

## COMPASS ADJUSTMENT BY GPS AND TWO LEADING LINES

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### **Abstract:**

*The deviation card of a magnetic compass previously adjusted on two leading lines was created using a GPS receiver.*

*During compass adjustment, the vessel follows magnetic courses using a gyroscopic compass and taking into account magnetic declination and gyro error. A satellite compass may also be used for the same purpose. However, some vessels are equipped with neither gyroscopic nor satellite compass. In these cases, a minimum of five leading lines must be considered, or a minimum of five bearings of conspicuous and distant points or sun's azimuths must be taken. This renders adjustment and creation of the deviation card more laborious and time consuming. In order to expedite this process, a reliable and practical method was developed to create the deviation card with courses over ground given by a GPS receiver instead of the headings provided by a gyroscopic or satellite compass. For this purpose, the stream factor, which, as in the ECDIS, also includes other factors such as wind and waves, was considered. The method is valid for all vessels, but is primarily intended for ships equipped with only magnetic compasses to indicate the heading. Actual compass adjustment is carried out following two leadings lines that coincide with or are close to magnetic cardinal courses separated by 90°. Subsequently, courses over ground are taken when the vessel follows the main (four cardinal and four quadrantal) compass headings to create the table of residual deviations.*

### **Keywords:**

*compass, adjustment, magnètic.*

## INTRODUCTION

The objective of this paper is the proper creation of the deviation card of a magnetic compass previously adjusted on two leading lines using a GPS receiver.

Compass adjustment is necessary for the proper working of the magnetic compass, i.e. the first nautical equipment mentioned in SOLAS V19. For many years, this procedure has remained stagnant. However, with the emergence of new technologies, magnetic compass applications and its adjustment techniques have once again become subjects of research. In Spain, for example, there are four doctoral thesis focused on the improvement of magnetic compass performance: Gaztelu-Iturri, 1999; Gea, 2003; Martinez-Lozares, 2009; Arribalzaga, 2016. This article follows this line of research in an attempt to simplify the process of compass adjustment as well as enhance efficiency, focused in small crafts that have a magnetic heading indicator only.

The paper is divided in seven sections. In section 1, the compass adjustment of small crafts is explained. In sections 2, 3 and 4, the proposed method is approached, developed and discussed respectively. In section 5, the application of the method is exposed, according to the obtained results in section 4. Section 6 is focused on the determination of necessary leading lines for the method's application, taking as example, the fieldwork carried out in the Barcelona harbour. Finally, in section 7, conclusions are drawn.

### 1. INTRODUCTION TO COMPASS ADJUSTMENT

The process of compass adjustment has two phases: actual compass adjustment or compensation, and creation of the table of residual deviations or deviation card. The present work is mainly concerned with the latter. However, before this method is presented, the following sub-sections explain how compensation is carried out in vessels with only magnetic compasses to indicate heading.

#### 1.1. DEVIATION EQUATION [2] [6]

The deviation equation commonly applied is

$$\Delta = A + B \cdot \sin \zeta' + C \cdot \cos \zeta' + D \cdot \sin 2\zeta' + E \cdot \cos 2\zeta' \quad [i],$$

where

$\Delta$  is the deviation,  $\zeta'$  is the compass course ( $\zeta$  indicates the magnetic course; see section 2) and A, B, C, D, E are the approximate coefficients, with the exact coefficients being the sines of the approximate ones.

Course deviation consists of three parts: constant deviation, A, which does not depend on the course; semicircular deviation,  $B \cdot \sin \zeta' + C \cdot \cos \zeta'$ , which depends on the course, and quadrantal deviation,  $D \cdot \sin 2\zeta' + E \cdot \cos 2\zeta'$ , which depends on twice the course. These deviations are called semicircular and quadrantal deviations because they are repeated with different sign every  $180^\circ$  and  $90^\circ$ , respectively, where  $180^\circ$  corresponds to half a circle (semicircle) and  $90^\circ$  to a quarter of a circle (quadrant).

The semicircular deviation depends mainly on the hard irons of the ship (which have permanent magnetism) and is corrected with magnets. On the other hand, the constant

and quadrantal deviations depend solely on and are corrected with the soft irons, which do not have permanent magnetism but are induced according to their orientation within the earth's magnetic field.

Considering  $\zeta' = 0^\circ, 90^\circ, 180^\circ, 270^\circ$ , the expressions of the deviations on the cardinal courses are obtained as follows:

$$\Delta_n = A + C + E \quad \Delta_e = A + B - E \quad \Delta_s = A - C + E \quad \Delta_w = A - B - E$$

Therefore, the deviations on the cardinal headings depend on the constant (coefficient A), semicircular (coefficients B, C) and part of the quadrantal (coefficient E) deviation, with the semicircular one being the main deviation.

## 1.2. COMPENSATION DEVICES

The semicircular deviation of compasses in small crafts is compensated with a device that fits the position of longitudinal and transversal magnets by turning one screw for the longitudinal magnets and one for the transversal ones with an anti-magnetic screwdriver. The polarity of magnets is indicated with colors: red for the north pole ends and blue or green for the south pole ends.

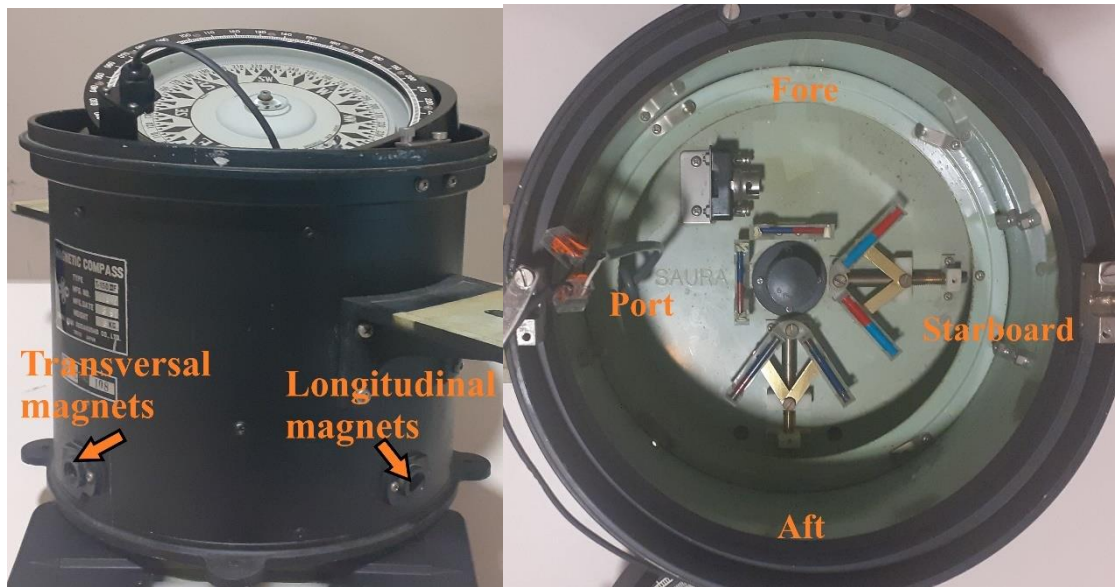
As an example, Figures 1 and 2 show a small binnacle comprising a longitudinal magnet (LM) on the port side with its north (red) end to aft and a transversal magnet (TM) on the fore side with its north (red) end to starboard. In addition, there are two magnets arranged in V-shape in the diametrically opposite position to the LM, and two more in the opposite position to the TM; that is, opposite poles are located at the apex and ends of the V, meaning that the latter form a magnet (V magnet). By adjusting the distance between the V ends with a screwdriver-operated device, the magnetic moment of the V magnet is changed because the distance between its poles is varied [1] [2]. Next, the magnets are arranged in such a way that each V magnet and its corresponding simple magnet have opposite polarities, and the magnetic moment of the V magnet at maximum distance between its ends is twice that of the simple magnet, such that

