Local Spatial Distortion Caused by Simple Geometrical Figures

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DRAFT

Abstract

Dynamic distortion of the visual field has been shown to affect perceptual judgment of visual dimensions such as size, length and distance. Here, we report four experiments demonstrating that the different aspects of a triangle differently influence judgments of distance. Specifically, when the base of the triangle faces the centre of the display, participants consistently underestimate and overestimate the distance of a small dot from the unmarked centre of the display relative to conditions in which the vertex of the triangle faces the centre. When the dot is close to the figure, the distance of the dot to the centre is underestimated. Conversely, when the dot is close to the figure, the distance to the centre is overestimated. The effect is replicated when the internal distances are equalized and when ellipses are used instead of triangles. These results support a ripple model of spatial distortion in which local curvature acts to attract or repel objects. In conclusion, we suggest some implications of our findings for theories of perceptual organization.

Keywords: visual space, distortion, Gestalt, geometric figures, perceptual grouping

Effects of Local Spatial Distortion Caused by Simple Geometrical Figures

Although there is a great deal of evidence that globally, the representation of visual space is non-Euclidean, the issue of which, of many non-Euclidean geometries best explains visual judgments remains open. While some authors (e.g. Luneburg, 1947) have proposed that visual space is hyperbolic (possessing negative Gaussian curvature), others have suggested that it is elliptic (possessing positive Gaussian curvature; e.g. Caelli, Hoffman & Lindman, 1978). At the same time, other investigators (e.g. Foley, 1972; Cuijpers, Kappers & Koenderink, 2002; Shipley, 1957) have suggested that the geometry of visual space cannot be defined globally because it is contingent on task or stimulus configuration (also see Wagner, 2006).[1]

A classic demonstration of the local nonlinearity of visual geometry was given by Bartlett (1951). Participants were briefly shown a dot located close to the centre of an A4 sheet of paper and were asked to draw it on an unmarked sheet. They then showed their drawing to the next participant who was asked to do the same. After a number of sequential reproductions, the dot invariably wandered off in the direction of one of the corners of the sheet and settled some small distance away from the corner. A study of the “wandering dot” phenomenon (Stadler, Richter, Pfaf & Kruse, 1991) provided a quantitative description of the field distortion reported by Bartlett. Participants were asked to reproduce the position of one of 609 dots drawn on an invisible rectangular grid on a sheet of A4 paper. Vector field decomposition of the data confirmed the presence of a gradient field possessing fixed-point attractors near the corners of the sheet. The authors did not speculate on the nature of the field apart from stating that the dynamic behaviour of this simple visual context contradicted Gibson’s (1979) direct perception account which neglects the contribution of inner dynamics and self-organizing tendencies of visual perception. The above evidence suggests that in

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[1] It should be noted that the current study investigated spatial relationships in the fronto-parallel plane and not in 3D space. Further, it does not offer a view on the global geometry of visual space.

perception, the geometrical properties of visual space vary locally with the changes in stimulus configuration.

Gestalt psychologist and art theoretician Rudolf Arnheim (1960) has provided a number of observations which suggest that shape and relative position of objects affect their metric relationship with other objects (as well as the whole display). Describing a simple display of a black disc placed off-centre within a square, Arnheim stated that the relations between visual figures are governed not only by the properties of figures themselves but also by a number of invisible and dynamic “directed tensions” or “psychological forces”. He pointed out the dynamic nature of perceptual organization by describing the nonlinear transformations of perceptual field (e.g. compression) caused by the changing relationship between elements. An experimental test of his hypothesis (p. 14) showed that the disc was primarily “attracted” to the corners of the frame and somewhat less to its sides – confirming Bartlett’s findings. In order to describe the forces responsible for attracting the disc towards the corners (and sides), Arnheim speculated that these must operate on the entire field, that is, that changes in any one part of the image must affect the whole.

Figure 1. Rhomboids and bow ties display. A: When figures are equidistant, they tend to group along their bases creating a chain of rhomboids. B: Only after inter-base distance is increased threefold do triangles start to group along their vertices forming “bow ties”.

Whereas the above evidence indicates that the field or frame can produce nonlinear judgments, thus far no studies have investigated the possibility that objects themselves could distort judgment due to local differences in shape. For instance, would the vertices of a triangle affect local distance judgments differently relative to those made with reference to triangle base? Circumstantial evidence in the form of “rhomboids vs. bow-ties” grouping display illustrates this vividly. When figures are equidistant, figure pairs are strongly grouped along triangle bases and are perceived as rhomboids (see Figure 1A). Only when the between-base distance is three times that between vertices, do triangles begin to group along vertices creating “bow-tie” groups (Figure 1B). Although the effect depends on the whole display, the question we pose is whether it could be captured in terms of point distances between triangles. In other words, when the triangles face each other with their bases, they appear closer to each other relative to mutually pointing triangles. Can this difference be measured?

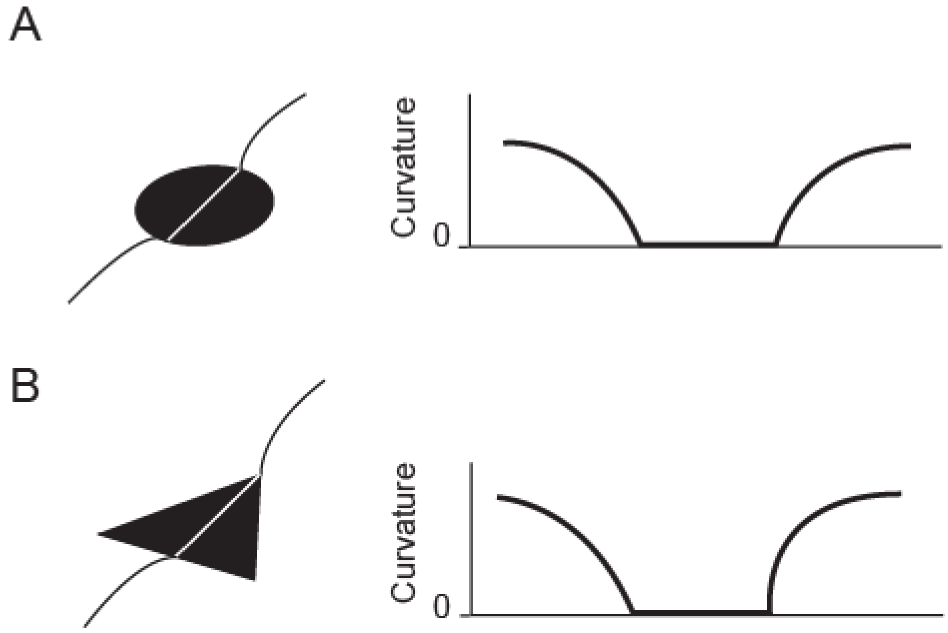

Figure 2. Schematic representation of local distortion created by simple geometrical figures. A: A disc is symmetrical on all sides. Consequently it distorts the surrounding space symmetrically (geodesic on the right). B: A triangle vertex distorts the surrounding space more relative to the base resulting in longer distance judgments (geodesic on the right). This model admits only positive curvature. The abrupt onset of the curvature is not computationally possible—it is shown for effect only.

The next question is: what can cause local difference in shape to affect the grouping distance so drastically? The above observation indicates that for the purpose of grouping two linearly equal intervals are not perceptually equal. It is clear that this effect depends not only on the distance between points of measurement but on the way in which the figures' local properties interact with surrounding space, possibly by distorting it. For instance, MacLeod and Willen (1995) described two-dimensional visual space as an “elastic sheet undergoing spatially continuous deformations that introduce errors in spatial judgment” (p. 51). One way of operationalizing this is to represent the differences in perceived distance as measurable differences in distortion of the space between them. This distortion can conveniently be represented as increased curvature of the space adjacent to the figure. In the simplest algebraic topological model of distortion we imagine a heavy object being dropped onto a thin rubber sheet. As shown in Figure 2A, a symmetrical object such as disc possesses no local figural differences. Consequently, the space around it is distorted equally in all directions. This is illustrated by the geodesic (line describing minimum distance across the display) which is distorted equally in all directions. By contrast, this is different for a triangle (Figure 2B). The curving of the space surrounding the vertex is greater relative to the distortion caused by the base. The greater curvature of the space between two vertices results in a longer perceived distance between the two figures. The rate of change in distortion (greatest positive curvature) is greatest very near the vertex and flattens out at longer distances. Perceived distance is greater near the vertex because of the cumulative effect of integrating across the positive curvature generated by the vertex. The cumulative effect of distortion increases up to a point and then starts to lessen as its contribution to the geodesic diminishes.

