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A Personal Response to the Euromath Project: Towards the Specification of an IT Infrastructure for the European Mathematical Community

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graphical user interfaces and Boolean Algebra search options. The closest existing systems are those for selecting mathematical software (eg. GAMSIC - [5]). Once the required information has been found, should we provide the user with the technology to do something with it, more than just accessing it {as in the case of mathematical software libraries)?

The creation of personal and group databases is strongly connected with the idea of producing large hypertext networks, called Intermedia [17]. This more clearly focusses a potential problem that the creation of a network of objects contains information in the links as well as in the nodes [10].

2.4 Online Encyclopedia

1 again approve of the idea of creating an online encyclopedia of mathematical concepts. The most natural medium is clearly hypertext. I am not aware of a sizeable system having been implemented, only small systems such as in [26]. It would be a massive project to produce one from sratch (taking hundreds or perhaps thousands of man years). The best book source is probably [15]. I wonder if the authors are thinking along the same lines (yet)?

An online encyclopedia could be used as a front-end to a database network and could also tie into computer aided learning material (see later).

2.5 Algebraic Manipulation

There are several powerful and usable systems currently on the market (eg. Axiom, Derive, Macsyma, Maple, Mathematica and REDUCE). It would be a large (and not necessarily successful) task to attempt to produce a new and better system. An interesting approach might be to classify the functionality required and offered by algebraic manipulator in a similar way to mathematical software algorithms. This might help to centralise and coordinate research in the area. There is a need for a standard interface language.

3 Other Specified Infrastructure Components

3.1 Theorem Proving

The use of theorem-provers by mathematicians is increasing. A generally avaialble theorem-prover supporting standard mathematical logics and theories with a graphical user interface would be of benefit. The need for compatability with formal languages is also clearly important.

3.2 Numerical Computation

A significant proportion of numerical computation is based on using large libraries of proven numerical algorithms. These libraries need to be made accessible and usable to

the general mathematical community. The current research in this area is the development of knowledge-based front-ends, with the long-term goal of creating problem solving environments [11].

I do not feel sufficiently experienced to comment on the use of mathematical programming languages for the computational modelling of mathematical problems, but this is clearly another important area.

3.3 Visualisation

Many mathematicians need to be able to represent and interact with graphical information, including data, functions and images. The science related to this area is called Visualisation or VISC (Visualisation In Scientific Computing). A foundational article on the subject is [20].

3.4 Computer-Aided Learning

Education plays an important role in the working lives of most mathematicians. IT in the form of Computer Aided Learning (CAL) seems destined to play a greater role in mathematical education due to the needs for: increased computer literacy; larger classes; cost reduction; and automation. Computers may even prove to be a more effective medium for education for aspects of the curriculum such as the repetitive acquisition of skills.

The use of the computer terminal for multi-media appears to be a likely future development [2]. Work is underway to produce such material at the first year undergraduate level [14], for which it appears to be particularly suited. There are moves to develop a standard syllabus at this level [3].

Modules of mathematical CAL 'Courseware' could be made available within the mathematician's working environment as an educational or reference resource (linked to the database and the encyclopedia).

4 Infrastructure Design Criteria

The design criteria I would identify are:

Component Identification - I propose the utility of the metaphor of building different system components which correspond to different mathematical processes or modes of working. I have attempted to identify separate components in the classification I have used above.

Integration - the separate components need to be integrated within a single integrated computer environment (eg. using a blackboard architecture).

Accessibility - information needs to be made accessible and manipulatable.

Standards Setting - standards cannot be prescribed too far in advance, but with thought and planning, the overall movement of the technological community can be anticipated.

Maintenance - the need to maintain any implemented system has already been identified by the Euromath Project.

Extendability - versions of 'the system' need designed in such a way that new and more sophisticated components can be easily integrated at a future date. For this reason, we need to have a clear idea of where we are heading.

User Interface - all components need to be implemented with sophisticated graphical user interfaces.

5 Future Trends

5.1 Supporting Mathematical Research

It is hard to define what mathematical research is and how it is best done. People seem to need space and time to think to do it. The classification of Mathematical Knowledge (eg. in creating an online encyclopedia) may help in identifying connections (analogies, models, etc.) between hitherto diverse and unrelated areas. If such hypertext systems are user modifiable, they may actually provide a new medium for mathematical research. Creativity [13] and problem-solving heuristics [18] are not well understood and may never be satisfactorily automated. However, it may be possible for IT to support them in some (more direct) way.

5.2 Modern Technologies

The development and use of new technologies may alter the infrastructure specification. Some such possible technologies are:

Long-Distance Working - the development of technologies to support the formation of working communities separated by physical distance.

Parallel Computing - mathematical programming itself may be affected by the increased power provided by certain kinds of new computer architectures.

Virtual Reality - currently only a highly speculative area [4].

5.3 Knowledge-Based Systems

The development of an IT infrastructure could be viewed as the creation of a massive knowledge-based system incorporating computer networks and reserach communities. This may be a useful metaphor during specification and give insight into long-term objectives. Knowledge acquisition methodologies (such as KADS [7]) could play an important role in this process, due to the recurrence of the 'knowledge acquisition bottleneck' in various guises.

5.4 Artifical Intelligence

I somewhat reluctantly mention AI because I see it, along with IT, as very much as the servant of Mathematics, rather than its master. Computers seem to be much better at doing the boring repetitive tasks, leaving people to concentrate in the intelligent tasks [21]. However, attempts at generating artificial intelligence may help us to understand more clearly what doing Mathematics and being a mathematician is all about in order to support the process with different technologies.

6 Conclusions

A substantial investment is required to produce an IT infrastructure for working mathematicians in which the depth and power of existing Mathematical Knowledge is made accessible and the tasks of mathematicians are most helpfully supported. The Euromath Project represents a valiant start, but there is much that still needs to be achieved.

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