \begin{cases} \sigma \frac{a_2-x}{a_2-a_1^{'}} & \text{if } a_1^{'} \leq x \leq a_2 \\ 0 & \text{if } x = a_2 \\ \sigma \frac{x-a_2}{a_3^{'}-a_2} & \text{if } a_2 \leq x \leq a_3^{'} \\ \sigma & \text{otherwise} \end{cases}$$

where $a_1^{'} \leq a_1 \leq a_2 \leq a_3 \leq a_3^{'}$ and TIFN is denoted by $\tilde{A}_{TIFN} = (a_1, a_2, a_3; a_1^{'}, a_2, a_3^{'})$

Definition 2.12: Non negative GTIFN: A GTIFN
$\tilde{A}^i_{GTIFN} = ((a_1, a_2, a_3; \omega), (a_1^{'}, a_2, a_3^{'}; \sigma))$ iff $a_1^{'} \geq 0$.

Definition 2.13: Equality of two GTIFN: A GTIFN $\widetilde{A}^i_{GTIFN} = ((a_1, a_2, a_3; \omega_1), (a_1^{'}, a_2, a_3^{'}; \sigma_1))$ and $\widetilde{B}^i_{GTIFN} = ((b_1, b_2, b_3; \omega_2), (b_1^{'}, b_2, b_3^{'}; \sigma_2))$ are said to be equal iff $a_1 = b_1$,$a_2 = b_2$,$a_3 = b_3$,$a_1^{'} = b_1^{'}$,$a_3^{'} = b_3^{'}$,$\omega_1 = \omega_2$ and $\sigma_1 = \sigma_2$.

Definition 2.14: α-cut set: α-cut set of a GTIFN $\widetilde{A}^i_{GTIFN} = ((a_1, a_2, a_3; \omega), (a_1^{'}, a_2, a_3^{'}; \sigma))$ is a crisp subset of R which is defined as follows

$A_\alpha = \{x : \mu_{\tilde{A}_i}(x) \geq \alpha\} = [A_1(\alpha), A_2(\alpha)] = [a_1 + \frac{\alpha}{\omega}(a_2 - a_1), a_3 - \frac{\alpha}{\omega}(a_3 - a_2)]$

Definition 2.15: β-cut set: β-cut set of a GTIFN $\widetilde{A}^i_{GTIFN} = ((a_1, a_2, a_3; \omega), (a_1^{'}, a_2, a_3^{'}; \sigma))$ is a crisp subset of R which is defined as follows

$A_\beta = \{x : \vartheta_{\tilde{A}_i}(x) \leq \beta\} = [A_1^{'}(\beta), A_2^{'}(\beta)] = [a_2 - \beta(a_2 - a_1^{'}), a_2 + \beta(a_3^{'} - a_2)]$

Definition 2.16: (α, β)-cut set: (α, β)-cut set of a GTIFN $\widetilde{A}^i_{GTIFN} = ((a_1, a_2, a_3; \omega),$ $(a_1^{'}, a_2, a_3^{'}; \sigma))$ is a crisp subset of R which is defined as follows

$$A_{\alpha,\beta} = \{[A_1(\alpha), A_2(\alpha)]; [A_1^{'}(\beta), A_2^{'}(\beta)]\}, \alpha + \beta \leq \omega, \sigma, \alpha\epsilon[0, \omega], \beta\epsilon[\sigma, 1]$$

Definition 2.17: Addition of two GTIFN: Let two $\widetilde{A}^i_{GTIFN} = ((a_1, a_2, a_3; \omega_1),$ $(a_1^{'}, a_2, a_3^{'}; \sigma_1))$ and $\widetilde{B}^i_{GTIFN} = ((b_1, b_2, b_3; \omega_2), (b_1^{'}, b_2, b_3^{'}; \sigma_2))$ be GTIFN, then the addition of two GTIFN is given by

$$\widetilde{A}^i_{GTIFN} \bigoplus \widetilde{B}^i_{GTIFN} = ((a_1 + b_1, a_2 + b_2, a_3 + b_3; \omega), (a_1^{'} + b_1^{'}, a_2 + b_2, a_3^{'} + b_3^{'}; \sigma))$$

where $\omega = min\{\omega_1, \omega_2\}$ and $\sigma_1 = min\{\sigma_1, \sigma_2\}$.

Definition 2.18: Subtraction of two GTIFN: Let two $\widetilde{A}^i_{GTIFN} = ((a_1, a_2, a_3; \omega_1),$ $(a_1^{'}, a_2, a_3^{'}; \sigma_1))$ and $\widetilde{B}^i_{GTIFN} = ((b_1, b_2, b_3; \omega_2), (b_1^{'}, b_2, b_3^{'}; \sigma_2))$ be GTIFN, then the subtraction of two GTIFN is given by

$$\widetilde{A}^i_{GTIFN} \ominus \widetilde{B}^i_{GTIFN} = ((a_1 - b_3, a_2 - b_2, a_3 - b_1; \omega), (a_1^{'} - b_3^{'}, a_2 - b_2, a_3^{'} - b_1^{'}; \sigma))$$

where $\omega = \min\{\omega_1, \omega_2\}$ and $\sigma_1 = \min\{\sigma_1, \sigma_2\}$.