However, this model assumes only positive curvature which in turn implies that the presence of multiple objects increases the “pressure” on visual space without a compensatory relief. Finite elasticity of the visual field would mean that the difference between aspects shown in Figure 1 would diminish with the increase in the number of triangles—and “pressure”. Visual inspection confirms that it does not happen when the size of the triangles is kept constant. Locally, the difference in the perceived distance is maintained irrespective of the number of triangles. In other words, in this model, the presence of positive curvature is not offset by the presence of negative curvature. As such, the model cannot account for the fact that Gestalt “forces” and “tensions” are not affected by the overall “mass” of the objects present in the display.

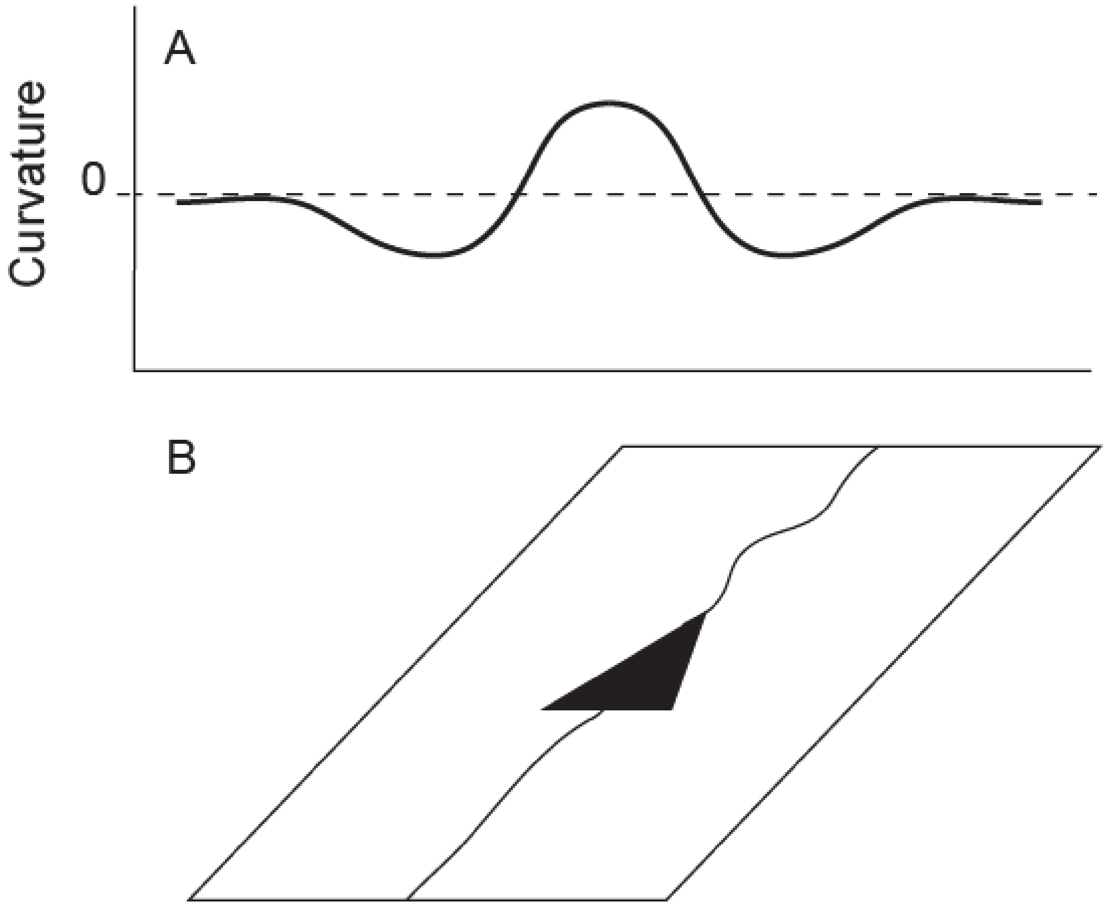

Figure 3. A ripple model of spatial distortion. A: The model assumes that the positive and negative curvature balance out locally. B: Hypothetical effects on different aspects of a triangle.

An alternative model of distortion addresses this problem. Rather than generating only positive curvature, objects create “ripples” or “kinks” in the surrounding space which contain both positive and negative curvature. This is illustrated in Figure 3. In this model, positive and negative curvature are equalised locally (Figure 3A) while still distorting the surrounding space. The hypothetical effects of the ripple model on different aspects of a triangle are shown in Figure 3B.

The aim of the current research was to investigate the local field distortions that might be created by simple geometrical figures within the theoretical framework provided above. In contrast to the experiments by Bartlett (1951) and Arnheim (1960), which describe *internal* distortion occurring within a (rectangular) frame, we wished to investigate *external* distortions that might be expected to propagate outwards from a figure. We hypothesized that these local distortions would affect subjects’ distance judgments and that triangle vertices would distort the surrounding space more relative to triangle bases. The current study consisted of four experiments. In Experiment 1, participants were asked to estimate the distance between two triangles. In one condition, the horizontally oriented triangles faced each other with their vertices and in the other, with their bases. Experiment 2 investigated the effects of local distortion caused by different aspects of a single triangle and Experiment 3 controlled for the centre of mass. Finally, in Experiment 4, elliptic discs were used in order to generalize the findings of Experiments 2, and 3.

Experiment 1

The aim of Experiment 1 was to investigate how judgments of distance in visual space might be affected by the different aspects of equilateral triangles. Following the above argument, we hypothesized that varying the distance between two triangles would produce differences in estimates of distance depending upon the aspect of the triangle (vertex or base). We would expect the region between the triangles to be more distorted when they face each other with their vertices. As the triangles move apart, the distortion of the space between vertices should increase more relative to the space between bases. The shortest path (or geodesic) joining two vertices would gradually become longer relative to that joining the bases, disproportionately affecting the distance judgment. If this were found, it would provide evidence that the presence of simple geometrical figures can distort distance judgments in contexts that do not fall under the category of visual illusions. It would also allow a more systematic investigation of the effects of local field nonlinearities.

Method

Participants. 5 participants (2 female; average age 26 years and 5 months) took part in Experiment 1. Two participants had corrected-to-normal vision.

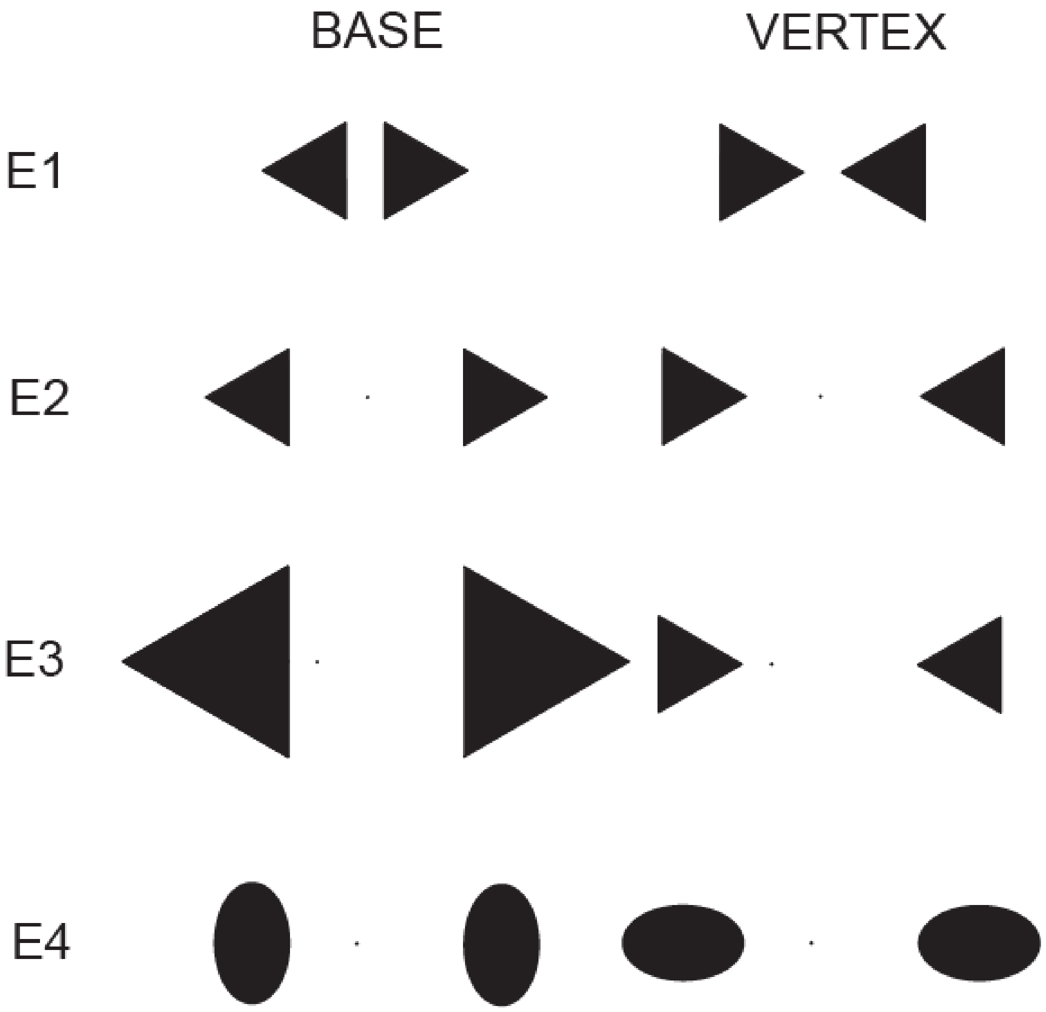

Figure 4. Stimuli presented in Experiments 1, 2, 3 and 4. See text for details.

