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ARTICLES

Compressional Alfvén cross–field surface waves in inhomogeneous dusty plasmas

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Compressional Alfvén surface waves in an inhomogeneous dusty plasma are studied. The inhomogeneity is modeled by two distinct regions of dusty plasmas with different ion densities. The stationary external magnetic field is along the interface between the two plasmas. The dispersion properties of cross-field surface waves, impossible in dust-free plasmas, are obtained for the constant dust charge case. The existence of the surface waves is due to an imbalance in the electron and ion Hall currents in a dusty plasma. © *1999 American Institute of Physics.* [S1070-664X(99)00207-4]

I. INTRODUCTION

Low-frequency magnetohydrodynamic waves play an important role in many physical processes in laboratory and space plasmas.[1] In particular, Alfvén and magnetoacoustic waves are used for the heating of fusion plasmas,[2] and have been successfully invoked in theoretical models for various astrophysical phenomena such as solar and stellar winds and coronae, micropulsations in the Earth's magnetosphere, etc.[3] It is well known that in regions with strong nonuniformity or discontinuity localized compressional or shear Alfvén surface waves (SWs) can exist.[4] The electromagnetic energy of the SWs is confined to the regions where such nonuniformities exist.[5]

Stable discontinuous structures with sharp interfaces can be formed in dusty space and laboratory plasmas (planetary rings, voids, etc.).[6–9] The dust grains are usually negatively charged due to electron attachment, and they can carry a large proportion of the negative charge, and so affect the dispersion properties and damping of the Alfvén SWs propagating obliquely to the external magnetic field in such structures.[10–12] In this paper the special case of cross-field propagation of compressional Alfvén SWs in highly structured dusty plasmas is studied. This case reveals some interesting physics since the existence of transversely propagating Alfvén SWs, impossible in dust-free plasmas,[13] is caused by the dust grains. As is shown in this paper, this becomes possible because of the dust-induced imbalance of the electron and ion Hall currents.[14] This leads to the possibility of wave field localization within the region of sharp discontinuity, and, hence, to the existence of pure surface wave solutions.

II. WAVE EQUATIONS

The inhomogeneous dusty plasma structure is modeled by two regions of distinct particle (electrons, ions and dust grains) densities with a planar interface between them at $x=0$. The region of SW field localization near the interface is of the order of the SW skin depth. It is usually small compared to the sizes of both plasma regions but much larger than that of the transition layer between the two plasmas. Thus, we can treat the structure as two semi-infinite plasmas (regions 1 and 2 for $x>0$ and $x<0$, respectively) separated by a sharp interface. The external magnetic field $\mathbf{B}_0$ is directed along the z axis, which is in the plane of the interface. For simplicity, we shall consider cross-field (along the y axis) surface wave propagation.

The perturbations associated with the SWs are taken to be in the form $A(\mathbf{r},t)=A(x)\exp[i(k_y y-\omega t)]$, where k_y is the wave number and ω is the eigenfrequency. We assume that the frequency of the waves is below the ion cyclotron frequency Ω_i but significantly above the dust cyclotron frequency Ω_d, so the dust grains can be taken to be stationary.

The dust grain size is assumed to be much smaller than the Debye length and the intergrain distance, and the dust is treated as a continuous stationary background of negative charge. In the steady state, we have $en_{i0}-en_{e0}+q_d n_{d0}=0$, where $n_{\alpha 0}$ is the steady-state number density of the species $\alpha=i,e,d$ for the (singly ionized) ions, electrons and dust grains, respectively. Here, $q_d=Z_d e$ is the (negative) charge of the dust grain, and $e>0$ is the magnitude of the electron charge. The charge number on the grain, Z_d, is assumed to

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be negative and constant, i.e., we neglect the effects of grain charging that have been of recent interest.[15] The case of positively charged dust grains ($Z_d > 0$) is also briefly considered.

