A New Approach to Design Switching Strategy for the Buck Converters

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***Abstract*— In this paper, a novel method is developed to control switched DC-DC Buck converters. The circuit dynamic is described as an affine linear switched system. Utilizing switched systems theory, a switching state-feedback law is derived to asymptotically stabilize the desired equilibrium point and also minimize a guaranteed cost. The efficiency of the proposed method is illustrated by simulation which verifies the improvement of the obtained results compared with the literatures.**

Keywords— ***Switched Systems, Buck Converters, Switching Strategy, Bilinear Matrix Inequality.***

I. INTRODUCTION

Power electronics converters have been widely used for more than three decades [1]. The Buck convertor is one of the usual circuit topologies that frequently employed in power electronic systems applications, for instance in battery powered appliances because of its light weight, compact size and high efficiency [2]. Buck circuit is a switching-mode power converter and the output voltage features are organized by designing a control law for deciding the position of the switches. The control subsystem of the Buck converter arranges the switching strategy to achieve output voltage regulation with desirable transient behavior in the presence of output load variation [3].

The dynamical model of the Buck circuit similar to other switching-model convertors is derived by the conventional state-space averaging technique and is used to design switching rule for the circuit transistors [4]. Pulse width modulation (PWM) and the sliding mode methods are the traditional procedures to design controller for the Buck convertor [5]. The mentioned approaches to obtain control laws are based on simplified models which involve the average behavior of the system (neglecting switching modeling) and linearizing around particular operational point. So, the converter is stable around the operating point, but may be unstable in the presence of sizable disturbances or parameter variations [6]. Therefore, the models which incorporate the hybrid nature of the switching-mode circuit are the best choice to design the switching controller. Hence, researchers recently have utilized switched systems theory to design control subsystem for stabilization of the DC-DC converters [7],[8].

Switched systems are composed of a number of subsystems and a rule that organize the switching among them [9]. One of the fundamental problems in the research of switched systems is to construct a switching signal that makes switched system asymptotically stable. In [8], a switching rule was developed to deal with switched converters. The switching controller takes any trajectory of the switched affine system to a desired point by minimizing a quadratic cost. Although, the obtained design methodology in [8] is simple; however, it is based on one quadratic Lyapunov function which makes the proposed sufficient conditions conservative.

Composite Lyapunov function constructed from multiple functions is natural choice for the stability analysis and stabilization of the switched systems. In [9], for practical reason, multiple quadratic functions are used to form a composite Lyapunov function to obtain stabilizing switching function for non-affine linear switched system.

In this paper, motivated by the form of the dynamical models of the Buck converter, the result of [9] is generalized to cope with affine switched linear systems. Furthermore, a guaranteed performance is assured by the suggested switching strategy. The suggested approach is used to control the Buck converter. The simulation results are compared with the results of [8] to illustrate the effectiveness of the proposed method. It should be noticed that the derived results can be applied to control the general types of the converters.

The paper is organized as follows: In section 2, a switched system model is derived for the Buck converter. Sufficient conditions for guaranteed cost control design for the affine switched system are introduced in section 3. In section 4, simulation results are presented to demonstrate the applicability of the proposed approach.

Notation: The notation $P > 0$ $(P \geq 0)$ means that P is real symmetric and positive define (positive semi-definite). The superscrip^T stands for matrix transposition. The set of all non- negative vectors λ that satisfies $\sum_{i=1}^{N} \lambda_i = 1$ is shown by Λ. $A_\lambda = \sum_{i=1}^{N} \lambda_i A_i$ is the convex combination of a set of matrices $\{A_1, \dots, A_N\}$. Each of $I[k_1, k_2]$ signifies the set of integers between k_1 and k_2.

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II. PROBLEM FORMULATION

A schematic diagram of the typical Buck converter, is shown in figure1. ; where the resistor R_0 is the load and the inductor current and the capacitor voltage are denoted by i_L and v_c, respectively.