- i) For no distance between ends of the V magnet, the resulting magnet is the simple magnet.
- ii) For the maximum distance between ends of the V magnet, a magnet with the magnetic moment of the simple magnet but the polarity of the V magnet is obtained.
- iii) For the middle distance between the ends of the V magnet, the resulting magnet is null.
- iv) For intermediate distances, magnets with magnetic moments lower than that of the simple magnet and polarities depending on the distance from the midpoint between ends are obtained.

The quadrantal deviation can be compensated by installing soft iron correctors, such as small spheres or cylinders, or boxes where several soft iron plates can be placed. However, as this is not a common practice, this paper does not consider the compensation of this deviation. It must be remembered, though, that the effect of the quadrantal deviation is always included in the residual deviations.

**Figure 1 (left). Compensating device: exterior view.**

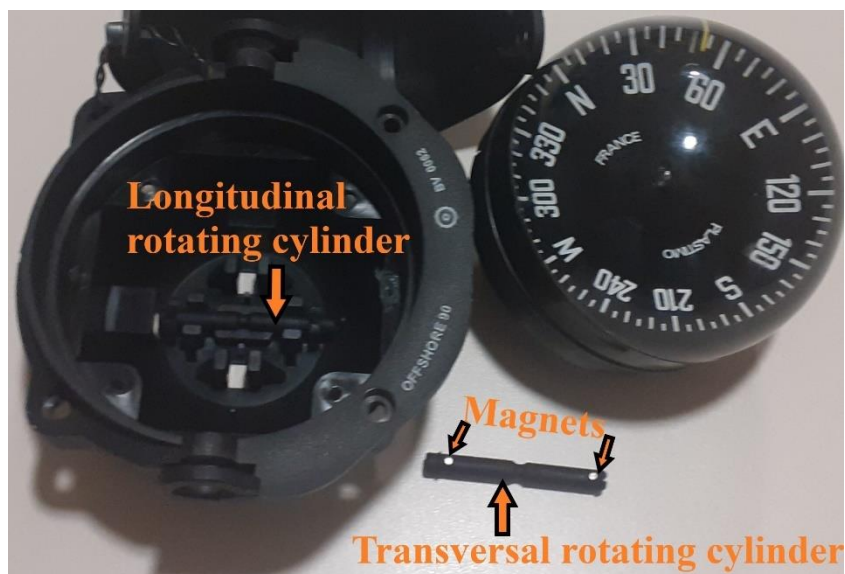
**Figure 2 (right). Compensating device: internal arrangement of the corrector magnets.**



Source: Compass of the Barcelona School of Nautical Studies.

Another very common device consists in placing one (or two) longitudinal and one (or two) transversal magnets that can rotate around their centers, as shown in Figure 3. In this case, the horizontal component of the magnetic moment of the magnets is used to compensate.

**Figure 3. Compensating device with rotating magnets.**



Source: The authors.

Notice that the longitudinal magnets are inside the transversal rotating cylinder and the transversal magnets are inside the longitudinal rotating cylinder.

### 1.3. COMPENSATION METHOD WITH TWO LEADING LINES

The ship follows a leading line that matches the magnetic east or west, or that is close to one of these headings, and the position of the resulting longitudinal magnet is fitted by turning the starboard screw to nullify the deviation, making the compass heading coincide with the magnetic course of the leading line.

Analogously, the ship follows a leading line that matches or is close to the magnetic north or south, and the position of the resulting transversal magnet is fitted by turning the fore screw to make the compass heading be the magnetic course of the leading line.

### 1.4. RESIDUAL DEVIATIONS

According to sub-section 1.3, when the deviations on two magnetic cardinal courses separated by  $90^\circ$  are nullified, the effect of hard irons (reflected in coefficients B, C) is minimized. However, this effect is not completely eliminated because the deviations on the cardinal courses also depend on the constant deviation (coefficient A) and part of the quadrantal one (coefficient E) (see sub-section 1.1). In addition, the effect of soft irons is not compensated, either. Consequently, residual magnetic effects remain after the compensation. [2] [6]

In small vessels, the procedure exposed in section 1.3 is usually enough to comply with the requirement in [3], i.e. the deviation on any course must not exceed  $4^\circ$  for ships with a length less than 82.5 m.