Definition 2.19: Multiplication by a scalar: Let $\widetilde{A}^i_{GTIFN} = ((a_1, a_2, a_3; \omega),$ $(a_1^{'}, a_2, a_3^{'}; \sigma))$ and k is a scalar then $k\widetilde{A}^i_{GTIFN}$ is also a GTIFN and is defined as

$$k\widetilde{A}^i_{GTIFN} = \begin{cases} ((ka_1, ka_2, ka_3; \omega), (ka_1^{'}, ka_2, ka_3^{'}; \sigma)) & \text{if } k > 0 \\ ((ka_3, ka_2, ka_1; \omega), (ka_3^{'}, ka_2, ka_1^{'}; \sigma)) & \text{if } k < 0 \end{cases}$$

where $0 < \omega, \sigma \leq 1$.

Definition 2.20: Multiplication of two GTIFN: Let two $\widetilde{A}^i_{GTIFN} = ((a_1, a_2, a_3; \omega_1),$ $(a_1^{'}, a_2, a_3^{'}; \sigma_1))$ and $\widetilde{B}^i_{GTIFN} = ((b_1, b_2, b_3; \omega_2), (b_1^{'}, b_2, b_3^{'}; \sigma_2))$ be GTIFN, then the multiplication of two GTIFN is given by

$$\widetilde{A}^i_{GTIFN} \bigotimes \widetilde{B}^i_{GTIFN} = ((a_1b_1, a_2b_2, a_3b_3; \omega), (a_1^{'}b_1^{'}, a_2b_2, a_3^{'}b_3^{'}; \sigma))$$

where $\omega = \min\{\omega_1, \omega_2\}$ and $\sigma_1 = \min\{\sigma_1, \sigma_2\}$.

Definition 2.21: Division of two GTIFN: Let two $\widetilde{A}^i_{GTIFN} = ((a_1, a_2, a_3; \omega_1), (a_1^{'}, a_2, a_3^{'}; \sigma_1))$ and $\widetilde{B}^i_{GTIFN} = ((b_1, b_2, b_3; \omega_2), (b_1^{'}, b_2, b_3^{'}; \sigma_2))$ be GTIFN, then the multiplication of two GTIFN is given by

$$\widetilde{A}^i_{GTIFN}/\widetilde{B}^i_{GTIFN} = ((\tfrac{a_1}{b_3}, \tfrac{a_2}{b_2}, \tfrac{a_3}{b_1}; \omega), (\tfrac{a_1^{'}}{b_3^{'}}, \tfrac{a_2}{b_2}, \tfrac{a_3^{'}}{b_1^{'}}; \sigma))$$

where $\omega = \min\{\omega_1, \omega_2\}$ and $\sigma_1 = \min\{\sigma_1, \sigma_2\}$.

3. Generalized Intutionistic Fuzzy Laplace transform

Theorem 3.1: Let $f(x)$ be a generalized intuitionistic fuzzy number valued function on $[a, \infty]$ and it represented by $(f_1(x, \alpha; \omega), f_2(x, \alpha; \omega); g_1(x, \beta; \sigma), g_2(x, \beta; \sigma)$ where $\alpha\varepsilon[0, \omega], \beta\epsilon[\sigma, 1], 0 \leq \omega, \sigma \leq 1$. Assume $f_1(x, \alpha; \omega), f_2(x, \alpha; \omega), g_1(x, \beta; \sigma)$ and $g_2(x, \beta; \sigma)$ are Riemann-integrable on $[a, b]$ for every $b \geq a$, and assume there are four positive function $M_1(\alpha)$,$M_2(\alpha)$,$N_1(\beta)$ and $N_2(\beta)$ such that,
$\int_a^b |f_1(x, \alpha; \omega)|dx \leq M_1(\alpha)$,$\int_a^b |f_2(x, \alpha; \omega)|dx \leq M_2(\alpha)$,$\int_a^b |g_1(x, \beta; \sigma)|dx \leq N_1(\beta)$ and $\int_a^b |g_2(x, \beta; \sigma)|dx \leq N_2(\beta)$ for every $b \geq a$. Then $f(x)$ is a intutionistic fuzzy Riemann-integrable on $[a, \infty)$ and the intutionistic fuzzy Riemann-integral is a intutionistic fuzzy number.

