HEALTH CARE SYSTEM DYNAMICS MODELIZATION FOR COVID-19

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(Dated: June 19, 2020)

Abstract

The COVID-19 pandemic has had a major impact worldwide and the health care systems have had to adapt to a new challenging and alarming situation. In this paper, a compartment model that adjusts the hospital dynamics during COVID-19 is presented, with the main objective of making short-term predictions and help inform the ongoing and future response to COVID-19. The fitting of this model has been carried out with France and Belgium data, and it has been analyzed in the time period from 17th of March until June 16th.

I. INTRODUCTION

COVID-19 has meant a new public health crises threatening the world. This virus was originated in bats and was transmitted to humans through yet unknown intermediary animals in Wuhan, Hubei province, China in December 2019. [6] The disease is highly transmitted by inhalation or contact with infected droplets and the incubation period ranges from 2 to 14 days. The symptoms are usually fever, cough, sore throat, breathlessness, fatigue, malaise among others. The disease is mild in most people, and many people is asymptomatic, but in some cases (usually the elderly and people with previous pathologies), it may progress to pneumonia, acute respiratory distress syndrome and multi organ dysfunction. [6]

This new virus outbreak has challenged the medical and public health infrastructure, collapsing the system in many countries. Health care organizations are facing challenges in efficiently accommodating increased patient demand with limited resources and capacity and in consequence they have had to take extraordinary measures to be able to tackle the issue.

Mathematical methods are a powerful tool that can help to face this problem by making future predictions and providing information on patient demand and hospital capacity. Compartmental models are a technique used to simplify the mathematical modelling of infectious diseases where the population is divided into compartments, assuming that each individual in the same compartment has the same characteristics.[5]

In this project, we have designed a compartmental model with the objective of modelling the hospital dynamics during this pandemic and make short-term predictions in the number of hospital patients, people in ICUs, recoveries and deaths. In this compartment model, a fitting of the cumulative cases of COVID-19 is done, and from there, different hospital compartments have been created to simulate the dynamics of the health care system.

To describe the cumulative cases of COVID-19 we employ the Gompertz growing function. This function enables us to analyze the dynamics of the spreading of COVID-19 to make short-time predictions of the new cases for the successive days. The Gompertz equation reads:

$$N(t) = Ke^{-log(\frac{K}{N_0})e^{-at}} \tag{1}$$

where the parameter K corresponds to the final number of cases, N0 is the initial number of cases for the definition of the origin of time, and parameter a is the rate of decrease in the initially exponential growth. [4]

Once the cumulative cases are correctly fitted, the different compartments in the model allow us to make a representation of the hospitalization dynamics of the infected people.

II. COMPARTMENTAL MODEL

A. Model Description

In the attempt of obtaining a compartment model that fits correctly to real data, we have designed and tested out several models, some of them more successfully than others. Nonetheless, we have finally met a compartment model which seems to reproduce fairly well the real behaviour and will be described below.

The model is based on four general compartments. The first one would be **new infected (NI)**, which represent the total cumulative number of infected people by COVID-19 considering asymptomatic cases, mild cases and cases with severe symptoms. The second compartment is **hospitalized patients (H1,H2, ICU)**, in which both people in intensive care units and people in hospitalization units are taken into account. The third is **recovered patients (RR)**, in which we consider infected people who have been hospitalized and overcome the disease (in this compartment, mild cases of infected people that are not hospitalized are not taken into account). The fourth and last compartment is the **deaths (DD)**, where we consider the cases of hospitalized patients that die in hospital. This compartments are related between them by different parameters and rates that regulate the global dynamics of the system. Figure 1 describes the basic scheme of the compartment model.

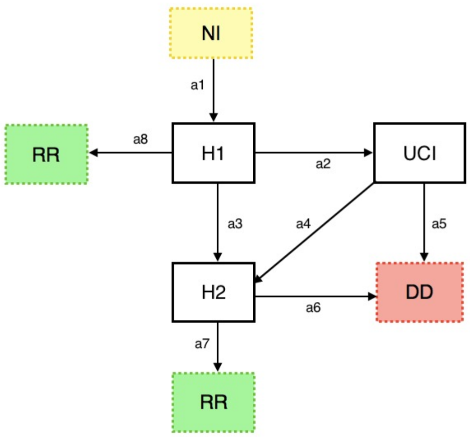

FIG. 1: Model description. The yellow square represents new infected, white squares represent hospitalized patients, green squares are recovered patients and the red one is the deaths. Arrows are used to mark the different transition rates between compartments (a_i)