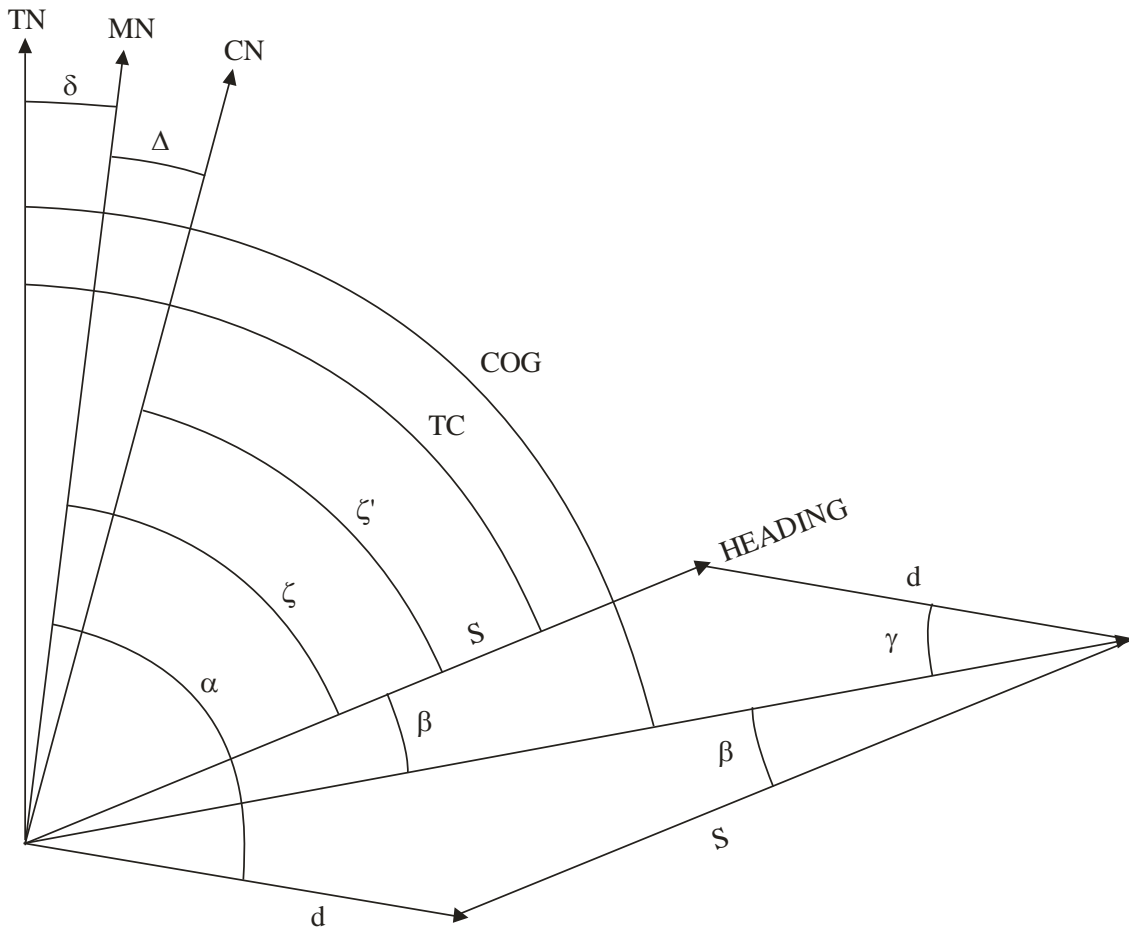
However, the residual deviations must be determined and reflected on the table (deviation card) attached to the compass adjustment certificate, which must also be checked regularly on board [4]. The proposed method is used to create this table, avoiding following more leading lines and/or taking bearings of conspicuous and distant points of land or sun's azimuths, as indicated in [2] and [6], respectively.

## 2. APPROACH OF THE METHOD

Coefficients A, B, C, D, E of the deviation equation [i], as well as the set and drift of the current, are determined from the courses over ground indicated by the GPS when following the eight main compass courses (the four cardinal and the four quadrantal ones).

The method is based on the triangle of speeds and types of courses, as shown in Figure 4. References TN, MN, CN correspond to the true, magnetic and compass north, which are the origins of the true (TC), magnetic ( $\zeta$ ) and compass course ( $\zeta'$ ), respectively;  $\delta$  is the magnetic declination;  $\Delta$ , the deviation, and S, the vessel's speed through the water. The parameters of the current are set ( $\alpha$ ) and drift (d), where  $\alpha$  is expressed as a magnetic course. Also,  $\beta$  is the course difference due to the current:  $\beta = \text{COG} - \text{TC}$ .

**Figure 4. Types of courses and triangle of speeds.**



Source: The authors.

### 3. DEVELOPMENT OF THE METHOD

According to Figure 4, by the law of sines,

$$\frac{S}{\sin \gamma} = \frac{d}{\sin \beta} \Rightarrow \sin \beta = \frac{d}{S} \cdot \sin \gamma$$

But  $\beta + \gamma = \alpha - \zeta$ , since they are opposite angles of a parallelogram. Then,

$$\sin \beta = \frac{d}{S} \cdot \sin(\alpha - \zeta - \beta)$$

Developing,

$$\sin \beta = \frac{d}{S} \cdot \sin(\alpha - \zeta) \cdot \cos \beta - \frac{d}{S} \cdot \cos(\alpha - \zeta) \cdot \sin \beta$$

$$\sin \beta \cdot \left[ 1 + \frac{d}{S} \cdot \cos(\alpha - \zeta) \right] = \frac{d}{S} \cdot \sin(\alpha - \zeta) \cdot \cos \beta \Rightarrow \tan \beta = \frac{\frac{d}{S} \cdot \sin(\alpha - \zeta)}{1 + \frac{d}{S} \cdot \cos(\alpha - \zeta)}$$

Since  $(a + b) \cdot (a - b) = a^2 - b^2$ , multiplying the numerator and denominator by  $1 - \frac{d}{S} \cdot \cos(\alpha - \zeta)$ , we obtain 1

$$\tan \beta = \frac{\frac{d}{S} \cdot \sin(\alpha - \zeta) \cdot \left[ 1 - \frac{d}{S} \cdot \cos(\alpha - \zeta) \right]}{1 - \frac{d^2}{S^2} \cdot \cos^2(\alpha - \zeta)}$$

Because  $d^2$  is much smaller than  $S^2$ , the denominator can be considered 1. Also, since  $\beta$  is a small angle, its tangent can be replaced by its sine, which in turn can be replaced by  $\beta \cdot \sin 1^\circ$ . Thus,

$$\beta \cdot \sin 1^\circ = \frac{d}{S} \cdot \sin(\alpha - \zeta) - \frac{d^2}{S^2} \cdot \sin(\alpha - \zeta) \cdot \cos(\alpha - \zeta)$$

Let  $\omega = \frac{d}{S} \cdot \csc 1^\circ$ ,

$$\beta = \omega \cdot \sin(\alpha - \zeta) - \omega^2 \cdot \sin(\alpha - \zeta) \cdot \cos(\alpha - \zeta)$$

where

$$\begin{aligned} \sin(\alpha - \zeta) &= \sin \alpha \cdot \cos \zeta - \cos \alpha \cdot \sin \zeta \\ \cos(\alpha - \zeta) &= \cos \alpha \cdot \cos \zeta + \sin \alpha \cdot \sin \zeta \end{aligned}$$

Now let

$$\left. \begin{aligned} \omega \cdot \cos \alpha &= x \\ \omega \cdot \sin \alpha &= y \end{aligned} \right\} \Rightarrow \begin{cases} \omega \cdot \sin(\alpha - \zeta) = -x \cdot \sin \zeta + y \cdot \cos \zeta \\ \omega \cdot \cos(\alpha - \zeta) = x \cdot \cos \zeta + y \cdot \sin \zeta \end{cases}$$