Furthermore, we have:
$\int_a^\infty f(x)dx = (\int_a^\infty f_1(x, \alpha; \omega)dx, \int_a^\infty f_2(x, \alpha; \omega)dx; \int_a^\infty g_1(x, \beta; \sigma)dx, \int_a^\infty g_2(x, \beta; \sigma))dx)$

Theorem 3.2: Let $f(x)$ be a continuous intutionistic fuzzy valued function. Suppose that $f(x) \odot e^{-px}$ is improper fuzzy Rimann-integrable on $[0, \infty)$, then $\int_0^\infty f(x) \odot e^{-px}dx$ is called intutionstic fuzzy Laplace transforms and is denoted by:
$\mathbf{L}[f(x)] = \int_0^\infty f(x) \odot e^{-px}dx$ (p is > 0 and integer)
We have
$\int_0^\infty f(x) \odot e^{-px}dx = (\int_0^\infty f_1(x, \alpha; \omega) \odot e^{-px}dx, \int_0^\infty f_2(x, \alpha; \omega) \odot e^{-px}dx; \int_0^\infty g_1(x, \beta; \sigma) \odot e^{-px}dx, \int_0^\infty g_2(x, \beta; \sigma) \odot e^{-px}dx)$
Also by using the definition of classical Laplace transform
$l[f_1(x, \alpha; \omega)] = \int_0^\infty f_1(x, \alpha; \omega)e^{-px}dx$,$l[f_2(x, \alpha; \omega)] = \int_0^\infty f_2(x, \alpha; \omega)e^{-px}dx$,
$l[g_1(x, \beta; \sigma)] = \int_0^\infty g_1(x, \beta; \sigma)e^{-px}dx$ and $l[g_2(x, \beta; \sigma)] = \int_0^\infty g_2(x, \beta; \sigma)e^{-px}dx$
Then we get
$\mathbf{L}[f(x, \alpha, \beta; \omega, \sigma)] = [l[f_1(x, \alpha; \omega), l[f_2(x, \alpha; \omega); l[g_1(x, \beta; \sigma), l[g_2(x, \beta; \sigma)]$

Theorem 3.3: Let F and G are continuous intutionistic fuzzy valued function and c_1, c_2 are constants. Then $\mathbf{L}[(c_1 \odot F(x)) \oplus (c_2 \odot G(x))] = (c_1 \odot \mathbf{L}[F(x)]) \oplus (c_2 \odot \mathbf{L}[G(x)])$

Theorem 3.4: Let F is continuous intutionistic fuzzy valued function on $[0, \infty)$ and $\lambda\epsilon R$. Then $\mathbf{L}[\lambda \odot F(x)] = \lambda \odot \mathbf{L}[F(x)]$

Theorem 3.5: Let f is continuous intutionistic fuzzy valued function and $q(x) \geq 0$ is real valued function. Suppose that $(f(x) \odot q(x)) \odot e^{-px}$ is improper fuzzy Riemann-integrable on $[0, \infty)$, then
$\int_0^\infty (f(x) \odot q(x)) \odot e^{-px}dx$

$= (\int_0^\infty q(x) f_1(x,\alpha;\omega) \odot e^{-px} dx, \int_0^\infty q(x) f_2(x,\alpha;\omega) \odot e^{-px} dx; \int_0^\infty q(x) g_1(x,\beta;\sigma) \odot e^{-px} dx,$
$\int_0^\infty q(x) g_2(x,\beta;\sigma) \odot e^{-px} dx)$

Theorem 3.6: Let f is continuous intutionistic fuzzy valued function and $\mathbf{L}[f(x)] = F(p)$, Then
$\mathbf{L}[e^{ax} \odot f(x)] = F(p-a)$
where e^{ax} is real valued function and $p - a > 0$.

Theorem 3.7: Let $f : R \to E$ be a function and denote
$f(x) = (f_1(x,\alpha;\omega), f_2(x,\alpha;\omega); g_1(x,\beta;\sigma), g_2(x,\beta;\sigma)$ for each $\alpha\epsilon[0,\omega], \beta\epsilon[\sigma,1], 0 < \omega, \sigma \leq 1$ then

(1) If f is (i)-gH differentiable then $f_1(x,\alpha;\omega)$,$f_2(x,\alpha;\omega)$ and $g_1(x,\beta;\sigma)$ and $g_2(x,\beta;\sigma)$ are differentiable functions and
$f^{'}(x) = (f_1^{'}(x,\alpha;\omega), f_2^{'}(x,\alpha;\omega); g_1^{'}(x,\beta;\sigma), g_2^{'}(x,\beta;\sigma))$
(2) If f is (ii)-gH differentiable then $f_1(x,\alpha;\omega)$,$f_2(x,\alpha;\omega)$ and $g_1(x,\beta;\sigma)$ and $g_2(x,\beta;\sigma)$ are differentiable functions and
$f^{'}(x) = (f_2^{'}(x,\alpha;\omega), f_1^{'}(x,\alpha;\omega); g_2^{'}(x,\beta;\sigma), g_1^{'}(x,\beta;\sigma))$

Theorem 3.8: Let $f^{'}(x)$ be an integrable fuzzy valued function, and $f(x)$ is the primitive of $f^{'}(x)$ on $[0,\infty)$. Then
$\mathbf{L}[f^{'}(x)] = p \odot \mathbf{L}f(x) -^h f(0)$, when f is (i)-gH differentiable
and $\mathbf{L}[f^{'}(x)] = (-f(0)) -^h (-p \odot \mathbf{L}[f(x)])$ when f is (ii)-gH differentiable.