The differential equations describing the dynamics of hospitalizations, deaths and recoveries are:

$$\frac{dH1}{dt} = a_1 NI - (a_2 + a_3 + a_8)H1 \tag{2}$$

$$\frac{dH2}{dt} = a_3 H1 + a_4 ICU - (a_6 + a_7)H2 \tag{3}$$

$$\frac{dICU}{dt} = a_2 H1 - (a_4 + a_5)ICU \tag{4}$$

$$\frac{dRR}{dt} = a_8 H1 + a_7 H2 \tag{5}$$

$$\frac{dDD}{dt} = a_5 ICU + a_6 H2 \tag{6}$$

The model starts with the daily total number of infected, which is adjusted with the Gompertz function based on data from each country. From the total infected, only a percentage suffer from severe symptoms and need hospitalization. Within the general compartment of the hospital, we distinguish 3 sub-compartments:

1. **First stage of hospitalization (H1)**: this first sub- compartment that acts as a distributor: all infected people who need hospitalization go to H1, and from there, depending on how the disease progresses in each case, patients either recover with rate a8 or are sent to H2 or ICU with rates a_2 and a_3.

2. **Second stage of hospitalization (H2)**: In this sub-compartment there are patients that don't recover in their first days of hospitalization but who don't develop severe enough symptoms to go to the ICU and they are in the process of recovery. There are also patients who have entered the ICU but have overcome the complications of the disease and no longer need intensive care. The rate of leaving ICU and entering H2 is a_4.

3. **ICU**: In this sub-compartment there are the fraction of patients in H1 who need intensive care in order to overcome the disease.

Regarding the recoveries, patients in the hospital can recover in both H1 and H2 sub-compartments. In the initial state of hospitalization, there is a probability that symptoms subside and the patient can be discharged quickly, so patients in H1 recover with rate a_8. If this is not the case, the patient either goes to ICU or to the second state of hospitalization. In H2 patients recover with rate a_7.

Finally, regarding the deaths compartment, the model considers the possibility of dying in H2 and in the ICU with rates a_5 and a_6.

B. Model Adjustment

When implementing this model, we seek to get the best adjustment to the real data. Firstly, we do the fitting on the cumulative cases, and then we find the parameters described previously in the compartment model that provide a better fitting of the number of hospitalizations, recoveries and deaths to the real data.

Code was implemented in MATLAB. Some of the details are highlighted below.

First of all, we carry out the fitting of the reported cases to obtain the best adjust of Gompertz function, which will be used to predict the infected cases later on. Real data is acquired from official sources from France and Belgium [2] [1], updated every day. It should be noted that real data is smoothed by calculating the average of seven days (three days before, three days after, plus the concerning one) for the purpose of avoiding the effects of possible head count unbalances, such as disproportionate increases over the weekend.

To find the parameters that provide the best fitting to real data, we performed the following mathematical optimization technique. The goal was to make a good selection of the best parameter combination from some set of available alternatives.

Optimization problems arise in many quantitative disciplines and the development of solution methods has a huge variety range. In our case, to get the set of available alternatives of parameter combination we performed the latin hypercube sampling (LHS), that is a statistical method for generating a near-random sample of parameter values from a multidimensional distribution. This method is directly implemented from a MATLAB function [3] which allows us to choose how many combinations of a group of desired parameters (eight in our case) are going to be calculated, so that we can finally select the most accurate set.

Therefore, an objective function is required as a way to attain the so called optimal solution. Our objective function is a quadratic error function of the form:

$$E^2 = \frac{(X_{real} - X_{aprox})^2}{X_{realmax}} \tag{7}$$

So starting from real and known initial conditions, we test 20.000 LHS possible parameters combinations in our compartment model settled on round numbers, and compare the results obtained with the real values using the objective function described above and finally we choose the distribution that gives the lowest error and best fits reality.

III. RESULTS

A. Parameters value adjustment

In this section we will focus on the application of the model in France and Belgium, as well as comparing the different parameters obtained in each case. Figure 2 and 3 show the real temporal evolution in the number of hospitalizations, ICUs, recoveries and deaths and predictions made with the compartment model for France and Belgium.