Stimuli. A stimulus display consisting of a pair of identical horizontally oriented black-filled equilateral triangles was presented against a white background with stimuli centred with the reference to the monitor screen (see Figure 4). The area of each triangle was 173 mm^2 (side = 20 mm, height = 17.3 mm) with eight equal distance increments from 4mm to 32 mm between triangles. This led the shortest display to subtend a horizontal visual angle of 3 degrees, and the longest, 5 degrees and 12 minutes from a viewing distance of 80 cm.

The vertical visual angle was approximately 1 degree and 24 minutes. The average luminance of the background was 0.0359 cd/cm^2, whereas the value for the stimulus was 0.00002 cd/cm^2. The experiment was controlled by Superlab experimental software. Experiment 1 and all subsequent experiments were run from a Dell Latitude notebook connected to a 19-inch Mitsubishi Diamond Plus 91 monitor. The horizontal length of the monitor was 341 mm and the maximum length of the stimulus in Experiment 1 was 66.6 mm or under 20% of the overall length. The length of the unused space on each side was 137 mm—over twice the length of the stimulus.

Design and procedure. The order of presentation was randomized in advance within four 24-trial blocks using random number tables. There were two levels of aspect (vertex, base) and eight levels of distance (4 to 32 mm), resulting in 16 conditions.

Participants were tested in a sound-attenuated room, with constant, low-level ambient illumination provided by a ceiling light. The participants were shown an example of an experimental display (minimum and maximum distance) and instructed to estimate verbally the distance between the two triangles using the category scaling method, which produces reliable subjective judgment functions (Stevens, 1975 p. 146). Participants were shown examples of the minimum and maximum distance in both configurations (bases and vertices) for two seconds each and instructed as follows: “Your task is to report how far apart the triangles are by assigning a number to your estimate. Assume the maximum distance between the figures is divided into 100 equal units. Assign, to each distance, a number of units that seems appropriate to you. Then assign numbers to successive distances in such a way that they reflect your impression of their magnitude. You may use whole numbers only. Try to make each number match the distance that you see.” The stimulus was shown at the beginning only because no benefit is gained by its repeated presentation (Kahnemann & Beatty, 1967; Stevens, 1975, p. 141).

Each trial was preceded by a visual prompt (a finger pointing at a button) signalling to the participants that they could initiate the next trial by pressing the space bar. Constant viewing distance was maintained by means of a chinrest the participants were instructed not to move their head during the experiment. Each display lasted two seconds and was succeeded by a quasi-random mask which covered the entire screen for two seconds and was followed immediately by the subsequent trial. The experimenter recorded participants' estimates on prepared response forms. No practice trials were given and the experiment was run in a single block without breaks. Each condition was presented 8 times, giving 128 trials per participant. According to Stevens (1975), this number of trials is sufficient for stable judgment curves. Experimental conditions were fully randomized. Experimental session lasted between 15 and 20 minutes.

Results and discussion

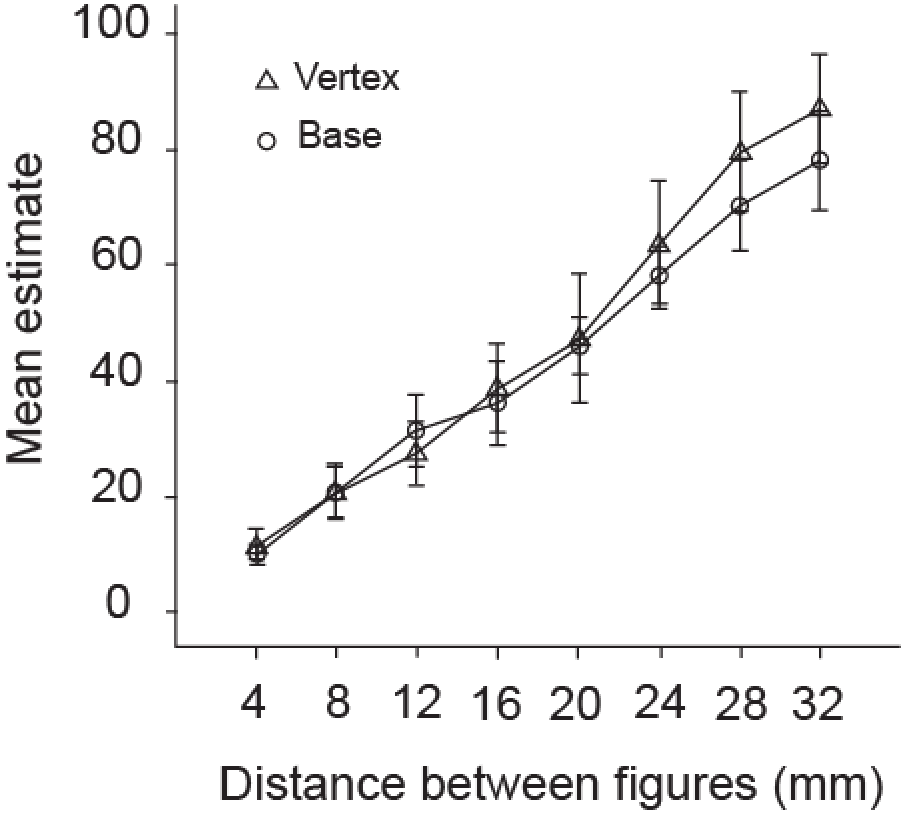

Figure 5. Mean distance estimates (± 1 SEM) as a function of physical distance and figure aspect in Experiment 1.

Mean estimates were analysed by means of a 2 x 8 repeated-measures analysis of variance (ANOVA) with terms aspect and distance. As in all subsequent experiments, a Greenhouse-Geisser correction was applied to control for the violation of the sphericity assumption. The main effect of aspect was non-significant. As shown in Figure 5, there was a highly significant main effect of distance [$F(7, 28) = 125.91$, $MSE = 52.90$, $p < .001$], as well as a significant interaction between aspect and distance [$F(7, 28) = 4.27$, $MSE = 11.80$, $p = .003$]. The former effect reflected the increase in estimate size as a function of increase in distance between triangles. The absence of a main effect of aspect indicated that overall, there was no difference in judged distance between the two triangles. However, the observed interaction suggested a gradual relative increase in judged distance in the vertex condition.

Simple main-effects analysis revealed a significant difference at 12 mm (mean difference = 3.88 units, standard error = 1.35, p = .045) with vertex estimates being shorter, and at 28 mm (mean difference = 9.25 units, standard error = 2.81, *p* = .030). This was caused by vertex estimates being relatively longer.