The linearized equations for the wave motion are

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \tag{1}$$

$$\nabla \times \mathbf{B} = \mu_0 e(n_{i0}\mathbf{v}_i - n_{e0}\mathbf{v}_e), \tag{2}$$

$$\nabla \cdot \mathbf{E} = \frac{1}{\epsilon_0} e(n_i - n_e), \tag{3}$$

$$m_i \frac{\partial \mathbf{v}_i}{\partial t} = e(\mathbf{E} + \mathbf{v}_i \times \mathbf{B}_0), \tag{4}$$

$$0 = \mathbf{E} + \mathbf{v}_e \times \mathbf{B}_0, \tag{5}$$

$$\frac{\partial n_\alpha}{\partial t} + \nabla \cdot (n_{\alpha 0}\mathbf{v}_\alpha) = 0, \tag{6}$$

where n_{e0} and n_{i0} are the equilibrium electron and ion densities, $\mathbf{v}_e$, $\mathbf{v}_i$, n_e, and n_i are the electron and ion velocity and density perturbations, $\mathbf{E}$ and $\mathbf{B}$ are the electric and magnetic fields of the wave, respectively, and the subscript $\alpha = e,i$ corresponds to electron or ion components. In (1)–(6), the electron inertia and displacement current are neglected.[11]

From (1)–(6), one can easily obtain for B_{zj}, E_{yj}, and E_{xj} the following equations:

$$\frac{\partial^2 B_{zj}}{\partial x^2} - \kappa_j^2 B_{zj} = 0, \tag{7}$$

$$E_{yj} = \frac{-iV_{Aj}}{\omega_{pi}(j)\Omega_i\sigma_j}\left(\frac{\partial B_{zj}}{\partial x} + k_y \xi_j \frac{\Omega_i}{\omega} B_{zj}\right), \tag{8}$$

$$E_{xj} = -\frac{i}{k_y}\frac{\partial E_{yj}}{\partial x} - \frac{\omega}{k_y V_{Aj}}\frac{\Omega_i}{\omega_{pi}(j)} B_{zj}, \tag{9}$$

where

$$\kappa_j^2 = k_y^2 - \omega\Omega_i^2\sigma_j / V_{Aj}^2, \tag{10}$$

and $V_{Aj} = c\Omega_i/\omega_{pi}(j)$ is the Alfvén velocity, $\omega_{pi}(j)$ is the ion plasma frequency, $\sigma_j = \omega(1-\Omega_i^2\xi_j^2/\omega^2)/(\Omega_i^2-\omega^2)$, $\xi_j = 1-\delta_j(1-\omega^2/\Omega_i^2)$, and $\delta_j = n_{e0}(j)/n_{i0}(j)$ is the proportion of negative charge carried by electrons in the dusty plasma. In the absence of dust, $\delta_j = 1$. Here, $j = 1,2$ corresponds to the plasma layer considered.

III. SURFACE WAVE SOLUTIONS

We look for solutions of (7) in the form

$$B_{zj} = \sigma_j A_j \exp[(-1)^j \kappa_j x], \tag{11}$$

$$E_{yj} = \frac{-iV_{Aj}}{\omega_{pi}(j)\Omega_i}\left[(-1)^j\kappa_j + k_y\frac{\Omega_i}{\omega}\xi_j\right] A_j \exp[(-1)^j\kappa_j x], \tag{12}$$

which describe SW fields decaying in both regions away from the interface. Using the boundary conditions $E_{y1} = E_{y2}$ and $B_{z1} = B_{z2}$ at the interface $x = 0$, we obtain

$$\frac{n_{i0}(1)\sigma_1}{n_{i0}(2)\sigma_2}\left(\kappa_2 + k_y\frac{\Omega_i}{\omega}\xi_2\right) = -\kappa_1 + k_y\frac{\Omega_i}{\omega}\xi_1, \tag{13}$$

which together with (10) gives the dispersion relation of the SWs.