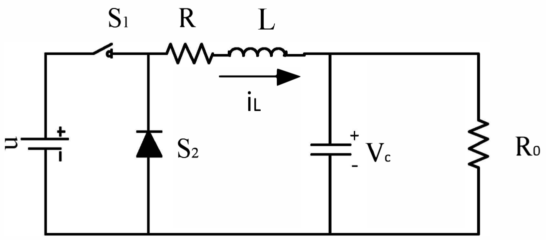

Figure 1. Schematic of Buck converter

The dynamical model of the Buck converter can be written as the following equation:

$$\dot{x}(t) = A_\sigma x(t) + B_\sigma u(t) \tag{1}$$

Where $x(t) = [i_L(t)\ v_c(t)]^T$ is the state vector , $u(t) = u$ is the control input that is supposed to be constant for all $t > 0$. The switching function $\sigma(t): t > 0 \to K = \{1,2\}$ selects at each instant of time $t > 0$, one subsystem among two available ones presented in the following matrices:

$$A_1 = A_2 = \begin{bmatrix} -R/L & -1/L \\ 1/C_0 & -1/C_0R_0 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 1/L \\ 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

The set of all attainable equilibrium points are calculated as follows:

$$X_e = \{(i_e, v_e): v_e = R_0 i_e, 0 \le i_e \le u/(R_0 + R)\} \tag{2}$$

which is a line segment practically defined only by the load since $R \ll R_0$.

It's obvious that the buck converter represented in the form of switched affine system (1) that comprises two affine subsystems. At each time, only one of the subsystems is active and the decision of which subsystem is active is the control variable that is determined by the switching rule.

The control design problem is to obtain a switching strategy $\sigma(\mathbf{x}(t))$ such that ideally, the following performance index is minimized:

$$\min_\sigma J_c = \int_0^\infty R_0^{-1} [v_c(t) - v_e]^2 + \rho R [i_L(t) - i_e]^2 dt \tag{3}$$

where, $x_e = [i_e, v_e]^T \in X_e$ is an attainable equilibrium point and $\rho \ge 0$ is a weight parameter. Furthermore, the set of all equilibrium points x_e which can be achieved by the suggested switching strategy, i.e. $x(t) \to x_e$ when $t \to \infty$ is determined. The performance index J_c in (3) represents the weighted sum of the energy of the deviation signal of each state variable from the chosen equilibrium point. Solving the mentioned control problem is difficult because of non-continuous nature of the switching function $\sigma(x(t))$. Instead, as usual, the upper bound of the performance index J_c is minimized. So, the resulting guaranteed cost control problem is to investigate the switching function $\sigma(x(t))$ that renders the equilibrium point x_e asymptotically stable and the following general guaranteed cost holds:

$$\int_0^\infty (x(t) - x_e)^T Q (x(t) - x_e) dt < (x_0 - x_e)^T P (x_0 - x_e) \tag{4}$$

in which ,the x_0 denotes the initial state.

III. CONTROLLER DESIGN

In this section, we utilize the concept of composite Lyapunov functions to extract a procedure to determine switching signal for affine switched systems with constant input which is applicable for Buck converters. Moreover, this switching rule is obtained such that the guaranteed cost (4) is minimized.

Theorem 1: Consider the switched affine system (1) with constant control input $u(t) = u$ for all $t \ge 0$ and let positive integer J and $x_e \in R^n$ be given. If there exist $\lambda \in \Lambda$, real scalars $\beta_{ij} \ge 0$ for $i \in K, j \in I[1, J]$ and symmetric positive definite matrices $P_j \in R^{n \times n}$ for $j \in I[1, J]$ such that

$$A_i^T P_j + P_j A_i + Q - \sum_{K=1}^{J} \beta_{jK} (P_j - P_K) < 0 \tag{5}$$

$$A_\lambda x_e + B_\lambda u = 0 \tag{6}$$

Then, the switching strategy

$$\sigma = \operatorname{argmin}_{i \in K} \xi^T \mathrm{P}(A_i x_e + B_i u) \tag{7}$$

with $\xi = x - x_e$ and $P \le P_j$ for all $j \in I[1,J]$, makes the equilibrium point $x_e \in R^n$ globally asymptotically stable and the guaranteed cost (4) holds.