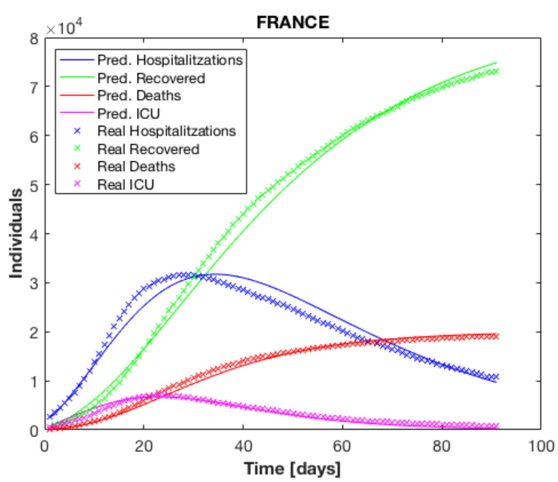

FIG. 2: Daily hospitalizations, ICUs, cumulative discharges and deaths are represented from 17/03/2020 to 15/06/2020 for France.

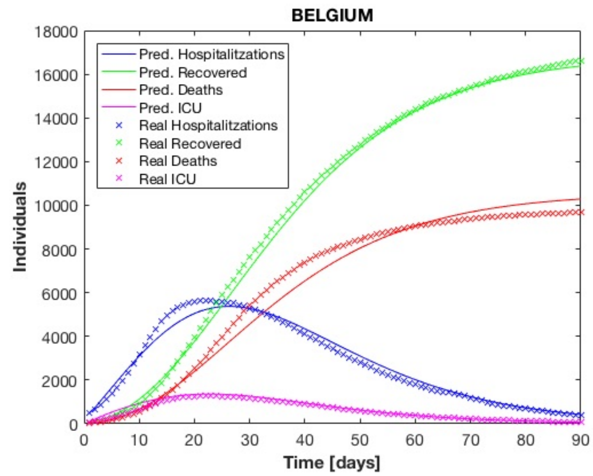

FIG. 3: Daily hospitalizations, ICUs, cumulative discharges and deaths are represented from 17/03/2020 to 15/06/2020 for Belgium.

In the following table, the results of the obtained values for the parameters describing the dynamics of the model (a_i) are collected:

	France	Belguim
a_1	0.70	0.56
a_2	0.37	0.32
a_3	0.41	0.48
a_4	0.067	0.094
a_5	0.06	0.068
a_6	0.001	0.032
a_7	0.03	0.074
a_8	0.24	0.07

Regarding France, we find that the rate of hospitalization of infected individuals is of 0.70.

Once hospitalized, the rate of patients entering ICU is of 0.37 and the rate of patients going to the second stage of hospitalization is 0.41. So we can see that there is a higher probability of going to second hospitalization stage than entering ICU. In the first stage of hospitalization (H1), the rate of recovery is 0.24 and no death possibility is considered.

For patients in ICU, the rate of overcoming complications and entering the second state of hospitalization is 0.067 and the rate of dying is 0.060. So it is observed that in general the stay in the ICU is quite long and the likelihood of dying or improving are quite the same.

Once in the second stage of hospitalization (H2), the rate of recovery is of 0.031 and the rate of dying is 0.0011, which means that few people are on edge of life in this compartment and the probability of obtaining discharge is higher.

On the other hand, in the case of Belgium we find that the rate of hospitalization of infected individuals is of 0.56, a much lower value than the one obtained for France, which means that less new infected individuals require hospitalization.

Once hospitalized, the rate of patients entering ICU is of 0.32 and the rate of patients going to the second stage of hospitalization is 0.48. In the first stage of hospitalization (H1), the rate of recovery is 0.07. From this values we can conclude that the behaviour of first stage hospitalization are similar to the one in France with the difference in recovery rate: in Belgium less people will recover from H1.

For patients in the ICU, the rate of overcoming complications and entering the second state of hospitalization is 0.094 and the rate of dying is 0.068, so it is observed that in general the stay in the ICU is quite less long than the one in France, with a second difference concerning that in Belgium, patients will last longer in ICU before dying than recovering.

Once in the second stage of hospitalization (H2), the rate of recovery is of 0.074 and the rate of dying is 0.032. In this case, individuals in Belgium have more chance to die in H2 than in France but also exists a higher probability of recovery.

On the whole, we could say that the implemented compartment model can be successful for different countries, each one of them with their own optimum parameters that will denote some differences in the hospitalization dynamics. So, by analyzing the rates, we should be able to learn how the dynamics evolve and identify limits and restrictions.

B. Predictions

The final step in this study is the prediction part. Once the proper behaviour of the compartment model have been proved, we could carry out some short-term predictions using previous known data.

We have performed a set of 5 day predictions from the 30th of May until the 15th of June. Predictions are performed taking the real data until a determined day and from it we run the Matlab model to obtain the predicted results for the 5 following days. This has been done for several intervals of days.