We consider the case when the plasma is highly structured, i.e., when the densities n_{j0} in both plasma regions differ significantly from each other. Namely, for $n_{i0}(2) \gg n_{i0}(1)$, Eq. (13) reduces to

$$-\kappa_1 + k_y\Omega_i\xi_1/\omega = 0. \tag{14}$$

Equating the squares of the κ_1's from (10) and (14), we obtain

$$\left(1 - \frac{\Omega_i^2\xi_1^2}{\omega^2}\right)\left(k_y^2 - \frac{\omega^2\Omega_i^2}{V_{A1}^2(\Omega_i^2-\omega^2)}\right) = 0. \tag{15}$$

The solutions of (15) yield the SW dispersion relations, provided that the coefficients κ_1 and κ_2 are real and positive.

Setting the first bracket of (15) to zero gives

$$\omega = \Omega_i\xi_1, \tag{16}$$

which leads to a solution

$$\omega = \Omega_m = \Omega_i(1-\delta_1)/\delta_1, \tag{17}$$

$$\kappa_2 = \left[k_y^2 - \frac{\Omega_i^2(\xi_1^2-\xi_2^2)}{V_{A2}^2(1-\xi_1^2)}\right]^{1/2}, \tag{18}$$

and $\kappa_1 = k_y$. This SW mode (mode 1) has the feature $\sigma_1 = 0$, i.e., $\mathbf{B} = 0$, and $E_y \neq 0$.

In the absence of dust ($\delta_1 = 1$), the mode (17) vanishes. If $\delta_1 = \delta_2$, i.e., the proportion of electrons is the same in both regions, then $\xi_1 = \xi_2$ and from (18) it follows that $\kappa_2 = k_y$. The solution is then a valid SW solution with amplitudes decaying on both sides away from the interface. If, however, $\xi_1 > \xi_2$, it follows from (18) that there is a value of k_y at which $\kappa_2 = 0$ and the SW solution ceases to be valid. It is of interest to note that Ω_m is the cutoff frequency for the right hand circularly polarized wave propagating parallel to the magnetic field in a uniform dusty plasma. Another apparent solution of (16) is $\omega = \Omega_i$, however in this case $\sigma_1 = (2\delta_1 - 1)/\Omega_i$, and (10) and (14) cannot both be satisfied, so this solution is extraneous.

A second solution of (15), obtained by setting the second bracket to zero, is given by $k_y = \omega\Omega_i/V_{A1}(\Omega_i^2-\omega^2)^{1/2}$, or

$$\omega = \Omega_i\frac{\alpha}{(1+\alpha^2)^{1/2}}, \tag{19}$$

where κ_1 is given by (10) and we define $\alpha = k_y V_{A1}/\Omega_i$. The inverse skin depths can be expressed as

$$\kappa_1 = k_y\xi_1(1+1/\alpha^2)^{1/2}, \tag{20}$$

$$\kappa_2 = k_y\{1 - r^{-1}[1-\xi_1^2(1+\alpha^{-2})]\}^{1/2}, \tag{21}$$

where $r = n_{i0}(1)/n_{i0}(2)$. This mode (mode 2) has the feature $E_y = 0$ and $B_z \neq 0$. For the case $n_{i0}(2) \gg n_{i0}(1)$, we find that $\kappa_2^2 > 0$, and mode 2 can exist as a SW, only if $\xi_1 > \xi_c$, where