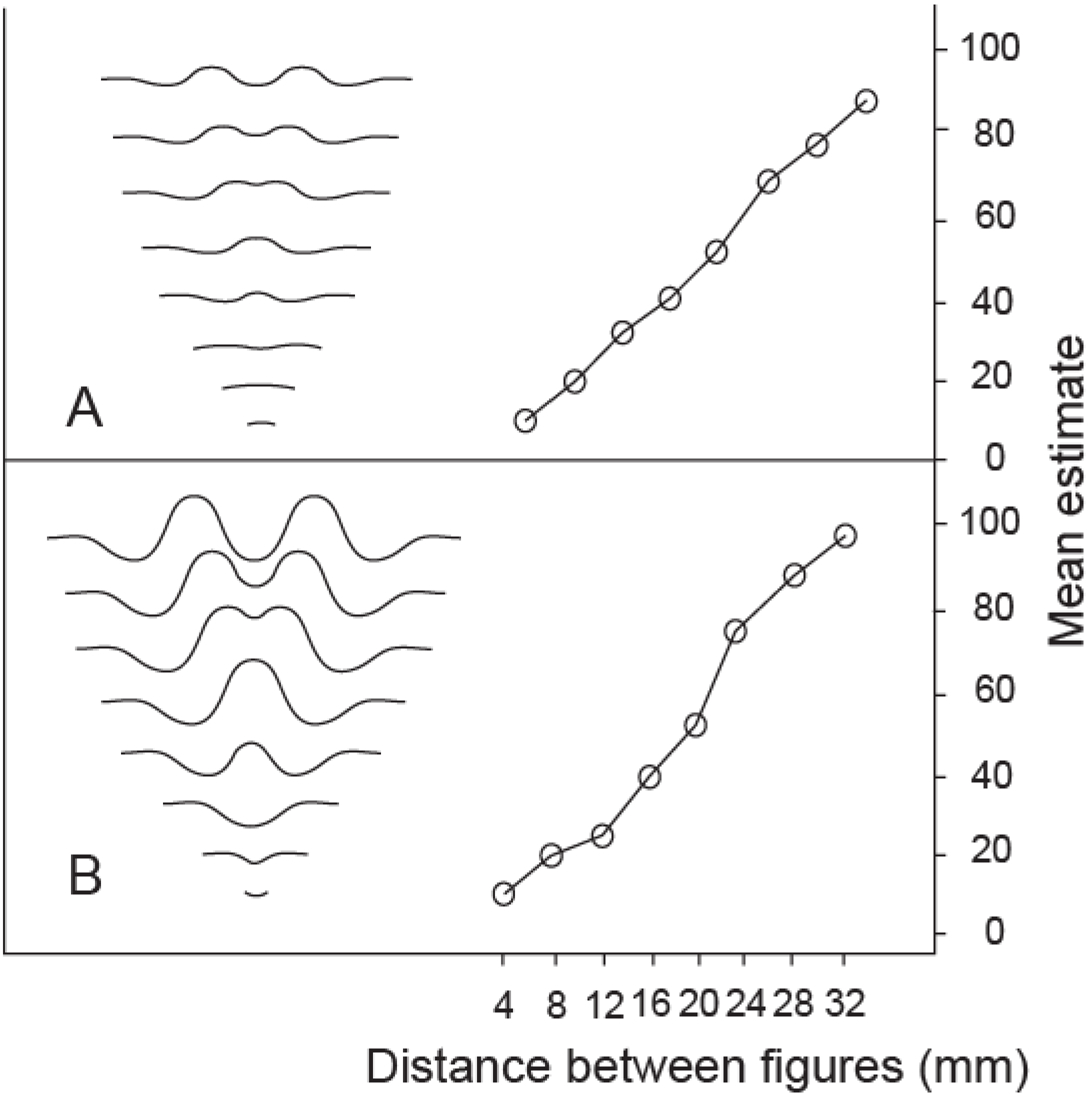

Figure 6. Hypothesised effects of a ripple model of space distortion on distance estimates. A: The base condition. B: The vertex condition. See text for details.

Figure 6 illustrates how the observed effects could be explained in terms of distortions in an elastic medium. As can be seen, the observed findings are incompatible with a simple "inflated-sheet" model which would predict a steady increase in difference between the conditions. Rather, the functions are best explained by a ripple model. When vertices are close, the curvature of the space is initially negative. Initially, the effect is too small to detect statistically, but the cumulative effect of this negative curvature manifests itself by 12 mm where the vertex condition leads to the figures seeming perceptually closer to each other than in the base condition.[2] At some point after 12 mm, the curvature of the space becomes positive and the cumulative effect of this positive curvature does not become manifest until 28 mm. The relatively abrupt increase in estimates at this distance is caused by the appearance of the second ripple at that distance. These results generally agreed with our hypothesis according to which, the vertex of a triangle distorts the local field more than does its base. This is confirmed by the trend for vertex estimates to be longer at larger distances.

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[2] We cannot exclude the possibility that the reversal at 12 mm was caused by the relative inflation of the space in the base condition (see Figure 6A).

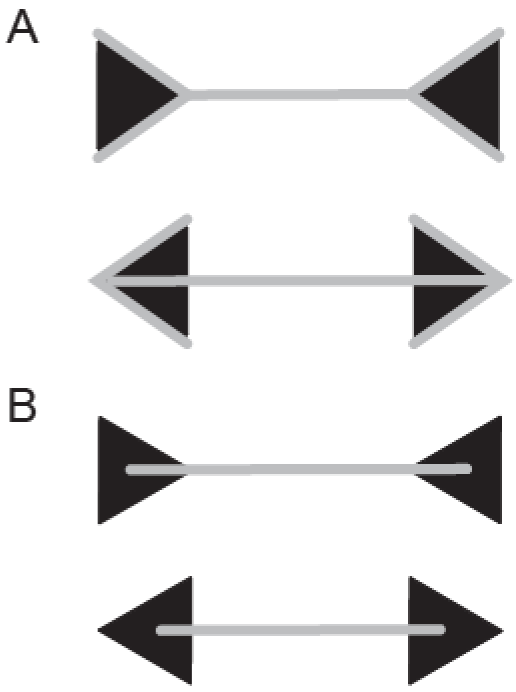

Figure 7. Alternative explanations for the observed effect. A: Müller-Lyer illusion—a vertex facing display appears longer. B: Asymmetrical location of the centroid makes the distance between two centres longer in a vertex-facing display.

There are two alternative explanations for the observed effect (see Figure 7). First, the display could have evoked a Müller-Lyer type illusion (Müller-Lyer, 1889) with the result of the vertex-facing display appearing longer (top of Figure 7A).[3] In a similar vein, the centre of mass (or centroid; Friedenberg & Liby, 2008) of an equilateral triangle is located closer to its base relative to the opposing vertex and has been shown to attract attention (Baud-Bovy & Soechting, 2006; Zhou, Chu, Li & Zhan, 2006). Thus, ostensibly, the observed result could be explained in terms of a larger inter-centroid distance in vertex-facing displays (top of Figure 7B). However, the absence of a main effect of aspect indicates that the difference

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[3] It is conceivable that the Müller-Lyer effect is caused by the spatial distortion described in the present study (Watson, 1978). However, a general treatment of this and other optical illusions is outside the scope of the paper.

between vertex and base estimate functions was due to local spatial distortion and not to either of the above accounts.

In order to examine the effects of aspect and distance on estimate variability, mean standard deviation (SD) scores of distance judgments were subjected to a 2 x 8 repeated-measures ANOVA. If the observed lengthening of vertex estimates were accompanied by increased response variability, the observed effect could not be considered a veridical record of the underlying spatial distortion. There was a highly significant main effect of distance [$F(7, 28) = 11.04$, $MSE = 11.90$, $p < .001$], indicating that estimate variability increases with distance. The main effect of aspect and the aspect by distance interaction did not achieve significance.

It is worth noting that estimates did not span the full range of the scale in the direction of the maximum value. This could not be explained in terms of framing by screen edges for two reasons. First, the horizontal extent of the screen was 341 mm and the maximum extent of the display was just over 66 mm. Thus, the edge of the display was very far from the screen edge. Second, framing would tend to accentuate internal distance by warning the observer that the maximum distance has been reached and one would expect overestimates and not underestimates. An explanation could be that visual space is compressed in memory—a number of classical studies show that spatial representations are distorted in memory (e.g. Taylor, 1961). Objects can also expand in memory (Baldwin & Shaw, 1895), making the space between them appear smaller. Alternatively, under conditions of uncertainty, participants could assume a conservative criterion and prefer to err on the side of caution.

In conclusion, the results of Experiment 1 offered qualified support for a measurable effect of local differences in figure aspect on subjective estimates of distance. The observed effects were compatible with a ripple model of spatial distortion. The next question was whether the presence of the ripple could be detected in the vicinity of a single figure.

Experiment 2

The findings of Experiment 1 indicated that different aspects of an equilateral triangle produced differences in subjective distance estimates. As the distance between triangles increased, estimates in the vertex condition increased relative to the estimates in base condition. The absence of a main effect of aspect indicates that the effect was not caused by a strategic choice of anchors. In other words, participants did not use different reference points for different aspects. The most plausible explanation of the observed effect is that figures locally distort surrounding perceptual space. However, the nature of the local distortion can only be inferred indirectly from the joint contribution of the two objects. If our hypothesis, namely, that different aspects of a triangle create different local distortion, is correct, a single figure should affect distance judgments in a similar way. Consequently, in Experiment 2 participants were asked to judge the distance of a dot from the unmarked centre of the display. The distance between the triangles was kept constant. It was assumed that this task would expose the shape of the distortion induced by a single object, and at the same time, the task did not require participants to explicitly refer to the object in order to arrive at their estimate (distance was judged between the dot and the centre). We hypothesized that distance judgments would be affected by local differences in spatial distortion with vertex distorting distance judgments more relative to base. More specifically, we predicted that the observed pattern of distortion would be compatible with the ripple model.

Method

Participants. 15 volunteers (6 female; average age 30 years and 5 months) took part in Experiment 2. Seven participants had corrected-to-normal vision.

Stimuli. The experimental display comprised two horizontally oriented black-filled equilateral triangles (area 173 mm^2) with distance between a triangle and the (unmarked) centre of the display of 14 mm (see Figure 4). To exclude the effects of scanning direction on subjective estimates (e.g. Brodie & Dunn, 2005; Brodie & Pettigrew, 1996) a small dot was placed on an equivalent number of trials either to the left or right and at one of six distances (2, 4, 6, 8, 10 and 12 mm) from the unmarked centre of the display. The display subtended a horizontal visual angle of 4 degrees and 29 minutes and a vertical angle of 1 degree and 24 minutes from the viewing distance of 80 cm.