Proof: A composite Lyapunov candidate function $V_{\min}(\xi)$ is built from the quadratic functions $V_j(\xi) = \xi^T P_j \xi$ as follows:

$$V_{min}(\xi) = \min \{V_j(\xi): j \in I[1,J]\}$$

where $P_j = P_j^T > 0$, $j \in I\ [1,J]$. For abbreviation, let $J_{min}(\xi) = \{1,2, \dots, J_0\}$ for a certain integer $J_0 \le J$. Then,

$V_{min}(\xi) < \xi^T P_k \xi$ for $k > J_0$. This fact can be written as: $\xi^T(P_j - P_k)\xi \leq 0$, For all $j \leq J_0$ and $k \in I[1, J]$. This inequality is equivalent to

$$P_j - P_k \leq 0, \quad j \leq J_0, \; k \in I[1, J] \qquad (8)$$

The time derivative of the $V_{\min}(\zeta)$ along trajectory of the switched system (1) is obtained as follows:

$$\dot{V}_{min}(\xi) = \min\{\dot{V}_j(\xi), \quad j \in J_{min}(\xi)\}$$

wherein, $V_j(\xi)$ is obtained as follows:

$$\dot{V}_j(\xi) = \dot{x}^T P_j \xi + \xi^T P_j \dot{x}$$
$$= 2\xi^T P_j(A_\sigma x + B_\sigma u)$$
$$= 2\xi^T P_j(A_\sigma x + B_\sigma u) + \xi^T(A_\sigma^T P_j + P_j A_\sigma)\xi$$

The choice of $\sigma = \operatorname{argmin}_{i,j} \xi^T P_j(A_i x_e + B_i u)$, Leads to:

$$V_j(\xi) = \min_{i \in K}[2\xi^T P_j(A_i x_e + B_i u)] + \xi^T(A_\sigma^T P_j + P_j A_\sigma)\xi$$

$$= \min_{\lambda \in \Lambda}[2\xi^T P_j(A_\lambda x_e + B_\lambda u)] + \xi^T(A_\sigma^T P_j + P_j A_\sigma)\xi$$

Now, let choose λ such that:

$$A_\lambda x_e + B_\lambda u = 0 \qquad (9)$$

and also for all $i \in K$ the following holds:

$$A_i^T P_j + P_j A_i < -Q \qquad (10)$$

then, $\dot{V}_j(\xi) < -\xi^T Q \xi$ and consequently the following holds:

$$\dot{V}_j(\xi) < -\xi^T Q \xi \qquad (11)$$

It's clear that if the above inequality holds, asymptotic stability of x_e would be achieved. By S-procedure [10], inequalities (8) and (10) can be combined which yields to the following inequality:

$$A_i^T P_j + P_j A_i + Q - \sum_{K=1}^{J} \beta_{jK}(P_j - P_K) < 0$$

in which, $\beta_{ij} \geq 0$ are real scalars. Moreover, integrating both sides of(11) from $t = 0$ to $t = \infty$ and setting $V_{min}(\xi(\infty)) = 0$,the relation (4) is obtained. The relation (4) is obtained. ■

Remark 1. The presence of the product of scalar variables and matrix variables in (5) brings bilinear terms in these matrix inequalities. For the considered two mode Buck converter, these conditions are simple to be figured out. Nonetheless, in complicated problems the path-following method proposed in [11] can be utilized to solve the feasibility BMI problem.

Remark 2. The switching function in (7) is linear and then its implementation is straightforward in practical applications. Although the Theorem 1 in [8] provides a linear switching law, the design conditions to be handled are much more stringent than the design conditions introduced in this paper. In the other word, the proposed design technique noticeably yield less conservative results compared with [8].

IV. Simulation Results

In this section, the suggested procedure is applied to obtain switching rule for a typical Buck converter. The circuit is simulated by SIMULINK® software and then, the results is compared with the approach introduced in [8]. The considered Buck converter has the nominal values that are equal to u=100V, R=2Ω, L=500 μH, C_0=470 μF and R_0=50Ω. Analogous to [7], the value of the weight matrix parameter ρ in (3) is set to zero, so the following cost weight matrix is obtained:

$$Q = \begin{bmatrix} 0 & 0 \\ 0 & 1/R_o \end{bmatrix}$$

The path following method [11] is employed to solve the bilinear matrix inequalities obtained in (5) and (6) using YALMIP® toolbox [12] and then, the switching strategy in (7) is simply implemented. The phase plane trajectories of the closed-loop system are shown in figure 2. These curves arising from zero initial condition towards the desired load voltages varying from 10 to 70 volts.