$$\xi_c = \alpha/(1+\alpha^2)^{1/2}, \tag{22}$$

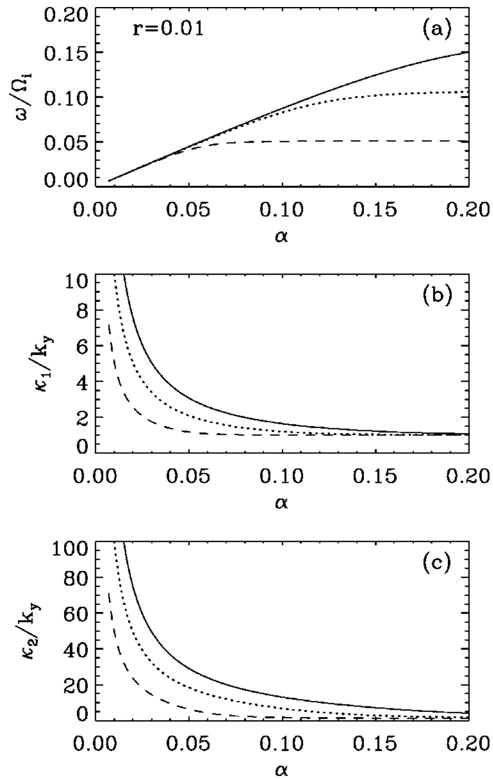

FIG. 1. (a) Normalized surface wave frequency for density ratio $r=0.01$, plotted against the normalized wave number $\alpha=k_yV_{A1}/\Omega_i$. Solid curve for $\delta_1=0.85$, dotted curve for $\delta_1=0.9$, dashed curve for $\delta_1=0.95$. (b) Corresponding normalized inverse skin depth in the lower density plasma. (c) Corresponding normalized inverse skin depth in the higher density plasma.

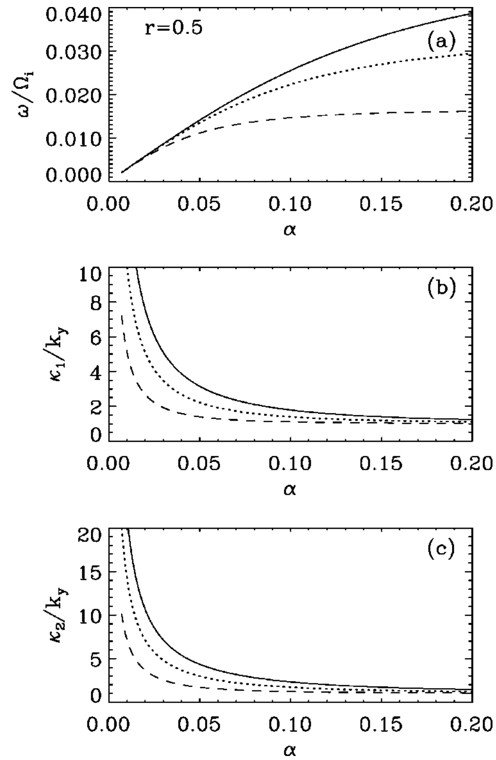

FIG. 2. As for Fig. 1, but with $r=0.5$.

which also defines a critical value for α, namely,

$$\alpha_c\approx\frac{1-\delta_1}{(2\delta_1-1)^{1/2}}, \tag{23}$$

such that mode 2 exists only if $\alpha<\alpha_c$. For small α, we have $\alpha_c\approx\xi_1$.

In the limit of zero density, i.e., a vacuum, in region 1, $V_{A1}\rightarrow\infty$ formally, and $\kappa_1=k_y$ from (10). Also $\alpha\rightarrow\infty$, and so the SW must be in mode 1 with frequency Ω_m. This result can be verified from the SW dispersion relation for oblique propagation on a surface separating a dusty plasma and a vacuum, in the limit $k_z=0$.[11] Thus mode 1 is associated with a very low density in region 1, and the existence of the SW in mode 2 depends on a non-zero density in region 1. Thus care must be taken in assessing the validity of the analytic solutions (17) and (19) for non-zero $n_{i0}(1)$. In fact, a numerical solution of (13) for a range of small but nonvanishing values of r reveals that only one SW mode exists for a given value of α. For $\xi_1<\xi_c$ [very low density $n_{i0}(1)$], the wave is in mode 1, while for $\xi_1>\xi_c$ the wave is in mode 2. Figure 1 shows the SW frequency normalized to Ω_i, and the corresponding inverse skin depths normalized to k_y, as a function of α, for $r=0.01$ and $\delta_1=0.85$, 0.9 and 0.95. The corresponding values of ξ_1 are ≈ 0.15, 0.1, and 0.05, respectively. We have assumed for simplicity that $\delta_2=\delta_1$. For the small values of α considered here, $\xi_c\approx\alpha$. We note that the wave is in mode 2 for small α, with ω given by (19), until $\alpha\approx\xi_1$. From (20) and (21), the inverse skin depths for mode 2 are