Proof: For arbitrary fixed $\alpha\varepsilon[0,\omega], \beta\epsilon[\sigma,1], 0 \leq \omega, \sigma \leq 1$ we have

$p \odot \mathbf{L}f(x) -^h f(0)$
$=(pl[f_1(x,\alpha;\omega)] - f_1(0,\alpha;\omega), pl[f_2(x,\alpha;\omega)] - f_2(0,\alpha;\omega); pl[g_1(x,\beta;\sigma)] - g_1(0,\beta;\sigma),$
$pl[g_2(x,\beta;\sigma)] - g_2(0,\beta;\sigma))$

Now, $pl[f_1(x,\alpha;\omega)] - f_1(0,\alpha;\omega) = l[f_1^{'}(x,\alpha;\omega)]$
$pl[f_2(x,\alpha;\omega)] - f_2(0,\alpha;\omega) = l[f_2^{'}(x,\alpha;\omega)]$
$pl[g_1(x,\beta;\sigma)] - g_1(0,\beta;\sigma) = l[g_1^{'}(x,\beta;\sigma)]$
$pl[g_2(x,\beta;\sigma)] - g_2(0,\beta;\sigma) = l[g_2^{'}(x,\beta;\sigma)]$

Therefore,
$p \odot \mathbf{L}f(x) -^h f(0) = (l[f_1^{'}(x,\alpha;\omega)], l[f_2^{'}(x,\alpha;\omega)]; l[g_1^{'}(x,\beta;\sigma)], l[g_2^{'}(x,\beta;\sigma)])$

by linearity property of l

$p \odot \mathbf{L}f(x) -^h f(0) = l[f_1^{'}(x,\alpha;\omega), f_2^{'}(x,\alpha;\omega); g_1^{'}(x,\beta;\sigma), g_2^{'}(x,\beta;\sigma)]$

In fuzzy sense we can write
$p \odot \mathbf{L}f(x) -^h f(0) = \mathbf{L}[f^{'}(x)]$

Similarly, we can write

$(-f(0)) -^h (-p \odot \mathbf{L}[f(x)])$

$=(-f_2(0,\alpha;\omega)+pl[f_2(x,\alpha;\omega),-f_1(0,\alpha;\omega)+pl[f_1(x,\alpha;\omega);-g_2(0,\beta;\sigma)+pl[g_2(x,\beta;\sigma)],$
$-g_1(0,\beta;\sigma)+pl[g_1(x,\beta;\sigma)])$

This is equivalent to:
$(pl[f_2(x,\alpha;\omega)-f_2(0,\alpha;\omega),pl[f_1(x,\alpha;\omega)-f_1(0,\alpha;\omega);pl[g_2(x,\beta;\sigma)]-g_2(0,\beta;\sigma),$
$pl[g_1(x,\beta;\sigma)]-g_1(0,\beta;\sigma))$

Therefore,
$(-f(0))-^h(-p\odot\mathbf{L}[f(x)])=(l[f_2'(x,\alpha;\omega)],l[f_1'(x,\alpha;\omega)];l[g_2'(x,\beta;\sigma)],l[g_1'(x,\beta;\sigma)])$
by linearity property of l we have
$(-f(0))-^h(-p\odot\mathbf{L}[f(x)])=l[f_2'(x,\alpha;\omega),f_1'(x,\alpha;\omega);g_2'(x,\beta;\sigma),g_1'(x,\beta;\sigma)]$
In fuzzy sense we can write
$(-f(0))-^h(-p\odot\mathbf{L}[f(x)])=\mathbf{L}[f'(x)]$

4. Application

4.1. **Application 1:** An electric circuit consists of a resistor of resistence $R\Omega$ in series with a capacitance C farads, a generator of E volts, and a key. At time $t=0$ the key is closed. Assuming that the charge on the capacitor is a Generalized Intuitionistic fuzzy number($=\tilde{Q}_0$) at $t=0$, find the charge and current at any later time. Assume R,C,E to be constant.

Solution: The initial value problem is

$R\frac{dQ}{dt}+\frac{Q}{C}=E(t)$ with $Q(0)=\widetilde{Q}_0=((q_1,q_2,q_3;\omega),(q_1',q_2,q_3';\sigma))$

Case I: Corresponding to (i)-gH differentiability system applying generalized intutionistic fuzzy laplace transform we have

$R[s\mathbf{L}[Q^i(t)]\ominus Q^i(0)]+\frac{1}{C}\mathbf{L}[Q^i(t)]=\mathbf{L}[E(t)]$

with initial conditions

$$Q_1(0,\alpha;\omega)=q_1+\frac{\alpha l_{\tilde{Q}_0}}{\omega}$$
$$Q_2(0,\alpha;\omega)=q_3-\frac{\alpha r_{\tilde{Q}_0}}{\omega}$$
$$Q_1'(0,\beta;\sigma)=q_1'+\frac{\beta l_{\tilde{Q}_0}'}{\sigma}$$
$$Q_2'(0,\beta;\sigma)=q_3'-\frac{\beta r_{\tilde{Q}_0}'}{\sigma}$$

where $l_{\tilde{Q}_0}=q_2-q_1$,$r_{\tilde{Q}_0}=q_3-q_2$,$l_{\tilde{Q}_0}'=q_2-q_1'$ and $r_{\tilde{Q}_0}'=q_3'-q_2$