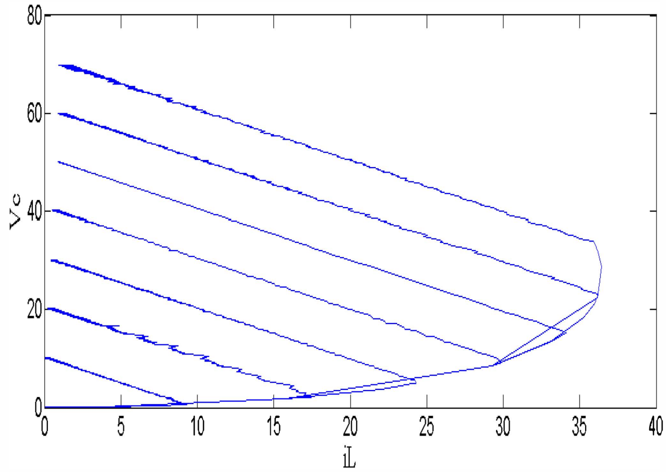

Figure 2. Trajectories of the converter states towards different points of the load voltages

The performance index J_c in (3) is calculated to compare the suggested approach with the results of the method in [8]. The values are summarized in the Table 1 for three different desired equilibrium points. The converter outcome resulting from the proposed controller improved up to 20% with respect to the results obtained by the approach of [8] which approves the better performance of the suggested scheme compared with [8].

Furthermore, to clarify the improvement of the converter output, time response of the states are shown in Figures 3 to 5 for $v_e = 10V$, $v_e = 20V$ and $v_e = 30V$; respectively. It's obvious that the transient behavior of the proposed converter is faster than the converter introduced in [8]. In Figure 3, the output settling times of the proposed converter and the one in [8] are respectively equal to 1.107ms and 1.608ms, and in Figure 4, the mentioned quantities are 1.091ms and 1.455ms for the designed converter and the rival one in [8]; which proves the enhanced output of the suggested scheme which verifies the improved output of the devised approach.

TABLE I. COMPARISON OF THE RESULTS FOR DIFFERENT EQUILIBRIUM POINTS

Performance index J		Desired equilibrium point
[8]	Proposed method	
0.0066	0.0051	$v_e = 30$
0.003	0.002	$v_e = 20$
0.00062	0.00051	$v_e = 10$