The obtained results of the predictions are compared to real data and the quadratic error for each of the 5 days is calculated. The errors obtained from the diferent predictions are represented in a Boxplot graphic. A boxplot is a method for graphically depicting groups of numerical data through their quartiles, which are position measurements used to quantify how concentrated the numerical variable is thet order the numeric variable from least to greatest and divide into groups with the same number of observations. This plot provides the information of the minimum and maximum values of the quartiles, its median, and of the extreme values in the range of typical observations, and of the atypical data. With this data, it is enough to see at a glance important aspects of the distribution. Box plots may also have lines extending from the boxes (whiskers) indicating variability outside the upper and lower quartiles.

The use of Boxplot allows us to get an idea of how accurate our predictions are. In Figures 4 and 5 the Boxplot of the errors obtained from the predictions for France and Belgium are represented.

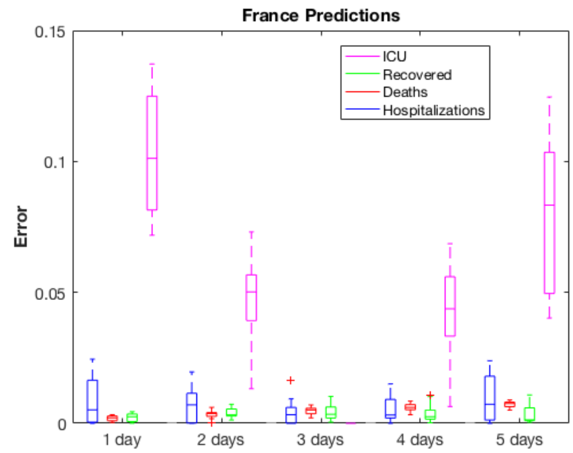

FIG. 4: France prediction errors for ICU, recovered, deaths and hospitalizations of 1, 2, 3, 4 and 5 days ahead.

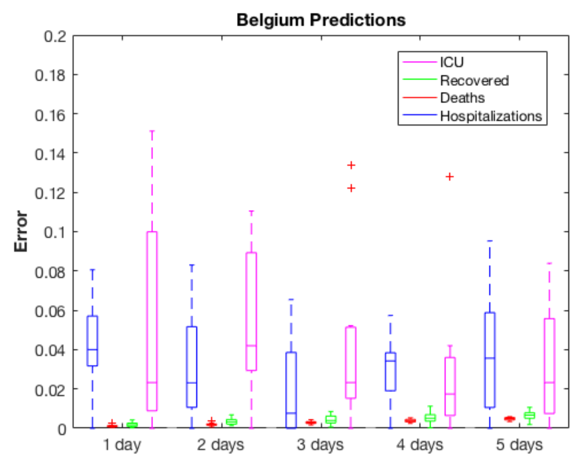

FIG. 5: Belgium prediction errors for ICU, recovered, deaths and hospitalizations of 1, 2, 3, 4 and 5 days ahead.

In the case of France it is observed that the predictions in the numer of hospitalizations, deaths and recoveries is nearly 0 and its variation is also very small. In the case of hospitalizations, the mean error is also near to 0 bus its variation is a little bit higher, and finally in the case of the ICUs the value of the error is between 0.05 and 0.1 depending on the prediction day and its variation is also arround 0.05.

In the case of Belgium, deaths and recoveries also have very small error and variation. Hospitalizations have slightly higher error ans variation than in the case of Belgium, but it harldy gets to 0.05. Finally, as in the case of France, ICUs have a higher error variation but in this case the mean value of the erros is slightly lower (its higher value is near 0.04 while for France it is 0.1).

IV. CONCLUSIONS AND DISCUSSION

In light of the above, it can be said that the designed compartment model enables us to get valid information about the dynamics of the hospitalizations during the COVID-19 pandemic. This compartment model enables us to gain insight of the evolution of the health system during this pandemic in different countries, as it allows to find the optimal values of the parameters that fit best in each country. The knowledge of the parameters that describe the model provides useful information about the hospitalization rate, the demand for ICU beds, and the rate of mortality and recovery and also information on the near future, and therefore, challenges such improving hospital efficiency and giving attention to everyone needing it can be met.

However, the model has some limitations: as we have seen the fitting in general is good enough but the ICUs do not adjust as well as the other compartments. In the study of other models we tested the possibility of separating in the ICUs into two subcompartiments, so that one had a faster mortality and the other a slower one, but we failed to reduce the error.

Therefore, in general we can say that although more variations of the model could be tested to try to minimize the error, this model allows to obtain good enough results that fit well to the real data.

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