In crisp sense this can be written as

$$R\frac{dQ_1(t,\alpha;\omega)}{dt}+\frac{Q_1(t,\alpha;\omega)}{C}=E(t)$$
$$R\frac{dQ_2(t,\alpha;\omega)}{dt}+\frac{Q_2(t,\alpha;\omega)}{C}=E(t)$$
$$R\frac{dQ_1'(t,\beta;\sigma)}{dt}+\frac{Q_1'(t,\beta;\sigma)}{C}=E(t)$$
$$R\frac{dQ_2'(t,\beta;\sigma)}{dt}+\frac{Q_2'(t,\beta;\sigma)}{C}=E(t)$$

with same initial condition.
After solving the above equations we get

$$Q_1(t,\alpha;\omega) = l^{-1}\{\frac{l\{E(t)\}+R(q_1+\frac{\alpha l_{\tilde{Q}_0}}{\omega})}{Rs+\frac{1}{C}}\}$$
$$Q_2(t,\alpha;\omega) = l^{-1}\{\frac{l\{E(t)\}+R(q_3-\frac{\alpha r_{\tilde{Q}_0}}{\omega})}{Rs+\frac{1}{C}}\}$$
$$Q_1^{'}(t,\beta;\sigma) = l^{-1}\{\frac{l\{E(t)\}+R(q_1^{'}+\frac{\beta l_{\tilde{Q}_0}^{'}}{\sigma})}{Rs+\frac{1}{C}}\}$$
$$Q_2^{'}(t,\beta;\sigma) = l^{-1}\{\frac{l\{E(t)\}+R(q_3^{'}-\frac{\beta r_{\tilde{Q}_0}^{'}}{\sigma})}{Rs+\frac{1}{C}}\}$$

Case II: Corresponding to (ii)-gH differentiability system applying generalized intutionistic fuzzy laplace transform we have

$$R[(-Q^i(0))\ominus(-s\mathbf{L}[Q^i(t)])] + \frac{1}{C}\mathbf{L}[Q^i(t)] = \mathbf{L}[E(t)]$$

with initial conditions

$$Q_1(0,\alpha;\omega) = q_1 + \frac{\alpha l_{\tilde{Q}_0}}{\omega}$$
$$Q_2(0,\alpha;\omega) = q_3 - \frac{\alpha r_{\tilde{Q}_0}}{\omega}$$
$$Q_1^{'}(0,\beta;\sigma) = q_1^{'} + \frac{\beta l_{\tilde{Q}_0}^{'}}{\sigma}$$
$$Q_2^{'}(0,\beta;\sigma) = q_3^{'} - \frac{\beta r_{\tilde{Q}_0}^{'}}{\sigma}$$

where $l_{\tilde{Q}_0} = q_2 - q_1$,$r_{\tilde{Q}_0} = q_3 - q_2$,$l_{\tilde{Q}_0}^{'} = q_2 - q_1^{'}$ and $r_{\tilde{Q}_0}^{'} = q_3^{'} - q_2$

In crisp sense this can be written as

$$R\frac{dQ_1(t,\alpha;\omega)}{dt} + \frac{Q_2(t,\alpha;\omega)}{C} = E(t)$$
$$R\frac{dQ_2(t,\alpha;\omega)}{dt} + \frac{Q_1(t,\alpha;\omega)}{C} = E(t)$$
$$R\frac{dQ_1^{'}(t,\beta;\sigma)}{dt} + \frac{Q_2^{'}(t,\beta;\sigma)}{C} = E(t)$$
$$R\frac{dQ_2^{'}(t,\beta;\sigma)}{dt} + \frac{Q_1^{'}(t,\beta;\sigma)}{C} = E(t)$$

with same initial condition.
After solving the above equation we get

$$Q_1(t,\alpha;\omega) = l^{-1}\{\frac{l\{E(t)\}}{(\frac{1}{C}+Rs)} + \frac{R}{C}\frac{(q_3-\frac{\alpha r_{\tilde{Q}_0}}{\omega})}{(\frac{1}{C^2}-R^2s^2)} - R^2s\frac{(q_1+\frac{\alpha l_{\tilde{Q}_0}}{\omega})}{(\frac{1}{C^2}-R^2s^2)}\}$$
$$Q_2(t,\alpha;\omega) = l^{-1}\{\frac{l\{E(t)\}}{(\frac{1}{C}+Rs)} - R^2s\frac{(q_3-\frac{\alpha r_{\tilde{Q}_0}}{\omega})}{(\frac{1}{C^2}-R^2s^2)} + \frac{R}{C}\frac{(q_1+\frac{\alpha l_{\tilde{Q}_0}}{\omega})}{(\frac{1}{C^2}-R^2s^2)}\}$$
$$Q_1^{'}(t,\beta;\sigma) = l^{-1}\{\frac{l\{E(t)\}}{(\frac{1}{C}+Rs)} + \frac{R}{C}\frac{(q_2+\frac{\beta r_{\tilde{Q}_0}^{'}}{\sigma})}{(\frac{1}{C^2}-R^2s^2)} - R^2s\frac{(q_2-\frac{\beta l_{\tilde{Q}_0}^{'}}{\sigma})}{(\frac{1}{C^2}-R^2s^2)}\}$$