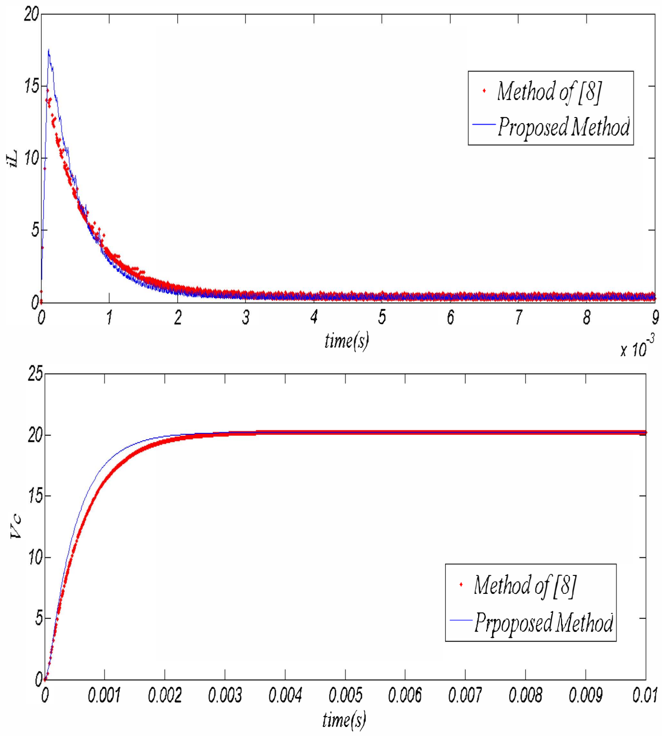

Figure 4. Time response of the output voltage and inductor current for ve =20 v

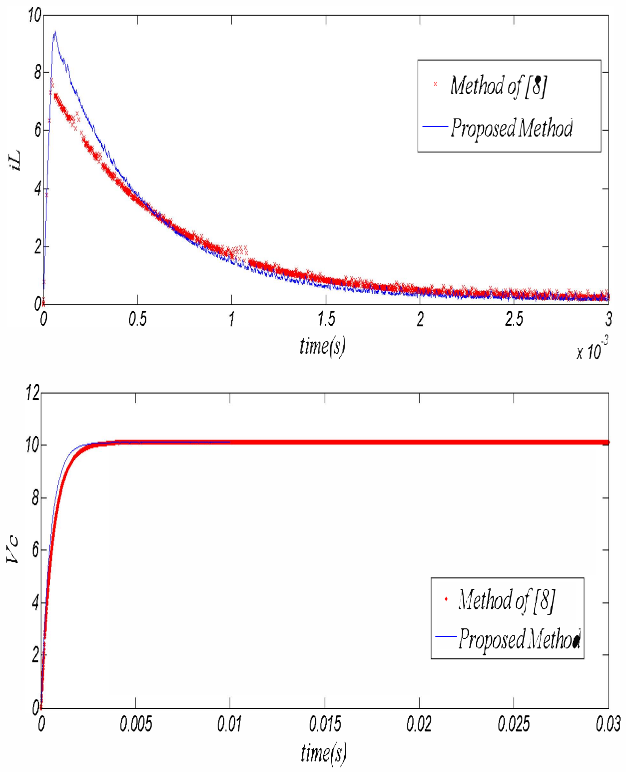

Figure 3. Time response of the output voltage and inductor current for ve=10 v

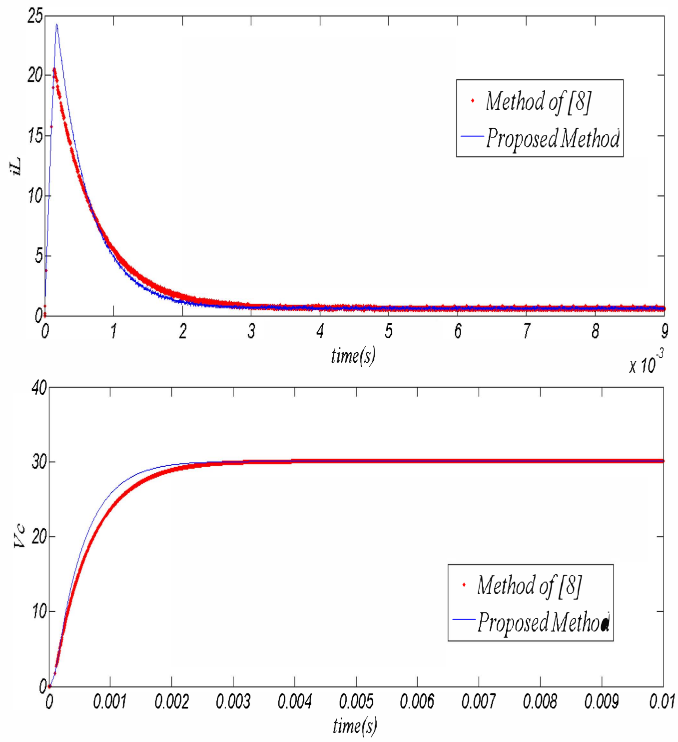

Figure 5. Time response of the output voltage and inductor current for ve=30 v

V. CONCLUSION

In this paper, an existing technique to obtain the stabilizing state-feedback switching law for autonomous switched systems extended to the affine switched systems with constant external input. Moreover, the switching strategy designed such that minimizes a quadratic guaranteed cost. A classical Buck converter simulated to show the simplicity and efficiency of the proposed design scheme. A comparison with the recent approach also discussed. Simulation results show that the proposed method improves the design performance criterion up to 20% with respect to rival approach. The application of the proposed method to design control subsystem for advanced converter circuits defines further research line.

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