Design and procedure. The design was a 2x2x6 repeated measures design with factors side (left, right), aspect (base, vertex) and distance from centre (2, 4, 6, 8, 10 and 12 mm). There were four trials per condition giving 96 trials per participant. Experimental trials were fully randomized and other stimulus presentation parameters were identical to those in Experiment 1. Participants were asked to estimate the distance of a dot from the centre in units from 0 (dot at the centre) and 100 (dot touching the figure) using the method of category scaling. As in Experiment 1, participants were tested in a sound-attenuated and lighting-controlled room. After being shown the maximum- and minimum-distance conditions from both aspects and orientations, they were given the following instructions: "Assume the distance between either figure and the centre of the display is divided into 100 equal units. Between the figures you can see a small dot. Your task is to judge the distance between the dot and the centre. Assign, to each distance, a number of units that seems appropriate to you. Then assign numbers to successive distances in such a way that they reflect your impression of their magnitude. You may use whole numbers only. Try to make each number match the distance that you see."

Each trial was preceded by a visual prompt signalling to the participants that they could initiate the next trial by pressing the space bar. Constant viewing distance was maintained by means of a chinrest. The participants were instructed not to move their head during the experiment. Each display lasted two seconds and was succeeded by a quasi-random mask subtending over 10 degrees of visual angle. The mask, which also lasted two seconds, was replaced by a visual prompt. Participants' estimates were recorded manually on another computer. There were no practice trials and the experiment was run in a single block. An experimental session lasted approximately 30 minutes.

Results and discussion

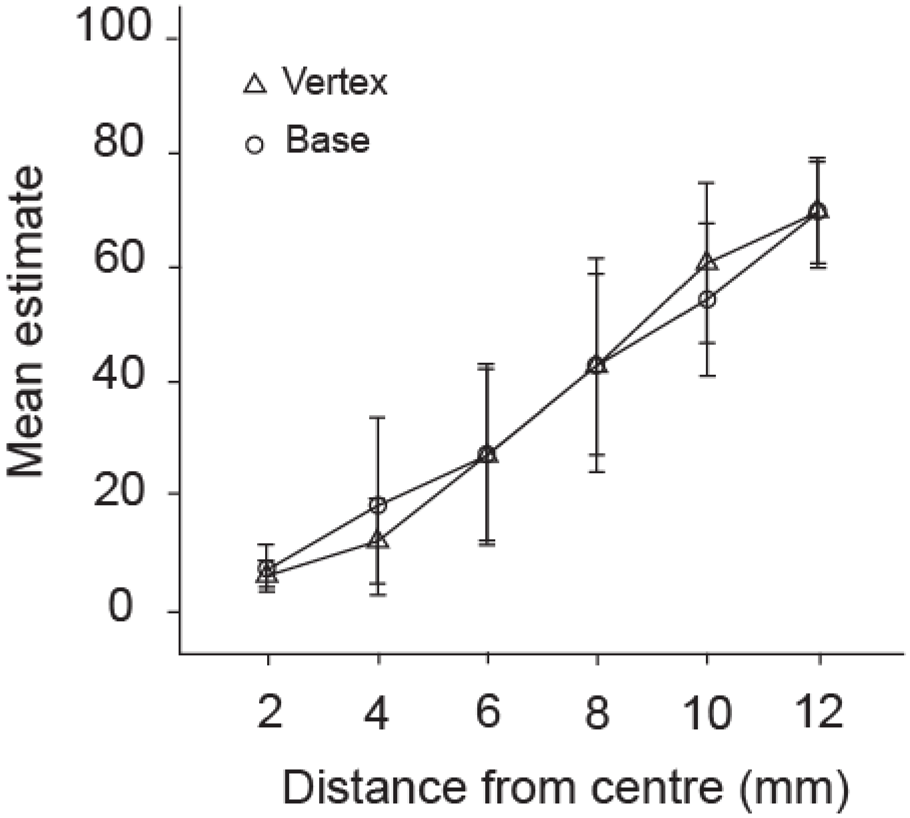

Figure 8. Mean distance estimates (± 1 SEM) as a function of physical distance and figure aspect in Experiment 2.

A repeated-measures ANOVA for factors side (left, right), aspect (base, vertex) and distance (2 to 12 mm) was carried out on mean estimates. As expected, the main effect of distance was highly significant [$F(5,70) = 330.67$, $MSE = 422.97$, $p < .001$] as was the effect of side [$F(1,14) = 23.32$, $MSE = 31.86$, $p < .001$].The latter was due to left judgments being slightly lower overall. There was a significant side by distance interaction [$F(5,70) = 3.20$, $MSE = 14.28$, $p = .012$] with left judgments being lower when the dot is close to the centre. Critically, as shown in Figure 8, the aspect by distance interaction was highly significant [$F(5, 70) = 9.90$, $MSE = 18.98$, $p < .001$], with other two interactions failing to reach significance. It should be noted that the vertex curve met the base curve at the minimum and maximum distance points suggesting a frame or edge effect; once the dot was close to the centre (or the triangle), the participants would correct their estimates to bring them in line with the base estimates. This is similar to the edge effect observed by Bartlett (1951) and Stadler, Richter, Pfaf & Kruse (1991). In these studies, dots never travelled all the way to the corners of the sheet but always stopped slightly before. A simple main-effects analysis revealed that the source of the interaction was significant difference between two aspect functions at 4 mm (mean difference = - 7.35 units, standard error = 1.30, p < .001) and 10 mm (mean difference = 4.57 units, standard error = 0.92, p < .001). All other differences were nonsignificant.

As in Experiment 1, an omnibus ANOVA for factors side (left, right), aspect (base, vertex) and distance, was performed on average SD scores, this time in order to test for the possibility that the observed distortion in the vertex condition was associated with greater response uncertainty caused by a single anchor. This revealed a highly significant main effect of distance with estimate variability peaking roughly halfway between the centre and the figure [$F(5,70) = 11.38$, $MSE = 70.71$, $p < .001$]. Interactions of side with aspect and distance reached significance [$F(1,14) = 5.80$, $MSE = 15.30$, $p < .05$ and $F(5,70) = 3.73$, $MSE = 26.01$, $p < .05$ respectively]. They reflected a 2-mm shift in the peak of the base SD function contingent on side. The three-way interaction was also significant [$F(5,70) = 3.36$, $MSE = 21.47$, $p < .05$]. This was caused by the vertex estimates being somewhat larger close to the figure, when the dots were presented on the right. Most relevant was the significant aspect by distance interaction [$F(5,70) = 5.32$, $MSE = 26.52$, $p < .01$] reflecting different variability profiles for the two aspects. Close to the figure, there was little difference in variability but as the dot approached the centre, base estimates became relatively more variable. This invalidates the argument according to which distortion in the vertex condition is due to the presence of a single reference point (in contrast with base which provides multiple reference points). Rather, the presence of multiple potential anchors made the judgment more variable as the dot approached the centre. Finally, there was no difference in variability between the two aspects close to the centre.

Experiment 3

The distance from the centre of an equilateral triangle to its apex is larger than the distance to the base. Although the results of Experiment 1 indicated that the centre of mass was not used in arriving at distance judgments, we wished to confirm this in a local distance estimate task. In addition, this allowed us to test directly the hypothesized effects of mass against those of aspect. If the observed difference were in any way caused by the differences in internal distance, the effect should be abolished when the internal distances are equalized in the two conditions. Consequently, in Experiment 3 stimulus size was adjusted so that the internal distance from the centre of the triangle to its base (in the base condition) equalled the internal distance from the centre to the vertex (in the vertex condition). This resulted in the triangle presented in the base condition being substantially larger than the triangle in the vertex condition. If the effect observed in Experiment 2 was caused by the aspect-related differences in spatial distortion, we expected it to be replicated under these conditions.

Method

Participants. 4 undergraduate students (1 female, average age 19 years and 5 months) participated in the experiment. 2 participants had corrected-to-normal vision.

Stimuli. Two horizontally oriented and centred black equilateral triangles were presented against the white background. In the base condition, the triangles were larger than in the vertex condition, (side = 38.7 mm; area = 649mm^2) so that the distance from the centroid of the large triangle to the centre of one of its sides equalled the distance from the centroid of the smaller triangle to one of its vertices (see Figure 4). The base display subtended a horizontal visual angle of 6 degrees and 46 minutes and a vertical angle of 2 degrees and 47 minutes. For the vertex display, the horizontal visual angle was 5 degrees and 32 minutes of arc and the vertical angle 1 degree 24 minutes, as in Experiment 2.