$$\kappa_1\approx\frac{k_y\xi_1}{\alpha}\quad\text{and}\quad\kappa_2\approx\frac{k_y\xi_1}{r^{1/2}\alpha} \tag{24}$$

for small α, in agreement with the numerical results. For $\alpha>\xi_1$, the wave is in mode 1, with the asymptotic frequency $\omega\approx\Omega_i\xi_1\approx\Omega_m$, and $\kappa_1\approx\kappa_2\approx k_y$.

For a smaller density difference between the two plasmas, the analytic solutions (17) and (19) may no longer be valid. However, numerical solutions (Fig. 2) for $r=0.5$ show the same overall qualitative dispersion characteristics as the large-density-difference case of Fig. 1. The major difference is that the frequency is reduced by roughly the density ratio.

IV. DISCUSSION AND CONCLUSIONS

We emphasize that the SWs considered here, which are compressional waves propagating transverse to the magnetic field, exist only in dusty plasmas and differ significantly from the usual obliquely propagating compressional and kinetic Alfvén SWs. For low frequencies ($\omega^2\ll\Omega_i^2$, but still satisfying $\omega\gg\Omega_d$), provided the density in region 1 is not too low, the perpendicularly propagating surface wave is in mode 2, with $\omega=k_yV_{A1}$ and $\kappa_1=\Omega_i(1-\delta_1)/V_{A1}$. In the absence of dust ($\delta_1=1$), this mode does not exist since the wave field is not localized in region 2 ($\alpha_c\rightarrow 0$, and $\kappa_2^2<0$). As the frequency increases, the resulting wave perturbations are in mode 1, with an upper limit frequency Ω_m (17), for low density in region 1. The existence of this mode is also possible only in the dusty plasma. For a low density in region 1, the limiting frequency can be of the order of Ω_i for small δ_1. Indeed, we note that for solution (17) to be valid, the electron proportion in plasma 1 cannot become too small. Otherwise, the frequency may become much larger than Ω_i and the effects of electron inertia neglected in (1)–(6) may come into play.

The conclusion on the impossibility of the existence of cross-field SWs in the dust-free case also follows from the analysis of SWs on a dust-free plasma-vacuum interface for a frequency range covering the ion cyclotron frequency,[16] and also by setting $k_z=0$ in the dispersion relation $\omega = V_A k_z[(k_z^2+2k_y^2)/(k_z^2+k_y^2)]^{1/2}$ for surface waves in the low-frequency ($\omega \ll \Omega_i$) dust-free case.[17] The presence of dust makes the localization of the field in region 2 possible ($\kappa_2^2>0$) and, as will be shown below, improves the field localization in the more rarefied region 1 by strongly affecting the skin depth (20). However, the frequency (19) and, hence, the phase velocity of the SWs at low frequency (mode 2) are dust density independent.