Consequently,

$$\begin{aligned} \omega^2 \cdot \sin(\alpha - \zeta) \cdot \cos(\alpha - \zeta) &= (-x \cdot \sin \zeta + y \cdot \cos \zeta) \cdot (x \cdot \cos \zeta + y \cdot \sin \zeta) = \\ &= -x^2 \cdot \sin \zeta \cdot \cos \zeta - x \cdot y \sin^2 \zeta + x \cdot y \cdot \cos^2 \zeta + y^2 \cdot \sin \zeta \cdot \cos \zeta = \\ &= x \cdot y \cdot \cos 2\zeta - \frac{1}{2}(x^2 - y^2) \cdot \sin 2\zeta \end{aligned}$$

Then,

$$\beta = -x \cdot \sin \zeta + y \cdot \cos \zeta - x \cdot y \cdot \sin 1^\circ \cdot \cos 2\zeta - \frac{1}{2}(x^2 - y^2) \cdot \sin 1^\circ \cdot \sin 2\zeta$$

Substituting  $\zeta$  by  $\zeta'$ ,

$$\beta = -x \cdot \sin \zeta' + y \cdot \cos \zeta' - x \cdot y \cdot \sin 1^\circ \cdot \cos 2\zeta' - \frac{1}{2}(x^2 - y^2) \cdot \sin 1^\circ \cdot \sin 2\zeta'$$

The deviation is the difference between the magnetic course and the compass course, i.e.  $\Delta = \zeta - \zeta'$ . Likewise, the magnetic course is the difference between the true course and the magnetic declination, i.e.  $\zeta = TC - \delta$ . Hence,  $\Delta = TC - \delta - \zeta'$ , and TC is the difference between the course over ground and  $\beta$ :  $TC = COG - \beta$ . Therefore,

$$\Delta = COG - \beta - \delta - \zeta'$$

And the deviation equation is

$$\Delta = A + B \cdot \sin \zeta' + C \cdot \cos \zeta' + D \cdot \sin 2\zeta' + E \cdot \cos 2\zeta'$$

Thus,

$$COG - \beta - \delta - \zeta' = A + B \cdot \sin \zeta' + C \cdot \cos \zeta' + D \cdot \sin 2\zeta' + E \cdot \cos 2\zeta'$$

Now let the pseudo-deviation ( $\Psi$ ) be defined as the difference between the course over ground and the compass course:  $\Psi = COG - \zeta'$ . Then,

$$\Psi = A + B \cdot \sin \zeta' + C \cdot \cos \zeta' + D \cdot \sin 2\zeta' + E \cdot \cos 2\zeta' + \beta + \delta$$

$$\begin{aligned} \Psi &= A + B \cdot \sin \zeta' + C \cdot \cos \zeta' + D \cdot \sin 2\zeta' + E \cdot \cos 2\zeta' - x \cdot \sin \zeta' + \\ &+ y \cdot \cos \zeta' - x \cdot y \cdot \sin 1^\circ \cdot \cos 2\zeta' - \frac{1}{2}(x^2 - y^2) \cdot \sin 1^\circ \cdot \sin 2\zeta' + \delta \end{aligned}$$

Particularizing for the four cardinal compass courses, we obtain

$$\Psi_n = A + C + E + y - x \cdot y \cdot \sin 1^\circ + \delta \quad [\text{ii}]$$

$$\Psi_e = A + B - E - x + x \cdot y \cdot \sin 1^\circ + \delta \quad [\text{iii}]$$

$$\Psi_s = A - C + E - y - x \cdot y \cdot \sin 1^\circ + \delta \quad [\text{iv}]$$

$$\Psi_w = A - B - E + x + x \cdot y \cdot \sin 1^\circ + \delta \quad [\text{v}]$$

Consequently,

$$[\text{i}] + [\text{ii}] + [\text{iii}] + [\text{iv}] \Rightarrow \Psi_n + \Psi_e + \Psi_s + \Psi_w = 4A + 4\delta$$

$$A = \frac{1}{4}(\Psi_n + \Psi_e + \Psi_s + \Psi_w) - \delta \quad [\text{vi}]$$

$$[\text{ii}] - [\text{iv}] \Rightarrow \Psi_e - \Psi_w = 2B - 2x \Rightarrow B = \frac{1}{2}(\Psi_e - \Psi_w) + x \quad [\text{vii}]$$

$$[\text{i}] - [\text{iii}] \Rightarrow \Psi_n - \Psi_s = 2C + 2y \Rightarrow C = \frac{1}{2}(\Psi_n - \Psi_s) - y \quad [\text{viii}]$$

$$[\text{i}] + [\text{iii}] - [\text{ii}] - [\text{iv}] \Rightarrow \Psi_n + \Psi_s - \Psi_e - \Psi_w = 4E - 4 \cdot x \cdot y \cdot \sin 1^\circ$$

$$E = \frac{1}{4}(\Psi_n + \Psi_s - \Psi_e - \Psi_w) + x \cdot y \cdot \sin 1^\circ \quad [\text{ix}]$$

And particularizing for the four quadrantal compass courses, we obtain



$$\Psi_{ne} = A + B \cdot \frac{\sqrt{2}}{2} + C \cdot \frac{\sqrt{2}}{2} + D - x \cdot \frac{\sqrt{2}}{2} + y \cdot \frac{\sqrt{2}}{2} - \frac{1}{2}(x^2 - y^2) \cdot \sin 1^\circ + \delta \quad [x]$$

$$\Psi_{se} = A + B \cdot \frac{\sqrt{2}}{2} - C \cdot \frac{\sqrt{2}}{2} - D - x \cdot \frac{\sqrt{2}}{2} - y \cdot \frac{\sqrt{2}}{2} + \frac{1}{2}(x^2 - y^2) \cdot \sin 1^\circ + \delta \quad [xi]$$

$$\Psi_{sw} = A - B \cdot \frac{\sqrt{2}}{2} - C \cdot \frac{\sqrt{2}}{2} + D + x \cdot \frac{\sqrt{2}}{2} - y \cdot \frac{\sqrt{2}}{2} - \frac{1}{2}(x^2 - y^2) \cdot \sin 1^\circ + \delta \quad [xii]$$

$$\Psi_{nw} = A - B \cdot \frac{\sqrt{2}}{2} + C \cdot \frac{\sqrt{2}}{2} - D + x \cdot \frac{\sqrt{2}}{2} + y \cdot \frac{\sqrt{2}}{2} + \frac{1}{2}(x^2 - y^2) \cdot \sin 1^\circ + \delta \quad [xiii]$$