Design and procedure. The design was a 2 x 2 x 6 repeated measures design with factors side (left, right), aspect (base, vertex) and distance from centre (2, 4, 6, 8, 10 and 12 mm). There were four trials per condition giving 96 trials per participant. Experimental trials were fully randomized. The task consisted in estimating the distance of a dot from the unmarked centre of the display using category scaling. As in previous experiments, testing took place in a sound-attenuated and lighting-controlled room. Each trial was preceded by a visual prompt signalling to the participants that they could initiate the next trial by pressing the space bar. Constant viewing distance was maintained by means of a chinrest and the participants were instructed not to move their head. The instructions were identical to those used in Experiment 2.

Each display lasted two seconds and was followed by a quasi-random mask subtending over 10 degrees of visual angle. The mask, which also lasted two seconds, was succeeded by a visual prompt. Participants' estimates were recorded manually. There were no practice trials and the experiment was run in a single block. An experimental session lasted approximately 30 minutes.

Results and discussion

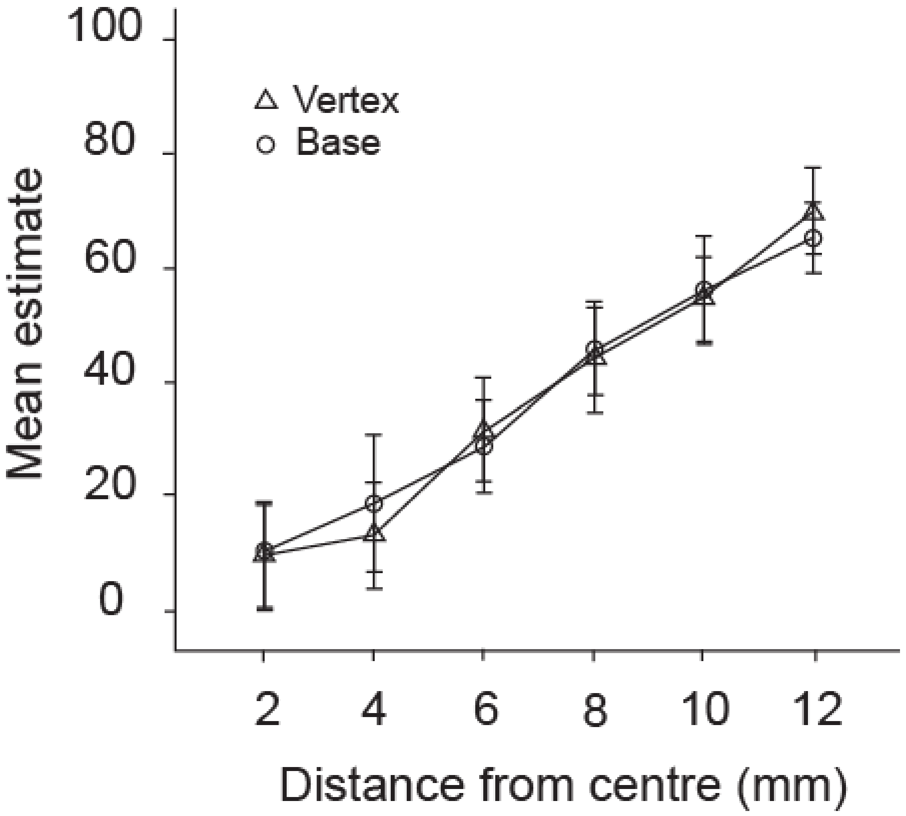

Figure 9. Mean distance estimates (± 1 SEM) as a function of physical distance and figure aspect in Experiment 3.

As in all previous experiments, the main effect of distance was highly significant [$F(5, 15) = 136.18$, $MSE = 96.52$, $p < .001$]. Again and critically, there was a significant interaction between aspect and distance [*F*(5, 15) = 5.83, *MSE* = 14.52, *p* < .01] with the shape of the interaction observed in Experiment 2 preserved (see Figure 9). The two conditions are identical over the central three points and the vertex estimates are lower than the base ones when the dot was 4 mm away from the centre and higher when it was 2 mm away from the vertex of the triangle. There was also a significant three-way side by aspect by distance interaction [*F*(5, 15) = 3.01, *MSE* = 9.61, *p* < .05]. Although the above-described pattern of effects was present at both left and right, it was more prominent when the stimuli were presented to the right. Again, a simple-effects analysis was carried out in order to establish the source of the interaction. The only significant differences were observed at 4 mm (as in Experiment 2; mean difference = -5.63 units, standard error = 1.41, *p* = .028) and 12 mm (mean difference = 4.69 units, standard error = 0.94, *p* = .015). It should be noted that unlike in Experiment 2, where one locus of difference was at 10 mm, here, a significant difference was observed at 12 mm. This is addressed in general discussion. In addition, the results of the variability analysis were very similar to those reported in Experiment 2. Briefly, both aspect functions described an inverted-U profile.

Experiment 4

The aim of Experiment 4 was to generalize the findings of Experiments 2 and 3 to objects not possessing fixed-point attractors (vertices). To this purpose, two elliptic discs were used. Since ellipses have sides of different length, it was possible that the participants would use the outer edge when making their judgment. If this were the case, the “long-side” estimates should be noticeably larger in all distance conditions resulting in a significant main effect of aspect. However, if the observed effect is caused by local distortion, the interaction observed in the previous experiments should be present, albeit not as strongly. This is because the local gradient field created by the short side of an elliptic disc is not as pronounced as that caused by a vertex of a triangle.

Method

Participants. 23 participants (eight female) took part in Experiment 4. The average age was 28 years and 3 months. 13 participants had corrected-to-normal vision.

Stimuli. Experimental display consisted of two black filled elliptic discs. The area of each disc was 282 mm^2 (axis ratio 1.6:1). From a viewing distance of 80 cm, the entire display subtended 5 degrees and 25 minutes in the “long” condition and 4 degrees and 9 minutes in the “short” condition (Figure 4).

Design and procedure. As in Experiments 2 and 3, the design was a 2 x 2 x 6 repeated measures design with factors side (left, right), aspect (short, long) and distance from centre (2, 4, 6, 8, 10 and 12 mm). There were four trials per condition giving 96 trials per participant. Experimental trials were fully randomized. As in previous experiments, participants were asked to judge the distance of a dot from the unmarked centre of the display using category scaling. Testing took place in a sound-attenuated and lighting-controlled room. Participants were shown the maximum- and minimum-distance conditions in both orientations. Each trial was participant-initiated and preceded by a visual prompt. Constant viewing distance was maintained by means of a chinrest and the participants were instructed not to move their head. The instructions were identical to those used in Experiments 2 and 3.

Each display lasted two seconds and was succeeded by a quasi-random mask identical to that used in previous experiments. The mask of two seconds duration was succeeded by a visual prompt. Participants’ estimates were recorded manually. There were no practice trials and the experiment was run in a single block. An experimental session lasted approximately 30 minutes.

Results and discussion

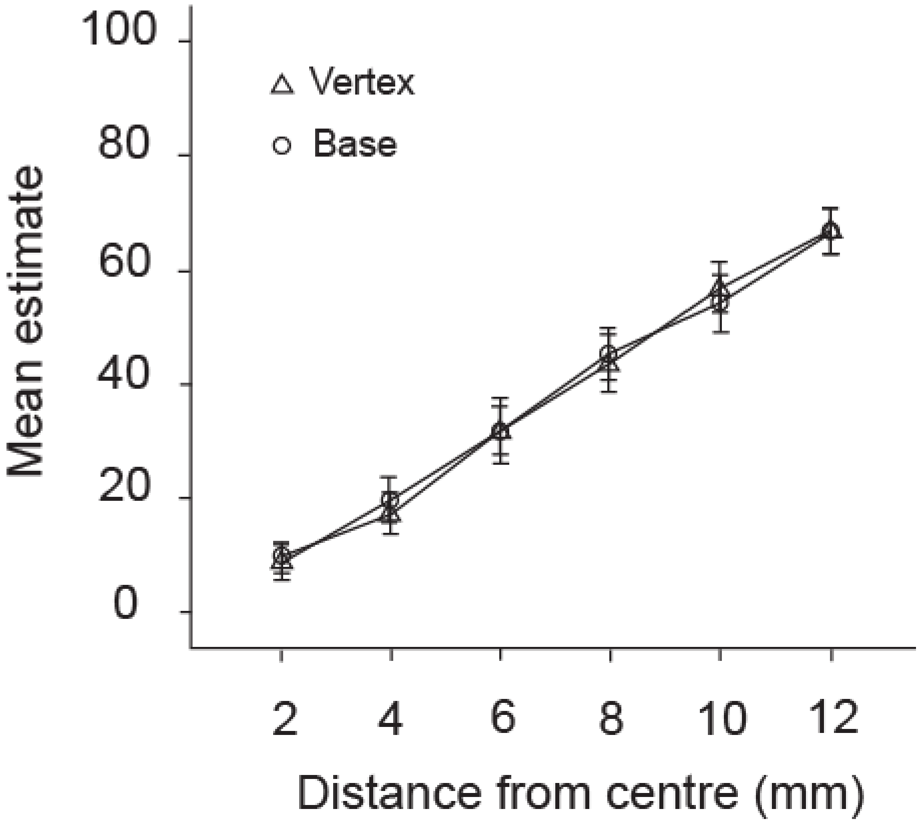

Figure 10. Mean distance estimates (± 1 SEM) as a function of physical distance and figure aspect in Experiment 4.