$$Q_2'(t,\beta;\sigma) = l^{-1}\{\frac{l\{E(t)\}}{(\frac{1}{C}+Rs)} - R^2 s\frac{(q_2+\frac{\beta r'_{\tilde{Q}_0}}{\sigma})}{(\frac{1}{C^2}-R^2s^2)} + \frac{R}{C}\frac{(q_2-\frac{\beta l'_{\tilde{Q}_0}}{\sigma})}{(\frac{1}{C^2}-R^2s^2)}\}$$

Numerical example:

If $R=4\Omega$, $c=0.25f$, $E(t)=4e^{2t}$ and $Q(0)=\widetilde{Q}_0=((5,6,7;0.7),(4.5,6,7.5;0.2))$ then find $Q(t)$ after $t=2$ seconds.

Solution: If we consider (i)-gH differentiable then the solution is given by
$Q_1(t,\alpha;0.7)=(\frac{14}{3}+\frac{10}{7}\alpha)e^{-t}+\frac{1}{3}e^{2t}$
$Q_2(t,\alpha;0.7)=(\frac{20}{3}-\frac{10}{7}\alpha)e^{-t}+\frac{1}{3}e^{2t}$
$Q_1'(t,\beta;0.7)=(\frac{17}{3}-\frac{15}{2}\beta)e^{-t}+\frac{1}{3}e^{2t}$
$Q_2'(t,\beta;0.7)=(\frac{17}{3}+\frac{15}{2}\beta)e^{-t}+\frac{1}{3}e^{2t}$

TABLE 1. Value of $Q_1(t,\alpha;0.7), Q_2(t,\alpha;0.7), Q_1'(t,\beta;0.2), Q_2'(t,\beta;0.2)$ for different α,β at $t=2$

α	$Q_1(t,\alpha;0.7)$	$Q_2(t,\alpha;0.7)$	β	$Q_1'(t,\beta;0.2)$	$Q_2'(t,\beta;0.2)$
0	18.8309	19.1016	0.2	18.7633	19.1693
0.1	18.8503	19.0823	0.3	18.6618	19.2708
0.2	18.8696	19.0630	0.4	18.5603	19.3723
0.3	18.8889	19.0436	0.5	18.4588	19.4738
0.4	18.9083	19.0243	0.6	18.3573	19.5753
0.5	18.9276	19.0050	0.7	18.2558	19.6768
0.6	18.9469	18.9856	0.8	18.1543	19.7783
0.7	18.9663	18.9663	0.9	18.0528	19.8798
			1	17.9513	19.9813

4.2. **Application 2:** Consider an electrical LR circuit with AC source:

$\frac{dI(t)}{dt}=-\frac{R}{L}I(t)+v(t)$, $0\le t\le 1$,

subject to the initial condition $I(0)=\widetilde{u}=((u_1,u_2,u_3;\lambda),(u_1',u_2,u_3';\eta))$

Solution:
Case I: Corresponding to (i)-gH differentiability system applying fuzzy Laplace transform we have

$s\mathbf{L}[I^i(t)]\ominus I^i(0)=-\frac{R}{L}\mathbf{L}[I^i(t)]+\mathbf{L}[v(t)]$

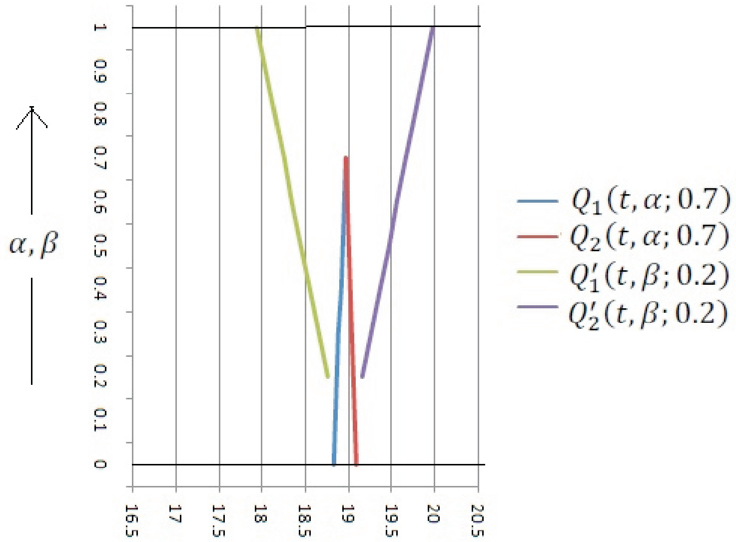

FIGURE 1. Graph of $Q_1(t,\alpha;0.7), Q_2(t,\alpha;0.7), Q_1^{'}(t,\beta;0.2), Q_2^{'}(t,\beta;0.2)$ for different α, β for $t = 2$

with initial condition

$$
\begin{aligned}
I_1(0,\alpha;\lambda) &= u_1 + \frac{\alpha l_{\tilde{u}_0}}{\lambda} \\
I_2(0,\alpha;\lambda) &= u_3 - \frac{\alpha r_{\tilde{u}_0}}{\lambda} \\
I_1^{'}(0,\beta;\eta) &= u_2 - \frac{\beta l_{\tilde{u}_0}^{'}}{\lambda} \\
I_2^{'}(0,\beta;\eta) &= u_2 + \frac{\beta r_{\tilde{u}_0}^{'}}{\lambda}
\end{aligned}
$$