Therefore,

$$[x] - [xii] \Rightarrow \Psi_{ne} - \Psi_{sw} = (B + C - x + y) \cdot \sqrt{2} \quad [xiv]$$

$$[x] + [xii] \Rightarrow \Psi_{ne} + \Psi_{sw} = 2A + 2D - (x^2 - y^2) \cdot \sin 1^\circ + 2\delta \quad [xv]$$

$$[xi] - [xiii] \Rightarrow \Psi_{se} - \Psi_{nw} = (B - C - x - y) \cdot \sqrt{2} \quad [xvi]$$

$$[xi] + [xiii] \Rightarrow \Psi_{se} + \Psi_{nw} = 2A - 2D + (x^2 - y^2) \cdot \sin 1^\circ + 2\delta \quad [xvii]$$

$$[xv] - [xvii] \Rightarrow \Psi_{ne} + \Psi_{sw} - \Psi_{se} - \Psi_{nw} = 4D - 2(x^2 - y^2) \cdot \sin 1^\circ$$

$$D = \frac{1}{4}(\Psi_{ne} + \Psi_{sw} - \Psi_{se} - \Psi_{nw}) - \frac{1}{2}(x^2 - y^2) \cdot \sin 1^\circ \quad [xviii]$$

$$[xv] + [xvii] \Rightarrow \Psi_{ne} + \Psi_{sw} + \Psi_{se} + \Psi_{nw} = 4A + 4\delta$$

$$A = \frac{1}{4}(\Psi_{ne} + \Psi_{se} + \Psi_{sw} + \Psi_{nw}) - \delta \quad [xix]$$

$$[xiv] + [xvi] \Rightarrow \Psi_{ne} - \Psi_{sw} + \Psi_{se} - \Psi_{nw} = 2(B - x) \cdot \sqrt{2}$$

$$B = \frac{1}{2\sqrt{2}} \cdot (\Psi_{ne} + \Psi_{se} - \Psi_{sw} - \Psi_{nw}) + x \quad [xx]$$

$$[xiv] - [xvi] \Rightarrow C = \frac{1}{2\sqrt{2}} \cdot (\Psi_{ne} + \Psi_{nw} - \Psi_{se} - \Psi_{sw}) - y \quad [xxi]$$

$$[vii] + [xx] \Rightarrow 2B = \frac{1}{2} \left[ \Psi_e - \Psi_w + \frac{1}{\sqrt{2}} \cdot (\Psi_{ne} + \Psi_{se} - \Psi_{sw} - \Psi_{nw}) \right] + 2x$$

$$B = \frac{1}{4} \left[ \Psi_e - \Psi_w + \frac{\sqrt{2}}{2} \cdot (\Psi_{ne} + \Psi_{se} - \Psi_{sw} - \Psi_{nw}) \right] + x \quad [xxii]$$

$$[viii] + [xxi] \Rightarrow C = \frac{1}{4} \left[ \Psi_n - \Psi_s + \frac{\sqrt{2}}{2} \cdot (\Psi_{ne} + \Psi_{se} - \Psi_{sw} - \Psi_{nw}) \right] - y \quad [xxiii]$$

$$[vi] + [xix] \Rightarrow A = \frac{1}{8} (\Psi_n + \Psi_{ne} + \Psi_e + \Psi_{se} + \Psi_s + \Psi_{sw} + \Psi_w + \Psi_{nw}) - \delta \quad [xxiv]$$

It is observed that unknowns  $x, y$  cannot be found. Hence, it is only possible to determine coefficients  $B, \dots, E$  if  $d, \alpha$ , on which  $x, y$  depend, are known previously. Therefore, it is convenient to express  $B, \dots, E$  as a function of the current parameters:

$$x = \omega \cdot \cos \alpha = \frac{d}{S} \cdot \csc 1^\circ \cdot \cos \alpha \cong 57.3 \cdot \frac{d}{S} \cdot \cos \alpha \quad [xxv]$$

$$y = \omega \cdot \sin \alpha = \frac{d}{S} \cdot \csc 1^\circ \cdot \sin \alpha \cong 57.3 \cdot \frac{d}{S} \cdot \sin \alpha \quad [\text{xxvi}]$$

$$x \cdot y \cdot \sin 1^\circ = \frac{d^2}{S^2} \cdot \csc^2 1^\circ \cdot \cos \alpha \cdot \sin \alpha \cdot \sin 1^\circ = \frac{d^2}{S^2} \cdot \csc 1^\circ \cdot \frac{1}{2} \sin 2\alpha \cong 28.65 \cdot \frac{d^2}{S^2} \cdot \sin 2\alpha \quad [\text{xxvii}]$$

$$\frac{1}{2}(x^2 - y^2) \cdot \sin 1^\circ = \frac{1}{2} \left( \frac{d^2}{S^2} \cdot \csc^2 1^\circ \cdot \cos^2 \alpha - \frac{d^2}{S^2} \cdot \csc^2 1^\circ \cdot \sin^2 \alpha \right) \cdot \sin 1^\circ$$

$$\frac{1}{2}(x^2 - y^2) \cdot \sin 1^\circ = \frac{1}{2} \cdot \frac{d^2}{S^2} \cdot \csc 1^\circ \cdot (\cos^2 \alpha - \sin^2 \alpha) \cong 28.65 \cdot \frac{d^2}{S^2} \cdot \cos 2\alpha \quad [\text{xxviii}]$$

Then, substituting the expressions of  $x$ ,  $y$ ,  $x \cdot y \cdot \sin 1^\circ$ ,  $\frac{1}{2}(x^2 - y^2) \cdot \sin 1^\circ$  according to [xxv], [xxvi], [xxviii], [xxvii] in expressions [xxii], [xxiii], [xviii], [ix], respectively, we have

$$B = \frac{1}{4} [\Psi_e - \Psi_w + \frac{\sqrt{2}}{2} \cdot (\Psi_{ne} + \Psi_{se} - \Psi_{sw} - \Psi_{nw})] + 57.3 \cdot \frac{d}{S} \cdot \cos \alpha \quad [\text{xxix}]$$

$$C = \frac{1}{4} [\Psi_n - \Psi_s + \frac{\sqrt{2}}{2} \cdot (\Psi_{ne} + \Psi_{nw} - \Psi_{se} - \Psi_{sw})] - 57.3 \cdot \frac{d}{S} \cdot \sin \alpha \quad [\text{xxx}]$$

$$D = \frac{1}{4} (\Psi_{ne} + \Psi_{sw} - \Psi_{se} - \Psi_{nw}) - 28.65 \cdot \frac{d^2}{S^2} \cdot \cos 2\alpha \quad [\text{xxxii}]$$

$$E = \frac{1}{2} (\Psi_n + \Psi_s - \Psi_e - \Psi_w) + 28.65 \cdot \frac{d^2}{S^2} \cdot \sin 2\alpha \quad [\text{xxxiii}]$$