As in all of the above experiments, the main effect of distance was highly significant [*F*(5, 110) = 723.53, *MSE* = 64.83, *p* < .001], reflecting the task-imposed increase in estimates with distance. Importantly, the interaction between aspect and distance was also highly significant [*F*(5, 110) = 5.05, *MSE* = 10.63, *p* < .001]. Inspection of Figure 10 confirms that distance judgments described the pattern familiar from Experiments 2 and 3 although the effect was more pronounced on the right [*F*(5,110) = 3.10, *MSE* = 11.96, *p* < .05]. Simple main-effects analysis showed that as in Experiments 2 and 3, vertex estimates were significantly shorter relative to base estimates at 4 mm away from the centre (mean difference = - 2.48 units, standard error = 0.72, p = .002). Although the difference at 10 mm mirrored that observed in Experiment 2, it failed to reach significance (*p* = .099). The results of the SD analysis were almost identical to those reported in Experiment 3.

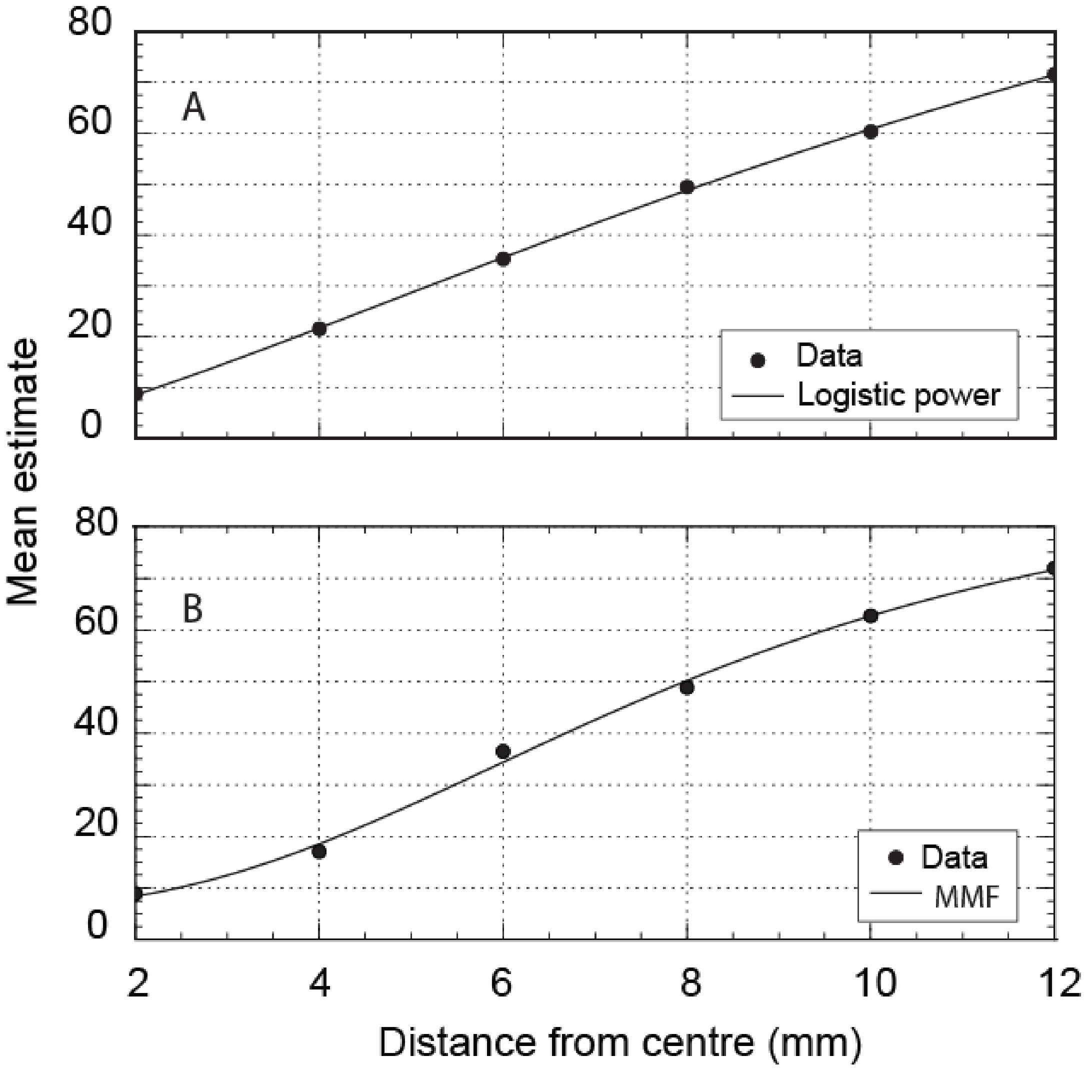

Figure 11. Curve fitting results for aggregate data from Experiments 2, 3 and 4. A: The best fit for the base function is provided by a sigmoid logistic power function of the form $y = a/(1+(x/b)^c)$; $r^2 > .99$. B: The vertex function was best described by a sigmoid MMF model ($y = (ab+cx^d)/(b+x^d)$; $r^2 > .99$).

If the above results reflect the same underlying process, aggregated data from Experiments 2, 3 and 4 (including 42 participants) should amplify the observed effects. Indeed, the results of an ANOVA for factors aspect and distance with experiment as a between-subjects variable, performed on the pooled data, revealed a highly significant main effect of aspect [$F(5,195) = 657.63$, $MSE = 44.75$, $p < .001$] and importantly, aspect by distance interaction [$F(5,195) = 14.89$, $MSE = 5.62$, $p < .001$] replicating the pattern of effects described above. Significant distance by experiment and aspect by distance by experiment interactions [$F(10, 195) = 6.73$, $MSE = 44.75$, $p < .001$ and $F(10, 195) = 4.06$, $MSE = 5.62$, $p < .001$, respectively], reflected the between-experiment changes in judgment functions illustrated in individual figures.

To examine the general trend in the data, base and vertex estimate functions were subjected to a curve fitting procedure. As can be seen in Figure 11, both were best described by sigmoid models (logistic power and MMF respectively; both $r^2 > .99$). This suggested that both base and vertex distances were distorted with the vertex function being more so. However, the nonlinearity in the base function could not be established conclusively since a linear fit was almost as good.

General Discussion

Despite circumstantial evidence suggesting that the visual field is a dynamic phenomenon subject to non-linear distortions, no research thus far has investigated the possibility that distortion could be caused by simple geometrical figures. In this study we investigated the effects of the shape of simple geometric figures on judgments of distance. We hypothesized that simple geometrical figures distort the surrounding space and that these distortions influence subjective estimates of distance. In Experiment 1, we showed that two equilateral triangles facing each other with their vertices were perceived as lying farther apart than two triangles facing each other with their bases. If this effect had been caused by strategic differences in the choice of anchor, we could have expected a significant main effect of aspect. This effect was not found.

In the base condition, the participants could have chosen the centre of mass or the vertex as their anchors. Yet, the only significant effect was the interaction between aspect and distance, which is suggestive of nonlinear changes in the space between the two figures. A reasonable explanation involves a disproportionate lengthening of the geodesic between the figures caused by the distortion of the field. Experiments 2, 3, and 4 employed a task which allowed the effects of a single figure to be investigated. The results indicated that estimates of distance of a small dot from the centre of the display were affected by the aspect of a figure (equilateral triangle in Experiments 2 and 3 or elliptic disc in Experiment 4). Specifically, distance estimates were distorted when a triangle vertex (or the short side of an ellipse) faced the centre of the display relative to the condition in which triangle side or long side of the ellipse faced the centre. The effect was maintained when the conditions were controlled for internal distance/centre of mass (Experiment 3). The most important finding of these experiments was the differential effect of dot position with regard to triangle vertex (or short ellipse side).

