STABILITY OF AXISYMMETRIC MULTIPOLE CONFIGURATIONS WITH CLOSED MERIDIONAL FIELD LINES

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Abstract: The stability of axisymmetric equilibria having meridional field lines only is investigated. In order to obtain stability, a hard core must be placed inside the plasma. The pressure distribution can be chosen such that absolute stability is obtained. However, a low pressure layer extends up to the wall.

We consider axisymmetric equilibria with closed meridional magnetic field lines produced by toroidal currents. By using a stream function $\psi(r,z)$ the magnetic field $\underline{B}$ can be written

$$\underline{B} = -\underline{e}_\theta \times \nabla\psi/r \qquad (1)$$

in cylindrical coordinates r, Θ, z. ($\underline{e}_\theta$ is the unit vector in the Θ direction). The toroidal current $\underline{j} = j\underline{e}_\theta$ is related to the pressure distribution $p(\psi)$ by the relation

$$p'(\psi) = j/r\,. \qquad (2)$$

For equilibria of this type the following form of the energy principle has been given in ref. [1]

$$\delta W = \frac{1}{2\pi}\left(L' + \frac{V'}{\gamma p}\right)f^2 + \int d\chi\left[\frac{1}{r^2B^2J}\left(\frac{\partial X}{\partial\chi}\right)^2 + p'JDX^2\right] \geq 0 \qquad (3)$$

with

$$L' = 2\pi\int d\chi\, J/B^2\,, \quad V' = 2\pi\int d\chi\, J\,, \quad J = 1/B\cdot|\nabla\chi|$$

$$D = -\frac{2}{rB}(\underline{e}\cdot\underline{\kappa})\,, \quad f = \left(2\pi\int d\chi\, JDX\right)/\left(L' + V'/\gamma p\right)$$

Thereby χ = const. are trajectories orthogonal to the flux surfaces ψ = const., $\underline{\kappa}$ is the vector of curvature of the $\underline{B}$-lines, and $\underline{e}_\psi = \nabla\psi/|\nabla\psi|$. From (3), Bernstein et al. have derived a necessary stability criterion and in two limiting cases (low β and almost circular field lines) necessary and sufficient criteria.

It is possible to derive further stability criteria in the general finite β case and to show that stable equilibria exist. Expanding δW around the center line of the flux surfaces one can show that the plasma is always unstable unless a current carrying hard core is placed inside the plasma. Since the plasma should be separated from the hardcore we assume that the plasma starts with $p = 0$ on a $\underline{B}$-line at some distance from the hardcore. From (3) it is obvious that we have stability in the region immediately behind since $p > 0$. Towards the wall the pressure must decrease and therefore ∇p must reverse direction on some flux surface. According to $\nabla p = \underline{j} \times \underline{B}$, ∇p reverses if $\underline{j}$ or $\underline{B}$ reverses. However, it can be shown that the second case would be unstable, and therefore j and $p(\psi) = j/r$ must change sign.

Except for very singular situations one can further show that the average curvature of the field lines must be positive:

$$\frac{1}{2}\int JD\,d\chi = \int \kappa\frac{J}{rB}\,d\chi \geq 0\,. \qquad (4)$$

If X is decomposed into $X_I + X_{II}$ with $\int X_I X_{II}\,d\chi = 0$ and $\frac{\partial X_{II}}{\partial\chi} = 0$ then (3) splits into the two independent criteria

$$\delta W_I = \int d\chi\left[\frac{1}{r^2B^2J}\left(\frac{\partial X_I}{\partial\chi}\right)^2 + p'JDX_I^2\right] \geq 0 \qquad (5)$$

and

$$\delta W_{II} = X_{II}^2\left[\frac{2\pi\gamma p\left(\int d\chi\, JD\right)^2}{V' + \gamma p L'} + p'\int JD\,d\chi\right] \geq 0 \qquad (6)$$

Inequality (5) can be fulfilled by making $|p'|$ smaller than a certain upper limit. If $D \geq 0$ in the whole region of ascending pressure, then this limitation applies only to the region with $p' \leq 0$. From (6), which is the criterion given in ref. [1], no restriction follows for the region with $p' \geq 0$ because of (4). On the other hand, the pressure in the region with $p' \leq 0$ may not drop to zero again within a finite distance from the outer wall. It may however decay rather fast towards the wall (in a straight cylinder $p \sim 1/r^{4\beta}$). Since finite restivity and other diffusion effects will enforce a low pressure layer extending to the wall even in cases where an ideal MHD equilibrium would be separated from this by a vacuum region, the low pressure layer obtained here will be no serious disadvantage.

[1] I.B. Bernstein, E.A. Frieman, M.B. Kruskal and R.M. Kulsrud; Proc. Roy. Soc. (London) A 244, 17 (1958)