Coefficients  $B$ ,  $C$  can also be obtained by substituting the expressions of  $x$ ,  $y$  according to [xxv], [xxvi] in the expressions [vii], [xx] and [viii], [xxi], respectively:

$$B = \frac{1}{2} (\Psi_e - \Psi_w) + 57.3 \cdot \frac{d}{S} \cdot \cos \alpha \quad [\text{xxxiiii}]$$

$$B = \frac{1}{2\sqrt{2}} \cdot (\Psi_{ne} + \Psi_{se} - \Psi_{sw} - \Psi_{nw}) + x$$

$$B = 0.3536 \cdot (\Psi_{ne} + \Psi_{se} - \Psi_{sw} - \Psi_{nw}) + 57.3 \cdot \frac{d}{S} \cdot \cos \alpha \quad [\text{xxxv}]$$

$$C = \frac{1}{2} (\Psi_n - \Psi_s) - 57.3 \cdot \frac{d}{S} \cdot \sin \alpha \quad [\text{xxxvi}]$$

$$C = 0.3536 \cdot (\Psi_{ne} + \Psi_{nw} - \Psi_{se} - \Psi_{sw}) - 57.3 \cdot \frac{d}{S} \cdot \sin \alpha \quad [\text{xxxvii}]$$

#### 4. DISCUSSION OF THE METHOD

Expressions [xxix], [xxx], [xxxiii], [xxxiv], [xxxv], [xxxvi] are not reliable for calculating coefficients B, C because not knowing with precision the d/S ratio can lead to a considerable error.

Nevertheless, it is observed that at a sufficient speed, the  $d^2/S^2$  ratio is very small, and can therefore be neglected in expressions [xxxii], [xxxiii]. For example, for an already quite critical ratio of  $S/d = 5$  (i.e. a ship sailing at 5 knots with a 1 knot current, or a ship sailing at 10 knots with a 2 knot current), the maximum error in the calculation of coefficients D, E would be  $1^\circ$ . This maximum error is shown below for the S/d ratio:

**Figure 5. Maximum error of coefficients D, E for the S/d ratio.**

S/d	5	6	7	8	9	10	11	12
Max. error: $28.65 \cdot d^2/S^2$	1.1460	0.7958	0.5847	0.4477	0.3537	0.2865	0.2368	0.1990

Source: The authors.

These results show that the effect of the current can indeed be neglected in expressions [xxxii], [xxxiii]. Therefore,

$$D = \frac{1}{4}(\Psi_{ne} + \Psi_{sw} - \Psi_{se} - \Psi_{nw}) \quad [xxxvii]$$

$$E = \frac{1}{4}(\Psi_n + \Psi_s - \Psi_e - \Psi_w) \quad [xxxviii]$$

In summary:

i) Coefficient A can be determined from the pseudo-deviations without error. The magnetic declination must also be applied (see [vi], [xix], [xxiv]).

ii) Coefficients D, E can be determined solely from the pseudo-deviations but with a small error which is negligible for sufficiently high speeds, i.e. for a ratio of S/d equal or greater than 8, which causes an error less than  $0.5^\circ$  for each coefficient. Assuming a drift of 1 knot or less, the minimum speed is 8 knots, and assuming a drift of 0.5 knots or less, the minimum speed is 4 knots.

iii) Coefficients B, C cannot be calculated with the pseudo-deviations because a considerable error can be made.

On the other hand, coefficients B, C can be determined from coefficients A, E (calculated from the pseudo-deviations) because

$$\Delta_{e,w} = A \pm B - E \Rightarrow B = \pm(\Delta_{e,w} - A + E) \quad [xxxix]$$

$$\Delta_{n,s} = A \pm C + E \Rightarrow C = \pm(\Delta_{n,s} - A - E) \quad [xl]$$

where  $\Delta_{e,w}$  and  $\Delta_{n,s}$  can be considered the deviations on the leading lines which have been nullified. Therefore,

$$B = \pm(-A + E) \quad (+ \text{ leading line towards E; } - \text{ leading line towards W})$$

$$C = \pm(-A - E) \quad (+ \text{ leading line towards N; } - \text{ leading line towards S})$$

Simplifying,

$$B = (-1)^{[\text{TB}_1/180 + 1]} \cdot (A - E) \quad [\text{xli}]$$

$$C = (-1)^{[\text{TB}_2/180 + 1.5]} \cdot (A + E) \quad [\text{xlii}]$$

where

[x] is the integer part of x, and  $\text{TB}_1$ ,  $\text{TB}_2$  are the true bearings, expressed in circular form (from  $0^\circ$  to  $360^\circ$ ), of the leading line close to the E or W and the leading line close to the N or S, respectively.

## 5. APPLICATION OF THE METHOD

The complete procedure to compensate the compass and create its table of the residual deviations is as follows:

1. The vessel must follow a leading line coincident with or close to the magnetic east or west, and the deviation must be nullified. Next, the vessel must continue following the leading line, and the course over ground indicated by the GPS must be noted down to determine the pseudo-deviation, i.e.  $\Psi_{e,w} = \text{COG} - \zeta'$ .
2. *Idem* on a leading line coincident with or close to the magnetic north or south.
3. The vessel must follow the other six main compass courses, and the course over ground indicated by the GPS must be noted down to determine the pseudo-deviations.
4. Coefficients A, D, E, B, C must be calculated using expressions [vi], [xxxvii], [xxxviii], [xli], [xlii], respectively. Coefficient A can also be obtained by using [xix], [xxiv].
5. The table of the residual deviations must be created by applying the deviation equation [i] to different compass courses, for example every  $15^\circ$ , as is the case of the deviation card model of the Spanish regulation [5].

## 6. DETERMINATION OF LEADING LINES

Two leading lines that coincide with or are as close as possible to magnetic cardinal courses separated by  $90^\circ$  must be found. However, these leading lines cannot always be determined on the chart (because one or both points are not on the chart), or if they can be, they may be at a considerable distance. Alternatively, points that determine the required leading lines can be selected despite not appearing on the chart, and then their positions can be obtained from *Google Maps* and verified *in situ*, as done by Moncunill and González La Flor in the outer northern area of the Barcelona harbour.