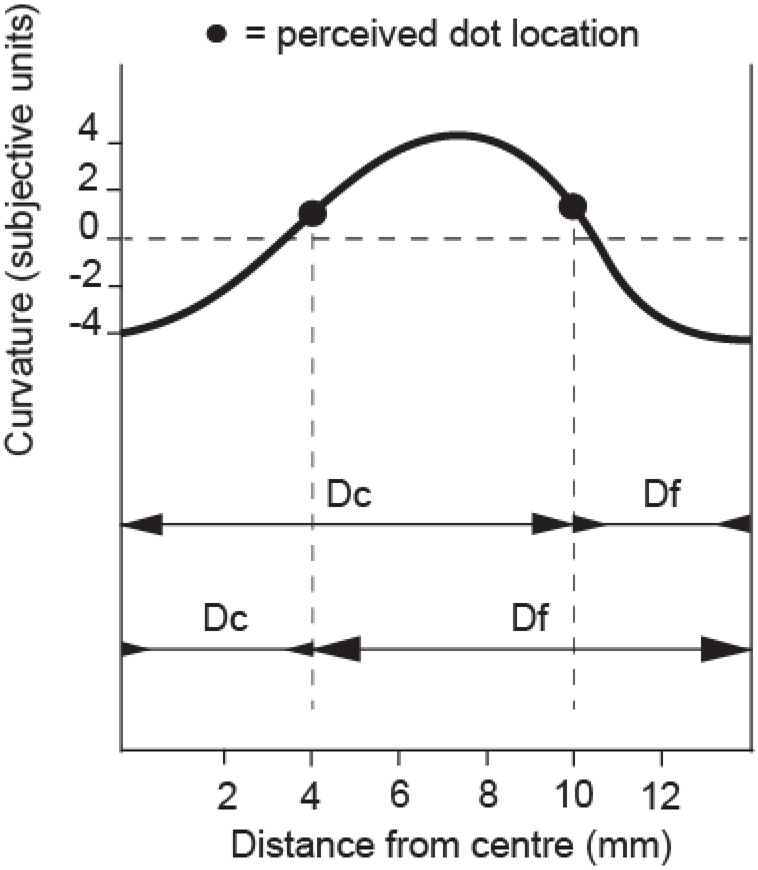

Figure 12. Inferred shape of the local spatial distortion caused by the vertex and its effect on distance judgments. The quasi-Gaussian curve represents a geodesic connecting the centre and the figure. The ripple is skewed towards the vertex leading to a small asymmetry in estimates at 4 and 10 mm. Arrowhead size reflects the perceived length of segments. The scale on the ordinate marks the extent of the curvature in subjective units.

Does the shape of the vertex function observed in Experiments 2, 3 and 4 (Figure 10B) tell us anything about the shape of the underlying perceptual space? If we discount the terminal points (2 and 12 mm), the observed effects suggest a localized disturbance of the surrounding space consistent with a Gaussian ripple in the surrounding space. The observed pattern of effects is consistent with a bisection model of the form Dtot = Dc + Df, where Dtot represents the total distance, Dc distance from the centre and Df distance from the figure (see Figure 12). When the dot is half-way between the figure and the centre, curvatures in Dc and Df balance out leading to close-to-linear estimates. As it approaches the centre, the observer has to integrate over the positive curvature created by the vertex, which leads to the overestimate of Df and consequently to the underestimate of Dc. Conversely, when the dot is close to the figure, the curvature causes overestimation of Dc and underestimation of Df.

Simple main-effects analyses performed on the aggregate data set indicated that the ripple was not completely symmetrical. A large difference (4.58 units, $p < .001$) was observed at 4 mm. At the same time, a somewhat smaller difference (2.38 units, $p < .001$) was observed at 10 mm. This is suggestive of the presence of asymmetry or skew in the ripple in the direction of the vertex (see top of Figure 12). The space close to the vertex is more curved which causes a large overestimation of Df and underestimation of Dc at this point. On the other hand, the space close to the centre is less curved, leading to a somewhat lower overestimate of Dc at 10 mm. The difference in curvature between centre—10 mm and the vertex—4mm segments, amounts to approximately two subjective units.

It is worth noting that estimates at 10 mm were both less distorted and more variable relative to those at 4 mm, aggregate data notwithstanding. Specifically, in Experiment 2, a difference was observed at 12 mm and in Experiment 4, the difference at 10 mm was observed but failed to reach significance. The explanation might lie in the fact that the centre—10 mm segment was less distorted as well as the fact that Dc component possessed only one visible anchor (dot)—unlike Df which was based on two visible anchors (dot and figure). This might have made long Dc estimates less salient and stable.

The distortion could not have been created by the centre itself. This would be conceivable if one assumed that the distortion were caused for instance by attentional modulation (Logan, 1996). However, if that were the case—namely, that attentional focus on the centre caused the curving of the surrounding space—the effect would have been observed in the base condition as well. The absence of centre-generated distortion in the base condition is a clear indication that the sole source of the observed effect was the vertex (or the short side of an ellipse). This has implications for models of perceptual space. Rather than distorting the surrounding space smoothly (as shown in Figure 2), the vertex creates a static ripple or kink in the field. This extends away from the vertex in the form of a raising gradient which then subsides creating a Gaussian profile. Thus, when a distance close to the centre is estimated, the ripple enlarges the perceptual distance from the figure, repelling the dot. The same if attenuated effect occurs in the opposite direction.

To recapitulate, the results of the present study are not compatible with a simple elastic sheet model. Rather, the vertex of a triangle (or a short side of an ellipse) creates a Gaussian ripple in the surrounding space which has a dual effect—it both attracts and repels. This interpretation is consistent with the field theory of perceptual organization which views objects as actively interacting with the visual space. In agreement with observations by Bartlett (1951) and Arnheim (1960), the field is distorted by visual objects, and these distortions play a role in perceptual organization. The findings evoke the metaphors used by Arnheim—those of attraction and repulsion. This is precisely what we observed—when close to the vertex, the dot is “attracted” to it, and it is “repelled” when close to the centre. Importantly, these results have been obtained with regular visual objects and do not depend on an idiosyncratic arrangement of stimulus elements characteristic of visual illusions. In other words, these effects occur in normal perception.

The observed differences in judgments are stable if subtle. In Experiment 1, the space bounded by two vertices was perceived as being roughly 10% longer than the same space in the side condition at the physical distance of 32 mm. The difference was smaller in

Experiments 2, 3 and 4. Importantly, the difference decreased in line with the increasingly stricter constraints, adding further support to our hypothesis. To illustrate, the difference caused by the change in aspect of a single equilateral triangle (Experiments 2 and 3) was about 5%--about half the difference observed in Experiment 1. The difference was even smaller in Experiment 4 due to change in the physical properties of the stimulus. Yet, the pattern of effects is consistent and reliable and as such, we believe, it is worthy of further investigation. It should also be noted that the effects of local distortion were observed in judgment variability scores. Generally, the most salient effect was the inverse U profile consistent with the increase in uncertainty halfway between the figure and the centre. Although no conclusions could be reached without further testing, the finding suggests that a parametric model of visual space must provide a thorough account of the effects of spatial distortion on response variability.

Our results are consistent with Watson’s (1978) proposal that visual space can be locally non-Euclidean. Watson proposed that the geometry of visual space changes locally depending on the relationship between the objects that occupy it. He hypothesized that lines and curves in visual space introduce a “force field” which distorts perceived geometrical relations with regard to the Euclidean geometry. He contrasted two approaches to this “force-field” theory. The first one assumes that perceptual distortions affect objects but not the underlying visual space which remains Euclidean (p. 142). Second, and following the failure of this model to account for visual illusions, Watson suggested that objects themselves affect the basic geometry of visual space, resulting in changes in distance between objects or lines. Using the assumptions of Riemannian geometry, he demonstrated that some well-known visual illusions (e.g. Müller-Lyer and Poggendorff illusions) can be explained by treating visual space as a smooth elastic manifold which is distorted by stimuli. In this differential-geometric framework, the effects of spatial distortion decrease with the distance from a line or figure and contractions in one portion of the field must be compensated by expansion in another (p. 146).

In this context, our results may reflect dynamic interactions producing a gradient landscape which is not directly perceivable but has permanent and stable effects on perception (e.g. Aksentijevic, Elliott & Barber, 2001). This landscape consists of attractors (troughs or basins), flat regions and transition regions (hyperbolic paraboloids or saddles) that are created by two figures when these are sufficiently close. Critically, the configuration of the landscape changes dynamically with the change in size, relative position and number of objects. The degree of distortion created by a figure depends on its size and shape. Larger figures affect more of the surrounding space and different features contribute differently to the local differences in distortion. These two factors interact to produce different grouping solutions.

In conclusion, we report four experiments which demonstrate for the first time the effects of spatial distortions created by simple geometrical figures (triangles and ellipses) on distance judgments. Our findings indicate that different aspects of a figure create different local gradients in the surrounding space. They support the idea that at least in two dimensions, visual space is locally nonlinear and that its extrinsic geometry (Fernandez & Farell, 2009) is affected by the mass and shape of figures embedded in it. Future research will investigate the interaction between mass and shape, effects of saddle asymmetries imposed by non-identical figures as well the effects of spatial distortions on the propagation of attention. Incidentally, there is evidence that the distortions in a number of illusions (Zöllner, Poggendorff and Müller-Lyer) critically depend on the presence of corner junctions (e.g. Day, 2006) and that errors on a Müller-Lyer shaft bisection task increase close to the angles (Prebedon, 2000). Thus, one of the future directions of this research will be to systematically relate the strength of this illusion to the degree of field distortion

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