In crisp sense we can write

$$
\begin{aligned}
\frac{dI_1(t,\alpha;\lambda)}{dt} &= -\frac{R}{L} I_2(t,\alpha;\lambda) + v(t) \\
\frac{dI_2(t,\alpha;\lambda)}{dt} &= -\frac{R}{L} I_1(t,\alpha;\lambda) + v(t) \\
\frac{dI_1(t,\beta;\eta)^{'}}{dt} &= -\frac{R}{L} I_2(t,\beta;\eta)^{'} + v(t) \\
\frac{dI_2(t,\beta;\eta)^{'}}{dt} &= -\frac{R}{L} I_1(t,\beta;\eta)^{'} + v(t)
\end{aligned}
$$

with same initial condition.

Solution: The solution can be written as

$$I_1(t,\alpha;\lambda) = l^{-1}\left\{\frac{v(t)}{(s+\frac{R}{L})} + s\frac{(u_1+\frac{\alpha l_{\tilde{u}_0}}{\lambda})}{(s^2-\frac{R^2}{L^2})} - \frac{R}{L}\frac{(u_3-\frac{\alpha r_{\tilde{u}_0}}{\lambda})}{(s^2-\frac{R^2}{L^2})}\right\}$$

$$I_2(t,\alpha;\lambda) = l^{-1}\left\{\frac{v(t)}{(s+\frac{R}{L})} - \frac{R}{L}\frac{(u_1+\frac{\alpha l_{\tilde{u}_0}}{\lambda})}{(s^2-\frac{R^2}{L^2})} + s\frac{(u_3-\frac{\alpha r_{\tilde{u}_0}}{\lambda})}{(s^2-\frac{R^2}{L^2})}\right\}$$

$$I_1(t,\beta;\eta)^{'} = l^{-1}\left\{\frac{v(t)}{(s+\frac{R}{L})} + s\frac{(u_2-\frac{\beta l_{\tilde{u}_0}^{'}}{\lambda})}{(s^2-\frac{R^2}{L^2})} - \frac{R}{L}\frac{(u_2+\frac{\beta r_{\tilde{u}_0}^{'}}{\lambda})}{(s^2-\frac{R^2}{L^2})}\right\}$$

$I_2(t,\beta;\eta)' = l^{-1}\{\frac{v(t)}{(s+\frac{R}{L})} - \frac{R}{L}\frac{(u_2-\frac{\beta l'_{\tilde{u}_0}}{\lambda})}{(s^2-\frac{R^2}{L^2})} + s\frac{(u_2+\frac{\beta r'_{\tilde{u}_0}}{\lambda})}{(s^2-\frac{R^2}{L^2})}\}$

Case II: Corresponding to (ii)-gH differentiability system applying fuzzy Laplace transform we have

$(-I^i(0)) \ominus (-s\mathbf{L}[I^i(t)]) = -\frac{R}{L}\mathbf{L}[I^i(t)] + \mathbf{L}[v(t)]$

with initial condition

$I_1(0,\alpha;\lambda) = u_1 + \frac{\alpha l_{\tilde{u}_0}}{\lambda}$
$I_2(0,\alpha;\lambda) = u_3 - \frac{\alpha r_{\tilde{u}_0}}{\lambda}$
$I'_1(0,\beta;\eta) = u_2 - \frac{\beta l'_{\tilde{u}_0}}{\lambda}$
$I'_2(0,\beta;\eta) = u_2 + \frac{\beta r'_{\tilde{u}_0}}{\lambda}$

In crisp sense we can write

$\frac{dI_1(t,\alpha;\lambda)}{dt} = -\frac{R}{L}I_1(t,\alpha;\lambda) + v(t)$
$\frac{dI_2(t,\alpha;\lambda)}{dt} = -\frac{R}{L}I_1(t,\alpha;\lambda) + v(t)$
$\frac{dI_1(t,\beta;\eta)'}{dt} = -\frac{R}{L}I_1(t,\beta;\eta)' + v(t)$
$\frac{dI_2(t,\beta;\eta)'}{dt} = -\frac{R}{L}I_2(t,\beta;\eta)' + v(t)$

with same initial condition.