On January 16, 2020, the sailboat APHRODITE sailed the coastal waters between the Besòs river mouth and the northern entrance to the Barcelona harbour, corresponding to a distance of about 5 nautical miles. The voyage actually began and ended at El Masnou marina (where the sailboat has her mooring), about 6 miles NE of the Besòs river mouth. The meteorological conditions were as follows: on the way out, in the morning, predominant onshore wind of force 2 on the Beaufort scale and rippled sea with a 0.5 m swell from E; on the way back, in the afternoon, SW wind of force 3 and smooth sea; good visibility and almost clear sky throughout the day. We sailed all the time, with the occasional use of the engine.

The following leading lines, on which the points can be clearly distinguished, were observed:

i) Close to the W, southernmost chimney stack of former thermal power plant by the Besòs river (point A) with the Tibidabo church (point B): IMAGE 1 (Figure 6).

ii) Close to the W, second northernmost chimney stack of the Besòs combined cycle plant (Endesa towers) (point C) with the Tibidabo church (point B): IMAGE 2 (Figure 7).

iii) Close to the W, the Glòries tower (former Agbar tower) (point D) with the Collserola telecommunications tower (point E): IMAGE 3 (Figure 8).

iv) Close to the N, the *Aigües del Besòs* tower (point F) between two emblematic buildings: IMAGE 4 (Figure 9). A ship's position was taken as the second point when the ship was following the leading line (point G).

v) Close to the N, the *La Catalana de Gas* tower (point H) with the Glòries tower (point D): IMAGE 5 (Figure 10).

**Figure 6. IMAGE 1: Leading line A–B, close to the W.**



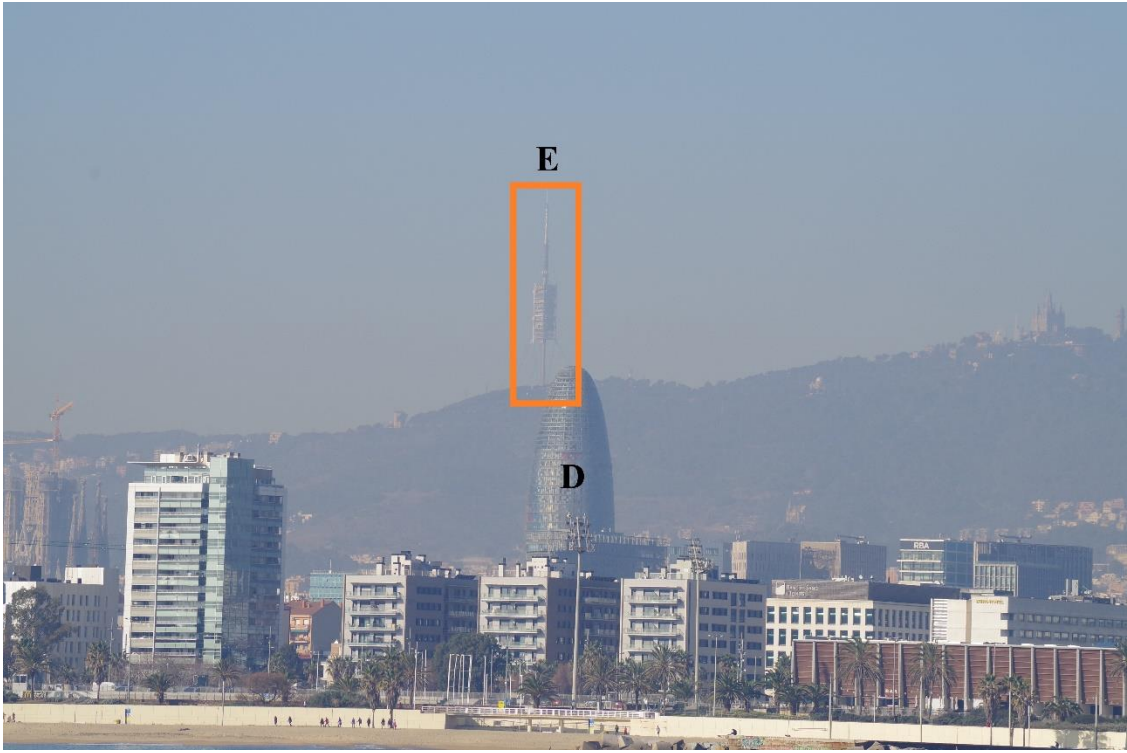
Source: The authors.

**Figure 7. IMAGE 2: Leading line C–B, close to the W.**



Source: The authors.

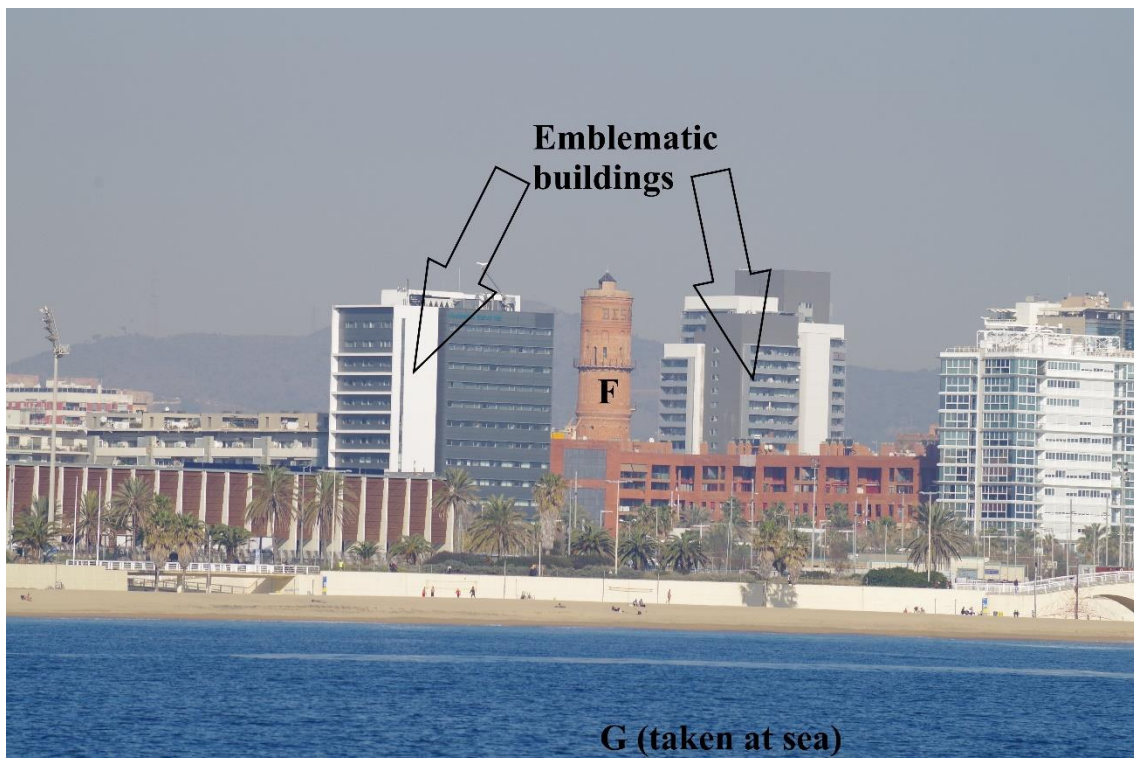
**Figure 8. IMAGE 3: Leading line D–E, close to the W.**



Source: The authors.