Furthermore, we see from (14) that for cross-field propagation of SWs in a plasma with negatively charged dust grains, only solutions with positive wavenumber k_y can exist. That is, SWs can only propagate across the external magnetic field in the positive y direction. We can also consider positively charged dust grains ($Z_d>0$ and $\delta>1$), such as may occur in a plasma with strong radiative photoionization of the grains. In that case, the same SW solutions are found as for the negatively charged grains of the same $|Z_d|$, and for the same $|k_y|$, but for $k_y<0$. This reflects the nonreciprocal nature of waves in dusty magnetized plasmas, i.e., the dispersion characteristics of the forward and backward propagating waves differ from each other, with negative (positive) dust introducing a cutoff in the right (left) hand polarized wave.[11,12] In fact, for SWs propagating at an angle to the external magnetic field, nonreciprocity is strongest for the case considered here, namely, with $\mathbf{k}\perp\mathbf{B}_0$. On the other hand, low frequency Alfvén SWs in a dust-free plasma,[4] which propagate obliquely to $\mathbf{B}_0$, are bidirectional. Ion cyclotron effects in a dust-free plasma also introduce nonreciprocity.[10,16]

We have found that the presence of dust not only leads to the possibility of field localization in region 2, but also essentially improves its localization in region 1. In the dust-free case, the skin depth κ_1^{-1} is much greater than the wavelength, so that the perturbation fields cannot be localized at the interface in region 1. Physically, this occurs because of a magnetic field-induced gyrotropy of the medium, and is analogous to the problem of SWs in the Voigt geometry.[13] Plasma gyrotropy is governed by the ratio $\epsilon_{xy}/\epsilon_{xx}$ (for the present geometry), where ϵ_{xx} and $i\epsilon_{xy}$ are components of the dielectric tensor of the magnetoplasma.[18] In this case, we have[11]

$$\frac{\epsilon_{xy}}{\epsilon_{xx}} = \frac{\omega}{\Omega_i} + \frac{\Omega_i(1-\delta_1)}{\omega}. \tag{25}$$

In the absence of dust ($\delta_1=1$), for low frequencies, we have $\epsilon_{xy}\to 0$ because the electron and ion Hall currents are equal for $\omega \ll \Omega_i$.[4] Since $\kappa_1/k_y=\epsilon_{xy}/\epsilon_{xx}$, the waves are not localized. In the presence of charged dust, which can account for a significant part of the negative charge of the plasma ($\delta_1 <1$), the mismatch of the electron and ion number densities leads to a difference in their respective Hall currents.[14] In this case, ϵ_{xy} is finite and the ratio $\epsilon_{xy}/\epsilon_{xx}$ becomes significant for $\omega \ll \Omega_i$. The skin depth is thus reduced, i.e., the dust particles lead to an improved localization of the electromagnetic field near the interface, as is apparent from the inverse skin depths shown in Figs. 1 and 2. The reduction of the SW skin depth in plasma 1 due to the dust is characterized by the factor $\gamma=\Omega_i^2\xi_1/\omega^2$, which can be large for low frequencies ($\omega^2 \ll \Omega_i^2$). Similar improved localization occurs for impurity-containing or non-neutral plasmas.

An interesting physical feature of oblique Alfvén SWs is that of local Alfvén resonance.[11,14] To briefly discuss this problem, we refine the sharp boundary model by inserting a transition layer $-a<x<a$ in which $n_{0i}=n_{0i}(x)$, where $n_{0i}(x)$ is a suitable density profile. The condition for local Alfvén resonance is that the x component of the wave number goes to ∞, and the SW energy is converted into the shear Alfvén wave. We see that since (10) does not contain singularities, this cannot occur for perpendicular propagation. Moreover, the shear Alfvén wave does not exist if $k_z=0$.[4] Thus the compressional Alfvén surface wave discussed here can propagate undamped by local Alfvén resonant absorption.

To conclude, we have found that the existence of low-frequency compressional Alfvén cross-field flute-like SWs is possible only in the presence of charged dust particles. Therefore, the detection of such low-frequency waves in highly structured inhomogeneous plasmas can imply the presence of charged dust. The SW properties investigated here can be used for the diagnostics of dusty plasmas. For example, the SW field structure and dispersion properties can be used to determine the dust density. On the other hand, since dust grains and sharp interfaces occur in many space and laboratory plasmas, study of the low-frequency SWs can be useful in understanding the formation, evolution, and physical properties of the plasma structures.

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