Solution: The solution can be written as
$I_1(t,\alpha;\lambda) = l^{-1}\{\frac{lv(t)}{(s+\frac{R}{L})} + \frac{(u_1+\frac{\alpha l_{\tilde{u}_0}}{\lambda})}{(s+\frac{R}{L})}\}$
$I_2(t,\alpha;\lambda) = l^{-1}\{\frac{lv(t)}{(s+\frac{R}{L})} + \frac{(u_3-\frac{\alpha r_{\tilde{u}_0}}{\lambda})}{(s+\frac{R}{L})}\}$
$I_1(t,\beta;\eta)' = l^{-1}\{\frac{lv(t)}{(s+\frac{R}{L})} + \frac{(u_2-\frac{\beta l'_{\tilde{u}_0}}{\lambda})}{(s+\frac{R}{L})}\}$
$I_2(t,\beta;\eta)' = l^{-1}\{\frac{lv(t)}{(s+\frac{R}{L})} + \frac{(u_2+\frac{\beta r'_{\tilde{u}_0}}{\lambda})}{(s+\frac{R}{L})}\}$

Numerical example:

If $R = 1$ Ohm,$L = 1$ Henry,$v(t) = sint$ and $u(t=0) = ((\frac{24}{25}, 1, \frac{101}{100}; 0.6), (\frac{23}{25}, 1, 102100; 0.3))$ then find $I(t)$ at $t = 0.5$ sec.

Solution: If we consider (ii)-gH differentiability concept then the solution is written as

$I_1(t,\alpha;0.6) = \frac{1}{2}(sint - cost) + \frac{1}{2}e^{-t} + (\frac{24}{25} + \frac{1}{15}\alpha)e^{-t}$
$I_2(t,\alpha;0.6) = \frac{1}{2}(sint - cost) + \frac{1}{2}e^{-t} + (\frac{101}{100} - \frac{1}{60}\alpha)e^{-t}$
$I'_1(t,\beta;0.3) = \frac{1}{2}(sint - cost) + \frac{1}{2}e^{-t} + (1 - \frac{4}{15}\beta)e^{-t}$
$I'_2(t,\beta;0.6) = \frac{1}{2}(sint - cost) + \frac{1}{2}e^{-t} + (1 + \frac{1}{15}\beta)e^{-t}$

TABLE 2. Value of $I_1(t,\alpha;0.6), I_2(t,\alpha;0.6), I_1^{'}(t,\beta;0.3), I_2^{'}(t,\beta;0.3)$ for different α,β at $t=0.5$

α	$I_1(t,\alpha;0.6)$	$I_2(t,\alpha;0.6)$	β	$I_1^{'}(t,\beta;0.3)$	$I_2^{'}(t,\beta;0.3)$
0	0.3899	0.4202	0.3	0.3657	0.4263
0.1	0.3940	0.4192	0.4	0.3495	0.4304
0.2	0.3980	0.4182	0.5	0.3333	0.4344
0.3	0.4020	0.4172	0.6	0.3171	0.4384
0.4	0.4061	0.4162	0.7	0.3010	0.4425
0.5	0.4101	0.4152	0.8	0.2848	0.4465
0.6	0.4142	0.4142	0.9	0.2686	0.4506
			1	0.2524	0.4546

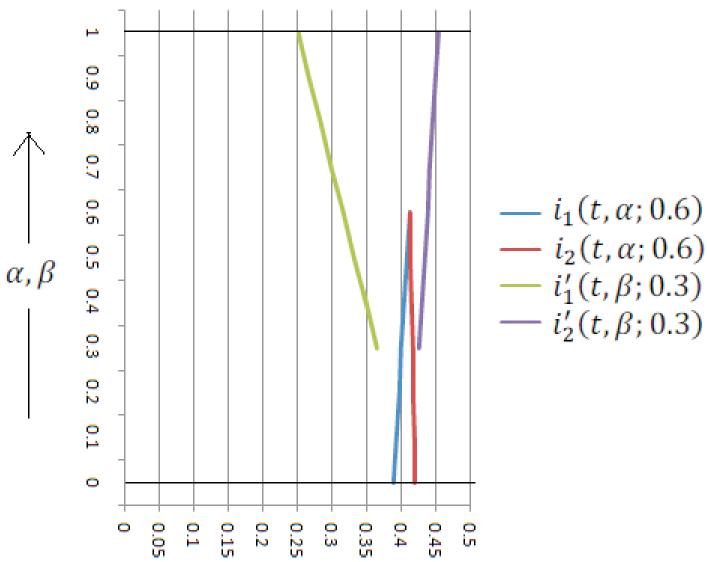

FIGURE 2. Graph of $I_1(t,\alpha;0.6), I_2(t,\alpha;0.6), I_1^{'}(t,\beta;0.3), I_2^{'}(t,\beta;0.3)$ for different α,β for $t=0.5$

5. Conclusion and future research

In this paper we solve the first order linear generalized intutionistic fuzzy differential equation by Generalized intutionistic fuzzy laplace transform method. We apply this procedure in two different imprecise electrical circuit problem. In future we can solve n-th order generalized intutionistic fuzzy linear or nonlinear differential equation by Generalized intutionistic fuzzy laplace transform and apply in different models of science and engineering with uncertainty.

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