**Figure 9. IMAGE 4: Leading line G–F, close to the N.**



Source: The authors.

**Figure 10. IMAGE 5: Leading line H–D, close to the N.**



Source: The authors.

Since the sailboat has a magnetic heading indicator only, it was impossible to determine exactly the true bearing of each leading line. Therefore, the positions of points A,...,H, must be taken to obtain them.

These positions, which were taken from *Google Maps* and verified *in situ*, are shown in the table below. They were rounded to the fourth decimal place of sexagesimal degree (in latitude, 0.0001° corresponds to 11 m). Differences between both positions (*Google Maps* and verified *in situ*) are due to the impossibility to access the central point of the buildings. As for the position provided by the GPS (point G), it is shown as it appears on the screen, i.e. rounded to the third decimal place of sexagesimal minute (0.001' = 1.852 m).

**Figure 11. Position of the points of the obtained leading lines.**

Point	Google Maps	<i>In situ</i> verification
A: Three Besòs chimney stacks	41.4266° 2.2345°	41.4263° 2.2342°
B: Tibidabo church	41.4220° 2.1187°	41.4217° 2.1192°
C: Endesa towers	41.4191° 2.2296°	41.4185° 2.2303°
D: Glòries tower (Agbar tower)	41.4038° 2.1895°	41.4037° 2.1896°
E: Collserola tower	41.4170° 2.1143°	41.4180° 2.1151°
F: <i>Aigües del Besòs</i> tower	41.4060° 2.2117°	41.4061° 2.2119°
G: Sea, following the leading line	* * *	41°23.319'N 2°12.828'E
H: <i>La Catalana de Gas</i> tower	41.3838° 2.1926°	41.3837° 2.1925°

Source: The authors.

The true bearing (TB) of each leading line can be determined in three different ways:

- i) Plotting the points on the chart and drawing the leading lines.
- ii) Using dead reckoning formulae.
- iii) Using the loxodrome equation.

With the help of a spreadsheet, it was finally decided to use the loxodrome equation, from which it follows that

$$\cos TB_{\text{SEMICIRCULAR}} = \frac{M_2 - M_1}{\sqrt{(M_2 - M_1)^2 + (\lambda_2 - \lambda_1 + 360 \cdot k)^2}},$$

where

$TB_{\text{SEMICIRCULAR}}$ , i.e. the arccosine provided by calculators and spreadsheets, is between 0° and 180°;  $M_1, M_2$  are the meridional parts, expressed in sexagesimal degrees, of the closest point of the leading line and the furthest one, respectively;  $\lambda_1, \lambda_2$  are the positive (E) or negative (W) longitude, expressed in sexagesimal degrees, of the closest point of



the leading line and the furthest one, respectively, and  $k$  is the integer (0, 1 or  $-1$ ) such that the absolute value of  $\lambda_2 - \lambda_1 + 360 \cdot k$  is inferior or equal to  $180^\circ$ . In this case,  $k = 0$ .

Likewise, with  $\varphi$  being the latitude of a point, expressed in sexagesimal degrees, and positive or negative depending on whether it is N or S, respectively, its meridional part (M) is

$$M = \frac{180^\circ}{\pi} \cdot \ln \tan\left(45^\circ + \frac{1}{2} \varphi\right)$$

Finally, the true bearing expressed in circular form (from  $0^\circ$  to  $360^\circ$ ) is  $TB_{\text{SEMICIRCULAR}}$  if  $\lambda_2 - \lambda_1 + 360 \cdot k$  is positive, or  $360^\circ - TB_{\text{SEMICIRCULAR}}$  if it is negative.  $TB_{\text{SEMICIRCULAR}}$  was obtained from a cosine and not a tangent (using the most common formula in Navigation) because, unlike the tangent function, the cosine function is continuous such that errors in the spreadsheet are avoided with the arccosine.

The obtained true bearings, rounded to half sexagesimal degree, are shown in the following table:

**Figure 12. True bearings of the obtained leading lines.**

A-B	C-B	D-E	G-F	H-D
267	271.5	283	355	353.5

Source: The authors.

The practical value of the magnetic declination (i.e. rounded to half sexagesimal degree) for the indicated day, taking the breakwater end at the north mouth of the Barcelona harbour (41.3577, 2.1854, *Google Maps*) as the reference position, is  $1^\circ$ .

According to the *World Magnetic Model* WMM (2019-24), as used by NOAA (<https://www.ngdc.noaa.gov/geomag/calculators/magcalc.shtml>),  $\delta = 1.23^\circ\text{E} \pm 0.34^\circ$ .

According to the *International Geomagnetic Reference Field* IGRF-12 (2015) Model, as used by *Natural Resources Canada* (<https://geomag.nrcan.gc.ca/calc/mfcal-en.php>),  $\delta = 1.107^\circ$ .

It is worth recalling that the magnetic declination varies very slowly, both in time and space (except near the geographical and magnetic poles), and therefore the value of  $1^\circ$  is valid for the entire area considered.

Hence, the magnetic bearings are

**Figure 13. Magnetic bearings of the obtained leading lines.**

A-B	C-B	D-E	G-F	H-D
266	270.5	282	354	352.5

Source: The authors.

The two closest to the magnetic bearings N and W are selected:

G–F: *Aigües del Besòs* tower between two emblematic buildings: IMAGE 4 (Figure 9)

C–B: second northernmost chimney stack of the Besòs combined cycle plant (Endesa towers) with the Tibidabo church: IMAGE 2 (Figure 7)

In addition, the leading line C–B has the advantage over A–B that it is closer to G–F.

## 7. CONCLUSIONS

The proposed method enables a rigorous and practical compensation to be carried out on vessels that have a magnetic heading indicator only. In order to perform the actual compass adjustment, two leading lines as close as possible to magnetic cardinal courses separated by 90° must be known, and to create the deviation card, a GPS receiver is used.

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